VOLUME 1

SIMPLE RESERVING METHODS

VOLUME 2

MORE ADVANCED METHODS

CLAIMS RESERVING MANUAL

CONTENTS

VOLUME 2 — MORE ADVANCED METHODS

INTRODUCTION (1989)

The Institute of Actuaries has prepared and published this Claims Reserving Manual in the hope that it will help both actuaries and others interested in claims reserving practices.

Background

The reserve for outstanding and IBNR claims is an important item in the financial statement of a general insurance company, and actuaries are now being used increasingly to help set the amounts or comment on their adequacy.

An informal survey of the traditional methods in use in the UK market showed, not unexpectedly, a large variety of relatively simple methods, with a common theme based on the "chain ladder" or "link ratio" approach. There was also considerable ambiguity in the terminology used. A need was identified for a comprehensive claims reserving manual which would describe and classify many of the methods which are found to be in use, and set out clear definitions and assumptions.

Contents

The Manual is structured in short sections, to provide easy access to the numerous topics. Volume 1 begins with the insurance background, material on the basic purposes of claims reserving, the types of data available, and the internal and external influences which it is important to consider in setting reserves. This volume then goes on to give an extensive and structured review of the methods available and commonly in use at the present time. There are detailed comments on the assumptions involved, the dependence on the data, and the different approaches available.

Volume 1 deals with arithmetic or deterministic methods. Volume 2 includes more advanced methods involving probabilistic and statistical concepts. The test for the inclusion of a method in Volume 2 is that it has been used by a practitioner with responsibility for advising on the setting of reserves, or commenting on their adequacy, and that he or she finds it helpful. The fact that a method may contain weaknesses from a theoretical statistical viewpoint may be commented upon, but will not prevent its publication.

The standpoint of the Manual is that claims reserving is a practical subject which requires informed judgement. The advantage of a method should be that it helps to interpret the data and apply such judgement.

The readership

Volume 1 of the Manual is intended to be understandable to all who are involved in the process of claims reserving — insurers, actuaries, accountants, or consultants — whatever the level of their technical knowledge. To this end it presents reserving through a series of detailed arithmetical examples with careful definition of the market terminology.

Volume 2 is for practitioners with some familiarity with more advanced statistical ideas, but the aim of the presentations and examples is to give all the readers of Volume 1 an intuitive understanding of the methods by way of a summary overview.

The change in approach from Volume 1 is established by an opening article in Volume 2 which looks briefly at chain ladder methods from a statistical point of view.

Notation

Although Volume 1 avoids any specialised mathematics it is useful to describe each method by a symbolic shorthand which is devised in the context of each method as it is developed. A comprehensive system of notation covering all methods would have been cumbersome and incomprehensibly complex. However the symbols used conform to a general structure which is set out in the Glossary of Notation which appears at the end of Volume 1. Each of the more advanced methods described in Volume 2 requires its own specific mathematical notation.

Format

The Manual is loose-leaf, for ease of updating and inclusion of additional material. It is expected that the first update will be one year after first publication.

General comment

The Manual describes methods; it does not discuss the suitability of any method, nor the level of caution at which a reserve should be set, for any specific purpose such as tax or solvency. Hence the essential inter-relationship between assets and liabilities is ignored except for a short section on discounting. Within the chosen limited framework, intimately related actuarial subjects such as allocation of capital, return on capital, premium rates, investment strategy and release of profit are completely ignored, and the reader must make due allowance for this.

Throughout the Manual the phrase "claims reserve" has been used. In certain situations accounting terminology will use the phrase "claims provision" to mean the same thing.

Acknowledgements

G F Chamberlin of the Centre for Insurance and Investment Studies at The City University Business School prepared the first full draft of Volume 1 and was followed very closely by W W Truckle who carried out the revisions for the final stage. R J Verrall of the Department of Actuarial Science and Statistics at The City University acted as editor of Volume 2 within the terms of reference given to him by the Institute. The Institute wishes to thank these gentlemen and several others who also helped from time to time.

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Section 1
1997 UPDATE OF MANUAL

Introduction

The Claims Reserving Manual was first published by the Institute of Actuaries in 1989. Since then, actuaries have in increasing numbers been gaining experience of reserving in a wide variety of contexts. Along with this broadening of experience has been the carrying out of research, both among practitioners and in the universities, into reserving methodology and the theoretical principles underlying the methods.

The Faculty and Institute of Actuaries jointly arranged for the existing Manual to be reviewed in the light of these developments. The outcome of this review is that the Manual has been updated to incorporate additional material in each volume, principally by the inclusion in Volume 1 of the new Section 2 and the new Section E13, and in Volume 2 of two additional papers, précis of a number of other papers that have not been selected for inclusion in full, and a short introductory section giving a description of stochastic models. In addition, Volume 2 now includes a précis of each of the papers that appear in full, and also a disk incorporating spreadsheet programs to illustrate the application of methods described in two of the papers published in Volume 2.

Acknowledgements

The original version of the Manual, published in 1989, was largely the work of G F Chamberlin, W W Truckle and R J Verrall, as acknowledged at the end of the Introduction (1989). The 1997 revision of the Manual was carried out by a working party comprising J M Taylor (editor), J A Lowe and D H Reid (assistant editors), S J Brickman, A B English, G E Lyons, J R Orbell and H Vignalou. The Faculty and Institute of Actuaries wish to thank these persons and others who have helped in the preparation of the updated version.

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Section 2
INTRODUCTION TO RESERVING

[2A]
DESCRIPTION OF CLAIM PROCESS

The insurance background and the influences that need to be taken into account in the reserving process are described in some detail in later sections. In this Introduction we shall give a brief overview of what the claims process in insurance entails, and how the problem of reserving arises.

We may start by supposing that we have a "risk" situation, associated with an insurance "cover". The essential features are that there is a person or corporate body, whose financial condition is directly affected by the occurrence of certain "events" occurring over a defined period of time. An obvious example would be an individual with Private Car Damage insurance who would suffer repair costs following an accident causing damage to their vehicle.

If an insurance cover exists, an event occurring under the cover will give rise to an insured loss, which subsequently becomes a claim on the insurer.

Typically there will be some delay between the event giving rise to the claim, and the ultimate settlement of the claim with the insured. In the case of a Motor claim, this delay from event to settlement may stretch from a number of days to several years, depending upon factors such as complexity or severity of the claim.

Other significant dates are involved in this process. Following the occurrence of the event, a significant date would be the date upon which that event becomes known to the insurer. Whilst notification would normally occur quite soon after the event, there can be circumstances where a considerable period of time may elapse between the occurrence of an event and the notification of a claim to an insurer (for example, when a ship is damaged in harbour, but the damage becomes evident only when she is dry-docked at some later date).

Claim Reserves

The delay between event and settlement dates means that the insurer must set up "reserves" in respect of those claims still to be settled. The reserves required at any time are the resources needed to meet the costs, as they arise, of all claims not finally settled at that time. The insurer must be able to quantify this liability if it is to assess its financial position correctly, both for statutory and for internal purposes.

Throughout this Manual we are dealing with claims reserves in respect of events which have already occurred. This is distinct from future claims, arising from risks covered by the insurer over the remaining period of the policy, where the insured event has not yet occurred. Such liabilities are covered through the mechanism of an Unearned Premium or Unexpired Risk reserve, which are outside the scope of this Manual.

We are concerned then with reserves in respect of claims which have occurred as at a particular date — which we shall call the "valuation date". We can distinguish two categories of such claims:

- claims for which the event has occurred, and which are already known and reported to the insurer;

- claims for which the event has occurred, but which have not yet been reported to the insurer.

Reserves relating to the former category are normally referred to as "Known (or Reported) Claims" reserves; the latter as "Incurred But Not Reported (IBNR)" reserves.

This definition of IBNR applies usually in relation to Direct Insurance: in the context of the London Market, the usage of "IBNR" will normally include, in addition to reserves for unreported claims, allowance for any future deterioration in amounts outstanding on claims already reported. The latter element is normally called "IBNER" (incurred but not enough reserved).

A number of variations on the type of claims reserve are needed because of various features including:

- a claim may involve a number of partial payments separated over an extended period of time, culminating in its final settlement;

- claims may be settled prematurely by an insurer, and then require to be reopened for further payments or recoveries that subsequently come to light (this may occur on more than one occasion for a particular claim);

- the insurer may share the liability for the claim with other insurers, either by reinsurance or by co-insurance.

Thus the need may arise for reserves for re-opened claims, and for reserves net of reinsurance or co-insurance.

From this description, it is clear that reserves represent an attempt, at a point in time, to attribute a financial value to those payments still to be made in respect of a set of incurred losses, as yet unsettled. This cannot therefore be quantified with precision, but must be the subject of estimation. Varying assumptions about future influences on the outcome of those losses will lead to greater or smaller estimates in a given context, leading to the idea of strength of reserves. In consequence, information will often be needed as to the likely adequacy of a given estimate of reserves. This may in turn involve careful examination of the methods by which estimates are reached, and the assumptions on which they are based.

The description of claim events given above is somewhat idealised. In many instances, it will be possible to identify a finite event or occurrence which gave rise to a claim in the way described. However, in others, the question of defining the relevant event can be one of considerable difficulty and argument. We need only consider the situation with claims for compensation for employment-related disease, where the dates of the relevant event, or indeed its definition, can be difficult to determine. The need for modification and extension of the picture included here will be addressed in certain specific areas outlined below.

Need for Reserves

The need to calculate reserves arises in a number of different circumstances, of which the following are examples:

- assessing the financial condition of an insurer, since movements in reserves over a period are key to assessing its progress;

- pricing insurance business in the sense of estimating the future cost of claims on risks yet to be taken on (by extrapolation of past paid and reserved claim cost);

- assessing the solvency of an insurer, in terms of its ability to meet its liabilities (requiring assessment of likely upper limits of outstanding claim cost);

- putting a value on the net worth of an insurer, particularly for purposes of sales or acquisitions;

- commutations and reinsurance to close: that is, putting a financial value on the run-off of a portfolio of insurance business.

The strength that is appropriate for the reserves may vary from one of these circumstances to another.

Reserving Methods

Until the early 1970s, the approach to reserves commonly related solely to the area of reserves for known claims. The practice was for each claim to be individually assessed by a claims official at an early stage of its existence, and possibly at subsequent stages. These individual "case" estimates would then be aggregated to form a total reserve for outstanding claims. With the passage of time, and the increasing ability to subject the results of this process to statistical scrutiny, it was found that other methods or approaches to reserving might be more appropriate. Nevertheless, it is still the case that, in some areas of business, particularly where the numbers of claims are relatively small, or where they are particularly complex, case estimation is employed, possibly in conjunction with other methods.

However, in many instances the volume of claims is such that it would be impractical or too expensive to assess each claim individually and, in such cases, an alternative approach is required. Furthermore, in the case of IBNR reserves, specific claim files are not available for examination. In such cases, an alternative method of reserving is required.

The case estimator — claims official — requires access to details of policy cover, claim event and subsequent history before making an assessment of each claim. Similarly, in using statistical estimation methods, the reserver requires access to relevant data relating to the group of claims for which a reserve is required.

The more obvious of such data would normally relate to claim cost levels and perhaps numbers of claims settled in the recent past — together with some information such as the numbers of claims in the group unsettled at the present time. It might however be that, in some instances, the claims reserving specialist has available not only what might be called "hard fact" relating to actual past settlement experience, but additional "softer" information relating to the claims concerned. This may include more generalised market and economic information.

As will be discussed in Section 2B below, reserving methods would normally involve the application of a series of assumptions to underlying data from which a reserve would be evaluated. It is crucial, if use is to be made of soft data, that this aspect is fully reflected in the methodology employed and reports written. Indeed, in some methodologies, it might be felt that the nature of factual data employed (for example, assuming ratios of claims paid and outstanding to premiums for past claim years), is too questionable to lead to valid results. This is when the "softer" data may be of some value, even if they relate to more general market conditions.

In reviewing methods of reserving to be employed, an important initial point of consideration will be the nature and quality of data upon which the method will be based, together with the extent to which the use of a particular model is likely to be valid for those data.

For this reason, it is crucial that the individual carrying out the reserving exercise should be as familiar as possible with the underlying business concerned. If that person is situated within an insurance operation, then they should maintain close contact with underwriters and claims management. If not, then it is still important to understand as closely as possible the origins of whatever data sources are to be used.

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RESERVING METHODOLOGY — GENERAL

Introduction

This section gives a general overview of the methodology used in reserving. Most of the comments that follow could equally well apply to any situation where one is constructing a model, fitting it to past observations, and using it to infer results about future statistics of interest.

Whatever the purpose of reserving, in essence it involves the following steps:

1. Construct a model of the process, setting out the assumptions made.

2. Fit the model, using past observations.

3. Test the fit of the model and the assumptions, rejecting or adjusting it.

4. Use the model to make predictions about future statistics of interest.

By "model" we refer to an artificial creation whose function is to represent what is important in the process under consideration, but to omit aspects not considered relevant to the particular area of understanding. The design of an appropriate model thus involves a process of selection from among many possible models. Normally, as an aid to understanding, we would try to select as simple a model as possible — indeed, the model can be regarded as a deliberate simplification of the underlying process itself.

There is a very close analogy with the underlying scientific method, which involves the successive refinement and replacement of models to improve their validity and accuracy for the purpose in hand.

Unfortunately, in the real world of claims reserving, restricted availability of data often places severe limitations on the models that can be applied in practice. It is clearly inappropriate to use a model which cannot be supported by the data available.

We are thus led to a situation where, in some practical instances, although a particular model may be felt to have a number of disadvantages, the available data and knowledge of the claims environment do not support any further refinement of this model.

The alternative is to revert to an earlier stage of model development, and to produce an alternative approach based on somewhat differing assumptions as to the underlying nature of the claim process. This might be used to create an alternative version of the original reserving exercise. This process may in turn be repeated, each such development producing alternative estimates of reserves.

In practice, a consequence of the limited availability of information may be that we have no alternative but to introduce a fifth step to those described above, as follows:

5. Apply professional judgement and experience to choose a number.

In other contexts, particularly those where direct access to the underlying claims data files can be made, it may be possible to develop apparently differing approaches to a point of broadly similar estimate values. The more elaborate models to which this development gives rise are among those contained in Volume 2 of the Manual. Even here, however, judgement as to the choice of model and the value to be chosen to represent the reserve is still largely at the reserver's discretion.

Ultimately, our objective is to formalise, as far as possible. the use of numeric or other available information, together with the judgement of the reserver, through the use of appropriate models.

Section 2E discusses at some length the elements that make up professionalism and judgement, the need for which cannot be emphasised too strongly.

The following sections describe some of the general considerations that affect the steps outlined above, and some of the more specific points that arise when performing such a process in the context of reserving.

Constructing a Model

The type of model constructed will depend on the purpose of the exercise. Estimates of future claims for pricing purposes may be required at a very detailed level (by type of car or geographical location, for example), and to be produced quarterly or monthly. Reserves for high level management information or statutory purposes may be required only by broad classification or on an annual basis.

Similarly, the model must reflect the data that are available and their limitation. Some types of information may not be adequately recorded (details of claims settled at no cost, for example). In other instances, past data may not be available for a sufficient period of time historically, or be of too small a volume, to form a credible basis for fitting the model. When data of an adequate quantity or quality are not available, it may be possible to supplement the available company-specific data by reference to industry-wide data. Section E13 refers to some of the possible sources of these additional external data. This section also sets out techniques for projecting data beyond the available experience, by using regression or graphical techniques. Data availability and limitations are also discussed further in Volume 1 Section B.

The foundations of any model are the assumptions that underlie it. Ideally, any modelling process should start with a clear statement of the assumptions being made. The model can then be fitted to the data, and the assumptions tested. Some simple reserving methods do not set out explicitly the assumptions underlying the method. This may make it hard or impossible to gauge the appropriateness of that particular model.

There is no easy classification of the types of model that can be used for reserving, but there are various distinctions that can be made. In the first instance, one can make the distinction between models that do not appear to start from a formal set of assumptions. and those which rigorously set out the foundations on which the model is built.

Many of the methods in Volume 1 fall into the former category, and those in Volume 2 the latter. To produce valid reserve estimates using methods that fall into the first category, it is essential that the user should consider the implicit and explicit assumptions being made, and endeavour to test their validity or otherwise. Just because a method jumps straight to step 4, does not mean that steps 1, 2 and 3 can be forgotten! The apparent simplicity of the arithmetic involved in performing some of the simpler methods in Volume 1 should not distract the user from the complexity of the underlying assumptions being made when applying these techniques.

A second general distinction can be made in the approach taken in constructing a model. Many of the simpler models do not, on the surface in any event, start from a base point that considers the underlying process influencing the future claims payments (cars crashing, houses burning down, people receiving compensation for injury, and so on). Instead, at a crude level, they are simply taking one aggregate set of values (the past claims experience), and making some estimate from this as to how the progression of values will continue (the future claims experience). Step 3 of the reserving process (testing and adjusting the model), will, of course, bring information about the nature of the claims into the process, as will step 5.

By contrast with this initially arithmetic approach, other models begin with a recognition of the underlying nature of the risks, perhaps starting with assumptions about the frequency and severity of claims, and modelling how these may change over time.

A final distinction that can be made is between deterministic models and stochastic models. The future claims payments predicted by a reserving model are random events. We can never know with certainty what the future payments will be. The best that any model can do is to produce an estimated value of those payments. Deterministic models just make assumptions about the <u>expected</u> value of future payments. Stochastic models also model the <u>variation</u> of the future payments. By making assumptions about the random component of a model, stochastic models allow the validity of the assumptions to be tested statistically. They therefore provide estimates not just of the expected value of the future payments, but also of the size of the possible variation about that expected value.

All the models in Volume 1 are deterministic. Several of the models in Volume 2 are stochastic. The introduction to Volume 2 describes the general nature of stochastic models, and sets out some of their strengths and weaknesses.

Fitting and Testing the Model

Fitting a model may very often be a simple arithmetic process. Testing a model is not so straightforward. A model can be broken down into a number of parameters. For example, the basic chain-ladder model for an n ´ n triangle of data may be summarised as n different "levels" for each year of origin, and n different development factors for the n years of claims development, making $2n$ parameters in all. Other models may have more or fewer parameters for the same sized triangle of data.

The more parameters a model has, the better it may appear to "fit" the observed data, but the less stable it will become for predicting future values. As an extreme example, if one had, say, twenty data points, one could fit a model with twenty parameters that would "fit" the data perfectly. The model would, however, be completely unstable. A small change in any of the data points may result in a large change in the parameters fitted and the future values predicted. There is always a compromise between having enough parameters to fit the data adequately, but few enough to produce a model with a certain amount of stability and predictive power.

The amount of data available for reserving is often very limited. It is important therefore to appreciate the limitations of an over-parameterised model. This may be done, for example, by examining the effect on the model, and the predicted results, of small changes in the data to which the model is being fitted. We know the data are just one realisation of some random process, so a given data point could reasonably be a little larger or smaller than in fact was the case. For a model to be acceptable, ideally, the predicted future payments should not be greatly affected by small changes in the observed data. The user should therefore check carefully whether the results are heavily influenced by a few data points.

Testing the assumptions of a model can take many forms. Whilst there are a variety of statistical techniques that can be used for this purpose, the approach in practice may often be more pragmatic. This is particularly so for the methods in Volume 1, where "soft" information may be used as a guide in choosing development factors, for example. This is not easily amenable to explicit testing. Many of the simpler, chain-ladder based, models are based on broad assumptions about the stability of the types of business, and the speed of settlement of claims, for example. The user of a reserving model should attempt to validate that these assumptions apply to the particular class of business being considered. Ideally, this should be done in a quantitative fashion but, as a practical compromise, it may have to take the form of a qualitative assessment. For example, this may involve a review of the types of policy written, or a visual examination of whether settlement rates appear to have remained stable.

Throughout the text of Volume 1, the reader's attention is drawn to some of the assumptions underlying the different methods. An awareness and questioning of these and other implicit assumptions should be kept in mind whenever one applies the methods in practice. Section B of Volume 2 discusses in a little more detail the requirements of a "good" model, and some of the more general techniques for testing models in general and stochastic models in particular.

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RESERVING IN CONTEXT

So far, reserving has been presented in terms of points of principle, removed from the particular features of specific classes of general insurance business. As reserving must in the end be a practical application, it is worth setting out the main characteristics of some specific reserving situations, as they in turn will affect the choice of appropriate reserving method. The examples given are not intended to be exhaustive, but indicate the range of characteristics found in general insurance.

Reference should be made to a suitable general insurance primer for basic details of policy coverage found in different classes of business.

Private Car

The main classifications for analysing data are:

cover type
- third party
- comprehensive

peril
- fire
- theft
- windscreen
- third party bodily injury
- third party physical damage
- accidental damage

In general this business is amenable to statistical treatment, as there will usually be a large number of similar policies, generating a large volume of claims.

Claims are mainly in respect of property damage, with a smaller number of liability claims. Third Party bodily injury claims are subject to the greatest uncertainty, since

- the size of claims can vary enormously, and for seemingly similar situations

- claims can remain outstanding for 7–10 years in some cases

- claims inflation is difficult to extrapolate, as judicial inflation in particular tends to be fairly erratic in its development.

Creditor

The claims should be split between the main perils (accident, sickness and unemployment), as the frequency and duration of claims may differ markedly, and vary according to the economic cycle.

If sufficient information is available, it is possible to use an annuity-type approach to reserving. Each outstanding claim is reserved according to the number of months the claim has already been running.

Experience can differ markedly by scheme, reflecting different selling practices and customer bases.

Mortgage Indemnity Guarantee

Following a period of severe market losses, the standard insurance cover changed during 1993/4. The new cover required the insured (the mortgage lender) to retain part of the loss on a coinsurance basis, together with the imposition of a maximum indemnity per property. Whilst this will serve to spread the loss more evenly between the insured and the insurer, the basic claim characteristics remain unaltered, namely:

- claims arise from a combination of repayment default and loss of property value. Whilst there is a steady underlying level of default due to marital breakdown and individual financial problems, the main underlying cause is economic. Sudden sharp rises in interest rates, and increasing unemployment, are two such influences;

- the cost of claims is heavily geared to house price deflation;

- the delays in claim settlements reflect the degree of forbearance of the lender following the borrower's first falling into arrears, the attitude of the courts to granting repossession orders and, finally, the length of time it takes to sell the property in a depressed market.

New forms of cover continue to evolve, and there is currently considerable discussion about the possibility of substituting some form of aggregate excess of loss contract for the cover described above. Under this, the insurer would be able to cap its liability, both in terms of the aggregate losses retained by the insured before the contract comes into play, and in terms of the upper limit of aggregate losses then payable by the insurer.

Because, under existing arrangements, cover normally extends over the whole life of a loan, the main reserving problem concerns the Unexpired Risk Reserve — i.e. the reserve needed if the cost of claims yet to arise on existing loans seems likely to exceed the Unearned Premium Reserve. The determination of reserves for outstanding or IBNR claims for this class of business is relatively straightforward, compared to that of Unexpired Risk.

Catastrophes

Most catastrophes arise from natural causes (windstorms, earthquakes, floods, etc.). They will usually give rise to a large number of related individual losses occurring within a short period of time defining that event.

Other catastrophes may be due to man-made causes (explosions, air crashes, etc.). These may give rise to a relatively small number of claims, but ones of exceptionally high cost, both in terms of material damage and of liability.

If we confine ourselves to property insurance, the main thing to note is their atypical features:

- reporting patterns are a function of the exact nature of the catastrophe, as the speed with which the insured reports the claim will depend on the seriousness of the loss, whether it was possible or practical to report losses initially, and any emergency procedures set up by the insurer;

- settlement patterns are also likely to be different for each catastrophe, particularly in the first few days, being dependent on the insurer's ability to cope with the problem.

In view of the singular behaviour of each natural catastrophe, it is preferable to try to project each such event separately. This may take the form of a curve-fitting exercise. Alternatively, it may be possible to refer to the daily development pattern of previous catastrophes, subject to the problems in the immediate aftermath as the insurer gears up to tackling the problem. An assessment of the insurer's aggregate exposure may be utilised in this initial period. In the extreme case, it may be necessary to rely on any external market comment or assessment of total insured loss available at that time.

Reinsurance and the London Market

The London Market specialises in writing those risks that are too large for the smaller direct insurers to handle. A significant part of this business relates to overseas risks. In view of the size of the risks involved, much of the business is written on a co-insurance basis, spreading the risk amongst several insurers. The London Market is also a major centre for the acceptance of reinsurance business, again on an international basis.

A given piece of London Market business could therefore range from a very specific property insurance on one risk, to an excess of loss reinsurance contract covering an insurance company's entire world-wide general liability business. In practice, the common features of both Reinsurance and London Market business are:

- the data available to the insurer of this business are limited particularly for retrocession business;

- numbers of claims and individual claims information will often not be available, particularly for proportional business. Hence, any reserving methods requiring such data are not applicable;
- classification of the business is difficult, and the description and nature of the cover may need careful interpretation;

- sub-dividing the data into too small groups, to improve homogeneity, can give rise to excessive volatility as the statistical significance of the development model becomes increasingly suspect;

- the development of claims data is generally medium/long-tailed. Hence, IBNR often forms a significant element of the reserve amounts;

- the length of tail also means that projections are usually based on incurred claims data, rather than on paid claims, since the latter may require development factors to ultimate which are too large and sensitive to be reliable. This is particularly so in the early periods of development, when little or no claim payment may have been made for some classes;

- differing underlying currencies and inflation rates may distort aggregated data;

- There may be a "spiral" effect for catastrophe losses, due to the number of retrocession contracts written by all the companies making up the market. This means that such losses can cycle round the market, particularly for those years where there is an active retrocession market;

- Latent claims are even more difficult to deal with, as different cedants may use different triggers to determine reinsurance coverage. This will have a knock-on effect on the reinsurer's own reinsurance coverage.

Latent Claims

Most of the methods examined in the Claims Reserving Manual are dependent on having historic data which can be modelled statistically, in order to project the total liabilities still outstanding. However, there are circumstances where this is not possible, latent claims being a prime example. These are losses which may lie dormant for several years after the originating event before manifesting themselves. Industrial diseases and environmental pollution are typical sources of such claims.

Even when the latent hazard becomes a reality and claims are being settled and paid, as is now the case with asbestosis, standard projection methods are still not applicable. The normal course of events for latent claims is that, when the nature of the hazard becomes apparent. claims start to be reported at roughly the same time, irrespective of the underwriting year to which they relate. Amounts then increase rapidly.

This is completely different from the normal course of events, where regular development by "development year" might be expected. In the case of latent claims, however, large movements occur across each of the affected diagonals of the data when displayed in the familiar triangle format. Conventional "triangulation" methods of projection will not work in this situation.

Another problem with latent claims is that the historic data may not exist. Even when they do exist, they give little indication of what will happen in the future and, hence, how claims will develop. This is because the claims are dependent on the results of court actions, and these are highly unpredictable. The court actions determine whether coverage applies, if it is deemed to apply then who is liable, when coverage is deemed to apply, and the amount of liability (which may include considerable amounts for punitive damages). Also, expenses are very high covering, inter alia, lawyers' contingency fees, and are often incurred even when coverage is found not to apply.

Although the previous development of claims data may not be of much use in forecasting future development, there are alternative approaches that might be tried. These might include the following:

Share of total market: tracking losses

This involves initially determining what the total loss will be world-wide and the sources of the loss.

For example, for pollution in the USA, this may be done by site, first for known sites and secondly for not-yet-known sites. Assessment of the size of loss by site and the probability that insurers will have to pay (“win-factors”) have to be determined, followed by how the losses are aggregated and spread by year and between insurers and reinsurers. This is dependent on court actions to date and will vary state by state.

The individual insurer or reinsurer then has to follow through the alternative insurance, reinsurance and retrocession linkages to obtain its own ultimate forecast figures. This might be done via a decision tree, based on tracking the total market loss through the consecutive layers of insurance, reinsurance and retrocession. The total market loss will have been estimated by splitting the exposure into four markets (US, Lloyd's, London Market and Others), considering retained/insured percentages and share of the insured amounts by market at each level.

Share of total market: relativities

Again, the total market loss is considered. However, for this simpler approach the percentage of the total market is then estimated, using suitable market parameters (for example, by considering the insurer's premium income for the relevant classes of business relative to the total premium income for those same classes/insurers over the whole market).

Exposure

The book of business written is examined to determine, for each contract written, what the possible exposure is to the various latent hazards, and somehow estimating numbers of losses to each contract. This must take into account numbers of reinstatements, aggregation issues, occurrence and aggregate excesses and limits, and so on.

Market Practice

This is based on the assumption that, as the results are so unpredictable, the best that can be done is to be in line with the market. Therefore, if the comparable market is in general using an IBNR of (say) 1.5 times reported outstanding claims for asbestos-related claims, then this would be used as a benchmark. A higher multiplier might be considered as placing an unnecessary strain on the resources of the company; a lower multiplier might be questioned by the company's auditors, unless it could be proved that the company's relative exposure differed from that of the whole market.

Further Research

Much work is currently being undertaken worldwide in researching approaches to reserving for latent claims, particularly in the USA. To date, however, there is no universally accepted approach to these problems. Each case must therefore be considered carefully on its own merits by the reserver, bearing the above points in mind.

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[2D]
COMMONLY USED SIMPLE RESERVING METHODS

Introduction

The following section lists all the reserving methods found in Volume 1. It briefly describes the model underlying each method, together with its advantages and disadvantages, and refers to where more detailed discussions and examples can be found.

The main purposes of this section are:

a) while reading through the Manual, to act as a simple summary to help recap and consolidate the knowledge gained to date,

b) having read the Manual, to help identify a shortlist of possible methods to apply to a given reserving situation, and to act as a quick reference point for those wanting to refresh their memory about a particular method.

Consequently some of the terminology used may not be familiar until subsequent sections have been read.

Methods of estimating ultimate liability which do not explicitly allow for inflation

In the following descriptions, unless otherwise stated, "ultimate liability" is taken to mean the ultimate loss from a particular claim year, however defined. The total incurred claims cost can then be found by adding across all claim years and deducting the paid to date.

Method and	Description of the Model	General Comments

References

1. Grossing up (general) D6 El–4, 11–12 F3,7	Ultimate liability = paid* at delay *d* / grossing up factor for delay *d* [various ways in which the grossing up factor can be derived]	Simple. Assumes a stable run-off pattern, but is susceptible to error if this is not the case (e.g. if inflation is varying rapidly), particularly for the most recent years.
2. Grossing up (case paid at F5–6	Ultimate liability = + (case reserves at delay *d* / grossing up factor for delay *d*) [various ways in which the grossing up factor can be derived]	As l. Tests the adequacy and consistency of case reserves if used with data grouped by report year.
3. Link ratio (general) D6 E5–12 F4–7	Ultimate liability = paid* at delay *d* ´ link ratio at delay *d* ´ link ratio at delay *d* + 1 ´ ... ´ last link ratio [various ways in which the link ratios can be derived)	As 1.
4. Link ratio (basic chain ladder) E8	As 3. with the link ratio at delay *d* derived from the run-off triangle as the sum of column *d* + 1 divided by the sum of column *d* excluding the last entry.	As 1. Trends and anomalies in the data are not allowed for if operated blindly. Produces a single rigid estimate, without any indication of how to look for possible variations.
5. Loss ratio (general) D6–7 G2, 10–12	Ultimate liability = premium ´ ultimate loss ratio [various ways in which the ultimate loss ratio can be derived, and in the way that premium and loss ratio can be defined]	Very simple. Requires little information. May be the only method if claims data are scanty or unreliable. Ignores the claim development pattern to date. Prejudges answer if used naively.
6. Loss ratio (step-by-step) G9	Ultimate liability = paid* at delay *d* + premium ´ (loss ratio for delay *d* + 1 + loss ratio for delay *d* + 2 + ... + loss ratio for last delay) [various ways in which the loss ratios can be derived, and in the way that premium and loss ratio can be defined]	Simple. Insensitive to data in the most recent years. Loss ratios are based on the observed claims development pattern to date, rather than chosen arbitrarily.

7. Bornhuetter Ferguson D6 G3–8	Ultimate liability = paid[1] at delay *d* + (premium ´ ultimate loss ratio ´ proportion of the ultimate liability which will emerge in future) [various ways in which the ultimate loss ratio can be derived, and in the way that premium and loss ratio can be defined]	Credibility approach between statistical estimate and predetermined figure. Insensitive to data in the most recent years. Prejudges answer to some extent.
8. Average cost D8 H1–5	Ultimate liability = ultimate number of claims ´ ultimate average cost per claim [various ways in which the number of claims and average cost per claim can be derived]	Makes use of extra information. Easier to detect and allow for changes in the development pattern of numbers. Doesn't allow for changes in the number of zero claims or in the company's definition of settled. Difficult to apply if claims are relatively few in number.

[1] incurred claims (paid plus case reserves) may be used as an alternative to paid claims only, and can give a further perspective on the estimating of ultimate liability. However, whereas methods based on paid data require only a stable settlement pattern, those based on incurred data also require a stable reporting pattern and consistency in the setting of case reserves.

Methods of estimating ultimate liability which explicitly allow for inflation

These methods can be used when inflation is varying rapidly.

Method and References	Description of the Model	General Comments
9. Inflation J2	Most of the above methods can be based on historical payments inflated to current money terms. The resulting projected payments are increased in accordance with expected future inflation.	Need to choose appropriate past inflation rates.
10. Bennett & Method A J3	Ultimate (report year) liability = paid at delay *d* + number of claims reported ´ (inflation adjusted average payments in delay *d* + 1 + inflation adjusted average payments in delay *d* + 2 + ... + inflation adjusted average payments in last delay)	Need to choose appropriate past inflation rates.
11. Separation J4	Ultimate liability = paid at delay *d* + number of claims ´ (proportion of payments in delay *d* + 1 ´ index of average payments in payment year *a* + *d* + 1 + proportion of payments in delay *d* + 2 ´ index of average payments in payment year *a* + *d* + 2 + ... + proportion of payments in last delay ´ index of average payments in payment year *a* + last delay)	Past inflation rates derived from data — no need to choose appropriate rates. Also derives any other calendar year effects within the data. Relatively complicated. Unstable.

Methods of estimating IBNR values directly

Method and References	Description of the Model	General Comments
12. Simple ratios (general) I3–4	IBNR value = previous period IBNR value ´ current value of a quantity related to IBNR / previous period value of the same quantity [various quantities can be used]	Very simple. Requires little information. May be the only method if claims data are scanty or unreliable. Insensitive to changes in the relationship between IBNR and the chosen quantity. Extremely limited applicability.
13. Simple ratios (Tarbell) I5	As 12. with the quantity defined as the product of the number of claims reported in the last 3 months and the average size of claims reported in the last 3 months.	Very simple. Too crude for medium or long tailed business.
14. Average cost I6	IBNR value = number of IBNR claims ´ average cost per IBNR claim	Makes use of additional information. Difficult to apply if claims are relatively few in number.
15. Loss ratio (step-by-step) I7–8	IBNR value = IBNR emerged at delay *d* + premium ´ (IBNR emergence/premium at delay *d*+1 + IBNR emergence/premium at delay *d*+2 + ... + IBNR emergence/premium at last delay) [various ways in which IBNR emergence can be defined]	More suited to long tailed business.

Other methods

Volume 1 also includes methods

for tail fitting	E13
for estimating reserves for reopened claims	K2–3
for estimating reserves for claims expenses	K4–5
for estimating reserves allowing for discounting	L2–4

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Section A
THE INSURANCE BACKGROUND

Preamble

Before embarking on the methods and techniques for claims reserving, which make up the greater part of the Manual, it is important to establish the background to the work. Why is claims reserving such a vital topic in General Insurance, and what purposes does it serve in the industry? What are the characteristics of the main classes of business to which the reserving relates? And what is the place of the claims reserve within the technical reserves as a whole?

The present section provides answers for these questions, but in summary form only. The Manual is not, and cannot be, a study of the whole of general insurance. The crucial point to establish is that the methods do not operate in a vacuum. In themselves, they are but abstractions. The reserver should take as a starting point the concrete world of business which the methods are intended to serve, and keep such a view in mind. Claims reserving methods are of little value unless they become good practice as well as good theory.

