

EXAMINATION

6 September 2006 (am)

Subject CT4 (103) — Models (103 Part) Core Technical

Time allowed: One and a half hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 6 questions, beginning your answer to each question on a separate sheet.*
5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

A1 A manufacturer uses a test rig to estimate the failure rate in a batch of electronic components. The rig holds 100 components and is designed to detect when a component fails, at which point it immediately replaces the component with another from the same batch. The following are recorded for each of the n components used in the test ($i = 1, 2, \dots, n$):

s_i = time at which component i placed on the rig
 t_i = time at which component i removed from rig

$$f_i = \begin{cases} 1 & \text{Component removed due to failure} \\ 0 & \text{Component working at end of test period} \end{cases}$$

The test rig was fully loaded and was run for two years continuously.

You should assume that the force of failure, μ , of a component is constant and component failures are independent.

(i) Show that the contribution to the likelihood from component i is:

$$\exp(-\mu(t_i - s_i)) \cdot \mu^{f_i} \quad [2]$$

(ii) Derive the maximum likelihood estimator for μ . [4]
 [Total 6]

A2 The price of a stock can either take a value above a certain point (state A), or take a value below that point (state B). Assume that the evolution of the stock price in time can be modelled by a two-state Markov jump process with homogeneous transition rates $\sigma_{AB} = \sigma$, $\sigma_{BA} = \rho$.

The process starts in state A at $t = 0$ and time is measured in weeks.

(i) Write down the generator matrix of the Markov jump process. [1]

(ii) State the distribution of the holding time in each of states A and B. [1]

(iii) If $\sigma = 3$, find the value of t such that the probability that no transition to state B has occurred until time t is 0.2. [2]

(iv) Assuming all the information about the price of the stock is available for a time interval $[0, T]$, explain how the model parameters σ and ρ can be estimated from the available data. [2]

(v) State what you would test to determine whether the data support the assumption of a two-state Markov jump process model for the stock price. [1]
 [Total 7]

- A3** (i) Define the following types of a stochastic process:
- (a) a Poisson process
 - (b) a compound Poisson process; and
 - (c) a general random walk
- [3]
- (ii) For each of the processes in (i), state whether it operates in continuous or discrete time and whether it has a continuous or discrete state space. [2]
- (iii) For each of the processes in (i), describe one practical situation in which an actuary could use such a process to model a real world phenomenon. [3]
- [Total 8]

A4 The credit-worthiness of debt issued by companies is assessed at the end of each year by a credit rating agency. The ratings are A (the most credit-worthy), B and D (debt defaulted). Historic evidence supports the view that the credit rating of a debt can be modelled as a Markov chain with one-year transition matrix

$$X = \begin{pmatrix} 0.92 & 0.05 & 0.03 \\ 0.05 & 0.85 & 0.1 \\ 0 & 0 & 1 \end{pmatrix}$$

- (i) Determine the probability that a company rated A will never be rated B in the future. [2]
- (ii) (a) Calculate the second order transition probabilities of the Markov chain.
- (b) Hence calculate the expected number of defaults within the next two years from a group of 100 companies, all initially rated A. [2]

The manager of a portfolio investing in company debt follows a “downgrade trigger” strategy. Under this strategy, any debt in a company whose rating has fallen to B at the end of a year is sold and replaced with debt in an A-rated company.

- (iii) Calculate the expected number of defaults for this investment manager over the next two years, given that the portfolio initially consists of 100 A-rated bonds. [2]
- (iv) Comment on the suggestion that the downgrade trigger strategy will improve the return on the portfolio. [2]
- [Total 8]

A5 A motor insurance company wishes to estimate the proportion of policyholders who make at least one claim within a year. From historical data, the company believes that the probability a policyholder makes a claim in any given year depends on the number of claims the policyholder made in the previous two years. In particular:

- the probability that a policyholder who had claims in both previous years will make a claim in the current year is 0.25
- the probability that a policyholder who had claims in one of the previous two years will make a claim in the current year is 0.15; and
- the probability that a policyholder who had no claims in the previous two years will make a claim in the current year is 0.1

- (i) Construct this as a Markov chain model, identifying clearly the states of the chain. [2]
 - (ii) Write down the transition matrix of the chain. [1]
 - (iii) Explain why this Markov chain will converge to a stationary distribution. [2]
 - (iv) Calculate the proportion of policyholders who, in the long run, make at least one claim at a given year. [4]
- [Total 9]

A6 (i) Explain the difference between a time-homogeneous and a time-inhomogeneous Poisson process. [1]

An insurance company assumes that the arrival of motor insurance claims follows an inhomogeneous Poisson process.

Data on claim arrival times are available for several consecutive years.

- (ii) (a) Describe the main steps in the verification of the company's assumption.
- (b) State one statistical test that can be used to test the validity of the assumption. [3]
- (iii) The company concludes that an inhomogeneous Poisson process with rate $\lambda(t) = 3 + \cos(2\pi t)$ is a suitable fit to the claim data (where t is measured in years).
 - (a) Comment on the suitability of this transition rate for motor insurance claims.
 - (b) Write down the Kolmogorov forward equations for $P_{0j}(s, t)$.

- (c) Verify that these equations are satisfied by:

$$P_{0j}(s,t) = \frac{(f(s,t))^j \cdot \exp(-f(s,t))}{j!}$$

for some $f(s,t)$ which you should identify.

[Note that $\int \cos x \, dx = \sin x$.]

- (d) Comment on the form of the solution compared with the case where λ is constant.

[8]

[Total 12]

END OF PAPER