EXAMINATIONS

10 April 2000 (pm)

Certificate in Derivatives: Mathematics and Basic Principles

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.*

2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 8 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formula sheet and an electronic calculator.

1 (i) Describe briefly options on futures. [3]

(ii) Distinguish between

(a) the payoff to the holder and
(b) what is delivered

when a call option on a futures contract is exercised. [2]

(iii) A UK company needs to convert US $1 million into sterling in slightly over three months time. The $:£ spot and forward exchange rates in US$ per £ are quoted below:

Spot	1.6080
30 day forward	1.6060
60 day forward	1.6040
90 day forward	1.6020

The company is contemplating entering into a forward contract to sell $1 million after 90 days.

(a) Calculate how much (in pounds) the company would receive in 90 days' time from the contract.

(b) Calculate the payoff if the spot exchange rate in 90 days time were 1.6500.

(c) Write down an expression for the payoff from the short position in the forward contract, as a function of the delivery price and spot price (pounds per dollar) in 90 days time. Sketch a graph for this payoff function. [6]

(iv) As an alternative to the futures contract in (ii) above, a bank offers the company a short position in a range forward contract. Under this contract, if the spot price is less than $X_1 = 1.55$ the company sells its dollars at the rate of X_1. If the spot price is greater than $X_2 = 1.65$ then the company sells its dollars at the rate of X_2. If the spot rate is between X_1 and X_2, then the company sells its dollars at the prevailing spot rate.

Write down an expression for the payoff from this range forward contract as a function of the spot, minimum and maximum delivery prices (expressed as pounds per dollar) in 90 days time. Sketch this payoff function. [5]

[Total 16]

2 The Group of Thirty report entitled "Derivatives: Practices and Principles" recommends that dealers and end-users should measure credit risk exposure on derivatives in two ways namely "current" and "potential" exposure.

(i) Explain the reasoning behind the recommendation. [6]

(ii) Describe the two measures of "potential" exposure commenting on their uses. [8]

[Total 14]

3 With reference to the Black Scholes formula and using the notation of the examination handbook and the formula sheet:

(i) Derive an expression for $N'(x)$. [2]

(ii) Show that $\frac{\partial d_1}{\partial S} = \frac{\partial d_2}{\partial S}$ and that $\frac{\partial c}{\partial S} = N(d_1)$. [3]

(iii) Show that $\frac{\partial c}{\partial t} = -rXe^{-r(T-t)}N(d_2) - SN'(d_1)\frac{\sigma}{2\sqrt{T-t}}$.

c is the price of a European call option.

You may assume that $SN'(d_1) = Xe^{-r(T-t)}N'(d_2)$. [4]

[Total 9]

4 Distinguish between an exchange traded option on the shares of a company and a warrant issued by the company on the same stock. [10]

5 A manufacturer of jewellery is considering entering into a forward contract on the price of gold. Let F_t and S_t represent the forward and spot prices of gold at time t respectively. The manufacturer has a special relationship with its bank such that it is able to borrow and invest cash at a risk free interest rate of r with continuous compounding. It is also able to initiate and close the forward contract without any dealing cost. At any time the cost of storing gold is a proportion c of the spot price.

Explain by general reasoning and arbitrage arguments why the forward price should be given by

$$F_t = S_t e^{(r+c)(T-t)}$$

where T is the time at which the forward contract matures. [8]

6 (i) Define what is meant by a continuous stochastic process X_t. [3]

(ii) X and Y are two stochastic processes adapted to the same **P**-Brownian motion W_t, with

$$dX_t = \sigma_t dW_t + \mu_t dt \text{ and}$$

$$dY_t = \rho_t dW_t + v_t dt$$

(a) Show that

$$d(X_t + Y_t) = dX_t + dY_t$$

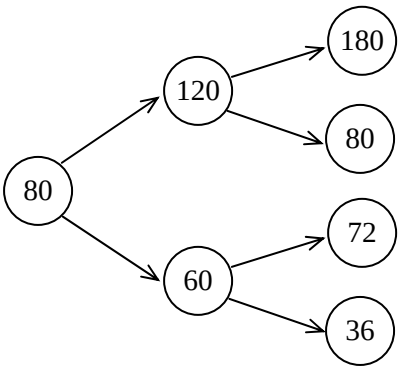
(b) Show that

$$d(X_t Y_t) = Y_t dX_t + X_t dY_t + \rho_t \sigma_t dt \quad [9]$$

[Total 12]

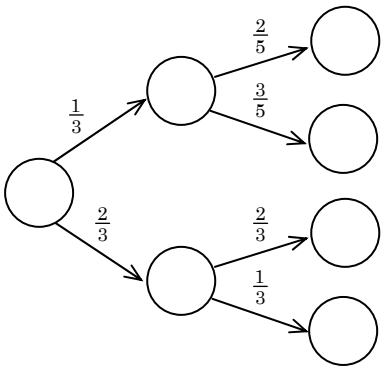
7 Figure 1 shows a process S on a tree. The number at each node is the value of the process at that node. Figure 2 is the measure **Q** on the same tree.

Figure 1



Tree with price process S

Figure 2



The measure **Q**

(i) Show that $E_{\mathbf{Q}}(S_j \mid \mathcal{F}_i) = S_i$ for all $i \le j$. [8]

(ii) Comment on the result in (i) above. [2]

[Total 10]

8 (i) Let:

S be a volatile stock whose price and volatility at time t is S_t and σ_t respectively;

B be a non-volatile cash bond whose price at time t is B_t; and

X be a claim on S at time T (i.e. $X = f(S_t)$ for some function f).

Consider a portfolio consisting of ϕ_t units of the stock S and ψ_t units of the cash bond at time t.

(a) Explain and define what it means for the portfolio to be self-financing.

(b) Define what is meant by a replication strategy for the claim X.

(c) Explain how the replication strategy can be used to value X. [12]

(ii) Given further:

- The stock price process S_t is exponential Brownian with parameters μ and σ under a measure **P**.
- Short term interest rates on cash are r p.a.
- The discounted cash process $D_t = B_t^{-1} = e^{-rt}$.
- The discounted stock process $Z_t = B_t^{-1} S_t$.
- The discounted claim is $B_T^{-1} X$ on S.
- There are no transaction costs for either the stock or the cash bond and both types of asset class can be traded instantaneously with either long or short positions at the price quoted.

(a) Derive an expression for the stochastic differential equation for Z_t using the fact that the stochastic differential equation for the exponential Brownian motion process S_t is

$$dS_t = S_t [\sigma dW_t + (\mu + \tfrac{1}{2}\sigma^2)\, dt]$$

(b) Describe how the Cameron-Martin-Girsanov theorem, the Tower Law and the Martingale Representation Theorem can be used to construct a replication strategy for X. [9]

[Total 21]