EXAMINATIONS

April 2004

Certificate in Derivatives: Mathematics and Basic Principles

EXAMINERS' REPORT

QUESTION 1

Syllabus: 5.1 and 5.2

Reading: Hull Ch 10

This question was simple bookwork, and was generally well answered. Some candidates lost marks in part (ii) for not writing out the derivative formula, or not using an arbitrage argument.

(i)

The portfolio $\pi = (\phi, \psi)$ of stock and bond accumulates to

$$\pi_{t+1} = \begin{cases} \phi s_1 + \psi b e^{r\Delta t} \text{ if } S_{t+\Delta t} = s_1 \\ \phi s_2 + \psi b e^{r\Delta t} \text{ if } S_{t+\Delta t} = s_2 \end{cases}$$

This must be the same as the derivative payoffs, hence we require:

$$f(s_1) = \phi s_1 + \psi b e^{r\Delta t}$$

and $f(s_2) = \phi s_2 + \psi b e^{r\Delta t}$.

These two equations can be solved simultaneously to give

$$\phi = \frac{f(s_2) - f(s_1)}{s_2 - s_1}$$

and $$\psi = \frac{1}{b} e^{-r\Delta t} \left(\frac{f(s_1)s_2 - f(s_2))s_1}{s_2 - s_1} \right)$$

(ii)

The current value of the portfolio is $V_t = \phi s + \psi b$. [*Some further algebraic manipulation can be done here using the result in (i).*]

This must be equal to the value of the derivative. Consider a market maker considering to buy or sell the derivative for a price *P* different from V_t.

Anyone could sell or buy (respectively) the derivative from the market maker in arbitrary quantity, and buy or sell (respectively) the portfolio $\pi = (\phi, \psi)$ at the same time.

At the current time t, the market-maker's counterparty would have a net cash flow surplus of $V_t - P$.

This surplus could be invested risk free and, since after the time interval Δt, the cash flows from the portfolio exactly match the cash flows in respect of the derivative, the cash flow surplus can accumulate to a risk free profit of $(V_t - P)e^{r\Delta t}$.

The only way this risk free profit is zero is if $V_t = P$.

QUESTION 2

Syllabus: 4.2, 4.3, 7.1, 7.2

Reading: Hull Ch 8 & 12

This question was also simple bookwork, and was averagely well answered. In part (ii), several responses were too vague, along the lines of "American = European with extra features, so must be worth more". In part (iii), as with most questions involving graphs, many candidates produced answers that were either too imprecise or showed little knowledge. The examiners were seeking (a picture of reasonable quality showing) the relationship between European and American put option values for a non-dividend paying equity.

(i)

Put price $P = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$

where

S_0 = current price of the stock, K = strike, r = risk free interest rate, T = time to exercise in years

and $d_1 = \dfrac{\ln\left(\dfrac{S_0}{K}\right) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$, $d_2 = d_1 - \sigma\sqrt{T}$

with N(..) being the cumulative Normal distribution and σ the stock price volatility.

(ii)

[*Note: there are several ways of proving part (ii). What follows is only one example.*]

As S_0 / K becomes smaller, i.e. for very low stock price compared with the strike price, $\ln(S_0 / K)$ becomes more negative, so $-d_1$ and $-d_2$ approach 1, hence the put value $P \approx Ke^{-rT} - S_0$.

Hence $P \approx Ke^{-rT} - S_0 < K - S_0$ for $r > 0$

and so the put price is lower than the intrinsic value.

[*In fact this is true for all values of S_0 just less than K and lower, as the graph below shows.*]

