EXAMINATIONS

September 1999

Subject D — Actuarial Mathematics

Paper One

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
2. *Begin your answers to Parts One, Two and Three on a separate sheet.*
3. *Mark allocations are shown in brackets.*
4. *Attempt all 15 questions.*

Graph paper is not required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.

PART ONE

For questions 1–8 indicate in your answer booklet which one of the answers A, B, C or D is correct.

1 A life insurance company holds Zillmerised net premium reserves for whole life assurance policies, where the sum assured is 1 and the initial expense is I. Premiums are payable annually in advance throughout life. The reserve after t years, for a policy sold to a life aged x at outset, is:

I $\quad {}_tV_x + I(1 - {}_tV_x)$

II $\quad (1 + I)\,{}_tV_x - I$

III $\quad {}_tV_x - I\,\dfrac{\ddot{a}_{x+t}}{\ddot{a}_x}$

A I and II only are correct
B II and III only are correct
C I only is correct
D III only is correct [2]

2 Which one of the following correctly defines the sickness function s_x?

A $\quad 52.18 \int_0^1 \dfrac{l_{x+t}\, z_{x+t}\, dt}{l_x}$

B $\quad 52.18 \int_0^1 \dfrac{l_{x+t}\, \bar{z}_{x+t}\, dt}{l_x}$

C $\quad 52.18\, \dfrac{\int_0^1 l_{x+t}\, z_{x+t}\, dt}{\int_0^1 l_{x+t}\, dt}$

D $\quad 52.18\, \dfrac{\int_0^1 l_{x+t}\, \bar{z}_{x+t}\, dt}{\int_0^1 l_{x+t}\, dt}$ [2]

3 On 1 October 1998 a pension scheme member was aged exactly 43 and had earned £40,000 over the preceding 12 months.

The salary scale s_x is defined such that for an individual aged exactly x on 1 January and for any integer $t > 0$:

$$\frac{s_{x+t}}{s_x} = \frac{\text{Expected earnings between ages } x+t \text{ and } x+t+1}{\text{Expected earnings between ages } x \text{ and } x+1}$$

Final salary is defined as the earnings received in the 12 months immediately prior to retirement. Salaries are increased annually on 1 January.

Which one of the following is the expected final salary for this member, given that he intends retiring on 31 December following his 62nd birthday?

A $\quad 40{,}000 \, \frac{s_{61.25}}{s_{42}}$

B $\quad 40{,}000 \, \frac{s_{61}}{s_{42}}$

C $\quad 40{,}000 \, \frac{s_{61}}{(.25)s_{41.25} + (.75)s_{42.25}}$

D $\quad 40{,}000 \, \frac{s_{61.25}}{(.25)s_{41.25} + (.75)s_{42.25}}$ [2]

4 $\bar{a}_{x|y}$ is equal to

I $\quad \int_0^\infty v^t \, (1 - {}_tp_x) \, {}_tp_y \, dt$

II $\quad \int_0^\infty v^t \, {}_tp_{xy} \, \mu_{x+t} \, \bar{a}_{y+t} \, dt$

III $\quad \bar{a}_x - \bar{a}_{xy}$

A I and II only are correct
B II and III only are correct
C I only is correct
D III only is correct [2]

5 A particular 10-year unit-linked insurance contract has the following profit signature before reserves are set up:

$$(-1, -1, +1, +1, -1, 0, -1, +1, -1, +1)$$

If positive sterling reserves are set up to zeroise negative cashflows then the profit signature, ignoring interest, becomes:

A (–1, 0, 0, 0, 0, 0, 0, 0, 0, 0)
B (–2, 0, 0, 0, 0, 0, 0, +1, 0, 0)
C (–2, 0, 0, 0, 0, 0, 0, 0, 0, +1)
D (–1, –1, 0, 0, 0, 0, 0, 0, 0, +1) [2]

6 Let T_x and T_y be random variables representing the exact future lifetimes of two lives aged x and y respectively, and assume that T_x and T_y are independent.

$\mu_{\overline{x+t:y+t}}$ is given by

A $\lim_{dt \to 0} \left[\frac{1}{dt} \times \text{Prob}\{T_x \le t + dt \text{ OR } T_y \le t + dt \,|\, T_x > t \text{ AND } T_y > t\} \right]$

B $\lim_{dt \to 0} \left[\frac{1}{dt} \times \text{Prob}\{T_x \le t + dt \text{ AND } T_y \le t + dt \,|\, T_x > t \text{ AND } T_y > t\} \right]$

C $\lim_{dt \to 0} \left[\frac{1}{dt} \times \text{Prob}\{T_x \le t + dt \text{ OR } T_y \le t + dt \,|\, T_x > t \text{ OR } T_y > t\} \right]$

D $\lim_{dt \to 0} \left[\frac{1}{dt} \times \text{Prob}\{T_x \le t + dt \text{ AND } T_y \le t + dt \,|\, T_x > t \text{ OR } T_y > t\} \right]$ [3]