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[A1]
PURPOSE OF CLAIMS RESERVING

Claims reserving in General Insurance is an activity which is critical to the success of the insurer. Its basic purpose is to estimate the cost of claims ultimately paid out of the business written to date. As such, it is concerned with the outcome of future events, and must therefore remain part art and part science. The contribution of the actuarial profession is, and will continue to be, to strengthen the scientific part of the analysis to the utmost degree possible. To this end, the present Manual is dedicated. This systematic attempt to improve the reliability of claims estimates in general insurance is both a worthwhile task in terms of the profitability of the industry, and a necessary one in terms of helping to ensure its continuing solvency.

From such a general statement, one must turn to the particular. What are the specific aspects of the business which are affected by, or indeed founded upon, the claims reserving figures? At least five such aspects can be picked out. While they are related, each has its own special significance.

a) Accounting to Shareholders — Annual returns under Companies Act.
b) Accounting to the Inland Revenue — Returns for tax purposes.
c) Insurance regulation and solvency control.
d) Ratemaking — Financial basis for writing future business.
e) General Management Control — Claims control, market strategy, etc.

The remainder of this section will be given to expanding the needs and requirements for claims reserving under each of these main headings.

Accounting to Shareholders

We are here concerned with the annual returns which every company must make under the Companies Act. The information so disclosed will be of vital interest to shareholders, current and prospective, and to stockmarket analysts, investment managers and others. The key questions which they will be seeking to answer concern essentially the profitability of the company and its future prospects in the insurance sector. Its strength vis a vis takeover activity may also be at issue, and if times are bad its continuing viability as an independent entity. This is not the place to go into a full discussion of company analysis, however. Suffice to say that, for a general insurance company, the largest single balance sheet item will very frequently be the reserve for outstanding claims. Because of its size, a comparatively minor variation in the value set upon the reserve may have disproportionately large consequences for the declared profitability of the company.

To give an idea of the magnitude of the figure, at the end of 1985 the largest insurers in the UK set their claims reserves in the region of 80–110% of the written premium income for the year. Their net trading profits over the previous decade, however, had averaged only approximately 5–10% of the written premiums.

These figures tend to emphasise the intrinsic paradox which underlies all general insurance. To a large extent, the costs of the business lie in the future, and are unknown in their precise extent. The claims reserve is the main reflection of such future costs, yet in the balance sheet it has of necessity to be stated as a precise amount. The uncertainty which is the essence of the business cannot be welcomed as a formal element in the company's financial statements to shareholders and others.

What, then, is the solution to the paradox? An accountant's view might be that the *best estimate* of the outstanding claims must be made. This might be defined, perhaps, as the position in which there is a 50% chance that the estimated amount will be exceeded by the actual out-turn. But the course is an insecure one, since an adverse out-turn could soon push the company towards insolvency. More satisfactory would be to take as the claims reserve a figure sufficiently large, that is fairly unlikely to be exceeded by the actual cost — say with a chance of 10% only, rather than 50%. Such conservatism will dampen the amounts immediately available for distribution to shareholders, but is likely to be in the better long-term interests of the company.

The question cannot be answered with any finality, however. It will depend on the particular circumstances of the evaluation, and on the expert opinion present. What is important is that there should be conscious consideration. The matter of the reliability of the claims reserve and the protection to be afforded against possible adverse experience should be addressed explicitly by those concerned.

Accounting to the Inland Revenue

It might be thought that the Companies Act returns (as discussed in a) above) would suffice for tax purposes as well. That is not necessarily the case: the Inland Revenue are not concerned with profitability or even solvency as such, but rather as to whether tax requirements as laid down by statute and regulation have been properly met. The tax regime controlling the insurance companies is a complex one, and a specialist subject in its own right. For reserving purposes, it is sufficient to note that there is no hard and fast right to tax exemption for the whole of the claims reserve declared in the annual company return. What may appear as common prudence to the finance director or the policyholder may be deemed overprovision by the tax inspector.

In contrast with paragraph a) above, the tax authorities may wish to require that allowable reserves be established only with strict regard to the "best estimate" principle. Margins to allow for possible adverse circumstances may well be ruled out of court, as being a device adopted for the postponement of taxation properly due. This may seem unfortunate and negative from the insurer's point of view, but one should appreciate that a different, legitimate stance can be taken by the tax inspector. The position may well need to be developed to its conclusion by cases at law — there is already some history in this regard.

Returns to the Insurance Supervisory Authority

Insurance regulation and solvency control are important matters which have been high on the agenda since the collapse of the Vehicle & General in the early 1970s. By now, there is a well-established system of regulation, based on the obligatory provision by the insurers of annual returns to the insurance supervisory authority. These returns require greater detail than that brought out by the Companies Act returns. In particular, claims reserves have to be shown broken down both by class of business and by year of origin. (This has resulted in the building up of a considerable bank of statistics. Their usefulness for general analysis, however, is marred by lack of consistency in the data-class definitions by different insurance companies and by the limited availability to the public at large.)

The supervisory authority is concerned essentially to protect the interests of the *policyholder*. Hence its emphasis is not on profitability or tax integrity, but on solvency itself. It follows that the conservative view of reserves described in a) above is entirely appropriate when compiling figures for the returns. The contrast with the best estimate view required for tax purposes could not be more evident — while the supervisory authority is looking for ample reserves to support solvency, the Revenue is demanding a paring down of those same reserves so as to maximise taxable income.

The existence of two contradictory requirements on the part of the Government apparently poses a dilemma when it comes to reserving. The truth that emerges, however, is a highly relevant one. It is that there can be no absolute *right value* for a claims reserve. The value will depend on the purpose for which the reserve is needed, and even then there will be room for a margin of error. Probability and uncertainty will always be present in an honest appraisal.

Ratemaking

Every company must establish and maintain a sound financial basis for writing future business, by sound underwriting and by setting premiums at an appropriate commercial level having regard to the three major elements of estimated claims costs, expenses and investment income. If premium rates are set and kept too low, then eventual insolvency will follow. If they are too high, then market share will be consistently lost to more aggressive competitors. The information that will allow a realistic setting of the rates comes from past and present experience. The level of the market as a whole needs to be looked at, in conjunction with the particular experience of the company. For the most recent view of the latter, the claims reserve on each past tranche of business will be needed. This, taken together with the claims already paid out, will provide an estimate of the incurred claims costs so that any inadequacy or oversufficiency in the premium rate will be detected at the earliest opportunity. The claims reserve is thus an essential part of the control mechanism which every efficient insurer requires.

What sort of estimate will be appropriate in this case? Should it be conservative in nature, or use the best estimate principle? The latter is more likely to be correct, since we are here considering the company very much as a going concern. In general, it cannot afford to be over-cautious in its ratemaking, or it will lose market share. Of course, the prevailing conditions of the market must also be taken into account. In a hard market, comfortable margins can be built in, whereas in a soft market they will be pared to the bone, with some policy lines even becoming effective loss-leaders. Thus, reserve estimates do not absolutely determine premium rates, but they are a vital input. As such, their realistic assessment is a key task.

General Management Control

Apart from the ratemaking process, claims reserving is vital to many other aspects of management control. An essential matter will be to monitor the company's premium writing capacity in relation to its free reserves after providing for expected claims and other costs. Other problems to be tackled will be those of claims control, market strategy and the identification of the relative profitability of different lines of business. As with ratemaking, it will be right to take reserves on a best estimate rather than a conservative basis. The matter of discounting will also be an important one to face. Although it has not been customary in the industry to discount estimates of outstanding claims, the evidence suggests that for purposes of management control it is a very desirable practice. The reason is that unless discounting is applied the pattern of future financial flows will be distorted, and give a different view of the relative profit and loss on given lines of business.

Another point at issue will be the subdivision of the data from the various classes of business. Modern data systems should allow the insurance manager to obtain finer detail concerning the lines under their control, and perhaps to isolate subclass characteristics which may enhance profit or loss. But there will be a limit to the process, in that statistical estimation of the reserves for very small classes becomes unreliable. (The matter is taken further in B2, dealing with the grouping of data.)

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[A2]
TYPES OF BUSINESS — THE PRIMARY MARKET

General insurance embraces a wide variety of contracts and covers. For practical reserving purposes it may be prudent to be aware of the effects of these variations, but the present discussion is confined to the chief types of business and their main characteristics, and how the reserving process may be affected by them. To begin with, it is useful to have a general classification of the field. The categories specified for solvency returns, in respect of all companies (primary and reinsurance) other than Lloyd's, make a good starting point. They are:

1) Accident & Health
2) Motor Vehicle
3) Aircraft
4) Shipping
5) Goods in Transit
6) Property Damage
7) General Liability
8) Pecuniary Loss
9) Non-proportional Treaty Reinsurance
10) Proportional Treaty Reinsurance

Classes 2) 3) & 4) include both physical damage and liability aspects. Classes 9) & 10) apply specifically to treaty reinsurance business. Facultative reinsurance is placed in with the direct business in Classes 1) – 8).

In general, the main classes will not be homogeneous in the range of risks they cover, although this will vary with the particular business mix of the company in question. An important issue with all classes, therefore, is whether it is necessary to subdivide for reserving purposes, and if so how the subdivision should be made. In some respects the returns require the main categories to be subdivided into risk groups.

Other dimensions are also important, which cut across the above classification. Thus, it is useful to separate personal and commercial lines; and to distinguish the primary, or direct, market from reinsurance written at Lloyd's (which is subject to special treatment under the returns) or on the London Market. In this section, we shall consider briefly the main primary classes listed above, with reference to the direct insurers. The next section (§A3) then looks at reinsurance and the London Market.

1) Accident & Health

Formerly designated "Personal Accident" in the statutory classification, this is essentially a personal rather than commercial type of insurance. Typical examples in the class would include holidaymakers taking out travel insurance, or families buying cover for private medical treatment. From the statistical point of view, personal lines have useful characteristics — i.e. a large number of policies are issued on relatively similar risks, which gives homogeneity to the class. However, as with life assurance the pattern of claims can be upset by atypical individual policies for particularly large sums insured.

2) Motor Vehicle

In spite of its ready familiarity, motor insurance is not a simple type. Thus, a UK comprehensive policy will cover property damage, third party liability, and possibly consequential loss as well. If homogeneity of data is the aim, the reserver may wish to analyse comprehensive business separately from third party only. They may also wish to isolate the physical damage from the liability element in the comprehensive class. Such refinements are not always possible in practice, however, and it is more important to make the most of the available data than to chase theoretical perfection.

Another source of heterogeneity in motor insurance is the difference between private cars and commercial vehicles. This is of great importance, and it would be usual to analyse the two groups separately. Again, there is the wide range in the vehicle types which can be covered, from motor scooters through private saloon cars to buses and heavy duty goods wagons. Such categories as motor cycles (private or commercial) and car or lorry fleets (commercial) may well need separate treatment.

In general, motor business is amenable to statistical treatment, with a large number of similar policies entering the reckoning. As such, it makes a good test ground for the development of systematic reserving methods.

3)–5) Aircraft/Shipping/Goods in Transit

Like motor, these classes of insurance are hybrid types, comprising both physical damage and general liability. They are particularly (though not exclusively) associated with Lloyd's and the London Market. Since there are key differences in procedure, e.g. the slip system tends to be used, and accounts are drawn up on a 3-year rather than a l-year basis, the group is better left until the next section (§A3).

6) Property Damage

This is a major class of insurance, in which the central cover is given against damage by fire. But the cover will normally be extended to many other perils, such as explosion, storm, flood, theft and riot. The important characteristic from a reserving point of view is that the run off of claims will be relatively brief in elapsed time. Thus, within 24 months of the accident year end one would expect the great majority of the outstanding claims to have been settled. The reason is that property damage is very evident in its nature, and relatively straightforward to assess. Classes of business with such a short run-off period are commonly described as *short tail* lines.

Within property damage, it will be necessary to separate out the personal from the commercial business, as the two types have very different characteristics. Once again, the personal business, mainly householders' policies, will consist of a large number of relatively similar units. These will be amenable to statistical treatment. The commercial side, however, may be more difficult to encompass, since the buildings and plant insured are likely to make a very heterogeneous collection. Reserving is most likely to be based on individual estimates for the various claims in question, as at the accounting date. Such case estimates will be made by expert assessors, either company employees or external loss adjusters.

However, this does not mean that statistical methods are ruled out for commercial property reserving. In the first place, the case estimates may need to be adjusted for bias. Second, an analysis of the claim size distribution and its development over time may give added insight about the incidence of large claims.

7) General Liability

For the reserver, it is in the liability class that the most profound problems are likely to arise. If property damage is taken as the typical short tail line, then general liability exemplifies the *long tail* side. Nowadays it is not uncommon to find liability run-offs extending for 15, 20 or even 25 years and more. The most notorious example is that of industrial disease claims resulting from exposure to asbestos. At the time that much of the insurance was written, the danger was unknown. But the subsequent claims have been upheld at law, particularly in the USA, and have resulted in a serious drain on the free reserves of the insurers concerned.

Apart from the emergence of previously unknown causes, other influences consistently work to extend the liability tail. Thus, the litigation required to establish liability in disputed cases may be a long drawn out process. Again, it may take years for the effects of bodily impairment or disease to become fully apparent. Until the ultimate condition of the injured party is known, damages cannot be properly assessed. During this time, the insurer must keep an appropriate reserve on the books. A case estimate, based on the most recent information, can be used. But given the timescales involved, a more realistic assessment is likely to come from combining the case estimate data with techniques of statistical projection.

Moving to the subdivision of liability business, employers' liability is likely to be analysed separately. Then the remaining aspects such as public and product liability will be taken together, in what has to be admitted is a very heterogeneous subclass. Professional indemnity, if such cover is given, will need to be treated as a further separate category for reserving purposes.

8) Pecuniary Loss

Pecuniary Loss is a very heterogeneous class of business. The main risk classes include mortgage indemnity guarantee, fidelity insurance, and the unemployment peril in creditor insurance. It may also include consequential loss (e.g. following fire damage to property), according to individual company practice.

In general, the experience is likely to be affected by the state of the overall economy. Claims may therefore exhibit behaviour consistent with that generated by a catastrophic event, rather than as an accumulation of independent risks. Setting reserves in some classes can be complicated by a lack of information provided by the insured, and by the long drawn out nature of the claim trigger (for example, in mortgage indemnity individual claims may not manifest themselves until 2–3 years after the borrower first falls into arrears with repayments).

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[A3]
TYPES OF BUSINESS — REINSURANCE & THE LONDON MARKET

Coming to reinsurance as opposed to direct writing, commercial operations are focused on the London Market. This is a distinct market from that of the direct insurers, but the separation is not absolute. At its centre, the London Market has the unique institution of Lloyd's. The broking, underwriting and accounting system which has evolved at Lloyd's gives the market its modus operandi, and distinguishes it clearly from the practices of the main direct writing offices. But Lloyd's itself should *not* be equated with the London Market, of which it is only a part. Institutions other than Lloyd's which typically participate in the market are:

Specialist reinsurance companies, both UK and foreign
Reinsurance subsidiaries of large broking firms
Home foreign departments of the large direct writing companies
Overseas branches and subsidiaries of foreign companies

A simple but incomplete definition of the London Market might be that it comprises all business which comes to be placed through the agency of Lloyd's brokers, using the Lloyd's slip system. This would be fine except that it does not allow for the considerable amount of business placed directly between reinsurers and other companies, without the agency of a broker.

Another point is that though the London Market is particularly associated with reinsurance, it also underwrites an appreciable amount of direct business. Direct marine and aviation insurance, comprising hull, cargo and liability covers are typically placed at Lloyd's, or with other London market firms. Lloyd's syndicates write a good deal of motor business, and indeed may take on direct risks in any of the other main insurance categories already described.

With these provisos, we may look at reinsurance, and its main types as transacted in the London Market. Three levels of classification are needed:

I) Type of Primary Business requiring Reinsurance

a) Marine
b) Aviation
c) Non-Marine (i.e. everything else)

II) Type of Reinsurance Cover

a) Facultative
b) Proportional Treaty
c) Non-Proportional Treaty (Excess of Loss/Stop Loss)

III) Length of Claims Run-off

a) Short Tail
b) All Other (including Medium and Long Tail)

The three classification levels are discussed very briefly below. For a full exposition of reinsurance on the London Market and the system by which it is written, the reader is referred to Craighead's papers and Kiln's book (details in §O).

I) Type of Primary Business requiring Reinsurance

Unfortunately, it is difficult to generalise about the sub-types under this heading. The mix of business will vary a great deal between different syndicates and reinsurance firms, and each will develop its own groupings for analysis. The three main subheads of Marine, Aviation and Non-Marine are essential in that they must be distinguished under the system of Lloyd's Audit Codes. That is the traditional division of the market, with different underwriters working in each area.

II) Type of Reinsurance Cover

The basic technical types of reinsurance are complex. Thus, facultative reinsurance can itself be proportional, or relate to an excess layer of loss on a given risk. It can comprise fleet covers (in aircraft, shipping or motor) as well as individual risks. Proportional treaties can be for quota share on a full portfolio, or on designated lines of business only. They can be in favour of a direct office, or of another reinsurer. Non-proportional treaties can be for excess of loss protection on given classes of business, and written in a number of distinct layers. They can be applied as a further safeguard to existing proportional treaties, and can also take the form of Stop Loss contracts on a whole portfolio. The position is thus an elaborate one, and the following is probably a minimum classification for the Lloyd's syndicate or London Market reinsurer:

a) Direct written business
b) Facultative reinsurance
c) Proportional treaties
d) Excess of loss/Stop loss treaties

Individual excess of loss risks are perhaps better taken as b), facultative business, than under d). In addition, Craighead (1979) recommends that excess of loss treaties should be split according to their origin, i.e. whether from the London Market or from the direct writing companies.

III) Length of Claims Run-off

Of major importance for reinsurance reserving is the length of the claims run-off. Whether the business is marine, aviation or non-marine, and irrespective of its technical form, it is likely to contain both property and liability elements. The general rule is that property damage will lead to a short tail in the run-off, and liability to a medium or long tail. Thus, given a particular reinsurance contract, it will always be useful to estimate the split of the risk between the short tail and the long tail elements. The claim amounts which actually emerge can then, if possible, be monitored over time to test their adherence to the original long/short estimate.

(It should be noted that reinsurance, of its nature, will lead to longer run-offs for all classes than will the writing of direct business. The reason is simply that there are more steps in the chain to be completed before accounts can be finally settled. Delays in the original reporting of claims, in the communications between broker and underwriter, and in the completion of complicated settlements across international boundaries, all add to the effect.)

Taking all the above classifications together there will be a number of separate reserving categories for each portfolio. Other distinguishing factors, for example currency, may increase the number of categories. However, practical limitations may make it necessary or justifiable to amalgamate some of the categories.

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[A4]
NOTE ON TECHNICAL RESERVES

While the subject of the Manual is Claims Reserving, other kinds of technical reserve will be encountered in General Insurance. Some of these are effectively special aspects of the main claims reserve, e.g. the IBNR reserve, and as such fall within the scope of the Manual. Others, however, in particular the unearned premium reserve, are outside its ambit. For convenience, this note briefly distinguishes the various types of technical reserve.

Reserve for known outstanding claims

At any given accounting date, there will be a number of claims on the books which have not yet been settled, or at least not finally settled. The insurer's estimated liability in respect of such claims may be referred to as the reserve for known outstanding claims. It forms part of the overall claims reserve. (See also note on p. A5.1).

IBNR Reserve

IBNR stands for *Incurred but not reported.* It refers to claims whose date of occurrence lies in the period on or before the accounting date, but which for some reason have not yet reached the insurer's books. Damage and liability can take time to become manifest, and there will be delays in the reporting and recording of claims even under the best of circumstances. Hence the need for the IBNR reserve, which relates to the claims which are effectively hidden from view. Apart from outstanding claims, IBNR forms the other main portion of the overall claims reserve.

Unearned Premium Reserve

At the accounting date, for each policy which remains open on the insurer's books, a part only of the contracted risk period is likely to have elapsed. If a premium was payable, say, on 1 October for one year's cover, then nine months will remain when the accounts are drawn up on 31 December. A proportionate part of the premium must be retained as a reserve to cover the period of risk from 1 January onward. (In this case, ignoring inflation, the portion would be 75%, less some allowance for initial expense.)

Such a reserve, for risk periods subsequent to the accounting date is known as the *unearned premium reserve* (UPR).

Unexpired Risk Reserve

The UPR, being based on the premium, may be inadequate if the premium itself is insufficient to cover the cost of the risk and expenses. Hence an upward adjustment may be needed, and this increment is known as the *unexpired risk reserve*.

Catastrophe Reserve

With an event such as a severe earthquake or hurricane, a large number of connected claims for personal accident, property damage, consequential loss and general liability will inevitably arise. In such cases, the normal provision for future claims, based on the concept of independent events, may be entirely inadequate. Hence, where the type or geography of the risks written indicates the insurer's susceptibility to catastrophe loss, an additional cushioning of the reserves may be considered.

Fluctuation Reserve

It is in the nature of things that an insurer's claims experience will fluctuate from year to year. Even without the occurrence of an identifiable catastrophe, random variation may throw up one, or even a series, of lean years. For protection against such an out-turn, the insurer may wish to establish a fluctuation reserve. In practice there may be little conceptual difference between a fluctuation reserve and a catastrophe reserve; and it is unusual for companies to show them explicitly in the UK where they are not allowable for tax purposes.

Claims Equalisation Reserve

For certain defined categories of business, UK insurance companies have from the end of 1996 been required to hold a fluctuation reserve, knows as a *claims equalisation reserve* (CER). The maximum amount of the CER, and the amounts to be transferred to and from the CER, are specified by statute. A transfer to CER is treated as a deduction from pre-tax profits, while a transfer from CER is treated as part of the taxable income of the company in the year in which it is made.

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[A5]
NOTE ON TERMINOLOGY

Rather unfortunately, there are several terms of central importance in claims reserving which can be given quite different meanings by different users. It is important to be aware of the ambiguities, and to be sure of the meaning intended in a given context. The chief problems arise with the terms *Outstanding Claims, IBNR, Incurred Loss* and *Chain Ladder Method,* and are described below. (The list is not supposed to be exhaustive.)

Outstanding Claims

The term has often been used to refer to claims on the insurer's books by the accounting date, but not by then settled — in other words, to the known outstanding claims. When this practice is adopted, *IBNR* claims are specifically excluded. The totality of claims for which reserves must be held then consists of *Outstanding Claims* plus *IBNR Claims*, although since the term *Outstanding Claims* is often used to refer to this totality there is obviously scope for confusion.

Unfortunately there is no universally agreed nomenclature. In the Manual we will use the following terminology —

a) *Open Claims* means claims which have been reported to the insurer and which are not yet settled.

b) *IBNR Claims* means claims which have been incurred but not yet reported to the insurer in question (see next paragraph).

c) *Outstanding Claims* means the total of a) and b).

IBNR

A further ambiguity arises with the term *IBNR* itself. It is often used as such to refer to the reserve as well as to the group of *IBNR* claims. That in itself is no problem: at the accounting date one has a set of open claims and an open claims reserve, plus a set of hidden claims and an *IBNR* reserve. But the ambiguity arises once one begins to consider the progress of claim settlements subsequent to the accounting day. Taking the group of open claims, the actual payments will not precisely match the reserve previously set — there will be a development, which may be upward or down. One definition of *IBNR* is such as to ignore this development, and it is the sense that will usually be taken in the Manual. The alternative definition of *IBNR*, however, deliberately *includes any development in the open claims.* In other words, the meaning of the *IBNR* reserve becomes:

IBNR = Estimated ultimate loss on all outstanding claims
less Reserve at accounting date for open claims

This seems less natural than the first definition, but it has real point in some circumstances. It is used in this sense in the London Market with particular reference to reinsurances. The Manual will specify where appropriate the sense in which the term *IBNR* is being used in the particular context.

Incurred Loss

There are two distinct definitions of the term Incurred Loss depending upon the context. The first arises when one is considering the insurer's portfolio of business as a whole, or perhaps a given class within the portfolio. Interest lies in the progress of the portfolio or the class over the course of the accounting year, and *Incurred Loss* is defined as:

Incurred Loss = Claims paid during course of year
less Claims reserve held at 1 January
plus Claims reserve established at 31 December

The second use arises equally naturally when one looks at a particular tranche of business which has its origin in a given policy year or a group of claims originating in a given accident year (normally referred to in actuarial work as a *cohort).* It is interesting to follow the progress of the cohort, and at each subsequent accounting date to assess the losses attaching to it. The estimate, omitting the IBNR element, is again called the *Incurred Loss.* It is:

Incurred Loss = Amounts paid to date on settled or partly settled claims
plus Reserve held for open claims

In the Manual, *Incurred Loss* will be used exclusively in this second sense. That is because the reserving methods discussed are very generally applied on a cohort by cohort basis. The overall picture for the portfolio or class is later found by adding up the parts, and is less commonly in question.

Chain Ladder Method

The term is a very familiar one to claims reserving practitioners. The problem is that it is sometimes used in a particular sense, and at other times very generally. In the latter case, it describes a wide range of reserving methods, which operate through comparing the claims development of cohorts of different years of origin, and which tend to employ triangular arrays of data. This usage leaves something to be desired. It obscures the important question as to what data are actually being used — many different possibilities exist. Also, it tends to suggest:

a) that a triangular array must be used, and
b) anything that is in triangular form must necessarily be a chain ladder.

Neither of these propositions is true.

In its particular sense, *Chain Ladder Method* is used to describe one means, and one means only, for evaluating the triangular array (see §E8).

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Section B
DATA & FORECASTING

Preamble

This section introduces some of the main building blocks for claims reserving. To begin with, there is the important idea of making a projection of past experience into the future. Since the future never takes the trouble to conform properly with the past, any projection whatsoever will be subject to error. One needs, therefore, to understand the principles which can lessen the likely degree of error, and so bring credibility to the work.

Apart from those principles which make for stability, there is the matter of the data themselves and the actual methods of forecasting. These are not intrinsically difficult matters, but there is a fair amount of detail to be mastered. On the data side, a number of different quantities can be used in the projections, or as supporting evidence — not only claim amounts, but such items also as claim numbers, premium income and loss ratios. They can often be displayed in different ways in the search for pattern and regularity, and the concept of the development table is particularly important here. Then there is the question of data validation, and of how the classification of the risk groupings is to be made.

On the forecasting side, there are some surprisingly simple methods available. It is straightforward, almost intuitively obvious, to look for the average or trend which is present in a sequence of figures. The really vital question to ask is whether the available evidence supports the continuation of such average or trend into future periods. Although far more elaborate types of projection can be devised, it is these simple foundations on which they rest, and which should therefore first be thoroughly understood.

Contents

[B1]
THE PROJECTION OF PAST EXPERIENCE

In claims reserving, the aim is to estimate the future claims experience which is to be expected on the business written to date by the insurer. As a first approach, the values set as case reserves on open claims by the claims handling staff may be used. However, some variation in these before final settlement of a claim is likely to take place, and by definition such values cannot cover the IBNR component of the required reserve. Thus it is usually necessary to go beyond the case reserves. The approach which then emerges, quite naturally, is to look at the insurer's past history of claims experience and to project this forward to the future years.

Taking this approach, the first need will be for suitable historical data. Ideally they will consist of such items as the number of claims reported and the number settled, and the amounts paid out by way of settlement. There will be information on the premiums written or earned, and perhaps other measures of risk exposure such as the number of units covered (e.g. households or motor cars). The data will be classified according to the class or sub-class of business involved, and also by the year of origin (i.e. accident or underwriting year). For each class and year, and for each data element, there should then be a series of figures showing the development with time up to the current date. In addition, figures showing the development of the case reserves themselves may be available, and can also be used as a basis for projection.

The second need will be for a method of projection, and very many of these have by now been devised. They range from the use of simple arithmetic on the familiar triangular arrays of data to the employment of highly sophisticated mathematical and statistical techniques. From the number of different methods available, the problem is to select that which is most appropriate in the given circumstances of each particular case. The Manual's main purpose is to describe the methods, together with their advantages and disadvantages.

Given that the data are available (and there will often be gaps and deficiencies), and that the skill needed for the projection is to hand, a leading question has now to be faced. That is, to what extent is it actually justifiable to project forward the experience of the past on to the future years of development? There can perhaps be no final philosophical answer to such a question. However, the theoretical understanding of statistics and probability, borne out in practical experience shows at least that it is *reasonable* to make such projections.

The projections, however, cannot be done arbitrarily. A systematic approach needs to be adopted. To begin with, for example, the reserver should scrutinise the data with which he or she is presented, or which he or she intends to collect. Apart from the obvious point of its validity and consistency, the stratification of the data into the main business classes and the risk subgroups will be of great importance too. For each subgroup, the larger it is and the more homogeneous the risks it contains, the greater the degree of statistical stability will be. Generally speaking, however, the desiderata of size and homogeneity tend to work against one another, as shown more fully in §B2. The reserver must find, and be prepared to justify, a suitable compromise in the risk classification to be adopted.

A further point of major importance will be to examine the *influences* which have shaped the claims pattern in the past, and how these may currently be changing. Such factors as the volume of business or the rate at which claims are handled, the level of inflation or the legislative climate, can all affect the position. If such influences are properly understood, then significant shifts in the experience may be detected promptly in advance, and taken into account by adjusting the projections. (A fuller discussion of the main influences on the claims pattern follows in §C.)

Above all, in claims reserving it is not sufficient just to take the data and blindly apply the first projection method which comes to hand. At each step, intelligence has to be applied. There are key questions which need to be answered afresh each time a new projection is to be made. A checklist now follows:

a) What historical data are available to the reserver, and how far can confidence be placed in its reliability?

b) To what extent is the homogeneity of the groups in the risk classification satisfactory?

c) What conditions have shaped the past experience, and what significant changes in them can be detected which may affect the future out-turn?

d) What methods of projection are proposed, and are these properly suited to the given circumstances?

To ignore these points is to ignore the whole essence of the work.

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[B2]
DATA GROUPINGS: PRINCIPLE OF HOMOGENEITY

The underlying principle of insurance is statistical in nature. In the business of taking over the risks of others, the insurer is best protected by taking on a sufficient number of similar, but independent, risks. The proportion actually becoming claims, and the amounts payable, can then be predicted within manageable margins. Hence an adequate premium can be set with some confidence in advance of the risk period itself. This is a result of what is popularly known as "the law of large numbers", but which appears in statistical theory as the necessary relationship between the variance of a sample and its size.

As with insurance at large, so it is for claims reserving in particular. Stability in projections is to be sought by aiming to work with data groupings each containing a sufficient number of similar but independent risks on the assumption that they determine the characteristics of the resulting claims. The question is, how far should the classification of business be taken in order to produce such individual risk groups? To begin with, there are the main types of business, such as Motor, Property, Liability and so on. These are reflected in the supervisory authority classification, and must therefore be observed for the purpose of statutory returns. Such an initial classification will be desirable also from the point of view of reserving. But the heterogeneity of many of these main classes is such as to make further subdivision essential.

To take the example of Motor, it will certainly be necessary to separate out Private from Commercial business. Private Motor can then be further classified into:

Motor Car Comprehensive
Motor Car Non-comprehensive
Motor Cycle

Again, the division seems necessary, given the different risk combinations covered by comprehensive and non-comprehensive business, and the different characteristics of motor cycle riders as a class from those of car owners.

The subdivision could again go further within each of these three categories. Thus we might use distinguishing features, say, of geographical area, make of car or cycle, age of driver, and so on. The further we take the classification in this way, the greater the homogeneity in each of the resulting risk groups. But the stage can soon be reached where the individual groups lose their statistical credibility. That is, their size (or lack of it) is such as to produce an unacceptably high variance, at least so far as claims reserving is concerned.

In practice, the ideal of homogeneity is not to be pursued with too much rigour. Indeed, often it cannot be so pursued. The data themselves may not permit very much subdivision — e.g. the required fields may not have been inserted into the data-base records in the first place. Again, the time available for the work of reserving will not be unlimited, and to multiply the sub-groups multiplies the work to be done. A sense of proportion has at all times to be kept.

To return to the Private Motor class of business, most insurers would be unlikely to go much further than the 3-way split suggested above. Indeed, some might treat Comprehensive and Non-comprehensive together as a single risk class. Strictly speaking, this is not a desirable combination — the two classes have a quite distinct risk profile and claims run-off. Thus, bodily injury and other third party claims will be the main element on the non-comprehensive side, while the comprehensive will have a more even split between third party and physical damage. Hence, although the overall length of the tail may be the same in both cases, the comprehensive business will show the stronger early development. This will be especially true in the first two years or so, during which time virtually all of the physical damage is likely to have been settled, but comparatively few of the major liability claims.

The justification for taking the two groups together can only be that the proportion of comprehensive to non-comprehensive is reasonably stable, and thought likely to continue so in the future. The patterns may be upset if there is a sharp change in business volume during the course of a particular year of origin.

Generally speaking, with personal lines business, lack of homogeneity would not be expected as a problem. The number of policies will often be large, with the individual amounts at stake relatively small, thus providing good conditions for statistical treatment. But in the commercial lines, where each risk taken on will have its own special characteristics, and where there may be relatively few policies issued, the homogeneity of a class will frequently be in doubt. The problem may be exacerbated by the existence of unique risks for very large sums insured.

The solution adopted may vary with the class of business involved. Thus, in commercial property, the answer may be to rely more on the case estimates than any statistical projection, or at least to use the case estimates as the main source of data for adjustment. In commercial liability, however, such is the length of the tail that case estimates may not help a great deal, except for the older years where some development has already taken place. The reserver will therefore be thrown back on statistical projection, but without the comfort of a firm underpinning. The need for a full and intelligent assessment of the conditions and influences surrounding the business will be all the more important, and no projection should be regarded as sound without such an assessment to back it up.

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[B3]
THE CLAIMS DEVELOPMENT TABLE

This section looks at the main claims data that will be needed for the work of reserving, and the format they are likely to take. Suppose that the risk classification is already established, with proper regard to the constraints of size and homogeneity in the subgroups. We then wish to examine the data for a particular group, say as at the 31 December for the current accounting year. What form will the information from the insurer's data-base take?

To begin with, there will be the claim amounts paid out during the course of the accounting year just past. Let us say the total is given as £5,769,000, rounded to the nearest £1,000. In itself, the figure is not very informative, although it can for example be set against comparable amounts for previous years. Even this is scarcely sufficient for the purposes of projection — there is no information on such vital matters as the length of the business run-off, the relative age of the claims being settled, or the true relationship to premium income. What is needed is an analysis of the claims figure by period of origin of the business.

Analysis by Origin Period

The origin period itself needs some attention. It is most commonly taken as a year, but can also be a quarter or even a month for rapidly changing lines. Again, it is common to take *accident year* as the origin for the business. Accident year is the term used to refer to the calendar year in which the occurrence giving rise to the claim took place. It is perhaps the most natural origin, but is not invariably used. Thus, in reinsurance work the origin is more often the *underwriting year*, i.e. the calendar year in which the policy covering the risk was written or renewed. Such a definition enters for the simple reason that it is the normal accounting basis in reinsurance. Finally, the *report year*, the year in which the claim is first registered in the insurer's books, can also be used in some reserving analyses.

Taking accident year as the origin, suppose the overall claims figure breaks down in the following way:

Yr 1	23,000	Yr 5	1,007,000
Yr 2	148,000	Yr 6	1,536,000
Yr 3	422,000	Yr 7	1,889,000
Yr 4	744,000		

£5,769,000

Year 1 is the earliest accident year for which claims are still being paid out. Then the other years follow in succession until Year 7, which is the year just past. The breakdown is informative, for example in indicating the probable length of the run-off. But more still can be learned by building up the picture with similar claims information from earlier years of account. For example, suppose information is available from the 5 previous years. Then a whole table can be drawn up, which might appear as follows (figures in £000s):

Year of Payment

		2	3	4	5	6	7
	<0 †	650	340	110	19	-	-
	0	478	395	272	110	20	-
	1	744	501	442	288	127	23
Yr of	2	1001	854	568	565	347	148
Origin	3		1113	990	671	648	422
	4			1265	1168	800	744
	5				1490	1383	1007
	6					1725	1536
	7						1889
	Total	2873	3203	3647	4311	5050	5769

† Individual data are not known for accident years earlier than Year 0. <0 implies that aggregated data are being given for these years.