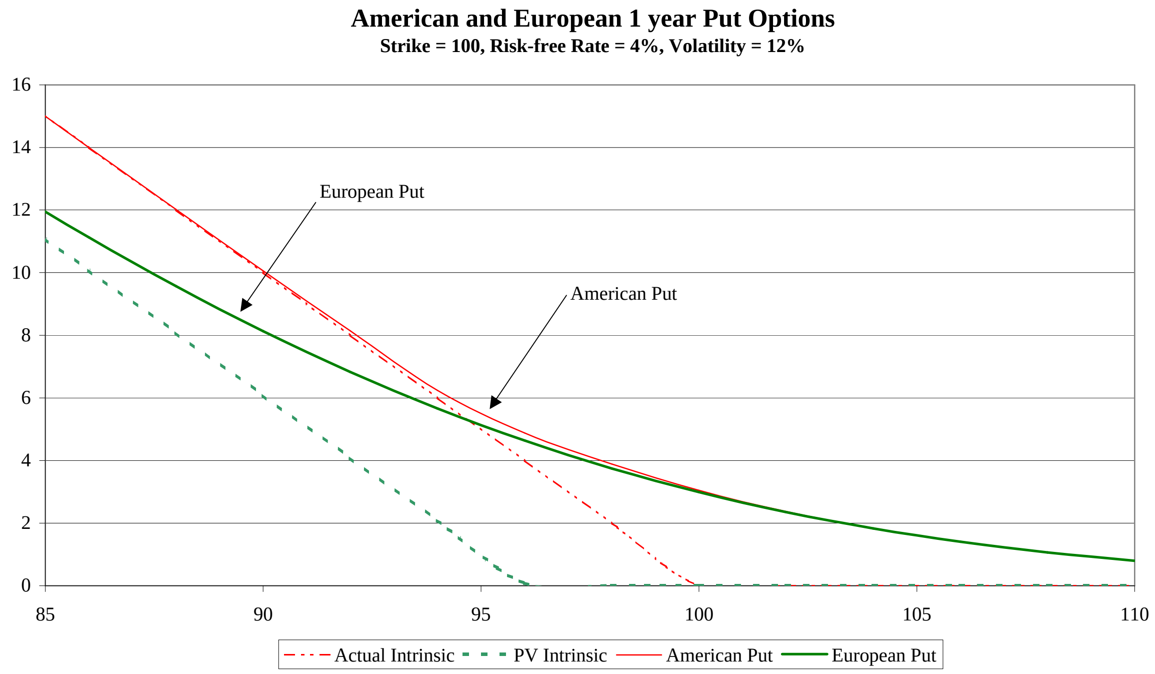
The American option can never be lower than intrinsic value, because it can be exercised immediately ...

... so the American option must be worth more than the European.

[*It is correct to state that, for very large S, the prices of the two are almost indistinguishable.*]

(iii)

[*The required graph is shown below for a particular example. Marks were given for the relative positions of the lines which are important.*]



[*Note: as time to expiry decreases, the left-hand dotted line approaches the right-hand dotted line. For r = 0, which also the case for margined options on futures, they co-incide.*]

QUESTION 3

Syllabus: 6.2

Reading: Hull Ch 11 / B&R Ch 3

This question was mostly another simple bookwork question on Ito's Lemma, which was well answered. Part (ii) caused a surprising amount of trouble for some candidates. However, if the reciprocal relationship of the two currencies was seen, the answer was easy to derive and really just an extension of the ideas in part (i).

(i)

Ito's Lemma for a function $G(x, t)$ based on the geometric process given is:

$$dG = \left(\frac{\partial G}{\partial t} + \mu x\frac{\partial G}{\partial x} + \tfrac{1}{2}\sigma^2 x^2 \frac{\partial^2 G}{\partial x^2}\right)dt + \sigma x\frac{\partial G}{\partial x}dW_t$$

(a) Putting G = ln x into Ito

=> $\dfrac{\partial G}{\partial t} = 0,\ \dfrac{\partial G}{\partial x} = \dfrac{1}{x},\ \dfrac{\partial^2 G}{\partial x^2} = -\dfrac{1}{x^2}.$

=> $dG = \left(\mu x\dfrac{1}{x} - \tfrac{1}{2}\sigma^2 x^2 \dfrac{1}{x^2}\right)dt + \sigma x\dfrac{1}{x}dW_t$

=> $dG = \left(\mu - \tfrac{1}{2}\sigma^2\right)dt + \sigma dW_t$

which is also Brownian, although normally rather than log-normally distributed.

(b) Putting G = x^2 into Ito

=> $\dfrac{\partial G}{\partial t} = 0,\ \dfrac{\partial G}{\partial x} = 2x,\ \dfrac{\partial^2 G}{\partial x^2} = 2.$

=> $dG = \left(\mu x.2x + \tfrac{1}{2}\sigma^2 x^2.2\right)dt + \sigma x.2x.dW_t$

=> $dG = \left(2\mu + \sigma^2\right)Gdt + 2\sigma G dW_t$

which is geometric Brownian like x.

(ii)

If x is the value of A in terms of B, then $G = 1/x$ is the value of B in terms of A.

