EXAMINATIONS

September 1998

Subject D — Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Begin your answers to Parts One, Two and Three on a separate sheet.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 16 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

D2—S98

PART ONE

For questions 1–8 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 A set of graduated rates obtained by fitting a mathematical formula with three unknown parameters is being tested by the calculation of standardised deviations of the graduated rates from the observed rates. The following characteristics of the pattern of deviations have been noted.

I many more positive deviations than expected
II a large number of runs of deviations with same sign
III many more deviations of small absolute value then expected

Which of these features would indicate that the graduated rates were unsatisfactory?

A I and II only
B II and III only
C I only
D III only [2]

2 The effect of including duplicate policies in a mortality investigation is to

A bias the estimated mortality rates
B violate the principle of correspondence
C increase the size of the standard error of the estimated rates
D change the formula used to estimate the rates [2]

3 In an investigation of mortality the number of deaths at age x during the period of the investigation, θ_x, has been recorded where x is age next birthday on the 1 October following the date of death. The census $P_x(t)$ of those alive with age label x at time t after the start of the period of investigation has also been recorded. If θ_x and $P_x(t)$ are to satisfy the principle of correspondence, then age label x for $P_x(t)$ must be defined as

A age next birthday at time t
B age next birthday on 1 October immediately preceding time t
C age next birthday on 1 January immediately following time t
D age next birthday on 1 October immediately following time t [2]

4 In 1995 the Continuous Mortality Investigation Committee published the results of an investigation into the mortality experience of male permanent assurance policyholders for the calendar years 1988-90 sub-divided by age and by smoking status. An extract from this publication for policies with durations 2 and over shows

	Smokers		*Non-Smokers*	
Age group (nearest ages)	*Actual deaths, A*	*A/E**	*Actual deaths, A*	*A/E**
–30	23	1.45	55	0.90
31–45	118	1.06	133	0.65
46–60	483	0.90	356	0.50
61–75	297	0.92	266	0.55
76–	126	1.00	73	0.64
All ages	1047	0.94	883	0.56

* E = Expected Deaths using AM80 Ultimate Mortality Table

(i) The A/E values for “All ages” are

A standardised mortality ratios
B indirectly standardised mortality rates
C area comparability factors
D directly standardised mortality rates [1]

(ii) The results displayed are illustrative of

A temporary initial selection
B time selection
C anti-selection
D class selection [1]

[Total 2]

5 Using the component method of population projection, the number of people projected to be alive in the population at the end of a calendar year n, who are aged x last birthday at the end of the year can be written

$$^{n}P_{x} = {}^{n-1}P_{x-1}\,(1 - {}^{n}q_{x-½}) + I - E$$

where $^{n}q_{x-½}$ is the probability that lives aged on average x – ½ at the start of year n, die during year n.

Possible definitions of I and E (all relating to lives aged x last birthday at the end of year n) are:

I(1) the number of people migrating into the population during year n;

I(2) the number of people migrating into the population during year n, who survive to the end of the year;

E(1) the number of people migrating out of the population during year n;

E(2) the number of people migrating out of the population during year n, who survive to the end of the year.

Which one of the following gives the correct definitions of I and E?

A I(1) and E(1)
B I(1) and E(2)
C I(2) and E(1)
D I(2) and E(2) [2]

6 For a certain type of short term insurance contract, company X charges higher premiums to people of a certain group A than to people of another group B. Company Y charges the same premium to both groups. Company Y's premium is intermediate in size between the two premium rates of company X, and is commensurate with the average claim cost of the two groups of lives. Independent studies have shown, however, that the claim experience of the two groups are not statistically distinguishable from each other. Assuming that neither company changes its premium rates in the future, which one of the following is most likely to happen in the long run?

A Both companies will become insolvent.
B Company X will become insolvent while company Y will prosper.
C Company X will prosper while company Y will become insolvent.
D Both companies will prosper. [2]

7 Which one of the following is the GM(1,3) function as used by the CMI Committee for some of its graduations of the 1979–82 life office mortality data? P, Q, R, and S are the constants which were fitted as part of the graduation process.

A $P + Q \,.\, R^x \,.\, S^{x^2}$

B $P \,.\, x + Q + R^x + S^{x^2}$

C $P^x + Q + R \,.\, x + S \,.\, x^2$

D $P \,.\, x + Q + R \,.\, S^x$ [1]

8 A set of mortality rates for n ages has been graduated using a graphical method. The serial correlation coefficient with unit lag $\hat{r}_1$, has been calculated. If the graduation is satisfactory, which of one the following statements is true?

