EXAMINATION

17 September 2008 (am)

Subject CT4 — Models
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 You work for a consultancy which has created an actuarial model and is now preparing documentation for the client.

List the key items you would include in the documentation on the model. [4]

2 The classification of stochastic models according to:

- discrete or continuous time variable
- discrete or continuous state space

gives rise to a four-way classification.

Give four examples, one of each type, of stochastic models which may be used to model observed processes, and suggest a practical problem to which each model may be applied. [4]

3 Compare the advantages and disadvantages of the Binomial and the multiple-state models in the following situations:

(a) analysing human mortality without distinguishing between causes of death
(b) analysing human mortality when distinguishing between causes of death

[5]

4 In the village of Selborne in southern England in the year 1637 the number of babies born each month was as follows

January	2	July	5
February	1	August	1
March	1	September	0
April	2	October	2
May	1	November	0
June	2	December	3

Data show that over the 20 years before 1637 there was an average of 1.5 births per month. You may assume that births in the village historically follow a Poisson process.

An historian has suggested that the large number of births in July 1637 is unusual.

(i) Carry out a test of the historian's suggestion, stating your conclusion. [4]

(ii) Comment on the assumption that births follow a Poisson process. [1]
[Total 5]

5 An investigation into the mortality experience of a sample of the male student population of a large university has been carried out. The university authorities wish to know whether the mortality of male students at the university is the same as that of males in the country as a whole. They have drawn up the following table.

Age x	*Number of deaths*	*Expected number of deaths assuming national mortality*
18	13	10
19	15	12
20	14	14
21	20	12
22	12	8
23	8	5

Carry out an overall test of the university authorities' hypothesis, stating your conclusion. [5]

6 A portfolio of term assurance policies was transferred from insurer A to insurer B on 1 January 2001. Each policy in the portfolio was written with premiums payable annually in advance. Insurer B wishes to investigate the mortality experience of its acquired portfolio and has collected the following data over the period 1 January 2001 to 1 January 2005:

d_x numbers of deaths aged x

$P_{x,t}$ number of policies in force aged x at time t (t = 0, 1, 2, 3, 4 years measured from 1 January 2001)

Where x is defined as:

age last birthday at the most recent policy anniversary prior to the portfolio transfer + number of premiums received by insurer B.

(i) (a) State the rate interval implied by the above data.
(b) Write down the range of ages at the start of the rate interval. [2]

(ii) Give an expression which can be used to estimate the initial exposed to risk at age x, E_x, stating any assumptions made. [2]

The following is an extract from the data collected in the investigation:

x	d_x	$\sum P_{x,t}$	$\sum P_{x,t+1}$
39	28	10,536	11,005
40	36	10,965	10,745
41	33	10,421	10,577

where the summations are from t = 0 to t = 3.

(iii) Estimate q_{40}, stating any further assumptions made. [3]
[Total 7]

7 (i) Explain why, under Continuous Mortality Investigation investigations, the data analysed are usually based upon the number of policies in force and number of policies giving rise to claims, rather than the number of lives exposed and number of lives who die during the period of study. [2]

Suppose N identical and independent lives are observed from age x exact for one year or until death if earlier.

Define:

π_i to be the proportion of the N lives exposed who hold i policies (i = 1,2,3,….);

$\mathbf{D}_i$ to be a random variable denoting the number of deaths amongst lives with i policies

$\mathbf{C}_i$ to be a random variable denoting the number of claims arising from lives with i policies.

(ii) Derive an expression for the ratio of the variance of the number of claims arising compared with that if each policy covered an independent life. [4]

(iii) Explain how the expression derived in (ii) could be used in practice. [2]
[Total 8]

8 A No-Claims Discount system operated by a motor insurer has the following four levels:

Level 1: 0% discount
Level 2: 25% discount
Level 3: 40% discount
Level 4: 60% discount

The rules for moving between these levels are as follows:

- Following a year with no claims, move to the next higher level, or remain at level 4.
- Following a year with one claim, move to the next lower level, or remain at level 1.
- Following a year with two or more claims, move down two levels, or move to level 1 (from level 2) or remain at level 1.

For a given policyholder in a given year the probability of no claims is 0.85 and the probability of making one claim is 0.12.

