Mortality and Deprivation

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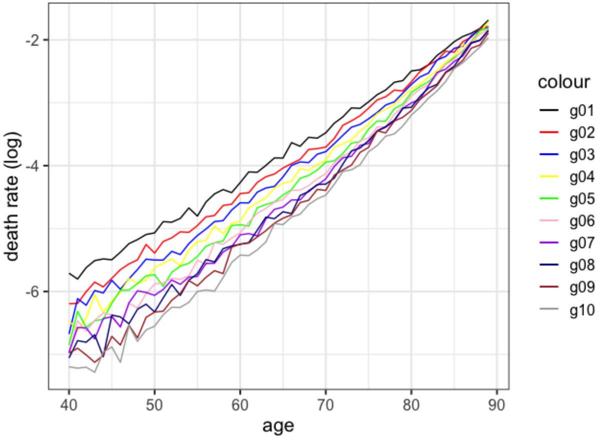
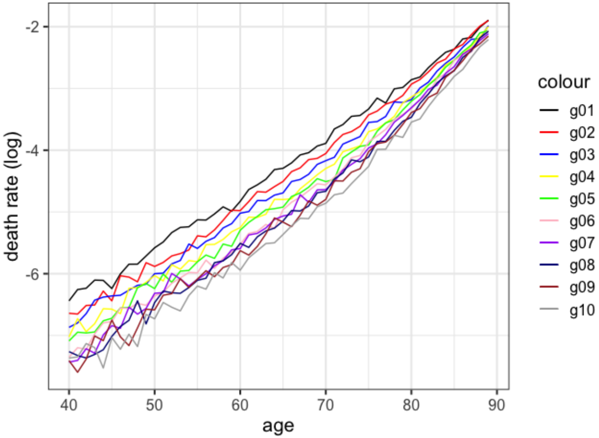
Crude death rates (males, 2017) by Socio-Economic Group



$$\frac{\text{number of deaths}}{\text{population size}}$$

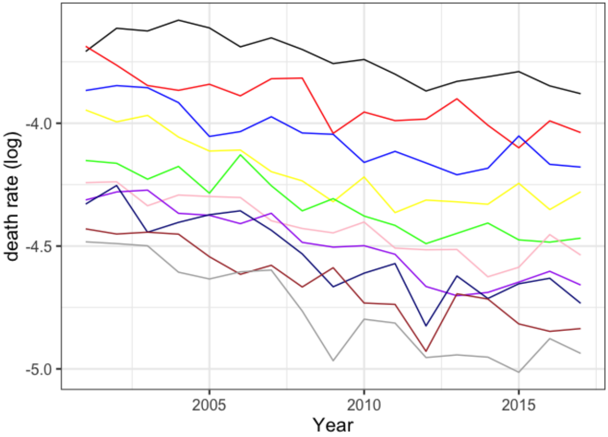
- roughly linear in age (Gompertz line)
- mortality differentials are decreasing with age

Crude death rates (females, 2017) by Socio-Economic Group



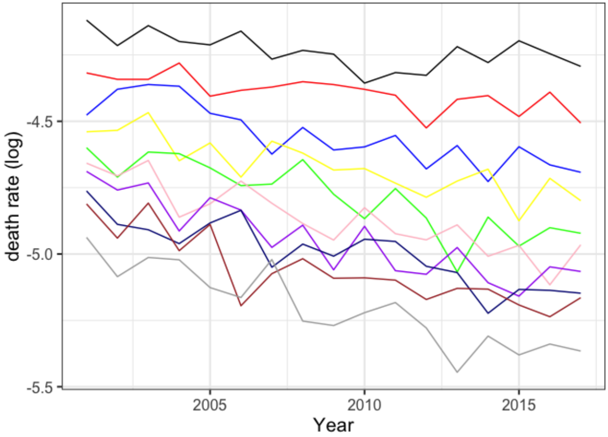
- similar shape as male log mortality, but lower level, slightly smaller differences
- again, mortality differentials are decreasing with age

