

# EXAMINATIONS

April 1999

## Subject D — Actuarial Mathematics

### *Paper One*

*Time allowed: Three hours*

#### **INSTRUCTIONS TO THE CANDIDATE**

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 14 questions.*

***Graph paper is not required for this paper.***

#### **AT THE END OF THE EXAMINATION**

*Hand in BOTH your answer booklet and this question paper.*

<p><i>In addition to this paper you should have available Actuarial Tables and an electronic calculator.</i></p>
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## PART ONE

For questions 1–7 indicate in your answer booklet which one of the answers A, B, C or D is correct.

- 1 Which of the following is equal to  ${}_3q_{2:\overline{3}}^2$ ?
- A  $1 - {}_3q_{2:\overline{3}}^1$
- B  ${}_3q_3 - {}_3q_{2:\overline{3}}^1$
- C  ${}_3q_3 - {}_3q_{2:\overline{3}}^1$
- D  ${}_3q_2 - {}_3q_{2:\overline{3}}^1$  [2]

- 2 Two lives, (x) and (y), are assumed to be independent with respect to mortality and are each assumed to be subject to a constant force of mortality of 0.015.

Which of the following is the probability that (x) dies more than 5 years after (y)?

- A 0.0750
- B 0.4291
- C 0.4304
- D 0.4639 [3]

- 3  $T_x$  and  $T_y$  are random variables denoting the complete future lifetimes of (x) and (y) respectively. The following statements have been made:

I  $E(\overline{a}_{\max(T_x, T_y)}) = \overline{a}_x + \overline{a}_y - \overline{a}_{xy}$

II  $P(\max(T_x, T_y) < t) = 1 - {}_t p_{xy}$

III  $E(v^{\min(T_x, T_y)}) = \overline{A}_{\overline{xy}}$

- A I and II only are correct
- B II and III only are correct
- C I only is correct
- D III only is correct [3]

- 4 Under the Manchester Unity model of sickness, you are given the following values:

$$\int_0^1 {}_t p_x \bar{z}_{x+t} dt = 0.1$$

$$\int_0^1 v^t {}_t p_x \bar{z}_{x+t} dt = 0.08$$

Which of the following is the value of  $s_x$ ?

- A 0.08  
 B 0.1  
 C 4.1744  
 D 5.218
- [2]

- 5 A life insurance company issues a number of 3-year unit-linked policies to lives each aged 50 exact.  $(PRO)_t$  is the expected profit at the end of policy year  $t$  per policy in force at the start of the year.  $(SCF)_t$  is the year-end sterling fund cash flow per policy in force at the start of year  $t$ . The values of  $(SCF)_t$  are as follows (in £'s):

year( $t$ )	1	2	3
$(SCF)_t$	+100	-40	-40

$(SCF)_t$  equals  $(PRO)_t$  since there is no allowance for reserves. Sterling reserves are to be set up at each year-end for each policy then in force to zeroise future negative cash flows. Assuming that the sterling fund earns interest at 5% per annum and that mortality is that of A1967–70 Ultimate and that sterling reserves are set up using these assumptions, which of the following gives the adjusted value of  $(PRO)_1$ ?

- A +25.07  
 B +25.62  
 C +26.17  
 D +26.24
- [3]

- 6** In a stationary population in which the only mode of decrement is death, which of the following gives a correct formula for the average age at death of individuals who die between the ages of 30 and 35?

A  $30 + \frac{T_{30} - T_{35} - 5l_{35}}{l_{30} - l_{35}}$

B  $30 + \frac{T_{30} - T_{35} - 5(l_{30} - l_{35})}{l_{30} - l_{35}}$

C  $30 + \frac{T_{30} - T_{35} - 5l_{30}}{l_{30} - l_{35}}$

D  $30 + \frac{T_{30} - T_{35}}{l_{30} - l_{35}}$  [3]

- 7** On the basis of A1967–70 mortality and 4% per annum interest, which of the following gives the value of  $a_{\overline{10}|50}$ ?

