

# EXAMINATIONS

April 1999

## Subject D — Actuarial Mathematics

*Paper Two*

*Time allowed: Three hours*

### **INSTRUCTIONS TO THE CANDIDATE**

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 17 questions.*

***Graph paper is not required for this paper.***

### **AT THE END OF THE EXAMINATION**

*Hand in BOTH your answer booklet and this question paper.*

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
--

## PART ONE

For questions 1–8 indicate in your answer booklet which one of the answers A, B, C or D is correct.

- 1** Mortality rates are to be projected to year  $T$  using the formula

$$q_{x,T} = q_{x,0} \cdot RF(x, T)$$

where  $q_{x,0}$  are the mortality rates in the base year, and

$$RF(x, T) = 0.4 + 0.6e^{-\beta T}$$

If 70% of all the expected future improvement in mortality occurs in the next 10 years, then  $\beta$  equals:

- A 0.0357  
 B 0.0693  
 C 0.1204  
 D 0.3401 [2]

- 2** A select mortality investigation has produced the following values of  $\hat{q}_{[x],r}^{\circ}$  where:

$[x]$  is the age nearest birthday at entry,  
 $r$  is the curtate duration at date of death.

$x$	$0$	$1$	$r$ $2$	$3$	$4+$
50	0.0054	0.0075	0.0093	0.0101	0.0119
51	0.0066	0.0083	0.0101	0.0115	0.0129
52	0.0079	0.0090	0.0114	0.0130	0.0151
53	0.0093	0.0098	0.0131	0.0149	0.0152
54	0.0095	0.0112	0.0150	0.0152	0.0173

The standard error of each estimate is 0.0002.

The best choice of the length of the select period for a table using these estimated rates would be:

- A 1 year  
 B 2 years  
 C 3 years  
 D 4+ years [3]

- 3 A ten year assurance policy includes an option at the end of the original term to extend the policy (at the normal premium rate for the then current age) for another ten years without any additional underwriting. This option is likely to give rise to the following forms of selection:

- I Anti Selection
- II Temporary Initial Selection
- III Time Selection

Which of the following statements about I, II and III is correct?

- A I and II only are correct
- B II and III only are correct
- C I only is correct
- D III only is correct [2]

- 4 A multiple decrement table has three decrements: death ( $d$ ), transfer ( $t$ ) and retirement ( $r$ ). Transfers and deaths take place continuously over each year of age, while retirement only occurs at the start of the year of age (i.e. at the birthday). An extract from this table is:

Age $x$	$(al)_x$	$(ad)_x^r$	$(ad)_x^d$	$(ad)_x^t$
61	7,547	871	85	50
62	6,541	1,287	91	35

- (i) The independent initial rate of retirement at age 61 is:

- A 0.1154
- B 0.1165
- C 0.1175
- D 0.1305 [2]

- (ii) The independent initial rate of mortality at age 62 is:

- A 0.0139
- B 0.0155
- C 0.0174
- D 0.0175 [2]

[Total 4]

- 5** In a mortality investigation, deaths are grouped by the following age definition:

$x$  = age next birthday at entry plus the nearest duration of the policy at the date of death.

Assuming that birthdays are uniformly distributed over the policy year, the exact age to which  $q$ -type (initial) estimates of mortality using this classification of deaths will apply is:

- A  $x - 1$
  - B  $x - 0.5$
  - C  $x$
  - D  $x + 0.5$
- [2]

- 6** A certain type of insurance contract is currently issued, by all companies in the market, to two risk types X and Y at the same rate of premium.

An insurance company believes that risk type X has a lower expected claim cost than risk type Y. This company has decided to change its premium rates so that, from now on, it will charge policyholders of type X and Y different premiums. The premium rate for type X will be decreased, and that for type Y will be increased.

All other companies are expected to continue to charge the same premium rate for both risks.

Ignoring expenses, which one of the following short term outcomes is this company expecting as a result of this action?

- A Sell more policies to risk type X with a lower margin for profit per policy
- B Sell more policies to risk type X with a higher margin for profit per policy
- C Sell more policies to risk type Y with a lower margin for profit per policy
- D Sell more policies to risk type Y with a higher margin for profit per policy

[2]

- 7 A life insurance company plans to use a standardised mortality ratio (SMR) to compare the mortality experience of its decreasing term assurance policies with that of a standard population of all its term assurance policies.

