

# EXAMINATIONS

April 1998

**Subject C — Statistics**

*Paper Two*

*Time allowed: Three hours*

## **INSTRUCTIONS TO THE CANDIDATE**

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions.*

## **AT THE END OF THE EXAMINATION**

*Hand in BOTH your answer booklet and this question paper.*

*In addition to this paper you should have available graph paper,  
Actuarial Tables and an electronic calculator.*

## PART ONE

For questions 1–5 indicate in your answer booklet which one of the answers A, B, C or D is correct.

- 1** The probability of a claim arising from a particular insurance policy over 1 year is  $q$  ( $0 < q < 1$ ). The distribution function for the size of a claim is  $F(x)$ . The probability that the amount paid exceeds  $L$ , given that a claim occurs is:
- A  $\frac{1 - F(L)}{q}$
- B  $q(1 - F(L))$
- C  $1 - F(L)$
- D None of the above. [2]
- 2** You are required to calculate the credibility premium for a certain risk, on which average annual aggregate claims over the last 5 years have been £1380. Average annual aggregate claims on the collective of risks over the last 5 years were £1247. In which of the following intervals could the credibility premium lie?
- I (£1200, £1300)
- II (£1300, £1400)
- III (£1400, £1600)
- A I only.
- B I and II only.
- C II only.
- D None of the above. [2]

- 3**  $\{S_1(t)\}_{t \geq 0}$  is a compound Poisson process with Poisson parameter 1 and mean claim amount 1.  $\{S_2(t)\}_{t \geq 0}$  is a compound Poisson process with Poisson parameter 2 and mean claim amount 2. The process  $\{S(t)\}_{t \geq 0}$ , where:

$$S(t) = S_1(t) + S_2(t)$$

is a compound Poisson process with:

- A Poisson parameter 3 and mean claim amount 5/3  
 B Poisson parameter 3/2 and mean claim amount 3  
 C Poisson parameter 3 and mean claim amount 3  
 D Poisson parameter 3/2 and mean claim amount 5/3 [3]

- 4** The number of claims  $N$  on a portfolio has a Poisson distribution with mean  $\mu$ . The claim amounts are independent and each has a gamma distribution with parameters  $\alpha$  and  $\lambda$ .

The moment generating function of the total claim amount  $S$  is:

- A  $\exp\left\{\mu\left[\left(\frac{\lambda}{\lambda-t}\right)^\alpha - 1\right]\right\}$   
 B  $\exp\left\{\mu\left[\left(\frac{\lambda-t}{\lambda}\right)^\alpha - 1\right]\right\}$   
 C  $\exp\left\{\mu\left(\frac{\lambda}{\lambda-t}\right)^\alpha - 1\right\}$   
 D  $\exp\left\{\mu\left(\frac{\lambda-t}{\lambda}\right)^\alpha - 1\right\}$  [2]

- 5** The random variable  $X$  has an exponential distribution with mean  $1/\lambda$ . The moment generating function of  $\min(X, M)$ , where  $M$  is a positive constant, is:

- A  $[\lambda - r \cdot \exp\{-(\lambda - r)M\}] / [\lambda - r]$   
 B  $[\lambda + r \cdot \exp\{-(\lambda - r)M\}] / [\lambda - r]$   
 C  $\lambda[1 - \exp\{-(\lambda - r)M\}] / [\lambda - r]$   
 D  $\lambda[1 + \exp\{-(\lambda - r)M\}] / [\lambda - r]$  [4]

## PART TWO

- 6** A claim amount distribution is normal with unknown mean  $\mu$  and known standard deviation £100. Based on past experience a suitable prior distribution for  $\mu$  is normal with mean £500 and standard deviation £20.
- (i) Calculate the prior probability that  $\mu$ , the mean of the claim amount distribution, exceeds £535. [1]
- (ii) A random sample of 10 current claims have a mean of £535. Determine the posterior distribution of  $\mu$ . Hence calculate the posterior probability that  $\mu$  exceeds £535 and comment briefly on your answer. [5]  
[Total 6]
- 7** Aggregate annual claims for a risk have a compound Poisson distribution with Poisson parameter  $\lambda$ . Individual claim amounts (before reinsurance) have an exponential distribution with mean  $\mu$ . The insurer effects individual excess of loss reinsurance with retention  $M$ .
- Derive an expression, not involving integrals, for the variance of the insurer's retained aggregate claims. [6]
- 8** At the start of 1997 a company employed 250 people for each of whom the probability of dying in the year was  $q$ . The number of deaths in the year had a Binomial(250,  $q$ ) distribution. The prior distribution of  $q$  was Beta with mean 0.015 and standard deviation 0.005. Given that 5 employees died in 1997, calculate the posterior distribution of  $q$ . [6]
- 9** In a Bayesian analysis of claims data, you ask a company claims expert to specify his prior knowledge about the claim rate for a certain class of business. He specifies that he expects the rate to be about 2.5 claims per year and that it is equally likely to be anywhere in the interval (2, 3), implying a uniform prior distribution on (2, 3).
- Determine parameters for a gamma prior distribution which reflect the experts stated opinion as regards its mean and variance. Draw a rough sketch of the resulting prior density function. [5]

