

# EXAMINATION

24 September 2008 (am)

## Subject CT8 — Financial Economics Core Technical

*Time allowed: Three hours*

### ***INSTRUCTIONS TO THE CANDIDATE***

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

***Graph paper is not required for this paper.***

### ***AT THE END OF THE EXAMINATION***

*Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.*

<p><i>In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.</i></p>
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**1** Two assets are available for investment. Asset 1 returns a percentage  $4B\%$ , where  $B$  is a Binomial random variable with parameters  $n = 3$  and  $p = 0.5$ . Asset 2 returns a percentage  $2P\%$ , where  $P$  is a Poisson random variable with parameter  $\mu = 3$ . Assume a benchmark return of  $3\%$ .

(i) Calculate the following three measures of investment risk for each asset:

- (a) variance
- (b) semi-variance and
- (c) shortfall probability [6]

An investor has £1,000 to invest in one of the assets.

(ii) Explain which asset the investor should choose assuming a utility function of the form:

$$u(x) = \begin{cases} -(1,060 - x)^2 & : x < 1,060; \\ 0 & : x \geq 1,060. \end{cases}$$

[2]  
[Total 8]

**2** (i) State the form of an equation that is appropriate to determine the relationship between the observed historical returns of a number of securities and a set of explanatory factors. Define all the terms you use. [2]

(ii) State the main features of:

- (a) macroeconomic factor models
  - (b) fundamental factor models and
  - (c) statistical factor models [6]
- [Total 8]

**3** (i) (a) Define Beta in the Capital Asset Pricing Model (CAPM)  
(b) Explain why Beta is used in pricing securities. [2]

In a market where the CAPM holds the following parameters are known:

Risk-free rate of interest =  $6\%$

Expected market rate of return =  $12\%$

Standard deviation of an efficient portfolio's returns =  $0.50$

Standard deviation of the market returns =  $0.7$

(ii) Calculate the expected return on the portfolio. [2]

- (iii) An investor is evaluating the risk and expected return of a portfolio of  $N$  securities.

Explain how many parameters need to be estimated if:

- (a) the evaluation is made using mean-variance portfolio theory without any assumed cross-sectional structure in the variances of the securities.
- (b) the CAPM is assumed to hold.

[3]

[Total 7]

- 4** One of your colleagues tells you that the work of Shiller conclusively proves that the stock-market overreacts.

(i) Outline the nature of Shiller's findings. [4]

(ii) Discuss whether you agree with your colleague. [4]

[Total 8]

- 5** State the defining properties of a standard Brownian motion. [4]

- 6** Consider an asset  $S$  paying a dividend at a constant instantaneous rate of  $\delta$ , a forward contract with maturity  $T$  written on  $S$  and a constant, instantaneous (continuously compounded) risk-free rate of  $r$ .

Derive the price at time  $t$  of the forward contract, using the no-arbitrage principle. [8]

- 7** Consider a one-period Binomial model of a stock whose current price is  $S_0 = 40$ . Suppose that:

- over a single period, the stock price can either move up to 60 or down to 30
- the continuously compounded risk-free rate is  $r = 5\%$  per period

(i) Show that there is no arbitrage in the market. [1]

(ii) Calculate the price of a European call option with maturity date in one period and strike price  $K = 45$  using each of the following methods:

- (a) by constructing a risk-neutral portfolio; and
- (b) by constructing a replicating portfolio

[9]

[Total 10]

- 8** (i) Write down the formula for the value at time  $t < T$  of a derivative payment due at time  $T$  in a market where the Black-Scholes formula applies, defining any notation that you use. [3]

A non-dividend paying stock has price  $S_t$  at time  $t < T$ . A derivative contract based on the stock pays \$1 at time  $T$  if and only if the stock price at time  $T$ ,  $S_T$ , is at least  $K$ .

Assume the following:

risk-free interest rate: 7% p.a. continuously compounded

stock price volatility: 25% p.a.

dividend yield: nil

$T = 18$  months

$S_0 = \$1.1$

- (ii) Derive a formula for  $V_0$ , the value of the derivative contract. [5]
- (iii) Calculate the value at time 0 of a derivative which delivers one unit of the stock at time  $T$  if and only if the stock price at time  $T$ ,  $S_T$ , is at least  $K$ . [3]
- (iv) Calculate the value of a derivative which delivers 150,000 shares of the stock if and only if the stock price at time  $T$ ,  $S_T$ , satisfies  $\$1.2 \leq S_T < \$1.5$ . [6]
- [Total 17]

- 9** (i) Explain why an investor might want to Vega-hedge a portfolio. [3]
- (ii) Derive a formula for the Vega of a European call option on a non-dividend paying stock in a market where the assumptions underpinning Black-Scholes apply. [4]
- (iii) (a) Determine the PDE satisfied by Vega by differentiating the Black-Scholes PDE.
- (b) Show that if a portfolio is Gamma-hedged, then its Vega satisfies the Black-Scholes PDE. [4]
- [Total 11]

- 10** State how the price at time  $t$  of a zero-coupon bond paying £1 at  $T$  (denoted by  $B(t, T)$ ) is related to:

- (a) spot rate curve  
 (b) instantaneous forward rate curve  
 (c) instantaneous risk free rate

Define all notation used. [4]

- 11** (i) Describe the Merton model for assessing credit risk. [5]

A company has just issued a zero-coupon bond of nominal value £8m with maturity of one year. The value of the assets of the company is £10.009m and this value is expected to grow at an average of 10% per annum compound with an annual volatility of 20%. The company is expected to be wound up after one year when the assets will be used to pay off the bond holders with the remainder being distributed to the equity holders. Shares in the company are currently traded at a market capitalisation of £2.9428m.

- (ii) Estimate the risk-free rate of interest in the market to within 1% p.a., stating any additional assumptions that you make. [10]

[Total 15]

**END OF PAPER**