EXAMINATIONS

April 1997

Subject A — Fundamentals of Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Begin your answers to Parts One, Two and Three on a separate sheet.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 16 questions.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–8 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Which of the following gives to the nearest integer the lower quartile of the future lifetime of a life aged 20 who is subject to the mortality of the ELT:12-Males life table?

A 43
B 59
C 63
D 79 [2]

2 Which of the following is equal to the function ${}_{m}|\ddot{a}_{x:\overline{n}|}$?

I $\dfrac{D_{x+m}}{D_x}(1 + a_{x+m:\overline{n-1}|})$

II $\ddot{a}_{x:\overline{n+m}|} - \ddot{a}_{x:\overline{m}|}$

III $\sum_{t=m}^{m+n} v^t {}_t p_x$

A I and II only are correct
B II and III only are correct
C I only is correct
D III only is correct [3]

3 Which of the following gives the value of $\ddot{s}_{50:\overline{10}|}$ using A1967–70 mortality and an effective rate of interest of 6% p.a.?

A 8.264
B 10.412
C 13.609
D 14.800 [3]

4 A one year term assurance is issued to a life aged 50 for a sum assured of £100,000 payable at the end of the year of death. Which of the following gives the standard deviation of the present value of the term assurance, using A1967–70 select mortality and 7% per annum interest?

A 4993
B 5244
C 5867
D 6452 [3]

5 Which of the following is the value of ${}_{10}P_{[25]:\overline{15}|}$, assuming A1967–70 mortality and 4% p.a. interest?

A 0.00103
B 0.06621
C 0.13117
D 0.17792 [3]

6 A company issues 20 identical policies to lives aged 60. The death strain at risk on each policy in the first year of the contract is £15,150. The insurer assumes that mortality follows the a(55) ultimate males mortality table. In the first year of the contract 2 lives die. Which of the following gives the mortality loss in the first year?

A 23,434
B 26,052
C 29,156
D 30,300 [2]

7 The reserve held for a policy at duration t for a life then aged 40, is ${}_tV$ = £15,000, assuming A1967–70 ultimate mortality and 8% p.a. interest.

In the year t to $t + 1$ no premiums are paid; expenses of £100 are incurred at time t. The benefits payable during the policy year t to $t + 1$ are £50,000 payable at $t + 1$ if the life dies during the year, and £1,000 payable at $t + 1$ if the life survives the year.

Which of the following gives the value of ${}_{t+1}V$?

A 14,044
B 15,021
C 15,043
D 16,128 [2]

8 The profit signature of a 3-year assurance contract issued to a life aged 60 is (–250, 150, 170).

The premium payable at the start of each year is £200. Mortality is assumed to follow A1967–70 ultimate mortality and there are no lapses.

Assuming that the office uses a risk discount rate of 15% per annum, which of the following gives the profit margin of the contract?

A 0.61%
B 0.62%
C 1.49%
D 1.51% [3]

PART TWO

9 Derive and simplify as far as possible an expression for the covariance of the present values of a whole life assurance and an n-year endowment assurance, each with unit sum assured payable immediately on death, both issued at the same time to the same life who is aged x at the issue date.

[Note: For random variables X, Y, $\mathrm{Cov}[X, Y] = E[XY] - E[X]E[Y]$] [6]

10 A life table with a select period of 2 years is based on rates of mortality which satisfy the following relationship:

$$q_{[x-r]+r} = \frac{1}{3-r} \times q_x \text{ (for all values of } x, \text{ and } r = 0, 1)$$

$q_{60} = 0.0195$, $q_{61} = 0.0198$, $q_{62} = 0.0200$ and $l_{63} = 100{,}000$.

(i) Calculate

(a) l_{62}

(b) $l_{[60]+1}$

(c) $l_{[60]}$ [4]

(ii) A select life aged 60 subject to the mortality table described in (i) above purchases a 3-year endowment assurance with sum assured £10,000. Premiums of £3,000 are payable annually throughout the term of the policy or until earlier death. The death benefit is payable at the end of the year of death.