The Development of Claims

Patterns for analysis are now beginning to emerge: eg, the volume of claims in the table is increasing steadily as the years progress. But the clearest picture will emerge if we directly compare the development pattern of claims for each successive year of origin. This can be done by examining the rows of the table above. The comparison is made much easier by shifting each row successively one place further to the left. The elements of the lower diagonal, for example, then form the first column of a new table, and so on for the other values. The top axis becomes, instead of payment year, the year of development for the business. The new table is as follows:

Year of Development

	0	1	2	3	4	5	6
<0				650	340	110	19
0			478	395	272	110	20

THE CLAIMS DEVELOPMENT TABLE

	1		744	501	442	288	127	23
Yr of	2	1001	854	568	565	347	148	
Origin	3	1113	990	671	648	422		
	4	1265	1168	800	744			
	5	1490	1383	1007				
	6	1725	1536					
	7	1889						

To read the table, take for example the origin year 2. By the end of the year itself, the claims paid out on the business originating in that year are £1,001,000. Then in the following year, a further £854,000 is paid, and so on, until in the most recent year claims are £148,000. The development years are labelled by the progression 0, 1, 2, 6 along the top of the table. The convention adopted is that development year 0 is just the origin year itself in each case. Then succeeding development years follow in natural sequence. Thus, for origin year 2, there have been five development years following it. The most recent year (the current accounting year) is therefore the development year 5. But for origin year 6, the most recent year is only development year 1, and so on. A useful relationship to note is that:

Year of Origin + Year of Development = Year of Payment

The relationship is quite general, and can be checked by applying it to the cells in the above table.

(The above convention, which is used throughout the Manual, is in common use. However it should be noted that in Lloyd's and the London Market, it is customary to label the development years 1, 2, 3 etc., i.e. starting with "1" instead of "0".)

Rows, Columns & Diagonals

Once the data have been put into this format, it is very suggestive of means for analysing and projecting the claims. Thus, ratios of values along the rows give the development pattern for each individual accident year, and regularities may soon become apparent. Down the columns, the ratios give the trend pattern from one accident year to the next, which again may be revealing. Lastly, the diagonals can be seen to relate to the position in succeeding calendar years, with the lowest diagonal representing the calendar year immediately past. (The sum of the values in this diagonal will take us back to the originally quoted claims figure of £5,769,000.)

The data array of claims of development year against origin year is thus a fruitful one. But there is a disorienting feature that the reserver may come across in practice. It is that the axes of the table can be arranged in different ways. Thus, the two main axes can be interchanged, or the order of the origin years can be reversed, so that the later years come at the top rather than the bottom. Also, some offices prefer to use payment years rather than development years as one of the axes (i.e. they revert to the earlier table above). All possible variations seem to be in use! However, the arrangement shown above is the one most commonly found in the literature, and it will be kept to throughout the Manual.

The Cumulative Claims Table

There is a further variation of the table which is often useful. Rather than looking at the year by year addition to the claims for each year of origin, we may be interested in the *cumulative* development. The cumulative figures are obtained simply by adding the values along each row. In the present table, this cannot be done for years earlier than Year 2, owing to the missing data. But for the years from 2 onward, the process yields the following array:

		Year of Development					
		0	1	2	3	4	5
	2	1001	1855	2423	2988	3335	3483
	3	1113	2103	2774	3422	3844	
	4	1265	2433	3233	3977		
Yr of	5	1490	2873	3880			
Origin	6	1725	3261				
	7	1889					

It will be observed that the data are now in the exact shape of a triangle. Such triangular form is widely used in claims reserving work. Though the form is appealing, it has its deficiencies. For example, the relationship with payment year data is not fully apparent. The diagonals other than the leading one are incomplete, and to improve the connection we need to return towards the parallelogram shape of the previous display.

Apart from the payment year relationship, the parallelogram of data has advantages from the projection point of view as well. Thus the given example contains a fair amount of information for development years 4, 5 and on, whereas in the strict triangle it is scanty indeed. Of course, it may be that data cannot be obtained at all for the earlier years of origin, in which case the triangle will have to suffice. But if the data can be found, the extension to parallelogram form may be well worth the effort.

In the claims reserving literature, a strong convention has arisen involving the use of triangular data arrays. In general, the Manual will follow the convention. But the reader should be aware that it is not an absolute requirement, and can often be dispensed with to good advantage.

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[B4]
DATA QUANTITIES

In claims reserving, a number of data items are commonly used in addition to the basic information on claim amounts paid out. These include case reserves, premium income, loss ratio, claim numbers and risk exposures as the main quantities. The information will mainly come from the insurer's data-base, but industry statistics may also be brought in. Availability of data is likely to differ between reinsurance and the direct market. In general reinsurance data will be less full and less up-to-date. In particular, claim numbers as opposed to claim amounts will often neither be known nor obtainable.

The present section describes the main data items, and gives some figures for illustrative purposes. Frequently the data can be set out in the tabular form of the previous section, showing development against year of origin, and this form is used where possible. However, it is well to note that some initial work has to be done to produce such tables or triangles — the data in their raw form are not always so conveniently presented.

Claims Paid

Amounts paid out on claims are by definition the central quantity for reserving purposes. If set out by period of origin, a development table or triangle results, as shown in §B3. The example data are repeated here for convenience, in their year by year form (but omitting years of origin earlier than Year 1):

		Year of Development						
		0	1	2	3	4	5	6
Yr of Origin	1	†	744	501	442	288	127	23
	2	1001	854	568	565	347	148	
	3	1113	990	671	648	422		
	4	1265	1168	800	744			
	5	1490	1383	1007				
	6	1725	1536					
	7	1889						

† Data not available

It is important to be clear as to the definition of the claims payment information. E.g. does it include expense directly attributable to the claims, such as litigation costs and loss adjusters' fees? Does it contain the partial payments on claims not yet fully settled? Are the figures gross or net of reinsurance, salvage and subrogation? Do they need adjustment perhaps because of some reporting or data processing delay? The bare figures given as example in the text do not fully convey the real life complications which the reserver must be ready to handle.

Case Estimates

Such estimates, usually made by personnel from the claims department, are a natural adjunct to the values for the paid claims. At the end of any accounting period, there is bound to be a number of claims still outstanding and the estimates will give a first approximation to their cost. For example, at the end of Year 7 (i.e. the current accounting year), the breakdown of the case estimates by year of origin might be:

≤ Yr 1	31,000	Yr 5	1,796,000
Yr 2	234,000	Yr 6	2,881,000
Yr 3	475,000	Yr 7	3,929,000
Yr 4	969,000		
			£10,315,000

In evaluating this information, the reserver should again be asking the relevant questions. E.g. is the likelihood of future inflation of claims cost taken into account in the estimates, and if so to what extent? Are the estimates intended to include a degree of conservatism? Do they have an allowance for direct claims expense? And so on. To gain a proper understanding, the reserver should seek contact with claims personnel, and ferret out the definitions of and underlying assumptions in the figures.

If data are available from past accounting years, it will be possible to build up a development table for the case estimates just as it was for the paid claims. Such a table might appear as follows (Year 1 data not available):

Year of Development

Yr of Origin	0	1	2	3	4	5
2	1776	1409	1029	606	384	234
3	2139	1701	1199	809	475	
4	2460	1971	1546	969		
5	3031	2549	1796			
6	3644	2881				
7	3929					

The table is similar in form to the paid claims table, but there is a difference in its status. It summarises sets of estimates made at points in time, i.e. the end of each accounting year. The paid claims data, on the other hand, are an accumulation of amounts through the years in question. The case estimates, of course, cannot be accrued in this way. But the values can be combined with those for the cumulative paid claims, to produce a quantity usually known as *incurred claims.* The latter is effectively an estimate of the ultimate loss to be experienced on a given year of origin from known claims at the accounting date, and makes no allowance for the IBNR.

Premium Income

The next data item of importance is the premium income. This provides the essential measure of the volume of business against which the claims are being paid out. It can also be seen as a first measure of the insurer's exposure to risk in the business class under consideration. It is not, however, a pure measure in that it does include a weighting for office expense in addition to the risk premium content. Also, the state of the insurance market will affect the relationship of premium rates to the quantum of pure risk — i.e. in a soft market, competitive pressures may force premiums down to the point where they are scarcely adequate to cover the risks underwritten. But in a hard market the opposite will be true, and the risk element will be well covered.

Premium income will, quite naturally, relate to the year of origin of the business. It will be extracted from the insurer's data-base as either the *earned* premium or the *written* premium for the years in question. The distinction is an important one, and is worth spelling out in detail. Thus, earned premium relates to all policy exposures on which the insurer is liable during a given period (normally a calendar year). E.g. for a policy renewed on 1 April, the earned premium for the current year will be 25% of the previous year's premium plus 75% of the current year's premium, and so on. Written premium, on the other hand, covers all premium income generated in the period in question, whether for new policies or renewals. In the case of the policy renewed on 1 April, the written premium will be 100% of the current year's premium.

The distinction between earned and written premium connects with the choice of either accident year or underwriting year for the claim development analysis. It is essential that the correct combination be used. (Actuarially speaking, it is a matter of correctly defining the exposed-to-risk.) Thus, where the origin for the claims development is accident year, the earned premium definition should preferably be used. On the other hand, for the origin as underwriting year, it is right to use the written premium.

The distinction often accords with the split between direct business and reinsurance. In direct insurance, the combination of earned premium with the accident year is most common. But in reinsurance there is usually little choice in the matter, and the data are often in such a form that only written premium with the underwriting year can be used.

Following our earlier illustration we give some example figures for premium, set out by the year of origin from Year 1 to Year 7:

Yr 1	4,031,000	Yr 5	6,590,000
Yr 2	4,486,000	Yr 6	7,482,000
Yr 3	5,024,000	Yr 7	8,502,000
Yr 4	5,680,000		

For direct business, the amount of the premium, whether earned or written, will usually be known for the end of the accounting year in question apart from an element of "pipeline" premiums relating to late notification of increments and cancellations. Unless these are significant it will not make sense, therefore, to draw up any kind of development table. But in reinsurance, it may take two or three years or even more before the premium is fully reported. Hence a development table can be drawn up, just as for the claim payments, and can be used in a similar way to project the final amount of the premium for any given year. (A further point is that a 3-year accounting system is the norm for reinsurance on the London Market. The system is well described in the London Market references in §O, and will not be further discussed in the Manual.)

Loss ratio

Loss ratio can be defined as the ratio of the ultimate amount of claim to the total premium for a given class of business. Thus it is not a primary data item — indeed it is what the reserver is effectively trying to forecast for the business on the years still open at the accounting date. However, in the past, underwriters may have established norms for the expected loss ratio on given classes of business. They may further be able to estimate how far such norms are likely to be stretched by the conditions more recently prevailing in the market. Such information provides the reserver with an initial set of guidelines against which to test the outcome of his or her projections. It also enables the reserver to extend the range of methods, e.g. in Bornhuetter-Ferguson and related techniques. Finally, where a sequence of values is available the loss ratio itself can be a subject for projection.

When defined as the ratio of ultimate claim to premium, the loss ratio is more precisely said to be the ultimate loss ratio. But there are other forms. Specifically, one may speak of the *paid loss ratio* and the *incurred loss ratio.* Such terms are used to denote the ratio of claim to premium as the business for a given year of origin develops. The paid loss ratio is just the amount paid to date on claims divided by the premiums. It rises from a low value in the early part of a development to reach the ultimate value once all claims for the year in question are settled. The incurred loss ratio is a similar quantity, but in which claim amounts paid to date are supplemented by the current value of claims outstanding.

Claim Numbers

A useful item, giving considerable further knowledge of the development of a year's business, is that of claim numbers. The numbers per se are of value, in giving a measure of the claim frequency. Also, when combined with the data on claim amounts, they enable the average cost per claim to be found. The reserver thus gains a fuller picture of the behaviour of the claims, and a first glimpse of the claim size distribution itself. Unfortunately, however, data on claim numbers are very often not available in the reinsurance field.

During the history of any given claim, there are certain distinct events which can be recognised:

a) Occurrence of the event giving rise to the claim.

b) Reporting of the claim to the insurer, and its recording in the insurer's data-base.

c) Settlement of the claim, either partially or in full.

Claims may thus be counted: i) as they are reported to the insurer and become established as open claims on the books, and ii) as they are finally settled and no longer represent any future liability to the insurer. For a given accounting year, the number of claims reported and the number settled can be defined, together with the number open at the beginning and end of the year. The simple relationship of these quantities is:

No. of claims open at 1 January
plus No. reported in year
less No. settled in year *equals* No. open at 31 December

The relationship can be used either as a check on the data, or to determine one of the quantities, if missing, from the values of the other three. In practice, doubt would most often attach to the number settled, which could be found or verified as:

No. of claims settled in year *equals* No. reported in year
plus No. open at 1 January
less No. open at 31 December

Having obtained the numbers for the current accounting year, the next step will be to divide these according to year of origin, whether this be accident or underwriting year. Then the numbers for preceding accounting years can be set alongside, and development tables produced as described for claim amounts in §B3. These tables will again show the development of business for each successive year of origin, and will be in the familiar triangular (or parallelogram) form. To begin with, three separate development tables can be produced:

a) No. of claims reported in each year, year by year basis.
b) No. of claims settled in each year, year by year basis.
c) No. of claims remaining open at end of each year.

For claims reported and claims settled, the figures can be added along the rows to give the cumulative position for each origin year. Hence two further tables result:

d) No. of claims reported, cumulative basis,
e) No. of claims settled, cumulative basis.

An example would be, for claims reported, year by year basis:

		Year of Development					
		0	1	2	3	4	5
	2	401	84	38	15	7	2
	3	513	90	41	28	12	
Yr of	4	665	88	50	34		
Origin	5	690	93	57			
	6	725	116				
	7	789					

Claims reported, cumulative basis:

		Year of Development					
		0	1	2	3	4	5
	2	401	485	523	538	545	547
	3	513	603	644	672	684	
Yr of	4	665	753	803	837		
Origin	5	690	783	840			
	6	725	841				
	7	789					

Extensive data can thus be developed where claim numbers are available. The interesting question arises as to how claim numbers are to be related to claim amounts when average costs per claim are being calculated. Some natural relationships exist with the quantities of paid and incurred claims, and with case reserves, but their handling requires a little care. The matter is dealt with in main section §H on average cost per claim methods.

With claim number data, as usual, there are some caveats. The figures can be complicated, for example, by claims which had been regarded as fully settled becoming reopened. This may occur, perhaps, because fresh symptoms develop in an injured claimant, or a new statute contains some retrospective effects. Again, some claims may prove to be null and void, and hence be closed with no payment by the insurer. How such circumstances are treated will affect the data and their proper interpretation. It is important, therefore, to know exactly what the claim numbers contain and what they exclude. Additional data to clarify such points as the reopened claims and those settled at nil may well be needed.

Measures of Exposure

As mentioned above, premium income can be regarded as a first measure of the risk exposure. But other measures can be used, and may become appropriate according to circumstance. The principal ones are as follows:

Total of sums insured at risk
Total of EMLs at risk (EML is Estimated Maximum Loss)
Number of policy or insured units earned/written

Different methods might be used to extract the required values from the insurer's data-base. A rough and ready technique for sums insured or EML will be to take the average of the values for the in-force policies at 1 January and 31 December of the year in question. Actuaries and statisticians will recognise this as an application of the census year method — as such, it can of course equally be applied to the policy or insured units. For the latter, however, a more rigorous method will be to extract the earned or written exposure on a policy by policy basis. Such an extraction requires more time and effort, but can be possible where policy files are held on an efficient modern computing system.

Of the above measures, sums insured or EMLs will be more appropriate for the commercial classes of business, particularly commercial fire. But with personal lines, where a large number of similar policies are written, the number of units will generally be better. Eg. in householders' insurance, it would be the number of houses insured for the year, or in motor the number of vehicles, and so on. As with premium income, the earned/written distinction will be important. If a policy terminates on 30 September, say, without renewal, or if a new policy commences on 1 April then in each case the earned exposure will be only .75 of a unit. But for the former policy the written exposure will be nil, while for the latter it will be unity.

Industry Data

Where a new line of business is being marketed, or where a new insurance company is being set up, there is no record of company experience on which to build. The best approach may therefore be to examine sources of data for the insurance industry as a whole.

One source which has grown in value in recent years is that of the insurance company returns to the supervisory authority. Since the 1970s, it has been obligatory for insurers to provide data on the main classes of business, and the main risk groups within class, in development table form. The problem, however, is that companies may use somewhat different definitions for the risk groupings. Also the business classes themselves are too broad to help with the analysis of particular types of policy or types of risk. Finally, it has to be remembered that the purpose of the returns is chiefly to demonstrate the solvency of the insurers. Hence they will not necessarily provide the most satisfactory data for management control of a new business line.

A further source of industry statistics is the Association of British Insurers (ABI). For example, in fire insurance, the ABI runs a market statistics scheme, whose main aim is to produce burning cost values by trade classification. Although the main use of such statistics is in underwriting and ratemaking rather than in reserving, they might assist an insurer able to extract data on its sums insured in forecasting the claims experience. These statistics are available only to those member companies of the ABI who have contributed the relevant data.

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[B5]
SIMPLE BREAKDOWNS OF THE CLAIMS PATTERN

The main types of data which are of use in claims reserving have been set out in §B4. It is worth looking at how these data elements connect together logically. A good way of doing so is to take three simple breakdowns which can be applied to the claims figures. The breakdowns are in any case useful to have in mind, both conceptually and practically speaking.

First Breakdown

We use the term *overall loss* to denote the full amount paid out on the group of claims in question, including if need be a component for expense. The first breakdown we wish to apply uses the information on numbers of claims:

$$\text{Overall Loss} = \text{Number of Claims} \times \text{Average Cost per Claim}$$

This formula can be taken to refer to the ultimate position reached on a given class and/or given year of business, or to the development at any point along the way. Normally, it will be used to determine the average cost per claim figures from the available data on losses and numbers of claims. A study of the movements in average cost per claim both by year of origin and year of development can give the reserver a fuller picture of the business being analysed.

Second Breakdown

The second breakdown brings in the exposure information:

$$\text{Overall Loss} = \begin{array}{c}\text{Measure of}\\ \text{Exposure}\end{array} \times \begin{array}{c}\text{Frequency}\\ \text{of Claim}\end{array} \times \begin{array}{c}\text{Average Cost}\\ \text{per Claim}\end{array}$$

The idea here is to go deeper into the claim number information, and replace it with a frequency measure on the earned or written exposure. The frequency will be expressed, e.g. as the number of claims per 100 exposure units. Both the ultimate position and a partial development can be referred to in this way. The information may reveal new characteristics of the business in question, particularly if it is set out in full development table form.

Third Breakdown

There will be those situations, particularly in reinsurance, where only the claims payment data are available and none on the number or frequency of claims. In such cases, a third breakdown comes into its own. This breakdown, which takes premiums as the starting point, is again a very simple one:

$$\text{Overall Loss} = \text{Premium Earned (or Written)} \times \text{Loss Ratio}$$

For direct business, the premium should have a known value soon after the end of the year of origin. The loss ratio (i.e. in paid loss ratio form) will then develop proportionately as the loss itself progresses towards the ultimate value. It is usually instructive to watch the loss ratio in development, since the comparison with other years of origin can be directly made. For reinsurance, the position is more complicated, since both terms on the right hand side are likely to show a development with time. But if the premium development has any regularity to it, the loss ratio will again be interesting to watch.

These three possible breakdowns of the loss should become familiar to the reserver as part of his or her conceptual basis, particularly when the reliability of a given method of projection is under the microscope. The point about the breakdowns is that the available data may sometimes point up trends or shifts in one or other of the components of the overall loss. In addition, analysis may show that a given reserving factor affects one component in particular, so that its final influence becomes easier to assess. The more that can thus be discovered about the anatomy of a given class of business, the better will be the chance of producing dependable reserving figures.

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[B6]
DATA SYSTEMS & VALIDATION

The main source of data for reserving purposes is likely to be the insurer's central computer system. The system will contain the main policy and claim files, together with the company's income and expenditure information. It is likely to be set up as an intricate data-base, with the record files indexed on certain key fields and inter-related with each other in a logical structure. The data-base will be completed by a suite of programs enabling data to be entered, modified and extracted in various forms. A high level query language may also be available, enabling on-line requests for information to be answered with some speed and efficiency.

It is important for the reserver to have a good knowledge of the company's data-base, and to be aware of its limitations. For example, there will be limits to the distinctions which can be made between different types of business — and this will affect the decision on the risk groups which are to be used in the reserving analysis. Other limitations will apply to the type of information which can be extracted. While data on claim payments, and probably also claim numbers, will be readily available, it may be impossible to establish the exact shape of the claim size distribution. If so, it will prevent the reserver from using some of the more complex methods, Reid's method (see Volume 2) being a case in point.

Sometimes certain data desirable for reserving may be made available, but only at a cost. It might, for example, prove necessary to read sequentially all the policy and claim records in the system, perhaps running into many millions of accesses. In such a case, either the expense or the time needed may be prohibitive. Sometimes sampling may be feasible and less costly.

It goes without saying that the reserver should know the exact definition of the data figures produced from the computer system. Thus, are the premiums recorded gross or net of commission? Do the claim number data include those claims which are settled without payment? Do the figures for paid claims include settlement costs, such as loss adjusters' fees and legal expenses? Each item will have its possible variations, and its true particulars must be known.

For reserving, the data situation may well be far from the ideal. Insurance data bases are usually designed in the first place to satisfy accounting and policy renewal purposes — the features desirable for statistical work may come a poor second. The position has been improved by the requirements of the returns to the supervisory authority, in which claims data have to be shown by year of origin, and therefore in a form suitable for reserving. But in general it may be most convenient to initiate the reserving analysis from data which are being produced anyway for the year-end accounts.

Data Reliability

It is essential that a thorough set of checks should be made on the reliability of data used for claims reserving. There is a number of aspects to this, which are dealt with in turn below.

Data Input. The computer software should be such as to incorporate a range of checks on all data that are input to the system. Examples are check digits in policy numbers, to help ensure that the correct record is being updated, and validation tests on the dates, currency codes, monetary amounts, etc. being entered.

Data Processing. Given some familiarity with the system, the reserver will be able to check on procedures used to extract the reserving data and arrange them in amenable form. He or she should ensure that all relevant records and business groupings have been included in the data, and check for deficiencies caused, say, by the late processing of reinsurance accounts.

Reconciliation of Data. Wherever possible, data should be reconciled with revenue accounts and details of policy and claim movements. An example would be to take for each year of origin the cumulative claims paid to the end of the current year, deduct the respective amount for the previous year, and check the result against the claims paid figure in the current year's accounts.

Other Checks. Further evidence can be gained in a number of ways. It can be useful to examine a sample of the claims files themselves, to throw more light on the anatomy of the business and the completeness of the data. Again, discussions with both claims staff and data processing staff may help to expand the picture and give advance warning of any new difficulties in the pipeline.

To sum up, the reserver's aim should be to examine critically each stage in the data production cycle. Nothing should be taken for granted, and efforts should be made to prove the degree of reliability of the data, to understand their content and test their reasonableness.

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[B7]
FORECASTING: SIMPLE AVERAGES & TRENDS

Methods of projection in claims reserving form the main subject of the Manual. But before discussing the particular methods, it is useful to do some basic groundwork. In projections of past experience into the future, the essential problem can be expressed as that of extending a time series. Thus, suppose a chronological sequence of claim amounts, claim numbers or development ratios is given. Taking the last, the data might read:

1.057 1.053 1.059 1.062 1.059 1.066 1.064

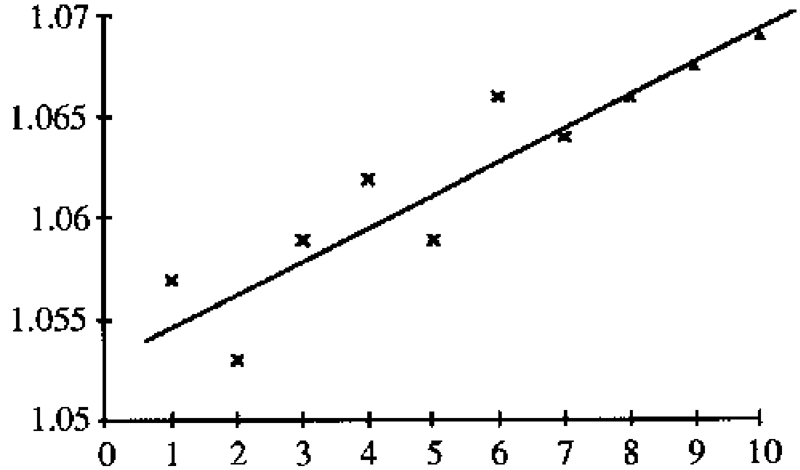
We now want to extend this sequence, say over the next 3 periods. How should this be done? There can be no foregone conclusion as to what is right, but two simple methods immediately suggest themselves:

i) To take an average
ii) To further a trend

Taking the first of these, the simple average of the 7 figures in the example works out at 1.060. Hence the extension would be:

1.060 1.060 1.060

Taking the second case, the trend, plotting the figures does suggest there could be a slight upward movement. A line can be fitted graphically:


1.07
1.065
1.06
1.055
1.05
0
1
2
3
4
5
6
7
8
9
10

This gives as the extension:

1.066	1.0675	1.069

which is an appreciably different result.

How should a choice be made? The assumption in both cases would be that there is a strong underlying pattern to the data, which is being disturbed by random variations about a mean. The difference is that in the first case, the mean is taken to be static, while in the second it is slowly increasing.

Given these distinct assumptions, the answer as to which one to use can only be found in the light of other knowledge. But if there is a proper appreciation of the business situation, then evidence for or against the real existence of a trend may be readily apparent. Eg. evidence from the claims department may support the hypothesis that settlement patterns are slowing down from year to year. Hence a trend in the loss development ratios is certainly to be expected, as against a static value.

Variations on Averaging

The choice between taking an average or a trend is perhaps the major one to be made. It is not the whole story, however. To take the example above, a simple average over the 7 years' figures was used. But many other variations would be possible within the theme of averaging. Some of the main ones are given below.

a) Curtailed Averaging Period

The use of *all* past data in compiling the average may be inapt. If the business and the influences on it are rapidly changing, then figures from as long ago as 7 years may be quite irrelevant. Hence a shorter averaging period should be chosen, say 3 years. The example figures would then yield the projection:

1.063	1.063	1.063

b) Exceptional Values Excluded

It might be more reasonable to exclude any aberrant figures from the average, particularly if these can be explained by known, exceptional influences. Again, the highest and lowest figures in any given sequence could be excluded as a matter of course, with the aim of producing a moderated value for any average.

The given example is fairly well behaved, and excluding the highest and lowest values leaves the same average of 1.060 as in the first trial above.

c) Weighted Averages

The past years' data can be given different weights, normally with higher weights for the more recent years. The rationale here is that the more recent the data, the greater the weight should be placed on their relevance for the future. In the given example, relative weights of:

1 2 3 4 5 6 7

could reasonably be used. The weighted average then works out at 1.0615, with a corresponding projection forward. (The weights have been chosen here in a simple arithmetic progression — but weights in geometric progression, or some other intrinsic relationship, could also be used.)

d) Claim or Exposure Weighted Averages

If the ratios given are, say, development factors on paid claim data, then they can be weighted according to the claim values from which they were originally derived. Thus, if the last three years' data only are used, and if the claim amounts concerned are:

800 1150 1300

then the weighted average comes to the value: 1.0635. This type of weighting is in fact commonly used in the chain ladder projection. As well as paid claim values, exposure measures for the years of origin can also be used as weights. Suitable measures might be the number of policy units exposed, or the earned or written premium for the year.

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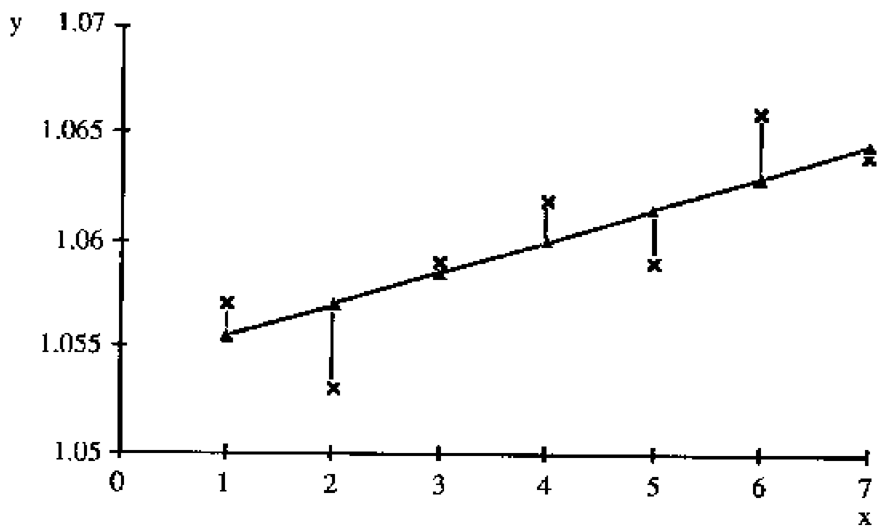
[B8]
MATHEMATICAL TRENDLINES

This section deals briefly with the mathematical aspect of fitting a trendline to a given set of data points. In the example of §B7 the trendline was fitted by eye, using a simple graph. A more satisfactory method, however, from the theoretical point of view, is to fit a mathematical line or curve. The most common standard adopted is to find that line which minimises the sum of the squares of the deviations of the observed points



Here, the line to be fitted is:

$$y = ax + b$$

where a, b are constants to be determined. It is taken that there are n points, with co-ordinates (x_i, y_i). The quantity to be minimised is thus:

$$\Sigma_i \, (ax_i + b - y_i)^2$$

Partial differentiation with respect to a, b respectively gives the equations:

$$\Sigma_i \, (ax_i + b - y_i) \, . \, x_i = 0$$

$$\Sigma_i \, (ax_i + b - y_i) = 0$$

Let $\bar{x}, \bar{y}$ be the mean values of the x_i and the y_i. The second equation immediately transforms to:

$$n\,.(a\bar{x} + b \;\; \bar{y}) = 0$$

Hence:

$$b = \bar{y} \;\; a\bar{x}$$

Substituting this value back into the first equation gives:

$$\Sigma_i \{a\,.(x_i \;\; \bar{x}) \;\; (y_i \;\; \bar{y})\}\,.\,x_i = 0$$

and hence also:

$$\Sigma_i \{a\,.(x_i \;\; \bar{x}) \;\; (y_i \;\; \bar{y})\}\,.(x_i \;\; \bar{x}) = 0$$

Thus a is found as:

$$a = \Sigma_i \{(y_i \;\; \bar{y}).(x_i \;\; \bar{x})\} \,/\, \Sigma_i (x_i \;\; \bar{x})^2$$

These formulae evaluated for the main example give a, b as:

a = .00168 b = 1.0533

Hence the trendline is:

y = .00168 x + 1.0533

Evaluating y for x = 8, 9, 10 gives the required projection:

1.0667 1.0684 1.0701

This compares with the values earlier fitted by eye of:

1.066 1.0675 1.069

The difference between the two sets of estimated values is not very great in this case, but can sometimes be quite marked. The advantage of the mathematical trendline is that it provides a fully reliable procedure for making the fit, i.e. one not subject to individual bias.

Fitting an Exponential

The assumption so far has been that any fitted trend should be a straight line. That is, the trend will show increasing values in arithmetic progression as the years pass by. But sometimes to assume a trend which progresses by *geometric* ratio may be more appropriate. The mathematical procedure is then to fit an exponential curve rather than a straight line to the data.

The simplest means for this is to convert the *y*-values on to a log scale, and then carry out the linear fit as before. In short period projections, the switch to the exponential may often not affect the results greatly. However, over a longer period, the influence of the geometric factor will very much become apparent.

This again highlights the importance of the choice of forecasting method. Even with relatively well-behaved data, such as those in the given example, appreciable differences in the results soon become apparent. The only way to make an informed choice of method is to be cognisant of the business conditions and influences which are currently making themselves felt. The assessment of such influences is taken up in the next main section, §C, of the Manual.

Deeper Waters

Beyond the simple functions dealt with above, there are many and more complex mathematical functions that can be used for trendline purposes. Indeed, the whole theory of Curve Fitting and Time Series Analysis can be brought into play if so desired. These subjects are touched upon in some of the methods described in later parts of the Manual, but are beyond the scope of the present section.

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Section C
COMPANY & EXTERNAL INFLUENCES

Preamble

If we lived in a world where nature and human activity were well behaved and gave no cause for upset or surprise, then claims reserving would be a simple matter scarcely requiring the services of the expert. One would need to assess the values of at most three quantities for each class of business: a) the exposure to risk, b) the frequency of claim, and c) the average loss per claim. Past and present trends for these factors could be assumed to hold equally in the future, and the known patterns could be projected forward with confidence.

However the real world is full of uncertainties so that projections are seldom straightforward. For protection, the reserver needs to acquire a knowledge of the influences which are most likely to disturb the picture. Only in this way can he or she hope to produce figures in which theory is properly tempered by reality. The present section outlines the main influences, both internal and external to the company, which usually need to be taken into account.

Contents

[C1]
CLASSIFICATION & GENERAL ANALYSIS

It is useful to have a systematic listing of the factors which may disturb the claim development pattern or the continuity of the loss ratio for a given class of business. With such a tool at hand, the reserver will be less likely to omit a relevant influence from the analysis.

To begin with, a clear distinction can be drawn between those factors over which a company has control, because they are part of its internal operation or its marketing, and those over which it has no control because they are part of the larger environment in which it must operate. We shall call these the company and external factors respectively. Going a stage further, on the company side we may distinguish:

a) Business Mix & Volume
b) Underwriting, Rating & Policy Conditions
c) Claims Handling & Definition

Of these, b) and c) are factors over which the company has the most control. Over a) it will have at least partial control through its marketing tactics and strategy. The factors are not all independent — underwriting policy will affect the mix of business achieved, for example — but they each contribute in a distinct way to the reserving problem.

On the external side, we may again distinguish three main different types of influence:

a) Inflation & Economic Factors
b) Legal, Political & Social Factors
c) Climate & Environmental Factors

Under a), investment conditions and currency exchange are important factors apart from inflation. Under b) such influences as legislation and the trend of court judgments are relevant, and c) covers such aspects as severe weather, catastrophes both natural and man-made, and the existence of latent hazards.

The remainder of this section, §C, gives a fuller description of the factors which can influence the claims projection under each of the six main headings above.

General Effects to Analyse

Before going on, however, the point arises as to how the effect of each of the influences may be analysed. A short comprehensive answer cannot be given, since so much will depend on the particular circumstances of the case. But there Are two general questions which can be put when examining any given factor. These seek to discover where the main influence of the factor is felt.

The first question asks whether the effect is chiefly on:

a) the Ultimate Loss (or Loss Ratio), or
b) the Claim Development Pattern.

This distinction is particularly important. Suppose a change in the claim development pattern is detected for the early years. The reserver will need to estimate to what extent this will be carried through to the ultimate position. He or she may, for example, find that the change is the result of an increase in inflation, which is expected to persist. The effect will clearly carry through to the ultimate amount to be paid out on the given business. On the other hand, the reserver may discover that a change in claims department staffing has caused an increase in the claims handling rate, so that the effect on the ultimate loss will be negligible.

The second question distinguishes an influence on:

a) the Number (or Frequency) of Claims, and
b) the Average Cost per Claim.

The distinction is again an important one. In general, at the reserving date, the information on the number of claims will be better developed than that on the average cost per claim. Hence it is factors of the latter type to which the claims reserving process has the greater sensitivity.

To give an example, a factor such as an increasing burglary rate will mainly affect the number of claims on a given subgroup of business. Its effect will quickly be detected in the increased number of reported claims per unit exposure, and so can easily be dealt with in the reserving process. But a factor such as an increasing level of court damages awards will operate more on the average cost per claim. Its effect on those reported claims which are still open at the reserving date may not be easy to assess, let alone its effect on claims of the IBNR variety.