Assuming that the ratio is to be calculated using mortality data split by age, which one of the following items of information is not required for the calculation?

- A The number of decreasing term assurance policyholders at each age.
  - B The mortality rate for all term assurance policyholders at each age.
  - C The number of all term assurance policyholders at each age.
  - D The total number of deaths of decreasing term assurance policyholders.
- [2]

- 8 Which one of the following statements is correct?

- A WA80 is the standard mortality table for whole life assurances.
  - B PMA80 is the standard mortality table for permanent assured lives (males).
  - C IM80 is the standard mortality table for immediate annuitants (males) on an amounts basis.
  - D PFL80 is the standard mortality table for life office pensioners (females) on a lives basis.
- [1]

## PART TWO

**9** Compare and contrast the data and methods used to construct the English Life Table No. 14 Males and the Permanent Assured Lives Males 1980 Table. [6]

**10** (i) List the reasons why it is beneficial to the life insurance industry for individual companies to participate in the investigations of the Continuous Mortality Investigation Bureau (CMIB). [3]

(ii) What additional advantages or disadvantages, if any, are afforded to individual companies who participate in the investigations? [1]

[Total 4]

**11** The period of a mortality investigation is the three calendar years 1994, 1995 and 1996. The following data have been tabulated:

(a) censuses of the numbers of lives aged  $x$  last birthday on 30 June 1994, 30 June 1995 and 30 June 1996, denoted by  $P_x(\frac{1}{2})$ ,  $P_x(1\frac{1}{2})$ ,  $P_x(2\frac{1}{2})$  respectively; and

(b) a count of the number of deaths during the period of the investigation aged  $x$  nearest birthday at the date of death, denoted by  $\theta_x$ .

(i) State the rate interval that you would use for this investigation. [1]

(ii) Derive a formula for the estimation of central rates of mortality. [5]

(iii) Determine the age to which the estimated mortality rate relates. [1]

State clearly any assumptions that you make. [Total 7]

**12** Using the observed experience data, the maximum likelihood estimate of the initial rate of mortality at age  $x$ ,  $\hat{q}_x$ , might be described as a “best estimate” of the underlying initial rate of mortality,  $q_x$ ; yet actuaries commonly replace this estimate by the graduated initial rate of mortality,  $\tilde{q}_x$ . The graduated rates are then used to construct mortality tables and to carry out financial calculations.

(i) Explain why maximum likelihood estimates,  $\hat{q}_x$ , might be considered to be best estimates, but are nevertheless unsatisfactory for actuarial calculations. [2]

(ii) What information, in addition to  $\hat{q}_x$ , is used to determine  $\tilde{q}_x$ , the graduated initial mortality rate for age  $x$ ? [2]

[Total 4]

**13** Describe briefly how diet and the taking of exercise can affect mortality. [3]

**14** Defining all symbols used, give appropriate formulae for making population projections using the component method. You should assume that projected numbers are required for each sex separately. [7]

**15** A life insurance company has some mortality data relating to its temporary assurance policyholders, which are to be collated in select form. The following is an extract from the data:

<i>Life</i>	<i>Date of birth</i>	<i>Date of entry</i>	<i>Date of exit</i>	<i>Reason for exit</i>
A	14 August 1959	1 April 1990	6 June 1998	Death
B	2 May 1960	1 October 1990	30 November 1998	Withdrawal
C	19 July 1960	1 August 1998	—	—

(i) Calculate the contributions made by each life to the central exposed to risk during the calendar year 1998. The contributions should be grouped by curtate duration and age last birthday, and the particular age and duration for each contribution should be clearly stated. The contributions should be given in days. You should assume that the day of exit does not count in the calculation of the exposed to risk, but that the day of entry does count. [5]

(ii) Repeat part (i) when contributions are grouped by age last birthday at entry and by curtate duration. [3]

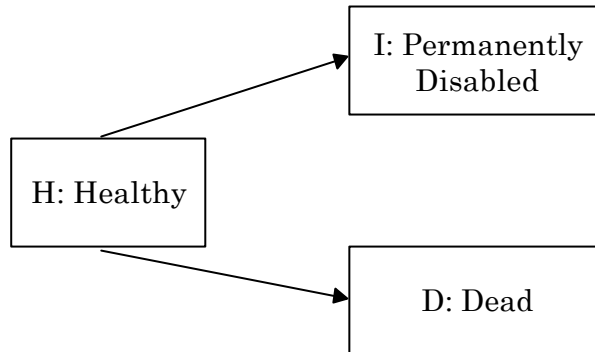
[Total 8]

## PART THREE

**16** An insurance company markets a policy which pays a sum assured on the occurrence of the first of the following two events:

- (a) total permanent disablement as the result of an accident; and
- (b) death.

