EXAMINATIONS

September 1998

Subject D — Actuarial Mathematics

Paper One

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Begin your answers to Parts One, Two and Three on a separate sheet.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 16 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–8 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Which of the following best approximates

$$\ddot{a}_{x|y}^{(m)} ?$$

A $\ddot{a}_{x|y} - \frac{m-1}{2m}\left(1 - \frac{D_x}{D_y}\right)$

B $\ddot{a}_{x|y} - \frac{m-1}{2m}$

C $a_y - a_{xy}$

D $\ddot{a}_x - \ddot{a}_{xy}$ [2]

2 An actuary is carrying out profit tests on a 4-year unit-linked policy. The year-end sterling fund cash flows, $(SCF)_t$, per policy in force at the start of policy year t, for $t = 1, 2, 3, 4$ are:

Year (t)	1	2	3	4
$(SCF)_t$	–100	130	30	–40

The profit vector $(PRO)_t$ equals $(SCF)_t$ since there is no allowance for reserves. Sterling reserves are to be set up at each year-end for each policy then in force to zeroise future negative cash flows. Assuming that the sterling fund earns interest at 5% per annum and that the rate of mortality $q_x = 0.01$ at each age, which of the following is the value of $(PRO)_2$ adjusted to allow for the sterling reserves?

A 120.00
B 122.73
C 123.14
D 125.64 [3]

3 The mortality and withdrawal experience of the whole-life policyholders of a life office is described by a double decrement table in which $(aq)_{[x]+t}$ and $(aw)_{[x]+t}$ are, respectively, the dependent rates of death and withdrawal for lives now aged $x + t$ who took out policies at age x.

A contract has sum assured S, payable immediately on death. The surrender value on withdrawal between exact durations t and $t + 1$ is SV_t, payable immediately on surrender. What is the approximate expected present value of the benefits?

A $\sum_{t=0}^{\infty} v^{t+\frac{1}{2}}{}_t(ap)_{[x]} \{S(aq)_{[x]+t} + SV_t(aw)_{[x]+t} (1 - (aq)_{[x]+t})\}$

B $\sum_{t=0}^{\infty} v^{t+\frac{1}{2}}{}_t(ap)_{[x]} \{S(aq)_{[x]+t} (1 - \frac{1}{2}(aw)_{[x]+t}) + SV_t(aw)_{x+t} (1 - \frac{1}{2}(aq)_{[x]+t})\}$

C $\sum_{t=0}^{\infty} v^{t+\frac{1}{2}}{}_t(ap)_{[x]} \{S(aq)_{[x]+t} + SV_t (aw)_{[x]+t}\}$

D $\sum_{t=0}^{\infty} v^{t+\frac{1}{2}} \{S(aq)_{[x]+t} + SV_t (aw)_{[x]+t}\}$ [2]

4 Under the Manchester Unity model of sickness

$z_x^{104/\text{all}}$ is:

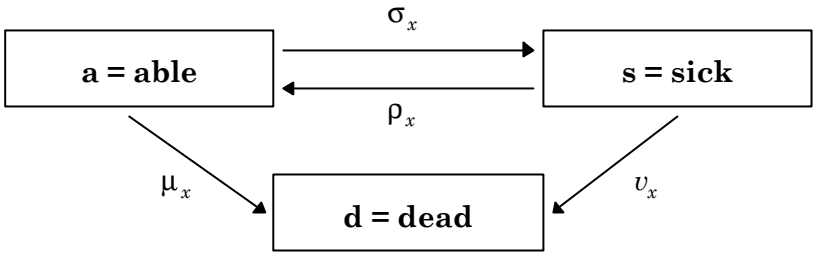
A $52.18 \dfrac{\int_0^1 l_{x+t}\, \bar{z}_{x+t}^{104/\text{all}}\, dt}{\int_0^1 \bar{z}_{x+t}^{104/\text{all}}\, dt}$

B $\dfrac{\int_0^1 l_{x+t}\, \bar{z}_{x+t}^{104/\text{all}}\, dt}{\int_0^1 l_{x+t}\, dt}$

C $52.18 \dfrac{\int_0^1 l_{x+t}\, \bar{z}_{x+t}^{104/\text{all}}\, dt}{l_x}$

D $52.18 \dfrac{\int_0^1 l_{x+t}\, \bar{z}_{x+t}^{104/\text{all}}\, dt}{\int_0^1 l_{x+t}\, dt}$ [2]

5 An employer recruits lives aged 25, all of whom are healthy when recruited. On entry the lives join a scheme which pays a lump sum of £20,000 immediately on death, plus an extra £10,000 if the deceased was sick at the time of death. The mortality and sickness of the scheme members are described by the following multiple state model, in which the forces of transition depend on age.



