

**2010 Examinations**

**SPECIMEN SOLUTIONS**

**Subject ST7 — General Insurance:  
Reserving and Capital Modelling**

**Specialist Technical**

- 1** (i) *Assumptions*  
 All yearly business  
 No reinsurance  
 Risks written uniformly across year  
 Risk is uniform across policy year

<b>Company</b>	<b>X</b>	<b>Y</b>	<b>Z</b>
<b>Assets</b>			
Total investments	125	3500	1000
Current Assets	5	80	30
Deferred Acquisition Costs	8	150	25
<b>Total Assets</b>	<b>138</b>	<b>3730</b>	<b>1055</b>
<b>Liabilities</b>			
O/S claims reserves	30	850	700
Additional URR	15	100	0
UPR	25	1000	125
Current Liabilities	11	100	40
Free Reserves	57	1680	190
<b>Total Liabilities</b>	<b>138</b>	<b>3730</b>	<b>1055</b>

- (ii) *Assumptions*
- assume GWP = GEP (i.e. business written in 2008 = business written in 2009)
  - assume AURR as at 31/12/2009 = AURR as at 31/12/2008
  - assume outstanding claims reserves include IBNR

*Loss Ratio = claims incurred/GEP*

Company X =  $(35 + 30 - 20)/50 = 90\%$

Company Y =  $(700 + 850 - 800)/2000 = 37.5\%$

Company Z =  $(150 + 700 - 750)/250 = 40\%$

*Expense Ratio = Acquisition Expense Ratio + Non Acquisition Expenses/GWP*

Company X =  $30\% + 5/50 = 40\%$

Company Y =  $15\% + 250/2000 = 27.5\%$

Company Z =  $20\% + 30/250 = 32\%$

*Underwriting Ratio = Loss Ratio + Expense Ratio*

Company X =  $90\% + 40\% = 130\%$

Company Y =  $37.5\% + 27.5\% = 65\%$

Company Z =  $40\% + 32\% = 72\%$

*For solvency ratio:*

*Solvency Ratio = Free Reserves/GWP*

Company X =  $57/50 = 114\%$

Company Y =  $1680/2000 = 84\%$

Company Z =  $190/250 = 76\%$

For return on capital employed:

Return on Capital employed =  $(\text{Earned Premium} - \text{Claims Incurred} - \text{Expenses} + \text{Investment Income}) / \text{Free Reserves}$

Company X =  $(50 - (35 + 30 - 20) - 5 - 15 + 3) / 57 = -21\%$

Company Y =  $(2000 - (700 + 850 - 800) - 250 - 300 + 100) / 1680 = 48\%$

Company Z =  $(250 - (150 + 700 - 750) - 30 - 50 + 16) / 190 = 45\%$

(iii) **Comments**

- Company X may have suffered from adverse claims experience due to its higher loss ratio compared to the other companies.
- Each company may be writing different classes or mix of business, each at a different point in their respective market cycle.
- Company X expense ratio is higher due to higher acquisition expense ratio.
- The company is smaller than Y and Z and it may be spending money to expand rapidly.
- Company X solvency ratio is higher than the other companies.
- This may be the result of a recent capital injection to expand the business.
- Company Z has the lowest solvency ratio, suggesting that the company is less financed than the other companies.
- Or it may have more stronger valuation basis for its assets and liabilities.
- Company Z return on capital employed is the highest, supported by a larger relative investment return compared to the other companies.
- Company Y and Z both have high returns on capital employed, supported by a good underwriting results.
- Relevant comment comparing profitability and solvency.

(iv)

- Investment Return =  $\text{Investment Income} / (\text{Current Assets} + \text{Investments})$
- This provides a comparison of the investment performance of the companies.
- Gross Claims Paid / Gross Outstanding Reserves
- This provides a comparison of the relative speed at which reserves are reduced by claims payments.
- Gross Outstanding Reserves / Gross Written Premium
- This provides a comparison of the relative strength of the outstanding reserves.
- Additional Unexpired Risk Reserve cfwd / UPR cfwd
- This provides a comparison of the relative profitability of the unexpired risk
- Current Assets / Current Liabilities
- This provides a comparison of the ability of each company to meet short Term liabilities without the need to realise investments.

Acceptable alternative valid ratios:

- Loss Ratio
- Expense Ratio
- Profit Margin
- Total Assets/Total Liabilities

**2 (i) Projecting incurred claims data only**

Projecting incurred claims data only is preferable to only projecting paid data. Incurred projections are useful as paid claims development will be less mature than incurred claims.

Paid projections, however, can help identify changes or inconsistencies in the strength of case reserves or possible redundancies.

