

## Summary 1

### ESTIMATION OF SHARE PRICE VOLATILITY

There are two generally used methods of estimating “sigma” – the volatility of share prices.

The purpose of this project is to test whether a proposed new method gives estimates of sigma that are consistent with accepted methods.

#### METHODS OF ESTIMATING SIGMA

##### Methods in use

##### 1. Historical prices method

- Daily log-share prices are obtained over an appropriate period.
- Sigma is estimated as the annualised sample standard deviation.

##### 2. Implied volatility method

- The theoretical price of a call or put option is calculated for different values of sigma using a formula approach (i.e. Garman-Kohlhagen).
- The value of sigma that reproduces the observed market price is identified (i.e. the value of sigma for which the formula gives a theoretical price equal to the observed market price).

##### Proposed new method

##### 3. Ratio method

- The ratio of the maximum to the minimum daily price over the last year is calculated.
- The value of sigma that leads to the same expected ratio is then adopted.
- This is done by performing a large number of simulations of a year of daily share prices. This is repeated for different values of sigma, until an appropriate level of sigma is found that results in an average ratio (across all simulations) equal to the historic observed ratio.

## Summary 2

### ANALYSIS

#### Model

- First a set of U(0,1) random variables was generated in a spreadsheet.
- Using the Box-Muller formula, these were used to simulate a series of N(0,1) random variables.
- One random variable was generated for each day of a year, and for 100 different runs.
- The N(0,1) random variables were converted into simulations of share price movements for a given mu and sigma. This was done using the lognormal model as given in the Instructions, where  $Z_t$  represents the independent N(0,1) random variables:

$$\text{Movement in share price} = \Delta S_t = S_t (\mu \Delta t + \sigma Z_t \sqrt{\Delta t})$$

- The maximum and minimum share prices in the year were determined for each of the 100 runs.
- The ratio  $R = \text{Max} / \text{Min}$  was calculated for each run and then averaged across all runs.

#### Results

- The average ratio R was tabulated for different combinations of mu and sigma.
- R was averaged across 5 sets of 100 runs to increase accuracy.
- The relationship between R, mu and sigma was then investigated.

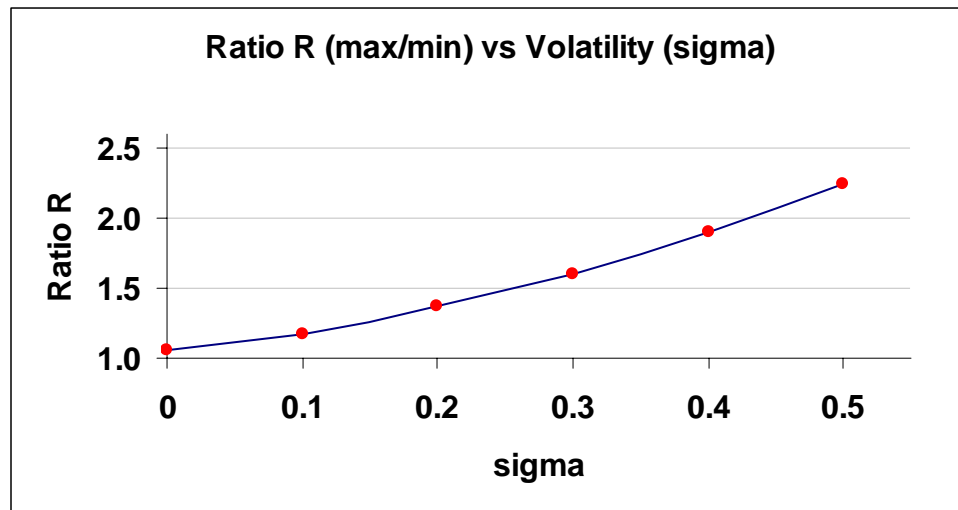
### Summary 3

## RESULTS

Table of averaged ratios (R)

mu	sigma					
	0.0	0.1	0.2	0.3	0.4	0.5
0.00	1.00	1.17	1.37	1.60	1.90	2.20
0.05	1.05	1.17	1.37	1.61	1.89	2.27
0.10	1.11	1.19	1.39	1.59	1.89	2.27

- For  $\sigma \geq 0.1$  the values appear to be relatively independent of  $\mu$ .
- Therefore the average can be taken across all  $\mu$ , and the relationship be considered between just R and  $\sigma$  (see graph below).



- The graph suggests that there is a relationship, but it is not linear.

## Summary 4

### TEST RESULTS

- To validate the proposed methodology, it was then tested against an implied volatility calculation using an observed market option price.
- A six month call option with current market price 43 was used.

#### Data

One-year maximum share price:	706
One-year minimum share price:	309
Current share price:	586
Strike price:	600
Risk-free rate:	3.5% per annum
Dividend rate:	2.7% per annum

*Source:* all Data is as provided in the Instructions.

*Assumption:* the risk-free rate and dividend rates are stated on a continuously compounded basis.

#### Estimates of sigma using proposed methodology

Ratio R:	2.28 (=706/309)
<b>Estimate of sigma:</b>	<b>0.5</b> (from graph, based on estimated R)

#### Option price calculation (Garman-Kohlhagen formula)

Calculated option price: 76 (based on sigma =0.5)  
 This is inconsistent with the actual market price of 43.  
 The sigma required as an input to the Garman-Kohlhagen formula to give a calculated option price of 43 is 0.3. So:

**Implied volatility: 0.3**

## Summary 5

### CONCLUSIONS

#### Suitability of the ratio method

- The proposed third method does not appear to give a reliable estimate of sigma.
- Using that estimate, the calculated option price is not consistent with the market.
- It should be noted that the calculation is based on a very limited test sample.

#### Possible reasons for the discrepancy

The ratio method is based only on the maximum/minimum values.

- It does not make use of all the available market data.
- It is disproportionately influenced by the extreme values.

The approach is based on the lognormal model. In reality:

- Share price movements are not lognormally distributed.
- The parameters  $\mu$  and  $\sigma$  are not constant over time.

#### Possible next steps

- Validate the implied volatility by looking at other traded options.
- Check the information given on the option and its underlying share.
- Use an alternative simulation approach that better reflects share price movements.
- Investigate the use of a ratio that uses highest and lowest share price values *after* elimination of extreme outliers (e.g. use 95<sup>th</sup> and 5<sup>th</sup> percentiles).
- Test the model on other sets of data, e.g. overseas equities.
- Test the model over different time periods.
- Increase the scale of the model in order to be able to use a larger number of runs without having to average across different sets of runs.