**10** Sixteen successive observations on a financial time series are as follows:

$t:$	1	2	3	4	5	6	7	8
$y_t:$	1.6	0.8	1.2	0.5	0.9	1.1	1.1	0.6
$t:$	9	10	11	12	13	14	15	16
$y_t:$	1.5	0.8	0.9	1.2	0.5	1.3	0.8	1.2

- (i) Plot the observations and from your plot state, with reasons, whether the data can be considered to be independent. [4]
- (ii) Given that estimates of the first four autocorrelation coefficients are:

$$\hat{\rho}_1 = -0.64, \quad \hat{\rho}_2 = +0.30, \quad \hat{\rho}_3 = -0.14 \text{ and } \hat{\rho}_4 = -0.25,$$

state, with reasons, what form of model from within the class of AR and MA models might be appropriate for this series. [4]

[Total 8]

**11** An insurance company has a group life insurance policy which covers 2 classes of employees. The sum insured payable in the event of death is twice the annual salary of the employee. Within each class of employee, annual salaries are assumed to be normally distributed, and the relevant data are given in the following table:

<i>Class of Employee</i>	<i>Number of Employees</i>	<i>Probability of Death</i>	<i>Mean Salary</i>	<i>Standard Deviation of Salaries</i>
1	1000	0.0001	10000	2000
2	500	0.0005	20000	3000

- (i) Derive the moment generating function of aggregate claims, stating any assumptions made. [6]
- (ii) Calculate the mean and variance of aggregate claims. [4]

[Total 10]

## PART THREE

- 12** The table below shows payments in cumulative form in units of £1,000 in respect of a non-life insurance portfolio. You may assume that payments are made in the middle of each calendar year and that all claims are fully settled by the end of development year 3. The insurer earns interest at 8% per annum effective on reserves and assumes that the rate of inflation, both in the past and in the future, is 5% per annum.

Use the inflation-adjusted chain ladder method to calculate the reserve, discounted for interest, which should be held at 1 January 1998 in respect of this portfolio.

		<i>Development Year</i>			
		0	1	2	3
<i>Year of origin</i>	1994	1500	3100	3950	4320
	1995	1750	2955	4010	
	1996	2375	4605		
	1997	1695			

[12]

- 13**  $\{S_n\}_{n=1}^{\infty}$  is a sequence of independent and identically distributed random variables, each with mean 5 and variance 25.  $S_n$  represents the aggregate claims from a risk in year  $n$ . The insurer intends to calculate the annual risk premium,  $\Pi$ , for this risk so that:

$$\Pr[S_n > \Pi] = 0.01$$

- (i) Assuming  $S_n$  has an exponential distribution, show that  $\Pi = 23.03$ . [2]
- (ii) Calculate the value of  $\Pi$  assuming  $S_n$  has a lognormal distribution. [6]
- (iii) Comment on the difference between your answers to parts (i) and (ii). [2]
- (iv) Assuming that  $S_n$  has an exponential distribution, calculate the value of:

$$P[(S_1 \leq 23.03) \text{ and } (S_1 + S_2 \leq 46.06)] \quad [5]$$

[Total 15]

**14** Claims occur according to a Poisson process with Poisson parameter  $\lambda$ . The claim size distribution has mean  $\mu$  and variance  $\sigma^2$ , and the premium loading factor is  $\theta$ .

(i) State briefly how these parameters affect the probability of ruin. [4]

On a portfolio of insurance policies, the average claim size is £2,000. There are 3 possible distributions under consideration for the claim sizes. These are:

I fixed claim size of £2,000

II exponential distribution

III gamma distribution, with standard deviation £2,500

The adjustment coefficients are denoted by  $R_I$ ,  $R_{II}$  and  $R_{III}$ , respectively.

(ii) Derive the equations for the adjustment coefficient for each of the above cases, I, II and III. [5]

(iii) Using the equations derived in (ii), expand the moment generating functions in order to show that

$$R_I \geq R_{II} \geq R_{III}. \quad [5]$$

(iv) Using the equations derived in (ii), show that the adjustment coefficient is an increasing function of  $\theta$  in all cases. [1]

(v) Discuss the results in parts (ii), (iii) and (iv) in relation to those in part (i), and explain how these results affect the choice of claim size distribution. [4]

[Total 19]