EXAMINATION

14 September 2005 (am)

Subject CT4 (104) — Models (104 Part)
Core Technical

Time allowed: One and a half hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 6 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 Describe the advantages and disadvantages of graduating a set of observed mortality rates using a parametric formula. [4]

2 A lecturer at a university gives a course on Survival Models consisting of 8 lectures. 50 students initially register for the course and all attend the first lecture, but as the course proceeds the numbers attending lectures gradually fall.

Some students switch to another course. Others intend to sit the Survival Models examination but simply stop attending lectures because they are so boring. In this university, students who decide not to attend a lecture are not permitted to attend any subsequent lectures.

The table below gives the number of students switching courses and stopping attending lectures after each of the first 7 lectures of the course.

Lecture number	*Number of students switching courses*	*Number of students ceasing to attend lectures but remaining registered for Survival Models*
1	5	1
2	3	0
3	2	3
4	0	1
5	0	2
6	0	1
7	0	0

The university's Teaching Quality Monitoring Service has devised an Index of Lecture Boringness. This index is defined as the Kaplan-Meier estimate of the proportion of students remaining registered for the course who attend the final lecture. In calculating the Index, students who switch courses are to be treated as censored after the last lecture they attend.

(i) Calculate the Index of Lecture Boringness for the Survival Models course. [4]

(ii) Explain whether the censoring in this example is likely to be non-informative. [2]

[Total 6]

3 A mortality investigation has been carried out over the three calendar years, 2002, 2003 and 2004.

The deaths during the period of investigation, θ_x, have been classified by age x at the date of death, where

x = calendar year of death − calendar year of birth.

Censuses of the numbers alive on 1 January in each of the years 2002, 2003, 2004 and 2005 have been tabulated and denoted by

$$P_x(2002),\ P_x(2003),\ P_x(2004) \text{ and } P_x(2005)$$

respectively, where x is the age last birthday at the date of each census.

(i) State the rate year implied by the classification of deaths, and give the ages of the lives at the beginning of the rate year. [2]

(ii) Derive an expression for the exposed to risk in terms of the $P_x(t)$ (t = 2002, 2003, 2004, 2005) which corresponds to the deaths data and which may be used to estimate the force of mortality, μ_{x+f} at age $x + f$. [4]

(iii) Determine the value of f, stating any assumptions you make. [3]

[Total 9]

4 An investigation was carried out into the mortality of male undergraduate students at a large university. The resulting crude rates were graduated graphically. The following table shows the observed numbers of deaths at each age x, d_x, and the $\mathring{q}_x$s obtained from the graduation, together with the number of lives exposed to risk at each age.

Age x	d_x	$\mathring{q}_x$	*Exposed-to-risk*
18	6	0.0012	5,200
19	8	0.0013	5,000
20	12	0.0015	4,800
21	8	0.0017	5,000
22	9	0.0019	3,800
23	6	0.0020	3,600
24	8	0.0021	3,200

(i) Test whether the overall fit of the graduated rates to the crude data is satisfactory using a chi-squared test. [5]

(ii) Comment on your results in (i). [1]

(iii) (a) Describe three possible shortcomings in a graduation which the chi-squared test cannot detect, and

(b) State a test which can be used to detect each one. [3]

[Total 9]

5 An investigation was carried out into the effects of lifestyle factors on the mortality of people aged between 50 and 65 years. The investigation took the form of a prospective study following a sample of several hundred individuals from their 50th birthdays until their 65th birthdays and collecting data on the following covariates for each person:

X_1 Sex (a categorical variable with 0 = female, 1 = male)

X_2 Cigarette smoking (a categorical variable with 0 = non-smoker, 1 = smoker)

X_3 Alcohol consumption (a categorical variable with 0 = consumes fewer than 21 units of alcohol per week, 1 = consumes 21 or more units of alcohol per week)

In addition, data were collected on the age at death for persons who died during the period of investigation.

In order to analyse the data, it was decided to use a Gompertz hazard, $\lambda_x = Bc^x$, where x is the duration since the start of the observation.

(i) Explain why the Gompertz hazard might be appropriate for analysing the mortality of persons aged between 50 and 65 years. [2]

(ii) Show that the substitution:

$$B = \exp(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3),$$

in the Gompertz model (where $\beta_0 \ldots \beta_3$ are parameters to be estimated), leads to a proportional hazards model for this particular analysis. [3]

(iii) Using the Gompertz hazard, the parameter estimates in the proportional hazards model were as follows:

Covariate	*Parameter estimate*	*Parameter*
Sex	β_1	+0.40
Cigarette smoking	β_2	+0.75
Alcohol consumption	β_3	–0.20
	β_0	–5.00
	c	+1.10

(a) Describe the characteristics of the person to whom the baseline hazard applies in this model.

(b) Calculate the estimated hazard for a female cigarette smoker aged 55 years who does not consume alcohol.

(c) Show that, according to this model, a cigarette smoker at any age has a risk of death roughly equal to that of a non-smoker aged eight years older. [6]

[Total 11]

6 Studies of the lifetimes of a certain type of electric light bulb have shown that the probability of failure, q_0, during the first day of use is 0.05 and after the first day of use the "force of failure", μ_x, is constant at 0.01.

(i) Calculate the probability that a light bulb will fail within the first 20 days. [2]

(ii) Calculate the complete expectation of life (in days) of:

(a) a one-day old light bulb
(b) a new light bulb

[7]

(iii) Comment on the difference between the complete expectations of life calculated in (ii) (a) and (b). [2]

[Total 11]

END OF PAPER