7 The population in a certain country is stationary, with l_0 births per annum, and is subject to the mortality represented by the life table l_x for all x from 0 to w. All people reaching age 65 are entitled to a pension payable annually in advance from age 65, which is guaranteed to be paid for 5 years certain and life thereafter. The annual amount of the pension payable to an individual is P. The total payments made in a year are:

A $P\, l_{65}[5 + {}_5p_{65}\, e_{70}]$

B $P\, l_{65}[5 + {}_5p_{65}\, \mathring{e}_{70}]$

C $P\, l_{65}[5 + {}_4p_{65}\, e_{69}]$

D $P\, l_{65}[5 + {}_4p_{65}(1 + e_{70})]$ [2]

8 A member of a pension scheme aged x exact expects to earn £30,000 over the next 12 months. He contributes 5% of salary per annum. Salary growth and decrements follow the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations.

Which of the following is an expression for the expected present value of his future contributions?

A $1{,}500\ \dfrac{\overline{N}_x}{D_x}$

B $1{,}500\ \dfrac{\overline{N}_x}{\overline{D}_x}$

C $1{,}500\ \dfrac{{}^s\overline{N}_x}{{}^sD_x}$

D $1{,}500\ \dfrac{{}^s\overline{N}_x}{{}^s\overline{D}_x}$ [2]

PART TWO

9 An insurance company issues a 3-year term assurance to a married couple, both aged exactly 60, under which the payment at the end of the year of the first death is £200,000. Premiums of £7,000 are payable annually in advance for 3 years or until the death benefit is payable, if sooner.

The office is considering the profit P under the contract, where P is the random variable:

P = present value of premium income – present value of benefit outgo

(i) Using English Life Table Number 12 — Males for the male and A1967–70 Ultimate rated down 5 years for the female, and interest of 8% per annum:

(a) calculate all four possible values which can be taken by P.

(b) calculate the mean and standard deviation of P. [8]

(ii) Comment on how you would expect the standard deviation of P to change if the assurance in question were a 3-year endowment assurance for the same sum assured. Assume that the premium charged is such that the mean of P is unchanged. You are not required to perform any calculations. [2]

[Total 10]

10 A member of a pension scheme who joined at age 45 exact is now aged 55 exact. While in the scheme he has earned total past salaries of £180,000 to date, and expects to earn £30,000 during the coming year.

Final salary is defined as the average earnings over the three years immediately prior to retirement.

Assuming all decrements and salary growth follow the Pension Fund Tables in the Formulae and Tables for Actuarial Examinations:

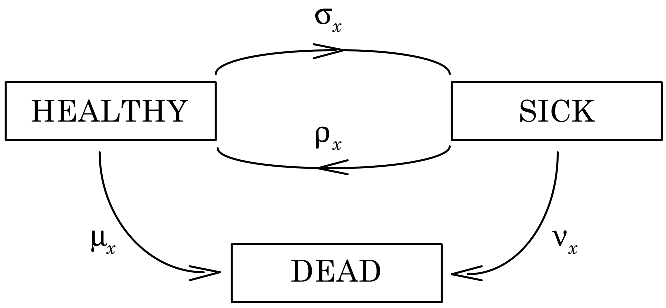
(i) calculate the present value of a pension based on 1/80th of career average salary for each year of service (counting fractions of a year). You should consider past and future service. [4]

(ii) calculate the present value of a pension based on 1/80th of final salary for each year of service (counting fractions of a year). You should consider past and future service. [4]

(iii) comment briefly on the relative values of the results in parts (i) and (ii). [2]

[Total 10]

11 An insurance company prices PHI policies using the following three-state model in which the forces of transition depend on age:



Level premiums are payable continuously. There is no death benefit, nor is there a waiting or a deferred period. Reserves are always positive under the normal premium basis.

(i) State, with brief reasons, what effect the following changes will have on the premium — whether the premium will certainly increase or certainly decrease or whether the direction of change, if any, is uncertain.

(a) An increase in the death rate from the healthy state and an increase in the recovery rate from the sick state.

(b) A fall in the death rate from the sick state and a fall in the recovery rate from the sick state.

(c) An increase in the death rate from the sick state and a decrease in the sickness inception rate.

(d) A fall in the premium rate of interest and an increase in the recovery rate from the sick state. [5]

(ii) Let $\mu_x = .01$ and $\sigma_x = .05$ for all x, $40 \le x \le 60$.

Calculate the occupancy probability ${}_5 p_{45}^{\overline{HH}}$. [2]

[Total 7]

12 (a) Express ${}_n q_{xy}^{\;2}$ in terms of single life and/or first death probability functions.

(b) Hence or otherwise, evaluate ${}_{20} q_{40:50}^{\;\;\;\;2}$ where $\mu_x = \dfrac{1}{80 - x}$ for $0 \le x < 80$.