Putting $G = 1/x$ into Ito

=> $\dfrac{\partial G}{\partial t} = 0,\ \dfrac{\partial G}{\partial x} = -\dfrac{1}{x^2},\ \dfrac{\partial^2 G}{\partial x^2} = \dfrac{2}{x^3}.$

=> $dG = \left(-\mu x\dfrac{1}{x^2} + \tfrac{1}{2}\sigma^2 x^2 \dfrac{2}{x^3}\right)dt - \sigma x\dfrac{1}{x^2}dW_t$

=> $dG = \left(-\mu + \sigma^2\right)Gdt - \sigma G dW_t$

Putting $d\tilde{W}_t = -dW_t$ still is a Wiener process, so:

$$dG = \left(-\mu + \sigma^2\right)Gdt + \sigma Gd\tilde{W}_t$$

so G is a geometric Brownian motion like x, but with growth $r_A - r_B + \sigma^2$.

[*Note: The appearance of the extra term σ^2 is highlighted in Siegel's paradox. It reflects the fact that the growth is still being measured with respect to currency B as the numeraire. Transforming numeraire to currency A has the effect of reducing the growth by σ^2, so it becomes $r_A - r_B$ as expected.]*

QUESTION 4

Syllabus: 1.4, 7.4

Reading: Hull Ch 14

This question was familiar territory to most, and averagely well answered, but few candidates obtained anywhere near full marks. The diagrams in part (ii), which could either be reproduced from Hull Ch 14 or derived from the Black-Scholes formula, caused the most trouble. Many knew only the Gamma relationships, and assumed that Vega was identical. Candidates are strongly encouraged to practise drawing diagrams relating to option prices and their sensitivities. Mostly these can be derived from first principles easily enough.

(i)

Gamma is the sensitivity of portfolio delta to changes in the commodity price. (Delta is the sensitivity of the portfolio value to changes in the commodity price.)

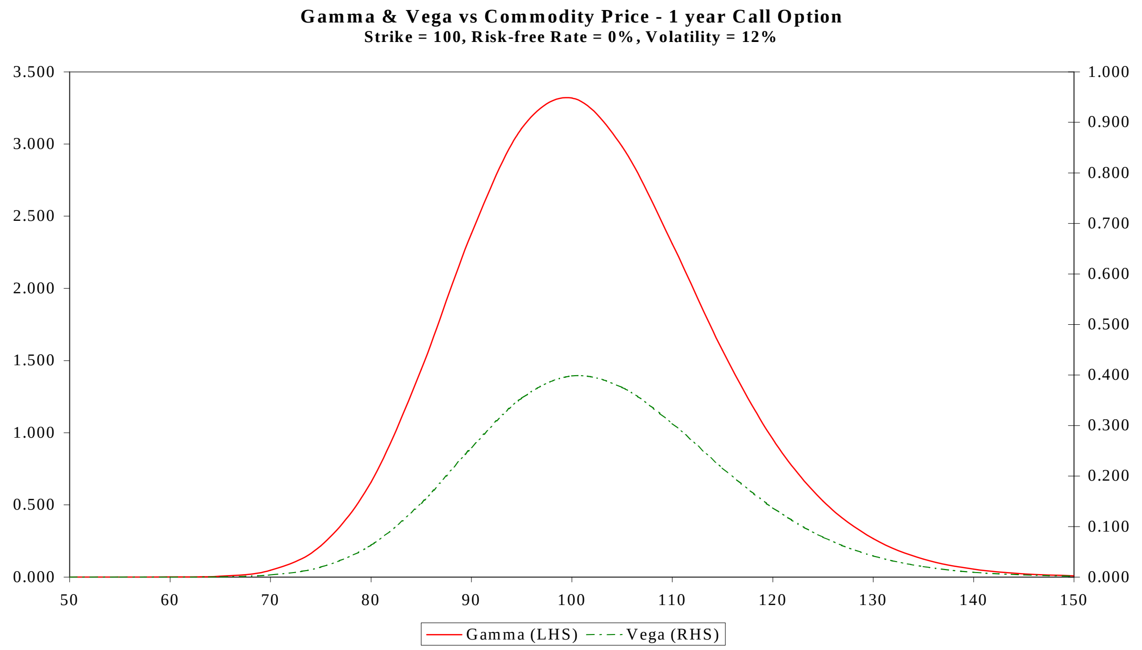
Vega is the sensitivity of the portfolio value to changes in the commodity price volatility.

A gamma hedge is where additional options are bought or sold which cancel out part of the gamma sensitivity of the portfolio, e.g. short positions offsetting long positions.

