Estimating the Relative Performance of Equities and Bonds Over the Next 25 Years

Jon Exley
Andrew Smith

Abstract:

This paper estimates the risk that equities will under-perform bonds over the next 25 years. Reasonable estimates are that, over the next 25 years:

- The probability that equity will under-perform bonds is 23.0%.
- The cost of guaranteeing an equity fund against underperformance relative to bonds over a 25-year period is 38.3% of the fund.
- The mean value of the guarantee in 25 years time is 8.8% of the bond fund, implying a discount rate of 5.7% below the bond return.

We conclude that there is a material risk that equities might under-perform bonds over the next 25 years. This is consistent with the fact that some investors hold long bonds as part of their portfolio.

1. Direct Data Analysis

We will start by considering the *probability* of underperformance.

The book "triumph of the optimists" by Elroy Dimson, Paul Marsh and Mike Staunton (Princeton University Press, 2002) provides historic returns over the last 100 years for a number of markets.

One of the quantities tabulated is the annualised return on equities in excess of bonds, for ten-year periods from 1900 to the present. This is a summary of their data, © 2002 Dimson, Marsh & Staunton and used with their kind permission. We have calculated the last decade returns from their tabulated 11-year and 1-year returns.

start date	Australia	Belgium	Canada	Denmark	France	Germany	Ireland	Italy	Japan	Netherlands	South Africa	Spain	Sweden	Switzerland	United Kingdom	United States
1900	11.3	0.8	6.1	0.6	2.5	6.1	0.6	-0.4	7.4	4.5	3.0	-1.2	14.7	#N/A	2.0	7.5
1910	9.0	4.5	8.6	12.6	9.3	8.4	0.6	6.4	13.8	7.8	4.8	0.6	3.4	-5.6	8.6	2.1
1920	10.1	6.3	7.6	-7.6	8.0	20.3	1.2	2.8	-7.6	-6.7	1.2	10.3	-3.5	4.2	0.9	6.9
1930	-0.2	-7.0	-4.2	1.3	-5.3	-2.7	-0.7	4.2	6.1	-2.4	7.3	-6.9	-5.3	-5.7	-3.1	-4.8
1940	3.1	6.4	4.9	0.9	18.0	13.3	1.2	22.2	14.6	3.8	5.5	-1.5	4.7	5.0	2.3	6.1
1950	16.3	10.1	14.8	7.8	13.8	20.0	1.6	17.0	30.8	17.2	4.6	7.2	15.8	8.5	16.4	18.3
1960	9.6	1.2	7.0	1.7	-0.9	0.8	15.8	-0.8	0.6	6.4	11.2	12.4	4.3	7.4	8.1	6.7
1970	1.0	-0.4	4.0	-0.6	-2.8	-4.4	5.3	-7.2	5.9	-1.7	9.7	-8.1	0.6	-4.1	3.1	1.0
1980	6.0	7.6	-0.8	5.0	6.2	8.9	3.9	11.7	10.7	10.2	12.5	11.5	19.0	6.4	7.3	3.6
1990	0.0	2.4	-1.2	-0.8	3.9	4.3	3.4	-2.1	-10.3	11.4	-2.7	3.8	3.6	8.3	1.7	8.1

It is clear from this data that there are many historic ten-year periods, over which equities have performed worse than bonds.

We can also evaluate longer periods. Equities under-performed bonds over the following 20-year periods:

Belgium: 1920-1940; 1930-1950
Canada: 1980-2000
Denmark: 1920-1940
France: 1960-1980
Germany: 1960-1980
Italy: 1960-1980
Japan: 1920-1940; 1980-2000
Netherlands: 1920-1940
Spain: 1900-1920; 1930-1950
Sweden: 1910-1930; 1920-1940; 1930-1950
Switzerland: 1910-1930; 1920-1940; 1930-1950
United Kingdom: 1920-1940

This represents 20 out of 143 observations. Of course these are not 143 independent observations because we have allowed overlapping periods. Furthermore, the world economies are by no means independent of each other. Nevertheless, the breadth of economies and periods with poor equity returns suggest that these are not isolated incidents that can be ruled out in future. There is a material probability, over 20 years, that equities might under-perform bonds in the next 20 years.