Crude death rates (males, age 65) by Socio-Economic Group



- clear differences between groups but some crossovers
- downward trend strongest for least deprived

Crude death rates (females, age 65) by Socio-Economic Group



- similar shape as for males, clear differences, but more crossovers
- again, different trends for different socio-economic groups

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Model for the Number of Death in Different Groups

For each period (calendar year) t, age x and socio-economic group i we assume for the number of deaths, D_{xti}:

$$D_{xti} \sim \text{Poisson}\left(m_{xti} E_{xti}\right)$$

where

E_{xti}: Central exposure-to-risk (mid-year population estimate)
m_{xti}: force of mortality

$$\text{Expected number of deaths } \mathrm{E}\left[D_{xti}\right] = m_{xti} E_{xti}$$

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Aims:

- compare different models for the force of mortality m_{xti}.
- identify common and group-specific parameters

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We define socio-economic groups with reference to the Index of Multiple Deprivation for England.

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Index of Multiple Deprivation

The IMD is a weighted combination of seven indices of deprivation:

- Income (22.5%)
- Employment (22.5%)
- Education (13.5%)
- Health (13.5%)
- Crime (9.3%)
- Barriers to Housing and Services (9.3%)
- Living environment (9.3%)

source: GOV.UK

IMD is calculated for about 33,000 small geographic areas (LSOA), ordered and split into ten deciles.

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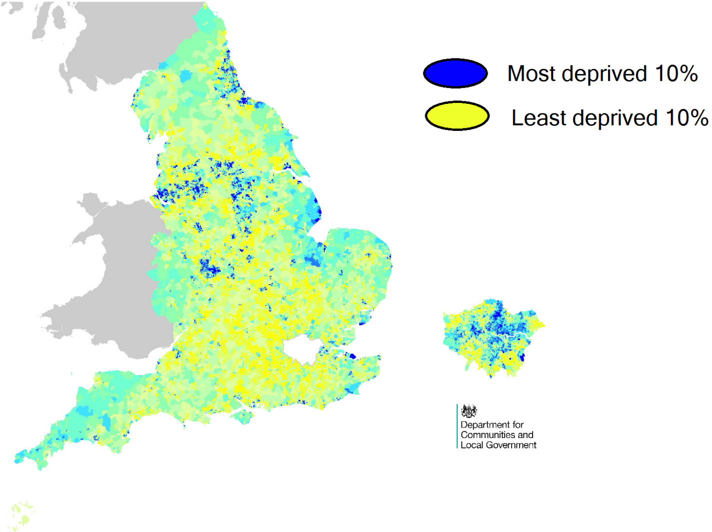
IMD areas


Most deprived 10%
Least deprived 10%
Department for
Communities and
Local Government

Data

- We consider mortality data in England for the ten IMD deciles (ranked in 2015).
- ages: 40-89, years: 2001-2017
- source: Office for National Statistics

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Well known fact: Mortality rates are higher in the most deprived areas compared to the least deprived areas

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Models

All considered models are variants of group specific Lee-Carter type models with the extension to a second age-period effect by Renshaw & Haberman (2003):

$$\log m_{xti} \quad = \quad \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$$

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$$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$$

Specific versions include models with:

common age effect : $\alpha_{xi} = \alpha_x$

fixed age effects : constant $\beta^1_{xi} = 1$ and linear $\beta^2_{xi} = x - \bar{x}$, where $\bar{x}$ is the mean age in the data set. (Plat, 2009)

non-parametric common age effects : $\beta^k_{xi} = \beta^k_x$ (Kleinow, 2015)

