

# EXAMINATIONS

April 1998

## Subject D — Actuarial Mathematics

### *Paper One*

*Time allowed: Three hours*

#### **INSTRUCTIONS TO THE CANDIDATE**

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 13 questions.*

***Graph paper is not required for this paper.***

#### **AT THE END OF THE EXAMINATION**

*Hand in BOTH your answer booklet and this question paper.*

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
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## PART ONE

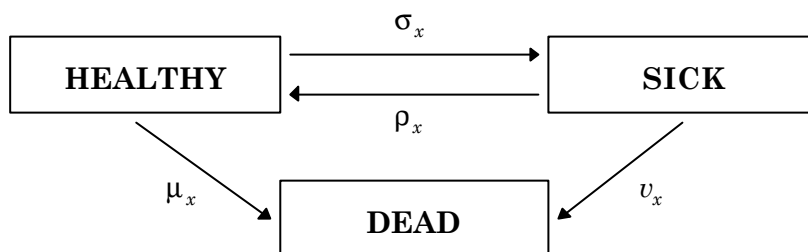
For questions 1–6 indicate in your answer booklet which one of the answers A, B, C or D is correct.

- 1 Two lives, each aged  $x$ , are subject to the same mortality table. According to the mortality table, and at a certain rate of interest,  $A_x = 0.4$  and  $A_{xx} = 0.6$ . Which of the following gives  $A_{xx}^2$  according to the same mortality table and interest rate?

- A 0.10
- B 0.15
- C 0.20
- D 0.25

[2]

- 2 A life office prices PHI policies using the following three-state model in which the forces of transition  $\sigma_x, \rho_x, \mu_x$  and  $v_x$  depend on age:



Level annual premiums are payable continuously.

Under the PHI policies a level benefit is payable during periods of sickness. Premiums are waived during periods of sickness. There is no death benefit, and there is no waiting period.

- (i) Under the normal premium basis it has been calculated that the reserves will always be positive.

Which of the following changes to the premium basis will certainly increase the premium rate?

- I a decrease in the sickness inception rate
- II a fall in the premium basis rate of interest
- III a fall in the death rate from the sick state

- A I and II only are correct
- B I only is correct
- C II and III only are correct
- D III only is correct

[3]

- (ii) A new PHI policy with a term of 35 years is to be issued to a healthy man aged 30 by the same life office. The PHI benefit is  $B$  per annum and  $P$  is the annual premium. There is no waiting period before benefits are payable.  $p_{x,t}^{g,h}$  is defined as the probability that a life who is in state  $g$  at age  $x$  is in state  $h$  at age  $x + t$  ( $t \geq 0$  and  $g, h = \text{healthy, sick or dead}$ ). In addition the office uses the following premium basis:

force of interest:  $\delta$ ;

expenses at the time of inception of a new claim:  $E$ .

there are no other expenses.

If premiums and benefits are assumed to be payable continuously, what is the correct formula for the expected present value of the profit on this contract?

- A  $\int_0^{35} e^{-\delta t} (Pp_{30,t}^{HH} - Bp_{30,t}^{HS} \sigma_{30+t} - Ep_{30,t}^{HS} \rho_{30+t}) dt$
- B  $\int_0^{35} e^{-\delta t} (Pp_{30,t}^{HH} - Bp_{30,t}^{SS} - Ep_{30,t}^{HS}) dt$
- C  $\int_0^{35} e^{-\delta t} (Pp_{30,t}^{HH} - Bp_{30,t}^{HS} \sigma_{30+t} - Ep_{30,t}^{HS} \sigma_{30+t}) dt$
- D  $\int_0^{35} e^{-\delta t} (Pp_{30,t}^{HH} - Bp_{30,t}^{HS} - Ep_{30,t}^{HH} \sigma_{30+t}) dt$  [3]

[Total 6]

- 3** An actuary is profit testing a 15-year endowment assurance policy. The sum assured is £25,000 payable on survival or at the end of the year of earlier death. If the policyholder surrenders then she will receive a return of premiums without interest at the end of the year of surrender.

A level premium of £1,500 per annum is payable annually in advance.

For a policy in force at the start of the eighth year the remaining details are as follows:

	(£)
Renewal expenses	35
Claim expenses per claim on death or surrender	75
Reserve at start of year, ${}_7V$	8,000
Reserve at end of year per survivor, ${}_8V$	9,300
Rate of interest	8% per annum
Dependent probability of death	0.02
Dependent probability of surrender	0.05

What is the profit expected to emerge at the end of the eighth year per policy in force at the start of that year.

