

**CERTIFICATE IN DERIVATIVES:
MATHEMATICS AND BASIC PRINCIPLES**

Examination Paper

April 1999

Time allowed: Three hours

INSTRUCTIONS TO THE CANDIDATE

1. *You have 15 minutes at the start of the examination in which to read the questions. You are strongly encouraged to use this time for reading only but notes may be made. You then have three hours to complete the paper.*
2. *You must not start writing your answers in the booklet until instructed to do so by the supervisor.*
3. *Write your surname in full, the initials of your other names and your Candidate's Number on the front of the answer booklet.*
4. *Mark allocations are shown in brackets.*
5. *Attempt all 9 questions, beginning your answer to each question on a separate sheet.*

AT THE END OF THE EXAMINATION

Hand in BOTH your answer booklet and this question paper.

In addition to this paper you should have available actuarial tables, derivatives formula sheet and an electronic calculator.

- 1** (i) Distinguish between spreads and combinations in the context of option trading strategies and name two different combination strategies. [4]
- (ii) Describe how a straddle can be constructed and develop formulae for the upside and downside break-even points. [6]
[Total 10]
- 2** (i) Define six elements of market risk. [6]
- (ii) Outline the method of measurement of market risk recommended by the Group of Thirty. [4]
[Total 10]
- 3** (i) Describe, using a long forward contract as an example, the conditions necessary for a credit loss to occur on a derivatives transaction. [3]
- (ii) Distinguish between the credit risk of over-the-counter (OTC) and exchange traded derivative contracts. [6]
[Total 9]
- 4** You are an equity options trader and wish to estimate the volatility of an equity security for use in your pricing model.
- (i) Describe two different methods of estimating volatility. [8]
- (ii) Discuss the issues which arise in deciding the volatility estimate which might be appropriate for use in your pricing model. [4]
[Total 12]
- 5** In practice, the probability distribution of stock prices is not exactly lognormal.
- Discuss the problems that arise if one uses the Black-Scholes model in pricing call and put options. [10]

6 Three call options on a share all have four months until expiry and have strike prices of 350p, 375p and 400p. The market prices of these options are 26p, 14p and 7p respectively. The current share price is 360p.

- (i) Explain how a long butterfly spread could be created using these options. [3]
- (ii) Draw a diagram of the pay-off profile for the long butterfly spread using the above data and calculate the maximum profit and loss from the trading strategy. [7]
- (iii) Describe the circumstances in which a trader might adopt a long butterfly spread trading strategy. [2]

[Total 12]

- 7**
- (i) State the no-arbitrage principle. [3]
 - (ii) An economy consists of three assets. At time $t = 0$, the assets all have market value 1. At time $t = 1$, the price of the assets will depend on which of two possible states the economy is in. The prices of the assets are given in the table below.

Table of Asset Prices

<i>Asset</i>	<i>Value at time $t=0$</i>	<i>Value at time $t = 1$</i>	
		<i>State 1</i>	<i>State 2</i>
1	1	1.05	1.05
2	1	1.03	1.07
3	1	1.09	1.04

Explain whether an arbitrage opportunity exists and, if so, how it could be exploited.

[5]

[Total 8]

8 A 1-year forward contract on an equity index with a continuous dividend yield of 3.5% per annum is entered into when the index is 3,250. The continuously compounded risk free rate of interest is 7% per annum.

- (i) Calculate the theoretical 1-year forward price for the contract. [3]
- (ii) Three months later, the index is at 3,900, the continuous dividend yield on the index is 3.1% per annum and the continuously compounded risk free interest rate is 6.5% per annum. Calculate the theoretical 9-month forward price at that time and the value of the forward contract in (i). [5]

- (iii) Explain the potential problems that can occur when using a published dividend yield to price forward contracts on an equity index. [4]
[Total 12]

- 9** (i) Explain the term “probability measure”. [2]
- (ii) Explain what is meant by saying that two probability measures P and Q are equivalent. [2]
- (iii) State the Cameron-Martin-Girsanov theorem and give a brief explanation of the theorem. [5]
- (iv) Let W_t be a P Brownian motion where X_t is the exponential Brownian motion with drift that satisfies the stochastic differential equation:

$$dX_t = X_t(\sigma dW_t + \mu_t dt)$$

Show that

$$X_t = X_0 \exp\left\{\sigma \tilde{W}_t + \int_0^t r_s ds - \frac{1}{2}\sigma^2 t\right\}$$

is a process that satisfies this stochastic differential equation for some Q Brownian motion \tilde{W}_t where r_s is a time dependent parameter. [8]
[Total 17]