Faculty of Actuaries **Institute of Actuaries**

EXAMINATIONS

September 1998

Subject A — Fundamentals of Actuarial Mathematics

Paper One

EXAMINERS' REPORT

PART ONE

1 A

2 D

3 B

4 A

5 C

6 C

7 A

PART TWO

8 The par yield y is given by

$$y\left\{\frac{1}{(1+f_{0,1})}+\frac{1}{(1+f_{0,2})^2}+\frac{1}{(1+f_{0,1})(1+f_{1,2})^2}\right\}+\frac{1}{(1+f_{0,1})(1+f_{1,2})^2}=1$$

Hence $y\{2.655250\} = 0.169806$

$y = 6.395\%$ per annum

9 $Xa^{(12)}_{\overline{5}|15.4\%} = 4.000$

$$a_{\overline{5}|15.4\%} = \frac{1-1.154^{-5}}{0.154} = 3.32065$$

$i^{(12)} = 0.144092 \quad \therefore \quad \dfrac{i}{i^{(12)}} = 1.06876$

$$\therefore \qquad X = \frac{4{,}000}{3.32065\times 1.06876} = 1{,}127.085$$

Total interest = $5 \times 1{,}127.085 - 4000 = 1635.425$

$\therefore$ flat rate = 8.2%

10 Price of £100 T-bill = $100\left(1-\frac{91}{365}\times 0.06\right) = 98.504$

Amount invested in 91 day deposit to give 100 would be:

$$100\left(1+\frac{0.0615}{365}\right)^{-91} = 98.479$$

$\therefore$ The deposit gives higher rate of return.

11 (i) (a) Accumulation from $t = 0$ to $t = 10$ is:

$$150A(0, 10) = 150e^{0.04\times 10} = 223.77370$$

Accumulation from $t = 10$ to 20 is:

$$223.77370\, A(10, 20)$$

$$A(10, 20) = e^{\int_{10}^{20} 0.001(t-10)^2 + 0.04\, dt}$$

$$\int_{10}^{20} 0.001(t-10)^2 + 0.04\, dt = \left[\frac{0.001(t-10)^3}{3} + 0.04t\right]_{10}^{20}$$

$$= 1.1\dot{3} - 0.4 = 0.3\dot{3}$$

$$\therefore \quad 223.7737\, A(10, 20) = 223.7737e^{0.7\dot{3}}$$

$$= 465.90$$

(ii) (a) $v(t)$ for $t \le 10 = e^{-0.04t}$

$\therefore$ we require $\int_{5}^{10} 10e^{-0.04t}\, dt$

$$= 10 \times \left[\frac{e^{-0.04t}}{-0.04}\right]_{5}^{10} = (-16.758 + 20.46827)\times 10$$

$$= 3.7103 \times 10 = 37.103$$

(b) $v(t) = e^{-0.04t}$

$\therefore$ we require $\int_0^{10} e^{-0.03t} \times e^{-0.04t}\, dt = \int_0^{10} e^{-0.07t}\, dt$

$$= \left[-\frac{e^{-0.07t}}{0.07} \right]_0^{10} = -7.09408 + 14.2875$$

$$= 7.19163$$

12 (i) $90a_{\overline{20}|} + 10(Ia)_{\overline{20}|}$ @ 8%

$$= 90 \times 9.8181 + 10\left(\frac{1.08 \times 9.8181 - 20 \times 0.21455}{0.08}\right)$$

$= 883.629 + 10 \times 78.90685$

$= 1672.6975$

(ii) Cap. o/s after 5 payments is:

6th payment = 150

$\therefore$ Cap. o/s $= 140a_{\overline{15}|} + 10\left(\frac{\ddot{a}_{\overline{15}|} - 15v^{15}}{0.08}\right)$

$$= 140 \times 8.5595 + 10\left(\frac{1.08 \times 8.5595 - 15 \times 0.31524}{0.08}\right)$$

$= 1762.7875$

Loan schedule:

	int.	cap.	cap. o/s
5			1762.7875
6	141.023	8.977	1753.8105
7	140.305	19.695	1734.1155

(iii) Last instalment = 290

$290 = (1 + i)$ cap. o/s

$\therefore$ cap. o/s $= 268.519$ This is all paid off.