Calculate the expected value of the present value of the profit or loss to the office on the contract, assuming an effective rate of interest of 6% p.a., and ignoring expenses. [4]

[Total 8]

11 Describe the benefit which has the present value random variable function given by Z below; T denotes the future lifetime of a life aged x.

$$Z = \begin{cases} \overline{a}_{\overline{n}|} & T \le n \\ \overline{a}_{\overline{T}|} & T > n \end{cases}$$ [3]

12 (i) State the conditions which are necessary for the retrospective policy value of a contract to be equal to the prospective policy value throughout the term of the policy. [2]

(ii) Consider a term assurance contract with a term of n years and a sum assured of £1 on a life aged x, payable immediately on death. Premiums are payable annually in advance. Assuming that the conditions referred to in (i) hold, prove that the retrospective and prospective policy values are equal at any integer duration, t. Ignore expenses. [4]

[Total 6]

13 A deferred annuity policy issued to a life age 50 has a 20-year deferred period, with premiums payable annually in advance throughout the deferred period, or until earlier death.

On death during the deferred period, the benefit payable at the end of the year of death is equal to the reserve that would have been required at that time had the life survived.

On survival to age 70, a whole of life annuity of £10,000 per year is payable annually in advance.

There are no expenses.

(i) Show that, if the annual premium is P, the reserve at duration t, $t = 1, 2, ..., 20$, is $P\ddot{s}_{\overline{t}|i}$ where i is the effective rate of interest assumed. [6]

(ii) Given that $i = .06$ and $\ddot{a}_{70} = 8.6$, calculate P. [2]

[Total 8]

PART THREE

14 (i) It is assumed that the future lifetime of a life aged x may be expressed as

$$T = K + S$$

where K is the curtate future lifetime, and S is independent of K and has uniform distribution on the interval (0, 1).

Show that ${}_{u}q_{x} = u.q_{x}$. [4]

(ii) Using the same assumption as in part (i), derive expressions in terms of mortality rates at integer ages for:

(a) ${}_{n}|{}_{u}q_{x}$ x, n integer, $0 < u < 1$

(b) ${}_{u}q_{x+n}$ x, n integer $0 < u < 1$

(c) ${}_{u}p_{x+r}$ x integer, $0 \le u < 1$, $0 < r < 1 - u$ [6]

(iii) (a) Sketch a typical curve of q_{x} for ages 0 to 60.

(b) State with reasons and with reference to your sketch of q_{x} at which ages the assumption in (i) above gives the closest approximation to the true values of ${}_{u}q_{x}$. [6]

[Total 16]

15 A life office issues a 20 year with-profit endowment assurance policy to a life aged 40. The sum assured of £20,000 plus declared reversionary bonuses are payable immediately on death, or on survival to the end of the term.

(i) Calculate the quarterly premium payable throughout the term of the policy if the office assumes that future reversionary bonuses will be declared at the rate of 1.92308% of the sum assured, compounded and vesting at the end of each policy year.

Basis:		
	Mortality:	A1967–70 select
	Interest:	6% per annum
	Initial expenses:	114% of the first premium and 2.5% of the basic sum assured
	Renewal expenses:	4% of each quarterly premium, excluding the first

[11]

(ii) The life office values its with-profits business using the net premium method, assuming an interest rate of 4% per annum and A1967/70 ultimate mortality.

Calculate the prospective policy value for the contract described above just before the 13th quarterly premium is payable, given that the total reversionary bonus declared up to that time is £600. [6]

[Total 17]

16 A life office issues a level premium term assurance policy with a term of 4 years to a life aged exactly 30 with a sum assured of £100,000. The death benefit is payable at the end of the year of death.

The annual office premium for the policy is £200.

The office assumes that it will earn interest on its investments at the rate of 6% per annum for the first two years and 8% per annum for the final two years of the term.

In addition, the office makes the following assumptions regarding mortality and expenses:

Basis: mortality — A1967–70 ultimate
initial expenses — £180 plus 40% of the premium
renewal expenses — 20% of each premium, excluding the first

The reserve set up at the end of each year per policy still in force is equal to 50% of the annual premium.

(i) Calculate the expected net present value of the profit for the policy, using a risk discount rate of 12% per annum. [9]

(ii) Show that the internal rate of return under the policy is approximately 34%. [3]

(iii) Explain how, if at all, your answers to (i) and (ii) would alter if the interest rate earned on investments had been assumed to be less than 6% per annum during the first year of the policy.

[3]

[Total 15]