A 6.823

B 7.175

C 7.796

D 7.828

[2]

## PART TWO

- 8 The random variable  $T_{xy}$  represents the time to failure of the joint-life status  $(x, y)$ .  $(x)$  is subject to a constant force of mortality of 0.01 and  $(y)$  is subject to a constant force of mortality of 0.02.  $(x)$  and  $(y)$  are independent with respect to mortality. Calculate the value of  $E[T_{xy}]$ . [5]

- 9 Calculate the annual premium for a policy which provides the following benefit in respect of two lives each aged exactly 55 at the issue date:

a lump sum of £50,000 payable immediately on the death of the second of the lives to die.

Premiums are payable annually in advance while at least one of the lives is alive.

Basis: mortality A1967–70 Ultimate  
interest 4% per annum  
expenses ignored [7]

- 10 (i) A Friendly Society issues a policy to a life aged exactly 50. The policy provides the following sickness benefit:

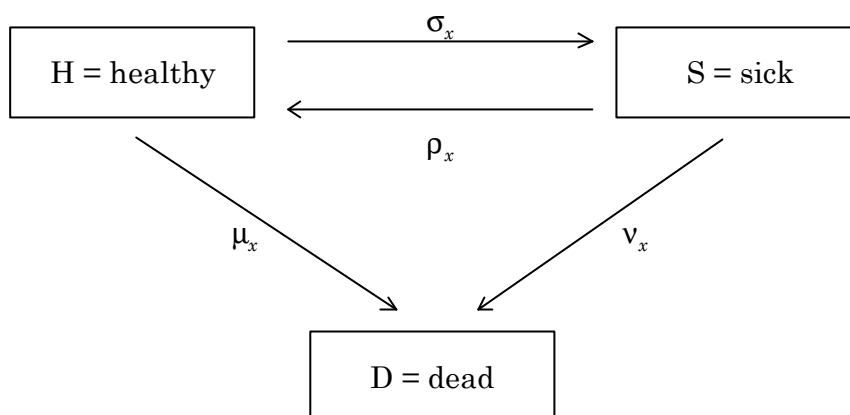
£10 per week for the first three months of sickness, reducing to £5 per week thereafter. Sickness benefit is not payable after age 60. There is no waiting period.

Premiums under the policy are payable weekly in advance for the term of 10 years or until earlier death. Any premiums falling due during periods of sickness are waived.

Calculate the weekly premium.

Basis: mortality English Life Table No. 12 – Males  
sickness Manchester Unity Sickness Experience  
1893/97 Occupation Group AHJ  
interest 4% per annum  
expenses 5% of each premium  
(Expenses continue even when premiums are waived). [6]

- (ii) The Friendly Society considers using the following multiple-state model, in which the forces of transition depend on age, to describe the mortality and sickness of its members.



Assume that a constant rate of sickness benefit is payable during sickness and that the benefit ceases to be payable after age 60.

If  ${}_t p_x^{AB}$  is defined as the probability that a life aged  $x$  who is in state  $A$  (where  $A$  may be  $H$ ,  $S$  or  $D$ ) will be in state  $B$  at age  $x + t$ , (where  $t \geq 0$  and  $B$  may be  $H$ ,  $S$  or  $D$ ), derive a differential equation for the probability that a member who is healthy at exact age 50 is sick at age  $50 + t$  ( $t \geq 0$ ).

[7]

[Total 13]

- 11** A life insurance company issues 2-year without-profit policies to the employees of a certain company, providing the following benefits:

- on death during the 2 years, a sum of £10,000;
- on withdrawal from the company within 2 years, a return of premiums paid without interest;
- on survival as an employee of the company to the end of 2 years, the sum of £500.

All benefits are paid at maturity or at the end of the year of claim, if earlier. There are no decrements from service other than those mentioned.

Calculate the level annual premium for the above policy issued to an employee aged exactly 50. Premiums are payable annually in advance for 2 years so long as the employee is in the service of the company at the dates at which premiums are due.