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[C2]
BUSINESS MIX & VOLUME

This section looks at the important aspects of the business mix and volume. The reserver should be aware of the general characteristics of mix and volume for each of the chosen subclasses of business, and how these may be changing over time. The consequences of such changes for the claims development pattern and the ultimate loss ratio can be very appreciable. Another related factor considered in this section is the new business proportion, i.e. in relation to renewals and the in-force.

Changes in Business Mix

Ideally, a class or subgroup of business for analysis will be chosen for the homogeneity of the risks which it covers. As seen in §B2, this lends stability to projections carried out by statistical means. But in practice the ideal is seldom attained, and more often the business groups will be amalgams of different elements.

A good example is the general liability group, which may contain a wide variety of public and product liability type contracts. In such a case, the claim development pattern can be much affected by changes in the balance between the different elements in the group, as can the ultimate loss ratio. We need to gain some idea as to how the balance in the group is changing, and what the likely effects will be.

As a simple example, take the case of a private motor account which is undifferentiated as between the comprehensive and non-comprehensive policies. Claims on the former policies will have a substantial physical damage element, and this will be largely paid off in the first two years of development. But the latter policies, being essentially third party type insurance, will have a greater proportion of settlements at two years or more. Hence if the balance of business in the account changes, the claim development pattern will also change. A projection for reserves which is based purely on the historical pattern will give an erroneous view, and must be corrected.

Sample figures, showing the percentage of the overall claims settled in the two time periods, follow. Comprehensive business is taken to outweigh non-comprehensive in the ratio 2:1.

	Comprehensive	*Non-Comp.*	*Whole Group*
Yrs 1–2	70%	40%	60%
Yrs 3+	30%	60%	40%

If a recent year's business shows claims development of £1m to the end of 2 years, the projection to ultimate on this basis will be just: £1m / .6 = £1.67m. Suppose now that non-comprehensive business has already increased to form 50% of the portfolio. The proportion of claims actually settled by the end of 2 years is in reality 55% only. Hence the correct projection to the ultimate will be: £1m/.55 = £1.82m.

In this example, business mix changes because of the mix in major policy types making up the group for analysis. There are other factors, however, which can also be important. Some cases in point would be:

Class	*Mix by*	*Example Categories*
Commercial property	Perils covered	Fire/Storm & Flood/Theft/ Consequential Loss
Employers' Liability	Industry profile	Manufacturing/ Distribution/ Service Industry
Professional Indemnity	Professional groups	Accountants/ Solicitors/ Doctors/ Architects
Householders'	Geographical area	Inner city/ Suburban/ Rural area
All Classes	Risk profile	% of poor or above average risks in the portfolio

Changes in Business Volume

Any substantial change in the volume for a given business group requires some analysis. It may come about, for example, from a recent and marked change in the premium rates or the underwriting standards. These will in turn affect the risk profile, changing the proportion of poor or above average risks on the books. Alternatively, it may be that the demand for a particular type of insurance cover has risen sharply, so that the business mix of the group is again altered.

There is an important technical effect of a change in business volume which should be noted. Suppose that the volume of a given line rises quickly during the year. The higher proportion of the business must then be written in the later months. The average duration in force at 31 December is thus reduced from that for a year where the business volume is constant, or only slowly rising. Since claims take time to be reported and settled, the proportion of claims settled by the end of the year must be less. Also, the proportion falling into the IBNR category will be increased. Adjustments to the projections will be needed, since to use the historical pattern alone would appreciably underestimate the final loss. Where the business volume is rapidly falling, the reverse effect will apply, i.e. the proportion of claims settled by the end of the year will be greater than normal. The use of strict historical patterns would then overestimate the final loss.

This "business acceleration" effect will operate whether accident or underwriting year is taken as origin. But the manner of operation will be somewhat different in the two cases, owing to the different patterns of exposure to risk which they employ.

Other points on business volume are:

a) Abrupt changes can affect the claims processing rate, causing either a logjam or a hiatus to develop in the work of the claims department. Either way, the claims settlement pattern is liable to be distorted.

b) Appreciable overall changes can make any business group more or less stable from a statistical point of view. This happens purely because the number of independent claims in the analysis is affected.

Changes in Business Volume

One aspect of business mix and volume is the proportion of new business to renewals. This can be a useful signpost. A high proportion of renewals denotes a stable portfolio, which may be expected to behave with some regularity in the future. But a business line with a high proportion of new policies is more likely to be volatile in its risk profile and claim development patterns.

In general, the new business in any class would be expected to show a different loss ratio than the renewals. Hence, an analysis of the in-force for the proportion of new business it contains can be of service to the reserver.

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[C3]
UNDERWRITING, RATING & POLICY CONDITIONS

This section deals with influences on the claims pattern and ultimate loss which arise from the conditions under which business is written. The main headings are:

Underwriting Standards	Deductibles
Rating Levels	Policy Limits & Retention
Policy Conditions	Levels

Underwriting Standards

A change in the underwriting standards for a given business class may be expected to change the risk profile for that class. Thus, if standards are relaxed, then risks that were formerly on the borderline or unacceptable will be taken on. This will produce a shift in the claims experience, through a higher claims frequency, or a higher average cost per claim, or perhaps both. Conversely, tightening the underwriting standards will push out some of the poorer risks, so reducing either or both of these quantities.

A further effect will operate on the volume of business. Other things being equal, a relaxation of the underwriting standard will tend to increase the volume, and vice versa. Hence the consequences discussed in §C2 should be considered. Again, with a heterogeneous business class, the underwriting change is likely to affect some lines only within the class, or to affect different lines in different ways. The business mix may therefore become an issue, as well as the volume.

These effects are easy to describe in theory, but will not be so easy to quantify in practice. The main point is for the reserver to be aware of their existence, and to distinguish those changes which are significant for reserving purposes from those of minor consequence only. It goes without saying that good lines of communication with underwriters will help the reserver to keep abreast of the position.

Rating Levels

A change in rating levels may be considered as part and parcel of an underwriting change, so that the points from the above paragraphs can again be relevant. There may, however, be different reasons for a rate level shift:

a) As part of a general shift in market rates, e.g. passing from a soft to a hard market.

b) As a tactical move by the insurer, changing the position of the insurer relative to the market as a whole.

The effect on the risk profile and business mix obtained will tend to be far less marked if a) is the case rather than b). It is the tactical shifts that will have the greater influence on the claim development pattern. However, shifts of the whole market will affect the loss ratio. By the same token, they may help to indicate the phase in the underwriting cycle which the market has currently reached. Such information will support any loss ratio projections which are being done, and will strengthen the reserver's perspective on market events.

Policy Conditions

A great variety of policy conditions can apply to the different lines of business, which it is scarcely possible to cover here. But a simple example may help, relating to the No Claims Discount (NCD) in motor business. In recent years, a form of policy with NCD protection has been offered by a number of insurers. The protection is granted on payment of a percentage addition to the premium, and has proved popular. Its adoption has, as expected in the portfolios concerned, yielded a larger proportion of relatively small claims with a short duration to settlement. Thus claim frequency, average cost and settlement pattern have all been affected. In such circumstances, most projections will require adjustment.

Deductibles

The introduction of a deductible to a policy type, or an increase in value of a deductible, will affect both frequency and average cost per claim. Frequency will be reduced, because a band of the smallest claims is eliminated. Average cost will be affected in a less certain way. It will tend to be increased by the loss of the smallest claims, but to be decreased by the operation of the deductible itself against the remaining claims in the group. Overall, the result will more often be an increase in average cost per claim. However, provided the change in the deductible is not a drastic one, the effect on the claim pattern may often be ignored in practice.

In normal circumstances, the most important question will not be the absolute value of the deductible — but whether it keeps pace with the claims inflation for the business class over the years.

Policy Limits & Retention Levels

Changes in policy limits will not affect the frequency of claim, but will influence the average cost. As with deductibles, the main question is the relationship of the policy limit to the claims inflation over the years. If the changes are *not* in line, then the percentage of claims reaching the limit can change markedly, altering the shape of the claim distribution and the development pattern.

Retention levels for reinsurance purposes again set a limit on claims, which may or may not move with the claims inflation. Hence similar effects to those noted above can occur. Where the analysis is being carried out gross of reinsurance, however, changes in retention levels will not be relevant. (The question of using figures gross v. net of reinsurance is treated in §D4.)

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[C4]
CLAIMS HANDLING & DEFINITION

In this section, we consider influences arising from the way in which claims are defined and handled by the insurer. These affect mainly, but not exclusively, the claim development pattern rather than the ultimate loss. The main headings are:

Claims Definition	Settlement Practice
Recording Procedures	Staffing Levels
Case Estimation Practice	

Claims Definition

The idea that "a claim is a claim is a claim" is not quite true. To begin with, insurers may give different treatment to multiple claims which arise from a single accident. Some will bring all such claims together, and count the accident as giving rise to a single, composite claim. Others will open a separate file for each individual claim that is made. From the reserving point of view, which system is used does not matter, so long as it is used consistently.

Another point arises in the treatment of claims which are either made in error or are spurious. Are these to be counted as proper claims to begin with, the formal rejection coming later on, or to be ignored in the first place? Again the practice does not matter, so long as it is consistently held to.

Next should be considered the separation of claims for special treatment. Insurers may sometimes use streamlined procedures for dealing with claims for relatively small amounts, especially in personal lines business (this may occur, for example, under block policies where claims up to a certain level are handled externally by a broker or other body). The cut-off point for such claims then becomes important. If it is raised, the claim numbers, average costs and durations to settlement will be changed both for the main group of claims and for the streamlined group.

Recording Procedures

The claims recording procedures used by an insurer can affect the reserving data all along the line. Particular points of importance are:

a) Initial reporting of a claim, and its recording in the data-base.
b) The approval and disbursement of claim payments.
c) Closing of claim files where no payment is made.
d) Re-opening of previously closed claim files.

The procedures may be subject to change from time to time, especially because of improvements in computing equipment and data processing methods. Much can be affected, for example, the effective cut-off date for a given claim category, or the time interval between the first reporting of a claim and its entry as a formal record to the insurer's data-base. Such changes can distort the figures provided for reserving purposes. As with the other influences on the claims pattern, the first requirement is for the reserver to ensure that he or she is informed of the changes that are taking place so as to be in a position to take any necessary corrective action.

Case Estimation Practice

An important input for the reserver is the case estimates on open claims provided by the claims department. This feature is dealt with in more detail in §F1. The main points to note are:

a) The reserver should know on what basis the case estimates are made. E.g. Do they allow for future claims inflation? Do they contain any implicit safety margin?

b) Consistency of practice over the years is more important than absolute accuracy. If the basis is changed at some point, say to become more or less conservative, then the figures will need to be adjusted prior to use in a projection.

Settlement Practice

As with other aspects of claims handling, the important point is the consistency of settlement practice. There is a number of features here. First is the degree of resistance put up to borderline claims, and the toughness of the negotiating stance where the amount of a large claim is in dispute. Although firm resistance may reduce claim costs, it will involve higher expense, particularly on the legal side — there is a trade off, which has to be resolved. All is well so long as the insurer's stance is constant, but if it changes the claims distribution and duration to settlement can be affected. One effect of increased resistance, whether or not it results in financial gain, will be to slow down the claim development pattern.

A second point is the use of the partial settlement. Particularly in such cases as compensation for industrial disease under employers' liability, the period to final settlement may be long drawn out. One or more interim payments may therefore be made to the claimant. Policy as to the amount and timing of these can be varied, again with an effect on the claim development pattern.

Another influence will be the practice with regard to claims remaining open, but on which there has been no action for a long time. From time to time, a closing off exercise may be carried out, in order to despatch as many such claims as possible. If the exercise is done regularly, there should be no undue distortion of the figures — but if spasmodic, then it could affect, for example, the consistency of case estimates.

Finally, there is the treatment of those claims which are closed without payment. Are such claims to be included in the number of claims settled, or not? The effect of the decision on both claim numbers and average costs in the group of settled claims can be very marked. Once again, the reserver needs to know what basis is being used, and to be satisfied that it is used consistently.

Staffing Levels

The adequacy of staffing levels in the claims department to deal with the volume of business can be important. If a surge of claims comes in, or if staff are below establishment for a time, a backlog can build up. The claims data produced for the given time period will be distorted, so that some correction may need to be made. There is also the point that quality and consistency of work may suffer where undue pressures are placed on the staff. Such effects may be detected in changes in claim settlement rates.

Another point on staffing is that if there is a high turnover, then it will be difficult to maintain the needed consistency of work over time.

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[C5]
INFLATION & ECONOMIC FACTORS

Inflation

Under modern conditions, inflation is a major influence on the ultimate loss experienced on any class of business. It is always necessary to consider how inflation is to be dealt with in the reserving calculations. Many methods (e.g. the separation method, Bennett & Taylor's Method "A" see §J) take explicit account of inflation in making the claims projection. Others (e.g. the year on year version of chain ladder) make no explicit allowance, but take any inflation already present in the data and project it into the future at the same rate by implicit means. The objection to the latter procedure, of course, is that it may not be right to assume that past rates of inflation will continue to apply. Thus, with the benefit of hindsight, we can say that it would have been wrong to use inflation rates from the 1970s in making claims projections during the early 1980s.

General procedure for making inflation adjusted projections is described with examples in §J. At this point, it is sufficient to observe that to treat inflation explicitly two basic questions have to be answered:

a) What historical rates of inflation are embodied in the data?
b) What rates are most likely to apply in the future?

In tackling the questions, it is important not to assume that RPI is the only way to measure inflation. It is *claims* inflation, not general price inflation, that we are addressing. Claims inflation, in fact, is likely to vary with the class of business. Thus, for motor repairs, claims inflation might be expected to keep pace with inflation of skilled manual earnings. For repair to buildings following fire, some index of construction costs might be used.

Again, court awards in damages cases are a law unto themselves so far as inflation is concerned — the right kind of specialised information should be sought. (Such inflation is sometimes termed "social inflation", since it depends mainly on attitudes and opinion in society rather than on strict economic factors. Price and wage inflation, in contrast, are types of economic inflation.)

The Underwriting Cycle

General economic conditions obviously affect the demand for insurance, and the level of premiums which the market is willing to pay. For the insurer, such conditions effectively make their mark through the underwriting cycle itself. As noted in §C3, a feel for the cycle and its current phase will be helpful to the reserver in making loss ratio projections.

Investment Conditions

Another economic factor of great importance to insurance, of course, is the state of the investment market and the terms on which insurance funds are invested. The rates of interest and dividend payable in the markets on short to medium term investments can affect profitability a great deal. From the reserving point of view, however, investment conditions only come into play when a decision to use discounted reserves has been taken. The norm in Britain is not to use discounting, although there are distinct advantages actuarially speaking. In the event that discounting is used, the chief influence on the eventual reserve will be not so much claims inflation itself, but rather the *gap* between inflation and the appropriate investment rate of interest.

Currency Exchange

The final major economic factor to be mentioned is currency exchange rates and their variation. This aspect particularly affects reinsurance and the London Market, with its large volume of international business. The Market does have three standard denominations for accounting purposes namely £ sterling, US$ and Canadian $ although risks are written in many different currencies. But risks underwritten or transferred can be such that while premiums are payable in one currency, claims are payable in quite another, non-standard currency. A common example would be of repairs to a ship while in waters foreign to its country of origin, or of a court case pursued in some foreign jurisdiction convenient for the insured's purposes

Classes of business showing such characteristics present grave difficulties for reserving. Not only are currency movements erratic and almost impossible to predict, but the proportion of claims in each given currency may be unstable into the bargain. The only rule that can be followed is, wherever possible, to separate out the business written in the different currencies. Failing that, the reserver must at least make clear the assumptions being made about the proportion of claims to be expected in each currency under review.

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[C6]
LEGAL, POLITICAL & SOCIAL FACTORS

This section deals with influences on the reserving position under the following headings:

Legislation	Attitudes to Compensation
Court Judgments	Trends in Behaviour & Awareness

Legislation

New legislation, or changes in laws and regulations, can cause discontinuity in the claims experience. An example of recent years has been the seat belt legislation, making it compulsory (with certain exceptions) for drivers and passengers to wear safety belts. The effect has been to reduce the severity of many of the injuries suffered in road accidents, thus making for a change in the claims pattern in motor business. The major part of the change would be expected in the third party injury claims, probably settled at a duration of two years plus.

At the time of a legislative discontinuity, the extent of the influence on the claims pattern may be difficult to predict. But some reasonable allowance can be made, and subsequently checked against the emerging experience. Also, as time passes, the presence of the discontinuity will come to be viewed as a historical fact. Hence when the affected experience is used in projections, any necessary adjustments can easily be made.

Generally speaking, legislation is so drafted as to apply to future events and occurrences only. Hence new legislation will not usually affect claims on the business written to date. This is a great help in claims reserving work. However, the reserver should be on the look out for Acts that may have a retrospective effect, or that may require to be put into immediate effect in new court decisions. In such cases, some adjustment to the reserves for business already written may be needed.

Court Judgments

Court judgments can be wide ranging in their effects on claims reserving. A good example is the ruling given in the High Court at Newcastle in 1983 on the subject of industrial deafness cases. Since 1973, there has been an obligation for manufacturers to provide earmuffs for all employees working in conditions of excessive noise. Any employer failing in the obligation is clearly liable for damages if an employee begins to suffer deafness as a result. One question the Court faced was as to whether any liability should be deemed to hold in similar cases occurring before 1973. The judgment given was that liability should apply in cases arising in the 10 year period prior to 1973, but not earlier. A practical back-stop was therefore set, preventing the re-activation of very old employers' liability contracts.

A further aspect of the Newcastle judgment was to establish the quantum of damages in industrial deafness cases to an amount in the region of £2,000. The effect for insurers was to set firm limits for this particular liability. Generally speaking, a reduction in reserves already set aside became possible.

Attitudes to Compensation

The level of damages awarded by the courts in compensation cases is an important factor for claims reserving. Because of changing attitudes in society, awards will tend to escalate in time, often by far more than the normal amount of price or wage inflation. As already mentioned in §C5 above, the phenomenon is sometimes called "social inflation". It is probable that this type of inflation will not be smooth, but will move by sudden steps upwards, as the result of particular judicial awards.

In compensation, the amount of damages is one aspect, but not the only one of importance. Courts also face the prior question as to what accidents and occurrences can actually be allowed to qualify for compensation. Lines have to be drawn to determine the conditions under which liability is established. But the position is not a static one, and in recent years there has been a move to widen the bounds a great deal. This is particularly true in the USA, and cases have been won for substantial damages even where injury has occurred entirely as a result of the plaintiff's own irresponsibility.

We can, therefore, detect a shift in society's view of the obligation borne to injured parties. Although the shift has not been as great in Britain as in the USA, there has been an undoubted raising of the stakes. Categories such as professional indemnity have been particularly affected, with premium rates increasing many times over in a short space of time. For the insurer the problem comes where the new rates of compensation are applied to business written at the old rates of premium. Reserves for the old years have to be strengthened to meet the new conditions, and the reserver must assess how far the trend seems likely to continue in future years.

A related point is that claim settlements negotiated out of court will be much affected by the current level and tenor of court awards.

Trends in Behaviour & Awareness

General trends in behaviour in society may have their effect on insurance claims. Connected to the above discussion on damage awards, there is the matter of what might be termed the "propensity to claim". As the general public become more aware of high court awards and new circumstances in which claims can be made, so the frequency of claim will rise. A good example comes from the area of industrial disease. Thus, there is by now a general awareness of the dangers of asbestos, and those who have worked with the substance will be far more likely to claim should the related symptoms appear. Also, apart from the action of claiming itself, the public may become more ready to take legal action if the claim is not fully satisfied.

The claims picture can clearly be affected by such trends as a general increase in the crime rate, or towards arson as a cause of industrial fires. Usually, such changes will be gradual, taking place over a number of years. They will therefore only gradually reflect themselves in the data and the projections. Such changes do not normally require any remedial action. It is the sudden shift which will throw out the estimates, and which the reserver should be on guard against.

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[C7]
CLIMATE & ENVIRONMENTAL FACTORS

The term "environmental" is here intended to include the man-made as well as the natural environment. The main headings are:

Vagaries of the Weather	Latent Hazards
Catastrophes	

Vagaries of the Weather

We can distinguish effects resulting from fluctuations in the weather a) between years, and b) within one given year. To take the variation between years first, this can result in different claim settlement patterns arising. An example would be in householders' insurance, which can be disturbed by claims for subsidence. Such claims can be very substantial ones, but they will be concentrated in years in which very long dry spells of weather occurred. One solution would be to remove all such claims from the data for the business group, and to treat them separately for reserving purposes.

On weather fluctuations within a given year, the greatest problems are produced by variations occurring just before the accounting date. 31 December is an unfortunate choice of accounting date from this point of view, since it comes towards the end of the Christmas holiday period. A sudden freeze in the latter part of December will produce a rash of claims, most of which will not be reported until the new year. Hence an IBNR liability arises which will be quite different from the pattern of those years in which there is no Christmas freeze. To establish the correct reserve, we need to have data on the claims pattern which normally follows a freeze, probably on a week by week basis. Annual or even quarterly figures will not be adequate.

Catastrophes

The reserver may need to give special consideration to any natural or man-made catastrophes, caused, for example, by hurricane, flood, earthquake or explosion, giving rise to claims on the insurer. The existence of any catastrophe that has occurred before the accounting date should be known to the reserver, and by the time the accounts have to be completed it will usually be possible to make a reasonable assessment of the gross cost of the claims incurred by the insurer. Often a large part of the gross cost will be recoverable from reinsurers, provided that the terms of the reinsurance contracts are satisfied and the reinsurers are able to pay. The need to consider making provision for bad debts in respect of possible non-recovery from reinsurers may be especially important in the context of catastrophes.

In the guidance given to British insurance companies on accounting for insurance business, it is stated that the potential requirement for an unexpired risks provision should be assessed on the basis of information available as at the balance sheet date. Claims events occurring after the balance sheet date in relation to the unexpired period of policies in force at that time need not therefore be taken into account in assessing the need for an unexpired risks provision if they were not capable of prediction at the balance sheet date. Where material, however, post balance sheet claims events should be disclosed in the notes to the accounts, together with an estimate of their financial effect. Making such an estimate may well be difficult in the case of a catastrophe that occurs very shortly before the date on which the accounts have to be finalised.

When the past claims experience is being used as a basis for forecasting the future, data relating to catastrophes will produce distortions. The usual approach is to try to eliminate from the data all claims resulting from catastrophes before making projections, and then to add on an allowance for the catastrophes that must be expected to occur in the future.

Latent Hazards

Perhaps more disturbing than immediate catastrophes, from the claims reserving point of view, is the possible existence of latent hazards. In recent years, the most notorious such hazard has proved to be asbestos. The problem is that an exposure to asbestos can result in debilitating or fatal disease perhaps 20 or even 30 years later on. But the danger was only fully demonstrated by research extending into the 1970s. Employers' liability rates on contracts written in the 1950s, 1960s and earlier could not be expected to contain provision for the hazard. Nevertheless, as time went by, the courts enforced awards based on these old exposures. The result is that substantial extra reserves have had to be set up by the insurers concerned.

One strategy, adopted particularly in the USA, has been to change liability contracts from an occurrence to a claims made basis. The latter type of contract may ease the reserving problem by limiting the IBNR provision. But it has its own disadvantages, and has not yet gained great popularity in Britain for UK risks.

In the meantime, the question that has to be faced is whether any more shocks comparable with asbestos are in the pipeline. Among chemicals, benzene has been mentioned as a possible long-term hazard, and there are other candidates. VDUs (Visual display units) are in such common use today that any proved capacity to cause disease would produce a very substantial liability for insurers.

Finally, environmental pollution has been a subject of some concern, particularly since the Love Canal case in the USA. The twentieth century has undoubtedly seen the disposal of harmful industrial wastes on a vast scale. If damage claims can be established against the companies concerned the eventual scale of liability could be very large indeed.

To sum up, it is quite impossible to make proper provision against hazards which are completely unknown. The point is to be aware of any impending threats, so that corrective action for reserving can be taken at the earliest reasonable time.

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Section D
DIMENSIONS OF CHOICE

Preamble

When embarking on the claims reserving exercise, a number of underlying choices have to be made. Often, they will be constrained by the availability of data, but on other occasions there will be considerable freedom. Again, the choices may not all be made consciously — they may be implicitly made through an office's established procedures for claims reserving. In this case, a periodic review of their appropriateness should still be made.

To make the choices clear, they are here brought out as a series of "either/or" dimensions. But often the right answer will not be "either/or" but "both". The reserver is likely to build up a fuller and more reliable picture if he or she approaches the problem in a number of different ways.

Contents

[D1]
CASE RESERVES v STATISTICAL METHODS

The distinctions made in this subsection, and throughout the whole of §D should be taken as practical pointers, *not* as hard and fast theoretical rules.

Case Reserves

Case estimation is, of course, the province of the claims office. An expert estimator will look at each individual claim, and make an assessment of its value, updating this as time goes by and new information comes in. On the other hand, statistical methods look at grouped data on sets of claims, and make the estimate by numerical manipulation.

This suggests that case reserves will be more apt for large claims and claims which have been open for a longer time.

Characteristics to look for: claims fewer in number but larger and more variable in size.

Statistical Methods

The corollary is that statistical methods will be more apt for smaller claims and for those very recently opened.

Characteristics to look for: claims larger in number but smaller and less variable in size.

Other Notes

By definition, IBNR claims *cannot* be case estimated. Hence it will always be necessary to use a statistical method.

Case estimates *can* be used as an input to a statistical method. Thus, the direct insurer's case reserves on ceded business will become part of the reinsurer's statistical input. And any set of case reserves can be adjusted for bias by a statistical method, given sufficient knowledge of previous years' run-offs.

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[D2]
SIMPLE STATISTICAL METHODS v MATHEMATICAL MODELLING & STOCHASTIC TECHNIQUES

When statistical methods are used, there is a wide range of possibilities. The Manual is largely concerned with exploring this range. One choice, made early on and sometimes without realising it, is in the degree of sophistication applied. But two divisions can readily be discerned:

Simple Statistical Methods

Data are charted numerically, and if necessary adjusted manually. Projection is done by intuitive methods, i.e. extrapolating by eye, or using simple averages and trendlines. Algebraic formulae, if used at all, are only for these averages and trends. Although a knowledge of mathematical statistics is not essential, the reserver should be trained to have an understanding of why different simple methods give different answers and what can be learned from them. The aim of this Manual is to provide that understanding.

Methods are "statistical" in the sense that they deal with numerical data, and treat claim sets en bloc (not as individual claims, by contrast with case reserving).

Claims reserving is frequently done in practice this way — in fact, more often than not. An important point is that claims reserves are often needed very quickly for accounts and reports — time is of the essence, and so sophistication may be quite out of place. The commercial imperative has to be obeyed, in preference to the academic ideal.

A final point is that simple methods lend themselves to making quick, commonsense adjustments for various biasing effects in the data — e.g. caused by a known change in the business mix, or the rate of claim settlement, say. Essentially, volume 1 of the Manual is devoted to methods which fall into this category.

Mathematical Modelling & Stochastic Techniques

The basic objection to the simple methods is that they pay no regard to the theoretical foundations. Close examination will show that even apparently intuitive projections have some underlying model on which they are founded. Hence, more can be known about the method, and its strengths and weaknesses, if this model is made *explicit.* (The chain ladder method, for example, has been particularly subjected to such criticism.)

In addition, an explicit model will show much about the conditions in which a particular method is valid, and where it will break down. The conditions for validity may sometimes be wider than previously suspected — e.g. the chain ladder works perfectly well *without* adjustment in conditions of constant inflation.

A further point is that the simple methods will invariably rely on first moments, i.e. the means of statistical distributions. There is no way of tackling the second and further moments — variance, kurtosis, etc. But the variance is especially important in measuring the confidence limits surrounding a given estimate. These are ascertained by explicit use of statistical techniques.

Here, "statistical" denotes the science and methodology of statistics, as taught for example in a degree course or the actuarial syllabus. This meaning goes far beyond the intuitive statistics of the simple methods.

The main disadvantage of the more elaborate methods is that they take more time to pursue. Hence they may prove to be impractical where an estimate is needed very quickly for business purposes. However some sophisticated methods may now be applied quite quickly by using modern computer facilities. Also, the reserver must be more highly trained, and have specific statistical or actuarial knowledge in addition to the insurance background. A trap to avoid is clearly that of indulging in mathematical sophistication for its own sake, without regard to the business needs.

The essentially mathematical and stochastic methods are treated in Volume 2 of the Manual.

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[D3]
ALL CLAIMS TOGETHER v SEPARATION OF LARGE AND/OR SMALL CLAIMS

Large Claims

Most reserving work is at present done without much regard for the claim size distribution. The only features of the distribution that regularly appear are the overall frequency of claims and the mean value.

The position is acceptable, provided the claim distribution is comparatively stable. But this assumption will be disturbed if, in a given year, one or more claims of exceptional size are encountered. (The effect will be different, according to whether the data are gross or net of reinsurance for excess of loss. The net data will be less affected, but will still show more claims at the upper limit, i.e. the retention level.)

The simple solution is to remove all claims exceeding a defined high limit, and treat them separately from the main group. This done, it will be natural to use case reserves, or adjusted case reserves, for the larger claims. Then the main group will be dealt with by some statistical method.

The principle can be taken further, if desired. Thus, claims can be stratified into a number of groups by size, and each group projected separately. Alternatively, a theoretical model with an explicit claim size distribution can be employed. Reid's method (see volume 2) is a good example of this line of approach.

Small Claims

Claims for comparatively trivial amounts, and recent claims where little case information is yet available, can be dealt with by using a standard cost approach. (A good example of this is the so-called "Fast Track" system, used in the USA.) The idea is that small, quickly settled claims can be allowed in effect to by-pass the main claim files and reserving records. They are valued by using an appropriate average cost per claim, a procedure that can result in useful savings of administrative effort.

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[D4]
FIGURES GROSS v NET OF REINSURANCE/CLAIMS EXPENSE/ SALVAGE & SUBROGATION

Reinsurance

One factor in the choice will be the comparative amount of reinsurance involved. For a large direct-writing company, the amount of reinsurance is likely to be relatively small. Hence it will be reasonable to use the gross figures, and make separate estimates for reinsurance recoveries. But for a smaller company, the proportion of ceded business to the total is likely to be much greater. The effect on reserve calculations will be very significant, and perhaps make net data the more realistic choice, in which case the consistency of the reinsurance programme will be a factor to be borne in mind

Another factor is the type of reinsurance. For proportional business, it would be usual to take the gross data. Reserves for the ceded losses can in this case easily be calculated from the gross figures.

For excess of loss business, the picture is more complicated. The advantage of gross data is that they are not subject to changes in the retention levels. But the projection, once made, must be converted to the net figure for use in financial statements. This might be done, for example, by taking as a credit on the reported claims:

$$S\ (kE - R\text{lim})$$

where summation is for all case estimates kE in excess of the retention limit Rlim. For IBNR, however, the position is more problematical, because historical data tend to be distorted by changes in the retention limits. One solution would be to set up an assumed distribution of the IBNR by claim size, and apply the retention limits accordingly.

If net data are used for excess of loss business, adjustment for changes in the retention limits must be made. One method would be to restate the losses in earlier years as if the current value of the limits applied then. It may be necessary to make allowance for the effect of inflation on the claims sizes and on the retention levels.

Claims Expense

Claims expense can be divided into 2 main categories: direct and indirect. Direct expense means such items as legal and loss adjustment expense which can be attributed to particular claims. Indirect expense covers the general overheads, e.g. in running the claims office as a whole, where it is unlikely that any direct attribution would be made.

The indirect expense, by its very nature, must be assessed by a separate route from the losses on the actual claims. But direct expense can very well be included as part of the loss figures, which is the usual position in the UK. However, in the USA, the losses are often assessed *net* of the direct expense, with separate estimations for the latter.

There is a good reason for the American practice, which relates to the legal system. Practices such as contingent fees, jury awards and punitive damages have led to the escalation of the legal expense element. In some cases, it becomes a very substantial part of the overall cost of the claim (perhaps 30% or even more). The significance for reserving is that the legal expense tends to increase more rapidly with development time than does the pure loss itself. Hence, unless such expenses are separated out, there will be a distortion to the projection, leading to the underestimation of the final overall liability.

It seems that in recent years some of the features of the American experience are beginning to be reflected in UK court decisions. Certainly there is a greater willingness to litigate, and legal expenses are increasing significantly. Hence it is likely that more attention will need to be paid to the separate examination of direct claims expense in the UK in the future.

Salvage & Subrogation

The more common practice is to develop reserving figures gross of salvage and subrogation. Then any justifiable allowances, based on evidence from the claims office, can be made subsequently. A fairly conservative view is likely to be taken of any supposed future recoveries in this direction.

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[D5]
ACCIDENT (OR UNDERWRITING) YEAR v REPORT YEAR COHORTS

Estimates can in fact be made without dividing claims into cohorts dependent on a time definition. E.g. assign all claims in a given class a standard value for reserving purposes. The value could be based on a distribution of claim size derived from past results.

But the recommended practice is undoubtedly to divide claims into cohorts with some given time-base. (Data should certainly be available in this form, because of the need to analyse data into cohorts for the returns to the supervisory authority.) The time-base is most usually the accident year, or report year of the claims. But on occasion underwriting year may be used (e.g. in the London Market), or even the settlement year.

Another variation is to use other periods than annual, perhaps half-yearly or quarterly. Even monthly periods can be used, if the data volume is sufficient, or the business type appropriate, e.g. storm damage to houses.

Accident Year Classification

<u>Advantage</u> All claims stem from the same exposure year, and so reflect the experience of that particular period. Variations can be related to the influences operating at that time, e.g. an uplift in business volume or a change in legislation. Adjusting the estimates for inflation will be straightforward.

For accounting purposes, the losses emerging can be compared with the actual charges made to the operations of that period (i.e. the accident year). Also, the classification is consistent with the requirements of returns to the supervisory authority.

<u>Disadvantage</u> The full number of claims in the cohort is not known. It will increase until all the IBNR claims are reported. Hence there is greater uncertainty in using average claim values, for example.

Report Year Classification

Advantage The number of claims is known from the outset. There is a fixed group of claims to be tracked during the run-off, so that statistical estimates have a more reliable base.

Disadvantage The claims in the cohort will have arisen from a number of different exposure periods, and the mix of ages may vary as report years progress, making comparisons less stable. Again, claim patterns can be affected by changes in the definition of report date in the office's data system. Finally, no exposure or premium base exists to underpin the loss development of the cohort.

Underwriting Year

Advantage Claims can be followed which arise from a particular rating series and the results used to test the adequacy of the premiums. Also the classification is often necessary for reinsurers, for whom claims are likely to be specifically related to the business written during a given contract year.

Disadvantage Data take longer to develop, because of the extended exposure period. IBNR emergence continues to disturb the number of claims in the cohort until the ultimate development is reached. Correction for inflation becomes more complex than with the accident year cohort.

Settlement Year

Advantage Can be used for short-tail lines, where claims are incurred, reported and settled within a short time interval.

Disadvantage Of little use for medium and long-tail lines. Claims in the cohort are too heterogeneous by their occurrence dates.

Report Year within Accident Year

Advantage Gives further refinement to estimates.

Disadvantage Can be done only where data volume is large. Requires a large amount of work with a large number of triangles to present and analyse the data.

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[D6]
LOSS RATIO v CLAIM DEVELOPMENT PATTERNS

The most common general method for reserving is to project the claim development patterns for a cohort of claims, defined by accident, policy or report year. Either the losses themselves can be used (paid or incurred), or an average-and-number of claims type projection.

Perhaps the chief problem that arises with such methods is that very little development information is available for the most recent year's business — yet it is this year that usually contributes the largest proportion of the total reserve. Again, for new lines of business, a very small amount of historical data will be available in order to establish the development patterns themselves. And for reinsurers, the data are often far too scanty to yield these patterns with any reliability.

One way round the problem is to use loss ratio methods. The office's underwriters and rate-makers will be taking in the premiums for each line of business with some expected loss ratio in mind. This loss ratio will be based on past experience of the business, and also the assessment of current trends. In the absence of further evidence, it therefore represents the best starting point for reserving. An initial estimate of the reserve can be calculated simply as:

Premium Income ´ Loss Ratio — Paid Loss to Date

The advantage of the method is that it gives a natural standard by which to assess the business as it develops. The objection is that the method is partly self-defeating, i.e. it assumes the answer before the run-off even begins. Hence regular monitoring of reserves established in this way is essential.

