Faculty of Actuaries **Institute of Actuaries**

EXAMINATIONS

April 1999

Subject C — Statistics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 12 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–4 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Which of the following is <u>*not*</u> true of the chain ladder method of estimating outstanding claims?

A In the inflation-adjusted method, past claims are adjusted for inflation using incremental data.

B The basic chain ladder method assumes that weighted average past inflation will be repeated in the future.

C Development factors are derived from incremental data.

D Inflation varies by calendar year of claim payment. [3]

2 The process $\{Z_t\}$ is white noise, and

$$Y_t = 1.2Y_{t-1} + Z_t \ .$$

Which of the following statements is/are true?

I Y_t is non-stationary.
II $Y_t - Y_{t-1}$ is non-stationary.
III $Y_t - Y_{t-1}$ will have the characteristics of an MA(1) process.

A I only.
B I and II only.
C I and III only.
D None of the above. [3]

3 Claims, X_i, from a portfolio of insurance policies are independent, identically distributed with an exponential distribution with mean 10. What is the value of

$$P(X_1 \le 15 \text{ and } X_1 + X_2 \le 30)?$$

A 0.1733
B 0.5545
C 0.6035
D 0.7022 [3]

4 Claims occur on a portfolio of insurance policies according to a Poisson process, at a rate l per annum. The density function of X, the claim size, is denoted by $f(x)$, and the insurer arranges excess of loss reinsurance with retention M. Let X_I denote the part of each individual claim paid by the direct insurer.

Which of the following is/are correct expressions for the variance of aggregate claims (net of reinsurance) paid per annum by the direct insurer?

I $\lambda \, Var(X_I)$

II $\lambda \int_{-\infty}^{M} x^2 f(x) dx$

III $\lambda \left[\int_{-\infty}^{M} x^2 f(x) dx + M^2 P(X > M) \right]$

A I and II only.
B I and III only.
C III only.
D none of the above. [3]

PART TWO

5 In one year, a portfolio of general insurance business generates 1000 claims with mean 200 and standard deviation 400. It is decided to model claim amounts using a generalised Pareto distribution with density function $f(x)$, where

$$f(x) = \frac{\Gamma(\alpha+2)\lambda^{\alpha}x}{\Gamma(\alpha)(\lambda+x)^{\alpha+2}} \qquad (x > 0)$$

Estimate α and *l* using the method of moments. [5]

6 Y_t is a moving average process given by

$$Y_t = aZ_t + bZ_{t-1} + cZ_{t-2} + dZ_{t-3}$$

where $\{Z_t\}$ are independent random variables, each with mean 0 and variance σ^2 and $a = 0.5$, $b = 0.1$, $c = 0$, $d = -0.4$.

Calculate the first three autocorrelations, ρ_1, ρ_2 and ρ_3. [5]

7 The table below gives incremental claims arising from a portfolio of general insurance policies over 4 years. Use these data to estimate the development factors of the chain ladder technique.

	Development Year			
Accident Year	*1*	*2*	*3*	*4*
1994	19856	7328	2748	3263
1995	25691	3947	1762	
1996	19200	7021		
1997	29113			

[5]

8 Total claims *X*, in any year for a specific insurance portfolio, are assumed to have a normal distribution with a fixed but unknown mean θ and with standard deviation £100,000. The mean θ is modelled as a prior normal random variable with mean £1,500,000 and standard deviation £400,000.

(i) Calculate the unconditional mean and variance of *X*. [3]

(ii) Given that the total claims in the past year amounted to £2,500,000 derive the revised figures for the mean and variance of θ. [3]

[Total 6]

PART THREE

9 The transition rules for moving between the three levels, 0%, 35% and 50%, of a No Claims Discount system are as follows:

If no claim is made in a year, the policyholder moves to the next higher level of discount or remains at 50%. When at the zero or 35% level of discount, the policyholder moves to (or remains at) the zero level of discount when one or more claims are made in the year. When at the maximum level of discount (50%), the policyholder moves to the 35% level of discount if exactly one claim is made during the year, and moves to the zero level of discount if two or more claims are made during the year.

It is assumed that the number of claims X made each year has a geometric distribution with parameter q such that

$$P(X = x) = q^x(1-q), \qquad x = 0, 1, 2, \dots .$$

The full premium is £350.

