# APPENDIX G

## SIMULATIONS TO TEST THE VALIDITY OF THE GRADUATION METHODS

From the variety of methods and formulae with which we have experimented in the past, we have chosen the 'pivotal' method and an exponential cubic formula as the basis for recent work. With some reservations, this approach has given reasonably satisfactory results. We have observed an appreciable tendency to wave-cutting and have been unsure whether this is an effect of the 'pivotal' method, a formula which is not entirely suitable, or due to the character of the data, *e.g.* the numbers of duplicates or the non-independence of experience at adjacent ages.

In order to investigate this subject, a series of simulation experiments has been conducted. In each experiment, an underlying 'true' curve of rates was postulated. For each simulation within the experiment, hypothetical data was generated by creating an 'observed' sickness rate at each age, equal to the 'true' rate plus a random deviation. These crude rates were then graduated by the method and formula chosen, and relevant statistics were recorded. The simulation was then repeated 500 times. The accumulated statistics for the whole experiment were then examined. Because the hypothetical data had been created under controlled conditions, theoretical values for these statistics were known, and it was possible to make comparisons. Further details and results are given in Tables G1 and G2. The distributions of the numbers of groups of signs are of particular interest.

Progressing through various experiments, the following conclusions were reached:

(a) If rates at different ages are stochastically independent, and if the true underlying curve is of the same form as the curve being fitted, then the results are very close to the theoretical. Thus, the 'pivotal' method is fully acceptable in producing results which for practical purposes are as good as could be obtained by an alternative (*e.g.* regression) method.

(b) When the formula for the underlying curve was changed to other plausible forms, the results were still very satisfactory. It seems, therefore, that our choice of graduation formula is able to cope, even if the underlying pattern of rates differs from it to some extent.

(c) When correlation exists between rates at adjacent ages, there is a change in the distribution of groups of signs, reflecting greater bunching of signs.

It appears, therefore, that a tendency of our graduation to show rather low numbers of groups of signs is unlikely to be due to an inappropriate method or formula, but could well be due to correlation. One cause of correlation is that

Table G1. *Graduation simulations*

| *Trial No.* | *Underlying curve of rates* |
|---|---|
| 1, 5, 6, 7 | $z_x = \exp(-4 + \cdot 8X - \cdot 4X^2 + \cdot 1X^3)$ |
| 2 | $z_x = \exp(-4\cdot 5 + \cdot 4X + \cdot 1 \times 2^X)$ |
| 3 | $z_x = \cdot 01 + \cdot 02X^2$ |
| 4 | $z_x = \cdot 08 \times 2^{-X} + \cdot 01 \times 3^X$ |
| | *Note*: $X = (X - 20)/10$ |

*Specimen underlying rates*

| | Formula | | | |
|---|---|---|---|---|
| Age | 1 | 2 | 3 | 4 |
| 25 | ·0250 | ·0156 | ·0150 | ·0739 |
| 30 | ·0302 | ·0202 | ·0300 | ·0700 |
| 35 | ·0346 | ·0269 | ·0550 | ·0802 |
| 40 | ·0408 | ·0369 | ·0900 | ·1100 |
| 45 | ·0530 | ·0532 | ·1350 | ·1700 |
| 50 | ·0821 | ·0821 | ·1900 | ·2800 |
| 55 | ·1632 | ·1396 | ·2550 | ·4747 |
| 60 | ·4493 | ·2725 | ·3300 | ·8150 |
| 64 | 1·3423 | ·5332 | ·3972 | 1·2608 |

claims data from some Offices becomes split between adjacent ages to adjust for the method of age classification used by those Offices. It was estimated that less than 8% of the total data is treated in this way. Another cause is the continuation of claims from one calendar year into the next, which will obviously be more pronounced with long-duration claims.

In each experiment, crude experience was simulated by allowing 10,000 exposed to risk at each age, 25–64, (n = 40), and assuming

$$\mu_2 = 10\mu_1$$

$$i.e.\ w_x = 10{,}000\,\{z_x + \sqrt{10z_x}\cdot R\}$$

where $R$ was an (approximately) normal random variable with zero mean and unit standard deviation.

