

EXAMINATIONS

September 1998

Subject A — Fundamentals of Actuarial Mathematics

Paper One

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 16 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
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PART ONE

For questions 1–7 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 $a_{\overline{q}|}^{(12)}$ calculated at a rate of interest of 8% per annum effective is 4.790052.

$\ddot{a}_{\overline{q}|}^{(12)}$ calculated at the same rate of interest is:

A 4.82087

B 5.17326

C 4.59335

D 4.99518

[2]

2 A unit sum of money will be invested for the next 3 years at a rate of interest of either 6% per annum effective with probability 0.4 or 5% per annum effective with probability 0.6. The rate of interest, once determined will be the same in each of the 3 years.

A The expected value of the accumulation is 1.17091 and the standard deviation of the accumulation is 0.2002.

B The expected value of the accumulation is 1.17098 and the standard deviation of the accumulation is 0.2002.

C The expected value of the accumulation is 1.17091 and the standard deviation of the accumulation is 0.01636.

D The expected value of the accumulation is 1.17098 and the standard deviation of the accumulation is 0.01636.

[3]

3 A series of payments is to be received annually in advance. The first payment will be £10. Thereafter, payments increase by £2 per annum. The last payment will be made at the beginning of the tenth year. Which of the following are equal to the present value of the annuity?

I $\sum_{t=0}^9 8v^t + 2 \sum_{t=0}^9 tv^t$

II $10\ddot{a}_{\overline{10}|} + 2(Ia)_{\overline{9}|}$

III $8\ddot{a}_{\overline{10}|} + 2(I\ddot{a})_{\overline{10}|}$

A I and II only

B II and III only

C I only

D III only

[3]

4 A bond with an annual coupon of 8% per annum has just been issued with a gross redemption yield of 6% per annum effective. It is redeemable at par at the option of the borrower on any coupon payment date from the tenth anniversary of issue to the twentieth anniversary of issue. The gross redemption yield on bonds of all terms to maturity is 6% per annum effective. Ten years after issue, the gross redemption yield on bonds of all terms to maturity is 10% per annum effective.

- A The bond is likely to be redeemed later than was assumed at issue and the gross redemption yield will be higher than assumed at issue.
- B The bond is likely to be redeemed later than was assumed at issue and the gross redemption yield will be lower than assumed at issue.
- C The bond is likely to be redeemed earlier than was assumed at issue and the gross redemption yield will be lower than assumed at issue.
- D The bond is likely to be redeemed earlier than was assumed at issue and the gross redemption yield will be higher than assumed at issue.

[3]

5 A loan has been issued which is repaid by a ten year decreasing immediate annuity. The loan is calculated at a rate of interest of 4% per annum effective. The first repayment is 100 and payments decrease by 10 per annum.

- A The interest component of the fifth instalment is 1.258 and the capital component is 58.742.
- B The interest component of the fifth instalment is 22.355 and the capital component is 37.645.
- C The interest component of the fifth instalment is 7.579 and the capital component is 52.421.
- D The interest component of the fifth instalment is 13.710 and the capital component is 36.290.

[3]

- 6** A particular investment project (*X*) involves investing £1m at outset and receiving £2m in 5 years time. An alternative project (*Y*) involves investing £1.8m at outset and receiving an income of £0.4m at the end of every year for 10 years. The investor calculates the internal rates of return (IRR) from both projects and also calculates the discounted payback periods at a rate of interest of 8% per annum effective from both projects.
- A The IRR is higher for project *X* and the discounted payback period is shorter.
- B The IRR is higher for project *X* and the discounted payback period is longer.
- C The IRR is lower for project *X* and the discounted payback period is shorter.
- D The IRR is lower for project *X* and the discounted payback period is longer. [3]

- 7** Let $v(t)$ denote the discount function associated with the force of interest function $\delta(t)$. Which of the following are equal to the present value of a continuous annuity payable for n years, under which the rate of payment at time t is t ?

I $\int_0^n t v(t) dt$

II $\int_0^n t e^{-\int_0^n \delta(r) dr} dt$

III $\int_0^n t e^{-\int_0^t \delta(r) dr} dt$

- A I and III only
 B II and III only
 C I only
 D III only [3]

PART TWO

- 8 The n year forward rate for transactions beginning at time t and maturing at time $t + n$ is denoted as $f_{t,n}$. You are given

$$f_{0,1} = 6.0\% \text{ per annum}$$

$$f_{0,2} = 6.5\% \text{ per annum}$$

$$f_{1,2} = 6.6\% \text{ per annum}$$

Determine the 3-year par yield. [3]

- 9 A customer borrows £4,000 under a consumer credit loan. Repayments are calculated to give an APR of 15.4%. Instalments are paid monthly in arrears for 5 years. Calculate the flat rate of interest. [4]

- 10 An investor is considering two investments. The first is a 91 day Treasury Bill, discounted at a simple rate of discount of 6% per annum. The second is a 91 day deposit at a rate of interest of 6.15% per annum convertible daily. Determine which provides the highest effective rate of return. [3]