(i) (a) Write down the transition matrix.

(b) Verify that the equilibrium distribution (in increasing order of discount), is of the following form, for some constant k:

$$\left(kq^2(2-q), kq(1-q), k(1-q)^2\right)$$

and express k in terms of q. [8]

(ii) The value of the expected premium in the stationary state, paid by "low risk" policyholders (with $q = 0.05$), is £178.51.

(a) Calculate the corresponding figure paid by high risk policyholders (with $q = 0.1$).

(b) Comment on the effectiveness of the No Claims Discount system.

[4]
[Total 12]

10 Claims occur on a general insurance portfolio according to a Poisson process. Individual claim amounts are either 1, with probability 0.8, or 10, with probability 0.2. The insurer uses a premium loading of 80% to calculate premiums and purchases excess of loss reinsurance with a retention of M $(1 \le M \le 10)$ from a company that uses a premium loading of 140%.

(i) (a) Derive the adjustment coefficient equation for the direct insurer.

(b) Calculate the smallest value of M that the insurer should consider, if the probability of ruin is to be less than 1.

(c) Calculate the adjustment coefficient (to 2 decimal places) if M = 5.

[10]

(ii) The same reinsurance company also offers proportional reinsurance with the same premium loading, such that the reinsurer pays a proportion, a , of each claim.

(a) Show that the insurer may either purchase excess of loss reinsurance with retention M, or proportional reinsurance with $\alpha = \dfrac{10-M}{14}$ for the same cost.

(b) Show that, if M = 5, the adjustment coefficient is smaller when proportional reinsurance is purchased than when excess of loss reinsurance is purchased at the same cost.

(c) Comment on the implications of (ii)(b). [8]

[Total 18]

11 The set of random variables $X_1, X_2, \ldots, X_n$ represents the loss to an office in successive years. Suppose the losses depend on a risk parameter q, which is itself modelled as a random variable, in such a way that the X_i are conditionally independent given q . Suppose that $E[X_i|\theta] = m(\theta)$ and that because of changing volumes of risk $Var[X_i|\theta] = s^2(\theta)(1.1)^{-i}$. Suppose that q is chosen independently from a distribution with $E[m(\theta)] = 2$, $Var[m(\theta)] = 1$ and $E[s^2(\theta)] = 1$, all measured in appropriate monetary units. Next year's loss X_{n+1} is to be estimated by minimising the linear expression

$$E\left[\left(X_{n+1} - \alpha_0 - \sum_{i=1}^{n} \alpha_i X_i\right)^2\right].$$

(i) Show that $E\left[X_i^2\right] = 5 + (1.1)^{-i}$ and determine the values of $E\left[X_i\right]$ and $E\left[X_i X_j\right]$, $i \neq j$. [7]

(ii) Show that the estimators of α_j, $j = 0,1, \ldots, n$ are given by

$$\hat{\alpha}_0 = \frac{2}{\kappa_n} \; ; \qquad \hat{\alpha}_j = \frac{(1.1)^j}{\kappa_n}, \; j = 1, 2, \ldots, n$$

where $\kappa_n = 1 + 11\left[(1.1)^n - 1\right]$. [10]

[Total 17]

12 (i) The random variable X has the lognormal distribution with density function $f(x)$ and parameters m and s. Show that for any real number $a > 0$ and any positive integer k

$$\int_0^a x^k f(x)dx = \exp\left(k\mu + \frac{k^2\sigma^2}{2}\right)\Phi\left(\frac{\log a - \mu - k\sigma^2}{\sigma}\right)$$

where Φ is the cumulative distribution function of the standard normal random variable. [6]

(ii) The amount, X, of a claim, in thousands of pounds, from an insurance portfolio has the lognormal distribution with mean 12.2 and standard deviation 16. Consider an excess of loss reinsurance policy with a retention of £28,000 so that the claim paid by the insurer (£'000) is given by X_I, where

$$X_I = \begin{cases} X & X \leq 28 \\ 28 & X > 28. \end{cases}$$

(a) Determine the probability that a claim involves the reinsurer.

(b) Calculate the mean and variance of the claims paid by the insurer.

(c) Given that a claim is referred to the reinsurer, what is the conditional expected value paid by the reinsurer. [14]

[Total 20]