

EXAMINATIONS

September 1999

Subject C — Statistics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
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PART ONE

For questions 1–3 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Which of the following is **NOT** true of the credibility factor, Z , in the Empirical Bayes Credibility Model 1?

- A Z must lie between 0 and 1.
 - B Z is an increasing function of n .
 - C Z is an increasing function of $E[s^2(\theta)]$.
 - D Z is an increasing function of $V[m(\theta)]$.
- [2]

2 The process Y_t is defined as below, where $\{Z_t\}$ is white noise.

$$Y_t = 0.2Y_{t-1} + Z_t - 0.5Z_{t-1}.$$

Which of the following statements is/are true?

- I Y_t is stationary.
- II $Y_t - Y_{t-1}$ is stationary.
- III $Y_t - Y_{t-1}$ will have the characteristics of an MA(1) process.

- A I only.
 - B I and II only.
 - C I and III only.
 - D None of the above.
- [3]

3 Which of the following statements is false?

- A A random sample of size n is taken from the Poisson distribution with parameter λ . The conjugate prior for λ is gamma.
 - B A random sample of size n is taken from the exponential distribution with parameter λ . The conjugate prior for λ is Poisson.
 - C A random sample of size n is taken from normal distribution $N(\mu, \sigma^2)$ where μ is unknown and σ is known. The conjugate prior for μ is normal.
 - D An observation is made of binomial distribution with parameter θ . The conjugate prior for θ is beta.
- [2]

PART TWO

- 4 Y_t is a time series defined by

$$Y_t = 0.5Y_{t-1} + Z_t + 0.2Z_{t-1}$$

where $\{Z_t\}$ are independent with mean 0 and variance σ^2 .

Derive the autocorrelations, $\rho_1, \rho_k (k \geq 2)$ of Y_t . [6]

- 5 Aggregate annual claims from a portfolio of general insurance policies have a compound Poisson distribution with Poisson parameter λ . Individual claim amounts have an exponential distribution with mean 1. The premium loading factor used to calculate the premium for these policies is 0.30. Given an initial surplus of 2, calculate the probability of ruin at the first claim. [6]

- 6 Let N_1 be the number of household contents insurance policies covered by a specific office in year 1. Suppose that $E[N_1] = m$ and $\text{Var}[N_1] = s^2$. At the end of year 1 each of the policies in force will either lapse, with probability q , or be renewed and remain in force throughout year 2, with probability $p = 1 - q$. Each policy lapse or renewal is independent of the others. Ignoring any new policies which might be added to the portfolio during year 2, let N_2 denote the number of policies in force in year 2. Determine expressions for $E[N_2]$, $\text{Var}[N_2]$ and $\text{Cov}[N_1, N_2]$ in terms of p, q, m and s^2 . [6]

- 7 Claims on a portfolio of household insurance policies have a Pareto distribution with mean £900 and standard deviation £1500. 150 claims are expected each year, and it is assumed that claims occur according to a Poisson process.

Calculate the percentage reduction in the mean of aggregate claims if a policy excess of £200 is introduced. [8]

- 8 The total loss to a general insurance office with respect to a particular portfolio over a month is represented by:

$$S = \sum_{i=1}^N X_i, \quad N > 0, \quad (S = 0 \text{ if } N = 0)$$

where N has a Poisson distribution with mean 2 and X_1, X_2, X_3, \dots is a sequence of independent and identically distributed random variables that are also independent of N . Their distribution is such that $P(X_i = 1) = P(X_i = 2) = \frac{1}{2}$, where X_i is measured in appropriate monetary units. A reinsurance contract has been arranged such that the amount paid by the reinsurer is $S - 3$, if $S > 3$ (and 0 otherwise).