- 11 The force of interest takes the following values:

$$\delta(t) = 0.04 \quad 0 < t \leq 10$$

$$\delta(t) = 0.001(t - 10)^2 + 0.04 \quad 10 < t$$

- (i) Calculate the accumulation of £150 from time $t = 0$ to time $t = 20$. [4]
- (ii) (a) A continuous payment stream of 10 per annum is received from time $t = 5$ to time $t = 10$. Calculate the present value of that payment stream at time $t = 0$.
- (b) A continuous payment stream is paid at rate $e^{-0.03t}$ from time $t = 0$ to time $t = 10$. Calculate the present value of that payment stream at time $t = 0$. [7]

[Total 11]

- 12** A loan is to be repaid by an immediate annuity. The annuity starts at a rate of £100 p.a. and increases by £10 per annum. The annuity is paid for 20 years. Repayments are calculated using a rate of interest of 8% p.a. effective.
- (i) Calculate the amount of the loan. [3]
 - (ii) Construct a loan schedule showing the capital and interest elements in and the amount of loan outstanding after the 6th and 7th payments. [5]
 - (iii) Find the capital and interest element of the last instalment. [2]
- [Total 10]

- 13** The annual rates of interest from a particular investment, in which part of an insurance company's funds is invested, are independently and identically distributed. Each year, the distribution of $(1 + i_t)$, where i_t is the rate of interest earned in year t , is log-normal with parameters μ and σ^2 .

i_t has mean value 0.07 and standard deviation 0.02, the parameter $\mu = 0.06748$ and $\sigma^2 = 0.0003493$.

- (i) The insurance company has liabilities of £1m to meet in one year from now. It currently has assets of £950,000. Assets can either be invested in the risky investment described above or in an investment which has a guaranteed return of 5% per annum effective. Find, to two decimal places, the probability that the insurance company will be unable to meet its liabilities if:
 - (a) All assets are invested in the investment with the guaranteed return.
 - (b) 85% of assets are invested in the investment which does not have the guaranteed return and 15% of assets are invested in the asset with the guaranteed return. [7]
 - (ii) Determine the variance of return from the portfolios in (i)(a) and (i)(b) above. [3]
- [Total 10]

PART THREE

- 14 (i) In an annuity the rate of payment per unit of time is continuously increasing so that at time t it is t . Show that the value at time 0 of such an annuity payable until time n , which is represented by $(\bar{I}\bar{a})_{\overline{n}|}$, is given by $(\bar{a}_{\overline{n}|} - nv^n) / \delta$ where δ is the force of interest per unit of time and

$$\bar{a}_{\overline{n}|} = \int_0^n e^{-\delta t} dt. \quad [4]$$

- (ii) A consortium is considering the purchase of a coal mine which has recently ceased production.

The consortium forecasts that:

- (1) the cost of reopening the mine will be £700,000, and this will be incurred continuously throughout the first twelve months
- (2) after the first twelve months the revenue from sales of coal, less costs of sale and extraction, will grow continuously from zero to £2,500,000 at a constant rate of £250,000 per annum
- (3) when the revenue from sales of coal, less costs of sale and extraction, reaches £2,500,000 it will then decline continuously at a constant rate of £125,000 per annum until it reaches £250,000
- (4) when the revenue declines to £250,000 production will stop and the mine will have zero value

Additional costs are expected to be constant throughout at £200,000 p.a., excluding the first year. These are also incurred continuously.

What price should the consortium pay to earn an internal rate of return (IRR) of 20% p.a. effective?

[10]

[Total 14]

- 15** On 1 April 1992 a large number of nominal £1 zero-coupon bonds were issued, to be redeemed at par on 1 April 1997. The issue price was £0.67 per £1 bond, and the table gives the prices at 1 April for each year until redemption in 1997.

The prices are used for both the sale and purchase of bonds, and fractions of bonds may be bought and sold.

<i>Date</i>	<i>1/4/92</i>	<i>1/4/93</i>	<i>1/4/94</i>	<i>1/4/95</i>	<i>1/4/96</i>	<i>1/4/97</i>
	67p	78p	81p	86p	95p	100p

Consider two investors A and B. Investor A invests £100 in bonds on 1 April each year from 1992 to 1996 inclusive; and investor B buys 100 bonds on 1 April each year from 1992 to 1996 inclusive.

For both investors construct a schedule to show the value of the fund at 1 April each year from 1992 to 1997 inclusive. From this calculate for the period from 1 April 1992 to 1 April 1997 for both investors:

- (i) the money weighted rate of return per annum
- (ii) the time weighted return per annum [12]

- 16** On 9 October 1997 an investor, not liable to tax, had the choice of purchasing either:

- (A) 7½% Treasury Stock at a price of £107 per £100 nominal repayable at par in 2006.
- (B) 2% Index Linked Treasury Stock 2006 at a price of £203 per £100 nominal. The RPI base figure for indexing was 69.5 and the index applicable to the next coupon (payable on 8 April 1998) was 158.5. (This may also be taken as the latest known value of the RPI on 9 October.)

Assume that both stocks are redeemable on 8 October 2006; and that both coupons are paid half yearly in arrear on 8 April and 8 October.

Assuming that the RPI will grow continuously at a rate of 2.5% per annum from its latest known value

- (i) Show that the real yield from stock (A) as at 9 October 1997 is 4% per annum effective. [5]
 - (ii) By considering the series of future receipts from stock (B), derive a formula for the present real value of £100 nominal of that stock. [6]
 - (iii) Calculate the real yield as at 9 October 1997 from stock (B). [2]
- [Total 13]