- A £419
  - B £433
  - C £468
  - D £477
- [3]

4 In order to value the benefits in a final salary pension scheme as at 1 January 1998, a salary scale,  $s_x$ , has been defined so that  $s_{x+t} / s_x$  is the ratio of a member's total earnings between ages  $x + t$  and  $x + t + 1$  to the member's total earnings between ages  $x$  and  $x + 1$ . Salary increases take place on 1 July every year. One member, whose date of birth is 1 October 1956, has an annual salary rate of £50,000 on the valuation date. The member's expected earnings during 1998 are approximately:

- A  $£50,000 \times \frac{s_{41\frac{1}{4}}}{s_{40\frac{1}{4}}}$
  - B  $£50,000 \times \frac{s_{41\frac{3}{4}}}{s_{40\frac{1}{4}}}$
  - C  $£50,000 \times \frac{s_{41\frac{3}{4}}}{s_{40\frac{3}{4}}}$
  - D  $£50,000 \times \frac{s_{41\frac{1}{4}}}{s_{40\frac{3}{4}}}$
- [3]

5 Upon age retirement between the ages of 60 and 65 a pension fund provides a pension of one eightieth of final pensionable salary for each year of scheme service. Final pensionable salary is defined as the average annual salary earned over the 36 months prior to retirement.

A member is now aged 40 exact and earned £35,000 over the last year.

Assuming an interest rate of 4% per annum and that all decrements and the salary scale follow the Pension Fund Tables of the Formulae and Tables for Actuarial Examinations, what is the expected present value to the nearest £100 of this member's future pension?

- A £32,300
  - B £49,100
  - C £51,900
  - D £53,400
- [2]

- 6** Two lives, each aged 30, are independent with respect to mortality, and are each subject to the mortality of the same life table. Given that the limiting age of the table is 100, and

$${}_tq_{30} = 0.01t \quad \text{for } 0 \leq t \leq 50$$

what is the value of  ${}_{10|10}q_{30:30}^1$ ?

- A 0.005
- B 0.080
- C 0.085
- D 0.405

[3]

## PART TWO

- 7** In a certain country, the membership of the police force (including those in retirement) has been in a stationary state for a number of years. Membership of the pension fund is compulsory for all members of the police force and all members work until they reach age 55, the compulsory retirement age. Each year 100 policemen and 25 policewomen reach retirement age, uniformly over the calendar year. Pensions are payable annually in advance to retired policemen and policewomen, commencing immediately on retirement, and are guaranteed to continue for 5 years certain (and life thereafter). The amount of pension payable to each pensioner is revised on each 1 January.

It is known that retired policemen experience the mortality of the English Life Table No. 12 — Males and that retired policewomen experience the mortality of the English Life Table No. 12 — Males with a deduction of 6 years from the age.

- (i) How many retired policemen and policewomen are there? [4]
- (ii) Calculate the total annual payments for 1997 to retired policemen and policewomen, given that the pension payment to each retired policeman and policewoman in 1997 was £10,000. [7]

[Total 11]

- 8**
- (i) What is the purpose of a reserve in respect of a single premium annuity contract? [1]
  - (ii) What is the likely effect on the expected net present value of the profit for such a contract if the reserving basis is strengthened? (Your answer should give reference to specific elements of the profit testing basis.) [3]

[Total 4]

**9** What is the expected present value of the future contributions of a member of a pension scheme, given the following information?

Member's age	41
Expected salary over the next year	£33,000
Contribution rate	5% of salary per annum
Interest rate	4% per annum

Salary growth and decrements are assumed to follow the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations. [2]

**10** An annuity policy provides the following benefits:

- an annuity of £20,000 per annum payable monthly in advance. It is guaranteed to last for 10 years and continuing thereafter until the first death out of the two policyholders (a woman aged 63 exact and her husband aged 66 exact);
- a reversionary annuity of £15,000 payable monthly in advance to the surviving spouse, commencing on the next payment date following the death of the first of the lives to die, or from the 10th policy anniversary if this is later.

Calculate the net single premium for this policy assuming mortality follows that of the a(55) ultimate mortality table (males or females as appropriate) rated down 3 years and using an interest rate of 6% per annum. [9]

**11** A non-profit endowment assurance policy with a term of 30 years is to be issued to a life aged 35 exact. The sum assured is £40,000 and will be paid immediately upon death or upon survival to age 65. Level premiums are payable weekly in advance for 20 years or until the death of the policyholder, if earlier.

The policy includes a premium waiver clause covering periods during which the policyholder is ill. Under this clause premiums must be paid by the policyholder for the first 12 months of sickness. If sickness persists for more than 12 months then premiums are waived until the policyholder recovers.