[*Note: the same applies to a vega hedge.*]

(ii)

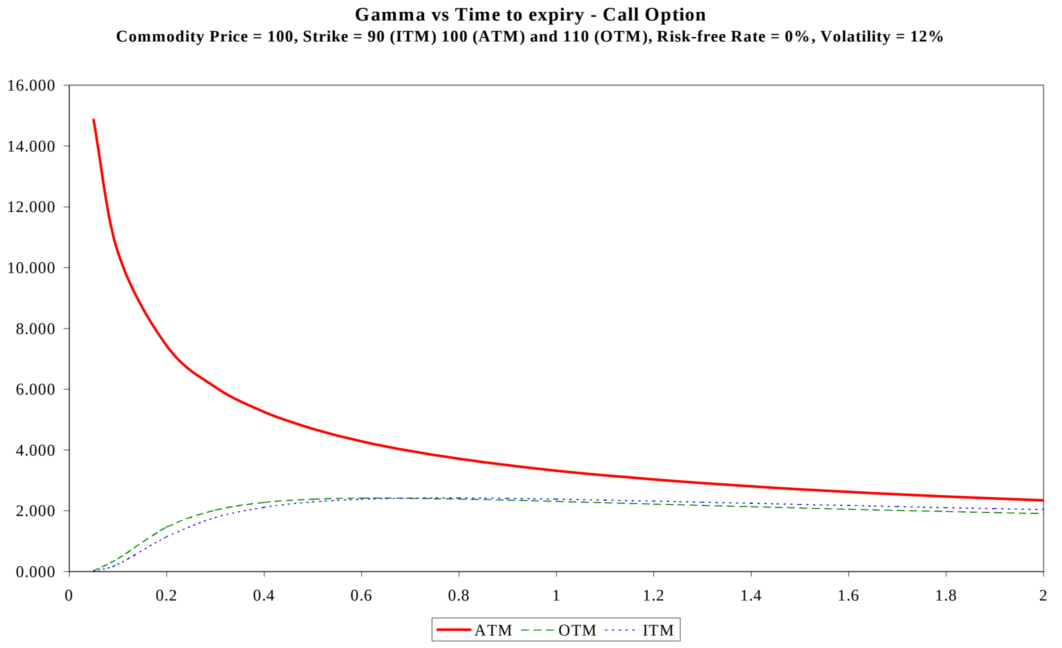
[*The diagrams given in this section are for a specific example. Total precision was not required – what was needed was a clear idea of the relationships involved.*]



<u>Gamma</u>

The curve of gamma against commodity price has a characteristic curve humped around the strike price (like a Normal distribution).

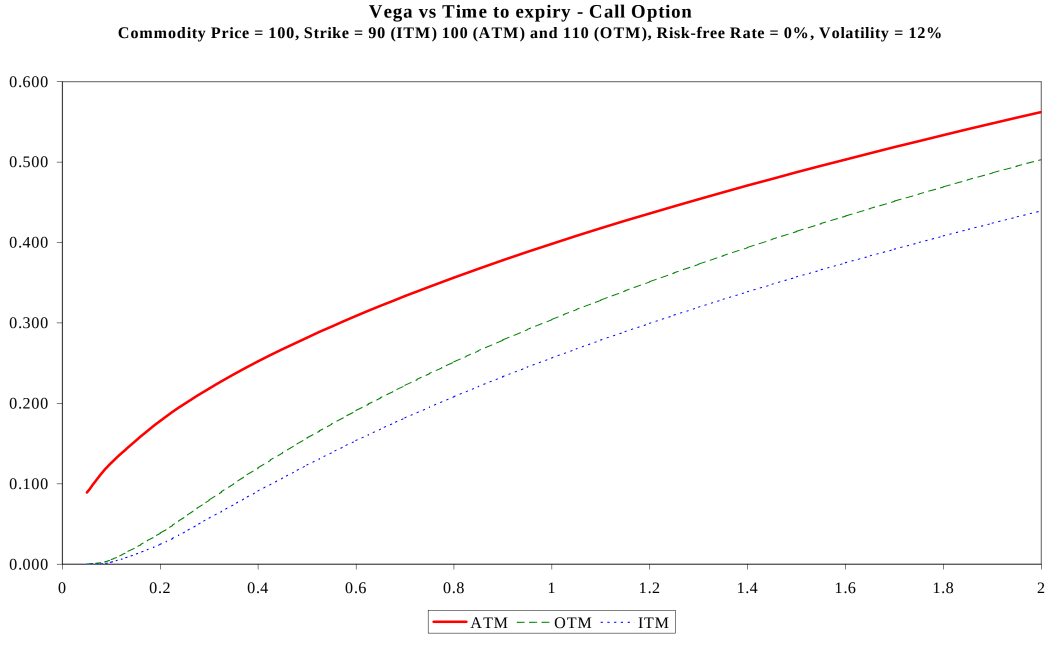
Gamma increases as time to maturity decreases, provided the option still has meaningful time value. Hence this effect is exacerbated for at-the-money options.