Combination Methods

An important group of methods combines the loss ratio approach with claim development analysis. For example, loss ratios could be used for the most recent accident years — perhaps the current one and the previous one or two. Then for all older accident years, the switch could be made to claim development factors.

A more sophisticated approach, allowing a gradual transfer from loss ratio to claim development, is at the heart of the Bornhuetter-Ferguson method (see §G). For example, if the paid loss after 3 years of development is estimated to be 75% of the ultimate figure, then a proportion of 25% remains unpaid. But this 25% can be estimated just as well, and perhaps better, as a proportion of the ultimate *loss ratio* figure. The split is in effect as follows:

Overall Loss	=	Loss Paid to Date	+	Loss Remaining
		(from known development)		(estimated from loss ratio)

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[D7]
PAID LOSS v INCURRED LOSS DEVELOPMENT

An important choice in claim development methods is whether to use paid or incurred loss data. In fact the answer is often to use both, i.e. in separate projections. Then the answers can be compared, and any anomalies brought to light. But some decision will still be needed as to which set of figures is the more reliable.

Paid Loss

Paid loss represents the actual payments made on the claims in the cohort. It thus has the advantage of being objective data. But it ignores information from other sources, in particular the case reserves. Such sources, although less objective, may still have their usefulness.

The chief problem with paid loss data is that claim settlement rates can vary from year to year, thus producing distortions in the projection. It is therefore important to make some assessment of payment patterns. For example, claim numbers closed at various stages of development can be calculated as a % of:

a) the number of claims reported (for report year cohorts), or
b) the estimated ultimate number of claims (accident year cohorts).

The stability (or lack of stability) of this percentage will be a useful indicator.

Incurred Loss

Incurred loss (i.e. paid loss + case reserves) makes use of the additional data from the case reserves. These lack full objectivity, it is true, but may be an absolutely vital component. E.g. in long-tail lines such as liability, the paid loss in the early years will be a small proportion only of the likely ultimate loss. It is therefore of little use on its own as an indicator, but with the addition of the case reserves becomes much more relevant.

A possible objection to incurred loss is that it uses an estimate of eventual loss as an *input* to the main estimating process. But this is hardly serious, since the case estimates are essentially *data* from the reserver's point of view.

A key problem with incurred loss is that standards of case estimating can change over the years, particularly as the claims office staff turn over, and as fresh sets of instructions are given to the estimators. Thus evidence should be sought on these points.

A useful point for reserving purposes is that case reserves do not have to be accurate. What is required above all is *consistency.* If the reserver knows that the case reserves are consistently overstated by 10% for commercial property and understated by 15% for liability, he or she can make the necessary correction. But the last thing to do is to ask the claims office to alter its practices, as the consistency would then be lost.

Summing Up

As stated at the beginning, the idea of a stark choice between paid and incurred loss data is deceptive. Often, the reserver should use both, in order to get the maximum information.

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[D8]
CLAIM AMOUNTS v USE OF CLAIM NUMBERS & AVERAGE COST PER CLAIM

Paid and incurred loss developments operate on the monetary amounts of loss associated with given cohorts. In some cases, especially in reinsurance, only this information will be available. But very often (in direct writing) numbers of claims are known as well as the monetary losses. Hence projections using average claim size can be developed.

Generally speaking, there is an advantage to be obtained by using average claim projections over relying on loss projections alone. The reserver has more information available, and so can obtain a clearer picture of the business. He or she can for example do separate analyses for settled claims and for reported claims (i.e. open and settled together) and may even be able to develop projections for the group of developing claims (i.e. open and IBNR together). These additional analyses will help towards the final decision-making on the level of reserves.

Thus many variations are possible using numbers and average claims. Another distinction is between using a cumulative basis for the average, and a year by year or incremental basis. The problem in using the year by year average is that the values are much more sensitive to change. At times, they can become quite erratic. The cumulative average, on the other hand, gives a more stable progression, but may fail to respond adequately to new conditions.

Other Notes

A general caveat for average claim methods is that they may be difficult to apply if the numbers of claims in the given cohorts are too low to give stability and credibility to the data.

There is a difference in applying the methods to report year as opposed to accident year data. For accident year cohorts, a projection of the ultimate number of claims will be needed. But for report year cohorts, no projection is needed. The ultimate number is already known — it is the number reported itself, which stays fixed by definition.

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Section E
THE PROJECTION OF PAID CLAIMS

Preamble

As a starting point for the simpler statistical methods, the projection of paid claim amounts is ideal. The idea underlying the method is a simple one, but it is quite fundamental. Thus, we can watch the claims for a given accident or report year developing to the ultimate value, and see the pattern that is established over the intervening years. The pattern can be expressed in terms of the proportion of the final amount which is paid out as the years progress. If subsequent accident or report years can be shown or assumed to follow a similar pattern, then we have a simple and direct means for arriving at the claims estimate.

When projecting claims in this way, there are two main techniques which can be followed. These are respectively the Grossing Up and Link Ratio methods, and on each a number of variations can be used. In fact, the two methods are opposite sides of the same coin, and will normally give very similar results. The skill comes in the choice of variation, and in the assessment as to how far the data conform to the basic assumption of a stable claim payment pattern. The methods are easy to follow in principle, and are illustrated in the text by means of an extended numerical example.

Contents

[E1]
THE GROSSING UP METHOD — INTRODUCTION

This method of treating the claims has been called the "Iceberg" technique (Salzmann 1984). The analogy is that the whole mass of the iceberg (or the ultimate value of the claims) is related by proportion to the visible part (the claims paid to date). Hence the unseen portion of the iceberg, which is the amount still to be paid, can be derived by subtraction. This is a good description, but the term "Grossing Up" is more common in Britain, and so will be used here.

Grossing up is best illustrated by means of a numerical example. Take the following table of paid claims data. (The figures are in £1,000s, but are not supposed to be based on any particular class or grouping of business.)

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

In the table as shown, the data are on a cumulative basis. In accordance with the format developed in §B3, the origin years are represented by the rows, and the development years by the columns. Origin is taken as accident year, and these years are listed down the left hand side from 1 to 6 (the current year). The development years from 0 to 5 are listed along the top of the table — year 0 being the accident year itself in each case.

The symbols *a* and *d* in the table are to denote "accident year" and "development year" respectively. In calendar time, we are standing at the end of year 6, and seeking a means for establishing the reserves at this date.

The first question to be asked is: How complete is the development of the paid claims after year $d = 5$? The data array itself gives no information, hence additional evidence is needed. If this shows that the development is effectively complete, all well and good. But it might be, for example, that more development years are needed for the completion of the run-off. In this case, some further information from earlier years will be needed, or some assumption about the remaining tail of claims must be made. Suppose, in the example, that the following information can be found from 3 earlier accident years:

		$pC_{(d=5)}$	*L-ult*	%
Earlier Yrs	1st	2969	3166	93.8
	2nd	3075	3257	94.4
	3rd	3200	3412	93.8
		9244	9835	94.0

where $pC_{(d=5)}$ represents the cumulative claims paid to the end of development year 5.

L-ult represents the ultimate liability.

The evidence points strongly to the pattern for paid claims at d=5 to stand at 94% of the ultimate liability. Thus, for accident year 1:

Estimated *L-ult* = 3483/.94 = 3705

Given this value for *L-ult*, we can now calculate the claim payment pattern for all development years of a=l:

d	0	1	2	3	4	5	*ult*
pC	1001	1855	2423	2988	3335	3483	3705
%	27.0	50.1	65.4	80.6	90.0	94.0	100

(% line indicates value of *pC* / *L-ult* e.g. 27.0% = 1001/3705)

Now, applying the basic assumption that this claim development pattern will hold in the subsequent years, we can gross up the claims to date from the later accident years a = 2, 3, . . 6. (These amounts are the figures appearing in the main diagonal of the original data array.)

a	6	5	4	3	2	1
pC	1889	3261	3880	3977	3844	3483
g	.270	.501	.654	.806	.900	.940
^*L-ult*	6996	6509	5933	4934	4271	3705

Here, g denotes the grossing factor obtained from the % line in the previous table. For each accident year we have used the formula:

^*L-ult* = *pC* / *g*, where ^ is the symbol for an estimate.

The estimated reserve V then follows by subtraction:

^*V* = ^*L-ult* – *pC*

The full figures are:

a	6	5	4	3	2	1
$\hat{L}$-*ult*	6996	6509	5933	4934	4271	3705
pC	1889	3261	3880	3977	3844	3483
$\hat{V}$	5107	3248	2053	957	427	222

It remains to accumulate the figures for all the accident years to arrive at the overall estimate:

Overall Values:	Σ*L-ult*	32,348
	Σ*pC*	20,334
	Reserve	12,014

It may also be useful to look at the *proportion* of the overall reserve which is attributable to each of the accident years:

a	6	5	4	3	2	1	Total
$\hat{V}$	5107	3248	2053	957	427	222	12,014
%	42.5	27.0	17.1	8.0	3.6	1.8	100.0

The estimated liability is heavily concentrated in the most recent two or three accident years. This is a common feature in much claims reserving. The implication is that the validity of the stable claim pattern assumption must be particularly scrutinised in relation to these latter years.

The reader may find it useful to review the calculations as a whole. They can be set out in an array of this form (pC^* denotes the amounts on the main diagonal):

	d						
a	0	1	2	3	4	5	*ult*
1	1001	1855	2423	2988	3335	3483	3705
2	1113	2103	2774	3422	3844		
3	1265	2433	3233	3977			
4	1490	2873	3880				
5	1725	3261					
6	1889						

$pC(a=1)$	1001	1855	2423	2988	3335	3483	3705
%	27.0	50.1	65.4	80.6	90.0	94.0	100
pC^*	1889	3261	3880	3977	3844	3483	
$\hat{g}$	.270	.501	.654	.806	.900	.940	
$\hat{L}\text{-}ult$	6996	6509	5933	4934	4271	3705	
pC^*	1889	3261	3880	3977	3844	3483	
$\hat{V}$	5107	3248	2053	957	427	222	

Overall Values:	$\Sigma L\text{-}ult$	32,348
	ΣpC^*	20,334
	Reserve	12,014

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[E2]
GROSSING UP—VARIATIONS 1 & 2

The objection to the example given in §E1 is that the grossing up is all based on the pattern of a single accident year, *a*=l. If this year is exceptional in any way, then a bias will be introduced into the grossing up factors. Also, there are bound to be variations from year to year, but data from a single year only can give no idea of the extent of the possible fluctuation. For these reasons, it may be better to bring in data from a number of years, so that an average can be taken. Alternatively, a conservative or worst-case estimate can be made based upon the least favourable grossing up factors observed in the set of data.

Variation 1 — Averaging

Suppose that full data from 3 earlier accident years become available. The claim development pattern for each can be found, just as was previously done for year *a*=l itself. The following table of values of *pC*/*L-ult* % might emerge:

d	0	1	2	3	4	5
1st yr	27.5	52.0	66.8	81.1	90.3	93.8%
2nd yr	28.1	51.7	66.0	82.3	91.2	94.4%
3rd yr	26.1	49.6	64.9	79.5	90.4	93.8%
Yr *a*=1	27.0	50.1	65.4	80.6	90.0	94.0%

The years show a strongly consistent pattern, but there is some variation as well. Using the additional information, we can construct a set of factors based on the average of the 4 years. These are as follows:

Avge	27.2	50.8	65.8	80.9	90.5	94.0%

They are close, but not coincident with, the original factors for year *a*=l. Applied to the paid claim figures (as in the previous example) they produce a slightly lower overall reserve:

a	6	5	4	3	2	1
*pC**	1889	3261	3880	3977	3844	3483

g	.272	.508	.658	.809	.905	.940
^L-ult	6945	6419	5897	4916	4248	3705
*pC**	1889	3261	3880	3977	3844	3483
^V	5056	3158	2017	939	408	222

Overall Values: Σ*L-ult* 32,130
Σ*pC** 20,334

Reserve 11,796

Variation 2 — Worst-Case Estimate

The lower the percentage figures are for the paid claims at any given point in the development, the greater will be the effect when grossing up takes place. Hence the worst-case estimate follows from selecting the *lowest* observed percentage in each column from the table of claim patterns. This gives:

Lowest	26.1	49.6	64.9	79.5	90.0	93.8%

Following which, the calculations for the estimated reserve take place as before:

a	6	5	4	3	2	1
*pC**	1889	3261	3880	3977	3844	3483
g	.261	.496	.649	.795	.900	.938
^L-ult	7238	6575	5978	5003	4271	3713
*pC**	1889	3261	3880	3977	3844	3483
^V	5349	3314	2098	1026	427	230

Overall Values: Σ*L-ult* 32,778
Σ*pC* 20,334

Reserve 12,444

Summary

We now have 3 possible estimates for the reserve:

Basis	*Value*
Accident Year 1 Pattern	12,014
Average of 4 Years	11,796
Worst Case from 4 Years	12,444

There is variation between the latter two which lie above and below the original estimate: while the lowest result is about 2% less than our first estimate of 12,014, the highest result is about 4% greater. It is a common situation in claims reserving to find a band of possible values within which the answer is likely to lie, although it must be borne in mind that the actual result could of course be even higher than the "worst case" estimate. Selection of the final value will depend on:

a) further analysis of the reliability of the various estimates, and
b) the degree of conservatism it is appropriate to include in the figure.

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[E3]
GROSSING UP — VARIATIONS 3 & 4

In spite of producing a variety of estimates, the method as used so far is still open to objection. In fact, the work has been based on information from old accident years. There has been no attempt to use the data from later years, as they appear in the paid claims triangle, apart that is from accident year 1 itself. But claims payment patterns may be changing, and the reserver should be abreast of the *current* situation. Hence it is time to drop the earlier accident years, and concentrate on what may be discovered from the paid claims triangle itself.

To recap, the triangle is:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

To make use of the information, one convenient way is to work back through the triangle, starting from the top right hand corner. This will be done in Variations 3 & 4.

Variation 3 — Averaging

For Year 1 the ultimate value of the claims is estimated at 3705, and analysis of its payment pattern gives the following set of figures:

d	0	1	2	3	4	5	*ult*
pC%	27.0	50.1	65.4	80.6	90.0	94.0	100

This can be applied to the latest development value for Year 2, i.e. 3844, attained at *d*=4. The appropriate grossing factor is 90.0%, giving a final estimated loss for Year 2 of:

$3844 / .900 = 4271$

Using this value, the whole payment pattern can be derived for Year 2 as well:

d	0	1	2	3	4	*ult*
pC	1113	2103	2774	3422	3844	4271
%	26.1	49.2	64.9	80.1	90.0	100

(Note: it is not necessary to write down the *pC*% value for *d*=5, although if needed it would be taken as 94.0% directly from the Year 1 figure.)

Coming to Year 3, we now have 2 different payment patterns to choose from. The vital value is that for *d*=3, and the available figures are 80.6% from Year 1, and 80.1 % from Year 2. The obvious step is to take an average, which gives 80.4% as the grossing factor. (80.35% could be used, but one decimal place will be quite sufficient in the example.) Hence the estimated final loss for Year 3 is:

$$3977 / .804 = 4947$$

This leads immediately to the payment pattern for Year 3, this time taken only to *d* = 3:

d	0	1	2	3	*ult*
pC	1265	2433	3233	3977	4947
%	25.6	49.2	65.4	80.4	100

We now have 3 values for the *pC*% at *d*=2: 65.4, 64.9, 65.4%. The average is 65.2%, which can be applied to the latest claims figure for Year 4, i.e. 3880. The process continues automatically until the whole triangle has been covered, and all claims projected to their ultimate values. It is most convenient to set the procedure out in a single display, as follows:

a \ *d*	0	1	2	3	4	5	*ult*
1	1001	1855	2423	2988	3335	3483	3705
	27.0	50.1	65.4	80.6	90.0	<u>94.0%</u>	
2	1113	2103	2774	3422	3844		4271
	26.1	49.2	64.9	80.1	<u>90.0%</u>		
3	1265	2433	3233	3977			4947
	25.6	49.2	65.4	<u>80.4%</u>			
4	1490	2873	3880				5951
	25.0	48.3	<u>65.2%</u>				
5	1725	3261					6628
	26.0	<u>49.2%</u>					
6	1889						7293
	<u>25.9%</u>						
							32,795

Overall Values:	$\Sigma L\text{-}ult$	32,795
	ΣpC^*	20,334
	Reserve	12,461

If this display looks somewhat elaborate, it may help to recall that the backbone of it is just the given triangle of paid claims values. The percentage values, and the final column to the right are the ones which have to be calculated.

As for the method of working through the display, this should be clear from the preceding development. Remember that we begin from the top right hand corner, and work down the leading diagonal. Each underlined % estimate on the diagonal is found as the average of the % values above it in the same column i.e. in the same year of development. Then it is applied as a grossing up factor to the current paid claims figure which is immediately beside it. This gives the estimate of the ultimate loss for the accident year, which is written in the final column of the display (e.g. for a = 6, 7293 = 1889/25.9%). With this figure, the other % values in the row can then be calculated, working in what may be called Arabic fashion from right to left. That done, we move to the next lower position on the leading diagonal, and repeat the process, until the work is finished on the lowest rank of the diagram.

Variation 4 — Worst-Case Estimate

As with Variations 1 & 2 above, a conservative or worst case estimate can be made using the data in the triangle. All that is necessary, when working down the columns of %s, is to choose the lowest value found rather than taking the average. E.g. in the above display in column d=2, use 64.9% instead of 65.2% as the choice. This new value will then appear in the 3880 cell, and be used for grossing up the Year 4 claims. The new procedure will of course affect the whole progress of the calculations, and the display must be redrawn. It appears as follows:

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
		27.0	50.1	65.4	80.6	90.0	94.0%	
	2	1113	2103	2774	3422	3844		4271
		26.1	49.2	64.9	80.1	90.0%		
	3	1265	2433	3233	3977			4965
a		25.5	49.0	65.1	80.1%			
	4	1490	2873	3880				5978
		24.9	48.1	64.9%				
	5	1725	3261					6780
		25.4	48.1%					
	6	1889						7586
		25.9%						
								33,285

Overall Values:	ΣL-*ult*	33,285
	ΣpC^*	20,334
	Reserve	12,951

The value obtained for the reserve is higher than that with simple averaging, by some 3.9%. A full comparison of the grossing up results appears in the next section.

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[E4]
GROSSING UP — COMPARISON OF RESULTS

The claims reserve for the given data has now been calculated by the grossing up technique in 5 different ways. The ways do not really have separate names in the literature, but it may help to list them here as follows:

1st Trial: Grossing up by top row claims pattern.
Variation 1: Earlier year claims patterns — Averaged.
2: Earlier year claims patterns — Worst case.
Variation 3: Arabic method, with averaging.
4: Arabic method, worst case factors.

(The term "Arabic method" is used to denote the technique of working through the claims triangle from right to left.)

The estimates obtained by the different grossing up techniques are summarised here:

		1st Trial	Variation			
			1	2	3	4
	6	5107	5056	5349	5404	5697
Accident	5	3248	3158	3314	3367	3519
Year	4	2053	2017	2098	2071	2098
	3	957	939	1026	970	988
	2	427	404	427	427	427
	1	222	222	230	222	222
Total estimate		12,014	11,796	12,444	12,461	12,951

The variation between the highest and lowest figures is now almost 10%. Which estimate should be chosen, or are there yet more calculations to be done? The "Top Row" method is certainly weak, because it depends on the claims pattern of a single accident year. Also, the "Earlier Year" methods use elderly data which have probably been superseded by now. Ceteris paribus, one would be likely to prefer the estimates from Arabic variations (Nos 3 & 4), since these use the most recent information. They place the estimate in the top half of the range which has so far been exposed, and hence err on the side of caution if at all. They are useful in that they begin to define a credible range for the reserve, with say £12,461 being taken as the best estimate and £12,951 as the conservative value.

But it is possible that more can be learnt about the data, and this will be explored in the following sections through the use of link-ratio methods. <>

[E5]
THE LINK RATIO METHOD — INTRODUCTION

The Link Ratio method is a close relation of the Grossing Up method just described. In a real sense, it is the reciprocal — the difference is effectively in the direction of working through the data triangle. In the Arabic version of grossing up, we worked from right to left, from projected final losses back to the vector of claims percentages. In the link ratio method, we work from left to right, using succeeding development ratios to build up towards the ultimate loss. The principle is most easily seen by working through the same example as in §E1–E4 above.

The basic triangle of paid claims is:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

The link ratios then operate along the rows, relating each value of paid claims to the value attained one development year later. Thus, working along the top row, the ratios are:

d	0	1	2	3	4
r	1.853	1.306	1.233	1.116	1.044

Hence, 1.853 = 1855 / 1001
1.306 = 2423 / 1855
......
1.044 = 3483 / 3335

The general formula is: $r_a(d) = pC_a(d+1) / pC_a(d)$

where $pC_a\ (d)$ represents in respect of accident year *a* the cumulative claims paid to the end of development year *d.*

and pC_a (d + 1) represents in respect of the same accident year *a* the cumulative claims paid to the end of development year d + 1.

The top row of the data triangle is not quite complete — as was seen in §E1, the data cover only the first 5 years of development following the year *a*=l. The ultimate loss was there estimated as 3705 for Year 1, using additional evidence from earlier accident years. With this same value, the link ratio needed from *d*=5 to ultimate is:

$$3705 / 3483 = 1.064$$

Moving on to year *a*=2 in the triangle, in a similar way we can develop the link ratios as:

d	0	1	2	3
r	1.889	1.319	1.234	1.123

where 1.889 = 2103/ 1113 etc.

Working through the whole triangle yields the following array of ratios:

		d					
		0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

(No ratio exists yet for year *a*=6.)

The ratios show a good degree of regularity — each column has values confined to a relatively narrow spread. So the essential hypothesis that the claims development pattern is similar from year to year is supported in this case. It remains only to project the ratios down the columns, and apply them to the succeeding accident years' claims data.

How shall the projections be done? Following the ideas of §B7, either averaging or trending methods could be used. A little later on, we will apply both of these techniques to the data. For the time being, however, let us take a conservative view. A worst case scenario is easily generated by using the *highest* values to appear in each column, and projecting forward with these.

The values in question are:

d	0	1	2	3	4	5
r	1.928	1.351	1.234	1.123	1.044	1.064

In order to project forward, we need to multiply up these ratios, starting from the right hand side:

d	0	1	2	3	4	5
f	4.010	2.080	1.539	1.247	1.111	1.064

Here:

$$1.111 = 1.064 \times 1.044$$
$$1.247 = 1.064 \times 1.044 \times 1.123$$
....
$$4.010 = 1.064 \times 1.044 \times 1.123 \times 1.234 \times 1.351 \times 1.928$$

In symbols, r is being used to denote the simple one-stage ratios, and f for the product of the r's. The mnemonic is:- "f is the ratio from the current claims to the final value L-ult." The related algebraic formulae are:

$$r_a(d) = pC_a(d+1) / pC_a(d) \text{ as before}$$

and

$$f_a(d) = r_a(d) \,.\, r_a(d+1) \; \ldots\ldots \; r_a(u-1)$$

from which it may be seen that:

$$f_a(d) = L_a\text{-}ult / pC_a(d)$$

i.e. $f_a(d)$ is the ratio from the current cumulative claims $pC_a(d)$ to the final ultimate value L-ult as given above.

The further relationship:

$$f_a(d) = r_a(d) \,.\, f_a(d+1)$$

may also be useful. (A full understanding of the algebra is not necessary in order to follow the methods described here.)

The final stage in the link ratio method is to use the f-ratios for multiplying up the given paid claims to their projected final values. The particular claims figures which are needed are of course those on the leading diagonal of the claims triangle. The calculations are as follows:

d	0	1	2	3	4	5
f	4.010	2.080	1.539	1.247	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483
^*L-ult*	7575	6783	5971	4959	4271	3706

The required reserves then follow by deduction of the paid claims to date:

^L-ult	7575	6783	5971	4959	4271	3706
*pC**	1889	3261	3880	3977	3844	3483
^V	5686	3522	2091	982	427	223

Overall Values: Σ*L-ult* 33,265
Σ*pC** 20,334

Reserve 12,931

For a conservative reserve, this compares with the £12,951 obtained by the relevant Arabic variation in §E3.

The whole link ratio process becomes clearer when set out as a full working array. The reader may check through the procedure, referring back if necessary to the details given above.

		d						
		0	1	2	3	4	5	*ult*
		1.853	1.306	1.233	1.116	1.044	1.064	
	1	1001	1855	2423	2988	3335	3483	3705
		1.889	1.319	1.234	1.123			
	2	1113	2103	2774	3422	3844		
		1.923	1.329	1.230				
	3	1265	2433	3233	3977			
a								
		1.928	1.351					
	4	1490	2873	3880				
		1.890						
	5	1725	3261					
	6	1889						
r		1.928	1.351	1.234	1.123	1.044	1.064	
f		4.010	2.080	1.539	1.247	1.111	1.064	
*pC**		1889	3261	3880	3977	3844	3483	
^L-ult		7575	6783	5971	4959	4271	3706	

pC^*	1889	3261	3880	3977	3844	3483
$^\wedge V$	5686	3522	2091	982	427	223

Overall Values:	$\Sigma L\text{-}ult$	33,265
	ΣpC^*	20,334
	Reserve	12,931

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[E6]
LINK RATIOS WITH SIMPLE AVERAGE

Having developed the example this far, it is a simple matter to work through the link ratios using averaged values. The idea will be to obtain what may be regarded as a best estimate in comparison with the conservative estimate derived above.

The changes which are necessary in the main array are minimal. It is only necessary to average the ratios in each column, instead of taking the highest of the values. Thus:

		d 0	1	2	3	4	5	*ult*
		1.853	1.306	1.233	1.116	1.044	1.064	
	1	1001	1855	2423	2988	3335	3483	3705
		1.889	1.319	1.234	1.123			
	2	1113	2103	2774	3422	3844		
		1.923	1.329	1.230				
	3	1265	2433	3233	3977			
a								
		1.928	1.351					
	4	1490	2873	3880				
		1.890	3261					
	5	1725						
	6	1889						
r		1.897	1.326	1.232	1.120	1.044	1.064	

The full calculations then follow with these values for *r* in place:

r	1.897	1.326	1.232	1.120	1.044	1.064
f	3.856	2.032	1.533	1.244	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483

^L-ult	7284	6626	5948	4947	4271	3706
pC*	1889	3261	3880	3977	3844	3483
^V	5395	3365	2068	970	427	223

Overall Values: $\Sigma L\text{-}ult$ 33,782
ΣPL^* 20,334

Reserve 12,448

There seems to be quite good agreement with the Arabic best estimate variation of £12,461. (There was also agreement on the worst-case estimates.) Does this lead to a strengthening of confidence in the estimates? Unfortunately, the answer is no, since the algebra behind the Grossing Up and the Link Ratio methods will show them to be near equivalents of one another. However, they are not quite identical, and certainly appear to give a different emphasis in the working. This point will become a little more apparent later on. It may be remarked that when we use the expression "worst-case estimate" we do not exclude the possibility that the actual result could be higher.

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[E7]
LINK RATIOS WITH WEIGHTED AVERAGE

To recap the position so far, we have the basic triangle of paid claims data:

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	†3705
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880				
	5	1725	3261					
	6	1889						

† estimated value

and from this we have calculated the one-stage link ratios:

		d 0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

We have then made a conservative estimate of losses by using the highest ratio from each column, and a best estimate by taking the average value of each column. Does this exhaust the possibilities, or could other options be open to us? In fact, plenty more can be done, and this will be the subject of the next three sections.

To begin with, evidence of changes in the business may suggest that it will be apt to place a heavy emphasis on the most recent experience. In this case, we can for example decide to use the last three years' data only, and take a weighted average with descending weights of 3, 2, 1. The last three years in this case refers to accounting years rather than accident or development years, and clearly involves taking the last three elements in each column of the triangle:

		d 0	1	2	3	4	5
	1	.	.	1.233	1.116	1.044	1.064
	2	.	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

In each column, the latest element will be given weight 3, the middle weight 2, and the earliest element weight 1. (For the column *d*=3, weights 3 & 2 only will be used.) This yields the following set of ratios:

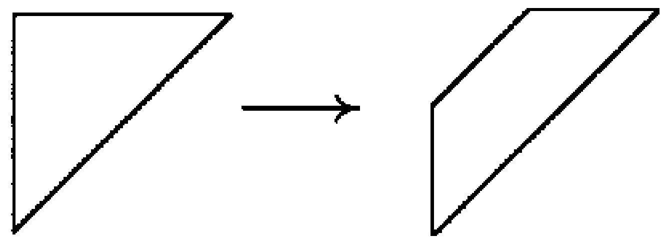
r	1.908	1.338	1.232	1.120	1.044	1.064

The full calculations then follow with these values for *r* in place:

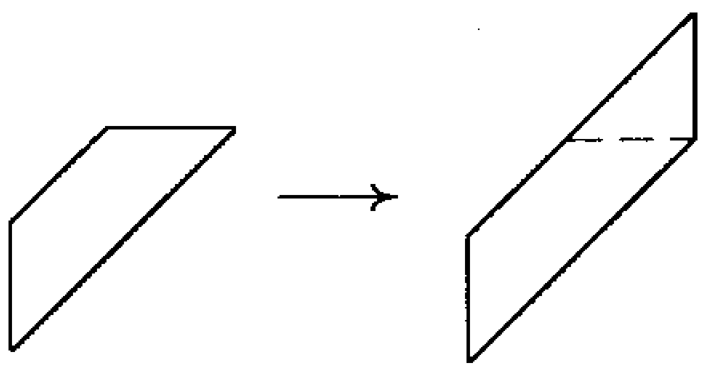
r	1.908	1.338	1.232	1.120	1.044	1.064
f	3.913	2.051	1.533	1.244	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483
^*L-ult*	7392	6688	5948	4947	4271	3706
*pC**	1889	3261	3880	3977	3844	3483
^*V*	5503	3427	2068	970	427	223

Overall Values:	Σ*L-ult*	32,952
	Σ*pC**	20,334
	Reserve	12,618

The liability here is about 1.5% higher than that from taking the simple average of the link ratios. The difference is not great because the data are relatively well behaved, and the triangle is of a modest size only. With a larger triangle, the exclusion of the data in the top left hand portion would be more likely to make a significant difference to the results. Indeed, in such cases it is right to question the continuing relevance of the figures in this part of the triangle. A trimming exercise on the data, leaving an array of the form shown below may well be in order. (Diagram overleaf.)



A corollary to this will be to seek additional data where possible in order to extend back the top right hand corner of the triangle. Such data will help improve the robustness of the later ratios in the link sequence:



We are here restating the point made in §B3.4, concerning the format of the data. The final "most desirable shape" for a claims reserving data triangle, in fact, may not be a triangle at all! If using recent data is the key, then the shape may be a parallelogram.

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[E8]
ORIGINAL WEIGHTINGS — THE CHAIN LADDER METHOD

Many other ways of weighting the ratios in the columns can be devised. But one of these stands out as being of particular importance from the mathematical point of view. This is the use of "original" weightings, by which is meant that each ratio in the column is weighted by the claims value from which it arises. Thus, in the first column of the example, the weights will be:

Ratio	Weight	Ratio × Weight
1.853	1001	1855
1.889	1113	2103
1.923	1265	2433
1.928	1490	2873
1.890	1725	3261
	6594	12525

Hence weighted average:

$$12525 / 6594 = 1.899$$

The figures in the Ratio × Weight column are familiar. Of course they must, from the definition of the link ratio itself, be just the paid claims figures from the second column of the data triangle. It follows that the weighted ratio in this case can be obtained simply by summing the second column, and dividing by the summed first column omitting its last element. A table will make this plain:

			d						
			0	1	2	3	4	5	*ult*
	1	†	1001	1855	2423	2988	3335	3483	3705
	2	.	1113	2103	2774	3422	3844		
a	3	.	1265	2433	3233	3977			
	4	.	1490	2873	3880				
	5	†	<u>1725</u>	<u>3261</u>					
	6		1889						
	Sum († ... †)								

6594 12525 Ratio = 1.899

A similar procedure gives the link ratios for the later columns of the triangle. Thus the second pair of columns yields:

		d 0		1	2	3	4	5	*ult*
	1	1001	†	1855	2423	2988	3335	3483	3705
	2	1113	.	2103	2774	3422	3844		
a	3	1265	.	2433	3233	3977			
	4	1490	†	<u>2873</u>	<u>3880</u>				
	5	1725		3261					
	6	1889							

Sum († .. †) 9264 12310 Ratio = 1.329

The full working for the chain ladder variation is:

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	<u>3335</u>	3483	3705
	2	1113	2103	2774	<u>3422</u>	3844		
a	3	1265	2433	<u>3233</u>	3977			
	4	1490	<u>2873</u>	3880				
	5	<u>1725</u>	3261					
	6	1889						
Σ–*e*		6594	9264	8430	6410	3335		
Σ		8483	12525	12310	10387	7179	3483	3705
r		1.899	1.329	1.232	1.120	1.044	1.064	
f		3.868	2.037	1.533	1.244	1.111	1.064	
*pC**		1889	3261	3880	3977	3844	3483	
^*L-ult*		7307	6643	5948	4947	4271	3706	
*pC**		1889	3261	3880	3977	3844	3483	
^*V*		5418	3382	2068	970	427	223	

In the above scheme:

Σ–*e* represents the sum of the items in the column excluding the last item
Σ represents the sum of all the items in the column

The values of r are calculated from $\Sigma\text{–}e$ and Σ as follows:

r for column $d = 0$ is equal to 12525/6594 = 1.899
for column $d =$ is equal to 12310/9264 = 1.329

and so on.

Overall Values:	$\Sigma L\text{-}ult$	32,822
	ΣpC^*	20,334
	Reserve	12,488

In this example the result is very close indeed to that obtained by the straight averaging method, i.e. £12,448.

Comments on the Chain Ladder Method

The term "chain ladder" is sometimes used in a general sense, to describe any method which uses a set of link ratios to evaluate the claim development pattern. But it is also used in a very particular sense, to refer to the "original weightings" technique shown above. The variation is an important one — it is commonly encountered in the literature, and often used as the starting point in describing a sequence of methods. However, its logical appeal does not necessarily mean that it gives the best statistical answer.

Indeed, from the point of view of practical reserving, the chain ladder variation is far from being the be-all and end-all. The main criticism is that it can be operated blindly to produce the answers without any further thought. Of course, any method can be handled in this way, but it is particularly true of the chain ladder in this particular form. It tends to produce a single rigid estimate, without any indication of how to look for the possible variations.

To pursue the point, when using the chain ladder algorithm, one calculates the averaged link ratios directly from the column sums, without needing to look at the individual accident year ratios. Information can thereby be lost to the reserver, and important features in the data can easily be missed. It should be a rule, if the chain ladder is used, also to calculate the individual accident year ratios. These should then be inspected carefully for anomalies, evidence of trends and so on. It is always important to examine the data critically and to use informed judgment.

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[E9]
LINK RATIOS WITH TRENDING

So far we have projected the link ratios by means of taking averages of various kinds. But suppose that a genuine trend is present in the data — in that case, a projection into the future will not be realistic unless it recognises and makes allowance for the trend. Let us look again at the example set of link ratios with this point in mind:

		d					
		0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123		
a	3	1.923	1.329	1.230			
	4	1.928	1.351				
	5	1.890					

Down each column, with the exception of $d = 2$, the figures do show some evidence of a trend to increase. Even with the limited amount of information available, it would not be unreasonable to hypothesise that a real trend is present (i.e. as opposed merely to statistical variation about a fixed mean). Knowledge of the business will be of assistance here.

