

**Subject CT5
Contingencies
Core Technical**

Syllabus

for the 2008 Examinations

1 June 2007

**The Faculty of Actuaries and
Institute of Actuaries**

Subject CT5 — Contingencies Core Technical Syllabus

Aim

The aim of the Contingencies subject is to provide a grounding in the mathematical techniques which can be used to model and value cashflows dependent on death, survival, or other uncertain risks.

Links to other subjects

Subjects CT1 — Financial Mathematics, CT3 — Probability and Mathematical Statistics and CT4 — Models: introduce techniques that will be drawn upon and used in the development of this subject.

Subject ST2 — Life Insurance Specialist Technical: uses the principles and techniques in this subject to help in the solution of life insurance problems.

Objectives

On completion of this subject the candidate will be able to:

- (i) Define simple assurance and annuity contracts, and develop formulae for the means and variances of the present values of the payments under these contracts, assuming constant deterministic interest.

1. Define the following terms:

- whole life assurance
- term assurance
- pure endowment
- endowment assurance
- critical illness assurance
- whole life level annuity
- temporary level annuity
- premium
- benefit

including assurance and annuity contracts where the benefits are deferred.

2. Define the following probabilities: ${}_n|_m q_x$, ${}_n q_x$ and their select equivalents ${}_n|_m q_{[x]+r}$, ${}_n q_{[x]+r}$.

3. Obtain expressions in the form of sums for the mean and variance of the present value of benefit payments under each contract above, in terms of the curtate random future lifetime, assuming that death benefits are payable at the end of the year of death and that annuities are paid annually in advance or in arrear, and, where appropriate, simplify these expressions into a form suitable for evaluation by table look-up or other means.
 4. Obtain expressions in the form of integrals for the mean and variance of the present value of benefit payments under each contract above, in terms of the random future lifetime, assuming that death benefits are payable at the moment of death and that annuities are paid continuously, and, where appropriate, simplify these expressions into a form suitable for evaluation by table look-up or other means.
 5. Extend the techniques of 3. and 4. above to deal with the possibility that premiums are payable more frequently than annually and that benefits may be payable annually or more frequently than annually.
 6. Define the symbols $A_x, A_{x:\overline{n}|}, A_{x:\overline{n}|}^1, A_{x:\overline{n}|}^{\overline{1}}, a_x, a_{x:\overline{n}|}, m|a_{x:\overline{n}|}, \ddot{a}_x, \ddot{a}_{x:\overline{n}|}, m|\ddot{a}_{x:\overline{n}|}$ and their select and continuous equivalents. Extend the annuity factors to allow for the possibility that payments are more frequent than annual but less frequent than continuous.
 7. Derive relations between annuities payable in advance and in arrear, and between temporary, deferred and whole life annuities.
 8. Derive the relations $A_x = 1 - d\ddot{a}_x, A_{x:\overline{n}|} = 1 - d\ddot{a}_{x:\overline{n}|}$, and their select and continuous equivalents.
 9. Define the expected accumulation of the benefits in 1., and obtain expressions for them corresponding to the expected present values in 3., 4., and 5. (note: expected values only).
- (ii) Describe practical methods of evaluating expected values and variances of the simple contracts defined in objective (i).
1. Describe the life table functions l_x and d_x and their select equivalents $l_{[x]+r}$ and $d_{[x]+r}$.
 2. Express the following life table probabilities in terms of the functions in 1.: ${}_n p_x, {}_n q_x, {}_{n|m} q_x$ and their select equivalents ${}_n p_{[x]+r}, {}_n q_{[x]+r}, {}_{n|m} q_{[x]+r}$.
 3. Express the expected values and variances in objective (i) 3. in terms of the functions in 1. and 2.

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4. Evaluate the expected values and variances in objective (i) 3. by table look-up, where appropriate, including the use of the relationships in objectives (i) 7. and 8.
 5. Derive approximations for, and hence evaluate, the expected values and variances in objective (i) 4. in terms of those in objective (i) 3.
 6. Evaluate the expected accumulations in objective (i) 9.
 7. Describe practical alternatives to the life table which can be used to obtain the evaluations in 4., 5., and 6.
- (iii) Describe and calculate, using ultimate or select mortality, net premiums and net premium reserves of simple insurance contracts.
1. Define the net random future loss under an insurance contract, and state the principle of equivalence.
 2. Define and calculate net premiums for the insurance contract benefits in objective (i) 1. Premiums and annuities may be payable annually, more frequently than annually, or continuously. Benefits may be payable at the end of the year of death, immediately on death, annually, more frequently than annually, or continuously.
 3. State why an insurance company will set up reserves.
 4. Describe prospective and retrospective reserves.
 5. Define and evaluate prospective and retrospective net premium reserves in respect of the contracts in objective (i) 1., with premiums as in (iii) 2.
 6. Show that prospective and retrospective reserves are equal when calculated on the same basis.
 7. Derive recursive relationships between net premium reserves at annual intervals, for contracts with death benefits paid at the end of the year of death, and annual premiums.
 8. Derive Thiele's differential equation, satisfied by net premium reserves for contracts with death benefits paid at the moment of death, and premiums payable continuously.
 9. Define and calculate, for a single policy or a portfolio of policies (as appropriate):
 - death strain at risk
 - expected death strain
 - actual death strain
 - mortality profit