(i) Write down the transition matrix of this No-Claims Discount process. [1]

(ii) Calculate the probability that a policyholder who is currently at level 2 will be at level 2 after:

(a) one year.
(b) two years. [3]

(iii) Calculate the long-run probability that a policyholder is in discount level 2.
[5]
[Total 9]

9 A company pension scheme, with a compulsory scheme retirement age of 65, is modelled using a multiple state model with the following categories:

1 currently employed by the company
2 no longer employed by the company, but not yet receiving a pension
3 pension in payment, pension commenced early due to ill health retirement
4 pension in payment, pension commenced at scheme retirement age
5 dead

(i) Describe the nature of the state space and time space for this process. [2]

(ii) Draw and label a transition diagram indicating appropriate transitions between the states. [2]

For *i*,*j* in {1,2,3,4,5}, let:

${}_t p_x^{1i}$ the probability that a life is in state *i* at age *x*+*t*, given they are in state 1 at age *x*

μ_{x+t}^{ij} the transition intensity from state *i* to state *j* at age x+t

(iii) Write down equations which could be used to determine the evolution of ${}_t p_x^{1i}$ (for each *i*) appropriate for:

(a) $x + t < 65$.
(b) $x + t = 65$.
(c) $x + t > 65$.

[6]
[Total 10]

10 In an investigation of reconviction rates among those who have served prison sentences, let *X* be a random variable which measures the duration from the date of release from prison until the ex-prisoner is convicted of a subsequent offence. The investigation monitored a sample of 100 ex-prisoners (who were all released on the same date) at one-monthly intervals from their date of release for a period of 6 months. Those who could not be traced in any month were removed from the sample at that point and not traced in subsequent months. Reconviction was assumed to take place at the duration that a prisoner was first known to have been reconvicted.

(i) Express the hazard rate at duration x months in terms of probabilities. [1]

The investigation produced the following data for a sample of 100 ex-prisoners.

Months since release	*Number of prisoners contacted*	*Number who had been reconvicted since last contact*
1	100	0
2	97	0
3	95	4
4	90	3
5	85	5
6	80	0

(ii) Calculate the Nelson-Aalen estimate of the survival function. [5]

A previous investigation found that the probability that a prisoner would be reconvicted within 6 months of release was 0.2.

(iii) Estimate confidence intervals around the integrated hazard using the results from part (ii) to test the hypothesis that the rate of reconviction has declined since the previous investigation. [6]

[Total 12]

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11 Consider the random variable defined by $X_n = \sum_{i=1}^{n} Y_i$ with each Y_i mutually independent with probability:

$$P[Y_i = 1] = p, \; P[Y_i = -1] = 1 - p \qquad 0 < p < 1$$

(i) Write down the state space and transition graph of the sequence X_n. [2]

(ii) State, with reasons, whether the process:

(a) is aperiodic.
(b) is reducible.
(c) admits a stationary distribution. [3]

Consider $j > i > 0$.

(iii) Derive an expression for the number of upward movements in the sequence X_n between t and $(t + m)$ if $X_t = i$ and $X_{t+m} = j$. [2]

(iv) Derive expressions for the m-step transition probabilities $p_{ij}^{(m)}$. [3]

(v) Show how the one-step transition probabilities would alter if X_n was restricted to non-negative numbers by introducing:

(a) a reflecting boundary at zero.
(b) an absorbing boundary at zero.

[2]

(vi) For each of the examples in part (v), explain whether the transition probabilities $p_{ij}^{(m)}$ would increase, decrease or stay the same.
(Calculation of the transition probabilities is not required.) [3]

[Total 15]

12 (i) Explain the meaning of the rates of mortality usually denoted q_x and m_x , and the relationship between them. [3]

(ii) Write down a formula for ${}_t q_x$, $0 \le t \le 1$, under each of the following assumptions about the distribution of deaths in the age range [x, x+1]:

(a) uniform distribution of deaths
(b) constant force of mortality
(c) the Balducci assumption

[2]

A group of animals experiences a mortality rate $q_x = 0.1$.

(iii) Calculate m_x under each of the assumptions (a) to (c) above. [8]

(iv) Comment on your results in part (iii). [3]
[Total 16]

END OF PAPER