Project 2

From: Investment.colleague@work

To: Student@CA2.exam

Subject: Estimating the volatility parameter (sigma)

As you know, our computer systems use the lognormal model for modelling the future behaviour of share prices. The simplest formulation of this model is through the equation $dS_t = S_t(\mu dt + \sigma dB_t)$. It is important for our applications that the value we use for the parameter σ is consistent with the market.

We already have two methods of estimating based on historical daily prices and option prices. However, we would like to have a third method to provide a final check on whether our calibrations are reasonable.

As well as publishing daily share prices, the *Financial Times*™ gives the maximum and minimum daily prices for the last year (*ie* the previous 250 trading days). According to our model the distribution of the ratio of these two values ($R = \max/\min$) should depend only on the values of the parameters μ and σ for the share. Unfortunately, we have not been able to derive an algebraic formula expressing the expected value of R in terms of the parameters μ and σ.

We would like you to investigate this relationship using computer simulations, to see whether we can use this as the basis for a third method of estimating σ. Ideally, we would like to have a graph showing the expected value of the ratio as a function of σ. We will be holding a group training session later this week at which we would like you to give a 15 minute presentation of the three methods and of your results.

To test this third method we would like you to use your results to estimate a value of σ for a share whose yearly maximum and minimum values are 706 and 309, and to calculate the theoretical price of a 6-month call option on this share with a strike price of 600. The current share price is 586, the risk-free rate is 3.5% and the dividend rate is 2.7%. (The current market price of the option is 43.)

I look forward to seeing your results and hearing your views on this third method.

Instructions to CA2 candidate

(i) Read the email above, which describes the background to this project.

(ii) Under this model the daily movements (ΔS_t) in the share price can be approximated using the equation $\Delta S_t = S_t(\mu\Delta t + \sigma Z_t\sqrt{\Delta t})$, where the Z_t's are independent values from the standard normal distribution.

Carry out the following investigation using an Excel spreadsheet.

(a) Use pseudo-random numbers to simulate 100 runs of the daily closing prices for the share for the coming year. Assume a starting price of 100 and 250 trading days, so $\Delta t = 1/250$. You should set up your spreadsheet so that you can specify and change the values of the parameters μ and σ. Your spreadsheet should calculate the ratio R for each run and calculate the average value of this ratio for the 100 runs.

(b) By varying the values of μ and σ, investigate how the average ratio R is related to these parameter values. Create a table showing estimates of the expected value of R for different combinations of the parameters. You should include at least the following parameter values:

For μ: 0, 0.05, 0.10

For σ: 0.1, 0.2, 0.3, 0.4, 0.5

(c) Create a graph showing as clearly as possible the relationship(s) you have found.

(d) Carry out the test calculations requested by your colleague in the penultimate paragraph of the email. Calculate the implied volatility and consider the reasons for any discrepancy in the two estimates of sigma.

Note that there are formulae relevant to this project in the Formulae and Tables for Actuarial Examinations.

(iii) Prepare a set of five pages of summary as preparation for your presentation. You should cover the following subjects:

Description of methods of estimating sigma

Outline the 3 different methodologies.

Analysis

Outline the investigations you carried out.

Results

Outline the results of your investigation, including your graph. Give the data you used in the test calculation, together with your calculated option price and the implied volatility.

Conclusions

State your conclusion about the suitability of the proposed third method. Give a list of reasons why your estimate of sigma may differ from the estimates derived from the other two methods, and suggest possible next steps.