

EXAMINATIONS

April 1998

Subject D — Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 16 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
--

PART ONE

For questions 1–9 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 You are given the following definitions:

q_x^d, q_x^w independent initial rates of mortality and withdrawal respectively at exact age x

$(aq)_x^d, (aq)_x^w$ dependent initial rates of mortality and withdrawal respectively at exact age x

relating to a double decrement model for a group of life assurance policyholders with decrements of death and withdrawal.

The following statements have been made.

I $(aq)_x^d \geq q_x^d$

II It is always the case that if q_x^w increases, $(aq)_x^d$ will increase.

III In practice it is usually the case that, because of the selective effect of the withdrawal decrement, if q_x^w is increased, q_x^d also increases.

Which of the following statements about I, II and III is correct?

- A I and II are correct
- B II and III are correct
- C I only is correct
- D III only is correct

[2]

2 The following statements have been made about the Continuous Mortality Investigation Bureau's projected mortality rates.

I $q_{[65]}$ on the IM80(B30) projection is greater than $0.75q_{[65]}$ in the base table.

II q_{65} on the IM80(C=1995) projection is less than $0.75q_{65}$ in the base table.

III The function $RF(x, t)$ used in the projection formula

$$q_{x,t} = RF(x, t) q_{x,0}$$

allows for greater improvement in mortality as age, x , increases.

Which of the following statements about I, II and III is correct?

- A I and II only are correct
- B II and III only are correct
- C I only is correct
- D III only is correct

[2]

3 $q_x^k = 1 - (1 - (aq)_x)^{\frac{(aq)^k}{(aq)_x}}$ could imply that:

- I each dependent decrement is uniformly distributed over the year of age
- II the force of decrement for each decrement is constant over the year of age
- III each independent decrement is uniformly distributed over the year of age

Which of the following statements about I, II and III is correct?

- A I and II only are correct
- B II and III only are correct
- C I only is correct
- D III only is correct

[2]

4 The following statements have been made about mortality in the UK.

- I Members of professional occupations experience lighter mortality than members of non-professional occupations.
- II The mortality rates among those with professional occupations have been improving at a faster rate than the mortality rates among those with unskilled manual jobs.
- III The difference in mortality between married and non-married people is generally greater for women than for men.

Which of the following statements about I, II and III is correct?

- A I and II only are correct
- B II and III only are correct
- C I only is correct
- D III only is correct

[2]

5 The following statements have been made about the statistical models underlying a process with a single decrement.

- I The Poisson model is an approximation to a more realistic model where the central exposed-to-risk is allowed to be a random variable.
- II When estimating an initial rate of mortality at age x the data must only consist of lives who have been observed from age x to age $x + 1$ or until earlier death.
- III Provided θ , the observed number of deaths, is greater than five we may assume that $\theta \sim N(E^c\mu, E^c\mu)$ where E^c is the central exposed-to-risk and μ is the underlying force of mortality.

Which of the following statements about I, II and III is correct?

- A I and II only are correct
- B II and III only are correct
- C I only is correct
- D III only is correct

[2]

6 Which of the following statements about the comparison of risk classification in general insurance with that in life insurance is correct?

- A The theoretical basis of risk classification differs in principal between general insurance and life insurance.
- B Competition between general insurers is almost entirely on price whereas life insurance offices compete only on the basis of the amount of cover they provide for a given premium.
- C In recent years life insurance contracts have tended to have shorter terms than general insurance contracts.
- D The emphasis on subdivision of potential risks into homogeneous risk groups is far more important for a general insurer than for a life insurer.

[2]

7 The following statements have been made about the calculation of multiple decrement rates.

- I The forces of decrement operating in a population of lives subject to multiple decrements are always independent.
- II None of the computational relationships between independent and dependent rates rely on the assumption of statistical independence of the underlying single decrements associated with the multiple decrement table.
- III In practice one would never graduate multiple decrement rates directly.

Which of the following statements about I, II and III is correct?

- A I and II only are correct
- B II and III only are correct
- C I only is correct
- D III only is correct

[2]

8 The following statements have been made about graduation.

- I In the chi-squared test of a graduation, a very low value of the test statistic is sufficient to indicate undergraduation.
- II In the (lag one) serial correlation test, a significant negative value of the test statistic indicates that the shape of the graduated rates is unsatisfactory.
- III In the grouping of signs test, a small number of groups but not a large number of groups of positive deviations is regarded as unsatisfactory.

Which of the following statements about I, II and III is correct?