Given some positive evidence then, how is the trend to be projected in the link ratios? The most straightforward way is to fit a least squares trendline to each column in turn. Taking the general formula for the line as:

$$y = bx + c$$

and the points for fitting as (x_i, y_i), $i = 1, 2 \ldots n$, the required solution for b and c is:

$$c = \bar{y} \;\; b\bar{x}$$

$$b = \{\Sigma x_i y_i \;\; n\bar{x}\,\bar{y}\} / \{\Sigma x_i^2 \;\; n\bar{x}\}^2$$

where $\bar{x}, \bar{y}$ are the mean values of the x_i and the y_i

The formulae are easiest to work with if the *x*-origin is chosen such that $\bar{x} = 0$. Then they reduce to:

$$c = \bar{y} \qquad b = \Sigma x_i y_i \,/\, \Sigma x_i^2$$

Calculations using these formulae now follow. Beginning with the first column of link ratios, we choose $x = 0$ for accident year 3 so that $\bar{x} = 0$.

a	x_i	y_i	x_i^2	$x_i\,y_i$
1	–2	1.853	4	–3.706
2	–1	1.889	1	–1.889
3	0	1.923	0	0
4	1	1.928	1	1.928
5	2	1.890	4	3.780
Σ	0	9.483	10	.113

Hence: $c = \bar{y} = 9.483 / 5 = 1.8966$
$b = \Sigma x_i y_i \,/\, \Sigma x_i^2 = .113 / 10 = .0113$

The projected ratio required for column 1 has $a = 6$ and $x = 3$, and is thus:

$$1.8966 + 3 \times .0113 = 1.931$$

For the second column of link ratios, we require $x = 0$ midway between accident years 2 and 3. So the calculations are:

a	x_i	y_i	x_i^2	$x_i\,y_i$
1	–1.5	1.306	2.25	–1.959
2	–.5	1.319	.25	–.660
3	.5	1.329	.25	.665
4	1.5	1.351	2.25	2.027
Σ	0	5.305	5	.073

Hence: $c = \bar{y} = 5.305 / 4 = 1.3263$
$b = \Sigma x_i y_i \,/\, \Sigma x_i^2 = .073 / 5 = .0146$

This time, two projected ratios are needed. They are for years a = 5 and a = 6, so that the respective x's are 2.5 and 3.5. The values are:

$$1.3263 + 2.5 \times .0146 = 1.363$$
$$1.3263 + 3.5 \times .0146 = 1.377$$

For the third column of link ratios, we need to put x = 0 for accident year 2.

So the calculations are:

a	x_i	y_i	x_i^2	$x_i\, y_i$
1	−1	1.233	1	−1.233
2	0	1.234	0	0
3	1	1.230	1	1.230
Σ	0	3.697	2	−.003

Hence: $c = 3.697/3 = 1.2323$
$b = -.003/2 = -.0015$

As before, the projected values follow. Three of them are needed, with x = 2, 3, 4:

$$1.2323 + 2 \times -.0015 = 1.229$$
$$1.2323 + 3 \times -.0015 = 1.228$$
$$1.2323 + 4 \times -.0015 = 1.226$$

Coming now to the fourth column of ratios, this can be projected immediately by inspection, using an increment of .007. The resulting values are:

1.130
1.137
1.144
1.151

We can now construct the full table of ratios:

		d					
		0	1	2	3	4	5
	1	1.853	1.306	1.233	1.116	1.044	1.064
	2	1.889	1.319	1.234	1.123	***1.044***	***1.064***
a	3	1.923	1.329	1.230	***1.130***	***1.044***	***1.064***
	4	1.928	1.351	***1.229***	***1.137***	***1.044***	***1.064***
	5	1.890	***1.363***	***1.228***	***1.144***	***1.044***	***1.064***
	6	***1.931***	***1.377***	***1.226***	***1.151***	***1.044***	***1.064***

The values in the upper triangle are the link ratios as found directly from the paid claims data. The ratios in the lower triangle are the projected values just obtained by the least squares method.

Before proceeding, one comment should be made. That is, it is probably unreasonable to infer a trend given only 2 data points, as has been done in the column $d = 3$. It may be better to use an average here, or just rely on the latest ratio value of 1.123. We will take the latter course, so that the lower triangle now reduces to:

		d					
		0	1	2	3	4	5
	2					1.044	1.064
	3				1.123	1.044	1.064
a	4			1.229	1.123	1.044	1.064
	5		1.363	1.228	1.123	1.044	1.064
	6	1.931	1.377	1.226	1.123	1.044	1.064

From this, the cumulative ratios are found simply by multiplying along each row in turn. This gives the set of values:

f	4.067	2.088	1.533	1.247	1.111	1.064

where:

$$4.067 = 1.931 \times 1.377 \times 1.226 \times 1.123 \times 1.044 \times 1.064$$
$$2.088 = 1.363 \times 1.228 \times 1.123 \times 1.044 \times 1.064$$

and so on.

The final step is to apply these cumulative ratios to the figures for the paid claims to date. This yields the estimates of the final losses and hence the required reserves:

d	0	1	2	3	4	5
f	4.067	2.088	1.533	1.247	1.111	1.064
*pC**	1889	3261	3880	3977	3844	3483
^*L-ult*	7683	6809	5948	4959	4271	3706
*pC**	1889	3261	3880	3977	3844	3483
^*V*	5794	3548	2068	982	427	223

Overall Values: Σ*L-ult* 33,376
Σ*pC** 20,334

Reserve 13,042

The value for the reserve is here almost 5% greater than that obtained with simple averaging, i.e. £12,448. This is quite a large difference compared with the differences we have seen so far, and if there is good evidence for the trending hypothesis, it will call into question the validity of the lower value.

It may be useful to set out the earlier part of the calculation again in summary form. This is done below, beginning from the basic triangle of paid claims data:

a \ *d*	0	1	2	3	4	5	*ult*
1	1001	1855	2423	2988	3335	3483	3705
2	1113	2103	2774	3422	3844		
3	1265	2433	3233	3977			
4	1490	2873	3880				
5	1725	3261					
6	1889						

We calculate the link ratios and extend them downwards in each column by fitting the least squares trendline (except for column $d = 3$):

a \ *d*	0	1	2	3	4	5
1	1.853	1.306	1.233	1.116	1.044	1.064
2	1.889	1.319	1.234	1.123	***1.044***	***1.064***

3	1.923	1.329	1.230	***1.123***	***1.044***	***1.064***
4	1.928	1.351	***1.229***	***1.123***	***1.044***	***1.064***
5	1.890	***1.363***	***1.228***	***1.123***	***1.044***	***1.064***
6	***1.931***	***1.377***	***1.226***	***1.123***	***1.044***	***1.064***

This then enables us to produce the key *f*-ratios by multiplying along each line:

f	4.067	2.088	1.533	1.247	1.111	1.064

The claims estimates are then obtained as on the previous page.

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[E10]
LINK RATIOS — COMPARISON OF RESULTS

The claims reserve for the given data has now been calculated by the link ratio technique in 5 different ways. To summarise what has been done, the following list may help:

Case
1) Worst case — selection of highest values
2) Link ratios with simple average
3) Weighted average of last 3 years only
4) Original weightings: the Chain Ladder method
5) Link ratios with trending

The values obtained by the different techniques are as follows:

		Worst Case	Simple Avge	W'td Avge	Chain Ladder	Trend
	6	5686	5395	5503	5418	5794
Accident	5	3522	3365	3427	3382	3548
Year	4	2091	2068	2068	2068	2068
	3	982	970	970	970	982
	2	427	427	427	427	427
	1	223	223	223	223	223
		12,931	12,448	12,618	12,488	13,042

The variation between the highest and lowest figures is just under 5%. In all this would be a comfortable range to be presented with in practice. The interesting features, *in this particular case,* are:

a) The chain ladder value is extremely close to that obtained from simple averaging of the ratios.

b) Using a weighted average based on the most recent data in the loss triangle produces only a small increase to the estimate.

c) The conservative estimate originally obtained, and that from trending the data, are very close in value.

As always, the final conclusion will depend on what further evidence is to hand — e.g. whether the apparent trend in the data is supported from other sources. But the range is well defined, and unless there is external evidence to support the trend it will be reasonable to set the best estimate at £12,488 (higher of chain ladder and simple averaging). Otherwise it would be prudent to set the estimate at a conservative value at £13,042.

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[Ell]
LINK RATIOS v GROSSING UP

Essentially, link ratio and grossing up methods are just opposite sides of the one coin. That this is so is partly confirmed by the close agreement of the estimates obtained. Under both techniques, we have arrived at a best estimate of approximately £12,500 and a conservative one of £13,000. The point can be even more forcibly demonstrated by taking the cumulative ratios from, say, the simple averaging method, and inverting them. The ratios and their reciprocals are:

f	3.856	2.032	1.533	1.244	1.111	1.064
$1/f$	.259	.492	.652	.804	.900	.940

Now look at the paid loss percentages which arose in the Arabic variation with averaging. They will be recalled as:

pC%	25.9	49.2	65.2	80.4	90.0	94.0%

The two sequences are identical. Mathematically, this is no surprise, from the very properties which the cumulative ratios f and the grossing up factors g have been given in the first place. These are such that the ultimate loss for any given origin year can be estimated as:

$$\hat{}L\text{-}ult = f \times pC$$

or

$$\hat{}L\text{-}ult = pC/g$$

Hence: f is an equivalent of $1/g$.

However, this equivalence for the single origin year does not mean that the two methods will give identical results in practice. The fact is that averaging the factors over a number of years will introduce at least small discrepancies. The average in one case is being taken of quantities which are the reciprocals of the quantities occurring in the other case. In general:

Reciprocal of {Average of A, B, C . . . }
$\neq$Average of {Reciprocals of A, B, C . . . }

But where A, B, C ... are close to each other in value, the difference is small.

Whether to use the Grossing Up or the Link Ratio techniques, then, is a rather academic question. What is more important is that they do have a different feel when in use. Grossing Up has the advantage in that it requires slightly fewer calculations. Also, it deals in quantities (the *g*-factors) which speak directly of the % of the losses incurred at each stage. The reserver who gains an insight into the way these percentages behave will certainly be gaining an understanding of the behaviour of the given class of business.

The Link Ratio methods, on the other hand, do bring out slightly more of the features of the data. This is because they examine the one-stage ratios r, as well as the final ratios f. The existence of the possible trend in the data, brought out by the last link ratio variation above, was not quite so apparent when using the grossing up technique alone. But there is an exception to this rule. Unfortunately, the chain ladder method, probably the most popular from the link ratio group, does more to conceal any clues or peculiarities in the data than any other variation here considered.

On balance, the reservers should choose between Link Ratio and Grossing Up techniques according to the one which gives them the best feel for it and what lies behind it.

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[E12]
PAID CLAIM PROJECTIONS & THE CLAIM SETTLEMENT PATTERN

The paid claim projection methods have the advantage of simplicity — they are dependent only on the assumption of the stability of the payment pattern for succeeding accident (or report) years. For the given example, the pattern is expressed through the set of percentages of the ultimate loss paid at any given development time:

d	0	1	2	3	4	5
pC%	25.9	49.2	65.2	80.4	90.0	94.0%

The reserver, however, should always examine the basic assumptions for their validity. For the paid claim projection, the question is always: Is the pattern elicited from the figures a stable one? What influences could be operating to affect it, and how seriously can they bias the results?

In fact, there are at least 3 major ways in which the paid claim pattern can be disturbed:

a) A speeding up or slowing down of the whole rate at which the settlement of claims occurs.

b) A change in the relative severities of the losses paid out on the early-settled and the late-settled claims.

c) Fluctuations in the rate of inflation as it affects the average payments made on claims generally.

Inflation, particularly as it manifested in the 1970s, can be an unsettling factor indeed. Much more will be said about ways of tackling it in §J of the Manual. But for the time being, it may be useful to illustrate the kind of effects which can be produced in the data by influences a) and b).

Change in the Claims Settlement Rate

Consider the problem of the speeding up of the settlement rate. Taking the data from the main example, let us suppose that all claims are normally paid out by the end of d = 8, but not before. The whole development period, including the original accident year, is thus 9 years long. Now say that the settlement rate speeds up proportionately so that the whole period contracts from 9 years to 8. How will the recorded claims pattern be changed?

Some crude calculations follow, which should be fairly self-explanatory. The idea is that l/8th of the paid claims in the second year move into the first year, 2/8ths of the payments in the 3rd year move forward into the second year, and so on. The first step, however, is to move from the cumulative values for paid claims to the increments which occur in each succeeding year. These increments are given by the row labelled ΔpC, where Δ is the usual symbol for "change in" or "increase of".

d	0	1	2	3	4	5	6	7	8
pC%	25.9	49.2	65.2	80.4	90.0	94.0	96.0	98.0	100
Δ*pC*%	25.9	23.3	16.0	15.2	9.6	4.0	2.0	2.0	2.0
–	.	(2.9)	(4.0)	(5.7)	(4.8)	(2.5)	(1.5)	(1.8)	(2.0)
+	2.9	4.0	5.7	4.8	2.5	1.5	1.8	2.0	
Δ*pC*%	28.8	24.4	17.7	14.3	7.3	3.0	2.3	2.2	
pC%	28.8	53.2	70.9	85.2	92.5	95.5	97.8	100	

In the table, the central line shows the losses which are being shifted forward. This enables the recalculation of the ΔpC%'s, which are then put back into cumulative form in the last line.

The direct comparison of the original and new claim patterns is:

d	0	1	2	3	4	5	6	*ult*
pC%	25.9	49.2	65.2	80.4	90.0	94.0	96.0	100
pC%	28.8	53.2	70.9	85.2	92.5	95.5	97.8	100

This can also be put in terms of the link ratios which would be given by such claim patterns:

d	0	1	2	3	4	5	6
r	1.900	1.325	1.233	1.119	1.044	1.021	1.042
r	1.847	1.333	1.202	1.086	1.032	1.024	1.022

Note that the ratios given here are the *one-step* ratios, r, rather than the final ratios. They have changed appreciably — but not out of all recognition. In general, the speeding up of the settlement rate has *reduced* the values of the ratios, but with the exception of the value for $d = 1$ and, marginally, at $d = 5$. It is to be expected that a slowing down of the settlement rate would have the opposite effect, i.e. a general increase in the one-step link ratios. (But anomalies could still occur in particular cases.)

Change in Early/Late Claims Relativity

Again, the effect will be illustrated by a simple example. Take the case where early payments increase in severity relative to the later ones. Say the first 20% of the claims increase in value by 20%, and the last 20% decrease by the same margin. A set of calculations involving change in the incremental claims ΔpC can again be made. These are as follows:

d	0	1	2	3	4	5	6	7	8
pC%	25.9	49.2	65.2	80.4	90.0	94.0	96.0	98.0	100
ΔpC%	25.9	23.3	16.0	15.2	9.6	4.0	2.0	2.0	2.0
–				(.1)	(1.9)	(.8)	(.4)	(.4)	(.4)
+	4.0	.	.						
ΔpC%	29.9	23.3	16.0	15.1	7.7	3.2	1.6	1.6	1.6
pC%	29.9	53.2	69.2	84.3	92.0	95.2	96.8	98.4	100

The direct comparison of the original and new claim patterns is:

d	0	1	2	3	4	5	6	*ult*
pC%	25.9	49.2	65.2	80.4	90.0	94.0	96.0	100
pC%	29.9	53.2	69.2	84.3	92.0	95.2	96.8	100

Again, we can make the comparison of the link ratios which would be given by such claim patterns:

d	0	1	2	3	4	5	6
r	1.900	1.325	1.233	1.119	1.044	1.021	1.042
r	1.779	1.301	1.218	1.091	1.035	1.017	1.033

Once more, the link ratios have been *reduced* systematically, but retain an affinity with the original pattern. The most surprising feature, perhaps, is the similarity of this pattern with the one obtained for the speeding up of settlement rates. It is worth bringing the figures together:

d	0	1	2	3	4	5	6	*ult*
pC%	28.8	53.2	70.9	85.2	92.5	95.5	97.8	100
pC%	29.9	53.2	69.2	84.3	92.0	95.2	96.8	100

d	0	1	2	3	4	5	6
r	1.874	1.333	1.202	1.086	1.032	1.024	1.022
r	1.779	1.301	1.218	1.091	1.035	1.017	1.033

The patterns are so similar that they could well be thought to have arisen merely from random variations in the data, rather than from quite different, systematically operating causes. The only means we have for knowing the difference is through the means of their construction.

The implication is that a study of claim development patterns and their change is not sufficient *on its own* to diagnose the causes of the changing. What, then, can be done if changes in the claim development pattern are suspected? A satisfactory reply is hard to give, but two possible tactics would be:

a) Weaker Option

Rely on the ability to observe correctly and follow any trends which may appear in the data. E.g. in the main example, there is a trend of increasing link ratios. This is the opposite of the effects demonstrated in the present section. Hence the observed trend could indicate either:

i) Slowing down of the claim settlement rate, or
ii) Later claims increasing in severity relative to earlier ones.

Since the effect of these factors on the link ratios can be very similar, it may not be fully necessary to distinguish the true cause of the change — at least, provided the trend is picked up sufficiently early on and properly monitored.

b) Stronger Option

Despite the above reasoning, a better alternative will be to seek further information so as to diagnose the cause of the variation. This could take the form of:

i) Evidence on possible trends from underwriters & claim managers, and/or
ii) Statistical information on claim numbers, claim severities, and frequency in relation to original exposure.

The aim would be particularly to monitor the settlement pattern through claim numbers and frequencies, and the early/late relativity plus inflation through claim severities. In this way, influences known to be operating, but not yet fully apparent in the paid claims figures, may be discovered. Then rational hypotheses about the quantity and direction of future change can be made, leading to the adjustment of the grossing up factors and link ratios. By such means, the paid claims projection method may yield an improved forecast of the ultimate losses, and a more reliable indication of the necessary reserves.

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[E13]
FITTING TAILS BEYOND THE OBSERVED DATA

Until now, it has been assumed that the reserver has enough historical data to determine the duration and development pattern of claims from the origin year (which may typically be the accident year, depending on the choice of cohort used). In practice, this may not always be the case (for example, the company may be new to the class of business). In such cases, it will be necessary to extrapolate the projected claim cost beyond the last development period covered by the base data.

Presented below are three approaches commonly found in practice for doing this. The methods shown have been applied to paid claims data. They may also be applied to incurred claims data, though Sherman's method may lead to problems if the individual link ratios straddle 1.

Graduation

The reserver uses the pattern of published industry figures to apply a trend to their own figures. Sources of such information may include

- Supervisory Returns
- Claims Run-off Patterns, published by General Insurance Study Group (taken from the Supervisory Returns)
- LIRMA and ABI annual summary of business performance
- RAA annual summary of performance (covering US reinsurance company data)

Consider as an example the data used in section E, with the link ratios from the chain-ladder method. Assume we have no further historic information on which to extrapolate future run-off (in previous sections it was assumed some historic information was available to produce a run-off factor of 1.064).

The chain-ladder ratios are:

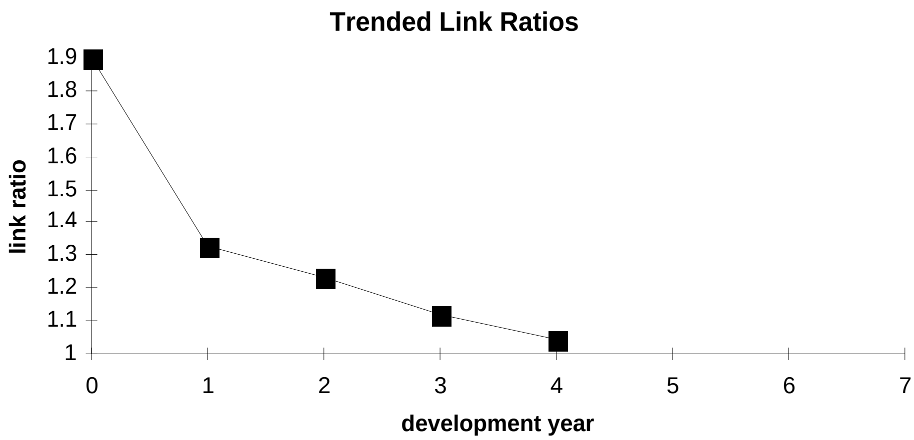
Dev Year	0	1	2	3	4
r	1.899	1.329	1.232	1.120	1.044

Suppose, for this particular class of business, the Claims Run-off Patterns publication shows a development factor from Development Year 5 to Ultimate of 10.1% for a similar class, then we could adopt a similar value for our own projection if we felt that it was likely to develop in the same way.

One problem with such data is that the classification used may not match that used by the insurer (both with respect to class and policy coverage), and the data quality may not be comparable.

Graphical

The actual development ratios for the above data are shown graphically below. It



is tempting to extrapolate the graph by fitting a curve, either mathematically (see Sherman's method below for a particular application) or by eye, as done below.

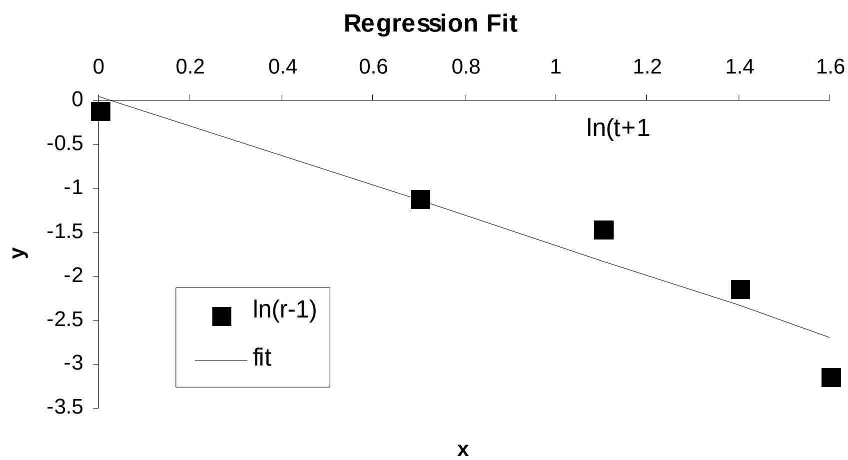
It is important, however, to check that the extrapolation is consistent with the data available. The extrapolation may seem to fit well but, if the actual outstanding claims contain (say) structured settlements, the extrapolation period would need to extend to at least the full term of those payments. Alternatively, if claims are near to some aggregate limit, there may be little scope left for further development of the size suggested by the graph.

Sherman[1]

The paper by Sherman recommends fitting a curve of the following form to the development ratios:

$$\boxed{\hat{r}_i = (1 + at^b)}$$

[1] see paper by R E Sherman



where *a*, *b* are parameters fitted using regression and *t* = development year

For example, with the data above (using linear regression fitted to the logs of $(r-1)$ against the logs of $(t+1)$):

$$a = 1.05$$
$$b = -1.7$$

The indicated tail factor here is 17.4% (using the product of factors fitted to development years 5 to 9).

Whilst this approach is conveniently simple to fit, it depends on

- all ratios being greater than one
- an arbitrary limit on the projection period, as it is unlikely that the product of the run-off factors will converge sensibly.

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Section F
CASE ESTIMATES & THE PROJECTION OF INCURRED CLAIMS

Preamble

At a given reserving date, more is usually known than the bare fact of the actual claim payments. For each class of business, there will be a number of claims still outstanding, and to these claims individual estimates will be attached by the claims office. Hence the reserver will have a further source of information towards producing a final figure for the liability. The question that arises will be as to the adequacy of these case estimates — if they are compounded with the paid amounts on settled claims, how close will the figure be to the ultimate loss on the business?

The quantity obtained by adding the case reserves to the paid claims is commonly called the "incurred claims". It turns out that the set of methods derived for projecting paid claims to the ultimate can be applied in just the same way to the incurred claims. Comparison of the results with the paid claim projections can be instructive. But this time, there are more possible disturbing influences at work — as well as the settlement pattern, the reporting pattern of the claims has to be considered. And as well as the adequacy of the case reserves, their consistency over time is of prime importance. One useful development of the projection techniques enables the reserver to assess this consistency. The example of the previous section is extended here to continue the illustration by numerical means.

Contents

F7. Adjustment of Incurred Claims Projection

[F1]
NATURE OF CASE ESTIMATES

The Manual is essentially devoted to the statistical aspect of claims reserving — dealing in terms of aggregate figures for the different classes of business. But there is another aspect, which concerns making estimates on the individual claims as they arise. It also is of great importance, but is properly the responsibility of the claims office, and so falls outside the scope of the present work. Clearly it is a wide field, requiring detailed and particular knowledge of the classes of insurance, and of the changing legal, social and economic influences which come to bear on them.

For the reserver using statistical/actuarial methods, this very detailed knowledge will not be a prerequisite, nor indeed practical to achieve in all the many classes of insurance. But though he or she will not be charged with making the individual case estimates, the reserver will often have to use these estimates as an input. Indeed, in certain classes of business, such as commercial fire or liability, it would be difficult or impossible to dispense with the case reserves.

In such classes, the simplest reserving method would be just to take the case estimates in toto as the reserve for reported claims. But one step on from this would be to make a percentage adjustment to the estimates. The adjustment would need to be determined from a study of actual past losses and the case reserves earlier made in relation to them — and hence the statistical work begins. The case reserves are also needed as a component of the incurred claims function, dealt with in §F2–F4.

Claims Office Practice

Case estimates or case reserves, then, serve as an important starting point for statistical work. It follows that the reserver must have a good understanding of the practice of the company's claims office in this respect. For example, what principles are followed in setting the individual estimates, and how are such influences as inflation accounted for? What level of adequacy is expected in the estimates, and how is consistency from year to year achieved? Indeed, is consistency achieved at all? Perhaps there have been significant changes in claims personnel, or in the guidelines issued to the claims inspectors. Such factors, if known to be operating, can change the interpretation to be put on the resulting case figures.

Another point to watch is that estimating practice can differ according to the class of business in question. Thus, it is commonly found that commercial fire figures tend to be overestimates of the eventual loss, but that in liability the reverse is the case.

Why should this be so? The answers to such questions can be revealing. Take the commercial fire class first. Property damage can be relatively straightforward to assess, but there will always be a possibility of a difference of opinion with the insured. If the estimator puts down a figure on the high side of the reasonable range, it is likely that he or she will eventually be able to settle for somewhat less. Thus to gain a *reduction* from the original estimate will show in a better light for the company than if the reserver were to start at a lower figure and suffer an *increase* at the time of settlement. That is not to say that such practices are always followed, or indeed followed deliberately at all, but there is a clear motivating factor at work.

With liability cases, other considerations entirely come in. The problem here is that many claims cannot be properly assessed, even for several years after their notification. This is particularly true with industrial diseases, where the symptoms may take a long time to reach full development. In the early years, there is just not enough information to go on. Further, the influence of economic and social inflation between the reserving and settlement dates can make the original estimates look quite inadequate. It is always a problem with the longer tailed classes of business.

The Treatment of Inflation

The point about inflation leads to a crucial distinction with regard to claims office practice. Two quite different estimating regimes can be followed:

a) To assess the claim for its loss value as if it were to be settled immediately

b) To assess it for loss as at the likely future date of settlement.

Of these, the first is by far the simpler. The estimator has only to assess the severity of the claim according to the most recent information, and in terms of the currency of the day. If the second is followed, then a number of complications are found to enter. In addition to the immediate severity, the estimator must also consider:

- The likely period to settlement.
- The rate of economic inflation in the interim.
- The influence of social & judicial trends.
- The possible effect of new legislation.

The complications are probably enough to make the alternative a) preferable in most cases although in practice alternative b) is often followed. It can well be argued that such influences as future inflation can better be dealt with by statistical methods operating on aggregates for the different classes of business, and should be left out from the individual claim figures. Be that as it may, the important matter for the reserver is to be fully aware of the practice applied in the company. He or she will then be able to put the correct interpretation on the case estimate figures when using them as an input to further statistical work.

Case Estimates & Case Reserves

Before proceeding, a point on terminology needs to be made clear. The terms "Case Estimates" and "Case Reserves" are used fairly interchangeably in practice. At times, however, it is useful to distinguish whether the amounts in question are gross or net of any partial payments on the claims. In the Manual, we shall generally use the following convention:

Case Estimate — for the full estimated loss on any claim still open at the accounting date.
Case Reserve — for the estimated liability remaining on such a claim.

If partial payments are nil, then reserve and estimate are equal. Where any payments have been made, the relationship is:

Case Reserve = Case Estimate - Partial Payments to Date

In the Manual, we shall mainly be interested in the case reserve figure, but the estimate may also be needed at times. Normally, the figures will be encountered as aggregated for a given class or subgroup of business.

Average Cost Reserving Systems

There is more to be said on the subject of case estimates. One point in particular concerns the use of average cost reserving systems to replace the case estimates. These are used for classes of business, mostly personal lines, where the large number of small claims means that the setting of individual reserves would be uneconomic. Such systems are described later on in the Manual, in §K2. For the present, we shall concentrate on the use made of case reserves in statistical work, in particular through their contribution to the incurred claims function.

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[F2]
THE INCURRED CLAIMS FUNCTION

The amount of *incurred claims,* as a rule, is defined simply as the addition of the *paid claims* and the *case reserves.* The general formulation is: $iC = pC + kV$, and for the particular case of *accident year a* at development time *d*:

$$iC_a(d) = pC_a(d) + kV_a(d)$$

where the class or subgroup of business is understood.

A point to note immediately about the incurred claims is that it is a hybrid function. But, hybrid or not, it brings together the most that is clearly known to date about how the claims are developing on the business in question. It has an element to cover the settled claims (paid claims), and one for those which are still outstanding (the case reserves). Is it not then, already the best estimate of the ultimate losses to be had, and why should any further work be required? The very name "incurred claims" suggests that the job is already done.

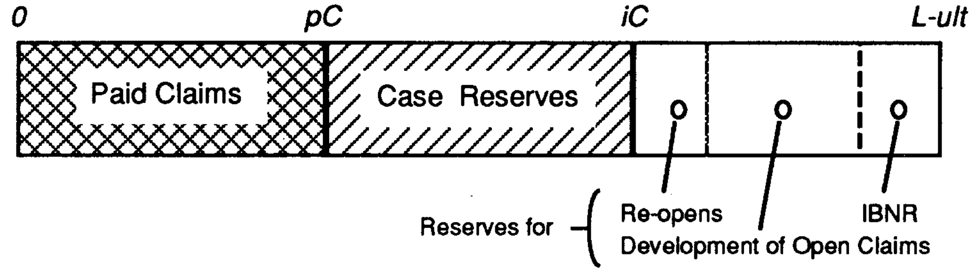
In fact, there are several reasons why the incurred claims (in most cases) cannot be accepted per se as the estimate of ultimate loss. These relate to the inadequacy of the case reserves to stand directly for the remaining liability, and can be stated as follows:

a) *Settled Claims.* Claims already settled can be reopened and further losses incurred. But the case reserves relate only to the outstanding claims, and so contain no allowance for reopens.

b) *Open Claims.* These claims, for which the case reserves have been established, are likely to undergo development between the reserving date and the settlement date. (This development can, of course, be in either direction.)

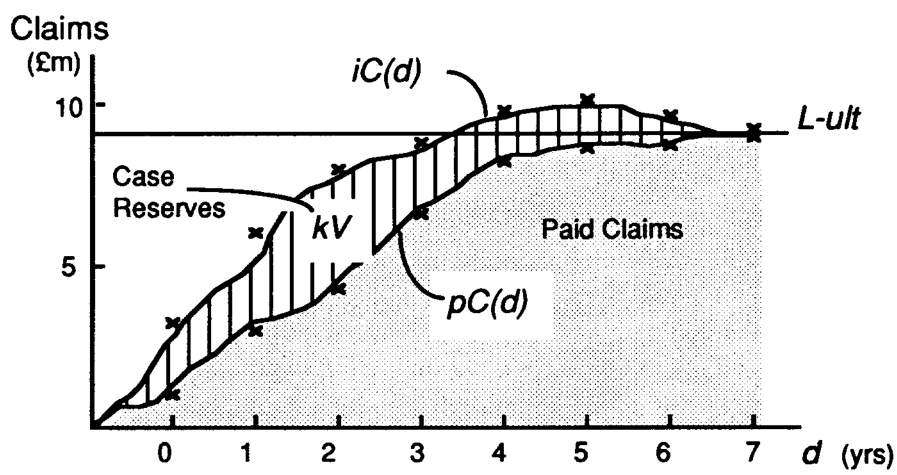
c) *IBNR Claims*. By definition, the IBNR at the reserving date are the claims which have not been reported. They cannot appear in the case estimates, but may still affect the final losses significantly.

To summarise diagrammatically:



It is possible to examine all these features individually, and to build up the ultimate loss by the addition of parts. In particular, the estimate for IBNR is likely to become available, as it will be needed for the statutory returns. But the overall approach will more often be to estimate the final loss directly. This has already been done in §E by projecting the paid claims, and parallel work with the incurred claims function can as easily be done (always provided the data are to hand).

A useful point to note in passing is that both paid claims and incurred claims functions must home in towards the ultimate loss as development time increases for a given accident year. Graphically:



The Incurred Claims Function

Taking the *iC*-function, the graph shows the increasing proportion of the final loss which it covers as time goes by. It is just for one accident year, but if subsequent years can be shown or assumed to follow a similar pattern, then the *iC*-function can be used as a means for making the claim estimates. The principle is the same as it was for the *pC*-function in §E. The only question is of the stability of the pattern as the accident years go by. For paid claims, this came down to the simple assumption of a stable claim settlement pattern. But incurred claims is a more complex function, and the assumption now becomes a 3-fold one, requiring:

a) A stable settlement pattern (as for *p=C)*
b) A stable claim reporting pattern
c) Consistency in the setting of case reserves.

Thus, curiously, although *iC* has appearance of being an improvement on *pC* (because it is a more complete estimate of the ultimate loss), there is more that can go awry with it. It is wiser to regard it as being neither better nor worse. Most important is the fact that *iC* can give a *new perspective* on the estimating of the ultimate loss and the required reserves.

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[F3]
INCURRED CLAIMS — GROSSING UP

In this section and the next one, we shall illustrate the projection of the incurred claims by means of a numerical example. Technically, the methods are exactly the same as those used for the paid claims in §E above, and all the variations explored there can equally well be applied. Hence we shall be content to exhibit one grossing up variation and one of the link ratio type. The data will be a straight extension of that previously used for the paid claims projections.

The new information that is needed is simply the triangle of the case reserves:

		d					
		0	1	2	3	4	5
	1	1776	1409	1029	606	384	234
	2	2139	1701	1199	809	475	
a	3	2460	1971	1546	969		
	4	3031	2549	1796			
	5	3644	2881				
	6	3929					

As for the paid claims, the figures are in £1,000s, and are given at each development interval for the succeeding accident years. The cumulative paid claim figures will also be needed, and these are repeated again for convenience:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

To produce the incurred claims figures, we simply add together the paid claims and case reserve triangles, element by element. This gives:

a	d = 0	1	2	3	4	5
1	2777	3264	3452	3594	3719	3717
2	3252	3804	3973	4231	4319	
3	3725	4404	4779	4946		
4	4521	5422	5676			
5	5369	6142				
6	5818					

As with the paid claims work, the first question that arises is as to the completeness of the development at the point d=5. Even for accident year 1, claims are still outstanding, so that some development *does* remain. However, the case reserves are a small proportion only (about 6.3%) of the incurred claims, and by this stage it may be right to assume that no IBNR reserve is needed. If so, then the incurred claims at d=5 may already provide the best estimate of *L-ult*. The point needs to be confirmed, and an examination of earlier years' data, if available, will help. We will take it, however, that no adjustment to the *iC* figure at $d=5$ is needed in order to produce the final loss.

The triangle of incurred claims can now immediately be evaluated using the Arabic variation of the grossing up method. We will use a straightforward averaging of the factors in each column, to give a best estimate of the liability. (Remember that the method is to work from right to left down the leading diagonal. For full details of the procedure, refer to §E3.)

	d = 0	1	2	3	4	5	*ult*
1	2777	3264	3452	3594	3719	3717	3717
	74.7	87.8	92.9	96.7	100.1	100.0%	
2	3252	3804	3973	4231	4319		4315
	75.4	88.2	92.1	98.1	100.1%		
3	3725	4404	4779	4946			5078
	73.4	86.7	94.1	97.4%			
4	4521	5422	5676				6103
	74.1	88.8	93.0%				
5	5369	6142					6987
	76.8	87.9%					
6	5818						7768
	74.9%						

Overall Values:	$\Sigma L\text{-}Ult$	33,968
	ΣpC^*	20,334
	Reserve	13,634

The figure for the reserve is appreciably higher than the likely range established under the *pC*-projections. (We had best estimates c. £12,500 and conservative figures at about £13,000.) Further consideration will be given to this discrepancy in §F6.