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some common period effects : $\kappa^k_{ti} = \kappa^k_t$ (Li and Lee, 2005 for common κ^1)

added cohort effects γ_{ci} or γ_c for cohort $c = t - x$

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Models (without cohort effect)

m1	$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$	(Renshaw & Haberman, 2003)
m2	$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$	(m1 with common β^2_x)
m3	$\log m_{xti} = \alpha_{xi} + \beta^1_{x}\kappa^1_{t} + \beta^2_{xi}\kappa^2_{ti}$	(Li and Lee, 2005)
m4	$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti}$	(Lee and Carter, 1992)
m5	$\log m_{xti} = \alpha_{xi} + \beta^1_{x}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$	(Kleinow, 2015)
m6	$\log m_{xti} = \alpha_{x} + \beta^1_{x}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$	(m5 with common α_x)
m7	$\log m_{xti} = \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$	(Plat, 2009)
m8	$\log m_{xti} = \alpha_{x} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$	(m7 with common α_x)
m9	$\log m_{xti} = \alpha_{xi} + \kappa^1_{t} + (x - \bar{x})\kappa^2_{ti}$	(m7 with common κ^1_t)
m10	$\log m_{xti} = \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{t}$	(m7 with common κ^2_t)
m11	$\log m_{xti} = \alpha_{xi} + \kappa^1_{t} + (x - \bar{x})\kappa^2_{t}$	(m7 with common κ^1_t and κ^2_t)
m12	$\log m_{xti} = \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$	(Cairns et al., 2006)

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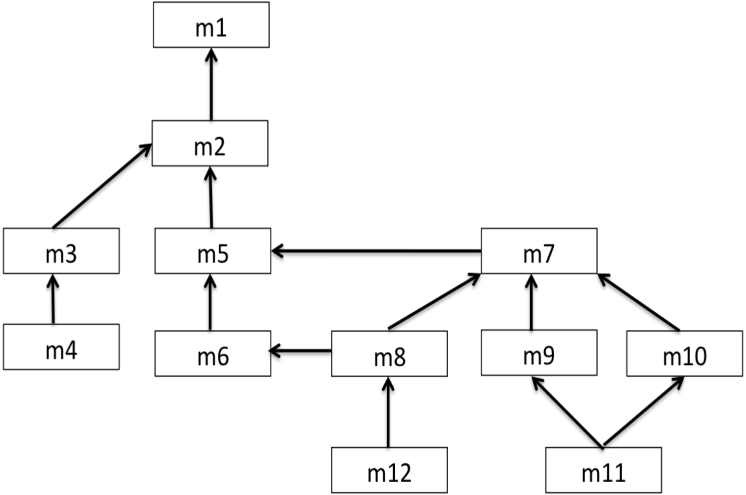
Models are nested



Estimation and Identifiability

- Maximum Likelihood estimation based on $D_{xti} \sim \text{Poisson}\left(\mu_{xti} E^{c}_{xti}\right)$ is applied to obtain estimated parameter values.
- Most suggested models have some identifiability issues, that is, different parameter values lead to the same fitted mortality rates m_{xti}, and, therefore to the same value of the likelihood function.
- To obtain unique parameter values we apply model-specific constraints.

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- To obtain unique parameter values we apply model-specific constraints.
- In a first step, models are ranked according to the Bayesian Information Criterion:

$$BIC = k \log(n) - 2 \log(\hat{L})$$

  where k represents the degrees of freedom, n is the sample size (number of years $\times$ ages $\times$ groups), and $\hat{L}$ is the likelihood value
- A smaller BIC indicates a better model

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Quantitative Comparison of Models

	females		males		
Model	$\log(\hat{L})$	BIC	$\log(\hat{L})$	BIC	$\log m_{xti}$
m1	-34398.54	85083.16	-35634.78	87555.64	$\alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$
m2	-34652.44	81600.86	-35900.31	84096.61	
m3	-34848.27	80689.64	-36065.10	83123.31	
m4	-35083.25	80571.50	-36293.44	82991.87	
m5	-35058.84	78423.59	-36242.45	80790.81	
m6	-35336.06	**75069.36**	-36702.57	**77802.39**	$\alpha_{x} + \beta^1_{x}\kappa^1_{ti} + \beta^2_{x}\kappa^2_{ti}$
m7	-35653.80	78726.80	-37422.19	82263.59	
m8	-37375.07	78260.70	-38213.32	79937.20	$\alpha_x + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
m9	-36104.58	78325.48	-37821.39	81759.10	
m10	-35746.95	77610.23	-37491.71	81099.75	
m11	-36760.83	78335.10	-38171.44	81156.32	
m12	-46822.86	96721.99	-41385.10	85846.45	