What about periods of 30 years? Here equities seem more likely to outperform, and the number of cases of underperformance are fewer. We note the following 30-year periods over which equities under-performed bonds:

Denmark: 1920-1950
Netherlands: 1910-1940; 1920-1950
Spain: 1930-1960
Sweden: 1910-1940; 1920-1950
Switzerland: 1910-1940

These figures suggest that at least out to 30 years, there is still a material probability that equities will under-perform bonds.

2. Model-Based Approaches

Analysis of historic data is one thing, but there may be good reasons why historic estimates are biased. The best known biases relate to survivorship and success biases. These biases exist because only the more successful economies have 100 years of uninterrupted data to analyse. Data exists, for example for stock exchanges in Prague and Moscow prior to their socialist revolutions. In the mid 1870's, many would have regarded the Russian economy, after 50 years of stability, as more robust than the USA still suffering the after-effects of a civil war. To exclude Russia's subsequent performance from the statistics is likely to give an overestimate of ex ante risk premiums. Furthermore, what we regard today as "emerging" economies have sometimes lacked stable governments or stock markets during the 20th century; we can only guess what the hypothetical returns on equity markets might have been for those economies, or which of our currently stable economies will in future hit various instabilities not reflected in their own historic data.

To take a more technical perspective, our historic estimates are based on unconditional probabilities, but for the next 25 years we should be using distributions conditional on information known today. To the extent that returns over separate 25-year periods are not independent, the conditional distribution over the next 25 years will probably be less dispersed than the unconditional distribution.

So what happens when we use a model-based approach?

The simplest approach is to use a random walk model, with returns log-normally distributed and independent from one period to the next. Our observed mixed distribution of 10-year returns, aggregated across 16 economies, is well captured by a lognormal distribution. The table below compares some relevant percentiles:

percentile	historical	lognormal
10%	-3.6%	-3.7%
20%	-0.6%	-0.9%
30%	1.0%	1.2%
40%	3.0%	2.9%
50%	4.5%	4.6%
60%	6.2%	6.4%
70%	7.6%	8.2%
80%	9.9%	10.5%
90%	14.0%	13.7%

Based on this distribution, we derive the following estimates of equity under-performance over various holding periods:

period (years)	probability
5	31.0%
10	24.1%
15	19.5%
20	16.1%
25	13.4%
30	11.2%
35	9.5%
40	8.0%
45	6.8%
50	5.8%

This was the simplest possible model. What about other actuarial models – what probabilities do these give that equities will under-perform bonds?

We have run Wilkie's 1995 model, Kemp's Random Walk model and Smith's Jump Equilibrium Model, using 5000 simulations from the freeware implementations listed in Smith (1996). Lee & Wilkie (2001) analyse a number of other models, and tabulate sample moments. We have interpolated these moments and applied lognormal approximations to estimate the probability of equity under-performance, for the Whitten & Thomas (1998) model, for Teeger and Yakoubov's model (Duval et al, 2000), for Cairns (2000) model and for Wilkie's 1995 ARCH model. We have added the Timbuk1 model here, using the parameters in Smith & Southall (2001). The model itself can be downloaded at http://www.timbuk1.co.uk. A simplified implementation of Barrie and Hibbert's (2001) model is available at their web site: http://www.barrhibb.com/actuaries.htm. We have run one proprietary model: B&W Deloittes' "The Smith Model". We are grateful to John Hibbert for supplying statistics for the "professional" version of his model.