Calculate the amount of the weekly premium for this policy using the following basis:

Interest: 4% per annum

Mortality: English Life Table Number 12 — Males

Sickness: Manchester Unity Sickness Experience 1893–97  
Occupation Group A, H, J

Expenses: Initial: £100

Weekly: 2.5% of each premium including the first, incurred at each premium payment date. Weekly expenses are not incurred during periods when the premium is being waived. [8]

## PART THREE

- 12** A life office is planning to issue a 3-year unit-linked endowment policy. The level premium is £2,000 payable annually in advance. 98% of each premium is allocated to purchase units at the offer price. Units are subject to a bid-offer spread of 5%. The annual management charge is 1.5% of the bid value of the units and is deducted at the end of each year immediately before the payment of any benefits due at that time.

On death before the end of the term of the policy the greater of £6,000 and the bid value of the units is paid out at the end of the year of death.

On early surrender, the bid value of the units is paid minus a penalty. The amount of the penalty depends upon the year of surrender as follows:

<i>Year</i>	<i>Penalty (£)</i>
1	350
2	150
3	0

Initial expenses are £200. Annual expenses of £20 per annum are incurred at the beginning of each policy year, and grow at the rate of 5% per annum from the date of issue of the policy.

It is assumed that assets in the unit fund will grow at 9% per annum while the rate of interest on sterling fund cashflows and negative sterling reserves will be 6% per annum.

For a new policyholder aged 60 exact the dependent rates of decrement are assumed to be:

<i>Age, x</i>	<i>Death rate</i> $(aq)_x^d$	<i>Surrender rate</i> $(aq)_x^s$
60	0.030	0.10
61	0.035	0.05
62	0.040	0.00

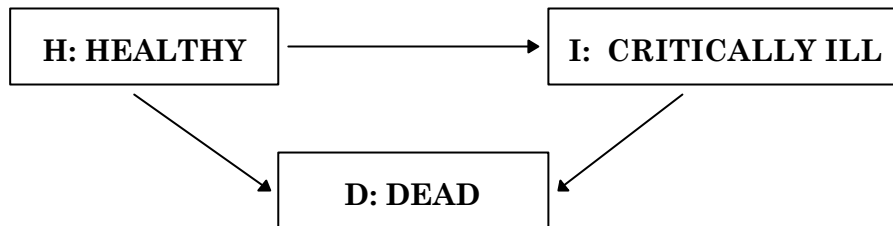
- (i) (a) Calculate the profit signature for this policy.
- (b) Using a risk discount rate of 15% per annum, calculate the expected net present value of the profit on this policy. [15]
- (ii) The office proposes to set up negative sterling reserves on this policy at the end of each year as appropriate, to ensure that the expected profit in each of policy years 2 and 3 is zero. Calculate the amounts of the reserves and the revised expected net present value of the profit. [6]
- (iii) Explain the difference between your answers to (i)(b) and (ii). [2]

- (iv) Explain briefly, with reasons, how the expected net present value of profit in (i) (b) would vary if the dependent rates of surrender were increased.

[2]

[Total 25]

- 13** A life office uses the following three-state model to calculate premiums for a three-year critical illness policy issued to healthy policyholders aged 56 at entry.



In return for a single premium of  $P$  payable at the outset, the office will pay benefits of:

£30,000 if the policyholder dies from the healthy state;  
 £20,000 if he is diagnosed as having a critical illness;  
 £10,000 if he dies from the critically ill state.

All benefits are payable at the end of the relevant policy year.

Let  $S_t$  represent the state of the policyholder at age  $56 + t$ , so that  $S_0 = H$ , and, for  $t = 1, 2, 3$ ,  $S_t = H, I$  or  $D$ . The transition probabilities are defined as follows:

$$p_{56+t}^{ij} = \Pr(S_{t+1} = j \mid S_t = i)$$

and their values are

$t$	$p_{56+t}^{HI}$	$p_{56+t}^{HD}$	$p_{56+t}^{ID}$
0	0.02	0.01	0.2
1	0.02	0.02	0.3
2	0.02	0.03	0.4

The transitions in the multiple state model are the only source of randomness.

- (i) One possible outcome for this policy is that a policyholder is diagnosed as having a critical illness in year 1 and then dies in year 3. The cashflows for this outcome are then:

$t$	<i>cashflow (£)</i>
0	$P$
1	-20,000
2	0
3	-10,000

- (a) Show that there are 10 possible outcomes, including the one above. [3]
- (b) For each outcome calculate the net present value at time 0 of the profit assuming a rate of interest of 7% per annum. (Ignore expenses.) [3]
- (c) Calculate the probability that each outcome occurs. [7]
- (ii) Using the results in part (i) or otherwise, calculate the mean and variance of the present value at time 0 of the profit per policy. [5]
- (iii) The office expects to sell 2,500 of these policies. The single premium is set at a level which will ensure that the probability that the office makes a profit is 0.99. Use the Normal approximation to calculate the amount of the single premium. [4]

[Total 22]