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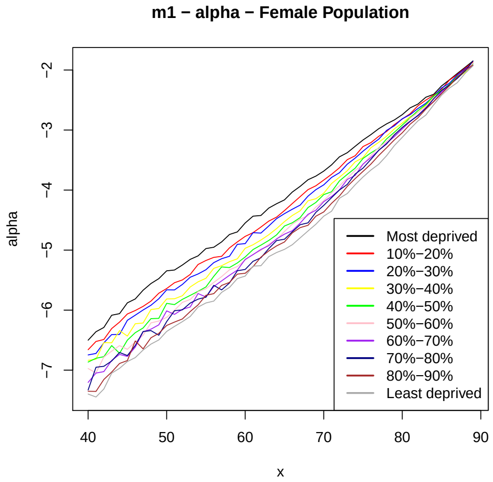
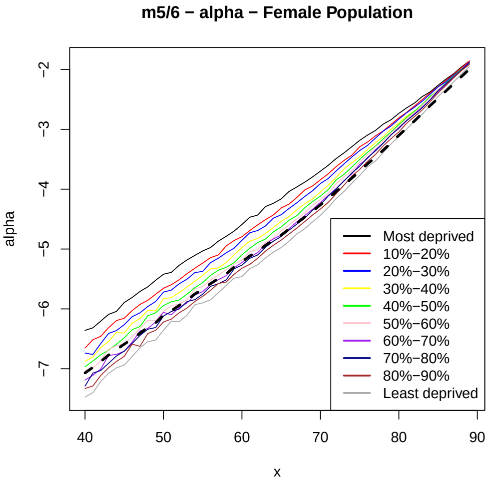
Parameter estimates - the most general model



- m1: $\alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$
- clear differences between socio-economic groups in basic age structure of mortality

Parameter estimates - common age effects



- m5: $\alpha_{xi} + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$
- m6: $\alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$
- estimates for m5 are very similar to those of m1
- dashed line is the common age structure α_x in m6

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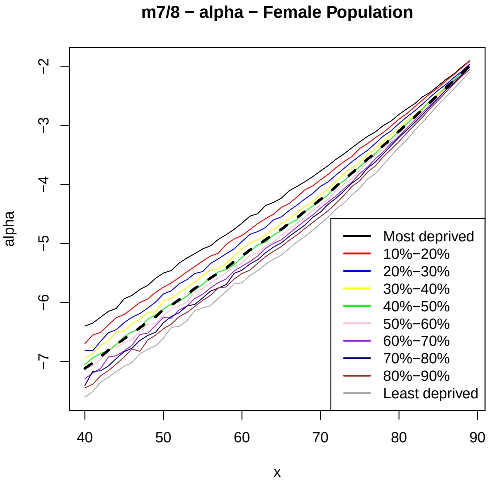
Parameter estimates - Plat model



- m7: $\alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
- m8: $\alpha_{x} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
- again, a very similar shape
- Summary: basic age structure is almost independent of chosen model

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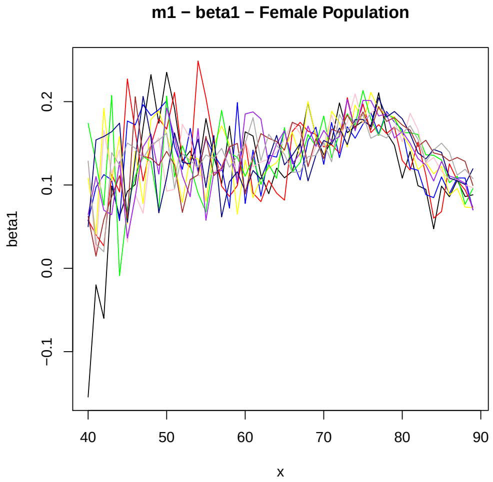
Parameter estimates - the most general model

