EXAMINATIONS

September 1999

Subject A — Fundamentals of Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

2. *Begin your answers to Parts One, Two and Three on a separate sheet.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 15 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–7 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 Given $e_{50} = 30$ and $\mu_{50+t} = 0.005$, what is e_{51}?

A 29.00
B 29.15
C 29.85
D 30.00 [3]

2 Which of the following is the best approximation to $A_{[60]:\overline{10}|}$?

Basis: Mortality: a(55) Males (select)
Interest: 6% per annum

A 0.5763
B 0.5843
C 0.5874
D 0.5894 [3]

3 $A_{[50]}$ evaluated at 10.25% p.a. and A1967–70 (select) mortality is 0.12495. Which of the following is the standard deviation of the present value of a whole of life assurance effected on a life aged exactly 50 with unit sum assured?

Basis: Mortality: A1967–70 (select)
Interest: 5% per annum

A 0.0285
B 0.1689
C 0.3535
D 0.4705 [3]

4 If deaths between consecutive integer ages in the A1967–70 (ultimate) table are assumed to be uniformly distributed over the year of age, then which of the following is the value of $q_{45.5}$?

A 0.0028051
B 0.0028068
C 0.0028071
D 0.0028089 [3]

5 Five lives all aged exactly 97 are independent and are subject to mortality rates at each age that are 75% of the corresponding ELT12-Males rates. Which of the following is the probability that the entire group of five lives survives to age 100 exactly?

A 0.001263
B 0.007814
C 0.009031
D 0.017925 [3]

6 A life insurance company sells a two-year with profits endowment policy to a life aged exactly 33. The basic sum assured of £10,000 is payable at the end of the year of death or on survival to age 35. Level premiums are payable annually in advance throughout the term of the policy and compound reversionary bonuses are declared at the start of each year. Ignoring expenses, what is the gross premium?

Basis: Mortality: A1967–70 (ultimate)
Interest: 14% p.a.
Bonuses: 5% p.a.

A £4485.56
B £4510.79
C £4521.03
D £4524.37 [3]

7 A life insurance company offers special annuity rates to those lives that can prove that they have smoked more than 20 cigarettes a day for the past ten years. The mortality basis for the special annuities is equivalent to that of the standard basis with an addition of 0.0192308 to the force of mortality for a standard life of the same age.

A life aged 60 exactly who qualifies for the special terms purchases an annuity payable monthly in advance for a premium of £50,000. What is the additional monthly annuity over and above the standard amount that is purchased?

Standard basis:

Mortality: a(55) Males (ultimate)
Interest: 4% per annum
Expenses: none

A £59.93
B £60.78
C £60.83
D £72.28 [3]

PART TWO

8 (i) A journalist states that mortality rates increase with age. Comment on this statement by referring to mortality rates from the A1967–70 (ultimate) table. [3]

(ii) Comment on whether or not the Makeham hazard function ($\mu_x = A + Bc^x$) gives an adequate approximation to the actual forces of mortality in the A1967–70 table. [2]

(iii) Prove that the expected present value of an annuity paid annually in arrears based on a Makeham hazard function is the same as that based on a Gompertz hazard function ($\mu_x = Bc^x$), but with a force of interest increased by A per annum. [3]

[Total 8]

9 Assuming that the force of mortality between consecutive integer ages is constant in the A1967–70 (ultimate) table, calculate the exact value of $\overline{A}_{50:\overline{2}|}$ using a rate of interest of 4% per annum. [7]

10 A ten-year endowment policy with a sum assured of £10,000 payable at the end of the year of death or on survival for ten years is issued to a life aged exactly 50. Premiums are paid annually in advance throughout the policy (or until earlier death).

(i) Calculate the premium.

Basis: Mortality: A1967–70 (ultimate)
Interest: 4% per annum [2]

(ii) An impaired life aged exactly 50 suffers the mortality of a life ten years older than a life of the same age subject to standard mortality. The life insurance company agrees to offer the policy to the impaired life for the standard premium, provided that the sum assured payable on death is reduced. The sum assured payable on survival remains at £10,000.

Calculate the amount by which the death benefit should be reduced. [3]

(iii) By how much would the survival benefit have to be decreased if it were desired that the death benefit remain at £10,000 for the impaired life (using standard premiums)? [3]

[Total 8]

11 (i) In a double endowment assurance policy, the unit sum assured is payable at the end of the year of death. On survival for the term of n years, a sum equal to twice the sum assured for the death benefit is payable. Write down an expression for the present value of this policy in terms of the random variable K_x, the curtate future duration of life for a life aged exactly x, and the constant rate of interest, i per annum. [2]

(ii) Write down an expression for the expected value of the assurance in (i) in standard actuarial notation. [2]