Our results are as follows:

Model	Prob{ equities < gilts } after 25 years
Whitten & Thomas	1%
Kemp	8%
Jump - Equilibrium	11%
Wilkie	18%
Cairns	25%
Wilkie ARCH	25%
Timbuk1	25%
The Smith Model	25%
Barrie & Hibbert (professional)	27%
Teeger - Yakoubov	29%
Barrie & Hibbert (public)	34%

Although the models by no means agree on the probability of equity under-performance, we can see that, with one exception, these models are all producing significant underperformance probabilities. The outlier, the Whitten & Thomas model, benefits from an equity risk premium of over 7% relative to bonds in geometric terms, an adventurous assumption which accounts for the low under-performance probability.

Typically, these models are producing higher probabilities of under-performance, compared to the historic data. This is to be expected – the models are often calibrated to frequent (at most annual) data sets and then scaled up to a 25-year period. Plainly the process relies on assumptions about the model structure, which may be difficult to verify empirically. However, the use of compounding to deduce 25-year outcome distributions should reduce sampling error, and does at least overcome some of the worst survivorship and success biases which occur in historic data. This is perhaps the dominant factor explaining why the models produce higher under-performance probabilities compared to historically observed frequencies.

Of course a common argument cited against the model based approach is that it may miss out subtle mean reversion effects in data. Perhaps equities can be relied upon to bounce back after any market correction. Mean reversion effects can indeed reduce the probability of underperformance over long periods, but it should be noted that several of the above models do include quite significant mean reversion and yet still produce material probabilities of underperformance – so mean reversion does not guarantee that underperformance probability will be low. Furthermore, in addition to the influences of survivor bias on this "bounce back" theory (i.e. if markets don't bounce back they disappear from the data), we would also in any event question some of the statistical methodologies that claim to discover mean reversion (as discussed in detail in another paper to be presented at this conference).

3.Investor Behaviour

The trouble with measuring risk in terms of probability alone, as we have done so far, is that we have no objective measure of what we mean by material. In most instances it is not the probability alone that determines what we regard as a "material" risk but the severity of the event – thus we may regard a 1% p.a. probability of a catastrophic event as material whilst regarding a 1% pa probability of less serious events as immaterial.

One way of approaching this issue is to consider an economy where market clearing prices can be observed. We might then note that prices exist for 25 year index linked bonds and 25-year conventional bonds. If the risk of equities under performing such bonds were immaterial, why would any investors hold these bonds instead of equities?

Of course it is sometimes argued that no individual investor would decide of his own free will to hold 25-year bonds. However, this would be a sad indictment of the insurance and pensions industry, given that investors hold substantial exposures to these assets through insurance and pensions products supposedly designed to meet the savings requirements of these individuals. After all what is a modern defined benefit company pension (subject to statutory indexation) if not an index linked corporate bond? What is the guarantee in a 25 year with profit endowment policy or guaranteed annuity if not a 25-year conventional bond?

We would argue that viewing equity underperformance risk as immaterial cannot in reality be reconciled with observed holdings of long bonds. Were the risk to be "immaterial", nobody would want to hold such bonds, the prices of bonds would then fall until the risk of equities under performing bonds was material enough for *some* investors to hold bonds in their portfolio. Therefore the risk of underperformance ceases to be immaterial (since it is large enough to justify *some* investors holding bonds – or buying the afore mentioned products without mis-selling).

A more rigorous way of developing these arguments further is through the use of utility theory and associated state price deflators. However, we will avoid straying into the mathematics here to avoid the criticism that our arguments rely on certain models or assumptions (although in reality the link between, for example, prices, utility theory and state price deflators makes surprisingly few assumptions).

Nevertheless, it is worthwhile picking up an easy counter example to highlight the dangers of using subjective measures of a "material" risk. This *counter example* uses the independent identically distributed lognormal random walk model of equity returns, with power utility. It goes as follows:

(1) If we assume a lognormal random i.i.d walk for equities, we get the simple effect whereby the *probability* of equity underperformance decreases as we increase the time horizon.
(2) Thus, on a pure probability measure of "risk", the "risk" of underperformance becomes lower the longer the time horizon (although as noted in the previous section, we believe that under most realistic models, a period of 25 years is insufficient for the term "immaterial" to be applied to this probability).
(3) However, if we have power utility, we find that despite the reduction in probability of underperformance, investor's portfolio preferences are independent of the time horizon. This is a well-known result, proved (for example) in Merton (1992)

The reason for this is that with power utility, although the *probability* of loss decreases, the worst outcomes become increasingly severe, and the two effects offset each other. So this counter example shows that even if the *probability* of equity underperformance over 25 years *was* immaterial (and from our analysis above, we would find it hard to define 25% probability as immaterial), it is a major step to assert that the "risk" is immaterial.