Projecting both bases can therefore reveal features of claim reserves that would otherwise be missed.

Need paid claims development if discounted reserves required

Would recommend that the client projects both paid and incurred data.

**Projecting net of reinsurance only**

Projections at a net level will be robust as long as the proportion of reinsurance recoveries remains stable.

It would be preferable to project at a gross level and apply the actual reinsurance program to the projected future claims

This may not be feasible in practice.

Alternatively project at gross level and analyse the trend in reinsurance to gross ratios for premiums, paid, incurred, outstanding claims in order to select reinsurance IBNR ratios.

In addition, understand change to the reinsurance programme and how this will impact the IBNR ratios selected.

**ELR using 3-year rolling average**

3-year rolling average is good where the historical selected ultimate loss ratios are volatile over time.

Using a weighted average by ultimate premium though would smooth out any volatility over the accident/underwriting year (as the years with the largest ultimate premiums will be more stable)

Gives credit to the account's own unique experience.

Easy to apply

but a fairly mechanical approach, which doesn't take into account any possible trends showing in the data/unusual years etc.

Claims and premiums are calculated on the same basis.

No account has been made to allow for premium rate changes and claims inflation.

Would recommend a weighted average where the number of prior year historical selected ultimate loss ratios is based on the data.

Would recommend loss ratios are adjusted to allow for premium rate changes and claims inflation.

Recommend use of market stats benchmarks, where available for a sense check – although need to ensure that the data is on a consistent basis.

**Exchange rate conversion**

This exchange rate conversion method would work on paid data but not on incurred.

The outstanding need to be treated in a different way depending on whether they are

- New claims in the quarter/year
- Prior claims with no movement in the quarter/year
- Re-stated claims with movement in the quarter (e.g. due to additional information or hyper-inflation)

For the first two type of outstanding claims the method is fine, for the last type of claim the entire outstanding amount would need to be converted into US \$'s.

The method assumes that the movement in exchange rates move in exactly offsetting ways to movements in inflation – this is not always the case in practice.

Dealing with exchange rates is always complicated. This approach does have its disadvantages but it is not unreasonable therefore no recommendation to change the current method.

- (ii) + Useful in exceptional circumstances e.g. a very new book of business with no prior data or extremely volatile own history – as is the case for this company.
    - + Underwriter has a good knowledge of the business
    - + May reflect market rate changes and inflation effects as well as trends in claims frequency and average cost
    - + Provides independent estimates
    - Underwriter’s estimates may be too optimistic and hence not representative of the actual loss ratios.
    - Need to check consistency of the basis used.
- This approach is reasonable, would recommend the use of market stats data as well, where available, as a sense check  
 Need to ensure consistency with the basis e.g. gross/net commission/reinsurance.

(iv)

<i>Year</i>	<i>Earned Premium \$000's</i>	<i>Earned Policy Years</i>	<i>Incurred Claims \$000's</i>	<i>Incurred Cumulative Development Factor</i>	<i>Selected Ultimate Loss Ratio</i>	<i>Premium Rate Increase for 2009 Level</i>	<i>Claims Cost Increase for 2009 Level</i>	<i>2009 On-Level Loss Ratio</i>	<i>Selected Ultimate Losses \$000's</i>
2004	11,750	1,150	8,765	1.000	75%	1.366	1.28	70.1%	8,236
2005	13,000	1,275	10,350	0.960	76%	1.368	1.22	67.5%	8,776
2006	12,500	1,125	9,235	0.940	69%	1.256	1.16	63.6%	7,951
2007	13,250	1,050	9,500	0.920	66%	1.106	1.10	65.8%	8,720
2008	15,250	1,125	11,250	0.975	72%	1.029	1.05	73.4%	11,201
2009	17,650	1,265	9,575	1.520	78%			68.3%	13,697

Where premium rate factor for 2004 is  $(17,650/1265)/(11750/1150) = 1.366$ , etc.

Claims cost increase for 2004 is  $1.05^5 = 1.276$ , etc.

2009 on level loss ratio is  $75\% \times (1.276/1.366) = 70.1\%$ , etc.