A $\hat{r}_1\sqrt{n} \sim t_{n-1}$

B $\hat{r}_1\sqrt{n} \sim N(0,1)$

C $\hat{r}_1 / \sqrt{n} \sim t_{n-1}$

D $\hat{r}_1 / \sqrt{n} \sim N(0,1)$ [2]

PART TWO

9 (i) In the context of the graduation of a set of observed rates of decrement explain what is meant by:

(a) intrinsic roughness
(b) smoothness
(c) adherence to data [5]

(ii) Using the concepts in (i) describe what is implied by saying that the observed rates have been overgraduated. [3]
[Total 8]

10 A colleague has recently completed the estimation of dependent rates of mortality, retirement and withdrawal as part of an investigation to revise the service table for the active lives of a large occupational pension scheme.

She has noticed that the estimated rates do not progress smoothly from age to age, and has asked your advice on the graduation of the rates to make the estimated service table fit for use in the valuation of the pension fund.

Write a short note describing how the graduation should be performed. You should indicate clearly the steps in the procedure and outline the methods of graduation that would be used, but a detailed description of the testing of graduated rates is **not** required. [5]

11 (i) In a mortality investigation the observed number of deaths at age x last birthday is d_x and the corresponding initial exposed to risk is E_x.

If q_x is the true initial rate of mortality at age x and θ_x is the random variable representing the number of deaths at age x last birthday, show that the standard error of the estimator

$$\hat{q}_x = \frac{\theta_x}{E_x}$$

is estimated approximately by

$$\frac{\sqrt{d_x}}{E_x}$$

stating clearly the assumptions you make in deriving this result. [3]

(ii) Describe the implications of this result for the estimation of the mortality rates for assured lives over the age range 18 to 110. [2]

(iii) Explain how the interval

$$\overset{\circ}{q}_x \pm \frac{2\sqrt{d_x}}{E_x}$$

might be used to assist in the graphical graduation of a set of observed mortality rates, $\overset{\circ}{q}_x$, where

$$\overset{\circ}{q}_x = \frac{d_x}{E_x}$$ [3]

[Total 8]

12 A life office sells term assurance policies direct to the public via newspaper advertisements. Policies are only available with 10 or 20 year terms.

You have been asked to carry out the first review of the mortality experience of these policies. The table shows the statistical summary of the mortality investigation, in which all rates of mortality are expressed per 1000 lives.

	All Policies		*10 year Policies*		*20 year Policies*	
Age	*Number in force*	*Central Mortality Rate*	*Number in force*	*Central Mortality Rate*	*Number in force*	*Central Mortality Rate*
–24	4360	1.02	4013	0.98	347	1.50
25–44	6407	1.76	5970	1.63	437	3.52
45–64	5047	10.42	4871	10.04	176	20.94
65–	2231	68.94	2192	68.50	39	93.52
Total	18045		17046		999	
Crude Death Rate		12.31		12.48		9.41
Directly Standardised Mortality Rate				12.09		19.03
Standardised Mortality Ratio (SMR)				0.983		1.665

In each case the "all policies" population has been used as the standard population.

(i) Write a note which describes and explains the observed values of the crude death rates, the directly standardised mortality rates and the standardised mortality ratios (SMRs). [6]

(ii) You have been asked to recommend which of these three summary statistics should be monitored on a regular basis. Give your recommendations, explaining the reasons for your choice. [3]

[Total 9]

PART THREE

13 (i) Why is it considered necessary to project the mortality of pensioners and annuitants but not the mortality of assured lives when constructing standard mortality tables? [2]

(ii) In the context of the CMIB method of projecting mortality rates, explain what is meant by:

(a) a base table
(b) a reduction factor
(c) a double entry table
(d) a single entry table [6]

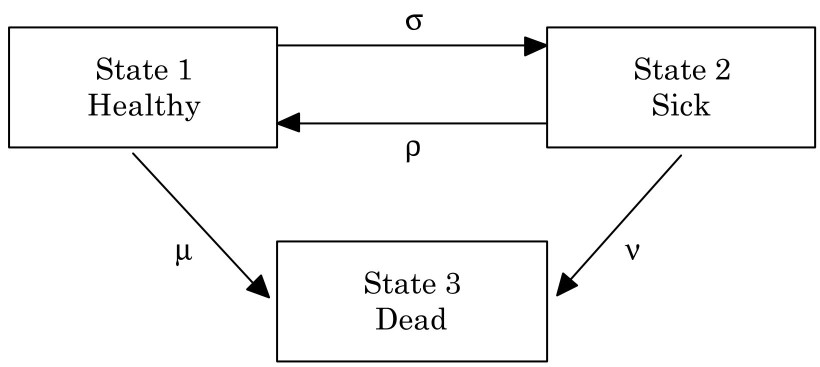
(iii) Two single entry tables from the projection of the 1980 series are WA80B35 and WA80C10.

