

# EXAMINATIONS

April 1997

## Subject A — Fundamentals of Actuarial Mathematics

### *Paper One*

*Time allowed: Three hours*

#### **INSTRUCTIONS TO THE CANDIDATE**

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 17 questions.*

#### **AT THE END OF THE EXAMINATION**

*Hand in BOTH your answer booklet and this question paper.*

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
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## PART ONE

For questions 1–8 indicate in your answer booklet which one of the answers A, B, C or D is correct.

- 1** Given below are three types of investment that can be modelled as cash flow processes. For which of these are the timing and amounts of all cashflows paid or received by a purchaser certain? Ignore the possibility of resale or default.

- I A zero-coupon bond with a 10 year term.
- II An index linked bond with a 10 year term.
- III An endowment assurance with a 10 year term.

- A I and II only
- B II and III only
- C I only
- D III only

[2]

- 2** At  $i = 0.062$ , which of the following is  $\ddot{a}_{10}^{(4)}$ ?

- A 7.401
- B 7.458
- C 7.571
- D 7.628

[2]

- 3** Let  $i_t$  denote the effective rate of interest in the year  $t$  to  $t + 1$ .

It is assumed that, for  $t = 0, 1, 2, \dots$

$$i_{t+1} = \begin{array}{ll} i_t + .02 & \text{with probability } 0.25 \\ i_t & \text{with probability } 0.5 \\ i_t - .02 & \text{with probability } 0.25 \end{array}$$

Given that  $i_0 = 0.06$ , which of the following gives the probability that an investment of 1 at time  $t = 0$  will accumulate to more than 1.2 at time 3?

- A 0.125
- B 0.375
- C 0.625
- D 0.875

[3]

- 4 A sum of £100 is accumulated at a nominal rate of discount of  $7\frac{1}{2}\%$  p.a. convertible quarterly for 1 year, and then at a nominal rate of interest of  $7\frac{1}{2}\%$  p.a. convertible quarterly for 1 year. Which of the following gives the accumulated amount of the investment after 2 years?
- A £100.00  
B £116.02  
C £116.19  
D £116.35 [3]

- 5 An equity pays annual dividends; the last dividend was £1 per share. What price would an investor pay for a share 6 months after the last dividend payment, if it is assumed that dividends will increase at  $4\frac{1}{2}\%$  compound per annum, and that the investor wishes to obtain a gross yield of 8% p.a. effective?
- A £22.22  
B £28.57  
C £29.86  
D £31.03 [3]

- 6 A company is considering a project with the following cash flows:

Initial investment	130,000
Income in 1st year	0
Income in 2nd year	80,000
Income in 3rd year	120,000

This income is expected to be received half-way through the year. Which of the following gives the internal rate of return for the project?

- A 18.80%  
B 23.08%  
C 24.03%  
D 33.27% [3]
- 7 A sum of £100 is accumulated at a simple rate of interest of 10% p.a. for  $2\frac{1}{2}$  years. The resulting amount is then accumulated at a simple rate of interest of 16% p.a. for a further  $T$  months. At the end of this period the accumulated value is £140. What is the value of  $T$ ?
- A 7.75 months  
B 9.00 months  
C 10.00 months  
D 11.25 months [2]

8 At time 0 the 1-year spot rate is 8% p.a., the 2-year spot rate is 9% p.a. and the 3-year spot rate is  $9\frac{1}{2}\%$  p.a. Which of the following gives the value of the 2-year forward rate from time 1?

- A 8.83%
- B 9.00%
- C 9.25%
- D 10.26%

[2]

## PART TWO

**9** A credit company offers a loan of £5,000. The loan is to be repaid over a 4 year term by level monthly instalments of £130, payable in arrears. What is the APR for the transaction? [4]

**10** An investor borrows £10,000 at an effective rate of interest of 15% per annum to finance a project. Income from the project is received at a level rate of £1,800 per annum, payable quarterly in arrear, for 20 years. Calculate the discounted payback period. [4]

**11** A loan of £20,000 was issued, and was repaid at par after three years. Interest was paid on the loan at the rate of 10% per annum, payable annually in arrears. The value of the retail price index at various times was as follows:

At the date the loan was made	245.0
One year later	268.2
Two years later	282.2
Three years later	305.5

Calculate the real rate of return earned on the loan. [5]

**12** A fund had a value of £100,000 on 1 July 1993. A net cash flow of £20,000 was received on 1 July 1994 and a further net cash flow of £35,000 was received on 1 July 1995. Immediately before receipt of the first cash flow the fund had a value of £115,000, and immediately before the second cash flow the fund had a value of £155,000. The value of the fund on 1 July 1996 was £180,000.

