EXAMINATIONS

September 1998

Subject A — Fundamentals of Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Begin your answers to Parts One, Two and Three on a separate sheet.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 15 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–7 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 What is the value of $A^{1}_{[60]:\overline{10}|}$?

Basis: mortality: a(55) females (select)
interest: 8% p.a.

A 0.0795
B 0.0861
C 0.4861
D 0.5302 [3]

2 A life office sells a two-year with profits endowment policy to a life aged exactly 60. The basic sum assured of £10,000 is payable at the end of the year of death or on survival to age 62. Level premiums are payable annually in advance and compound reversionary bonuses are declared at the start of each year. Ignoring expenses, what is the gross premium?

Basis: mortality: A1967–70 (ultimate)
interest: 10% p.a.
bonuses: 4.7619% p.a.

A £4,570
B £4,682
C £4,787
D £5,015 [3]

3 Let K_x be the curtate future lifetime of a life aged exactly x. Which of the following are true:

I $\text{Var}[a_{\overline{\min(K_x,n)}|}] = \text{Var}[\ddot{a}_{\overline{\min(K_x+1,n+1)}|}]$

II $\text{Var}[a_{\overline{\min(K_x,n)}|}] = {}^{1}\!/_{d^2}\left({}^{2}A_{x:\overline{n+1}|} - \left(A_{x:\overline{n+1}|}\right)^2\right)$

III $\text{Var}[a_{\overline{\min(K_x,n)}|}] = \text{Var}[\ddot{a}_{\overline{\min(K_{x+1}+1,n)}|}]$

(The prefix "2" indicates that the assurance is calculated at a rate of interest of $(1+i)^2 - 1$ per annum. The other functions are all based on an interest rate of i per annum.)

A I and II only are correct
B II and III only are correct
C I only is correct
D III only is correct [3]

4 At what age would it be expected that only 10 lives survive out of an initial population of 100 lives, all aged exactly 50, subject to the mortality of ELT: 12–Males?

A 84.6
B 85.6
C 88.9
D 89.0 [2]

5 An impaired life aged exactly 60 is subject to the mortality of a(55) males (select) with an addition of 0.03774 to the force of mortality. Which of the following is the expected present value at 4% p.a. of a whole of life assurance with sum assured £1,000 payable at the end of the year of death issued to the impaired life?

A £297.8
B £371.9
C £635.4
D £673.8 [3]

6 If the deaths between consecutive integer ages in the A1967–70 (ultimate) table are assumed to be uniformly distributed over the year of age, then which of the following is the value of ${}_{0.25}q_{50.25}$?

A 0.0011306
B 0.0011972
C 0.0011986
D 0.0012708 [3]

7 If the force of mortality between consecutive integer ages is assumed constant in the A1967–70 (ultimate) table, then which of the following is the exact value of $\bar{a}_{50:\overline{1}|}$ using a rate of interest of 4% p.a.?

A 0.95693
B 0.97831
C 0.97840
D 0.97847 [3]

PART TWO

8 Let $T_x = K_x + Y$, where T_x is a random variable denoting the complete future lifetime of a life aged exactly x, K_x is the corresponding curtate future lifetime and Y is an independent uniformly distributed random variable on the interval [0, 1). You may further assume that $Cov\left[Y, v^{T_x}\right] \approx 0$.

(i) Using the above assumption, prove that $\left(\overline{IA}\right)_x \approx \left(I\overline{A}\right)_x - \frac{1}{2}\overline{A}_x$. [4]

(ii) Calculate an approximate value for $\left(\overline{IA}\right)_{50}$ using A1967–70 ultimate mortality and interest of 4% p.a. [2]

[Total 6]

9 A life aged exactly 50 is issued with a whole of life policy with sum assured £10,000 payable at the end of the year of death and premiums of £220 payable annually in advance.

Given that $A_{[50]}$ evaluated at a rate of interest of 10.25% per annum is 0.12495, calculate the mean and variance of the initial net present value of the policy.

Basis: mortality: A1967–70 (select)
interest: 5% p.a. [8]

10 An impaired life aged exactly 30 suffers five times the force of mortality of a life of the same age subject to standard mortality.

(i) A two-year term assurance policy is issued to both the impaired life and a standard life with a death benefit of £10,000 payable at the end of the year of death.

Calculate the single premium payable for:

(a) the standard life and
(b) the impaired life

Basis:		
	standard mortality:	A1967–70 (ultimate)
	interest:	4% p.a.
	expenses:	none

[4]

(ii) A two-year endowment assurance policy is issued to both the impaired life and a standard life with a death benefit of £10,000 payable at the end of the year of death or on survival at the end of the two years.