For the avoidance of doubt here, we are not claiming that the simple lognormal random walk, or power utility are necessarily realistic. However, viewed as an elementary *counter example* the Merton result shows that establishing an immaterial probability of an event is not a *sufficient* condition for establishing an immaterial risk to investors.

Again, without straying into the mathematics, we will conclude this section with some general observations about the link between the existence of market clearing prices for bonds and equities:

(1) We all agree that equities are *expected* to out perform bonds
(2) If markets clear at a level where investors are holding 25 year index linked or conventional bonds, then these investors are, by definition, indifferent between holding a marginal £100 of these bonds or £100 of equities (otherwise they would have bought more equities or bonds at these prices).
(3) As good actuaries, we also agree that a price is a discounted present value of future cash flows.
(4) These three observations can be reconciled by using higher discount rates in the scenarios where equities do well, and lower discount rates in scenarios where they do badly.
(5) These discount rates reflect the risk preferences of actual investors, rather than subjective ideas about what is or is not a material probability of loss, or indeed what the probability is.

We will invoke some of these concepts further in our explanation of option pricing results.

4. Option Prices

Another approach to uncovering the joint impact of both frequency of risk and severity is to determine the price at which the risk can be bought or sold. This has the advantage of objectivity – although different parties might disagree over the probability of an event (as discussed above), and different parties may have different risk tolerances *and hold different allocations to equities and bonds in their portfolios*, if the market clears at a certain price for buying and selling the risk then everyone should at least agree on this price. In principle, working out a market price for risk of underperformance should be easy in the case of equity underperformance risk – the cost is just the cost of a put option. If the cost is high then we might call the risk material.

We immediately face an obstacle – the lack of option price data for 25-year periods. What we do have is option price data out to about 10 years. In this range, we have

- Many theories to explain option pricing, of which the vast majority are grounded in Black-Scholes' idea of no-arbitrage. Different variants involve allowances for time varying models, stochastic term structures, stochastic volatility, jumps, transaction costs, credit risk and other complex features.
- A practical world where everybody uses Black-Scholes to price these options, although often with some tweaks which compromise the theoretical purity of the model; for example, the use of smile effects to make implied volatility a function of strike. Typically, these smile effects become less marked for longer dated options.

What can we learn from this about 25 year options? We can take some comfort from the close correspondence between theory and prices in the "up to 10 years" maturity. Beyond this point, our best guess is to continue to apply the same theory. There is no reason why the application of Black-Scholes should suddenly fall off a cliff for maturities of more than 10 years.

One of the major challenges is credit risk. If we had negative equity returns over the next 30 years, a put option could theoretically be well in the money. But would the bank who sold the option still be around to meet its promise in such adverse conditions? Similarly, the authors of this paper were tempted to offer a wager to traditional actuaries who believe in risk-free equity out performance over the long term. But it is hard to organise this in a way which manages the credit risk – if the equity market fell for the next thirty years, the actuarial profession's reputation may not be riding high – and lawyers for insurers and pension funds would likely get hold of any actuarial assets long before we came to claim on our wager.

But lets disregard the credit risk for now, and assume the Black-Scholes formula applies. Actually, we should be using Margrabe's formula, a modification of Black-Scholes providing the better of two assets. Equivalently, when we consider put and call options on the equity fund, we need to look at options whose strike price is not constant but is instead indexed to the bond fund.

There are many possible choices of model parameters. For these calculations, we assume a 25-year distribution equivalent to an annual (geometric mean) out performance of 3% per annum for equities over bonds. We also assume a 20% annual standard deviation.

