

EXAMINATIONS

April 1999

Subject A — Fundamentals of Actuarial Mathematics

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
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PART ONE

For questions 1–7 indicate in your answer booklet which one of the answers A, B, C or D is correct.

- 1** Using A1967–70 mortality, which of the following gives the value of ${}_3|_2q_{45+1}$? [2]
- A 0.008927
B 0.008943
C 0.012683
D 0.012705
- 2** Which of the following is the value of $A_{60:\overline{5}|}$ using a(55) males mortality and interest at 4% per annum? [3]
- A 0.82110
B 0.82715
C 0.83357
D 0.83663
- 3** Which of the following is the present value of an endowment assurance policy issued to a life aged x with term n years, where T_x is the future lifetime of (x) ? [2]
- A $\begin{cases} v^{T_x} & T_x < n \\ v^n & T_x \geq n \end{cases}$
B $\begin{cases} v^n & T_x < n \\ v^{T_x} & T_x \geq n \end{cases}$
C $\begin{cases} 0 & T_x < n \\ v^n & T_x \geq n \end{cases}$
D $\begin{cases} v^{T_x} & T_x < n \\ 0 & T_x \geq n \end{cases}$

4 Which of the following formulae are correct?

$$\text{I} \quad (IA)_{x:\overline{n}|}^1 = \frac{\sum_{k=0}^{n-1} M_{x+k} - n \cdot M_{x+n}}{D_x}$$

$$\text{II} \quad (IA)_{x:\overline{n}|}^1 = \frac{\sum_{k=0}^{n-1} (k+1) \cdot C_{x+k}}{D_x}$$

$$\text{III} \quad (IA)_{x:\overline{n}|}^1 = (IA)_x - \frac{D_{x+n}}{D_x} \cdot (IA)_{x+n}$$

A I and II only.

B II and III only.

C I only.

D III only.

[2]

5 A life aged 50 effects a with profit whole life assurance with sum assured of £1,000 plus attaching bonuses, payable immediately on death. Assuming allowance for compound bonuses of 1.9231% per annum, vesting at the start of each policy year, which of the following gives the closest approximation to the single premium payable?

Basis: Mortality A1967–70 (ultimate)

Interest 6% per annum

Expenses none

A £384.50

B £388.40

C £392.11

D £395.87

[3]

6 Which of the following is equal to the expression:

$$\int_x^w {}_{s-x}p_x \mu_s ds + \exp\left(-\int_x^w \mu_s ds\right) \quad \text{where } x > 0 \text{ and } w > x$$

A ${}_s q_x$

B ${}_s p_x$

C 1

D 2

[2]

- 7 A one year term assurance is issued to a life aged 60 for a sum assured of £50,000 payable at the end of the policy year of death. Which of the following gives the standard deviation of the present value of the term assurance, using A1967–70 (select) mortality and 8% per annum interest?
- A 3,403
 B 3,777
 C 4,235
 D 5,522 [3]

PART TWO

- 8 Let W be the random variable representing the present value of an annuity of £100 per annum paid continuously to a life now aged exactly x until death. Find an expression for $\text{Var}(W)$ in terms of whole of life assurance functions. [4]

- 9 (i) Show that $({}_tV_x + P_x)(1 + i) = (p_{x+t})({}_{t+1}V_x) + q_{x+t}$ [3]
- (ii) On 1 January 1977 a life insurance company issued a number of whole life assurances to lives then aged exactly 30 subject to annual net premiums of £156.23 per policy. Each policy has a death benefit of £10,000 payable at the end of the year of death. The premium basis was such that $p_{50} = 0.9862$ and interest was 3½% per annum.