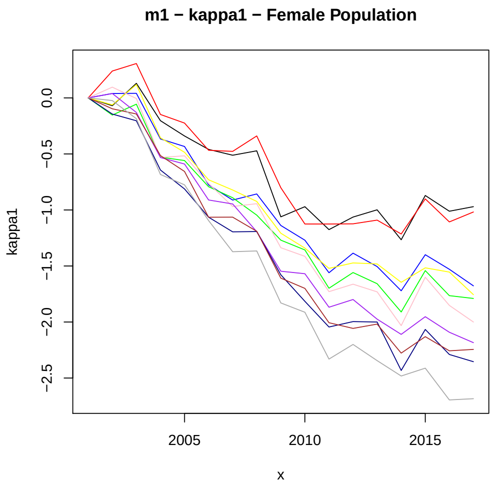


- m1: $\alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$
- no clear differences between groups
- suggests a common parameter
- but not constant as in m7 and m8

Parameter estimates - common age effects

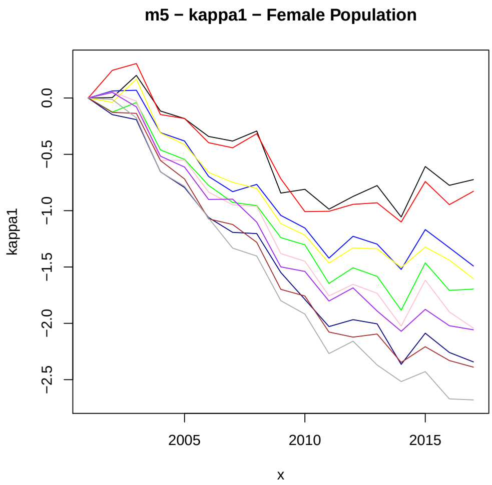
- m5: $\alpha_{xi} + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$
- m6: $\alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$
- shape of β^1 in m5 is similar to m1
- ... but for m6 the shape is very different
- note that β^1 is constant in m7 and m8

Parameter estimates - the most general model



- m1: $\alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti}$
- clear differences in the trend of mortality between groups
- least deprived show greatest improvements

Parameter estimates - common age effects



- m5: $\alpha_{xi} + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$
- again, different trends for different groups
- period effects are very similar to those in m1
- ... this suggests that projections would also look similar

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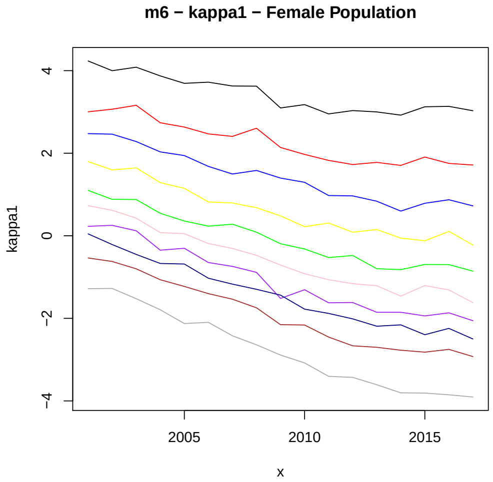
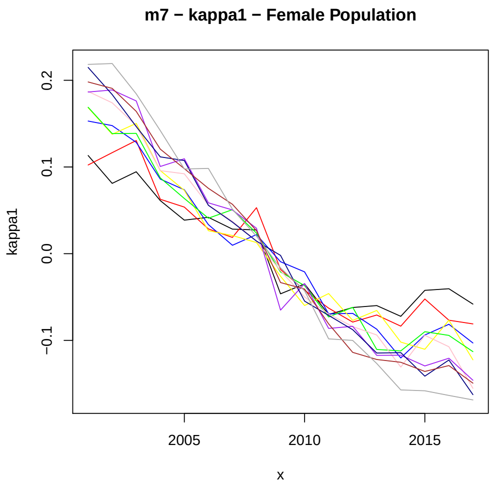
Parameter estimates - common age effects



