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Approximations, Estimates & Errors

Proxy Models Working Group

13 June 2013

AGENDA

- An introduction to Proxy Models
- Model Choice and Design
- Case Study
- A “Spooky” Result
- Closing Remarks
- Questions & Answers

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Part 1

An Introduction to Proxy Models

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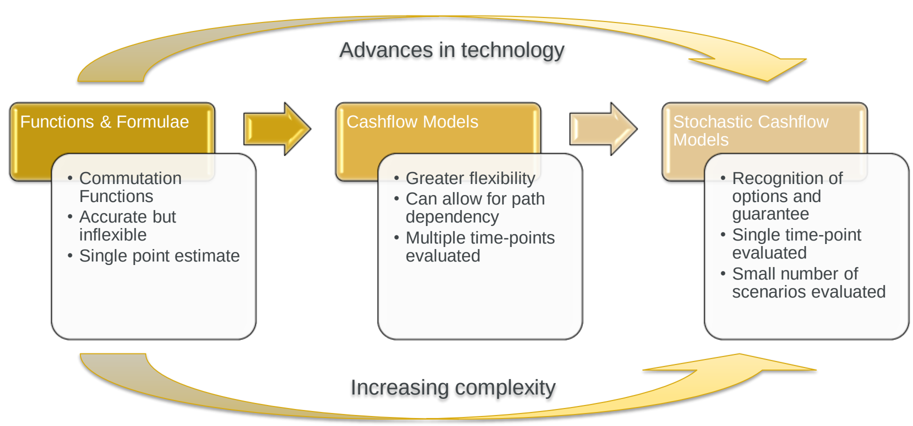
Background

A Brief History of Modelling Methods



- Where next?

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Background
The need for proxy models

Increasing regulatory and risk management demands

Many more scenario results required

A Return to Functions and Formulae

- Replicating formulae and other proxy models introduced
- Aim to reproduce the more complex cashflow model results
- Often less sophisticated than the functions and formulae originally discarded in favour of cashflow models

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What is a Proxy Model?
Types of Model

- In the wider sense of the word, all models are proxies

Models approximating reality

Deterministic	Deterministic Formula		Deterministic Cashflow Model	
Stochastic	Deterministic formula + formula for time value	Stochastic formula	Monte-Carlo cashflow model	Deterministic Cashflow model + formula for time value

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What is a Proxy Model?
How Accurate is Accurate?

- Monte-Carlo model results are often seen as the ‘right’ answer.
- Accuracy of any other proxy model is often measured against this baseline
- Are formula results any less valid than Monte-Carlo results?
  - Consider extent and level of parameter approximation
  - Consider the lengths gone to in order to produce the theoretical result
- Need to consider a wider range of models and tools calibrated to the same baseline.
- Could even reconsider choice of baseline.

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What is a Proxy Model?

For our purposes, proxy models are those models approximating a more complex model, often the Monte-Carlo cashflow model

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Part 2

Model Choice & Design

Choice of Model

Evaluating proxy models

- Use of the Model
- Quality of fit
- Ease of implementation and cost
- Speed of implementation
- Model stability
- Complexity – management acceptance
- Predictive versus Descriptive

Choice of Model
From 'Heavy' to 'Lite'

- Most proxy models can be classified as replicating formulae,
  - Consisting of Formula elements derived from basis functions,
  - With each formula element being multiplied by a coefficient
- Using this classification is useful
  - Can identify fundamental issues common to all models
  - Provides a common framework for comparison
- Models range from 'Lite' (e.g. polynomial) to 'Heavy' (e.g. cashflow)
- Where a model lies in that range depends on:
  - The degree of complexity of each element
  - The number of elements

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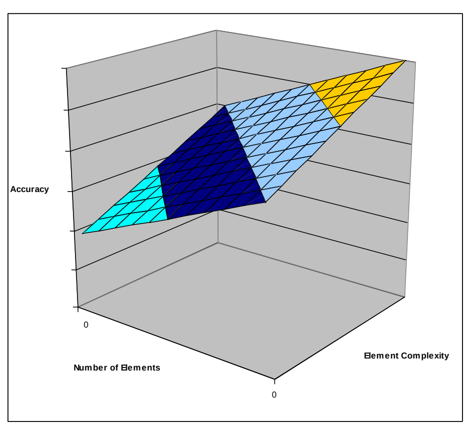
Model Design
Complexity versus 'Accuracy'