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[F4]
INCURRED CLAIMS — LINK RATIO METHOD

We now repeat the projection of the incurred claims data, this time using link ratios and making a cautious estimate by taking the *highest* of the ratios shown in each column. The display summarises the working (refer to §E5 for full details. Note the explanation below of a slight variation):

		d						
		0	1	2	3	4	5	*ult*
		1.175	1.058	1.041	1.035	.999	1.000	
	1	2777	3264	3452	3594	3719	3717	3717
		1.170	1.044	1.065	1.021			
	2	3252	3804	3973	4231	4319		4315
						.999		
		1.182	1.085	1.035				
a	3	3725	4404	4779	4946			5114
					1.034			
		1.199	1.047					
	4	4521	5422	5676				6249
				1.101				
		1.144						
	5	5369	6142					7340
			1.195					
	6	5818						8337
		1.433						
								35,072
r		1.199	1.085	1.065	1.035	.999	1.000	
f		1.433	1.195	1.101	1.034	.999	1.000	

Overall Values:	$\Sigma L\text{-}Ult$	35,072
	ΣpC^*	20,334

Reserve 14,738

———

(A slight variation has been introduced into the above array, in order to shorten the working a little. The values for r and f in the two lines under the main triangle have been found in the usual way. Then the values for f have been *written back* against the iC-figures in the main diagonal — these are the figures shown in italic. Multiplication immediately yields the loss estimates in the final column, which are summed for the overall loss. Finally, the total of paid claims to date is deducted to give the required reserve.)

The estimate for the reserve here obtained is a good deal higher than any found before. If we are to go by the incurred claims projections as opposed to the paid claims ones, we have both higher estimates and a wider range between the best and the conservative figures. A summary of the position is:

	Best Estimate	Conservative
pC-Projection	12,461	12,931
iC-Projection	13,634	14,738

In both cases, the best estimate is via the Arabic variation of grossing up with averaging of the factors. The conservative estimate is by link ratio method, taking the highest ratio in each column.

The difference between the two sets of figures is marked. Doubt is immediately thrown on the original range of £12,500–13,000 given by the paid claims methods. Perhaps the reserve should now be set rather higher, if the incurred claims projection is to be believed. What is really needed, though, is further investigation of the data, to find whether any systematic cause underlies the difference in the figures, and to say which set is the more reliable. To make the reconciliation is important, because if a similar answer can be found by two different routes, then the credibility of the result is much improved.

However, before engaging on this task, a new projection method is needed, which is introduced in the next sections (§§F5, F6). Then §F7 goes on to tackle the reconciliation proper.

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[F5]
GROSSING UP OF CASE RESERVES

By now, we have estimated the ultimate losses by projections of both the paid and incurred claims figures. But there is a third angle on the work, which brings case reserves themselves to the focus of attention. The idea is to compare the case reserves at each point of development with the true reserves actually needed at that time. If the development of an accident year to maturity follows a stable pattern, then the stability should be reflected in this relationship. It will become possible to project outstanding claims for the later accident years merely by applying a grossing up factor to the latest figure for case reserves.

The main conditions that need to hold are in the stability of the claim settlement and reporting patterns, and in the consistency of the case reserving standards. These are, in fact, no different from the assumptions required for the projection of the incurred claims function itself.

An immediate problem with the method is that *true* reserves are not in general known for the more recent accident years. They can only be found with certainty for those past years which have already reached full development. For such years we have:

$$V_a(d) = L_a\text{-}Ult - pC_a(d)$$

i.e. the true reserve is the ultimate loss less the claims paid to date at each stage. For the later years, some means of hypothecating the reserves must be found. It turns out that this can be done quite easily. Thus, given an initial estimate for the ultimate loss of the accident year, we can write:

$$hV_a(d) = \hat{\ } L_a\text{-}ult - pC_a(d)$$

letting d run through the various development stages. *(hV is* used as the symbol to stand for *hypothecated* reserve.)

As usual, the whole procedure is best illustrated by working a numerical example. Taking accident year 1 from the main example, the following information is known:

d	0	1	2	3	4	5
pC	1001	1855	2423	2988	3335	3483
kV	1776	1409	1029	606	384	234

The development is not complete, but as seen before (§F3) the case reserves at $d = 5$ may give a reasonable estimate of the outstanding claims. As with the incurred claims projection, we shall assume this to be so. Consequently, the ultimate losses are estimated at: (3483 + 234) = 3717, and from this figure the hypothecated reserves can be found by subtraction:

^L-ult	3717	3717	3717	3717	3717	3717
pC	1001	1855	2423	2988	3335	3483
hV	2716	1862	1294	729	382	234

The next step is to set out the proportions which the actual case reserves bear to these hypothecated figures:

kV	1776	1409	1029	606	384	234
hV	2716	1862	1294	729	382	234
%	65.4	75.7	79.5	83.1	100.5	100.0

which completes the work for accident year 1. For Year 2, the data are:

d	0	1	2	3	4
pC	1113	2103	2774	3422	3844
kV	2139	1701	1199	809	475

To begin work on these figures, we note that the reserve ratio kV/hV for Year 1 at $d = 4$ was 100.5%. The assumption of a stable pattern now allows us to apply this same factor to gross up the case reserves for Year 2 at $d = 4$. The result is:

$$475 / 1.005 = 473$$

(Since the factor is greater than 1, the process in this case actually results in a slight *reduction* to the reserves.) Now the estimated final losses for Year 2 must be:

$$3844 + 473 = 4317$$

i.e. adding the known paid claims at $d = 4$ to the grossed up case reserves. The full set of hypothecated reserves for Year 2 can now be derived, and again the case reserve proportions worked out:

d	0	1	2	3	4
^*L-ult*	4317	4317	4317	4317	4317
pC	1113	2103	2774	3422	3844
hV	3204	2214	1543	895	473
kV	2139	1701	1199	809	475
hV	3204	2214	1543	895	473
%	66.8	76.8	77.7	90.4	100.5

Moving on to Year 3, the data are:

d	0	1	2	3
pC	1265	2433	3233	3977
kV	2460	1971	1546	969

This time, at $d = 3$, we have two values for the reserve ratio kV/hV, which are 83.1% from accident year 1, and 90.4% from year 2. The average is 86.8%, which becomes our next grossing factor. Applied to case reserves of 969, it gives a grossed up figure of 1116. Hence the ultimate loss for Year 3 is put at (3977 + 1116) = 5093, and so the process continues.

The procedure should be clear by now, and it will be best to show the whole calculation set out in a double array. The upper part of the array is the familiar triangle of paid claims figures, while the lower part shows the main working.

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880			**[*pC*]**	
	5	1725	3261					
	6	1889						

		65.4	75.7	79.5	83.1	100.5	100.0%	
	1	1776	1409	1029	606	384	234	3717
		2716	1862	1294	729	382	234	
		66.8	76.8	77.7	90.4			
	2	2139	1701	1199	809	475		4317
		3204	2214	1543	895	473		
						100.5%		
		64.3	74.1	83.1				
	3	2460	1971	1546	969			5093
a		3828	2660	1860	1116			
					86.8%			
		65.4	78.5					
	4	3031	2549	1796				6122
		4632	3249	2242				
				80.1%				
		68.6				**[%]**		
	5	3644	2881			**[*kV*]**		7037
		5312	3776			**[*hV*]**		
			76.3%					
	6	3929						7833
		5944						
		66.1%						34,119

The lower array is built up around the triangle of the given case reserves. Its main cells each contain 3 values, of which the central figure is just the case reserve itself. The lower number is then the hypothecated reserve, and the upper number is the proportion which case reserves bear to it. Thus, cell (a=3, d=2) has kV's of 1546, hV's 1860, and proportion kV/hV = 83.1%. The kV's are given, while the other 2 figures are calculated.

The array may best be read by the Arabic technique of working backwards down the main diagonal. In any diagonal cell (except the first), the % figure is found as the average of the %s in the cells above it in its column. Thus, 80.1% in cell (a=4, d=2) is the average of 79.5, 77.7 and 83.1%. This proportion is then applied to gross up the case reserve figure to the hypothecated value, e.g. 1796/ 80.1% = 2242

The next step is to refer to the *pC* triangle above to find the corresponding paid claims — in this case 3880. The addition then gives the estimated final loss for the given accident year:

$$2242 + 3880 = 6122$$

This final loss is written in the extreme right hand column of the lower array, and then the hypothecated reserves are calculated backwards along the row. This again requires reference to the upper triangle of paid claims. Corresponding rows in the two triangles have to be matched, and the reserves found by subtraction. Thus:

at (a=4, d=l) 6122 - 2873 = 3249
at (a=4, d=0) 6122 - 1490 = 4632

The work on the row a = 4 is completed by calculating the proportions kV/hV. In this case:

$$2549/3249 = 78.5\%$$
$$3031/4632 = 65.4\%$$

Attention now moves to the next lower cell in the main diagonal, and the procedure is repeated until ending as usual in the bottom left hand corner of the array. The final step is to add up the loss estimates in the RH column, and deduct the paid claims to date. The summary of results is:

Overall Values:	$\Sigma L\text{-}Ult$	34,119
	ΣpC^*	20,334
	Reserve	13,785

The final figure for the reserves can be checked against the sum of the hV-figures in the leading diagonal.

Although the method is rather complicated to describe, it is not at all difficult to operate in practice. After a few repetitions the numbers almost find their own places, and the answers fall out with apparent ease. Therein, perhaps, lies the danger. Being taken with the elegance of the algorithm, one may forget to examine critically the results it is giving. With this projection, as much as with the simpler paid and incurred claims projections, it is necessary to shun the idea that the answer is automatically right. The point is pursued in the next section.

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[F6]
ADEQUACY & CONSISTENCY OF CASE RESERVES

When projections are made with incurred claims figures or using case reserves themselves (as in §§F3–F5), one question that should always be asked concerns the adequacy and consistency of the case reserves from year to year. But while the adequacy is certainly something the reserver should be concerned to know about, the really crucial aspect is the consistency. The projections of incurred claims and case reserves do assume this consistency, and if it is not fulfilled then the results can be thrown out of balance.

One useful aspect of the case reserve projection is that it can sometimes throw light on this question. Take the example just given in §F5. The figures for the case reserve grossing factors were almost lost in the welter of detail in the calculation array. It is useful to extract them to stand on their own:

		d					
		0	1	2	3	4	5
	1	65.4	75.7	79.5	83.1	<u>100.5</u>	*100.0%*
	2	66.8	76.8	77.7	<u>90.4</u>	*100.5%*	
a	3	64.3	74.1	<u>83.1</u>	*86.8%*		
	4	65.4	<u>78.5</u>	*80.1%*			
	5	<u>68.6</u>	*76.3%*				
	6	*66.1%*					

Here, the figures in the leading diagonal have been italicised because they have a different status from the other values. In fact, they are just the averages of the figures above them in their respective columns. It is the latter figures, in roman type, which hold the real interest. If each column is scanned, it can be seen that the last of them is noticeably higher than the ones above it in the column. The pattern is repeated all the way up the diagonal from (a=5, d=0) to (a=2, d=3). It is a feature which stands out from the data, and it needs further investigation.

It will be recalled that diagonals of the triangle represent payment years. Hence a noticeable change between adjacent diagonals is a change which can be identified by calendar time. In the present case, there is a distinct change in the kV/hV ratio, occurring at some point in the previous payment year but one — or to put it another way, between the reserving date of 2 years ago, and that of 1 year ago.

What has taken place? The data themselves cannot give the exact cause, though they give a strong pointer. Enquiries should be made of the claims office, and of those responsible for case estimating, to see whether any light can be thrown on the question. This might, for example, reveal the following facts:

> "For some time the office had been aware of under-reserving the open claims in this particular class of business. The underestimation was not thought serious, but then two years ago a new head was appointed to the department with a more punctilious attitude. They instituted changes soon after their appointment to bring the position into better balance, and these were put into effect by the claims staff. As a result, it is believed that case reserves are being set on average 5% higher than before the change."

This information is highly relevant. It means that the incurred claims and case reserve projections previously illustrated are being thrown out of line. But, supposing the 5% change in case reserving level to be correct, the projections can immediately be re-done with adjusted figures. What is needed is to increase the figures in the upper left part of the case reserve triangle by 5%, while leaving those in the two longest diagonals unchanged. This is done below:

		d 0	1	2	3	4	5
	1	*1776*	*1409*	*1029*	*606*	384	234
	2	*2139*	*1701*	*1199*	809	475	
a	3	*2460*	*1971*	1546	969		
	4	*3031*	2549	1796			
	5	3644	2881				
	6	3929					

INCREASE ITALICISED FIGURES BY 5%

		d 0	1	2	3	4	5
	1	1865	1479	1080	636	384	234
	2	2246	1786	1259	809	475	
a	3	2583	2070	1546	969		
	4	3183	2549	1796			
	5	3644	2881				
	6	3929					

Having adjusted the case reserves, we will now apply the grossing up procedure of §F5. The full calculation array appears on the next page.

d

		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880			**[*pC*]**	
	5	1725	3261					
	6	1889						
		68.7	79.4	83.5	87.2	100.5	100.0%	
	1	1865	1479	1080	636	384	234	3717
		2716	1862	1294	729	382	234	
		70.1	80.7	81.6	90.4			
	2	2246	1786	1259	809	475		4317
		3204	2214	1543	895	473		
						100.5%		
		67.9	78.6	84.3				
	3	2583	2070	1546	969			5068
a		3803	2635	1835	1091			
					88.8%			
		69.9	80.5					
	4	3183	2549	1796				6041
		4551	3168	2161				
				83.1%				
		70.8				**[%]**		
	5	3644	3881			**[*kV*]**		6871
		5146	3610			**[*hV*]**		
			79.8%					
	6	3929						7542
		5653						
		69.5%						
								33,556

Overall Values: $\Sigma L\text{-}Ult$ 33,556
ΣpC^* 20,334

Reserve 13,222

There has been a pronounced reduction in the final figure for the reserve. It is now £13,222 as compared with £13,785 in §F5, a reduction of 4.1%. It will be interesting to find whether the adjustment in case reserves will similarly affect the incurred claims projection, but that will be tackled in the next section §F7. In the meantime, it is worth drawing out the triangle of *kV/hV %s* from the above calculation array.

		d 0	1	2	3	4	5
	1	68.7	79.4	83.5	87.2	100.5	*100.0%*
	2	70.1	80.7	81.6	90.4	*100.5%*	
a	3	67.9	78.6	84.3	*88.8%*		
	4	69.9	80.5	*83.1%*			
	5	70.8	*79.8%*				
	6	*69.5%*					

Again, the main diagonal of averages is not of note. But, higher in the table, the figures show a very respectable picture. The former irregularity has disappeared, and consistency (at least, so far as it can be judged by this test) is restored to the case reserves. What is more remarkable now, perhaps, is the comparatively slow progress made towards the 100% adequacy goal in development years 2 & 3. It appears that there may be a strong influence here from the IBNR, and from late development in claims already reported.

To sum up, the adjustment to case reserves has been made in the light of good evidence. It has produced a more satisfactory result, both in the analysis of case reserve adequacy (the last triangle above), and in the final figure for the claims reserve. Hence it will be right to prefer the new estimate of £13,222 for the liability over the former one of £13,785.

Final Notes

a) The above working has proceeded by taking *averages* of the % figures in each column, i.e. when deciding on the value to place in the leading diagonal. But it would be possible also to make a cautious estimate by taking the *lowest* of the % figures in each case.

b) Again, it would be possible to use a trending method, or other of the variations given in §E for downward projection of the columns. (But the link ratio technique as such is not appropriate for case reserve projections.)

c) The table of proportions above, showing the relative adequacy of the case reserves at various stages of the claims development, is particularly useful with *report* year data. This is because there is no disturbance from the IBNR claims — the set of claims is, of course, fixed from the beginning. The proportions observed in the table are then a very direct test of the case reserving standards applied by the office.

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[F7]
ADJUSTMENT OF INCURRED CLAIMS PROJECTION

The adjustments made in §F6 have naturally had a marked effect on the results of the case reserve projection. But in the incurred claims function, the influence of the case reserves is lessened by the addition of paid claims. One might hypothesise, therefore, that the effect will not be so great. The best course is to make a trial by using the new data. To recap, the adjusted case reserve figures are:

		d					
		0	1	2	3	4	5
	1	1865	1479	1080	636	384	234
	2	2246	1786	1259	809	475	
a	3	2583	2070	1546	969		
	4	3183	2549	1796			
	5	3644	2881				
	6	3929					

To these we add the usual figures for the cumulative paid claims:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

which gives the adjusted triangle of the incurred claims:

		d					
		0	1	2	3	4	5
	1	2866	3334	3503	3624	3719	3717
	2	3359	3889	4033	4231	4319	
a	3	3848	4503	4779	4946		
	4	4673	5422	5676			

5	5369	6142
6	5818	

We tackle the evaluation as before (§ F3,F4), using grossing up and link ratio methods. The working is shown on the next two pages.

Grossing Up Method

The first trial is by the Arabic version of grossing up, using averaged values of the factors.

	d							
		0	1	2	3	4	5	*ult*
	1	2866	3334	3503	3624	3719	3717	3717
		77.1	89.7	94.2	97.5	100.1	100.0%	
	2	3359	3889	4033	4231	4319		4315
		77.8	90.1	93.5	98.1	100.1%		
	3	3848	4503	4779	4946			5057
a		76.1	89.0	94.5	97.8%			
	4	4673	5422	5676				6032
		77.5	89.9	94.1%				
	5	5369	6142					6847
		78.4	89.7%					
	6	5818						7517
		77.4%						
								33,485

Overall Values: $\Sigma L\text{-}Ult$ 33,485
ΣpC^* 20,334

Reserve 13,151 (Grossing Up, Best Estimate)

This brings out the required reserve at £13,151, which is a fair reduction from the former value of £13,634. The difference is one of 3.5%, which is almost as much as the reduction in liability of 4.1% given above in the case reserve projections.

Link Ratio Method

The second trial (shown in the table overleaf) is by the link ratio method, with a cautious choice of ratios in each column. (The reduced version of the link ratio display is again used, as in §F4.) The estimate of the required reserve now comes out at £13,645. Compared with the previous figure of £14,738, this is a reduction of some 7.4%.

d

		0	1	2	3	4	5	*ult*
		1.163	1.051	1.035	1.026	.999	1.000	
	1	2866	3334	3503	3624	3719	3717	3717
		1.158	1.037	1.049	1.021			
	2	3252	3804	3973	4231	4319		4315
						.999		
		1.170	1.061	1.035				
a	3	3848	4503	4779	4946			5070
					1.025			
		1.160	1.047					
	4	4673	5422	5676				6102
				1.075				
		1.144						
	5	5369	6142					7008
			1.141					
	6	5818						7767
		1.335						
								33,979
r		1.170	1.061	1.049	1.026	.999	1.000	
f		1.335	1.141	1.075	1.025	.999	1.000	

Overall Values:	$\Sigma L\text{-}Ult$	33,979
	ΣpC^*	20,334
	Reserve	13,645

(Link Ratio, Cautious Estimate)

Having adjusted the case reserves, and reworked the figures for the incurred claims projections, we can return to the question posed at the end of §F4: "Can there be any systematic reason for the divergence of the projected reserves as given by the paid claims and incurred claims methods respectively?" Clearly, we have uncovered one such systematic cause in the inconsistency of the given case reserves. This made for a distortion in the incurred claims projection, which has now been corrected. The full comparison of the paid and incurred claims figures is now:

	Best Estimate	Conservative
pC-Projection	12,461	12,931
iC-Projection	13,634	14,738
iC-Adjusted	13,151	13,645

While the pC and iC figures are still not coincident, the agreement is a great deal better. Also, the range of values suggested by the iC projections has narrowed to a more acceptable figure. Choices will still have to be made, or further investigations carried out. But a partial reconciliation is better than none at all. If no further evidence were forthcoming, one might take averages of the pC and iC-adjusted figures. Thus:

Best Estimate - 12,806 Conservative - 13,288

Final Note on the Incurred Claims

In projections, the incurred claims function has the habit of looking rather similar in general form to the paid claims. Certainly, it can be treated by just the same statistical techniques described in §E. But in fact it is a *hybrid,* and the reserver should not lose sight of this fact.

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Section G
METHODS USING LOSS RATIO & LOSS RATIO PROJECTIONS

Preamble

Introduction of the loss ratio into claims reserving methods at first sight seems paradoxical. If one were to know the loss ratio for a class of business with confidence, then the reserving procedure would become almost trivial. But of course the loss ratio is subject to uncertainty, just like other quantities used in claims reserving. Here again the past is no sure guide to the future. But though the reserver cannot have full knowledge of the future for the loss ratio, some familiarity with its past history and the current expectations of underwriters and ratemakers will be of great service.

This familiarity, in fact, should help to provide the reserver with a kind of standard, or benchmark, against which the results of other projections can be assessed. It should help to stabilise results where data are volatile, and provide a first guide to reserves where data are scanty or even non-existent. The loss ratio, and the techniques associated with it, thus form an important part of the reserver's toolkit. The only additional data element required for this work is the premium income (earned or written) for the class of business in question. Being a valid measure of the risk exposures it gives *scale* to the loss data, and hence enables the loss ratio benchmarking to begin.

Contents

[G1]
CONCEPT OF THE LOSS RATIO

The work so far has depended on the use of paid claims and case reserve data. At this stage we introduce a new element — the loss ratio — which brings considerable added scope to the methods.

The loss ratio is a simple concept, but a fundamental one in general insurance. If we take a class or subgroup of business and look at any given cohort, then once the development is complete the loss ratio can be found with certainty. It provides a natural way of summing up the result as a single figure.

Loss Ratio = Ultimate Losses / Related Premium

or $$\lambda = L\text{-}ult \,/\, P$$

where λ is used as the symbol for loss ratio, and P for the premium earned in relation to the losses.

If we now aggregate the cohorts, we can find the loss ratio for the whole class of business. It will be in such terms that the underwriter thinks when quoting, say, the loss ratio for the motor portfolio. (The one problem with the measure, of course, is that the most recent cohorts will not be fully developed, and so cannot contribute properly to it. The loss ratio is only known with certainty several periods *in arrear*.)

The definition given above requires a little more attention. While the meaning of the ultimate losses *L-ult* should be clear, the premium term *P* remains in question. Does it denote earned premium or written premium, or perhaps the premium in-force? Is it an office premium including commission and expense, or is it the pure risk premium only? There is no absolute answer, and different forms can be used at different times.

To begin with, if we are using accident year cohorts as the basis of study, then earned premium will be the correct measure. But if policy, or contract, year cohorts are in use (as is common in reinsurance and the London Market), then written premium will be indicated. The point is that the premium definition should correspond to the risk exposure period of the cohort. The rule is:

Accident Year Exposure — *Earned* Premium
Policy Year Exposure — *Written* Premium

The question of including expenses and/or commission in the premium is more tricky. It would be quite possible to work either with the pure or office premiums. However, the picture is a fuller one if commission and expense *are* included, and we shall take that to be the case in the Manual. (This is the usual practice in the British insurance industry, when quoting a loss ratio — but once again, the practice does differ in the London Market and reinsurance.)

Sources for the Loss Ratio

If we intend to apply loss ratio methods in reserving, the key question that arises is how to select the appropriate ratio for a given class or subgroup of business. There are a number of sources that might be used:

a) Data of past results for the given class.
b) Assumptions being used in the ratemaking process.
c) Opinion of underwriters and claims officials with knowledge of the business.
d) Market statistics for similar types of business, if available.

But whatever the source, it must be recognised that a forecast of some kind is effectively being made. If the class of business has a good stable record of loss ratio in the past 5 years, say, that is encouraging. But there is no guarantee that the level will be adhered to in future years. And more often, the loss ratio will be found to vary appreciably, as the underwriting cycle and other economic influences take their course.

Nevertheless, one has to make the best use of the evidence to hand, and take a rational view of the likely future course for the loss ratio. Then, as the months and years pass, the view must be updated as new influences make their mark, and old ones fade away or return. The importance of setting the loss ratio is that, at least for the time being, it will establish a benchmark against which the emerging loss development can be assessed. And it will give the reserver a standard, albeit a changeable one, to which to refer when other measures fail or cannot be applied.

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[G2]
NAIVE LOSS RATIO METHOD

The word "naive" is included in the title as a warning. The estimate of the claims reserve given by this method derives from the simplistic assumption that the loss ratio cannot lie. Unfortunately, the real world does not contain such certainties — but the method still gives a useful reference point against which to view other more sophisticated methods, such as the Bornhuetter-Ferguson in §G3.

To carry out the estimation, some data are needed, and we shall as usual begin from the paid claims figures of the main example. These are repeated here for convenience:

		d					
		0	1	2	3	4	5
	1	1001	1855	2423	2988	3335	3483
	2	1113	2103	2774	3422	3844	
a	3	1265	2433	3233	3977		
	4	1490	2873	3880			
	5	1725	3261				
	6	1889					

Beside these must be set, naturally enough, the accepted loss ratio for the given class of business. We shall take this to be 83%, supposing it to have been settled after a consideration of past data for the class. Finally, we need to know the earned premium for each of the accident years 1 to 6. This is given here, with the figures in £1,000s as usual:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486

(*aP* is being adopted as the symbol for earned premium. For written premium, applicable to the policy year case, we would write *wP*.)

To make the actual estimate is simplicity itself. We have only to multiply the earned premium figures by the loss ratio of 83% to obtain the ultimate losses for each accident year. Then deducting the paid claims to date gives the required reserve. The calculations are shown overleaf.

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	83%	83%	83%	83%	83%	83%
^*L-ult*	7057	6210	5470	4714	4170	3723
*pC**	1889	3261	3880	3977	3844	3483
^*V*	5168	2949	1590	737	326	240

Overall Values:	Σ*L-ult*	31,344
	Σ*pC**	20,334
	Reserve	11,010

Not surprisingly, the value now obtained for the claims reserve does not accord with the previous paid and incurred claims projections. But what interpretation is to be placed on the discrepancy? This cannot be answered without some further investigation.

The first need, clearly, will be to re-examine the value taken for the loss ratio. It was based on past data for the business in question. But accident years for which development is complete enough to yield a loss ratio will be relatively old by now. More up-to-date information must be sought. Consultation with underwriters may indicate, say, that the market has softened in the last few years, with a persistent tendency for loss ratios to increase. Further talks with ratemaking staff may show that current rates are being set with an implicit loss ratio closer to 90%. This new evidence suggests that the loss ratio should be *trended*, say by 1% p.a. from 84% in year *a*=l to 89% in year *a*=6. This allows the reserves to be recalculated as follows:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	89%	88%	87%	86%	85%	84%
^*L-ult*	7567	6584	5733	4885	4270	3768
pC	1889	3261	3880	3977	3844	3483
^*V*	5678	3323	1853	908	426	285

Overall Values:	Σ*L-ult*	32,807
	Σ*PL**	20,334

Reserve	12,473

The result is now in line with the best estimate projections of the paid claims (§E4, E10). This is a satisfactory result, but should not give grounds for complacency. It would not be unusual to find, in later years, that the loss ratio had run ahead rather faster than originally predicted.

The major criticism of the method, however, is that it completely ignores the pattern of claims development to date for the recent accident years. The estimate of overall losses depends *only* on the premium income and the stated loss ratio for the class of business. Important changes shown, or incipient in, the claims development patterns will not be acknowledged or made use of in any way. The method only comes into its own where the claims development data are either scanty, unreliable or missing altogether. The best examples would be in new lines of business, and in the very long-tailed liability classes.

Thus, for the latter, the most recent accident years will not have had time to produce meaningful development figures in the context of the full liability. But the loss ratio approach enables an initial estimate to be made. Of course, it must be modified as time passes and more becomes known about the development. Then in the later years, more reliance can be placed on claims development figures, and the loss ratio estimate gradually phased out. The Bornhuetter-Ferguson method in §G3 in fact gives an automatic means for achieving this end.

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[G3]
BORNHUETTER-FERGUSON METHOD — INTRODUCTION

For any claims projection method based effectively on the use of development factors such as the chain-ladder method, it is often the case that the projected result cannot be relied on with the degree of confidence that one would like. This is particularly likely for more recent underwriting years, where the development factor to project from the current to ultimate claim amount is relatively large and variable, owing to the present lack of claims development.

However, it may be possible to make use of an alternative ultimate figure, usually derived from an assumed loss ratio. This may simply be taken as a fixed rate (such as 100%) as a reasonable first estimate of that experience, or it may be derived from external market views and information.

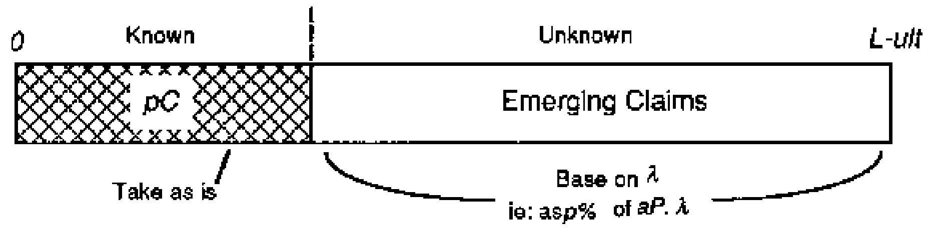
It is then possible to combine the original projected result with this alternative (a priori) value, using a weighted credibility approach. Under this, most weight is initially attached to the a priori value, gradually reducing to zero as the actual claims experience (and hence the projected value) develops towards its ultimate value. The Bornhuetter-Ferguson method adopts this principle.

The Bornhuetter-Ferguson method provides a nicely judged combination of the naive loss ratio method and the earlier paid/incurred claims projections. It is based on the idea of splitting the overall loss for each accident year into its past and future, or emerging, portions. These are then treated separately, according to their merits. The argument goes as follows:

> "As far as the past is concerned, the claims are already well known (paid claims method) or at least well estimated (incurred claims method). But the future is not well known, and the particular claims patterns and case reserves to date of the given accident year do not necessarily provide the right clue to it. It may be better to use a *more general* estimator, based on the overall loss ratio for the class of business in hand. This being done, and the two parts added together, we then have the most reliable estimate we can get for the overall losses, and hence for the required reserves."

The argument has much merit in it. Thus, taking the naive loss ratio method, we have already seen that it pays no attention to the actual claims development in the most recent accident years. Apparently, it flies in the face of reality. On the other hand, the claims development methods rely on the continuation into the future of the patterns for claim reporting and settlement, which seem to be indicated by the particular data in hand. A sudden shift in the pattern for the latest accident year in particular will throw the projections into disarray. The Bornhuetter-Ferguson method steers a safer course in these eventualities — its stability shows through well in the numerical example of §G8.

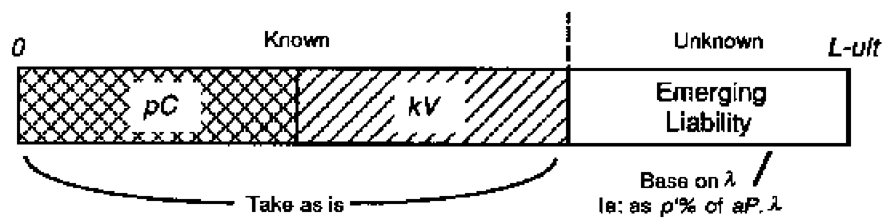
The Bornhuetter-Ferguson principle can equally well be applied using either paid or incurred claims as the base. With paid claims, the picture is as follows:



B/F on *pC*– Projection

The claim payments are split into 2 parts: those already made, and those which will emerge in the future. The first part has the known value *pC*, while the second is unknown. It is to be estimated as a proportion *p* of the final losses, which in turn are estimated by the simple application of the loss ratio to the earned premium.

With incurred claims, a larger part of the liability is taken as already known, by adding in the case reserves to the paid claims. But the unknown part of the liability is again found as a (smaller) proportion *p*' of the estimated final losses. The picture this time is:



B/F on *iC*– Projection

The main problem in B/F methods is just to determine the proportion *p* or *p*'. It turns out this can be done via the usual link ratio or grossing up methods applied to the triangle of paid claims or incurred claims data. We shall see that the correspondence is:

$$p = (1 - 1/f) \text{ or } p = (1 - g)$$

where *f* is the final link ratio, and g the grossing up factor, for a given accident year. (Since *f* and *g* can be applied equally for paid and incurred claims, the relations hold just as well whether *p* or *p*' is involved.)

Abbreviations

We are already using B/F as a shorthand for the Bornhuetter-Ferguson method. Further abbreviations to be used are:

$n\lambda$ — Naive Loss Ratio Method
$t\lambda$ — Trended Naive Loss Ratio Method

BF-pC — Bornhuetter-Ferguson Method applied to Paid Claims data
BF-iC — Bornhuetter-Ferguson Method applied to Incurred Claims

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[G4]
BORNHUETTER-FERGUSON ON INCURRED CLAIMS

The section is devoted to a numerical illustration of the B/F method. In the original paper (Bornhuetter & Ferguson 1972), the authors use a static loss ratio, and work with data in the form of *incurred* claims. They also use a link ratio/chain ladder approach in the first part of the exercise. We shall repeat these particular features here. The data will be the adjusted incurred claims triangle (§F7), with the premium figures and loss ratio (83%) from §G2.

The first stage is just to work out the link ratios themselves. It is the final ratios (f-values) that are needed for the method. Although a chain ladder approach is used below, any of the main link ratio variations could be substituted. Again, a grossing up method could be used equally well, with the g-factors substituting for the f-ratios. (Equivalence is: $1/g$ <—> f.) The working runs as follows:

		d 0	1	2	3	4	5	*ult*
	1	2866	3334	3503	3624	3719	3717	3717
	2	3359	3889	4033	4231	4319		
a	3	3848	4503	4779	4946			
	4	4673	5422	5676				
	5	5369	6142					
	6	5818						
Σ–*el*		20115	17148	12315	7855	3719		
Σ		25933	23290	17991	12801	8038	3717	3717
r		1.158	1.049	1.039	1.023	.999	1.000	
f		1.290	1.114	1.062	1.022	.999	1.000	

This array could be carried through to find the ultimate losses and reserve estimate, just as in the original demonstration of §E8. But these figures are not necessary for the B/F projection. It is the f-ratios which are the crucial output at this stage of the work.

The next step is to invert the f-ratios, and subtract the results from unity. The reason for doing this will soon become apparent.

f	1.290	1.114	1.062	1.022	.999	1.000

1/*f*	.775	.898	.942	.978	1.001	1.000
1 − 1/*f*	.225	.102	.058	.022	−.001	0

To explain, inverting the *f*-ratio gives the equivalent of a *g*- or grossing up factor. Now a *g*-factor simply shows the proportion that the claims in its column (whether paid or incurred) bears to the ultimate loss, as estimated. Hence $(1-g)$ shows the proportion that the *remaining* claims should bear to the ultimate figure. If we apply these $(1-g)$ factors, or $(1-1/f)$ which comes to the same thing, to the ultimate loss, then we have the remaining claims which should emerge in the future.

Now, at last, we are ready to bring in the loss ratio and premium data. These, of course, give us our benchmark estimates for the final losses, just as in the naive loss ratio method. To recap, the figures are:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	83%	83%	83%	83%	83%	83%
B-ult	7057	6210	5470	4714	4170	3723

These benchmark losses (symbol *B-ult*) are now to be used *as a base* for finding the remaining, or emerging, claims. This point is the crux of the B/F method, and where it distinguishes itself from the usual *pC* and *iC* projection methods. The calculations are straightforward:

a	6	5	4	3	2	1
B-ult	7057	6210	5470	4714	4170	3723
$1-1/f$	.225	.102	.058	.022	–.001	0
^*eV*	1588	633	317	104	–4	0

The result of multiplying the benchmark losses by the $(1-1/f)$ factors is called here the emerging liability, symbol ^*eV*. (The ^ mark as usual shows that the value is in the nature of an *estimate*.)

^*eV* is the liability still to emerge. It is to be contrasted with the liability already established, which is just the case reserves, *kV*. Adding the two parts together will give the whole required reserve:

a	6	5	4	3	2	1
*kV**	3929	2881	1796	969	475	234
^*eV*	1588	633	317	104	–4	0
CV	5517	3514	2113	1073	471	234

(The *kV*-values are taken from the case reserve data triangle — see §F3. The * indicates use of the main diagonal.)