Basis:

mortality	the independent rate of mortality is that of A1967–70 Ultimate
withdrawal	the independent rate of withdrawal is 0.05 at ages 50 and 51
interest	5% per annum
expenses	ignored

State clearly any assumptions that you make.

[11]

## PART THREE

- 12** 1,000 new staff members are recruited by a company each year at exact age 18. Recruitment is carried out uniformly over the year. New staff members are tested at the end of each of the first two years of service; those failing a test leave service immediately. The failure rates are 15% of those tested at the end of one year of service and 10% of those tested at the end of two years of service. Staff in service during the first two years are classified as probationary. Staff members become permanent members of staff immediately on successful completion of the test at the end of the second year of service.

60% of the staff who attain their 55th birthday retire immediately and all those who attain their 60th birthday retire immediately.

Staff do not leave for any reason other than failing a test, dying or retiring at age 55 or 60.

Mortality follows that of the English Life Table No. 12 — Males and the staff numbers have now reached a stationary condition.

- (i) Every 1 January the company makes a payment of £500 to each permanent staff member. Calculate the total annual amount of these payments. [3]
  - (ii) The company makes a payment of £1,000 to each staff member who completes 20 years of service and £2,000 to each member who completes 40 years of service. The payments are made immediately on completion of the relevant periods of service, and the periods of service include the probationary period. Calculate the total annual amount of these payments. [5]
  - (iii) The company makes a lump sum payment of £1,000 on the death of a probationary member and £10,000 on the death of a permanent member, paid immediately on death. Calculate the total annual amount of these payments. [6]
- [Total 14]

- 13** A life insurance company issues a 3-year unit-linked endowment assurance contract to a person aged exactly 62 under which level annual premiums of £2,000 are payable in advance throughout the term of the policy or until earlier death. 95% of the first year's premium is invested in capital units, at the offer price, and 95% of the subsequent years' premiums are invested in accumulation units, at the offer price.

There is a bid-offer spread in unit values, with the bid price being 95% of the offer price.

On the death of the policyholder during the term of the policy, there is a benefit payable at the end of the year of death of £8,000 or the bid value of the units allocated to the policy, if greater. On maturity, the bid value of the units is paid. Withdrawals are not permitted.

Annual management charges of 4% of the bid value of the policyholder's capital units and 0.75% of the bid value of the policyholder's accumulation units are deducted at the end of each year, immediately before payment of any death or maturity claim.

The life insurance company uses the following assumptions in its profit test of this contract:

mortality		A1967–70 Ultimate
expenses	initial	20% of the first premium
	renewal	3% of the second and third premiums
unit fund growth rate		9% per annum
sterling fund interest rate		7% per annum

- (i) Assuming that the life insurance company holds no sterling reserves and holds unit reserves equal to the full bid value of both types of unit, calculate the profit margin in respect of the policy, using a risk discount rate of 10% per annum. [12]
- (ii) Assuming that the life insurance company holds a proportion, calculated as  $A_{\overline{62+t:3-t}|}$  (assuming mortality as above and interest of 3% per annum), of the full number of capital units at time  $t$  ( $t = 0, 1, 2$ ), calculate the contribution to profit from the management charge of the capital unit fund at the end of the first year. [4]  
[Total 16]

**14** A pension scheme provides the following benefits:

- (i) on retirement, a pension commencing immediately at the annual rate of  $\frac{1}{45}$  of Final Pensionable Salary for each year of service, with fractions of a year to count proportionately. Final Pensionable Salary is defined as the member's average annual salary received during the last three years of service;
- (ii) on death in service, a lump sum of 4 times the annual rate of salary at the date of death.

Members may retire once they have attained age 60; members must retire on attaining age 65.

Data are available in respect of each member's salary in the year to the valuation date.

Develop commutation functions you would use for valuing the death benefits and the past and future service pension benefits, for a member aged 55 exactly, with exactly 5 years past service. State clearly any assumptions that you make and define carefully all the symbols that you use. [16]