Calculate the single premium payable for:

(a) the standard life and
(b) the impaired life [3]

(iii) Comment on the relative sizes of the impaired life and standard life premiums for the policies described in (i) and (ii) above. [3]

[Total 10]

11 An office issues a ten-year temporary increasing assurance policy payable immediately on death to a life aged exactly 50. The sum assured is £50,000 in the first year of the policy, £55,000 in the second year of the policy, £60,000 in the third year, etc.

Ignoring expenses, calculate the quarterly premium payable in advance.

Basis:		
	mortality:	A1967–70 (select)
	interest:	4% p.a.

[6]

12 In a certain mortality table, the force of mortality, μ_x, is assumed constant for all ages, x.

(i) Show that $\overset{o}{e}_x$ is constant for all x. [3]

(ii) Comment on whether you think that this is a suitable model for human mortality. [1]

[Total 4]

PART THREE

13 A life office is considering special ten-year endowment assurance policies to lives aged exactly 55. The basic sum assured is £20,000 payable at the end of the year of death or on survival to age 65. Simple reversionary bonuses are declared at the start of each year and a terminal bonus is paid on survival to age 65. In addition, at the end of each complete year that the policyholder survives, an amount of £1,000 is paid to the policyholder. Premiums are payable annually in advance throughout the term of the policy.

(i) Show that the premium for the above policy is £4,284.

Basis:	mortality:	A1967–70 (select)	
	interest:	4% p.a.	
	expenses:	Initial:	£500
		Renewal:	1% of each premium after the first
	bonuses:	Reversionary:	5% p.a.
		Terminal:	50% of basic sum assured

[7]

(ii) 100 of the special endowment policies commenced on 1 January 1997. The office holds reserves in respect of each policy equal to the retrospective policy value calculated on the same basis as in (i). Actual bonus declarations follow those assumed in the premium/reserving basis above. One of the policyholders died immediately after paying their first premium. No further deaths occurred during 1997.

Calculate the mortality profit or loss to the office on these 100 policies for the year 1997. [7]

(iii) Briefly explain how the profit or loss has arisen. [3]

[Total 17]

14 An office issues an endowment policy to lives aged exactly 62 with a sum assured of £10,000 payable at the end of the year of death or on survival to age 65. Premiums are payable annually in advance. The premium is set so that the expected net present value of future profits discounted at 10% p.a. is equal to 50% of the initial commission.

For each life in force at the start of the year, the office holds reserves equal to the sum of the premiums paid as at the end of the previous year.

(i) Calculate the annual premium.

Basis:	mortality:	A1967–70 (select)	
	interest:	4% p.a.	
	expenses:	Initial:	£100
		Renewal:	3% of every premium, including the first
	initial commission:	£200	

[10]

(ii) Explain (without further calculations) what the effect on the premium would be if:

(a) ultimate mortality rates were used
(b) a risk discount rate of 12% per annum were used. [4]

[Total 14]

15 A life aged exactly 40 purchases a special single premium deferred annuity. The annuity payments are to commence at age 60, and are payable monthly in advance for life. The amount of the first monthly annuity payment is to be £1,000, but once in payment the amount is to increase monthly in line with the rate of inflation.

There are no death benefits payable in the event of death during the deferred period.

(i) Show that the single premium is £41,706.

Basis:	mortality:	A1967–70 (select) before age 60 a(55) Males (ultimate) after age 60
	interest:	6% p.a.
	inflation:	1.9231% p.a.
	expenses:	Initial: £500 Claim: 1% of each annuity payment.

[6]

(ii) The office holds reserves in respect of the policy equal to the prospective gross premium policy values on the following basis:

Basis:	mortality:	A1967–70 (ultimate) before age 60 a(55) Males (ultimate) after age 60
	interest:	6% p.a.
	inflation:	1.9231% p.a.
	expenses:	Claim: 1% of each annuity payment

Calculate the reserve held in respect of the policy at the end of the 10th year, assuming that the life is still alive. [3]

(iii) At exact age 50, the policyholder requests that the policy be altered so that the annuity payable from age 60 is of a level amount of £2,000 per month in advance.

Using the same basis as in (ii) above plus an additional cost for altering the policy of £500, calculate the additional premium payable annually in advance by the policyholder for the remainder of the deferred period in order to effect the alteration. [6]

[Total 15]