(iii) Show that the variance of the assurance in (i) can be written as an expression of the form:

$$\alpha_1 \, {}^2A_{x:\overline{n}|} + \alpha_2 \, {}^2A_{x:\overline{n}|}^{\,1} + \alpha_3 \, (A_{x:\overline{n}|})^2 + \alpha_4 \, (A_{x:\overline{n}|}^{\,1})^2 + \alpha_5 \, A_{x:\overline{n}|} \, . \, A_{x:\overline{n}|}^{\,1}$$

where $\alpha_1 \ldots \alpha_5$ are constants, assurances with the prefix 2 are to be valued using a rate of interest of j where $v_j = v_i^2$ and the remaining assurances are to be valued using a rate of interest of i per annum. [4]

[Total 8]

12 A life insurance company sells a 25-year decreasing term assurance policy to a life aged exactly 30. The initial sum assured is £100,000 and this decreases by £4,000 at the end of each year. The sum assured is payable immediately on death. Premiums are payable quarterly in advance to age 55 or earlier death.

Calculate the quarterly premium.

Basis: Mortality: A1967–70 (select)
Interest: 4% per annum
Expenses: none [5]

PART THREE

13 A life insurance company sells a ten-year without profits endowment assurance policy to a life aged exactly 55. The basic sum assured is £50,000 payable immediately on death or on survival to age 65. Premiums are payable monthly in advance.

(i) Show that the monthly premium for the policy is £374.93.

Basis: Mortality: A1967–70 (select)
Interest: 4% per annum
Expenses: Initial: £500
Renewal: £1% of each premium
Claim: £250. [5]

(ii) The company holds reserves in respect of this policy equal to the prospective gross premium policy value using the premium basis. Calculate the reserve held in respect of the policy at the end of the fifth year of the policy assuming that the life is still alive. [4]

(iii) After 5 years the assured requests that the policy be altered to a with profits endowment assurance policy maturing after a further ten years. Simple reversionary bonuses are to be declared on the policy at the start of each year and a terminal bonus is payable on survival to age 70. Premiums will be paid monthly in advance until maturity or earlier death. The sum assured remains unchanged.

Calculate the monthly premium payable by the policyholder for the remainder of the term of the policy assuming the basis below.

Basis: Mortality: A1967–70 (select)
Interest: 4% per annum
Expenses: Alteration: £500
Renewal: £1% of each premium
Claim: £300
Bonuses: 5% p.a. (simple) reversionary bonuses
50% (of basic sum assured) terminal bonus on survival [7]

[Total 16]

14 A life insurance company sells single premium deferred annuities on the following basis:

Basis: Mortality: During deferred period: A1967–70 (select) less 5 years
After deferred period: a(55) Females (ultimate)
Interest: During deferred period: 6% p.a.
After deferred period: 4% p.a.
Expenses: Initial: £100
Annuity: 0.5% of each annuity payment

The annuities are payable annually in advance. Under the standard policy no death benefit is payable during the deferred period.

A life aged exactly 45 purchases such an annuity of £1,000 per annum to commence on her 65th birthday.

(i) Calculate the single premium. [5]

(ii) The company also offers an option whereby a benefit is payable on death during the deferred period. The death benefit is four times the amount of the annual annuity payment and is paid at the end of the year of death. In addition, if death occurs between the 63rd and 65th birthdays, a level annual annuity of £500 per annum is payable for ten years certain. The level annuity is payable annually in advance with the first payment made at the end of the year of death.

The interest rate basis for the level annuity component of the option is 4% per annum. The rest of the basis is as before.

Calculate the cost of the option to a life aged exactly 45. [7]

[Total 12]

15 On 1 January 1995 a life insurance company issued 100 three-year with-profit endowment assurance policies to a group of lives then all aged exactly 57. Compound reversionary bonuses were to be declared at the start of each year. No terminal bonus was payable. The sums assured of £10,000 for each policy were paid at the end of the year of death or on survival to age 60. Premiums were payable annually in advance throughout the term of the policy.

The premiums were set so that the expected net present value of future profits discounted at 10% p.a. was equal to £200.

For each policy in force at the start of the year, the reserve to be held was equal to a net premium policy value based on a rate of interest of 3% per annum and mortality using A1967–70 (ultimate), with an allowance of 4% per annum for the compound reversionary bonuses.

(i) Show that the reserves held for each policy in force at 31 December 1995 and 31 December 1996 were £3577.59 and £7322.93 respectively. [2]

(ii) Show that the annual premium is £3,482.

Basis: Mortality: A1967–70 (select)
Interest: 6% per annum
Bonuses: 4% per annum
Expenses: £50 at the start of each year [9]

(iii) By 31 December 1995, 10 of the original 100 policyholders had died. Calculate the mortality profit or loss to the office on the reserving basis in respect of the original 100 policies over the year 1 January 1995 to 31 December 1995. [4]

[Total 15]