$\therefore$ interest $= 290 - 268.519 = 21.4815$

13 (i) (a) Assets would accumulate to $950{,}000 \times 1.05 = 997{,}500$

$\therefore$ Probability = 1.00

(b) The guaranteed portion of the fund would accumulate to $0.15 \times 950{,}000 \times (1.05) = 149{,}625$.

$\therefore$ non-guaranteed portion needs to accumulate to $1{,}000{,}000 - 149{,}625 = 850{,}375$.

$\therefore$ we require probability that $(0.85 \times 950{,}000)\ (1 + i_t) < 850{,}375$

$$= \Pr((1 + i_t) < 1.053096)$$

$$= \Pr(\ln(1 + i_t) < \ln 1.053096)$$

$$= \Pr\left(\frac{\ln(1 + i_t) - 0.06748}{0.0186896} < \frac{\ln 1.053096 - 0.06748}{0.0186896}\right)$$

$= \Pr(Z < -0.842479)$ where $Z \sim N(0, 1)$.

$$1 - \Pr(Z < 0.842479)$$

$$= 1 - 0.79955 \approx 0.20$$

(ii) (a) Variance of return = 0

(b) Return from portfolio $= 0.15 \times 0.05 + 0.85 \times i_t$

$\therefore$ Variance of return $= 0 + 0.85^2 \times 0.0004$

$$= 2.89 \times 10^{-4}$$

14 (i) $(\bar{I}\bar{a})_{\overline{n}|} = \int_0^n t\, e^{-\delta t}\, dt$

$$= t \int_0^n e^{-\delta t}\, dt \,\Big|_0^n - \int_0^n \left(-\frac{e^{-\delta t}}{\delta}\right) dt$$

$$= \frac{t}{-\delta} e^{-\delta t} \,\Big|_0^n + \frac{\bar{a}_{\overline{n}|}}{\delta}$$

$$= \frac{n e^{-\delta n}}{-\delta} + \frac{\bar{a}_{\overline{n}|}}{\delta} = \frac{\bar{a}_{\overline{n}|} - n e^{-\delta n}}{\delta}$$

(ii) Present value of net revenue

$$v\,250{,}000(\bar{I}\bar{a})_{\overline{10}|} + v^{11}\left\{2{,}500{,}000\bar{a}_{\overline{18}|} - 125{,}000(\bar{I}\bar{a})_{\overline{18}|}\right\}$$

$v = \dfrac{1}{1.2}$ $\qquad (\bar{I}\bar{a})_{\overline{10}|} = \dfrac{\bar{a}_{\overline{10}|} - 10v^{10}}{\delta} = 16.3663$

$\bar{a}_{\overline{18}|} = \dfrac{i}{\delta} a_{\overline{18}|} = 5.2788$ $\qquad (\bar{I}\bar{a})_{\overline{18}|} = \dfrac{\bar{a}_{\overline{18}|} - 18v^{10}}{\delta} = 25.2450$

PV = 3,409,645.8 + 1,351,448.4 = 4,761,094.2

Present value of fixed costs and initial cost

$$500{,}000\bar{a}_{\overline{1}|} + 200{,}000\bar{a}_{\overline{29}|}$$

$\bar{a}_{\overline{1}|} = \dfrac{i}{\delta} a_{\overline{1}|} = 0.9141$ $\qquad \bar{a}_{\overline{29}|} = \dfrac{1 - v^{29}}{\delta} = 5.4571$

PV = 457,050 + 1,091,420 = 1,548,470

Maximum price to yield 20% p.a. is 4,761,094 – 1,548,470 = 3,212,624

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		Fund A		*Fund B*	
Year	*Price*	*New*		*New*	
92	67	100	100.00	67	67
93	78	100	216.42	78	156
94	81	100	324.74	81	243
95	86	100	444.79	86	344
96	95	100	591.33	95	475
97	100		622.46		500