2009 IER =  $(11750 \times 70.1\% + 13000 \times 67.5\% + \dots + 15250 \times 73.4\%) / (11750 + 13000 + \dots + 15250)$   
= 68.3%

2009 Selected IBF Ultimate =  $9575 + (1 - (1/1.520)) \times (68.3\% \times 17650) = 13,697$ .

**3** (i)

- Try to maximise investment return subject to meeting liabilities with chosen level of certainty.
- Match assets and liabilities by:
  - term
  - amount
  - nature
  - currency
- Motor property damage claims are mainly short tailed, so need liquid assets
  - need to hold cash on deposit, very short dated assets such as short dated government securities to match liability outgo
- Motor third party claims are longer tailed and costs are influenced by inflation
  - need to hold some longer dated real assets (index linked securities if available or low risk equities)
- Consider regulatory requirements:
  - restrictions on assets that can be held
  - prescription to hold assets
  - custodianship of assets
    - mismatching allowed
- Since company is small, need to have extra consideration of the level of uncertainty in reserves, so more secure, liquid assets required.
- A small company might consider collective investment vehicles (e.g. unit trusts, investment company shares)
- Investment likely to be in assets of small unit size (e.g. no direct property investment)
- Level of investment expenses of each asset type
- Tax efficiency of each asset type
- Availability of certain asset types
- Benchmarking against competition
- Availability of additional capital (e.g. parent company, shareholders)
- Diversification of assets held (within and between asset types)
- Size of the free reserves (in excess of solvency requirements)
  - As the company is small, the company is less likely to be able to accept the risk of investing in higher risk/reward investments (e.g. property).

- Expected growth plans and resultant needs to invest in the business
- Shareholders and management's attitude to risk

(ii)

- Since company is large, assuming larger free assets, potential scope for more aggressive investment strategy.
- Employers' liability claims are generally longer tailed and costs are influenced by inflation
  - Greater need to hold longer dated assets providing real returns
  - since better match by term for liabilities
- Equities and properties are an appropriate match.
- Index-linked bonds (if available) for security and inflation hedge.
- Potential investment in specialist areas such as large unit size, ventures, brokers, derivatives.
- Likely to handle investments in-house through specialist team of managers giving greater control over investment choice.

**4**

(i) ***Benefits***

*Marine Property*

To indemnify the insured against the value of the loss or damage to the marine hull (subject to limits or excesses).

Cover can also be for marine cargo and specie and marine freight.

*Marine Liability*

To indemnify the insured against a financial loss (subject to limits or excesses). Associated legal expenses may also be covered.

(ii) ***Insured Perils***

*Marine Property*

Perils of the sea/other navigable waters e.g. storm, tsunami

Fire

Explosion

Jettison

Theft of cargo

Spoilage and contamination of cargo

Piracy

Capsizing

Stranding

Collision (iceberg or other)

Actions of the sea (e.g. waves damaging vessel)

Running aground

Specie (valuables)

*Marine Liability*

Damage to 3rd party property

Injury to 3rd parties (including death)

Injury to employees (including death)

Errors and omissions

(iii) ***Exposure Measures***

Insured value of the hull/ship  
Tonnage of hull/ship  
Value of cargo  
Limits of liability

(iv) ***Claim Characteristics***

*Reporting delays:* claims usually reported when the vessel reaches a major port.  
(May be only a very small delay if claim takes place in the port.)

*Settlement delays:* could be long,  
especially if there is a dispute over legal liability or the amount that should be paid.

*Claim Amounts:* variable. Relatively small amounts for hull damage to small vessels; very large amounts for complete loss of a large vessel and its cargo. Liability claims very variable; legal expenses element can dominate.

*Claim Frequency:* infrequent for hull but more frequent for cargo

Accumulations of risk are possible

e.g. geographical concentration (storm/tidal wave); spillage of hazardous material

Moral hazard – frequency increases in bad economic conditions

Salvage and subrogation are often employed

Currency issues

(v) ***Risk Factors***

*Hull*

Level of cover / excesses and limits

Size/tonnage of vessel

Type of vessel

Condition of vessel

Age of vessel

Type of industry

Classification society

Engine type/manufacture

Country of build

Experience of captain and crew

Detention history

Areas sailed in (rough seas/war zones etc.) / locations visited

Tonnage of hull

Previous claims experience of ship

Previous claims experience of owner

Insured value / sum insured

*Cargo*

Level of cover

Value of cargo

Nature of cargo  
How packaged  
Where stored on ship (deck versus hold)  
Trade terms  
Trade routes taken  
Standard property insurance risk factors apply when warehoused at port

*Liability*

Number of passengers and crew  
Type of work undertaken by the insured (e.g. shipbuilder, marina operator)  
Limits of liability

- 5** (i) a Assessing solvency capital requirements
- b Allocating the capital held between classes, products or individual policies for:
- i. performance measurement
  - ii. pricing
  - iii. business planning and strategy setting
- c Reinsurance purchasing
- d Asset allocation studies
- e Studies of enterprise level risks such as credit risk and operational risk
- (ii) *Any five of the following sections:*

*Diversification in assessing solvency capital requirements*

This would be a “ground-up” exercise and could be at product level, class of business or whole company portfolio level.