In- and out-of-the-money options have lower time value, so the delta varies less. Hence the gamma is close to zero for these near expiry. However, at the extreme, an exactly at-the-money option (i.e. commodity price = strike price) with only a few hours to run has almost infinite gamma.

Vega

Like gamma, vega against commodity price has a characteristic curve humped around the strike price.



Vega decreases as time to maturity decreases.

Longer-dated options have more vega sensitivity (*although this can be deceptive because long-dated volatility tends to vary less than short-dated*).

At-the-money options have the most vega sensitivity, as that is where the time value is greatest.

(iii)

Hedging

Gamma and vega hedges are dynamic so will change over time and as the commodity price changes – if not exactly matched, the hedge will need rebalancing.

Gamma hedge will not work as time to maturity decreases because short-dated gamma can increase dramatically, so would need ever increasing amounts of longer-dated gamma.

Vega hedge will not work as time to maturity decreases because short-dated vega is a lot lower, so would need very large amounts of short-dated vega to hedge long-dated vega.

Long-dated and short-dated volatilities move differently – the latter varies more over time.

QUESTION 5

Syllabus: 2.1 – 2.3

Reading: Hull Ch 2

This question was well answered in general, especially parts (i) and to some extent (ii). Not enough thought went into part (iii). For example, too many candidates merely stated "use currency futures", repeating what was given in the question, instead of defining which contract to use and whether the company would buy or sell it.

(i)

Forward contracts are bipartite agreements, usually entered at zero initial cost ...

... with tailor-made size, dates and underlying asset(s)

Futures contracts are traded on an exchange ...

... with a specific standard size, set of dates and underlying asset(s)

Forward contracts settle at the first forward date ...

... replicating the economic effect of a real transaction

Futures contracts also settle at the first forward date ...

... but according to a margined formula of difference

(ii)

Let the variable S be the current price in pounds of 1 unit of the foreign currency (dollars); i.e. the current exchange rate, expressed in pounds per dollar.

Let K be the forward price agreed to in the contract.

Let T be the term of the forward contract: $T \sim 9/12$ of a year.

Let $r_{UK(T)}$ be the continuously compounded risk free rate in pounds and let $r_{US(T)}$ be the risk free rate in dollars.

Let f_t be the value of the forward contract at time t.

The two portfolios that enable us to price a forward contract on a foreign currency are:

A: One long forward contract plus an amount of cash equal to $Ke^{-r_{UK}(T-t)}$; and

B: An amount $S\,e^{-r_{US}(T-t)}$ of the foreign currency (US dollars).

Both of these portfolios will become worth the same as one unit of the foreign currency (ie, one dollar) at time T.

For arbitrage freeness, they must be equally valuable at time t.

Hence:

$$f_t + Ke^{-r_{UK}(T-t)} = Se^{-r_{US}(T-t)}$$

Forward contracts are entered into at zero cost, i.e. $f_t = 0$ ...

... which means that $K = Se^{(r_{UK} - r_{US})(T-t)}$.

(iii)

To hedge $ assets in pounds sterling (£), the company would need to buy sterling-dollar futures (i.e. sell dollars, buy pounds). Divide the total by the contract size to find how many contracts.

Some problems in using futures:

(a) variation margin needs to be paid in cash, could be large if £/$ rate moves a lot => cashflow problems

(b) might not get enough liquidity when required, if position is large

(c) need to rollover contracts at 3 month intervals, and dates probably won't match => trading cost (not too great, though)

(d) futures basis moves around vs forward market – might enter/exit the contracts at wrong basis and lose money

QUESTION 6

Syllabus: 8.3

Reading: Hull Ch 22

This was undoubtedly the hardest question for most. Half the candidates achieved less than 25% of the marks. Difficulties lay in failing to apply a simple concept (an option valuation exercise) to an unfamiliar problem (a callable bond), although Hull does cover this fully in Chapter 22; also, in poor understanding of bonds, which is disappointing from a set of actuarial students. Unbelievably, several candidates tried to value the bond at the risk-free rate, rather than at its given yield. Note that the owner of a callable bond has <u>*sold*</u> *an option.*

(i)

(a) Duration is the average maturity time of the present value of the bond's cashflows.

(b) Forward price volatility is standard deviation of percentage changes in forward prices. It is used in the formula for an option valuation based on prices, the assumption being (in Black's forward price option model) that bond prices follow a Brownian motion. The appropriate forward date will be the option expiry date.

