EXAMINATIONS

April 1998

Subject A — Fundamentals of Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Begin your answers to Parts One, Two and Three on a separate sheet.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 16 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–8 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Which of the following is equal to the function ${}_{k}|A^{1}_{x:\overline{n}|}$?

I $\dfrac{D_{x+k}}{D_x}(A_{x+k} - A_{x+k+n})$

II $\dfrac{D_{x+k}}{D_x}\sum_{h=0}^{n-1} v^{h+1} \cdot {}_{h}|q_{x+k}$

III $\sum_{h=k+1}^{n} v^{h} \cdot {}_{h-1}p_x \cdot q_{x+h-1}$

A I and II are correct
B II and III are correct
C I only is correct
D III only is correct [3]

2 Which of the following is the value of $A_{[35]:\overline{10}|}^{\ 1}$, using A1967–70 mortality, with interest at 6% p.a.?

A 0.00974
B 0.00995
C 0.01010
D 0.01053 [3]

3 Which of the following is the probability that a female aged exactly 62 will live to age 67 but die before reaching age 70, assuming a(55) select mortality?

A 0.05204
B 0.05225
C 0.08123
D 0.08180 [2]

4 Which of the following is $\frac{\partial}{\partial x}\ {}_tp_x$?

A $\quad {}_tp_x \,.(\mu_x - \mu_{x+t})$

B $\quad {}_tp_x \,.(\mu_{x+t} - \mu_x)$

C $\quad {}_tp_x \,.\mu_{x+t}$

D $\quad -\ {}_tp_x \,.\mu_{x+t}$ [2]

5 Which of the following is equal to ${}_{10}V_{45:\overline{25}|}$?

A $\quad \frac{1}{D_{55}}\left(M_{55} - M_{70} + D_{70} - \frac{(M_{45} - M_{70} + D_{70})(N_{55} - N_{70})}{(N_{45} - N_{70})}\right)$

B $\quad \frac{1}{D_{55}}\left(M_{55} - M_{70} - \frac{(M_{55} - M_{70})(N_{55} - N_{70})}{(N_{45} - N_{70})}\right)$

C $\quad \frac{1}{D_{45}}\left(M_{45} - M_{70} + D_{70} - \frac{(M_{45} - M_{70} + D_{70})(N_{45} - N_{55})}{(N_{45} - N_{70})}\right)$

D $\quad \frac{1}{D_{45}}\left(M_{45} - M_{70} - \frac{(M_{45} - M_{70})(N_{45} - N_{55})}{(N_{45} - N_{70})}\right)$ [3]

6 Which of the following is $\mathring{e}_{20:\overline{40}|}$?

A $\quad \int_{20}^{60} {}_sp_{20}\ ds$

B $\quad \int_{20}^{40} {}_sp_{20}\ ds$

C $\quad \int_{0}^{40} {}_sp_{20}\ ds$

D $\quad \int_{20}^{60} {}_sp_{0}\ ds$ [2]

7 A life aged exactly 50 effects a with profit whole life assurance policy with sum assured of £10,000 plus attaching bonuses payable at the end of the year of death. Compound reversionary bonuses vest at the end of each policy year, provided the policyholder is still alive at the time. Which of the following gives the single premium payable at the outset?

Basis:		
	mortality:	A1967–70 ultimate
	interest:	5% per annum
	compound reversionary bonus rate:	1.94175% of the sum assured each year

A £4,697
B £4,767
C £4,788
D £4,813 [3]

8 Which of the following is the value of $\overline{a}_{40:\overline{25}|}$, assuming mortality of ELT No. 12 (males) and interest of 6% p.a.?

A 11.603
B 12.420
C 14.900
D 15.826 [2]

PART TWO

9 Derive, and simplify as far as possible, an expression for z, in terms of standard actuarial notation, such that:

$$A_x = (1 - z)\, A^{1}_{x:\overline{n}|} + z\, .\, A_{x:\overline{n}|}$$ [3]

10 On 1 January 1985 a life office issued a number of 30 year pure endowment assurance contracts to lives then aged 35, with premiums payable annually in advance throughout the term or until earlier death. In each case, the only benefit was a sum assured of £20,000, payable on survival to the end of the term.

During 1996, 4 policyholders died out of the 580 policyholders whose policies were in force at the start of the year.

Assuming that the office uses net premium policy reserves, calculate the profit or loss from mortality for 1996 in respect of this group of policies.

