EXAMINATIONS

September 1997

Subject D — Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Begin your answers to Parts One, Two and Three on a separate sheet.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 19 questions.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–9 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Consider the following statements about risk classification in general insurance.

I The age of the policyholder is not used for risk classification.

II The aim of risk classification in general insurance is to separate policyholders into groups with the same expected claim frequencies.

III Competition between general insurers is based almost entirely on price and this generally leads to greater competitive pressures for the refinement of risk classifications than apply in life insurance.

Which of the following statements about I, II and III is correct?

A I and II are correct.
B II and III are correct.
C I only is correct.
D III only is correct. [2]

2 In the signs test of a graduation, if n is the total number of deviations and n_1 is the number of positive deviations, a test statistic which is expected to be approximately distributed $N(0, 1)$ for large n is:

A $\dfrac{n_1 - n/2}{\sqrt{n}}$

B $\dfrac{n_1 - n}{\sqrt{n/4}}$

C $\dfrac{n_1 - n/2}{\sqrt{n/4}}$

D $\dfrac{n_1^2 - n^2/2}{n}$ [1]

3 The following information relates to a continuing investigation of some assured lives. For $n = 0, 1, 2, ...$:

$N_{x,n}$ = number of lives observed to attain exact age x during calendar year $1995 + n$

$P_{x,n}$ = number of lives observed at 1 January $1995+n$ then aged x next birthday

$D_{x,n}$ = the number of deaths occurring among the $N_{x,n}$ lives while aged x last birthday

$\theta_{x,n}$ = the number of deaths occurring during calendar year $1995+n$ while aged x next birthday

Which one of the following is a maximum likelihood estimate of a q-type rate of mortality, according to the Binomial mortality model?

A $\dfrac{D_{x,n}}{N_{x,n}}$

B $\dfrac{q_{x,n}}{P_{x,n}}$

C $\dfrac{2.D_{x,n}}{N_{x,n} + N_{x,n+1} + D_{x,n}}$

D $\dfrac{2.q_{x,n}}{P_{x,n} + P_{x,n+1} + q_{x,n}}$ [3]

4 Consider the following statements concerning graduation by mathematical formula.

I The serial correlations test should not be used with the graduation produced.

II If the weighted least squares method is used to fit a non-linear formula, it is usually necessary to use iterative techniques to obtain the parameter estimates.

III The method is commonly used in constructing standard tables.

Which of the following statements about I, II and II is correct?

A I and II are correct.
B II and III are correct.
C I only is correct.
D III only is correct. [1]

5 Consider the following statements about the application of the cumulative deviations test to a set of graduated mortality rates.

I The test should be applied as a two-tailed test.

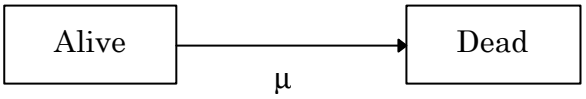
II The test should be appied with a continuity correction.

III In choosing the range of ages over which the test is performed, the runs in the signs of the deviations should be taken into account.

Which of the following statements about I, II and III is correct?

A I and II are correct.
B II and III are correct.
C I only is correct.
D III only is correct. [2]

6 Consider the following 2-state model of mortality:



in which

μ is constant;

the Maximum Likelihood Estimate of μ is $\mathring{m} = \frac{d}{v}$;

d is the observed number of deaths; and
v is the observed time spent in the "alive" state,

in respect of a group of observed lives.

The variance of the maximum likelihood estimator can be approximated by:

A $\mathring{m} / v$

B $\mathring{m} / v^2$

C d^2 / v^2

D $\mathring{m} / d$ [2]

7 Under the "Balducci Assumption", ${}_{(1-t)}q_{x+t} = (1-t)q_x$, $0 \le t < 1$,

${}_{(b-a)}q_{x+a}$ with $0 \le a < b \le 1$ is given by:

A $\dfrac{(b-a)q_x}{1-a.q_x}$

B $\dfrac{(b-a)q_x}{1-(1-b)q_x}$

C $\dfrac{(b-a)q_x}{1-(1-a)q_x}$

D $\dfrac{(b-a)q_x}{1-b.q_x}$ [3]

8 Consider the following statements concerning mortality investigations.

I An advantage of the policy year rate interval is that calculation of the age to which the rates apply does not require an approximation.