(c) As for (b) but for yields, so the assumption is that bond yields follow a Brownian motion.

(ii)

Duration is approximately equal to the sensitivity of price to yield:

$$D \approx -\frac{1}{P}\frac{dP}{dY}$$ [*This representation is* <u>*modified*</u> *duration.*]

So $dP / P \sim -D\, dy$

hence $dP / P \sim -D\, y\, dy / y$

Taking variances (and hence standard deviations) over a number of observations, as a first approximation the LHS gives the volatility of prices and the RHS gives $D\,y$ times the volatility of yields – this assumes D and y are constant, which for small variations they will nearly be.

[N.B. *The y term is important – missing it out in the calculation below gives a huge volatility!*]

(iii)

The current price of the non-callable 5-year bond is:

$$P = \left[\sum_{k=1}^{5} 4.\exp(-ky)\right] + 100.\exp(-5y)$$

where y = 4.70% is the cts yield (not the risk-free rate!), hence P = 96.465.

Now we have to work out the forward bond price in the risk-free world.

First, take off the discounted coupon due in one year's time, worth 4 x exp(-0.03) = 3.882, using the risk-free rate (<u>not</u> the bond yield!).

Hence forward bond price = (96.465 – 3.882) exp(0.03) = 92.583 x 1.03045 = 95.403.

The current duration is given as 4.41, so using this gives

fwd price vol = 4.41 x 4.70 x fwd yld vol = 4.15%.

In fact, we should use the forward duration, i.e. the duration in one year's time.

This will be roughly 80% of the current duration (amortising over 5 years) – this is not quite accurate, but will be close enough. (We get 3.53% this way, against a true value of 3.46%.)

This gives fwd price vol = 3.53 x 4.70 x fwd yld vol = 3.32%.

[*This is the value used below, although use of the current duration was also accepted since it was given in the question. If a candidate made a mistake in calculating the values above, gives marks below according to the application of those calculated values.*]

Now all we need is Black's model for a call option, which is a slight modification of Black-Scholes with forward price $F = Se^{rt}$.

Using our values, F = 95.403, X = 100, σ = 0.0332, t = 1 and r = 0.03.

Then $$d_1 = \frac{\ln\left(F/X\right) + \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}} = -1.4009$$

and $$d_2 = \frac{\ln\left(F/X\right) - \frac{1}{2}\sigma^2 t}{\sigma\sqrt{t}} = -1.4341$$

The cumulative normals for these values are 0.0806 and 0.0758 respectively.

Hence the call option value is:

$$C = \exp(-rt)[F\,\mathbf{N}(\mathrm{d}_1) - X\,\mathbf{N}(d_2)] = 0.106.$$

The callable bond price is therefore simply(!) the bullet less the option value, i.e. 96.359.

(iv)

As yields fall, both bonds will rise in price.

However, the short option position embedded in the callable bond will also increase in price, offsetting the rise. The option exerts a convexity effect as it nears its strike.

Hence the callable bond will rise in price more slowly than the bullet bond.

[*Note: If the callable option were American, the callable bond would be capped at 100, since if the price were over 100 the option would be exercised. In most cases, the option is European or Bermudan, so there is one date (or a few dates) when this would apply. Outside those periods, the option would exercise a drag effect as described.*]

QUESTION 7

Syllabus: 5.3, 6.1 – 6.5

Reading: B&R Ch 2 and 3

This question was on changes of measure in Brownian processes. It was long to write out in full, but ultimately involved little other than repeating bookwork from Baxter & Rennie. Candidates often found themselves lost in the middle parts (ii) and (iii), but then recovered to complete the slightly more straightforward part (iv), which took the candidate through the effect of changing measure on stochastic drift. Having seen the wide spectrum of answers given to part (iii), the examiners will look to frame such questions more precisely in future, but there still was little excuse for candidates listing features *of the Radon-Nikodym derivative, whereas the question had asked for* what was meant by it.

(i)

(a) The filtration **F** of the process *S* is the history of the process S, i.e: F_t is the path that the stock price *S* has taken up to time *t*, i.e., it is the set of all previous stock prices of *S* up to (and including) time *t*.

(b) Two measures **P** and **Q** are said to be equivalent if they operate on the same space and agree on what is possible (and on what is impossible).

Formally, if A is any event in the sample space

P(A) ≥ 0 if and only if **Q**(A) ≥ 0.