- Can increase complexity of a proxy model in two ways
  - Use more complicated or sophisticated formula elements
  - Increase the number of formula elements
- Normally associate increasing complexity with greater accuracy and slower runtime.
- Generally, Increasing element complexity leads to fewer required elements for same level of accuracy



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Model Design
Calibration

- First stage is determining Formula Structure
  - Deciding which ‘elements’ are to be included in the formula
- Second stage is determining coefficients of formula element
  - Optimising formula to a given dataset
- Various choices remain
  - Optimised components versus optimised whole
  - Regression or precise interpolation
  - Target calibration, e.g. minimax, least squares etc.
  - Domain over which model is to be used

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Model Calibration
Summary of options

Type of Proxy Formula	Determining Formula structure	Regression, Interpolation or both	Optimised components, whole or both
Replicating Polynomials	Choice and number of nomials	Both possible	Both possible
Radial Basis Functions	Choice of radial basis function	Both possible	Optimised whole
Commutation functions	Choice and number of commutators	Both possible	Optimised whole
Replicating Portfolios	Choice of assets	Regression	Optimised Whole

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Part 3

Case Study

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Introduction

Method

- Artificial with-profit liability model used to generate data
  - Cost of maturity guarantee in excess of asset share modelled
  - Over one thousand model points, various terms and moneyness of guarantees
- Nine risk factors
  - Three Insurance risks; persistency, expenses and mortality
  - Six market risks; UK & overseas equities, property, Credit, interest rates and inflation
- Simple scenario generator; normally distributed random variables
- Replicating polynomial proxy investigated

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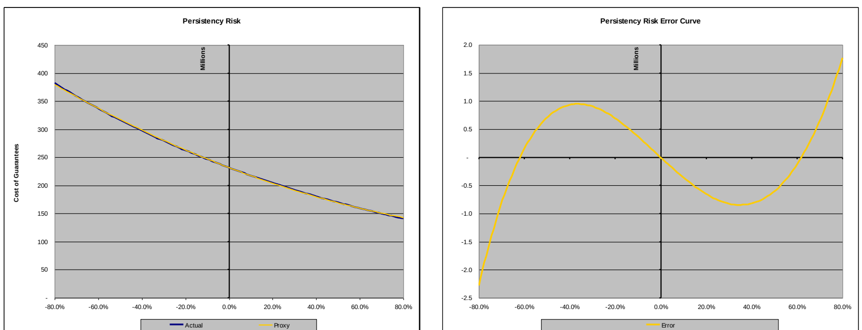
Determining Formula Structure
Marginal Risk Functions

- Consider variation in liability value with respect to lapse risk
- Determined least squares quadratic fit by precise interpolation



- Resulting Error curve is near optimal for a quadratic fit.

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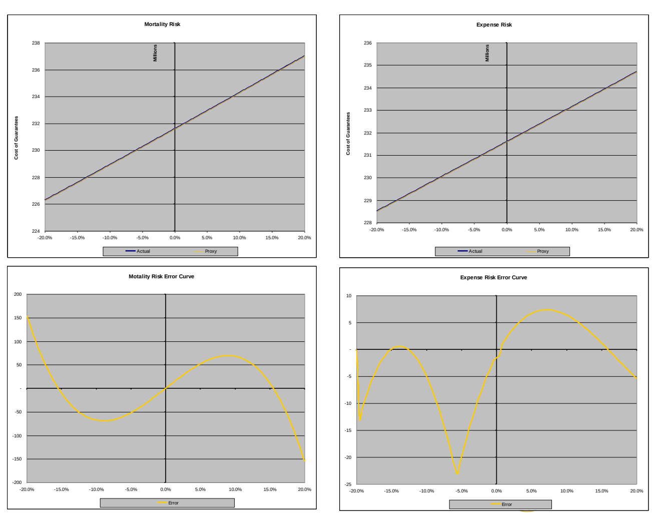
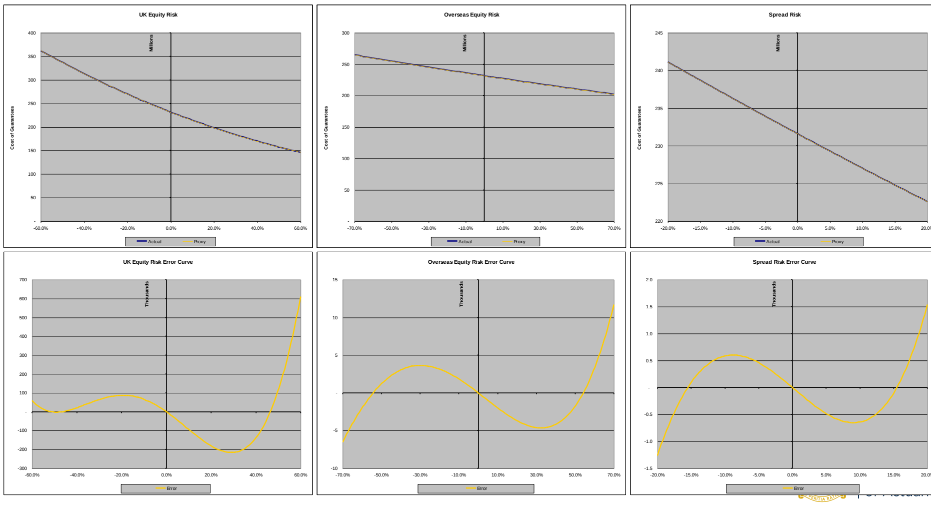
Determining Formula Structure
Marginal Risk functions – Insurance Risks