(i) Money weighted rates of return

A: $100\ddot{s}_{\overline{5}|} = 622.46$ $\qquad i \sim 7.4\%$

B: $67(1+i)^5 + 78(1+i)^4 + 81(1+i)^3 + 86(1+i)^2 + 95(1+i) = 500$

Try $i = 8\%$ LHS = 509.51 $\quad i = 7\%$ LHS = 495.56 $\quad i \sim 7.3\%$

(ii) Time weighted rates of return will all be equal to the time weighted return on the fund given by

$$67(1+i)^5 = 100 \quad \text{i.e. } i = 8.3\%$$

Otherwise:

$$\text{A: } (1+i)^5 = \frac{116.42}{100}\cdot\frac{224.74}{216.42}\cdot\frac{344.79}{324.74}\cdot\frac{491.33}{444.79}\cdot\frac{622.46}{591.33} \Rightarrow i = 8.3\%$$

$$\text{B: } (1+i)^5 = \frac{78}{67}\cdot\frac{162}{156}\cdot\frac{258}{243}\cdot\frac{380}{344}\cdot\frac{500}{475} \Rightarrow i = 8.3\%$$

16 (i) For the Treasury Stock if the RPI is assumed to grow continuously at a rate of 2½% p.a. then if i' is the real rate of return

$$107 = 3.75\left\{\frac{1}{[(1+i')(1.025)]^{½}} + \frac{1}{[(1+i')(1.025)]^{1}} + \ldots + \frac{1}{[(1+i')(1.025)]^{9}}\right\} + \frac{100}{[(1+i')(1.025)]^{9}}$$

$$= 7.50a_{\overline{9}|}^{(2)} + 100v^9 \text{ at } i \text{ where } (1+i) = (1+i')(1.025).$$

With $i = 7\%$ RHS = $7.5 \times 6.6273 + 54.39 = 104.10$

$i = 6\%$ RHS = $7.5 \times 6.9022 + 59.19 = 110.96$

By interpolation $i \sim 6\% + \dfrac{110.96 - 107}{110.96 - 104.10} \times 1\% = 6.6\%$

So $i' = \dfrac{1.066}{1.025} - 1 = 4\%$.

[N.B. As an alternative to interpolation, candidates may choose to evaluate

$$7.50a_{\overline{9}|}^{(2)} + 100v^9$$

at $(1 + i) = (1.04)(1.025) = 1.066$

but if they take this route there must be evidence that they have actually done the evaluation.]

(ii) The nominal and real values of the future receipts per £100 nominal may be found as follows:

Date	*Nominal value*	*Real value*
9/4/98	$\frac{158.5}{69.5} v^{½}$	$\frac{158.5}{69.5} v^{½} \frac{1}{(1.025)^{½}}$
9/10/98	$\frac{158.5}{69.5} (1.025)^{½} v$	$\frac{158.5}{69.5} v \frac{(1.025)^{½}}{(1.025)}$
. . . .	. . . .	. . . .
9/10/2006	$\frac{158.5}{69.5} (100)(1.025)^{8.5} v^9$	$\frac{158.5}{69.5} (100) \frac{(1.025)^{8.5}}{(1.025)^9} v^9$

Summing the real value of these payments gives

$$\frac{158.5}{69.5(1.025)^{½}} \{2a_{\overline{9}|}^{(2)} + 100v^9\} .$$

(iii) The real yield is found from the expression

$$203 = \frac{158.5}{69.5(1.025)^{½}} \{2a_{\overline{9}|}^{(2)} + 100v^9\}$$

with $i = 3\%$ RHS = 2.2526 {15.6881 + 76.6417} = 207.98

with $i = 3½\%$ RHS = 2.2526 {15.3474 + 73.3731} = 199.85

By interpolation

$$i \sim 3\% + \left\{\frac{207.98 - 203}{207.98 - 199.85}\right\}$$

= 3.3%.