Applying Black and Scholes' formula, we look at the price of a put option to protect an equity fund against out performance relative to bonds. According to Black-Scholes, an option to pay the difference would cost 38% of the fund. To the extent that this is a material cost relative to the fund, we would say that this indicates a material risk.

The expected option payout would be only 8.8% as a proportion of the bond fund at maturity. In other words, the average of (guarantee divided by bond total return) is 8.8%. Equivalently, the average is 24.5% as a proportion of the equity fund. This is consistent with equities being expected to out-perform in most scenarios, but of course the particular option we are considering only comes into the money only when equities have under performed, in which case the guarantee cost will be larger as a proportion of the equity fund than as a proportion of the bond fund.

We could look instead at call options to protect a bond fund against underperformance relative to equities. At first sight, we might think this would be much more expensive than the put option – after all, a bond fund is more likely to under-perform than the equity fund.

However, option-pricing theory gives us the same price for a call option as a put option. The reason is simple: put-call parity. We can see that, on maturity:

bond fund + call option = equity fund + put option

Since the funds start off with the same initial value, by hypothesis, the call option must be just as valuable as the put option. Even though the mean call option payout is much greater than for the put, the market also discounts the call options at a higher rate. This reflects the fact that the call options carry a great deal of market risk (they pay out when times are good and expire worthless when times are bad). As we saw in the previous section, we can reconcile the fact that the present value of £100 of equities is the same a the present value of £100 of bonds, despite all agreeing on a higher expected return for equities, by inferring that investors use a higher discount rate to discount payouts in the good times and a lower discount rate to discount payouts in bad times. Since the put options carry negative market risk and thus pay out in the bad times, we would expect their payouts to be discounted at a low discount rate. This is indeed what we see.

5. Implications for Investors

There have been two major strands to the arguments presented in this paper.

(1) Even if risk is defined simply as the probability of underperformance by equities relative to bonds, this probability is not immaterial over 25 years.

(2) Risk definitions based on probability alone are flawed in any event, as risk should take account of the quantum of loss and the circumstances in which loss occurs.

The arguments associated with (2) are more subtle and invoke equilibrium pricing to the extent that we argue that nobody would hold 25 year bonds (implicitly or explicitly) if the risk was immaterial. Equally though, the fact that investors clearly hold some equities as well as bonds suggests that although the risk is not immaterial, neither is the equity risk premium, and in equilibrium investors are prepared to hold some equities in exchange for the likelihood of better returns in the more benign future scenarios.

Thus we stress that we are not arguing that investors should hold 100% bonds to invest over 25 years – we are arguing that most investors should probably hold some combination of equities and bonds. Our argument is simply that in this plausible equilibrium framework it must be wrong to argue that the risk of equities under-performing bonds is immaterial.

However, although we would not advocate 100% bond investment for an individual's total portfolio we do of course argue that it might make sense to invest in 100% bonds in particular components of an individual's portfolio. A bond investment trust should always invest in bonds. Or consider an individual's allocation to equities. He might own shares in companies who themselves hold exposure to shares in other companies by investing their defined benefit pension funds in equities. This is inefficient. It is more efficient for the pension funds to hold 100% bonds and the individual to buy more equities himself if he wants to hold more equities. However, the logical assertion that company pension funds should invest 100% in bonds is not the same as saying that an individual should invest his whole portfolio in bonds. We have stressed that looking at an individual's total portfolio he should probably be holding some equities and some bonds.

To give a practical example of this, consider the Boots pension fund that recently moved to 100% bonds (see Ralfe, 2002 for an account). This does not mean that Boots' shareholders should all invest in 100% bonds – they have shares in Boots and probably hold shares in many other companies. The argument is that it is unnecessary to give them added exposure to equities via the pension fund. Also to the extent that scheme members want participation in equity returns, they should buy equities in other vehicles and regard their secure pension benefit as part of their implicit holding of bonds. However, once again the individuals should probably hold some equities and some bonds.

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