(a) Describe the mortality represented by each table. [3]

(b) At which age will the mortality rates in these two single entry tables be identical? [1]

[Total 12]

14 A population of lives between the integer ages x and $x + 1$ is modelled using the following sickness and death model



where σ, ρ, μ and ν represent the forces of transition between the three states.

During a period of investigation of three calendar years, the following data have been recorded for lives aged between x and $x + 1$.

W_1 = total waiting time in state 1 = 1237191 days
W_2 = total waiting time in state 2 = 4237 days
θ = number of healthy lives dying = 4
S = number of healthy lives falling sick = 9
R = number of sick lives recovering = 10
D = number of sick lives dying = 1

(i) Write down the likelihood function for the observed data. What assumptions are necessary to construct this likelihood function? [4]

(ii) Using these data, determine estimates of σ, ρ, μ and ν. [2]

(iii) Is there statistical evidence that ρ is greater than 1.250? Show all your calculations and explain your reasoning clearly. [6]

[Total 12]

15 The following data are available from a life insurance company, relating to the mortality experience of its temporary assurance policyholders.

$\theta_{x,d}$ number of deaths over the period 1st January 1993 to 30th September 1996, aged x nearest birthday at entry and having exact duration d at the next policy anniversary following the date of death.

$P_{y,e}(n)$ number of policyholders with policies in force at time n, aged y nearest birthday at entry and having curtate duration e at time n, where n = 1.1.1993, 30.9.1993, 30.9.1995 and 30.9.1996.

(i) Develop formulae for the calculation of both crude central select and crude initial select rates of mortality corresponding to the $\theta_{x,d}$ deaths, and derive the age and duration to which these estimates apply. You can assume that all months are of equal length. All other assumptions made should be stated. [11]

(ii) Use your formulae obtained in (i) to calculate an estimate of $q_{[40]+1}$ from the data given in the table

$P_{y,e}(n)$

				n	
y	*e*	*1.1.1993*	*30.9.1993*	*30.9.1995*	*30.9.1996*
40	0	3608	3218	3920	4012
40	1	2975	3111	2866	3382
40	2	4341	3959	3691	3115
41	0	4194	4260	3704	3429
41	1	3546	3184	3372	2990
41	2	2887	3890	3662	2845

$q_{x,d}$

x	*d*	$q_{x,d}$
40	1	25
40	2	48
41	1	37
41	2	41

[4]

[Total 15]

16 (i) The service table for the active members of a pension scheme has three causes of decrement

d = death
r = retirement
w = withdrawal, ie. leaving the scheme for reasons other than death or retirement.

The dependent rates of decrement at age x exact are $(aq)_x^i$ for $i = d, r, w$ and the total rate of decrement is $(aq)_x$.

(a) Show that if the independent and dependent forces of decrement for each cause at age x are assumed to be equal then

$$(ap)_x = p_x^d \; p_x^r \; p_x^w$$

where p_x^i for $i = d, r, w$ are the one year survival probabilities in the underlying single decrement tables. [3]

(b) If the force of decrement for each cause is constant over $(x, x+1)$ show that

$$(aq)_x^i = \frac{\mu_x^i}{(a\mu)_x}(aq)_x \quad \text{for } i = d, r, w$$ [3]

(c) Using the result obtained in (b) or otherwise, show that

$$q_x^i = 1 - (1 - (aq)_x)^{\frac{(aq)_x^i}{(aq)_x}} \quad \text{for } i = d, r, w$$

and that

$$(aq)_x^i = \frac{\log_e p_x^i}{\log_e p_x^d \; p_x^r \; p_x^w}\left(1 - p_x^d \; p_x^r \; p_x^w\right) \quad \text{for } i = d, r, w$$ [4]

(ii) The service table for the scheme described in (i) is

x	$(a\ell)_x$	$(ad)_x^d$	$(ad)_x^r$	$(ad)_x^w$
60	19863	297	3618	146
61	15802	253	2370	153
62	13026	228	1303	178

Making the same assumptions as in (i) calculate the values of $(ad)_{61}^d, (ad)_{61}^r$ and $(ad)_{61}^w$ for a revised service table with $(a\ell)_{61} = 10{,}000$. This revised table has independent initial rates of withdrawal which are 10% lower than in the current table, and independent initial rates of retirement which are 20% higher than in the current table. The independent initial rates of mortality are unchanged. [6]
[Total 16]