[8]

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PART THREE

13 A man aged 40 exact purchases a 25 year combined endowment assurance and sickness policy under which the following benefits are payable:

- on maturity at age 65 or immediately on earlier death a sum assured of £40,000;
- on sickness a benefit of £200 per week for the duration of sickness. The benefit ceases at age 65. There is no waiting or deferred period.

Premiums, which are waived during all periods of sickness, are payable weekly in advance until age 65 or earlier death.

(i) Show that the weekly premium is £34.37.

Basis:	Interest:	4% per annum
	Mortality:	English Life Table Number 12 — Males
	Sickness:	Manchester Unity Sickness Experience 1893–97 Occupation Group A, H, J

Initial expenses: £200

Renewal expenses: 3.5% of each premium, including the first. These are not incurred if the premium is waived. [7]

(ii) On the twentieth policy anniversary, the man wishes to alter the policy in the following manner. All sickness benefits (including the waiver of premium benefit) are to be removed. His wife, then aged 62 exact, is to be included on the policy which is to become a whole of life policy, with the death benefit to be payable immediately on the death of whichever of the lives dies second. The premium will remain unchanged in size but will now continue until the benefit is payable.

What is the maximum sum assured which the life insurance company can permit on the altered policy if it is not to incur an expected loss?

The office holds reserves for the original policy on the premium basis, incurs expenses of £200 in processing the alteration and uses the following basis for all calculations relating to the altered policy:

Basis:	Interest:	4% per annum
	Mortality:	a(55) ultimate (males or females as appropriate)
	Expenses:	4% of each premium

(You may assume all underwriting criteria are met.) [9]

[Total 16]

14 In a certain country the population has been stationary for many years, females being subject to A1967–70 Ultimate mortality and males subject to the same table, but with an addition to the age of 10 years. There are 100,000 female births each year.

On reaching their 20th birthday, all males are required to undertake military service for a period of 5 years, during which time they are subject to an extra force of mortality of 0.039221.

95% of males and 80% of females marry on their 30th birthdays and all others remain single throughout their lifetimes.

During military service, all soldiers receive a bonus from the State of £1,000 on each of their birthdays. This includes both the day of commencement and the day of completion of military service. At Christmas each year all married or widowed members of the population receive a bonus from the State of £500.

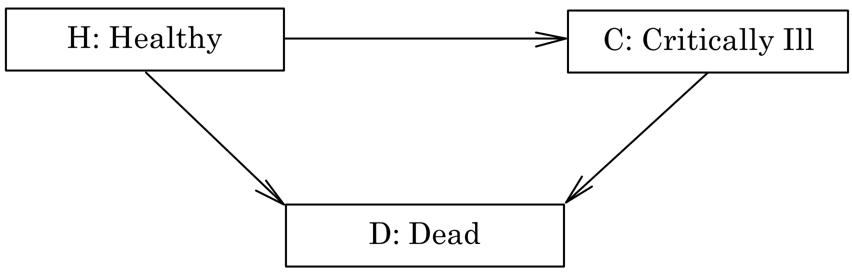
(i) What proportion of births is female? [7]

(Hint: The number of males marrying is equal to the number of females marrying.)

(ii) What is the annual cost of military bonuses? [4]

(iii) What is the annual cost of Christmas bonuses? [3]
[Total 14]

15 A life insurance company uses the following three-state model to calculate the annual premium for a 3-year critical illness policy issued to healthy policyholders aged exactly 57 at entry.



In return for premiums payable annually in advance for 3 years, the insurer will pay benefits of:

£50,000 if the policyholder dies from the healthy state;
£30,000 if the policyholder is diagnosed as having become critically ill;
£20,000 if the policyholder dies from the critically ill state.

All benefits are payable at the end of the relevant policy year.

Let S_t represent the state of the policyholder at age 57 + t, so that $S_0 = H$, and for $t = 1, 2, 3$, $S_t = H$, C or D. The transition probabilities are defined as follows:

$$\begin{aligned}
&\Pr(\text{critically ill at } t+1 \mid \text{healthy at } t) = 0.04 \\
&\Pr(\text{dead at } t+1 \mid \text{healthy at } t) = 0.02 \\
&\Pr(\text{dead at } t+1 \mid \text{critically ill at } t) = 0.20
\end{aligned}$$

(i) Calculate the probabilities of being in each state at times $t = 1, 2, 3$. [5]

(ii) Calculate the net present value of the expected profit on this contract by carrying out a profit test using the following assumptions:

Annual premiums:	£3,000
Interest on cash flows and reserves:	5% per annum
Risk discount rate:	10% per annum
Initial expenses:	£200 incurred on payment of the first premium
Initial commission:	40% of the first premium
Renewal expenses:	£40 at times $t = 1, 2$ whether healthy or not
Reserves	${}_0V = {}_3V = 0$ for $t = 1, 2$

$${}_tV = \begin{cases} 1 \times \text{annual premium if policyholder is healthy at time } t \\ 2 \times \text{annual premium if policyholder is critically ill at time } t \end{cases}$$

[13]
[Total 18]