- A I and II only are correct
 - B II and III only are correct
 - C I only is correct
 - D III only is correct
- [2]

9 A double decrement table is such that decrement 1 occurs only at exact age $x + k$ where k satisfies $0 \leq k \leq 1$, and decrement 2 is uniformly distributed over the year of age from x to $x + 1$ in the associated single decrement table. Consider the following relationships:

- I $(aq)_x^2 = q_x^2(1 - (1 - k)q_x^1)$
- II $(aq)_x^1 = q_x^1(1 - (1 - k)q_x^2)$
- III $(aq)_x^2 = q_x^2(1 - kq_x^1)$

Which of the following statements about I, II and III is correct?

- A I and II only are correct
 - B II and III only are correct
 - C I only is correct
 - D III only is correct
- [3]

PART TWO

- 10** You have been provided with the following figures relating to the AIDS epidemic in a particular country:

<i>Date</i>	<i>Number HIV Positive</i>
31 December 1992	4937
31 December 1993	5173
31 December 1994	5298
31 December 1995	5473
31 December 1996	5601

- (i) Describe how you would use a simple mathematical model to project the number of lives who are HIV positive on 31 December 1997. You are not required to fit any model. [3]
- (ii) Outline the limitations of using the model in (i) to project the number of HIV positive individuals beyond 31 December 1997. [3]
- (iii) It is further assumed that the number of HIV positives will be approaching a maximum level by the year 2010. Show how you would adapt the model to take into account the assumption that the maximum number of HIV positives will be 8000. [3]
- [Total 9]

- 11** (i) Define the Standardised Mortality Ratio (SMR) and show that it can be expressed as a weighted average of the ratio of the age-group specific mortality rates in the study population to those in the standard population. Describe the weights used in the calculation of this average. [3]
- (ii) You are a member of a committee responsible for monitoring the trend in assured lives mortality rates. You have been presented with the following ratios of actual to expected mortality rates on the basis of a standard table constructed twenty years ago, and the total expected deaths over the period 1991–1994 based on the same table.

<i>Age</i>	<i>Ratio of Actual to Expected Mortality Rates</i>		<i>Total expected deaths for the period 1991–1994 (thousands)</i>
	<i>1991 and 1992</i>	<i>1993 and 1994</i>	
15–44	1.70	2.00	10
45+	0.85	0.75	30
Total			40

- Calculate the SMR for each period. [2]

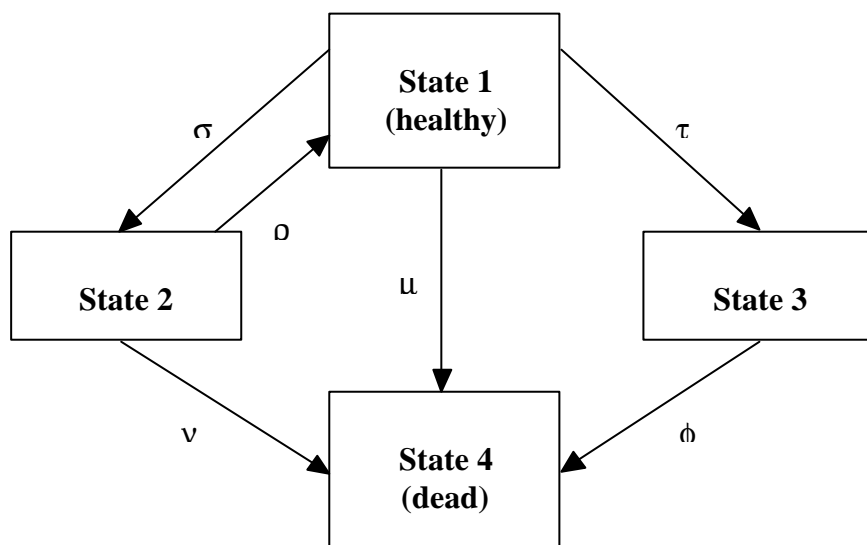
- (iii) The committee is in the habit of measuring the change in mortality by calculating a “Comparative Mortality Factor” (CMF). This is calculated “by applying the observed mortality rates to a standard population and then comparing the result to that obtained when the expected mortality rate is applied to the same population”. In this case the standard population was that upon which the standard table was based twenty years ago. The CMF for the two periods were 0.94 and 0.88 respectively which lead the committee to conclude that mortality is improving.

Explain the difference between the results of your SMR calculation and these figures. Indicate, with a reason, which you think provides the better results. [3]

[Total 8]

- 12** The following continuous-time Markov model is used for a population subject to a certain terminal disease (disease X) as well as other morbidity. State 2 represents lives which are sick but not suffering from X, and state 3 represents lives suffering from X.

The forces of transition ρ , ϕ , σ , μ , ν and τ may be assumed to be constant. The probability that a life in state i at exact age x is in state j at exact age $x + t$ is denoted by ${}_t p_x^{ij}$.