It would also be net of reinsurance with reinsurance being modelled either explicitly or implicitly.

Typically, the sum of the capital requirements for the various risks from each portfolio segment assessed separately would be expected (other things equal) to be more than the capital requirement from all risks analysed together for the whole portfolio (the sum of the the  $p^{\text{th}}$  percentiles is more than the  $p^{\text{th}}$  percentile of the sum).

A company may be exposed to operational risks associated with its corporate group structures and policies.

These risks are likely to be targeted by the group’s enterprise risk management process,

These risks may be a source of negative diversification effects i.e. require additional capital to be held.

In quantifying the diversification adjustment, assumptions will need to be made over the extent of applicable correlations.

This will involve understanding the various assumptions underlying the modelling of the risks in each portfolio.

This will involve understanding how they could interact when combined in order to assess the capital requirements of the overall portfolio.

Correlation can occur between risks in the same class and also between risks in different classes.

The allowance for correlation within the stochastic model can either be explicit within the modelling process or implicit through the use of correlation matrices.

#### *Diversification in capital allocation for performance measurement*

This “top-down” exercise will typically involve using the results from the capital assessment exercise for cascading the capital held by the company down to individual classes of business and products for performance measurement purposes.

As the capital allocated includes a diversification credit the diversification assumption in this case would be implicit, in terms of how much diversification benefit is allocated to each class.

The risk measure used may differ from that in assessing the capital requirement.

Although a group’s solvency capital requirement may be pitched against a target percentile in the tail of the underlying aggregate loss distribution, the allocation of the diversified capital down to individual classes of business or products for a company in the group may be made with reference to a lower percentile or with reference to various percentile-defined layers to prevent over-allocation to catastrophe type business.

Marginal capital method may be used (a “last in” method): here the capital allocation is made with reference to the marginal capital requirements of each segment.

The Shapley method may be used: The capital allocation is made with reference to an average of the marginal capital requirements assuming that the class under consideration is added to the overall portfolio first, second, third etc.

Capital allocation methods should have regard to the use to which the results will be put and consideration should be given to desirable properties of the results such as stability over time.

There is not necessarily one method which is best suited in all cases.

Typically, the results from several methods of allocation would be compared, and that the actuary would use his or her judgement when recommending or setting the final allocation.

A company may hold more capital than its capital model may suggest is needed.

The excess capital may be required to support the company's credit rating or be true surplus capital that the company may choose to hold to enable it to take advantage of business opportunities in the future (amongst other reasons).

Within a capital modelling exercise it is important to distinguish between the following:

- Total capital
- Economic capital
- Excess capital

The total capital available to a company is generally fixed at a single point in time. The stochastic model will ascertain the amount of economic capital. Economic capital will normally be allocated to each class of business in proportion to its contribution to the risk metric on a stand-alone basis.

The excess capital is the balancing item, allocated between classes of business pro-rata to its risk based capital or certain components of it depending upon the purpose of the exercise.

*Diversification in allowing for the cost of allocated capital for pricing*

In determining a "technical price" benchmark the actuary will be looking at ways for cascading the amount of capital allocated to a particular class of business or product down to the underlying individual policies in a way that reflects their relative risk profiles.

The pricing actuary should consider how the allocated capital compares to that needed on an underwriting year basis, which may be subject to different diversification effects.

For expanding or contracting portfolios, the capital needed to support the reserves that would be held until all claims from the specific underwriting year are fully paid may differ to the reserve risk component of the allocated capital, which would be based on the size of the total reserves brought forward.

Similar differences may arise with the underwriting risk component. Where material differences arise, the pricing actuary should discuss with the underwriter and the management of the company the implications of setting pricing loads with reference to the allocated capital.

The effect from diversification in pricing would be partly offset by the cost of any excess capital.

In determining a “technical price” benchmark the actuary should consider if the relevant load for the cost of capital should be based on the diversified or non-diversified capital requirement.

*Diversification in capital modelling investigations for business planning*

The modelling investigations made should allow for:

- expected changes in the mix of business by class,
- the marginal effects of adding new classes or products and
- the effectiveness of its reinsurance protection programme in reducing the volatility of the retained risks.

In cases where new classes of business or products are being considered, investigate the relevant additional solvency capital requirement against the corresponding diversification effects to the overall company’s portfolio. The assumptions made for the new class will require careful consideration. As a new class, external data or prior experience will be needed to parameterise the modelling assumptions.

Diversification effects should be quantified as accurately as possible and allow for interactions between classes and products as well as economic and territorial correlations.