<u>*or*</u>

If A is possible under **P**, then it is possible under **Q**, and vice-versa. Also, if A is impossible under **P** then it is impossible under **Q**, and vice-versa.

(ii)

[*The joint likelihood function quantifies marginal probability distributions at any time t, conditional on every history* $\boldsymbol{F}_s$ *for all times s < t. In a sense, it captures the likelihood of a path.*]

Consider an arbitrary point in time t_i with $0 < t_i \le T$. Let x_i be a possible point which a Brownian motion process could pass through at this point in time t_i. Ignoring other points through which the Brownian motion may have passed before time t_i, we can use the probability density function:

$$f_P^{(i)}(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

to measure the likelihood that Brownian motion process W_{t_i} at time t_i passes through x_i .

Next consider the arbitrary large set of points in time $\{t_0 = 0, t_1, t_2, \cdots, t_n = T\}$, and the set of paths which go through the set $\{x_1, x_2, \cdots, x_n\}$ at times $\{t_1, t_2, \cdots, t_n\}$. The joint likelihood of this is

$$f_P^n(x_1, x_2, \cdots, x_n) = \prod_{i=1}^n f_P^{(i)}(x_i)$$

Writing $\Delta x_i = x_i - x_{i-1}$ and $\Delta t_i = t_i - t_{i-1}$, and using the fact that Brownian motion increments $W(t_i) - W(t_{i-1})$ are mutually independent, we can write down

$$f_P^n(x_1, \cdots x_n) = \prod \frac{1}{\sqrt{2\pi\Delta t_i}} \exp\left(-\frac{(\Delta x_i)^2}{2\Delta t_i} \right)$$ (which can be further re-arranged).

[*Integrating* f_P^n *over any given subset* $A \subseteq R^n$ *gives the probability that the vector* $(W_{t_1}, W_{t_2}, \cdots, W_{t_n})$ *is in A. This measures the probability that the discrete points at times* $t_1, \cdots, t_n$ *along the paths that the continuous process* W_t *can take, are in the set A.*]

(iii)

The Radon-Nikodym (RN) derivative is defined on a given path over a given time interval and it represents the relative likelihoods of that path under the two probability measures.

In the limit, as the mesh for the time interval becomes infinitesimally granular, the RN-derivative measures the relative likelihood of the path under the two probability measures. The relative likelihood is measured in terms of the ratio of the two likelihood measures, or equivalently, the amount by which the likelihood under the new measure Q has grown relative to the old measure P.

Let ω be any path of the Brownian Motion over the interval (0,T].

Let $\{t_1, t_2, \cdots, t_n\}$ be an arbitrarily granular ordered mesh over the interval with $t_n = T$.

Let $x_i = W_{t_i}(\omega)$ be the values that the Brownian motion process takes on path ω at the meshed time points.

Symbolically, $\frac{dQ}{dP}(\omega) = \lim_{n\to\infty} \frac{f_Q^n(x_1, x_2, \cdots x_n)}{f_P^n(x_1, x_2, \cdots x_n)}$.

The set of paths agreeing with ω on the mesh $\{t_1, t_2, \cdots, t_n\}$

$$A = \{w' : W_{t_i}(\omega') = W_{t_i}(\omega); i = 1, 2, \cdots n\}$$

gets smaller and smaller as the mesh gets increasingly granular. In the limit, as the mesh gets infinitesimally granular, the set consists of just one path.

The RN-derivative evaluated over a given path over a given time period can be thought of as the ratio of the probabilities of the path ω being observed under the two probability measures in the limit as the set of all possible paths that agree with the chosen path on which to evaluate the RN-derivative becomes the single path set { ω }.

<u>*Alternative:*</u>

In other words, the RN-derivative evaluated over a given path over a given time period can be thought of as the rate of increase in probability of observing the path ω under the new measure Q, compared with the old measure P, in the limit as the set of all possible paths that agree with the chosen path ω on which to evaluate the RN-derivative becomes the single path set { ω }.

(iv)

The first step involves writing re-writing dX_t as

$$dX_t = \sigma_t \left(dW_t + \left(\frac{\mu_t - \nu_t}{\sigma_t} \right) dt \right) + \nu_t dt \, . \qquad \{A\}$$

Next set $\gamma_t = \left(\frac{\mu_t - \nu_t}{\sigma_t} \right)$. $\qquad$ {B}

The third step involves checking that γ_t satisfies a growth condition

$$E_P[\exp(0.5\int_0^T \gamma_t^2 dt)] < \infty \, . \qquad \{C\}$$

If this is the case, then the Cameron Martin Girsanov theorem tells us that there is an alternative measure **Q** (equivalent to **P**), such that $\tilde{W}_t = W_t + \int_0^t (\mu_s - \nu_s)/\sigma_s ds$ which is a Brownian motion under measure **Q**.

