

**ADVANCED CERTIFICATE IN DERIVATIVES:
THE MATHEMATICS, PRINCIPLES AND PRACTICE**

Examination Paper

April 1998

Paper Two

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 5 questions leaving sufficient space between each answer.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formula sheet and an electronic calculator.

1 Discuss the issues that an investment bank is likely to consider before approving a new derivative product for use by the firm and suggest how it might draw the line between new and variations on existing products. [20]

2 Consider a German government bond yield curve, let $P(t)$ be the price and $y(t)$ the yield of a zero coupon bond of term t , where $t = 1, 2, 3 \dots$ years. Assume intermediate points along the curve are not required.

(i) Derive formulae for $f(t)$, the one-year forward rate from $t - 1$ to t , and $g(t)$, the par coupon yield at time t , in terms of $P(t)$ for $t = 1, 2, 3, \dots$. The par coupon yield is the annual coupon on a bond priced at par (DM100 per DM100 nominal). [3]

(ii) (a) Calculate the values of the forward rates and par yields given the following zero coupon German government bond yields:

| <i>Maturity (years)</i> | <i>Spot yield</i> |
|-------------------------|-------------------|
| 1 | 4.20% |
| 2 | 4.70% |
| 3 | 4.90% |
| 4 | 4.95% |
| 5 | 4.90% |

(b) Sketch the zero coupon, par and forward rate yield curves on a single graph and explain the relationship between the shape of the par and zero coupon yield curves. [11]

(ii) Use your curve to value a DM 100 million 5-year interest rate swap, paying 6% fixed rate annually and receiving floating rate annually and suggest reasons why the market value of such a swap is likely to be different?

[7]

[Total 21]

3 Consider a Vasicek process with parameters $a = 0.2$, $b = 0.1$, $\sigma = 0.02$.

The binomial tree representation of the stochastic process followed by r has initial up and down steps $r_0 + u$ and $r_0 - u$ respectively, where r_0 is the initial interest rate.

(i) Using a time step $\Delta t = 0.5$ and assuming that $u = \sigma\sqrt{\Delta t} = 0.014$, find the risk neutral probabilities for the up and down steps. [5]

(ii) Construct the binomial tree using $\Delta t = 0.5$ and $r_0 = 7\%$ to value a 2-year annual 6% fixed coupon bond. [10]

(iii) Derive the same values using the Vasicek analytical formula for a zero coupon bond (given below) and compare your results. [4]

You may assume that $b - \frac{\sigma^2}{2a^2} = 0.095$.

The formula for a zero coupon bond of maturity t in Vasicek is:

$$P(t) = e^{(\alpha - \beta r)}$$

where

$$\beta = \frac{1 - e^{-at}}{a}$$

$$\alpha = (\beta - t) \left(b - \frac{\sigma^2}{2a^2} \right) - \frac{\sigma^2 \beta^2}{4a}$$

This is a decimal formula in which par = 1 rather than 100.

(iv) Discuss the problems you would encounter:

- (a) extending the binomial tree with the current time step and
- (b) increasing the size of the time step

[3]

[Total 22]

- 4** All institutions involved in derivatives activities face two types of liquidity risk: market liquidity risk and cash flow/funding liquidity risk.

Discuss the nature of these two types of liquidity risk making particular reference under the market liquidity risk heading to over-the-counter dealer markets and derivative instruments.

[21]

- 5**
- (i) Discuss the way in which transaction costs, lack of liquidity and lack of depth in securities markets and capital gains taxes reduce the performance of active managers who manage funds across international equity markets. [9]
 - (ii) Explain how futures contracts might be used to reduce these drags on portfolio performance when making short term switches between international equity markets. [7]

[Total 16]