All surviving scheme members retire at age 60 and leave the scheme regardless of their state of health. If $p_{x,t}^{gh}$ is defined as the probability that a life, who is in state g at age x ($g = a, s, d$), is in state h at age $x + t$ ($t \geq 0$ and $h = a, s, d$), then the expected present value, at force of interest δ, of the death benefit in respect of a single new recruit is:

A $\quad 10{,}000 \times \int_0^{35} e^{-\delta t}\{2p_{25,t}^{aa} + 3p_{25,t}^{as}\}\, dt$

B $\quad 10{,}000 \times \int_0^{35} e^{-\delta t}\{2p_{25,t}^{aa} \,.\, \mu_{25+t} + 3p_{25,t}^{aa} \,.\, \sigma_{25+t}\, \nu_{25+t}\}\, dt$

C $\quad 30{,}000 \times \int_0^{35} e^{-\delta t}\{p_{25,t}^{as} \,.\, \nu_{25+t} - p_{25,t}^{aa} \,.\, \mu_{25+t}\}\, dt$

D $\quad 10{,}000 \times \int_0^{35} e^{-\delta t}\{2p_{25,t}^{aa} \,.\, \mu_{25+t} + 3p_{25,t}^{as} \,.\, \upsilon_{25+t}\}\, dt$ [3]

6 A multiple decrement table is subject to two forces of decrement, a and b. The independent rates of decrement at age x are q_x^a and q_x^b respectively.

It is known that the dependent rates of decrement are:

$(aq)_x^a = 0.2$ and $(aq)_x^b = 0.05$.

Assuming a uniform distribution of the independent decrements over each year of age, what are the values of q_x^a and q_x^b?

A 0.20513 and 0.05555
B 0.20513 and 0.05573
C 0.20573 and 0.05555
D 0.20573 and 0.05573 [3]

7 Let T_x and T_y be random variables representing the exact future lifetimes of two lives age x and y respectively, and assume that T_x and T_y are independent. Then $\mu_{x+t:y+t}$ is:

A $\lim_{dt\to 0}\left\{\frac{1}{dt}\text{Prob}\{T_x \le t+dt \text{ AND } T_y \le t+dt \,|\, T_x > t \text{ OR } T_y > t\}\right\}$

B $\lim_{dt\to 0}\left\{\frac{1}{dt}\text{Prob}\{T_x \le t+dt \text{ OR } T_y \le t+dt \,|\, T_x > t \text{ OR } T_y > t\}\right\}$

C $\lim_{dt\to 0}\left\{\frac{1}{dt}\text{Prob}\{T_x \le t+dt \text{ AND } T_y \le t+dt \,|\, T_x > t \text{ AND } T_y > t\}\right\}$

D $\lim_{dt\to 0}\left\{\frac{1}{dt}\text{Prob}\{T_x \le t+dt \text{ OR } T_y \le t+dt \,|\, T_x > t \text{ AND } T_y > t\}\right\}$ [2]

8 A 5-year endowment assurance contract issued to female lives aged 70 exact has the following profit vector

(–414, +103, +145, +176, +199)

Assuming that ${}_t p_{70} = 1 - 0.04t$ for $t = 0, 1, 2, 3, 4$ the internal rate of return lies between:

A 9% and 10%
B 10% and 11%
C 11% and 12%
D 12% and 13% [3]

PART TWO

9 Calculate the annual premium for a policy which provides the following benefits in respect of two lives aged exactly 50 at the issue date:

(a) if both lives survive to age 65, an annuity of £10,000 per annum is payable monthly in advance throughout their joint lifetimes; after the first death, an annuity of £5,000 per annum is payable monthly in advance throughout the lifetime of the survivor;

(b) a lump sum of £50,000 is payable immediately on the first death, if this occurs within 15 years.