- (i) (a) Classify each state in this model as “absorbing” or “transient”.
 (b) Explain the implications of the term “Markov” used to describe the model. [3]

- (ii) Derive from first principles expressions for:

(a) $\partial / \partial t ({}_t p_x^{12})$ [4]

(b) $\partial / \partial t ({}_t p_x^{14})$ [4]

[Total 11]

- 13** The following data are taken from the records of a pension scheme between 1 January 1996 and 31 December 1997. The scheme allows retirement only on a member's exact birthday, between ages 60 and 65 inclusive. Both the lives shown were active members of the scheme at 1 January 1996.

	<i>Life 1</i>	<i>Life 2</i>
Date of birth:	1 August 1937	1 September 1936
Date of retirement:	1 August 1997	1 September 1997
Date of death:	1 October 1997	

For each life calculate the contribution to the following exposures to risk at as many ages as the data allow. Your answers should be expressed in life years (you can assume that all months are of equal length).

- (i) The independent initial exposed to risk of mortality, grouped by (integer) age x last birthday (note that for this purpose the data for pre-retired and retired lives are combined).
- (ii) The independent initial exposed to risk of retirement at exact (integer) age x .
- (iii) The dependent initial exposed to risk of mortality, grouped by (integer) age x last birthday suitable for estimating the dependent mortality rates in a double decrement model of mortality and retirement. [7]

PART THREE

- 14** A large life office has decided to investigate its non select mortality experience over the period 1 January 1992 to 30 June 1997.

Censuses of the number of in force policyholders classified by age next birthday at entry plus nearest duration at the census date are available at each 1 January from 1992 to 1997 and also at 30 June 1997. The number of deaths classified by age next birthday at entry plus curtate duration at the date of death are available for each inter-censal period.

- (i) State, with reasons, the rate interval you would use for this investigation.

State the age definition you would use, and indicate the exact age to which your estimated mortality rates would apply. [4]

- (ii) Derive an expression for the initial exposed to risk corresponding to your answer to (i). You may assume that 30 June occurs exactly half way through the calendar year, all other assumptions should be stated. [6]

- (iii) State with reasons how your answers to (i) and (ii) would be called into question by each of the following additional pieces of information:

- (a) Policies were effected on average three months before a birthday.
- (b) Policies were effected on average at the end of February in each calendar year.

You are not required to give any modified exposed to risk formulae. [4]
[Total 14]

- 15** (i) Explain what is meant by the following terms and give examples of each:

- (a) temporary initial selection
(b) time selection
(c) spurious selection [8]

- (ii) Discuss the factors which led to the choice of the select period for the AM80, AM80(5) and AF80 mortality tables. [3]

- (iii) In North America the select period for published mortality tables for assured lives often extends up to 15 years. Discuss whether or not you consider that a select period of this length might be justified. [3]
[Total 14]

- 16** (i) A set of assured lives mortality data consisting of the deaths, θ_x , and the initial exposed-to-risk, E_x , is to be graduated by reference to a standard table. The formula to be used is:

$$q_x = \alpha + \beta q_x^s$$

where q_x^s denotes a mortality rate in the standard table.

- (a) Derive two equations satisfied by the maximum likelihood estimates of α and β . [5]
- (b) The statistical assumptions underlying the derivation in (a) are likely to be violated in practice.

State which violation is most likely to affect the accuracy of the parameter estimates in practice, and indicate the most likely cause of the problem. [2]

- (ii) Suppose the method described in (i) has been used to graduate the following mortality rates.

(1) Age, x	(2) Initial exposed to risk, E_x	(3) Actual deaths, θ_x	(4) Graduated mortality rate, \hat{q}_x	(5) Expected deaths, $E_x \hat{q}_x$	(6) Standard deviation $\sqrt{E_x \hat{q}_x (1 - \hat{q}_x)}$	(7) Standardised Deviation $\frac{[(3) - (5)]}{(6)}$
50	35,000	161	0.004810	168.3	12.94	-0.5641
51	40,000	205	0.005340	213.6	14.58	-0.5898
52	45,000	260	0.005928	266.8	16.28	-0.4177
53	50,000	344	0.006579	329.0	18.08	0.8296
54	55,000	418	0.007300	401.5	19.96	0.8267
55	60,000	506	0.008097	485.8	21.95	0.9203
56	50,000	463	0.008977	448.8	21.09	0.6733
57	40,000	388	0.009948	397.9	19.85	-0.4987
58	30,000	318	0.011017	330.5	18.08	-0.6914
59	25,000	296	0.012194	304.8	17.35	-0.5072
60	20,000	251	0.013489	269.8	16.31	-1.1527
Total	450,000	3,610		3,616.8		

The observed value of the serial correlation coefficient (with lag one) between the ten pairs of values of the standardised deviations is 0.6344.

Perform five appropriate tests to determine whether or not the graduation is satisfactory.

Comment on your results. [11]
[Total 18]