Faculty of Actuaries **Institute of Actuaries**

EXAMINATIONS

September 1998

Subject A — Fundamentals of Actuarial Mathematics

Paper Two

EXAMINERS' REPORT

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PART TWO

8 (i)

$$(\overline{IA})_x = E(T_x\, v^{T_x})$$

$$= E((K_x + Y)\, v^{T_x})$$

$$= E[(K_x + 1)\, v^{T_x}] + E[(Y-1)\, v^{T_x}]$$

$$\approx (I\overline{A})_x + E(Y-1)\, E(v^{T_x}) \qquad \text{(using assumption)}$$

$$= (I\overline{A})_x + (½ - 1)\, \overline{A}_x$$

$$= (I\overline{A})_x - ½\overline{A}_x$$

(ii)

$$(\overline{IA})_{50} = (1.04)^{½}\, [(IA)_{50} - ½A_{50}]$$

$$= (1.04)^{½} \left[\frac{39164.062}{4597.0607} - ½\frac{1767.5555}{4597.0607}\right]$$

$$= 8.4920$$

(Approximations using 1.02 or $\dfrac{.04}{.039221}$ instead of $1.04^{½}$ are also acceptable.)

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$$g(K_{[50]}) = 220\ddot{a}_{\overline{K_{[50]}+1|}} - 10{,}000v^{K_{[50]}+1}$$

$$= 220\,\frac{1 - v^{K_{[50]}+1}}{d} - 10{,}000v^{K_{[50]}+1}$$

where $K_{[50]}$ = curtate future lifetime

$$E[g(K_{[50]})] = \frac{220}{d} - \left(10{,}000 + \frac{220}{d}\right) E(v^{K_{[50]}+1})$$

$$= \frac{220}{d} - \left(10{,}000 + \frac{220}{d}\right) A_{[50]}$$

$$= \frac{220}{.05} 1.05 - \left(10{,}000 + 220 \frac{1.05}{.05}\right) \times .31053$$

$$= 80.0514$$

(or use $220\ddot{a}_{[50]} - 10{,}000 A_{[50]}$)

$$\text{Var}[g(K_{[50]})] = \left(10{,}000 + \frac{220}{d}\right)^2 \text{Var}(v^{K_{[50]}+1})$$

$$= \left(10{,}000 + \frac{220}{d}\right)^2 ({}^2A_{[50]} - A_{[50]}^2)$$

$$= 213744400({}^2A_{[50]} - .09642888)$$

where the prefix "2" indicates that the assurance is evaluated at a rate of interest of $1.05^2 - 1 = .1025$

Using the given value:

$$\text{Var}[g(K_{[50]})] = 6{,}096{,}230$$

10 (i) **Standard life**

$$\text{Single premium} = \frac{C_{30} + C_{31}}{D_{30}} \times 10000$$

$$= £12.49$$

Impaired life

$$\text{Single premium} = (q_{30}^{*}\, v + p_{30}^{*}\, q_{31}^{*}\, v^2) \times 10000$$

$$= [(1 - p_{30}^5)\, v + p_{30}^5 (1 - p_{31}^5)\, v^2] \times 10000$$

$$= £62.26$$

(ii) (a) **Standard life**

$$\text{Single premium} = £12.49 + 10000\frac{D_{32}}{D_{30}}$$

$$= £9245.81$$

(b) **Impaired life**

$$\text{Single premium} = £62.26 + p_{30}^{5} \times p_{31}^{5} \times \frac{10000}{1.04^2}$$

$$= £9246.77$$

(iii) Premium for term assurance roughly reflects relative mortalities because the benefit is payable only on death. For the endowment the benefit will be paid in any event — it is only slightly greater for the impaired life because of the possible acceleration of the claim.

11 Let P be annual premium.

$$\text{PV premium} = P\ddot{a}^{(4)}_{[50]:\overline{10}|}$$

$$= P\left[\ddot{a}_{[50]:\overline{10}|} - \frac{3}{8}\left(1 - \frac{D_{60}}{D_{[50]}}\right)\right]$$

$$= P\left[8.230 - \frac{3}{8}\left(1 - \frac{2855.5942}{4581.3224}\right)\right]$$

$$= 8.0887P$$

$$\text{PV benefits} = 45000 \times 1.02 \times \left(A_{[50]:\overline{10}|} - \frac{D_{60}}{D_{[50]}}\right) + 5000 \times 1.02 \times (IA)^{1}_{[50]:\overline{10}|}$$

$$= 45000 \times 1.02 \times \left(.68347 - \frac{2855.5942}{4581.3224}\right)$$

$$+ 5000 \times 1.02 \times \left(\frac{39142.86 - 22644.604 - 14770.842}{4581.3224}\right)$$

$$= 2761.24246 + 1922.984377$$

$$= 4684.226837$$

$\Rightarrow \quad P = 579.1044$

$\Rightarrow \quad \frac{P}{4} = £144.78$

12 (i) $\mathring{e}_x = \int_0^\infty t \; {}_tp_x \; \mu_{x+t} \, dt$

$= \int_0^\infty {}_tp_x \, dt$ (integration by parts)

$= \int_0^\infty e^{-\int_0^t \mu_{x+s} ds} \, dt$

$= \int_0^\infty e^{-\mu t} \, dt$ which is independent of x

$[= \frac{1}{\mu}]$

(ii) The force of mortality varies considerably for humans and the expectation of future lifetime is certainly not the same for a new born infant and for a 99-year old.