- Least squares quadratic fit to mortality and expense risk determined in a similar fashion.
- Precise interpolation used throughout – three calibration nodes determine unique quadratic function for each.



Determining Formula Structure

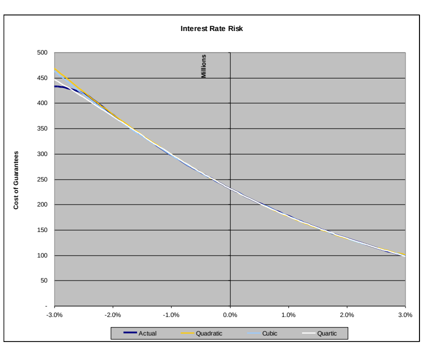
Marginal Risk functions – Market Risks

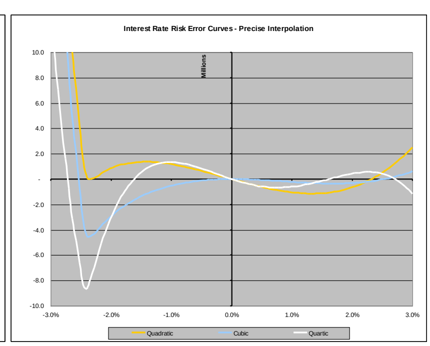


Determining Formula Structure

Marginal Risk Functions – Interest Rates

- Quadratic fit to interest rate risk is less than ideal





- Higher order polynomials tested but fit remains unsatisfactory at the lower end of the domain.

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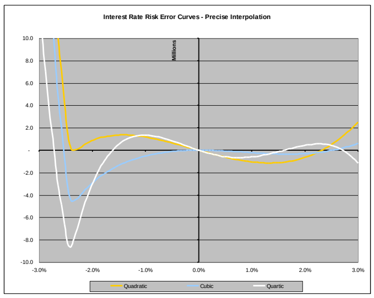
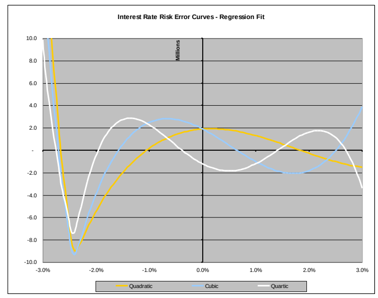
Determining Formula Structure
Marginal Risk Functions - Regression versus Interpolation

- Attempt to capture shape of whole curve using regression fit.





- Similar fit between regression and interpolation
- Improvement in fit is not sufficient to justify a higher order polynomial – quadratic retained for either regression or interpolation

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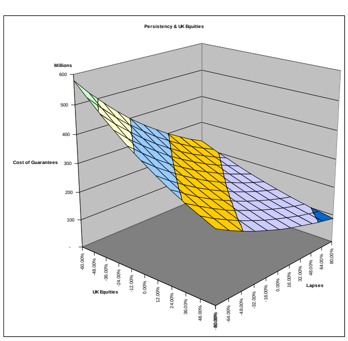
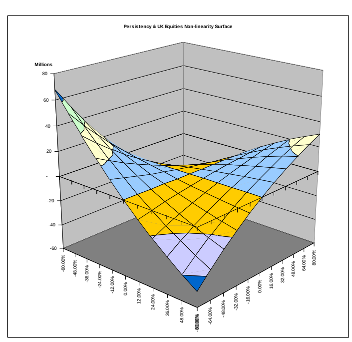
Determining Formula Structure
Non-linearity & Risk dependency structure