Basis: mortality: A1967–70 ultimate
interest: 4% per annum [6]

11 An insurance company issues a three-year decreasing term assurance policy to a life aged exactly 60.

The sums assured in respect of the first, second and third years of the policy are £10,000, £8,000 and £6,000, respectively. Death benefits are payable at the end of the year of death.

In addition, a claim expense is incurred at the time a benefit is paid. The initial level, at the outset of the policy, is £200; however, this figure increases in line with inflation.

Calculate the mean and standard deviation of the present value of the outgo for this policy.

Basis: mortality: A1967–70 select
interest: 8% per annum
expense inflation: 5% per annum [7]

12 In a mortality table with a one-year select period $q_{[x]} = a \,.\, q_x$ for all $x \geq 0$ and for a certain constant $0 < a < 1$.

(i) Let

$K_{[x]+t}$ = Curtate future lifetime of a person aged $x + t$ who became select t years ago.

K_x = Curtate future lifetime of a person aged x whose mortality is reflected by the ultimate part of the mortality table.

Explain whether the expected value of $K_{[x]+1}$ is less than, greater than, or equal to the expected value of K_{x+1}. [2]

(ii) Calculate $\overline{A}_{[45]:\overline{20}|}$ according to the following assumptions:

$a = 0.9$
Interest: 4% per annum
Ultimate mortality: ELT No. 12 (males) [7]
[Total 9]

13 A survival model is being constructed which is based on the assumption that there is a uniform distribution of deaths between integral ages.

(i) (a) Write down an expression for l_{x+t} in terms of l_x and l_{x+1} for $0 \leq t \leq 1$.

(b) Show that ${}_tq_x = t \,.\, q_x \qquad 0 \leq t \leq 1$.

(c) Show that if $0 \leq s, t \leq 1$ such that $s + t \leq 1$ then

$$ {}_tq_{x+s} = \frac{t \,.\, q_x}{1 - s \,.\, q_x} $$ [5]

(ii) Calculate the single premium payable by a life aged exactly 58¾ in respect of a two year pure endowment policy with sum assured of £3,000.

Basis: mortality: ELT No 12 (males)
interest: 7% per annum [4]
[Total 9]

14 A life office is profit testing a 3-year assurance contract issued to lives aged exactly 50. The office calculates that it can expect the following cash flows per policy in force at the start of each year:

Year	*Expected cash flow occurring at the end of the year* £
1	–70
2	35
3	40

(i) Using a risk discount rate of 8% per annum and A1967–70 ultimate mortality:

(a) Calculate the expected net present value of the policy.

(b) Calculate the expected internal rate of return of the policy, to one decimal place. [6]

(ii) Use your answer to (i)(b) to explain in words why the answer to (i)(a) is negative. [2]

[Total 8]

PART THREE

15 A life insurance company issues a 20 year endowment assurance policy to an impaired life aged exactly 45.

The sum assured is £60,000 and death benefits are payable at the end of the year of death.

The company assumes that the person will be subject to the following mortality:

First ten years: standard mortality with a constant addition to the force of mortality of 0.009569

Second ten years: standard mortality for a life three years older than the actual age

The contract is issued at the company's standard rate of premium, with premiums payable annually in advance until age 65 or earlier death. The death benefit is subject to a level debt of £X for the first 10 years of the contract and £0.9X for the second 10 years.

(i) Show that the standard annual premium is £2,260.04.

Basis: mortality: A1967–70 select
interest: 4% per annum
expenses: initial: 25% of the annual premium
renewal: 4% of each premium, excluding the first [3]

(ii) Calculate the debt, £X. [14]

(iii) Without doing any further calculations, explain how the size of the debt would change if the age rating used in the second half of the term had been five rather than three years. [2]

[Total 19]

16 A life insurance company issues a special 20 year endowment assurance policy to a life aged exactly 40.

The death benefit is £20,000 together with a return of 25% of the premiums paid. The payment is made 2 months after the policyholder's date of death.

The survival benefit is £50,000 and is payable without delay at exact age 60.

Premiums are payable annually in advance for 15 years, or until earlier death.

(i) Calculate the annual premium.

Basis: mortality: A1967–70 select
interest: 4% per annum
expenses: initial: 20% of first premium
renewal: 3% of each premium, except the first

[12]

(ii) After 15 years the policy is altered to a deferred annuity contract with no further premiums payable.

The annuity is payable monthly in advance from age 60 for the rest of the policyholder's life. A lump sum death benefit of £25,000 is payable on death during the deferred period. The payment would be made 2 months after the date of death.

Using a basis which is consistent with that used in (i) above, calculate the monthly amount of deferred pension payable from age 60. [7]

[Total 19]