EXAMINATION

16 September 2008 (am)

Subject CT6 — Statistical Methods
Core Technical

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Mark allocations are shown in brackets.*

4. *Attempt all 11 questions, beginning your answer to each question on a separate sheet.*

5. *Candidates should show calculations where this is appropriate.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator from the approved list.

1 Claim amounts on a portfolio of insurance policies have an unknown mean μ. Prior beliefs about μ are described by a distribution with mean μ_0 and variance σ_0^2. Data are collected from n claims with mean claim amount $\bar{x}$ and variance s^2. A credibility estimate of μ is to be made, of the form

$$Z\bar{x} + (1 - Z)\mu_0 .$$

Suggestions for the choice of Z are:

A $\quad \dfrac{n\sigma_0^2}{n\sigma_0^2 + s^2}$

B $\quad \dfrac{n\sigma_0^2}{n\sigma_0^2 + n}$

C $\quad \dfrac{\sigma_0^2}{n + \sigma_0^2}$

Explain whether each suggestion is an appropriate choice for Z. [4]

2 Write down the general statistical model for the run-off triangle claim data and explain the terms used. [5]

3 An insurer is considering whether to outsource its advertising. If it decides not to outsource, it expects to spend £1,400,000 next year on its advertising, and believes that this would result in a portfolio of 100,000 policies. It has received quotations from two different companies for outsourcing its advertising.

Company A would cost £2,100,000 per year, and believes that this would result in the business expanding to 125,000 policies.

Company B would cost £3,000,000 per year, and believes that this would expand the business to 140,000 policies.

At present, each policy returns a profit to the company of £30 per year; but this is not guaranteed in the future. The company has assessed that it will stay at this level with probability 0.6, but could reduce to £20 per year with probability 0.25, or increase to £40 per year with probability 0.15.

(i) Explain which of the three options can be immediately discarded. [1]

(ii) Determine the Bayes solution to the problem of maximising the profit to the company over the coming year. [5]
[Total 6]

4 An insurance company provides warranties for a certain electrical gadget. At the start of 2006 there were 4,500 gadgets under warranty, each of which has a probability q of suffering complete failure in 2006 (independently between gadgets). The prior distribution of q is beta with mean 0.015 and standard deviation 0.005. Given that 58 gadgets suffer a complete failure in 2006, determine the posterior distribution of q. [6]

5 The table below shows the cumulative values (in units of £1,000) of incurred claims on a portfolio of an insurance company:

	Development year		
Underwriting year	*1*	*2*	*3*
2005	3,541	7,111	9,501
2006	2,949	6,850	
2007	3,894		

The estimated loss ratio for 2006 and 2007 is 87% and the respective premium income (also in units of £1,000) is:

	Premium income
2005	11,041
2006	11,314
2007	12,549

Given that the total of claims paid to date is £20,103,000, calculate the reserve for this portfolio using the Bornhuetter-Ferguson method. [7]

6 Consider the ARCH(1) process

$$X_t = \mu + e_t\sqrt{\alpha_0 + \alpha_1(X_{t-1} - \mu)^2}$$

where e_t are independent normal random variables with variance 1 and mean 0. Show that, for $s = 1, 2, \ldots, t-1$, X_t and X_{t-s} are:

(i) uncorrelated. [5]

(ii) not independent. [3]

[Total 8]

7 (i) Let $U_1, U_2, \ldots, U_n$ be independent random numbers generated from a U(0,1) distribution. Write down the Monte Carlo estimator, $\hat{\theta}$, for the integral

$$\theta = \int_0^1 (e^x - 1)dx.$$ [1]

(ii) Determine the variance of the estimator $\hat{\theta}$ in (i). [4]

(iii) Calculate the smallest value of n for which the estimator $\hat{\theta}$ has absolute error less than 0.1 with probability 90%. [4]

[Total 9]

8 An insurer has issued two five-year term assurance policies to two individuals involved in a dangerous sport. Premiums are payable annually in advance, and claims are paid at the end of the year of death.

