

EXAMINATIONS

September 1997

Subject A — Fundamentals of Actuarial Mathematics

Paper One

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 15 questions.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

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| <p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p> |
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PART ONE

For questions 1–7 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Which of the following is **not** true:

A $i^{(p)} = p((1-d)^{-1/p} - 1)$

B $\delta = \log(1-d)$

C $v = 1-d$

D $d^{(p)} = p(1 - (1-d)^{1/p})$ [2]

2 Which of the following is the value of $(\bar{I}\bar{a})_{\overline{20}|}$ at an effective rate of interest of 5% p.a.?

A 100.93

B 104.67

C 107.26

D 113.70 [3]

3 The 1-year forward rates for transactions beginning at times $t = 0, 1, 2$ are f_t , where

$$f_0 = 0.06, \quad f_1 = 0.065, \quad f_2 = 0.07$$

Which of the following gives the par yield for a 3-year bond?

A 6.326%

B 6.478%

C 6.499%

D 6.955% [3]

4 Dividends payable on a certain share are assumed to increase at a compound rate of 3% per half-year. A dividend of £5 per share has just been paid. Dividends are paid half-yearly.

Which of the following gives the value of the share to the nearest £1, assuming an effective rate of interest of 8% p.a.?

A £515

B £526

C £555

D £558 [3]

- 5** A customer takes out a loan of £1,000 in a consumer credit transaction. The loan is repaid by instalments of £703.84 p.a. payable annually in arrear for two years. The APR of the transaction is:
- A 26.00% p.a.
 - B 26.10% p.a.
 - C 26.17% p.a.
 - D 26.20% p.a.
- [3]
- 6** An investment fund is valued at £1 million on 1 January. Income is received continuously during the following year at a constant rate of £0.09232 million. The fund is valued at £1.2 million on 31 December of the same year. The money weighted rate of return is:
- A 10.1% p.a.
 - B 10.2% p.a.
 - C 10.3% p.a.
 - D 10.5% p.a.
- [3]
- 7** The rate of interest in any year has a mean of 6% and standard deviation of 1%. The yield in any year is independent of the yields in all previous years. Which of the following gives the standard deviation of the accumulated value at time 12 of an investment of £10 at time 0?
- A 0.04326
 - B 0.65775
 - C 6.57750
 - D 14.2866
- [3]

PART TWO

8 The force of interest is given by:

$$\delta(t) = 0.05 + 0.001t + 0.0001t^2 \quad 0 \leq t \leq 10$$

- (i) Calculate the total at time 10 of the accumulated proceeds of an investment of £100 at time 0 plus an investment of £100 at time 5. [4]
- (ii) Calculate the equivalent constant force of interest earned on the transaction. [3]
- [Total 7]

9 An investor is considering two investments. One is a 3-month deposit account which pays a rate of return of 4% p.a. convertible half yearly. The second is a 3-month Treasury Bill. Calculate the annual simple rate of discount available from the Treasury Bill if both investments provide the same effective rate of return. [3]

10 An investment fund is valued at £1.2 million on 1 January 1996 and at £1.4 million on 25 March 1996. Immediately after the second valuation a payment of £0.2 million is paid into the fund. On 31 December 1996 the fund is valued at £1.8 million. Calculate the time weighted rate of return for the year. [2]

11 An insurance company issues an annuity of £10,000 p.a. payable monthly in arrear for 20 years. The cost of the annuity is calculated using an effective rate of interest of 10% p.a.

- (i) Calculate the interest component of the first instalment of the sixth year of the annuity. [3]
- (ii) Calculate the total interest paid in the first 5 years. [3]
- [Total 6]

12 The rate of interest earned in the year from time $t - 1$ to t is denoted by i_t . Assume $(1 + i_t)$ is lognormally distributed. The expected value of the rate of interest is 5%, and the standard deviation is 11%.

- (i) Calculate the parameters of the lognormal distribution of $(1 + i_t)$. [4]
- (ii) Calculate the probability that the rate of interest in the year from time $t - 1$ to t lies between 4% and 7%. [4]
- [Total 8]

PART THREE

- 13** A company is considering investing in the following project. The company has to make an initial investment of three payments, each of £105,000. The first is due at the start of the project, the second six months later, and the third payment is due one year after the start of the project.

After 15 years it is assumed that a major refurbishment of the infrastructure will be required, costing £200,000.

The project is expected to provide no income in the first year, an income received continuously of £20,000 in the second year, £23,000 in the third year, £26,000 in the fourth year and £29,000 in the fifth year. Thereafter the income is expected to increase by 3% per annum (compound) at the start of each year.

The income is expected to cease at the end of the 30th year from the start of the project.

The cash flow within each year is assumed to be received at a constant rate.

- (i) Calculate the net present value of the project at a rate of interest of 8% p.a. effective. [8]
 - (ii) Show that a discounted payback period does not fall within the first 15 years, assuming an effective rate of interest of 8% p.a. [5]
 - (iii) Calculate the discounted payback period for the project, assuming an effective rate of interest of 8% p.a. [5]
- [Total 18]

14 An investor purchases a fixed interest security. The security pays coupons at a rate of 10% p.a. half-yearly in arrear, and is to be redeemed at 110 in 20 years. The investor is subject to tax on the coupon payments at a rate of 25%.

- (i) (a) Show that the price paid by the investor to obtain a rate of return of 10% p.a. effective is £81.76%.
- (b) Calculate the volatility of the security at 10% p.a. effective for this investor at the purchase date. [7]
- (ii) The investor has two liabilities. The present value of the liabilities, at an effective rate of interest of 10% p.a. is equal to the present value of the investor's holding in the fixed interest security described above. The amount of each of the two liabilities is the same. The second liability is due in 10 years.
- (a) Show that the first liability must be due in 9.61 years in order that the volatility of the liabilities is equal to the volatility of the assets.
- (b) Calculate the convexity of the total of the two liabilities described above.
- (c) Without any further calculations, state with reasons whether the fund comprising the asset and liabilities described is immunised against small movements in the rate of interest. [10]
- [Total 17]

15 On 15 May 1997 the government of a country issued an index-linked bond of term 15 years. Coupons are payable half-yearly in arrears, and the annual nominal coupon rate is 4%.

Interest and capital repayments are indexed by reference to the value of a retail price index with a time lag of 8 months. The retail price index value in September 1996 was 200 and in March 1997 was 206.

The issue price of the bond was such that, if the retail price index were to increase continuously at a rate of 7% p.a. from March 1997, a tax exempt purchaser of the bond at the issue date would obtain a real yield of 3% p.a. convertible half-yearly.

- (i) (a) Derive the formula for the price of the bond at issue to a tax-exempt investor.
- (b) Show that the issue price of the bond is £111.53%. [12]

- (ii) An investor purchases a bond at the price calculated in (i) and holds it to redemption. The retail price index increases continuously at 5% p.a. from March 1997. A new tax is introduced such that the investor pays tax at 40% on any real capital gain, where the real capital gain is the difference between the redemption money and the purchase price revalued according to the retail price index to the redemption date. Tax is only due if the real capital gain is positive.

Calculate the real annual yield convertible half-yearly actually obtained by the investor.

[7]

[Total 19]