- Non-linearity is the difference between the combined impact of two or more risk factors and the sum of those same risk factors.
- Construct a combined risk surface by adding marginal risk functions
- Compare with actual combined risk surface to evaluate non linearity





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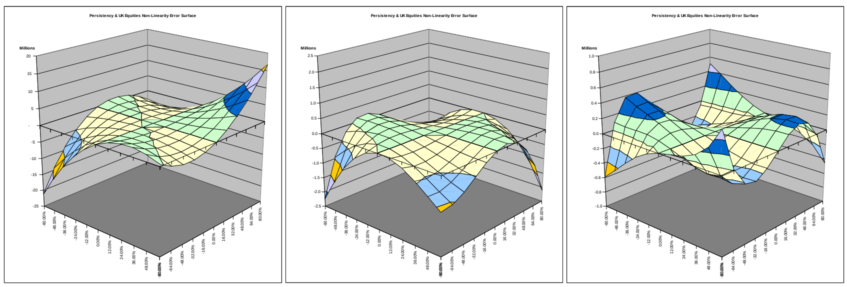
Determining Formula Structure
Non-linearity – Persistency & UK Equities

- Construct a two factor polynomial approximation to non-linearity
- A single XY cross term provides a poor fit to non-linearity



- Using a combination of terms in xy, x^2y and xy^2 the fit is improved
- Best fit is achieved with the addition of x^2y^2 term.

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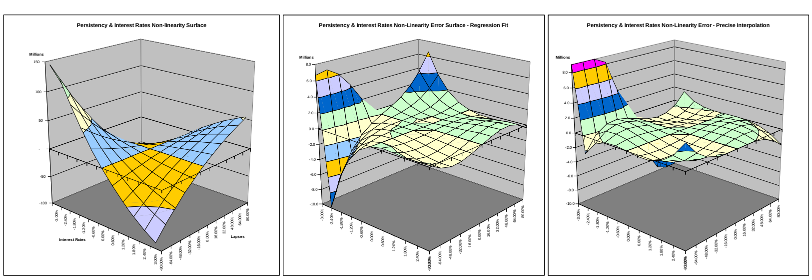
Determining Formula Structure
Non-linearity – Regression versus Interpolation

- Calibration can be performed by regression or interpolation
- Consider non-linearity between lapses and interest rates



- Error surface from regression fit has lower maximum error
- Interpolated fit is better for a large portion of the domain

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Determining Formula Structure
Summary

- Constant plus quadratics used for each of the nine risk factors
- Two factor non-linearity functions for risk pairings involving lapse risk or interest rates
- Expenses, mortality and inflation ignored for non-linearity
- Three factor non-linearity function included for combination of three largest risks; Lapses, UK Equities and Interest Rates

Formula component	No. of Components	No. of Elements
Constant	1	1
Quadratic Marginal Risk Functions	9	18
2-Factor 2^{nd} order non-linearity function	9	36
3-Factor 2^{nd} order non-linearity function	1	8
Total	20	63

Calibration

- Given a formula structure, the model can now be calibrated to different data sets
- Calibration methods include both regression and interpolation

Regression	Interpolation
• Random in-sample calibration scenarios • Number of calibration scenarios varied from 100 to 1000	• 63 calibration nodes for 63 formula terms • Nodes selected based on roots of Legendre polynomials

- Quality of fit measured using 4500 out of sample scenario results.
- Various metrics considered

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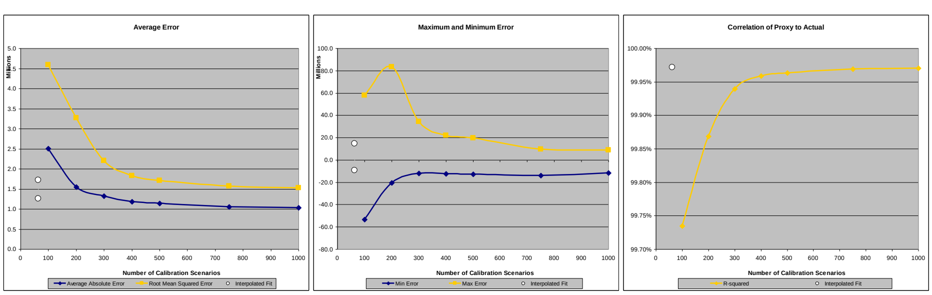
Results
Summary