Individual	*Annual Premium*	*Sum Assured*	*Annual Prob (death)*
A	100	1,700	0.05
B	50	400	0.1

Assume that the probability of death is constant over each of the five years of the policy. Suppose that the insurer has an initial surplus of U.

(i) Define what is meant by $\psi(U)$ and $\psi(U,t)$. [2]

(ii) Assuming U = 1,000

(a) Determine the distribution of $S(1)$, the surplus at the end of the first year, and hence calculate $\psi(U,1)$.

(b) Determine the possible values of $S(2)$ and hence calculate $\psi(U,2)$.

[8]

[Total 10]

9 A motor insurance company applies a NCD scale policy of discount levels

Level 0	0%
Level 1	20%
Level 2	50%

Following a claim-free year, the policyholder moves to the next higher level (or remains at Level 2). If one or more claims are made, the policyholder moves to the next lower level (or remains at Level 0).

The probability of a policyholder not making a claim in a policy year is $1 - p$.

The annual premium at Level 0 is £600.

(i) Derive, in terms of p, the expected proportions of policyholders at each discount level assuming that the system is in equilibrium . [7]

(ii) Calculate the average premium paid in the stable state for the particular values $p = 0.1$ and $p = 0.3$ and comment on the results. [3]
[Total 10]

10 From a sample of 50 consecutive observations from a stationary process, the table below gives values for the sample autocorrelation function (ACF) and the sample partial autocorrelation function (PACF):

Lag	*ACF*	*PACF*
1	0.854	0.854
2	0.820	0.371
3	0.762	0.085

The sample variance of the observations is 1.253.

(i) Suggest an appropriate model, based on this information, giving your reasoning. [2]

(ii) Consider the AR(1) model

$$Y_t = a_1 Y_{t-1} + e_t,$$

where e_t is a white noise error term with mean zero and variance σ^2.

Calculate method of moments (Yule-Walker) estimates for the parameters of a_1 and σ^2 on the basis of the observed sample. [4]

(iii) Consider the AR(2) model

$$Y_t = a_1 Y_{t-1} + a_2 Y_{t-2} + e_t,$$

where e_t is a white noise error term with mean zero and variance σ^2.

Calculate method of moments (Yule-Walker) estimates for the parameters of a_1 , a_2 and σ^2 on the basis of the observed sample. [7]

(iv) List two statistical tests that you should apply to the residuals after fitting a model to time series data. [2]
[Total 15]

11 Losses on a portfolio of insurance policies in 2006 are assumed to have an exponential distribution with parameter λ. In 2007 loss amounts have increased by a factor k (so that a loss incurred in 2007 is k times an equivalent loss incurred in 2006).

(i) Show that the distribution of loss amounts in 2007 is also exponential and determine the parameter of the distribution. [3]

Over the calendar years 2006 and 2007 the insurer had in place an individual excess-of-loss reinsurance arrangement with a retention of M. Claims paid by the insurer were:

2006: 4 amounts of M and 10 claims under M for a total of 13,500.
2007: 6 amounts of M and 12 claims under M for a total of 17,000.

(ii) Show that the maximum likelihood estimate of λ is:

$$\hat{\lambda} = \frac{22}{13{,}500 + \frac{17{,}000}{k} + 4M + \frac{6M}{k}}$$

[7]

(iii) The insurer is negotiating a new reinsurance arrangement for 2008. The retention was set at 1600 when the current arrangement was put in place in 2006. Loss inflation between 2006 and 2007 was 10% (i.e. $k = 1.1$) and further loss inflation of 5% is expected between 2007 and 2008.

(a) Use this information to calculate $\hat{\lambda}$.

(b) The insurer wishes to set the retention M' for 2008 such that the expected (net of re-insurance) payment per claim for 2008 is the same as the expected payment per claim for 2006. Calculate the value of M', using your estimate of λ from (iii)(a). [10]

[Total 20]

END OF PAPER