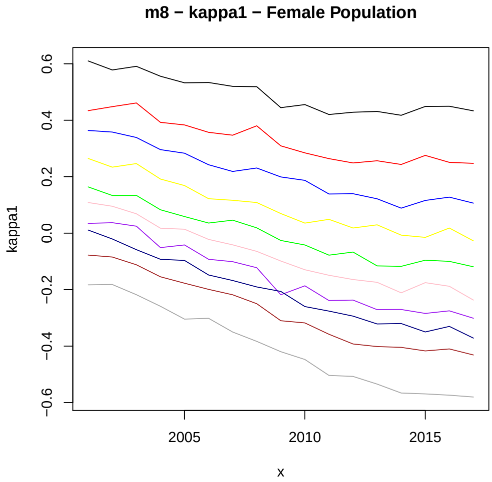
- m6: $\alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$
- since age effects are now common, the first period effects picks up differences in level of mortality
- we also see different trends

Parameter estimates - Plat model



- m7: $\alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
- differences in trend are clearly visible
- note that different constraints have been used (compared to m5 and m6)

Parameter estimates - Plat model



- m8: $\alpha_x + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
- similar to m6; common α_x leads to different levels
- trends are also different

Standardised residuals

Define Pearson's residuals

$$Z_{xti} = \frac{D_{xti} - \mathrm{E}\left[D_{xti}\right]}{\mathrm{std}\left[D_{xti}\right]} = \frac{D_{xti} - E_{xti}\hat{m}_{xti}}{\sqrt{E_{xti}\hat{m}_{xti}}}$$

where $\hat{m}_{xti}$ is the fitted death rate at age x in year t that we obtain from our various models.

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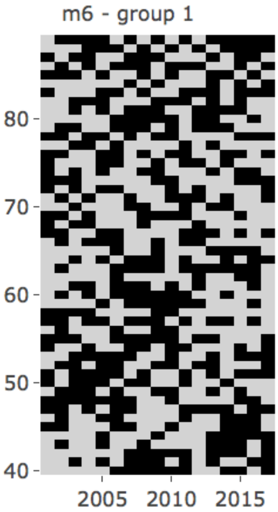
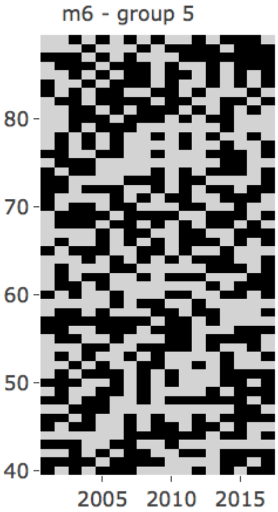
Standardised residuals - common age effects



- m6: $\alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$
- no obvious clusters or pattern
- good fit

Standardised residuals - common age effects



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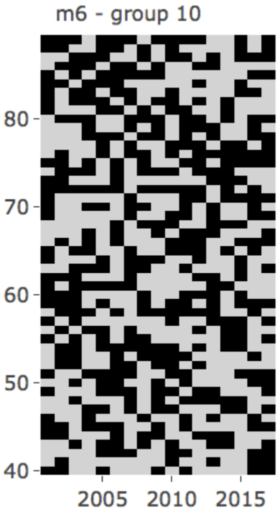

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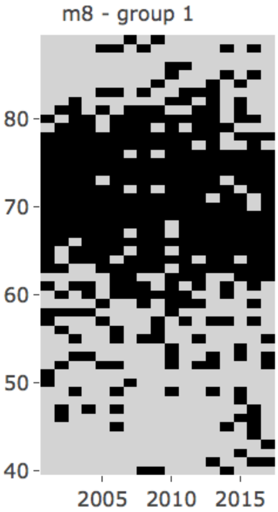
Standardised residuals - Plat model