Number of Calibration Scenarios	100	200	300	400	500	750	1000	63
Number of Test Scenarios	4500	4500	4500	4500	4500	4500	4500	4500
Average Absolute Error	2,509,057	1,547,157	1,327,213	1,187,143	1,136,247	1,049,534	1,026,928	1,257,291
Root Mean Squared Error	4,590,857	3,270,854	2,205,439	1,827,784	1,706,344	1,571,821	1,525,197	1,719,845
Std. Dev. Of Error	4,573,911	3,265,540	2,201,346	1,827,626	1,706,470	1,571,379	1,525,228	1,719,961
Min Error	- 53,401,858	- 20,473,378	- 12,026,350	- 12,294,987	- 12,893,071	- 13,876,837	- 11,810,128	- 9,135,754
Max Error	57,825,043	83,537,745	34,508,250	21,795,996	19,633,982	9,762,366	8,979,645	14,812,315
Average Absolute % Error	1.16%	0.75%	0.61%	0.53%	0.50%	0.46%	0.45%	0.55%
Root Mean Squared % Error	2.46%	2.56%	1.27%	0.87%	0.81%	0.72%	0.69%	0.80%
Std. Dev. % Error	2.17%	2.45%	1.12%	0.69%	0.63%	0.55%	0.52%	0.58%
Min % Error	-53.31%	-19.36%	-11.37%	-7.79%	-8.17%	-8.79%	-7.48%	-7.22%
Max % Error	38.27%	102.65%	35.57%	14.30%	13.49%	6.64%	5.67%	4.16%
R-squared	99.73%	99.87%	99.94%	99.96%	99.96%	99.97%	99.97%	99.97%

Results
Regression versus Interpolation

- Under all metrics tested, quality of regression fit improves as number of calibration scenarios increases



- Law of diminishing returns applies
- Interpolation fit achieves near optimum results

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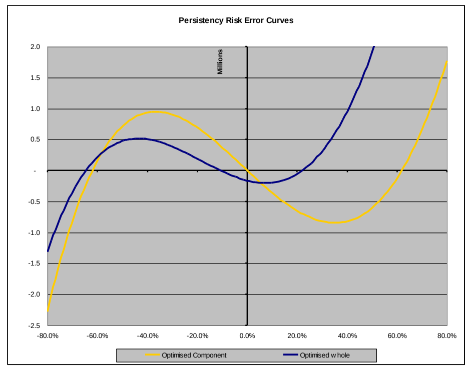
Results

Optimised Component versus optimised whole

- Recall the optimised lapse risk error curve
- Unchanged under interpolation fit as nodes have been selected to optimise components
- Regression fitting to a new data set optimises the whole formula
- Resulting marginal risk error curves are no longer optimal
- Chosen calibration method must reflect model use



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Results

1 in 200 risk capital

- Errors at the 99.5th percentile measured for various test datasets

ERRORS	Calibration Scenarios							
Test Scenarios	100	200	300	400	500	750	1000	63
5000	-0.93%	-0.60%	-0.47%	-0.58%	-0.46%	-0.69%	-0.75%	0.49%
10000	-1.22%	-0.58%	-0.73%	-0.89%	-0.83%	-0.39%	-0.46%	0.22%
15000	-0.35%	-0.42%	-0.56%	-0.60%	-0.41%	-0.57%	-0.61%	0.53%
20000	-0.25%	-0.43%	-0.57%	-0.62%	-0.42%	-0.61%	-0.67%	0.43%

- Observed errors at extremes are much smaller than expected
  - Size of errors appears uncorrelated to quality of fit
  - Errors reduce as number of scenarios increase
- Why?

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Part 4

A “Spooky” Result

Thought leadership Progress Community Sessional Meetings Education Working parties Volunteering Research Shaping the future Networking Professional support Enterprise and risk Learned society Opportunity International profile Journals

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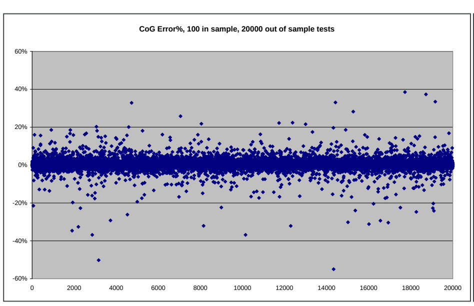
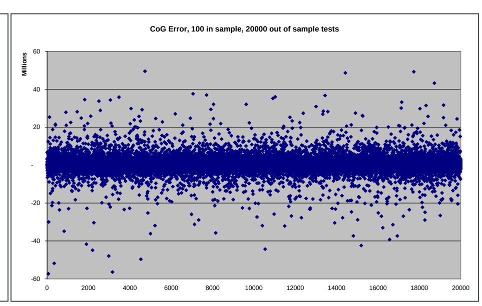
Scenario (In)accuracy

- Proxy models can be very inaccurate





- The fit here is poor by any conventional measure.