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- (iv) Describe the calculation, using ultimate or select mortality, of net premiums and net premium reserves for increasing and decreasing benefits and annuities.
1. Extend the techniques of (ii) to calculate the expected present value of an annuity, premium, or benefit payable on death, which increases or decreases by a constant compound rate. Calculate net premiums and net premium reserves for contracts with premiums and benefits which vary as described.
 2. Define the symbols $(IA)_x$, $(I\ddot{a})_x$, and $(Ia)_x$ and their select equivalents.
 3. Calculate the expected present value of an annuity, premium or benefit payable on death, which increases or decreases by a constant monetary amount. Calculate net premiums and net premium reserves for contracts with premiums and benefits which vary as described.
 4. Define with profits contract.
 5. List the types of bonus that may be given to with profits contracts.
 6. Calculate net premiums and net premium reserves for with profits contracts.
- (v) Describe the calculation of gross premiums and reserves of assurance and annuity contracts.
1. List the types of expenses incurred in writing a life insurance contract.
 2. Describe the influence of inflation on the expenses listed in 1.
 3. Define the gross future loss random variable for the benefits and annuities listed in (i) 1. and (iv).
 4. Calculate gross premiums using the future loss random variable and the equivalence principle. Premiums and annuities may be payable annually, more frequently than annually, or continuously. Benefits may be payable at the end of the year of death, immediately on death, annually, more frequently than annually, or continuously.
 5. Calculate gross premiums using simple criteria other than the equivalence principle.
 6. Calculate gross premium prospective reserves using the future loss random variable.
 7. Define and calculate the gross premium retrospective reserve.
 8. State the conditions under which, in general, the prospective reserve is equal to the retrospective reserve allowing for expenses.

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9. Prove that, under the appropriate conditions, the prospective reserve is equal to the retrospective reserve, with or without allowance for expenses, for all standard fixed benefit and increasing/decreasing benefit contracts.
 10. Derive a recursive relation between successive annual reserves for an annual premium contract, with allowance for expenses, for standard fixed benefit contracts.
- (vi) Define and use straightforward functions involving two lives.
1. Extend the techniques of objectives (i)–(v) to deal with cashflows dependent upon the death or survival of either or both of two lives.
 2. Extend the techniques of 1. to deal with functions dependent upon a fixed term as well as age.
- (vii) Describe methods which can be used to model cashflows contingent upon competing risks.
1. Explain how the value of a cashflow, contingent upon more than one risk, may be valued using a multiple-state Markov Model.
 2. Derive dependent probabilities from given transition intensities, using the Kolmogorov equations introduced in Objective (vii) of the Models Subject.
 3. Derive transition intensities from given dependent probabilities.
- (viii) Describe the technique of discounted emerging costs, for use in pricing, reserving, and assessing profitability.
1. Define unit-linked contract.
 2. Evaluate expected cashflows for whole life, endowment and term assurances, annuities, and unit-linked contracts.
 3. Profit test simple annual premium contracts of the types listed in 2. and determine the profit vector, the profit signature, the net present value, and the profit margin.
 4. Show how the profit test may be used to price a product.
 5. Show how the profit test may be used to determine reserves.
 6. Describe the construction and use of multiple decrement tables, including the relationships with associated single decrement tables.

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7. Use multiple decrement tables to evaluate expected cashflows dependent upon more than one decrement, including:
 - pension benefits
 - other salary related benefits
 - health and care insurance
 8. Describe practical alternatives to the multiple decrement table which can be used to obtain the evaluations in 7.
 9. Extend the techniques of 3., 6., and 7. to evaluate expected cashflows contingent upon risks other than human life.
- (ix) Describe the principal forms of heterogeneity within a population and the ways in which selection can occur.
1. State the principal factors which contribute to the variation in mortality and morbidity by region and according to the social and economic environment, specifically:
 - occupation
 - nutrition
 - housing
 - climate/geography
 - education
 - genetics
 2. Define and give examples of the main forms of selection:
 - temporary initial selection
 - class selection
 - time selection
 - spurious selection
 - adverse selection
 3. Explain how selection can be expected to occur amongst individuals taking out each of the main types of life insurance contracts, or amongst members of large pension schemes.
 4. Explain why it is necessary to have different mortality tables for different classes of lives.
 5. Explain how decrements can have a selective effect.
 6. Explain the theoretical basis of the use of risk classification in life insurance.

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7. Explain the impact of the availability of genetic information on risk classification in life insurance.
8. Explain the concept of a single figure index and its advantages and disadvantages for summarising and comparing actual experience.
9. Define the terms crude mortality rate, directly standardised and indirectly standardised mortality rate, standardised mortality ratio, and illustrate their use.

End of Syllabus