Premiums are payable annually in advance during the joint lifetime of the two lives for at most 15 years.

Basis:		
	Mortality	A1967–70 ultimate
	Interest	4% per annum
	Expenses	ignored

[10]

10 List the main assumptions used in valuing the benefits of a defined benefit occupational pension scheme. [4]

11 A pension scheme provides a pension of 1/60th of career average salary in respect of each full year of service, on age retirement between the ages of 60 and 65. A proportionate amount is provided in respect of an incomplete year of service.

A member aged 40 exact has 20 years of past service and total past salary of £260,000 and expects to receive a salary of £33,000 over the next year.

Calculate the total expected present value of the past and future service pensions on age retirement for this member, using the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations. [5]

12 In the context of an insurance policy providing benefits during sickness, explain briefly what is meant by:

(i) deferred period

(ii) s_x

(iii) $\bar{z}_x$, and

(iv) ${}_{m|n}z_x$ [4]

13 In the context of a life office establishing reserves for its in-force business, list four distinct ways of reducing policy values to allow for the incidence of expenses. In each case, state whether it applies to unit-linked or non-linked life assurance contracts (or to both), and state its advantages and disadvantages.

[8]

PART THREE

14 A life office sells a unit-linked endowment policy with a term of 3 years. The premium is £800 payable annually in advance. 50% of the premium is allocated to units in the first year, and 102% in the second and third years. The units are subject to a bid-offer spread of 5% and there is an annual management charge of 0.5% of the bid value of the units, which is deducted at the end of each policy year.

On death before the end of the term, a benefit of £2,500 or the bid value of the units, whichever is greater, is payable at the end of the year of death. On surrender or on survival to the end of the term, the bid value of the units is payable at the end of the policy year of exit.

The management charge is deducted before payment of any death benefit or surrender value due at that time.

The initial commission is 25% of the first year's premium, and renewal commission is 2.5% of the second and third years' premiums. Other initial expenses are £60 plus 50% of the amount of initial commission. Other renewal expenses are £25 per annum incurred on the second and third premium payment dates.

The life office uses the following pricing assumptions:

Age at entry	40
Independent rates of mortality	A1967–70 Ultimate
Independent rate of withdrawal	5% per annum
Rate of increase in value of assets underlying the unit funds	8% per annum
Rate of return earned on sterling cash-flows	6% per annum
Rate of expense inflation	Nil
Risk discount rate	12% per annum

(i) Assume the office does **not** set up sterling reserves. Calculate the profit margin of this endowment policy. [13]

(ii) Now assume the office **does** set up sterling reserves, to zeroise future negative expected cash flows.

(a) Calculate the expected sterling reserves which must be set up at the end of each year, per policy in force at the start of each year.

(b) Recalculate the profit margin allowing for the cost of setting up these sterling reserves. [5]

[Total 18]

15 For many years a society has recruited 100 new members uniformly over each year at age 20. Mortality is in accordance with the A1967–70 Ultimate table, and there are no other decrements. Members pay level premiums, annually in advance, starting on the date of entry, to secure annuities at age 65, when premiums cease. These annuities are for £1,000 per annum, payable continuously for 5 years certain and for life thereafter. No payment is made on death before age 65. Assume the society earns 4% interest and ignore expenses.

If the fund has now reached a stationary condition, what is the amount of the fund? [14]

16 A life office issues policies to lives age 58 exact. These policies provide the following benefits:

(a) On becoming permanently disabled before the 60th birthday, an annuity of £5,000 per annum payable weekly for life with a lump sum of £100,000 on death, and

(b) On death before the 60th birthday, if not previously permanently disabled, a lump sum of £50,000 with a further sum of £50,000 if death was due to accidental causes.

In either case the death benefit is payable immediately on death.

Calculate the annual amount of premium, payable weekly and ceasing on death, permanent disability or the 60th birthday, on the following basis:

Mortality:	The independent rates of mortality of lives who are not permanently disabled are those of the A1967–70 Ultimate table; 6% of these deaths are due to accidental causes. The permanently disabled are subject to the mortality of the English Life Table No 12 Males with the age rated up by 10 years.
Permanent disability:	A constant independent rate of 0.007.

All decrements are assumed to be uniformly distributed over each year of age.

Interest:	4% per annum
Expenses:	5% of all premiums [17]