EXAMINATIONS

April 1997

Subject C — Statistics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 15 questions.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–7 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 The number of claims N on a portfolio has a Poisson distribution with mean μ. The claim amounts are independent and each has a gamma distribution with parameters α and λ.

The moment generating function of the total claim amount S is:

A $\exp\left\{\mu\left[\left(\frac{\lambda}{\lambda - t}\right)^{\alpha} - 1\right]\right\}$

B $\exp\left\{\mu\left[\left(\frac{\lambda - t}{\lambda}\right)^{\alpha} - 1\right]\right\}$

C $\exp\left\{\mu\left[\left(\frac{\lambda}{t - \lambda}\right)^{\alpha} - 1\right]\right\}$

D $\exp\left\{\mu\left[\left(\frac{t - \lambda}{\lambda}\right)^{\alpha} - 1\right]\right\}$ [3]

2 The aggregate claims from a risk form a compound Poisson process. The Poisson parameter is 25 and individual claim amounts have an exponential distribution with mean 10. The premium for this risk is calculated using a loading factor of 15%. The insurer's initial surplus is 5.

The Lundberg upper bound for the probability of ruin for this risk is:

A 0.013
B 0.065
C 0.937
D 0.997 [3]

3 The random variables X and Y have the same mean and variance as each other. X has a lognormal distribution with parameters $\mu = 5$ and $\sigma = 1$. Y has a Pareto distribution with parameters α and λ.

The value of λ is:

A 626
B 726
C 826

D 926 [2]

4 Claims on a certain class of policy are classified as being of two types, I and II. Past experience has shown that:

25% of claims are of type I and 75% are of type II;
20% of type I claims require a company inspector, and
10% of type II claims require a company inspector.

Given that a claim required a company inspector, the probability that it was a type I claim is:

A 0.125
B 0.25
C 0.4
D 0.667 [2]

5 Claims occur on a portfolio of insurance policies according to a Poisson process at rate λ. Individual claim amounts have mean and variance μ and σ^2, respectively, and the insurer effects excess loss reinsurance with retention level M. The premium loading factors applied by the direct insurer and the reinsurer are θ and ζ, respectively.

Let $\psi(U)$ denote the probability of ultimate ruin for the direct insurer, given initial surplus U. Which of the following statements is/are true?

I The value of λ does not affect $\psi(U)$.
II If M increases, then $\psi(U)$ is certain to increase.
III If σ^2 increases, then $\psi(U)$ decreases.

A I only
B III only
C I and II only
D I and III only [3]

6 Random variables X and Y are independent and identically distributed as follows:

$$P(X = k) = P(Y = k) = pq^k, \qquad k = 0, 1, 2, ...$$

where $0 < p < 1$ and $q = 1 - p$.

The value of $E[X \mid X + Y = n]$ is:

A $nq/2$
B $n/2$
C $2nq/p$
D $(n + 1)/2$ [2]

 PLEASE TURN OVER

7 Aggregate annual claims from a risk have a compound binomial distribution. The number of claims each year has a binomial (50, 0.1) distribution and individual claim amounts have mean 100 and standard deviation 50.

The variance of the aggregate annual claims is:

A 11,250
B 45,000
C 57,500
D 62,500 [2]

PART TWO

8 Claims occur on a portfolio of insurance policies according to a Poisson process at rate λ. The insurer's initial surplus is U and the premium rate is calculated using a premium loading factor θ. All claims are for a fixed amount $d (> U)$. Show that the probability that ruin occurs at the first claim is:

$$1 - \exp\left[-\frac{1}{1+\theta}\left(1 - \frac{U}{d}\right)\right]$$ [4]

9 A household contents insurance portfolio consists of two independent groups of policies. For each group the distribution of the number of claims arising in a year, N, is given by:

$$P[N = n] = \binom{k+n-1}{n} p^k (1-p)^n \qquad n = 0, 1, 2, \ldots$$

For the first group, the parameters for N are $p = 0.8$ and $k = 80$. For the second group, the parameters for N are $p = 0.6$ and $k = 15$. Individual claims from the two groups have the same distribution, which has mean £450 and standard deviation £300.

Calculate the mean and variance of aggregate annual claims from this portfolio.

[7]

10 The no claims discount system operated by a motor insurer has seven classes. The class at the start of a year depends only on the class at the start of the previous year and the number of claims in the previous year. The rules for moving between classes are given by the following table.

Class	*Premium (% of full premium)*	*Class in following year after*			
		0	*1*	*2*	*^{3} 3*
		claims in current year			
7	100	6	7	7	7
6	75	5	7	7	7
5	65	4	6	7	7
4	55	3	5	7	7
3	45	2	5	7	7
2	40	1	4	6	7
1	33	1	4	6	7

On 1 January 1997 a group of 10,000 policyholders entered the system in class 6. For each policyholder the number of claims in a year has a Poisson distribution with mean 0.2. Calculate the expected number of policyholders in

each class on 1 January 1999, assuming no policyholders leave and no more join the group.