This was modified in trials 5, 6 and 7 to incorporate some degree of correlation at adjacent ages, by replacing $R$ by $R'$, a variable of auto-regressive form, derived as

$$R'_x = \{r\,R_{x-1} + (1-r)R_x\}\cdot c$$

where $r$ is the assumed correlation factor and $c$ is an adjustment to ensure that $R'$ has unit standard deviation.

The arbitrary trial values assigned to $r$, namely 0·1, 0·2 and 0·3 for trials 5, 6 and 7 respectively do not reflect any view as to the degree of correlation which may exist in practice in the *C.M.I.* data.

## Table G2. *Results of simulation experiments*

| Statistic | Experiment number 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1. No of '+' signs | | | | | | | |
| Mean – 20 | ·060 | ·054 | ·260 | –·188 | –·092 | –·070 | ·116 |
| Standard deviation | 2·01 | 1·89 | 1·83 | 2·02 | 2·01 | 2·01 | 2·04 |
| 2. Groups of signs | | | | | | | |
| Mean | 20·8 | 21·2 | 21·4 | 20·7 | 19·7 | 18·3 | 16·6 |
| Standard deviation | 3·2 | 3·1 | 3·0 | 3·2 | 3·1 | 3·2 | 3·0 |
| 3. Mean $\chi^2$ (36 df) | 40 | 40 | 39 | 42 | 41 | 41 | 42 |
| 4. Frequency distribution of groups of signs | | | | | | | |
| 8 | | | | | | 1 | 2 |
| 9 | | | | | | 0 | 1 |
| 10 | | | | | | 6 | 7 |
| 11 | | | | 1 | 1 | 3 | 5 |
| 12 | 1 | 1 | | 6 | 6 | 10 | 32 |
| 13 | 2 | 0 | 1 | 0 | 4 | 9 | 15 |
| 14 | 5 | 6 | 3 | 8 | 20 | 32 | 66 |
| 15 | 12 | 7 | 5 | 8 | 14 | 29 | 40 |
| 16 | 24 | 12 | 17 | 13 | 44 | 58 | 85 |
| 17 | 41 | 32 | 20 | 33 | 21 | 49 | 42 |
| 18 | 39 | 37 | 41 | 62 | 68 | 69 | 77 |
| 19 | 51 | 56 | 43 | 40 | 48 | 51 | 35 |
| 20 | 56 | 55 | 62 | 72 | 74 | 55 | 49 |
| 21 | 65 | 59 | 54 | 44 | 61 | 50 | 22 |
| 22 | 60 | 68 | 77 | 71 | 57 | 33 | 7 |
| 23 | 39 | 48 | 49 | 40 | 24 | 15 | 10 |
| 24 | 35 | 44 | 52 | 39 | 29 | 15 | 5 |
| 25 | 30 | 33 | 32 | 30 | 13 | 11 | |
| 26 | 20 | 19 | 22 | 18 | 7 | 3 | |
| 27 | 10 | 12 | 15 | 10 | 7 | 1 | |
| 28 | 7 | 5 | 6 | 3 | 1 | | |
| 29 | 2 | 4 | 0 | 2 | 1 | | |
| 30 | 1 | 0 | 1 | | | | |
| 31 | | 2 | | | | | |

*Notes*

1. If the number of signs obeyed the binomial distribution, the standard deviation would be $\sqrt{npq} = \sqrt{40 \times \frac{1}{2} \times \frac{1}{2}} = 3{\cdot}16$. The observed standard deviations are consistently smaller than 3·16. This reflects the fact that, in curve fitting, the curve tends to follow the statistical deviations. On the usual binomial assumption, the standard deviation of the mean would be $3{\cdot}16/\sqrt{500} = {\cdot}141$. The observed deviations of the mean from 20 are acceptable in comparison.
2. The distribution of the numbers of groups of signs depends on $n$ but mean values around 20·5 are expected. Although the mean, as observed, even for experiment 7, is not drastically low, it can be seen from the frequency distribution that the chance of getting a low value is greatly increased when rates at adjacent ages are co-related.
3. None of the mean $\chi^2$ values is much above the 'expected' value of 36.