On 31 December 1996 there were 130 such policies still in force. At that time, the policies had a total policy value of £338,325 on the net premium basis given above. There were 3 deaths during 1997, claims being paid at the end of the year. Calculate the mortality profit or loss for 1997. [5]

[Total 8]

- 10 In a certain survival model,

$${}_t p_{40} = \frac{6000 - 70t - t^2}{6000} \quad 0 < t < 50$$

Calculate the following:

- (i) q_{55} [3]
- (ii) μ_{60} [3]
- (iii) $\overset{\circ}{e}_{40:\overline{20}|}$ [2]

[Total 8]

11 (i) Describe, in words, the difference between e_x and e_x . [2]

(ii) Given:

$$e_x = \sum_{k=1}^{\infty} {}_k p_x$$

Show that:

$$e_x = p_x(1 + e_{x+1}) \quad [2]$$

(iii) Given the following values of e_x , calculate $P_{70:\overline{4}|}$ using 6% per annum interest.

Age x	e_x
70	12.2
71	11.6
72	11.1
73	10.6

[5]
[Total 9]

12 A man aged 45 exact wishes to effect a 20 year endowment assurance providing a level sum assured of £500,000 payable on maturity or at the end of the year of earlier death. Level premiums are to be payable monthly in advance until maturity, or until earlier death.

The man approaches two life insurance companies, A and B, for premium quotations. After medical examinations, the companies offer the following terms to him:

- (a) Company A offers the policy at a rate calculated assuming a constant addition of 5 years to his age for the term of the policy.
- (b) Company B offers the policy at a rate calculated assuming a constant addition of 0.009569 to the force of mortality over the term of the policy.

Both companies assume that normal mortality is represented by A1967-70 (ultimate) and use an interest rate of 4% per annum.

- (i) Ignoring expenses, calculate the premium which each company would charge. [9]
- (ii) As an alternative to (a), Company A offers to revise their quotation on the following basis:

The contract is to be offered at the normal rate of monthly premium but the death benefit is to be subject to a level debt throughout the term.

Calculate the debt.

[5]
[Total 14]

PART THREE

- 13** A life insurance company is to issue the following special 3-year temporary annuity policy to a life aged 75 exact.

Single Premium: £5,300

Annuity Payment: £1,800 payable annually in arrears for 3 years or until earlier death.

Death Benefit: The amount of the single premium less all annuity payments made up to the date of death, payable at the end of the year of death.

- (i) Calculate the profit signature using the following basis:

Mortality: a(55) Males (select)

Interest: 8% per annum.

Initial Expenses: £150

Regular Expenses: 1% of each annuity payment.

Reserves: £4,000 per policy in force at the end of the first year
£2,000 per policy in force at the end of the second year.

[9]

- (ii) Calculate the following amounts assuming a risk discount rate of 12% per annum.

(a) The net present value

(b) The break-even duration (i.e. the discounted payback period). [4]

- (iii) Without carrying out any further calculations, state with brief reasons what the effect on the net present value would be in each of the following cases:

(a) The office uses a higher interest rate than 8% per annum

(b) The office uses a higher risk discount rate than 12% per annum.

[4]

[Total 17]

- 14** A life insurance company issues a 20 year with profit endowment assurance to a life aged 40 exact.

The basic sum assured of £20,000, together with declared bonuses, is payable at maturity or immediately on death, if earlier. Bonuses vest at the end of each policy year.

Premiums are payable annually in advance throughout the term of the policy.

- (i) Show that the annual premium is £1,178.18.

Basis: Mortality:	A1967-70 (select)
Interest:	4% per annum
Reversionary Bonus Rate:	3 % per annum simple
Initial Expenses:	£100 plus 50% of the first premium
Renewal Expenses:	5% of each premium, excluding the first

[9]

- (ii) After 15 years, just before the 16th premium is due, the insurance company agrees to an alteration of the policy as follows:

- the original term is extended by 5 years
- the policy is converted into a without-profit contract with immediate effect
- the premium is to become £400 per annum with immediate effect.

The company has declared simple reversionary bonuses of 3.5% per annum over the last 15 years. Using a valuation basis which is consistent with the premium basis described in (i) above, calculate the revised level of sum assured.

[12]

- (iii) Explain why the above alteration may present an additional financial risk to the company.

[2]

[Total 23]