13 (i) **Premiums**

P = annual premium

$P\ddot{a}_{[55]:\overline{10}|} = P8.093$

Death and Endowment benefit

$$20000A_{[55]:\overline{10}|} + 20000 \times 0.05 \times (IA)^1_{[55]:\overline{10}|}$$

$$+ (10000 + 20000 \times .05 \times 10) \times \frac{D_{65}}{D_{[55]}}$$

$$= 20000 \times .68874 + 20000 \times 0.05 \frac{30518.593 - 15675.797 - 12587.316}{3638.3307}$$

$$+ (10000 + 20000 \times .05 \times 10) \times \frac{2144.1713}{3638.3307}$$

$= 13774.8 + 619.921658 + 11786.5663$

$= 26181.28796$

Survival

$1000a_{[55]:\overline{10}|}$ $= 1000\left(\ddot{a}_{[55]:\overline{10}|} - 1 + \frac{D_{65}}{D_{[55]}}\right)$

$= 7682.328315$

EPV Total benefits $= 33863.61628$

EPV Expenses $= £500 + P(8.093 - 1) \times .01$

$= £500 + .07093P$

Eqn of value:

$(8.093 - .07093)\, P = 33863.61628 + 500$

$\Rightarrow P = 4283.63$

(ii) Reserve at end of year 1 = ${}_1V$

$({}_0V + P - E)(1.04) = 20000 \times 1.05 \times q_{[55]}$

$+ (1 - q_{[55]})(1000 + {}_1V)$

$\Rightarrow {}_1V = [(4284 - 500)\ 1.04 - 21000 \times .00447362$

$- 1000 \times (1 - .00447362)] \div (1 - .00447362)$

$= 2858.68$

DSAR $= 21000 - (2858.68 + 1000)$

$= 17{,}141.32$

EDS $= .00447362 \times 100 \times 17{,}141.32 = 7668.38$

ADS $= 17{,}141.32$

Profit $= -9{,}472.94$, i.e. a loss of 9472.94

(iii) The expected number of deaths is $100 \times .00447362 = .447$.

The actual number of deaths is 1.

Because the benefit paid out on death is larger than the reserve released, the greater than expected number of deaths leads to a loss.

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P	E	I	*Ben.*	*In force*
P	$.03P + 300$	$.0388P - 12$	77.9347	$1.0088P - 389.9397$
P	$.03P$	$.0388P$	114.6756	$1.0088P - 114.6756$
P	$.03P$	$.0388P$	10000	$1.0088P - 10000$

${}_{t-1}V$	ΔV	$i\,{}_{t-1}V$	*In force profit*
O	$.99220603P$	0	$.01659397P - 389.9397$
P	$.97706488P$	$.04P$	$.07173512P - 114.6756$
$2P$	$-2P$	$.08P$	$3.0888P - 10000$

Profit signature	*Discounted @ 10%*
$.01659397P - 389.9397$	$.015085427P - 354.4906$
$.071176079P - 113.7818218$	$.058823156P - 94.03456$
$3.029581056P - 9808.278478$	$\underline{2.276169088P - 7369.10479}$
	$2.350077671P - 7817.63$

$2.350077671P = 100 + 7817.63$

$\Rightarrow P = £3369.09$

(i) (a) Claims accelerated $\Rightarrow$ <u>increase</u> in premium required.

(b) Discounted value of profit signature would decrease hence <u>increase</u> in premium required to maintain profit margin.

15 (i) EPV of benefit at age 60.

Initial annual rate of annuity = £12000

$$\text{EPV} = 1000 + {}_{1/12}p_{60} \times \frac{1}{1.06^{1/12}} 1.019231^{1/12} \times 1000$$

$$+ {}_{2/12}p_{60} \times \frac{1}{1.06^{2/12}} 1.019231^{2/12} \times 1000$$

$$+ \ldots$$

$$= 1000\left[1 + {}_{1/12}p_{60}\frac{1}{1.04^{1/12}} + {}_{2/12}p_{60} \times \frac{1}{1.04^{2/12}} + \ldots\right]$$

$$= 12000\ddot{a}_{60}^{(12)} \quad @\ 4\%$$

$$= 12000\left[\ddot{a}_{60} - \frac{11}{24}\right]$$

$$= 12000\left[12.625 - \frac{11}{24}\right] = 146000$$

EPV at age 40 $= 146000 \dfrac{D_{60}}{D_{[40]}}$ @ 6%

$$= 146000 \frac{1}{1.06^{20}} \frac{30039.787}{33518.794}$$

$= 40798.48299$

EPV expenses $= 500 + .01 \times 40798.48299$

$= 907.9848299$

Hence single premium payable in £41706

(ii) Reserve $= 12000 \times 1.01 \times \ddot{a}_{60}^{(12)} \times \dfrac{D_{60}}{D_{50}}$

$$= 12000 \times 1.01 \times \left(12.625 - \frac{11}{24}\right) \times \frac{30039.787}{32669.855} \frac{1}{1.06^{10}}$$

$= 75{,}712.08723$

(iii) EPV new benefit $=$ £$24000\, \ddot{a}_{60}^{(12)} \times \dfrac{D_{60}}{D_{50}}$

$$= 24000 \times \left(10.813 - \frac{11}{24}\right) \times \frac{1}{1.06^{10}} \times \frac{30039.787}{32669.855}$$

$= 127596.3802$

EPV expenses $= 500 + .01 \times 127596.3802$

$= 1775.963802$

EPV premium $= 127596.3802 + 1775.963802 - 75712.08723$

$= 53660.25677$

Let P be annual premium

$$P\ddot{a}_{50:\overline{10}|} = 53660.25677$$

$$\Rightarrow P = \frac{53660.25677}{7.599}$$

$$= £7{,}061.49$$