However if $\tilde{W}_t = W_t + \int_0^t (\mu_s - \nu_s)/\sigma_s ds$, $\qquad$ {D}

then

$$W_t = \tilde{W}_t - \int_0^t (\mu_s - \nu_s)/\sigma_s ds \qquad \{D`\}$$

from which it follows

$$dW_t = d\tilde{W}_t - \left(\frac{\mu_t - \nu_t}{\sigma_t} \right) dt \qquad \{E\}$$

and hence (substituting {E} into {A})

$$dX_t = \sigma_t \left(d\tilde{W}_t - \left(\frac{\mu_t - \nu_t}{\sigma_t} \right) dt + \left(\frac{\mu_t - \nu_t}{\sigma_t} \right) dt \right) + \nu_t dt = \sigma_t d\tilde{W}_t + \nu_t dt$$

which establishes the drift of the process *X* under **Q** is $\nu_t dt$ instead of $\mu_t dt$.

QUESTION 8

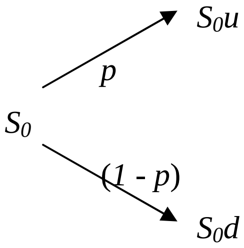
Syllabus: 6.5, 7.1.2

Reading: Hull Ch 12

What should have been a very straightforward application of a binomial tree process seemed to elude several candidates, although this may have had more to do with time pressures at the end of the examination.

(i)

Using u for up move and d for down move, $d = 1/u$



$$E(S_t) = S_0\left(pu + (1-p)d\right)$$

$$\begin{aligned}Var(S_t) &= S_0{}^2\left(pu^2 + (1-p)d^2 - \left(pu + (1-p)d\right)^2\right) \\ &= S_0{}^2\left(p(1-p)u^2 + p(1-p)d^2 - 2p(1-p)\right) \\ &= S_0{}^2\left(p(1-p)(u-d)^2\right)\end{aligned}$$

since $u.d = 1$.

Equating first and second moments [*key argument – several candidates missed this*]:

$$S_0e^{rt} = S_0\left(pu + (1-p)d\right) \qquad \{A\}$$

and $\sigma^2 S_0{}^2 t = S_0{}^2\left(p(1-p)(u-d)^2\right) \qquad \{B\}$

The solution to equation {A} is:

$$p = \frac{e^{rt} - d}{u - d}$$

Substituting into equation {B}gives:

$$\sigma^2 t = -(e^{rt} - u)(e^{rt} - d) = (u + d)e^{rt} - (1 + e^{2rt})$$

Multiplying through by u gives:

$$u^2e^{rt} - u(1 + e^{2rt} + \sigma^2 t) + e^{rt} = 0$$

This is a quadratic in u which can be solved in the usual way.

(ii)

$\sigma = 0.1$ and $t = 0.25$, so $u = \exp(0.05) = 1.051271$, $d = 1 / u = 0.951230$

and since $r = 0$, $p = (1 - d) / (u - d) = 0.48750$

t = 0	t = 0.25	t = 0.5	t = 0.75	
			116.183	*Node A*
		110.517		
	105.127		105.127	*Node B*
100		100		
	95.123		95.123	*Node C*
		90.484		
			86.071	*Node D*

(iii)

Notate the paths by U for up and D for down, in order.

The averages for each successful path are:

UUU = ¼ (100 + 105.1271 + 110.5171 + 116.1834) = 107.957 @ *Node A*

UUD = ¼ (100 + 105.1271 + 110.5171 + 105.1271) = 105.193 @ *Node B*

UDU = ¼ (100 + 105.1271 + 100 + 105.1271) = 102.564 @ *Node B*

DUU = ¼ (100 + 95.1229 + 100 + 105.1271) = 100.063 @ *Node B*

UDD = ¼ (100 + 105.1271 + 100 + 95.1229) = 100.063 @ *Node C*

with the remaining paths not exceeding 100 as an average. The payoff is the average less the strike of 100.

The probabilities of arriving at each node are:

Node A = $p^3 = 0.116$

Node B = $p^2(1 - p) = 0.122$

Node C = $p(1 - p)^2 = 0.128$

Hence the value of the Asian option

$= (0.116 \times 7.957) + (0.122 \times [5.193 + 2.564 + .063]) + (0.128 \times 0.063)$

$= 1.885.$

END OF REPORT