[8]

11 For the AR(2) process

$$Y_t = 2\alpha Y_{t-1} - \alpha^2 Y_{t-2} + Z_t, \qquad |\alpha| < 1$$

where $\{Z_t\}$ is white noise with mean zero and variance σ^2, derive formulae for the autocovariances γ_1 and γ_2 in terms of α and σ. [7]

12 Consider the stationary autoregressive process of order 1 given by

$$Y_t = \alpha Y_{t-1} + Z_t, \qquad |\alpha| < 1$$

where $\{Z_t\}$ denotes white noise with mean zero and variance σ^2.

Express Y_t in the form $Y_t = \sum_{j=0}^{\infty} a_j Z_{t-j}$ and hence, or otherwise, find an expression for $V(Y_t)$, the process variance, in terms of α and σ. [4]

PART THREE

13 (i) In the context of Empirical Bayes Credibility Theory Model 2, and with the usual notation:

(a) give a brief intuitive explanation of what $E[s^2(\theta)]$ and $V[m(q)]$ represent, and

(b) hence, or otherwise, state which of the following two formulae is correct for the credibility factor, Z, giving brief reasons for your answer

$$Z = \sum_{j=1}^{n} P_j \Bigg/ \left\{ \sum_{j=1}^{n} P_j + E[s^2(\theta)] / V[m(\theta)] \right\}$$

or, $$Z = \sum_{j=1}^{n} P_j \Bigg/ \left\{ \sum_{j=1}^{n} P_j + V[m(\theta)] / E[s^2(\theta)] \right\}$$ [6]

(ii) For the past five years an insurance company has insured 15 different chains of newsagents' shops against damage to their premises and stock from any cause. For chain i, $i = 1, 2, ..., 15$, and year j, $j = 1, 2, ..., 5$, the random variable Y_{ij} represents the annual claims and P_{ij} represents the number of shops in the chain. The sequence $\left\{ \{Y_{ij}; P_{ij}\}_{j=1}^{5} \right\}_{i=1}^{15}$ satisfies all the assumptions for Empirical Bayes Credibility Theory Model 2. The data for the first three chains in this collective are shown in the table below. Also shown for each of the first two chains is the credibility premium per shop for the coming year.

$Y_{ij}; P_{ij}$

Chain	*j = 1*	*2*	*3*	*4*	*5*	*Credibility premium per shop*
1	450; 2	220; 2	3700; 2	250; 2	380; 2	750
2	2500; 3	1140; 4	3600; 4	3900; 4	860; 5	733
3	4950; 9	39600; 9	14850; 11	29700; 12	9900; 14	

(a) Calculate the credibility premium per shop for the coming year for chain number 3.

(b) Explain carefully why the credibility premium per shop for the coming year is higher for chain 1 than for chain 2 even though the average annual claim per shop is lower for chain 1 than for chain 2. [11]

[Total 17]

14 Cumulative claim payments, in £000s, on a portfolio of general insurance policies for five underwriting years are displayed in the table below.

Underwriting year	*Development year*				
	0	*1*	*2*	*3*	*4*
1992	15417	19812	21424	22090	23174
1993	14918	20847	24141	24812	
1994	20990	27841	29010		
1995	21871	29427			
1996	30141				

The premium income, in £000s, for each underwriting year is given below:

Underwriting year	*Premium income*
1992	26906
1993	25147
1994	33198
1995	31204
1996	37387

(i) Use the separation method to estimate the annual claims inflation rate for each of the years 1992/93, 1993/94, 1994/95 and 1995/96. [10]

(ii) Assuming the future claims inflation rate will be 7% per annum, estimate the outstanding claims reserve which should be held at the end of 1996 in respect of this portfolio.

[5]
[Total 15]

15 The random variable S represents the annual aggregate claims for an insurer from policies covering damage due to windstorms. The insurer models S as follows:

$$S = \sum_{i=1}^{M} Y_i$$

where: M denotes the number of windstorms each year. M has a Poisson distribution with mean κ.

Y_i denotes the aggregate claims from the ith windstorm.

Y_i is modelled as follows: $Y_i = \sum_{j=1}^{N_i} X_{ij}$

where:

N_i denotes the number of claims from the ith windstorm. $\{N_i\}_{i=1}^{M}$ is a sequence of independent and identically distributed random variables, each with a Poisson distribution with mean λ.

X_{ij} denotes the amount of the jth claim from the ith windstorm. $\left\{\{X_{ij}\}_{j=1}^{N_i}\right\}_{i=1}^{M}$ is a sequence of independent and identically distributed random variables, each with mean μ, variance σ^2 and third central moment τ.

(i) (a) Derive expressions in terms of λ, μ, σ and τ for the mean, variance and third central moment of Y_i.

(b) Derive expressions in terms of κ, λ, μ, σ and τ for the mean, variance and third central moment of S. [8]

(ii) Now suppose that: $\kappa = 4$; $\lambda = 1000$

and X_{ij} has an exponential distribution with mean 1.

(a) Show that for any positive numbers x and M:

$$P[X_{ij} \le x + M \mid X_{ij} > M] = P[X_{ij} \le x]$$

(b) Calculate the mean, variance and third central moment of S and of S_R, where:

$$S_R = \sum_{i=1}^{M} \sum_{j=1}^{N_i} \max(X_{ij} - 2, 0)$$ [13]

[Total 21]