- m8: $\alpha_x + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
- pattern along the age dimension for group 1 (most deprived)

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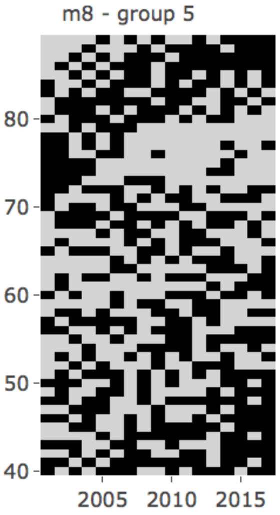
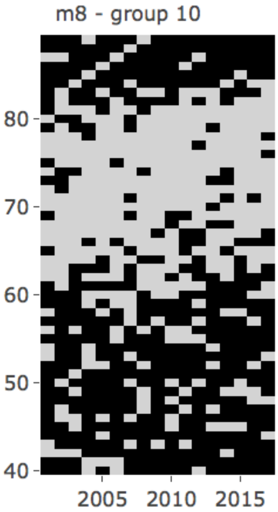
Standardised residuals - Plat model



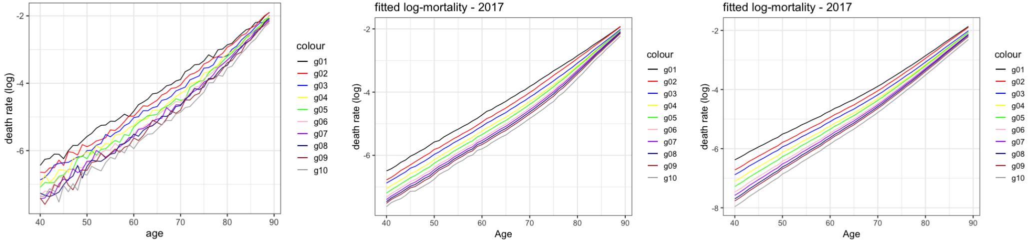
- m8: $\alpha_x + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
- good fit for group 5

Standardised residuals - Plat model



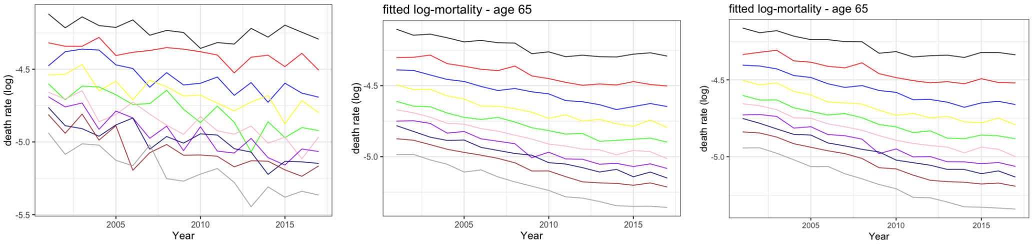
- m8: $\alpha_x + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti}$
- pattern along the age dimension for group 1 (most deprived) and 10 (least deprived)

Fitted Mortality Rates



- Empirical log mortality rates (left) in 2017 (females),
- fitted rates from models m6, $\alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$ (middle) and
- m8, $\alpha_x + \kappa_{ti}^1 + (x - \bar{x})\kappa_{ti}^2$ (right)
- fitted rates are similar, but m8 produces smoother rates

Fitted Mortality Rates



- Empirical log mortality rates (left) at age 65 (females),
- fitted rates from models m6, $\alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$ (middle) and
- m8, $\alpha_x + \kappa_{ti}^1 + (x - \bar{x})\kappa_{ti}^2$ (right)
- again, fitted rates look very similar, in particular, similar improvement rates between models
- ... but different improvement rates for different groups

