EXAMINATIONS

11 April 2002 (am)

Certificate in Derivatives: Mathematics and Basic Principles

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *Enter all the candidate and examination details as requested on the front of your answer booklet.*

2. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only, but notes may be made. You then have three hours to complete the paper.*

3. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*

4. *Mark allocations are shown in brackets.*

5. *Attempt all 7 questions, beginning your answer to each question on a separate sheet.*

Graph paper is required for this paper.

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet, with any additional sheets firmly attached, and this question paper.

In addition to this paper you should have available Actuarial Tables, Derivatives Formula Sheet and your own electronic calculator.

1 (i) State the Cameron Martin Girsanov Theorem for a $\boldsymbol{P}$-Brownian motion process W_t, defining all the symbols you use. [4]

(ii) Explain briefly what it means for a process M_t to be a $\boldsymbol{Q}$-Martingale, and state the Martingale Representation Theorem for a $\boldsymbol{Q}$-Martingale process, defining all the symbols you use. [3]

(iii) Explain the significance of the Cameron Martin Girsanov and the Martingale Representation Theorems for the martingale approach to valuing derivatives. [6]

(iv) Describe "Risk Neutral" valuations, and explain the significance of the Cameron Martin Girsanov Theorem for the "Risk Neutral" approach to valuing derivatives. [5]

[Total 18]

2 Consider a 30-day silver futures contract, and a 30-day silver forward contract of the same specification. On day k from today ($k = 0$ to 30), let the futures contract price be F_k, and the forward contract price G_k. Let δ_k represent the risk-free rate of interest on each day k.

(i) Demonstrate algebraically, using a process of daily re-hedging of a suitable portfolio, that, if $\delta_k = \delta$ (a constant) for all k, $F_0 = G_0$. [7]

(ii) It has been asserted that the prices of precious metals, as a "safe haven" in inflationary times, are strongly positively correlated with interest rates. Explain the effect this feature might have on your answer to (i) when interest rates are not constant, and list three other factors which might affect the relationship between the futures and forward prices. [4]

[Total 11]

3 Consider a non-dividend paying equity Quist. It is actively traded on a recognised exchange, with a current price of 238-240 (pence). The LIBOR interest rate curve stands at 6% per annum for all maturities. For those able to sell short stock they do not own, stock can be borrowed at a cost of 50 basis points (0.50%) per annum.

The trading market is populated by banks who fall into three main categories:

Category	*Borrowing rate*	*Can sell short?*
A	LIBOR	No
B	LIBOR + 50 basis points	No
C	LIBOR + 25 basis points	Yes

(i) Calculate the 1 year forward price(s) of Quist for each category of bank. [2]

(ii) Discuss briefly the implication of the above environment for options on Quist when applied to three key concepts which underlie the Black-Scholes model, namely: the riskless rate of interest, the existence of frictionless markets and zero-cost hedging. [3]

(iii) The implied volatility of 3-month at-the-money options on Quist is currently 25% per annum. Explain why the implied volatility on 3-month options with different strikes may not be the same, and suggest realistic values. [4]

(iv) Discuss briefly the implication of the above environment for market participants using the Black-Scholes model:

(a) to support market-making activity in Quist options;

(b) to calculate the delta hedge in Quist equity required to cover (or simulate) a call or put option on Quist. [4]

[Total 13]

4 Consider a discrete symmetric random walk $W_n(t)$ in which jumps of magnitude $1/\sqrt{n}$ (either up or down) occur at time intervals of $1/n$. There is an equal probability of each jump being up or down.

(i) Write down an algebraic description of the sequence $W_i(t)$ *for* $i = 0, 1, 2, \ldots n$ in terms of a set of independent binomial random variables. [2]

(ii) Define what is required for a process W_t to be a continuous **P**-Brownian motion process. [2]

(iii) $W_n(t)$ follows a symmetric random walk under **P**.
Show that $\lim_{n\to\infty} W_n(t)$ is a continuous **P**-Brownian motion process. [8]

(iv) Explain how the simple Brownian motion described above can be adapted to represent equity price behaviour. [4]

[Total 16]

5 A fund manager is considering adopting one of the following three option strategies (portfolios) based on a non-dividend paying equity, currently priced at 100:

(a) Buy 1 European straddle, strike 100

(b) Buy 1 European call, strike 100
Sell 2 European calls, strike 120

(c) Buy 1 European call, strike 120
Sell 1 European put, strike 80

All options have the same expiry date in three months' time.

(i) For each of the option strategies above, show graphically the value of the strategy against the equity price at expiry, allowing for all cash receipts and payments. Explain the way you have allowed for the option premiums. [6]

(ii) Show on your diagrams the approximate value of each strategy two weeks before expiry **and** three months before expiry. [6]

[Total 12]

6 An equity is currently priced at 80. Over each of the next two 3-month periods its price is expected to go up by 2½% or down by 5%. The risk-free interest rate is 4% per annum, continuously compounded.

(i) Calculate, using a risk neutral valuation, the value of a 6-month at-the-money European put, and estimate its current delta. [5]

(ii) Verify your answers to (i), using instead **only** a no-arbitrage argument. [8]

(iii) Calculate the value of a 6-month at-the-money American put. [5]

[Total 18]

7 (i) Define the purpose of the following, as they apply to the London Financial Futures and Options Exchange (LIFFE):

(a) Non Public Order members ("locals")
(b) options "Autoquote" [3]

(ii) Explain the working of the London Clearing House (LCH) in relationship to LIFFE. [4]

(iii) Describe briefly the London SPAN system of margining, and explain how it would operate with:

(a) inter commodity spreads
(b) equity options

traded on LIFFE. [5]

[Total 12]