*Diversification in capital modelling investigations for reinsurance purchasing*

The capital requirement of a particular class of business or product is often investigated alongside the relevant reinsurance protection requirement.

A company optimising its reinsurance protection programme must consider the effects of different reinsurance strategies to the company’s overall solvency capital requirement.

The modelling required will depend on the reinsurance being considered.

If the reinsurance covers just single classes then the assumptions within the modelling of the gross losses will need to be tailored to be able to evaluate the reinsurance recoveries.

If the reinsurance covers multiple classes then allocation back to the individual contributing classes will also need to be considered.

For whole account reinsurance protections, allocation back to individual class will also be important.

In addition to allocating the benefit of these policies, the stochastic model can also be used as a basis for allocating the reinsurance premium at a class level.

Depending on the type and amount of reinsurance purchased, the diversification of the retained (net) business will change.

Drivers of such change will include:

- Tail diversification (insurance risk)
- Mix of risk by type (e.g. less insurance risk but more credit and liquidity risks)

#### *Diversification in Asset Allocation Studies*

The economic scenario generator (“ESG”) used within the stochastic model will enable the impact of different asset allocations to be analysed.

The ESG will ensure the consistency of assumptions throughout the modelling process.

Using the stochastic model, the company can then assess the impact of its asset portfolio within the overall capital requirement.

Consider adequacy of liquidity in our portfolio

Consider investing in bonds for longer tail liabilities

Consider security profile of bonds, given the liability profile and the current investment environment

Consider equities/property for the longest liabilities or the excess capital

#### *Diversification in Enterprise Level Risks*

In addition to insurance risk, the company needs to consider risks such as liquidity, market, credit and operational risks.

Each of these needs to be modelled either implicitly or more likely explicitly. The impact of each risk on the overall capital requirement can then be assessed.

We then need to consider how to allocate any change in the economic capital assessment as a result.

- 6**
- (i) the occurrence and severity of claims  
the notification delays on individual claims  
legal changes that affect the size of awards  
legal changes that affect the heads of damage awarded  
changes in the litigiousness of society  
levels of claims inflation (which in turn is related to levels of price inflation and wage inflation in the economy)  
court rulings on liability or quantum of individual claims not foreseen by claims handlers or not in the historic data  
changes in the mix of claim types, either caused by an underlying change in claim type experience or by changes in the mix of business written  
changes in claims handling, either because of policy changes or because of external events (such as a catastrophe leading to claims handlers being over-stretched)  
the historic data only provides a limited sample  
the quality of data may have varied over time  
there are many ways of deriving an estimate of the claims reserve, and many judgements required within each method - the uncertainty introduced by this is known as "model error" or "model uncertainty"
  - (ii) the run-off pattern is the same for each origin period (as for the chain-ladder)  
incremental claim amounts are stochastically independent  
the variance of the incremental claim amounts is proportional to the mean  
incremental claims are positive for all development periods
  - (iii) (a) There can be mismatches between the type of model and the data to be used.  
For log-Normal models (because the log of the incremental movements are taken) any negative increments must be ignored.  
Generally, this is not a problem for paid claims triangles (unless there are significant salvage or subrogation recoveries),  
But the method often does not work well for incurred claims data where there are likely to be more instances of negative increments.  
The over-dispersed Poisson model is slightly more flexible in that individual negative increments for any development period are possible,  
as long as the development factor across the development period as a whole is greater than one.  
The Mack model is very flexible in its model form in that negative increments are allowed, as are development factors of less than one across a whole development period.
  - (b) The stochastic methods described above tend not to be suitable for certain types of claim, in particular latent claims, since they are only able to reflect the variability in the claims data available.

A possible approach around this is to use an exposure-based method whereby assumptions concerning the volatility around the number of future claims, and the average cost of future claims, are made.

- (c) As with best estimate reserving, sparse data sets can be problematical for stochastic methods, as can data peculiarities, such as missing or erroneous data.  
In particular, small changes in numbers can lead to significant changes in the distribution of outcomes, and the results can be quite sensitive to individual points.  
Coping with individual data peculiarities is a matter of individual actuarial judgement; judgement forms as important a part of stochastic reserving as best estimate reserving.
- (d) For some purposes, stochastic reserving is used extensively for determining the extreme tail of the distribution of possible outcomes. However, the distribution is parameterised on a finite amount of historic data, which may not be representative of the tail. In addition, most stochastic methods make some simplifying assumptions, which may be approximately correct for the bulk of the distribution of outcomes, but which may significantly break down at the extremes. It follows that great care needs to be taken when estimating the tail of a claims distribution.

## **END OF SOLUTIONS**