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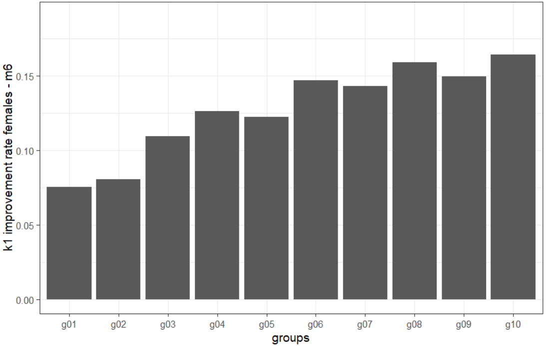
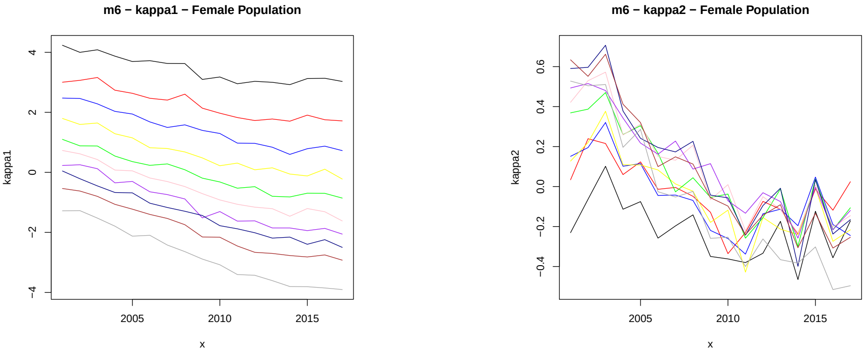
Improvement Rates for Leading Period Effect



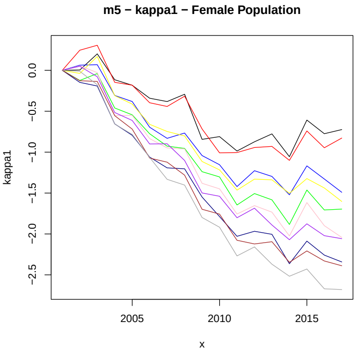
- m6: $\alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$ (females)
- large differences; from g1 to g10 the improvement rate doubles
- rescaled with common β_x^1
- differences are even greater for model m5

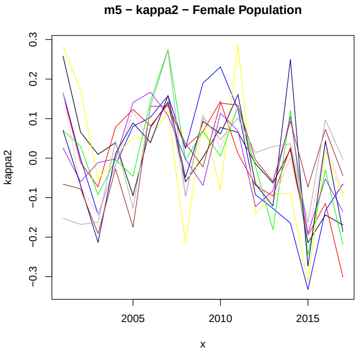
Projections are challenging - Model m6



- Projections require assumptions (expert judgement) about differences in level and improvement rates for leading period effect (left)
- and additional assumptions about κ^2

Projections are challenging - Model m5





- modelling of κ^2 seems easier, some correlated stationary processes look appropriate
- ... but the main issue about identifying reasonable assumptions (expert judgement) for leading period effects remains

Conclusions

- age effects are common to all ten socio-economic groups in England (as measured by the Index of Multiple Deprivation)
- ... but the fit is improved if the age effects β^1 and β^2 are not constant and linear functions of age
- However, the fitted rates look very similar for the CAE model and the Plat model with common alpha

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- However, the fitted rates look very similar for the CAE model and the Plat model with common alpha
- period effects are different ...
- ... different levels
- ... but also different trends; mortality differentials are increasing (although there is some evidence that improvement rates even in for the least deprived are slowing down)
- Therefore, models with common period effects (in particular κ^1) are not a good fit

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- However, the fitted rates look very similar for the CAE model and the Plat model with common alpha
- period effects are different ...
- ... different levels
- ... but also different trends; mortality differentials are increasing (although there is some evidence that improvement rates even in for the least deprived are slowing down)
- Therefore, models with common period effects (in particular κ^1) are not a good fit
- The challenge is, of course, to project period effects: what assumptions can we make about long term trends?

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