II An advantage of the life year rate interval is that no approximation is required in the calculation of the initial exposed to risk using the census method.

III If age in a mortality investigation is defined as "age next birthday on 30 June in the calendar year of death", it will not be necessary to calculate separately age at the time of death.

Which of the following statements about I, II and III is correct?

A I and II are correct.
B II and III are correct.
C I only is correct.
D III only is correct. [2]

9 Consider the following statements concerning select mortality investigations.

I If a calendar year rate interval is used, the estimate of the mortality rate for duration zero is likely to be biased.

II The investigation usually assumes that birthdays are uniformly distributed over the policy year, in order to obtain the ages to which the estimated select rates will apply.

III The force of mortality is typically a linear function of duration within the select period.

Which of the following statements about I, II and III is correct?

A I and II are correct.
B II and III are correct.
C I only is correct.
D III only is correct. [2]

PART TWO

10 Compare English Life Table No. 14 with the "80" series of standard tables, commenting on:

(i) the periods of investigation and the sources of data [1]

(ii) the classes of lives with which the tables were designed to be used [3]

(iii) the select periods in the published tables [2]

[Total 6]

11 (i) Calculate the ratio of the mortality rate in the A1967–70 Select table at duration zero to that in the English Life Tables No. 12 — Males for each of the following ages:

(a) age 20

(b) age 70 [1]

(ii) Comment on your results in (i). [2]

(iii) Repeat (i) but using A1967–70 Ultimate mortality instead of A1967–70 Select mortality. [1]

(iv) Comment on your results in (iii) as compared with your results in (i). [3]

[Total 7]

12 The following information relates to a population projection being carried out using the component method. The data refer to the male population of a medium-sized country.

$P(x,n)$ = population at 1 January 1997+n aged x last birthday, $x,n = 0, 1, 2, ...$

$M(x,n)$ = net number of emigrants from the population during the year 1997+n, aged x last birthday at 1 January 1997+n

$q(x,n)$ = independent probability that a life who attains exact age x during the year 1997+n, dies during that year

The following is a selection from the available data.

x	$P(x,0)$	$q(x,0)$	$q(x,1)$	$M(x,0)$	$M(x,1)$
40	743,795	.0074	.0076	11,244	11,955
41	829,412	.0080	.0079	9,430	7,682
42	688,577	.0088	.0085	12,925	10,091

Calculate the projected number of males in the population aged 42 last birthday at 1 January 1999 from the information given. State any other assumptions you make. You should give your answer to the nearest whole number. [4]

13 Explain briefly the influence of occupation upon mortality. [5]

14 Describe the method used to project the PMA(80) base tables into double entry tables. [4]

15 (i) Suppose that N lives are observed between exact ages x and $x + 1$. Some of these lives possess more than one life insurance policy; let the proportion with i policies be p_i ($i = 1, 2, 3, ...$). Let C_1 be the number of policies under which claims are made. Show that

$$E[C_1] = \sum_{i=1}^{\infty} ip_i . Nq_x$$

and $$\text{Var}(C_1) = \sum_{i=1}^{\infty} i^2 p_i . Nq_x(1 - q_x)$$ [4]

(ii) Suppose instead that the total number of lives observed is

$$\sum_{i=1}^{\infty} ip_i . N$$

and that each life possesses one life insurance policy. Let C_2 be the number of policies under which a claim is made. Show that

(a) $E[C_2] = E[C_1]$

(b) $$\frac{\text{Var}(C_1)}{\text{Var}(C_2)} = \frac{\sum_{i=1}^{\infty} i^2 p_i}{\sum_{i=1}^{\infty} ip_i}$$ [3]

(iii) State the consequence of the result in (ii)(b) above for the statistical testing of a graduation of crude rates of mortality based on observations of policies rather than lives.