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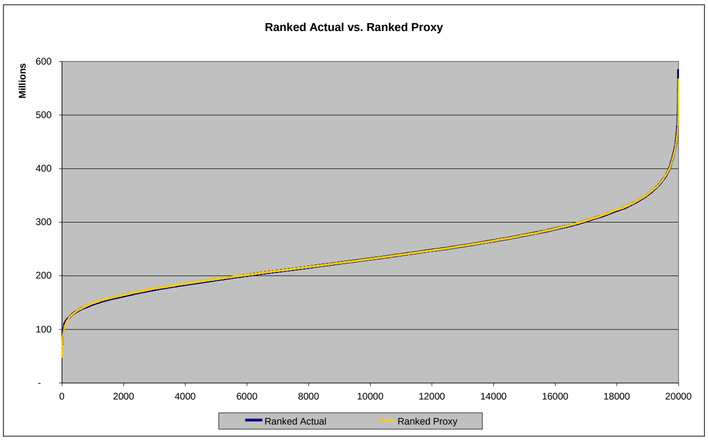
Capital Accuracy

- Capital value can still be accurate



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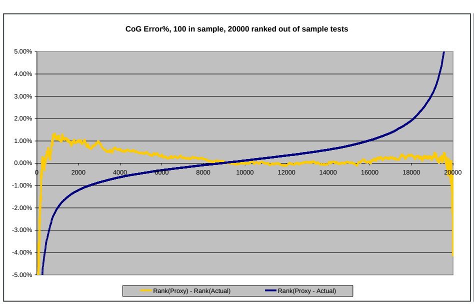
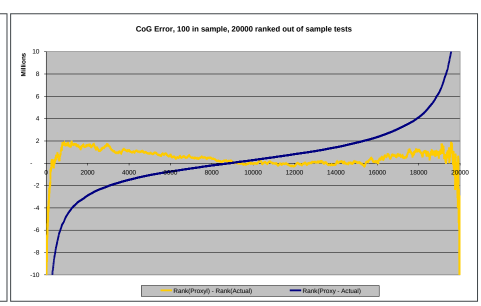
Model inaccuracy versus capital accuracy

- The working party nicknamed this the ‘*Spooky* Result’





- **How can the model be so inaccurate, but the capital result be so accurate?**

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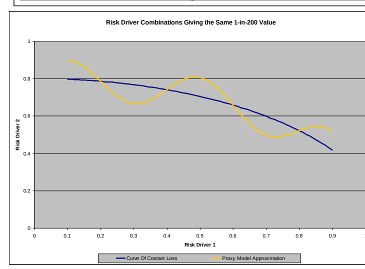
The “Curve of Constant Loss”

- In one risk dimension, errors increase at the extremes
  - A single point suffers error bias
- In multiple dimensions, a single point is replaced by a contour
  - “Curve of Constant Loss”
- The actual curve of constant loss and the proxy are different
  - Proxy can be greater or less than actual along the path
- <u>AT SPECIFIC POINTS</u>, the proxy could be off by up to 60%



When the result may apply

- Wish to minimise error bias along curve of constant loss
  - Desirable that E(error) = 0
- Error at the percentile result is dictated by errors in the scenario results lying outside the curve of constant loss
- Too few can introduce error bias
  - Statistical impact
  - All points limited to one region of the risk distribution
- Open question being explored: Is E(error)=0 a general requirement or need it only be satisfied along the relevant curve of constant loss?

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Part 5

Closing Remarks

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Closing Remarks

- Must think very carefully about how ‘accuracy’ is defined and the benchmarks for ‘accuracy’
- Due to the inaccuracy of some proxy models, individual scenario results should be used with caution
  - A ‘biting’ scenario derived from the proxy model may be wrong
  - Evaluating the biting scenario in the heavy model may lead to the incorrect capital result
- Ultimately, the key influence on the design and implementation of a proxy model is the use to which it will be put.

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Questions **Comments**

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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