(i) Show that the money weighted rate of return earned on the fund between 1 July 1993 and 1 July 1996 was 6.32% p.a. [2]

(ii) Calculate the time weighted rate of return p.a. earned on the fund between 1 July 1993 and 1 July 1996. [3]

(iii) Comment briefly on why these two values differ. [2]

[Total 7]

**13** A loan of £20,000 is being repaid by an annuity payable quarterly in arrears for 20 years. The annual amount of annuity is calculated at an effective rate of interest of 10% p.a.

(i) Calculate the quarterly payment under the annuity. [2]

(ii) Calculate the capital and interest instalments in the 25th payment. [5]

[Total 7]

- 14** An investor is considering purchasing an office block for £1,700,000. Rental income is received continuously. Rents are increased every 5 years. The last such increase took place 2 years before the purchase date, and set the current rate of rent of £100,000 p.a. The next increase is due 3 years after purchase; subsequent increases are due 8 years after purchase, 13 years after purchase and so on, with a final increase 38 years after the purchase date. The property reverts to its original owner after 43 years for no payment. The investor estimates that £270,000 will need to be spent to refurbish the offices 6 months after the initial purchase date and that no further refurbishment will be required.

The investor requires an internal rate of return of 10% p.a. Show that the level compound rate of rental increase is 5.55% p.a. [7]

## PART THREE

**15** In any year the yield on funds invested with a given insurance company has mean value  $j$  and standard deviation  $s$ , and it is independent of the yields in all previous years.

- (i) Derive formulae for the mean and the variance of the accumulated value after  $n$  years of a single investment of 1 at time 0. [6]
- (ii) Let  $i_t$  be the rate of interest earned in the  $t$ th year. Each year the value of  $(1 + i_t)$  has a lognormal distribution with parameters  $\mu = 0.08$  and  $\sigma = 0.04$ .

Calculate the probability that a single investment of £1,000 will accumulate over 16 years to more than £4,250. [6]  
[Total 12]

**16** A loan of nominal amount of £100,000 is to be issued bearing interest payable quarterly in arrear at a rate of 8% p.a. Capital is to be redeemed at £105% on a coupon date between 15 and 20 years after the date of issue, inclusive, the date of redemption being at the option of the borrower.

- (i) An investor who is liable to income tax at 40% and tax on capital gains at 30% wishes to purchase the entire loan at the date of issue. What price should she pay to ensure a net effective yield of at least 6% p.a.? [9]
- (ii) Exactly 10 months after issue the loan is sold to an investor who pays income tax at 20% and capital gains tax at 30%. Calculate the price this investor should pay to achieve a yield of 6% p.a. on the loan:
- (a) assuming redemption at the earliest possible date  
(b) assuming redemption at the latest possible date [6]
- (iii) Explain which price the investor should pay to achieve a yield of at least 6% p.a. [2]

[Total 17]

**17** Let  $\{C_{t_k}\}$  denote a series of cash flows at times  $t_k$  for  $k = 1, 2, \dots, n$ .

- (i) (a) Define the volatility of the cash flow series, and derive a formula expressing the volatility in terms of  $t_k$ ,  $C_{t_k}$  and  $v$ .
- (b) Define the convexity of the cash flow series and derive a formula expressing the convexity in terms of  $t_k$ ,  $C_{t_k}$  and  $v$ . [4]
- (ii) A loan stock issued on 1 March 1997 has coupons payable annually in arrear at 8% p.a. Capital is to be redeemed at par 10 years from the date of issue.
- (a) Show that the volatility of this stock at 1 March 1997 at an effective rate of interest of 8% p.a. is 6.71.
- (b) At 1 March 1997 an investor has a liability of £100,000 to be paid in 7.247 years. Calculate the volatility and convexity of this liability at 1 March 1997 at an effective rate of interest of 8% p.a.
- (c) On 1 March 1997 the investor decides to invest a sum equal to the present value of the liability in the loan stock, where the present value of the liability and the price of the loan stock are both calculated at an effective rate of interest of 8% p.a.

Given that the convexity of the loan stock at 1 March 1997 is 60.53, state with reasons whether the investor will be immunised against small movements in interest rates on that date. [10]

- (iii) Calculate the present value of investor's profit or loss at 1 March if, immediately after purchasing the loan stock the rate of interest changes to 8½% p.a. effective. [3]

[Total 17]