For financial calculations in connection with these policies, the company uses the following three state model. The possible transitions between the states are shown by the arrows.



The following functions are defined for this model:

$\mu_{x+t}^{ij}$  force of transition from state  $i$  to state  $j$  at exact age  $x + t$  (where  $i \neq j$ ),

${}_t p_x^{ij}$  the probability that a life in state  $i$  at exact age  $x$ , is in state  $j$  at exact age  $x + t$

where  $i = H, j = I, D$ ;  $x$  is an integer and  $t \geq 0$ .

The company maintains individual records for each policy giving the date of birth, date of purchase of insurance and if appropriate the date and reason for payment of sum assured or date of policy lapse.

- (i) (a) Describe the summary statistics which should be calculated in order that the forces of transition may be estimated from the available data. Detailed descriptions of how these statistics would be calculated are not required. [3]
- (b) Obtain, from first principles, an expression for the transition probability,  ${}_t p_x^{HI}$ , in terms of the given forces of transition only. State all the assumptions you make. [8]

- (c) If the estimated forces of transition for all  $x \geq 0$  are

$$\mu_x^{HI} = 0.003$$

$$\mu_x^{HD} = 0.001$$

determine the expected number of claims for permanent disablement in the next three years for a group of 500 policyholders now aged exactly 50. [4]

- (ii) The company is considering including an additional benefit which pays a level monthly income for life to those who are permanently disabled.

- (a) Describe how the multiple state model should be changed so that it can be used to cost this benefit, explaining why your changes are needed. [2]

- (b) Describe the additional summary statistics that would need to be calculated in order to estimate all the forces of transition in this new model. [2]

- (c) Write down or obtain expressions for

$$\frac{\partial}{\partial t} {}_tP_x^{Hj} \quad j = I, D$$

in respect of the transition probabilities in this new model. [3]  
[Total 22]

- 17** An investigation of the mortality of assured lives has provided the following data:

$\theta_x$  number of deaths during the investigation period aged  $x$  last birthday,

$E_x$  initial exposed to risk of death contributed during the investigation by lives while aged  $x$  last birthday,

$q_x^s$  mortality rate at age  $x$  according to the A1967/70 Ultimate mortality table (the standard table).

The following is an extract from the data:

Age $x$	$q_x$	$E_x q_x^s$
45	24	20.35
46	17	18.61
47	15	13.09
48	44	31.32
49	12	10.77
50	8	11.24
51	40	48.38
52	41	36.25
53	27	42.98
54	50	66.31
55	42	59.27
56	13	25.30
57	88	74.41
58	76	91.65
Total	497	549.93

- (i) Test the hypothesis that the true underlying mortality rates of the experience are equal to those defined by the A1967/70 Ultimate mortality table, using the following statistical tests:
- the chi-squared test,
  - the cumulative deviations test,
  - the grouping of signs test (Stevens Test). [8]
- (ii) State clearly the conclusions that you draw from your investigation. [2]
- (iii) The experience is to be graduated by reference to the standard table, using a range of possible functions, including the following:
- $\tilde{q}_x = q_x^s (\alpha + \beta x)$
  - $\tilde{q}_x = \alpha q_x^s + \beta$

where  $\tilde{q}_x$  is the graduated rate of mortality at age  $x$ .

- Explain why it might be necessary to graduate an experience several times, using different functions, in the way described here. [3]
  - Obtain and simplify the simultaneous equations required in order to find the values of the parameters  $\alpha$  and  $\beta$  for function (1), using a weighted least-squares method. [You are not required to solve the equations.] [4]
- (iv) It has been suggested that a more successful graduation would be achieved by using mathematical formulae which do not include any reference to a standard table. Discuss the merits of this suggestion. [4]

[Total 21]