Define S_I, S_R as aggregate claims paid by the direct insurer and reinsurer respectively. Calculate $E[S_I]$ and $E[S_R]$. [8]

9 A company has decided to introduce a No Claims Discount system for a certain class of business. This system will have 5 levels of discount: 0%, 15%, 30%, 50% and 75%. The rules for moving between these levels of discount will be:

- following a year with one or more claims, a policyholder will move down one level of discount, or remain at the 0% discount level
- following a claim free year, a policyholder will move up one level of discount, or remain at the 75% discount level

The company believes that the probability of a policyholder having a claim free year is 0.85. What proportion of policyholders at each level of discount should the company expect once these proportions have become stable? [8]

PART THREE

10 In 1996 an insurance company started issuing a new form of medical cover insurance. Cumulative claims, in £'000, for these policies have been:

<i>Origin Year</i>	<i>Development Year</i>		
	<i>0</i>	<i>1</i>	<i>2</i>
1996	950	1925	2410
1997	1225	2425	
1998	3000		

and premium income, also in £'000, in each year has been:

1996	3010
1997	4100
1998	8625

Assume that the first year of business has now run off completely and that inflation will be at the rate of 5% per annum.

Use the separation technique to determine the reserve for outstanding claims that should be held at the end of 1998. [11]

11 An aggregate claims process is compound Poisson with Poisson parameter λ . Individual claim amounts follow an exponential distribution with mean 200 and the company calculates premiums using a 20% loading factor. The company is considering proportional reinsurance of this portfolio with retention α from a reinsurer who calculates premiums using a 35% loading factor.

(i) Show that the adjustment coefficient, R , is given by

$$R = \max\left(0, \frac{7\alpha - 3}{600\alpha(9\alpha - 1)}\right)$$

and state the range of possible values of α such that the probability of ruin is less than 1. [8]

(ii) Find the value of α that maximises R . [4]

[Total 12]

12 The number of claims arising each month from a general insurance portfolio has a Poisson distribution, with unknown Poisson parameter λ . Claims are monitored over a period of 50 months, and an average of 210 claims per month are observed.

(i) It is suggested, based on knowledge gained from similar portfolios, that a suitable prior distribution for λ has mean 250 and variance 45. Using the conjugate prior distribution, determine the posterior distribution of λ and the Bayesian estimator of λ under quadratic loss. [6]

(ii) An alternative suggestion for estimating λ is to use the number of claims occurring on a single day, which is assumed to have a Poisson distribution with mean $\lambda/30$. It is suggested that the following prior distribution for λ should be used:

$$\begin{aligned}\lambda &= 230 \text{ with probability } 0.2 \\ &= 250 \text{ with probability } 0.5 \\ &= 270 \text{ with probability } 0.3\end{aligned}$$

If 7 claims were recorded on the most recent day for which data are available, determine the posterior distribution for λ , and hence find the Bayesian estimator of λ under quadratic loss. [6]

(iii) Discuss briefly the differences between the estimators in (i) and (ii), indicating which you think is preferable. [2]

[Total 14]

- 13** Claims occur on a portfolio of general insurance policies according to a Poisson process at an average rate of 200 per year. $S(t)$ is defined as aggregate claims on the portfolio up to time t , where t is measured in years.
- (i) Suppose that individual claim amounts have a gamma distribution, with parameters $\alpha = 200/155$ and $\beta = 1/155$.
- (a) Find the mean and variance of $S(1)$ and $S(2)$.
- (b) Find the mean and variance of $S(1) - S(2)$. [6]
- (ii) Suppose instead that individual claim amounts are one of two types, type I and type II. The probability of a type I claim is 0.75, and these claims have mean 100 and variance 1000. The expected claim size and the variance of claim sizes considered over the whole portfolio are the same as in (i).
- (a) Determine the mean and variance of type II claims.
- (b) Find the mean and variance of $S(1)$ and $S(2)$. [6]
- (iii) Describe briefly the risk the company would face if it used a single claim size distribution rather than distinguishing between the two claim types when modelling the distribution of aggregate claims. [2]
- [Total 14]