[2]
[Total 9]

16 (i) In the context of an investigation where several decrements are operating simultaneously, state the differences, if any, between the treatment of the various decrements in the computation of the initial exposed to risk for:

(a) an independent rate of decrement; and

(b) a dependent rate of a particular decrement [2]

(ii) Repeat parts (i) (a) and (b) in respect of the computation of the central exposed to risk. [1]

[Total 3]

PART THREE

17 You are required to investigate the mortality experience of a large portfolio of whole life and temporary assurance policies. The following information is available:

(A) the number of policies in force at each 1 January from 1 January 1980 to 1 January 1995 inclusive, subdivided by age nearest birthday and curtate duration on the relevant 1 January

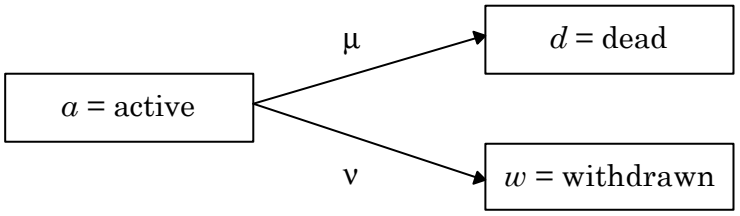
(B) for each calendar year from 1980 to 1994 inclusive, the number of deaths, subdivided by age nearest birthday at death and curtate duration at death

(i) State, with reasons, the parts of the policy data, or the special groups of lives, which you consider should be excluded or investigated separately. [5]

(ii) Derive formulae for crude estimates of m-type select mortality rates using the given data. Obtain the ages and durations to which your rates apply. You should define all symbols used and state all assumptions made. [8]

[Total 13]

18 The following multiple state model, with constant transition intensities μ and ν, is used to represent the mortality and withdrawal experience of life assurance policyholders between ages x and $x + 1$:



(i) State the assumptions underlying the model. [2]

(ii) Show that

(a) $${}_tp_x^{\overline{aa}} = e^{-(m+n)t}$$

(b) $${}_tp_x^{ad} = \frac{m}{m+n}\left[1 - e^{-(m+n)t}\right]$$ [10]

[Total 12]

19 (i) (a) State why crude mortality rates derived from observation of a particular population may be inappropriate for modelling the mortality of that population.

(b) Describe very briefly three approaches to producing graduated rates for a standard table at ages for which few deaths have been observed.

(c) List the various types of standard tables against which any new standard table for a class of select lives should be checked for consistency. [7]

(ii) The following information relates to an investigation of the mortality experience of a large group of lives.

Age	*Exposed to risk*	*Observed deaths*	*A1967–70 Ultimate*	*Expected deaths*	*Deviation*
x	E_x	q_x	q_x	$E_x q_x$	$q_x - E_x q_x$
30	7251	3	0.00065	4.71	–1.71
31	7344	3	0.00067	4.92	–1.92
32	7590	3	0.00070	5.31	–2.31
33	7830	5	0.00074	5.79	–0.79
34	7998	9	0.00079	6.32	2.68
35	8280	12	0.00086	7.12	4.88
36	8370	12	0.00094	7.87	4.13
37	8805	8	0.00103	9.07	–1.07
38	8856	6	0.00115	10.18	–4.18
39	8811	11	0.00129	11.37	–0.37
40	8730	18	0.00144	12.57	5.43
41	9024	9	0.00162	14.62	–5.62
42	9615	9	0.00183	17.60	–8.60
43	9510	18	0.00207	19.69	–1.69
44	9483	27	0.00234	22.19	4.81
45	9855	24	0.00264	26.02	–2.02
TOTAL	137,352	177		185.35	–8.35

It has been suggested that the standard table A1967–70 Ultimate could be used as a model of the mortality of this population. Apply the following tests, and state your conclusions regarding the validity of the suggestion. (You need not state the statistical bases of the tests.)

(a) cumulative deviations test, over the entire age range
(b) chi-squared test
(c) signs test
(d) grouping of signs test
(e) individual standardised deviations test [12]

[Total 19]