Overall Values:	ΣkV*	10,284
	Σ^eV	2,638
	Reserve	12,922

An alternative approach at this stage is to add the emerging liability to the incurred claims themselves. This then gives the estimate of the ultimate losses:

a	6	5	4	3	2	1
*iC**	5818	6142	5676	4946	4319	3717
^*eV*	1588	633	317	104	–4	0
^*L-ult*	7406	6775	5993	5050	4315	3717

Overall Values:	Σ*L-ult*	33,256
	ΣpC*	20,334
	Reserve	12,922

There are one or two slight puzzles here.

a) We now have two new sets of figures for the estimated final losses: i) the benchmark set, ii) the set found immediately above. Which are to be believed? Of course, the B/F method points us towards the second set. The benchmark figures were used along the way, but can now be dropped. If we took them completely to heart, we should just arrive back at the naive loss ratio method of §G2.

b) There is a negative figure in the set of values for ^*eV*, the emerging reserves, at *a*=2. That is really no problem. It arises because the incurred claims for the earlier accident year *a*=1 (i.e. at the point *d*=4) exceeds the ultimate loss for that year. The excess is then projected forward into the figures for year *a*=2. In this particular case, the value is trivial. Nevertheless, the liability is still reduced — it would be possible to take a cautious view by excluding any such negatives in the projection, just by setting them to zero.

Summary of the Method

To bring the whole procedure together, we now summarise the main steps in the Bornhuetter-Ferguson method using incurred claims:

i) The incurred claims data are set out in the triangular form, and projected using a link ratio or grossing up technique. (The chain ladder variation is used here, but is not obligatory.)

ii) If a link ratio method is used, the final ratios *f* are inverted (i.e. to produce the equivalent of a *g*-factor).

iii) The benchmark losses are found by multiplying the earned premium for each accident year by the chosen loss ratio.

iv) The emerging reserves are estimated by applying factor (1–1/*f*), or (1–*g*) as the case may be, to the benchmark losses.

v) The emerging reserves are added to the existing incurred claims data to give the estimate of final loss.

vi) The reserve is taken as the estimated final loss minus the paid claims.

Calculations in Full

		d 0	1	2	3	4	5	*ult*
	1	2866	3334	3503	3624	3719	3717	3717
	2	3359	3889	4033	4231	4319		
a	3	3848	4503	4779	4946			
	4	4673	5422	5676				
	5	5369	6142					
	6	5818						
Σ–*el*		20115	17148	12315	7855	3719		
Σ		25933	23290	17991	12801	8038	3717	3717
r		1.158	1.049	1.039	1.023	.999	1.000	
f		1.290	1.114	1.062	1.022	.999	1.000	
1/*f*		.775	.898	.942	.978	1.001	1.000	
aP		8502	7482	6590	5680	5024	4486	
λ		83%	83%	83%	83%	83%	83%	
B-ult		7057	6210	5470	4714	4170	3723	
1–1/*f*		.225	.102	.058	.022	–.001	0	
^*eV*		1588	633	317	104	–4	0	
*iC**		5818	6142	5676	4946	4319	3717	
^*L-ult*		7406	6775	5993	5050	4315	3717	

Overall Values:	$\Sigma L\text{-}ult$	33,256
	ΣpC^*	20,334
	Reserve	12,922

Key to Symbols

Σ	Column sum	r	One-step link ratio
$\Sigma{-}el$	Column sum – Last element	f	Final link ratio
aP	Earned premium	λ	Loss ratio
$B\text{-}ult$	Benchmark losses	iC	Incurred claims
eV	Emerging reserves	^	Estimate symbol
$L\text{-}ult$	Ultimate losses (B/F estimate)	pC	Paid claims

(Where a grossing up technique is used in preference to link ratios, g will substitute for $1/f$.)

Formulae

It may help also to put down the main relationships which are used in the calculations. These can be expressed in words and/or symbols as follows:

Benchmark Loss $\quad B\text{-}ult = \lambda . aP$
= Loss Ratio × Earned Premium

Emerging Liability $\quad \hat{}eV = (1-1/f).\ B\text{-}ult$
= B/F Proportion × Benchmark Loss

Estimated Ultimate Loss $\quad \hat{}L\text{-}ult = iC + \hat{}eV$
= Incurred Claims + Emerging Liability

Reserve Required $\quad CV = \hat{}L\text{-}ult - pC$
= Estimated Ultimate Loss – Paid Claims to Date

The first formulation given in the text, using Case Reserves to go direct to the full claims reserve value, is:

Reserve Required $\quad CV = kV + \hat{}eV$
= Case Reserves + Emerging Liability

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[G5]
BORNHUETTER-FERGUSON ON PAID CLAIMS

The B/F method is equally applicable to paid claims data as to the incurred. The working is very similar, indeed almost coincident with that used for the incurred claims. As usual, we illustrate by means of numerical example.

The first part of the exercise is to work through the triangle of paid claims by either a link ratio or grossing up method. Having used the former for the incurred claims, we will here choose grossing up by way of contrast. But any of the link ratio variations could equally well be used. The point is to determine apt values for the g-factors, i.e. the equivalent of $1/f$ in the link ratio case. The working is as follows, using the Arabic technique with simple averaging of the factors in each column. The last 2 lines summarise the resulting g and $(1-g)$ values.

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
		27.0	50.1	65.4	80.6	90.0	94.0%	
	2	1113	2103	2774	3422	3844		
		26.1	49.2	64.9	80.1	90.0%		
	3	1265	2433	3233	3977			
a		25.6	49.2	65.4	80.4%			
	4	1490	2873	3880				
		25.0	48.3	65.2%				
	5	1725	3261					
		26.0	49.2%					
	6	1889						
		25.9%						
g		.259	.492	.652	.804	.900	.940	
$1-g$		.741	.508	.348	.196	.100	.060	

The second stage is to bring in the benchmark estimates for the final losses, based of course on the loss ratio and earned premium data. The figures are the same as they were for the BF-*iC* projection.

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	83%	83%	83%	83%	83%	83%
B-ult	7057	6210	5470	4714	4170	3723

To these benchmark figures, we simply apply the factors (1−*g*) in order to bring out the remaining, or emerging, claims. The rationale is as before: the *g*-factor shows the proportion that the claims in its column bear to the ultimate loss, as estimated. Hence (1−*g*) shows the proportion that the *remaining* claims should bear to the ultimate figure. The working is as follows:

a	6	5	4	3	2	1
B-ult	7057	6210	5470	4714	4170	3723
1−*g*	.741	.508	.348	.196	.100	.060
^*eC*	5229	3155	1904	924	417	223

Overall Value: Σ ^*eC* 11,852

The result is here called the emerging claims, symbol ^*eC*. (The ^ denotes an estimate, as usual.) For each accident year, ^*eC* is the claims still to emerge. The summation over the accident years immediately gives the estimate for the full claims reserve.

Adding the emerging claims to the claims already established, i.e. the value *pC,* gives the final estimated losses by accident year:

a	6	5	4	3	2	1
*pC**	1889	3261	3880	3977	3844	3483
^*eC*	5229	3155	1904	924	417	223
^*L-ult*	7118	6416	5784	4901	4261	3706

Overall Values:	Σ*L-ult*	32,186
	Σ*pC**	20,334
	Reserve	11,852

Summary of the Method

To bring the whole procedure together, we now summarise the main steps in the Bornhuetter-Ferguson method using paid claims:

i) The paid claims data are set out in the triangular form, and projected using a grossing up or link ratio technique. (The Arabic variation is used here, but is not obligatory.)

ii) If a link ratio method is used, the final ratios f are inverted (i.e. to produce the equivalent of a g-factor).

iii) The benchmark losses are found by multiplying the earned premium for each accident year by the chosen loss ratio.

iv) The emerging claims are estimated by applying the factor $(1-g)$, or $(1-1/f)$ as the case may be, to the benchmark losses. Adding these claims together across the accident years gives the required reserve.

v) Finally, the emerging claims are added to the existing paid claims data to give the estimate of final loss.

Calculations in Full

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
		27.0	50.1	65.4	80.6	90.0	94.0%	
	2	1113	2103	2774	3422	3844		
		26.1	49.2	64.9	80.1	90.0%		
	3	1265	2433	3233	3977			
a		25.6	49.2	65.4	80.4%			
	4	1490	2873	3880				
		25.0	48.3	65.2%				
	5	1725	3261					
		26.0	49.2%					
	6	1889						
		25.9%						

g	.259	.492	.652	.804	.900	.940
aP	8502	7482	6590	5680	5024	4486
λ	83%	83%	83%	83%	83%	83%
B-ult	7057	6210	5470	4714	4170	3723
1–*g*	.741	.508	.348	.196	.100	.060
^*eC*	5229	3155	1904	924	417	223

Overall Reserve: Σ ^*eC* 11,852

It is usually worthwhile to complete the calculations to give the final losses aswell as the reserve itself. This is done below.

^*eC*	5229	3155	1904	924	417	223
*pC**	1889	3261	3880	3977	3844	3483
^*L-ult*	7118	6416	5784	4901	4261	3706

Overall Losses: Σ*L-ult* 32,186

Key to Symbols

g	Grossing up factor		
aP	Earned premium	λ	Loss ratio
B-ult	Benchmark losses	*pC*	Paid claims
eC	Emerging claims	^	Estimate symbol
L-ult	Ultimate losses (B/F estimate)		

(Where a link ratio technique is used in preference to grossing up, l/*f* will substitute for *g*.)

Formulae

The main relationships used in the calculations can be expressed in words and/or symbols as follows:

Benchmark Loss $\quad B\text{-}ult = \lambda .\, aP$
= Loss Ratio × Earned Premium

Emerging Claims $\quad \hat{}eC = (1-g).\, B\text{-}ult$
= B/F Proportion × Benchmark Loss

Reserve Required $\quad CV = \Sigma_a(\hat{}eC)$
= Sum of Emerging Claims by Accident Year

The estimate of the ultimate loss follows from the further relationship:

Estimated Ultimate Loss $\quad \hat{}L\text{-}ult = pC + \hat{}eC$
= Paid Claims + Emerging Claims

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[G6]
COMPARISON OF RESULTS

Having worked through the Bornhuetter-Ferguson method for both paid and incurred claims, it will be worthwhile to make a comparison of the numerical results. Following the order of the main text, let us begin with the BF-*iC* projection. Because this is a hybrid technique, we need to take into account 3 different projections:

a) Incurred Claims Projection (Link Ratio, Best Estimate)
b) B/F on Incurred Claims
c) Naive Loss Ratio Method

Figures for the projected losses and the required reserves are as follows:

		Projected Losses			% Divergence
		iC	BF-*iC* (from G4.3)	$n\lambda$ (from G2.2)	
Accident Year	6	7505	7406	7057	22.1
	5	6842	6775	6210	10.6
	4	6028	5993	5470	6.3
	3	5055	5050	4714	1.5
	2	4315	4315	4170	0.0
	1	3717	3717	3723	0.0
	Σ	33,462	33,256	31,344	9.7

		Required Reserves			% Divergence
		iC	BF-*iC* (from G4.2)	$n\lambda$ (from G2.2)	
Accident Year	6	5616	5517	5168	22.1
	5	3581	3514	2949	10.6
	4	2148	2113	1590	6.3
	3	1078	1073	737	1.5
	2	471	471	326	0.0
	1	234	234	240	0.0

Σ	13,128	12,922	11,010	9.7

The % divergence is measured as the relative divergence of the BF-*iC* loss (or reserve) value from the incurred claims result towards the naive loss ratio one. Formula:

$$[\hat{}L\{iC\} - \hat{}L\{BF{-}iC\}] \,/\, [\hat{}L\{iC\} - \hat{}L\{n\lambda\}]$$

The characteristics of the B/F method begin to emerge well from this comparison. Three points in particular can be made:

a) It yields an overall result intermediate between the incurred claims projection and the naive loss ratio estimate.

b) In the more developed accident years (a=1,2,3), it is very close to or coincident with the *iC* values.

c) In the less developed years (a=4,5,6), it diverges rather more from the *iC* projection in the direction of the naive loss ratio estimate.

These characteristics show clearly the general properties of the B/F method which are explored further in §G7. They will be found to be repeated to a large extent with the paid claims projection — but there is a very important difference of emphasis. The figures now follow:

		Projected Losses			% Divergence
		pC (from E3.2)	BF-*pC* (from G5.2)	$n\lambda$ (from G2.2)	
Accident Year	6	7293	7118	7057	74.2
	5	6628	6416	6210	50.7
	4	5951	5784	5470	34.7
	3	4947	4901	4714	19.7
	2	4271	4261	4170	9.9
	1	3705	3706	3723	5.6
	Σ	32,795	32,186	31,344	42.0

		Required Reserves			% Divergence
		pC (from E4.1)	BF-*pC* (from G5.2)	$n\lambda$ (from G2.2)	
Accident Year	6	5404	5229	5168	74.2
	5	3367	3155	2949	50.7
	4	2071	1904	1590	34.7
	3	970	924	737	19.7

2	427	417	326	9.9
1	222	223	240	5.6
Σ	12,461	11,852	11,010	42.0

The % divergence is measured as the relative divergence of the BF-*pC* loss (or reserve) value from the paid claims result towards the naive loss ratio one. Formula:

$$[\hat{}L\{pC\} - \hat{}L\{\mathrm{BF}\text{-}pC\}] \,/\, [\hat{}L\{pC\} - \hat{}L\{n\lambda\}]$$

It will be seen that the general pattern whereby the B/F method yields an intermediate result remains. But this time the divergence from the *pC* values is present in all the accident years. In the most recent years (a=5,6), it becomes far more pronounced in the direction of the naive loss ratio estimate. This difference of emphasis is only to be expected. The reason is that the BF-*pC* method refers all claims not actually paid by the reserving date to the benchmark calculation. But in BF-*iC*, only those additional reserves for claims beyond the case reserves are referred in this way. (The diagram in §G3 should make the point clear.)

The particular figures for the divergence of the B/F results from the *pC* and *iC* projections shown here are, of course, illustrative only. The exact values taken will depend completely on the data in hand, and the variations found in practice can be wide.

There is one last interesting point to be made. That is, while for any given accident year the B/F figures must be of an intermediate nature, it is not true in absolutely all cases of the overall result. This is because crossovers can occur, say, in the naive loss ratio and *pC* figures for projected claims by accident year, and these can sometimes push the B/F figure out of alignment. But it would be unusual to find such a result in practice.

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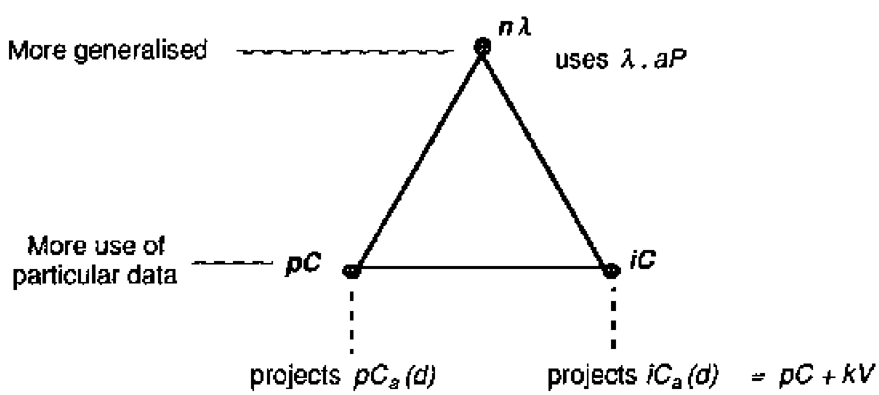
[G7]
TAKING STOCK OF THE METHODS (*pC*/*iC*/*n*λ/BF)

It is time to take stock of the position. We have by now used three different primary routes to reach the reserve estimate:

a) Paid claims projection
b) Incurred claims projection
c) Naive loss ratio method

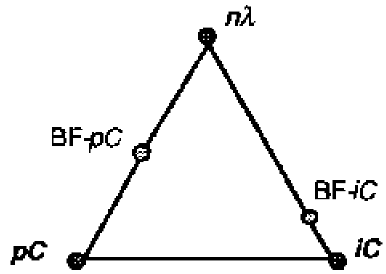
In addition, we have put together route c) with either a) or b) according to the Bornhuetter-Ferguson prescription.

Taking first the three primary routes, the difference between them is not fully characterised by the method of calculation — in fact, the calculations required for the *pC* and *iC* projections are identical in form. Rather, the difference is to be found in the data elements, i.e. the starting point for the method, and in the assumptions which underlie it.



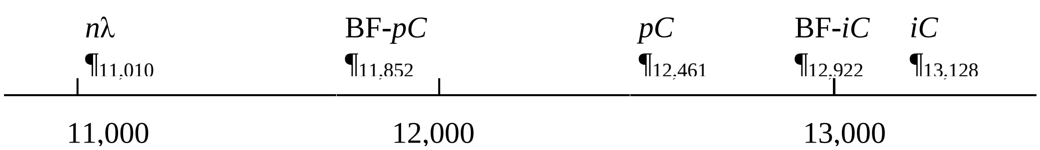
The diagram summarises the position. Note that while the *pC* and *iC* methods make use of developing data, dependent on both accident year and development period, the naive loss ratio method takes no account of these details. In a sense, it is on a loftier, more generalised plane.

Next we can place the B/F projections on the diagram. They try to make the best of both worlds, the generalised and the detailed, by combining the naive loss ratio method with either *pC* or *iC*. (The principle behind the combination is given in full in §G3.)



Note that the BF-*iC* point will be relatively closer to *iC* than will BF-*pC* be to *pC*. The exact positioning of the B/F points along the two lines will depend very much on the particular data distributions given, as already noted in the previous section.

Another useful way to illustrate the B/F divergence towards *n*λ from either *pC* or *iC* is to plot the reserve values obtained along an axis:



In this particular case, the BF-*pC* point divides the (*n*λ *pC*) range in the ratio 58:42. For BF-*iC* and the (*n*λ *iC*) range, the ratio is 90:10.

Choice of Estimate

Let us return to the central problem — given the widening range of estimates, how is the final choice to be made? As usual, hard and fast rules cannot be laid down. To begin with, the right course is to seek ways of reconciling the different estimates, to look for systematic reasons for the divergences observed. In the present case, the estimate most out of kilter is the naive loss ratio one.

An explanation for this, in fact, is already to hand. We have been working with *a fixed* loss ratio of 83%, which was shown to be a poor assumption in §G2. The 83% value was used, essentially, because the original B/F paper employs such a fixed ratio in its description. But there is nothing to prevent the reserver from applying B/F with a trended ratio, or a ratio varying in some other way — say, cyclically, to match the fluctuations of the underwriting cycle.

To illustrate this, let us bring in the trended ratio, moving from 84% in year *a*=l to 89% in year *a=6.* This brings out the losses as follows (as in §G2):

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	89%	88%	87%	86%	85%	84%
L-ult	7567	6584	5733	4885	4270	3768

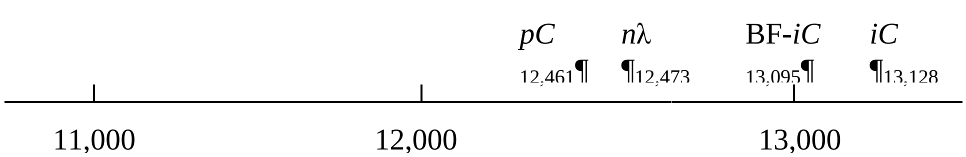
The resulting estimate for the full reserve is £12,473.

We can now bring these revised figures into the main B/F calculations as the benchmark losses. There is no need to rework the basic claims triangles, nor the calculation of the (l–l/*f*) factors, but the subsequent figures must be re-calculated. The results for the BF-*iC* projection are set out here:

a	6	5	4	3	2	1	
B-ult	7567	6584	5733	4885	4270	3768	
1–1/*f*	.225	.102	.058	.022	–.001	0	
							(from G4.2)
^*eV*	1703	672	333	107	–4	0	
*iC**	5818	6142	5676	4946	4319	3717	
							(from G4.3)
^*L-ult*	7521	6814	6009	5053	4315	3717	

Overall Values:	Σ*L-ult*	33,429
	Σ*pC**	20,334
	Reserve	13,095

It is hardly surprising to find that the BF-*iC* result is now even closer to the original *iC* value of £13,128. (This is because the loss ratio estimate itself has been brought much nearer to the *pC* and *iC* projections.) The values can again be plotted along an axis:



It would be possible to repeat the B/F calculations for the *pC's*, using the new benchmark losses. But since the loss ratio and *pC* estimates are already so close, this is hardly worthwhile. In the present example, we have reached the point where loss ratio and B/F calculations tell us little more than the original *pC* and *iC* projections. We are back to the range of approximately £12,500 to £13,000 for the best estimate of the required reserves.

Is this a typical result, throwing doubt on the usefulness of doing loss ratio and B/F calculations at all? The answer is emphatically *no*. Loss ratio methods, including particularly the B/F variation, are an important tool in the armoury. They come particularly into their own for the very long-tail classes of business, where the build up of paid and incurred claims in the early years is very slow. But there are more general reasons for their importance, which will come out in the remaining sections of this part of the Manual.

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[G8]
SENSITIVITY TESTING & CHOICE OF ESTIMATE

There is one feature of the projections which has been mentioned briefly before (§E1), but not brought out sufficiently. This is the fact that, when the overall claims reserve is analysed, it is the most recent accident years which make the dominant contribution. For example, the *pC* projection we have been using gives the following breakdown:

Total Reserve = 12,461

a	6	5	4	3	2	1
$\hat{V}$	5404	3367	2071	970	427	222
%	43.4	27.0	16.6	7.8	3.4	1.8

Here, 87% of the liability is concentrated in the most recent three accident years. A similar, but not quite coincident, pattern results if we look at the trended loss ratio method's results (from §G2):

Total Reserve = 12,473

a	6	5	4	3	2	1
$\hat{V}$	5678	3323	1853	908	426	285
%	45.5	26.6	14.9	7.3	3.4	2.3

Here again, 87% of the liability comes from the three latter accident years, though this time the emphasis on the last of all (year 6) is even greater.

The corollary which must be noted is that the projections will tend to be particularly sensitive to the actual claims data for the very recent years. They will be most sensitive of all to the figure in the *bottom left hand corner* of the data triangle. But there is a notable exception to this general rule. The B/F projections, in fact, are not sensitive at all to the lower left hand figure. This feature is worth demonstrating in detail.

Consider a projection in which, for the latest accident year, the figures are as follows:

	£		
Paid claims	30	Grossing up factor	30%
Earned premium	125	Loss ratio	80%

The pC method gives a projected ultimate loss for the accident year of:

$$pC / g = 30 / .3 = 100$$

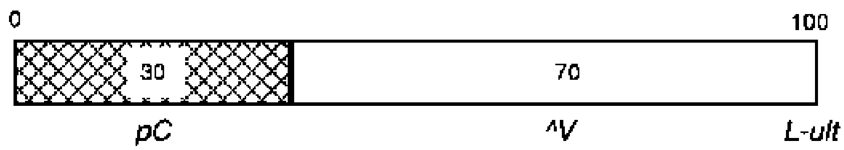
which agrees with the loss ratio method's value of:

$$\lambda \,.\, aP = .8 \times 125 = 100$$

The required reserve in both cases is:

$$(100 - 30) = 70$$

The position can be shown in a diagram:



Since pC and $n\lambda$ give the same answer, so also will BF-pC. The calculation is:

$$\hat{}V = (1 - g) \,.\, B\text{-}ult = (1 - .3) \times 100 = 70$$

Now consider what happens to the three estimates if, for some unexplained reason, the paid claims for the accident year come through as 33 instead of 30. It is assumed that every other quantity in the problem retains its former value. In particular, earned premium is still 125 and the g-factor is still 30%.

a) *pC Projection*

$$\begin{array}{llrcrcr} \hat{}L\text{-}ult & = & pC / g & = & 33 / .3 & = & 110 \\ \hat{}V & = & \hat{}L\text{-}ult - pC & = & 110 - 33 & = & 77 \end{array}$$

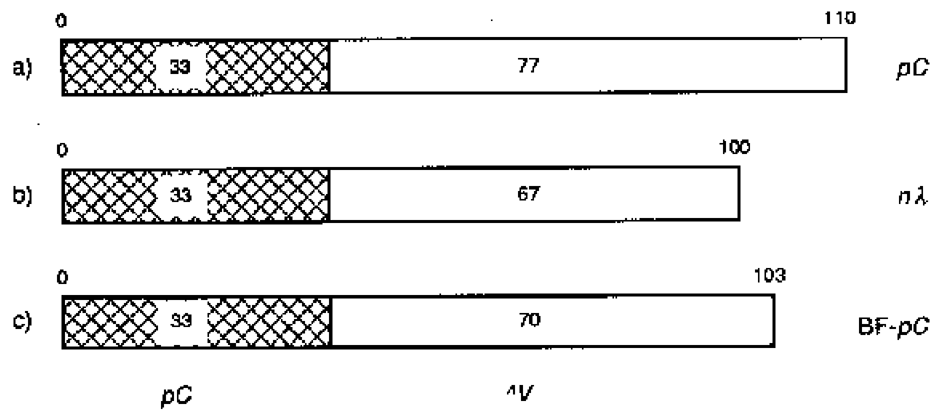
b) *nλ Estimate*

$$\begin{array}{llrcrcr} \hat{}L\text{-}ult & = & \lambda \,.\, aP & = & .8 \times 125 & = & 100 \\ \hat{}V & = & \hat{}L\text{-}ult - pC & = & 100 - 33 & = & 67 \end{array}$$

c) *BF-pC Projection*

$$\begin{array}{llrcrcr} \hat{}V & = & (1 - g) \,.\, B\text{-}ult & = & .7 \times 100 & = & 70 \\ \hat{}L\text{-}ult & = & \hat{}V + pC & = & 70 + 33 & = & 103 \end{array}$$

We now have three quite distinct answers! With an increase of 10% in the figure for paid claims, the pC method has increased the reserve for the year by the same proportion. The naive loss ratio method has *reduced* the reserve by a lesser amount (about 4%), while the BF-*pC* method has left the reserve exactly where it was before. The position can again be shown most clearly by a diagram:



This demonstrates the stability of the B/F method when data for the recent accident years are in a volatile state, or where there is doubt as to their true values. (In an accounting sense *pC* is a "hard" figure but in fact it is a random variable.) It is an extremely useful technique to apply in such circumstances.

Shifts in Payment Pattern & Loss Ratio

We have still not thrown enough light on how to choose the estimate, when given *pC* or *iC*, naive loss ratio and B/F values. But the example just given will be found to yield some clues. To begin with, we can look at the shift in the payment pattern of claims which is implicit in each method. The value to focus on here is the grossing factor *g*, of 30%. This will have been derived from the grossing up or link ratio methods applied to the earlier accident years. Given that the paid claims in the latest accident year now seem to be coming out on the high side, what response do the three methods make? That is, to what extent does each allow for a shift in the underlying payment pattern on the claims?

a) pC Projection

Assumes there is no shift at all. *g* remains static at 30%. This is the way the whole projection works.

b) nλ Estimate

Payment pattern is assumed to be speeding up. Effective *g*-value for the accident year changes to: 33/100 = 33%.

c) BF-pC Projection

Again, speed up in payment pattern assumed. But not so pronounced as *n*λ case. The factor is now: 33/103 = 32%.

A second way of looking at the reaction of the three methods is from the point of view of the loss ratio on the business. Taking the paid claims for the most recent accident year as a proportion of the earned premium, this has gone up from: 30/125 = 24% to the higher value of: 33/125 = 26.4%. Should it be assumed from this evidence that the ultimate loss ratio on the year will also increase? Again, the three methods give different answers:

a) pC Projection

Ultimate loss ratio will increase, from 80% to the value: 110/125 = 88%.

b) λA Estimate

In spite of first year increase in paid claims, the loss ratio overall will stay put at 80%.

c) BF-pC Projection

Loss ratio will increase, but not so much as suggested by the *pC* method. Its value will be: 103/125 = 82.4%

Using this analysis, a more general answer can now be given on the choice of estimates as between *pC*, naive loss ratio and BF-*pC*. This runs as follows:

i) Where the data are very stable, there is likely to be little to choose between the different estimates.

ii) Where the data patterns are shifting, the reserver should assess to what extent stability still remains in either: a) the payment patterns, or b) the loss ratio for the succeeding accident years.

iii) Where the evidence is of a stable payment pattern, the *pC* projection will be preferable. Where evidence supports a fairly constant loss ratio, the loss ratio method will be better.

iv) Where data are decidedly volatile, or scanty, particularly in the most recent accident years, a B/F projection will come into its own, and provide the firmest ground.

Very similar considerations apply to the choice between the *iC*, *n*λ and BF-*iC* methods. In this case, however, reporting patterns of claims and stability of case reserving enter the balance in addition to the payment patterns. This may have the merit of bringing in somebody else's judgment on case estimates. It can be seen that the central question to be asked in all cases is: How far are the assumptions underlying the chosen method likely to be satisfied in the data? The weight to be give to each method might depend upon one's judgment on ii), iii), and iv) above.

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[G9]
PAID LOSS RATIO — STEP-BY-STEP PROJECTION

There is more that can be learned from the study of the loss ratio in claims reserving. So far, we have only worked with an assumed final, or *ultimate*, loss ratio. This is the proportion of claims to premiums for the business in question when the development is complete. But the progression of the loss ratio towards its final value can also be studied, and it can be done with either the paid or the incurred figures. In the present section, we shall concentrate on the former, leaving the incurred for §G11.

The loss ratio progression can be derived from the usual data. It is only necessary to divide the developing claims for a given accident year by the value of earned premium for that year. In terms of symbols,

$$p\lambda(d) = pC(d)\,/aP$$

where $p\lambda$ is simply called the "paid loss ratio". To look at the numbers, we take the usual data triangle of paid claims and divide along each row in turn by the earned premium for that row. The results are:

aP	*a*	*d* = 0	1	2	3	4	5
4486	1	1001	1855	2423	2988	3335	3483
		22.31	41.35	54.01	66.61	74.34	77.64%
5024	2	1113	2103	2774	3422	3844	
		22.15	41.86	55.21	68.11	76.51%	
5680	3	1265	2433	3233	3977		
		22.27	42.83	56.92	70.02%		
6590	4	1490	2873	3880			
		22.61	43.60	58.88%			
7482	5	1725	3261				
		23.06	43.58%				

8502	6	1889
		22.22%

(Along the top row, the paid claims 1001, 1855 ... ... 3483 have been divided by the premium value of 4486 to give percentages 22.31, 41.35 ... ... 77.64%. In the second row, the elements have been divided by 5024, and so on.)

The display is informative, but for a clear view it is better to set out the ratios on their own:

		d 0	1	2	3	4	5
	1	22.31	41.35	54.01	66.61	74.34	
	2	22.15	41.86	55.21	68.11	76.51	77.64%
a	3	22.27	42.83	56.92	70.02		
	4	22.61	43.60	58.88			
	5	23.06	43.58				
	6	22.22					

It can be seen that ratios increase steadily along each row towards a final value of 80%+. This is in accord with our earlier assumption of an 83% loss ratio. Further, looking down the columns, there is clear evidence of an upward trend in almost all development periods. The exception is the column *d*=0, which shows a very stable pattern indeed. In period *d*=l, the upward trend is very approximately +.5%, while in *d*=2 it rises to +1.5%. In periods *d*=3,4, the trend is nearly +2%. The pattern is strongly suggestive. If trends so continue, annual increases of 2%+ in the final loss ratio would very much be expected.

How can the triangle be more formally evaluated? For periods *d*=1,2,3, the trends can certainly be projected, say by the least squares method. For *d*=4, this becomes more difficult since there are only two data values, and for *d*=5 no trend at all can be established. Hence direct trending down the columns will not of itself provide any answers for the movement in the ultimate loss ratio.

There is a simple answer to this problem. We move from overall values of the loss ratio to a step by step, or incremental, approach. That is, we calculate by how much the ratios *increase* at each successive development interval. This yields the following triangle:

		d 0	1	2	3	4	5
	1	22.31	19.04	12.66	12.60	7.73	
	2	22.15	19.71	13.35	12.90	8.40	
a	3	22.27	20.56	14.09	13.10		3.30%
	4	22.61	20.99	15.28			
	5	23.06	20.52				
	6	22.22					

In detail: for year *a*=1, $p\lambda$ at the end of period *d*=0 is 22.31%. Then in period *d*=1 it goes up to 41.35%, and the increase is 19.04%. Similarly, in *d*=2 it goes up to 54.01%, a further step of 12.66%. The same process carries on throughout to give the new triangle.

Examining the stepwise data, the columns from *d*=1 onward all show evidence of the rising trend. We can project them downward by fitting the least squares trendlines, according to the method described in §B8. The required calculations are given in the annex at the end of the section. (They have the same form as those shown in §E9). The results are as follows:

Col	*d*=1:	one value,	21.44
	d=2:	two values,	16.00, 16.86
	d=3:	three values,	13.37, 13.62, 13.87
	d=4:	trending not appropriate,	use 8.40
	d=5:	no trending possible,	use 3.30

One final matter still has to be settled. That is the value of the step from period *d*=5 to ultimate. If we take the standard loss ratio of 83% as previously used, this step must be just 83.00 – 77.64% = 5.36%. The full projection can now be put down, converting the previous triangle into a square:

	d						
		0	1	2	3	4	*ult*
	1	22.31	19.04	12.66	12.60	7.73	8.66%
	2	22.15	19.71	13.35	12.90	8.40	***8.66%***
	3	22.27	20.56	14.09	13.10	***8.40***	***8.66%***
a	4	22.61	20.99	15.28	***13.37***	***8.40***	***8.66%***
	5	23.06	20.52	***16.00***	***13.62***	***8.40***	***8.66%***
	6	22.22	***21.44***	***16.86***	***13.87***	***8.40***	***8.66%***

The figures in italics are the projected ones. In the final column, 8.66% is just the sum of 3.30 and 5.36%. This column summarises the two steps, *d*=4 →5 and *d*=5→*ult*.

The result is propitious — we now have a complete set of stepwise paid loss ratios for each accident year. Adding along the rows will give the projected final loss ratios. The work is reduced if we replace the upper left triangle of known loss ratios by their cumulative values to date.

		d							
		0	1	2	3	4	*ult*	*row sum*
	1						83.00	83.00
	2					76.51	8.66	85.17
a	3				70.02	8.40	8.66	87.08
	4			58.88	13.37	8.40	8.66	89.31
	5		43.58	16.00	13.62	8.40	8.66	90.26
	6	22.22	21.44	16.86	13.87	8.40	8.66	91.45

The projected final loss ratios are all that is needed to make the full estimate of losses and reserves. They are simply multiplied with the usual earned premium figures:

a	^λ	*aP*	^*L-ult*
1	83.00	4486	3723
2	85.17	5024	4279
3	87.08	5680	4946
4	89.31	6590	5886
5	90.26	7482	6753
6	91.45	8502	7775

Overall Values:	Σ*L-ult*	33,362
	Σ*pC**	20,334
	Reserve	13,028

The result is interesting for the steady upward progression it yields in the ultimate loss ratio. The trend is of the order of 2% p.a. or a little less. This is satisfying in that it confirms the reaction on first seeing the paid loss ratio figures set out on page G9.2.

Evaluation of the Method

This method is a particularly important one, as may be seen by the following argument. Thus, a strong objection to the usual claim development methods of §E–F is their sensitivity to the actual amount of paid claims for the latest accident year (see §G8). In contrast, the Bornhuetter-Ferguson method is not sensitive in this way. But it is dependent on the particular choice made for the loss ratio — an outdated value can easily spoil the estimates.

The virtue of the present method is that it largely avoids both these criticisms. First, in common with Bornhuetter-Ferguson, it does not depend on the bottom left hand element in the triangle. These latest year paid claims can take any value at all, and the projection will yield the same answer for the final reserve. Second, in common with the claim development methods, it does not depend on some arbitrary value for the loss ratio. Instead, it makes good use of the observed claims patterns of earlier accident years, applying them in a consistent way to forecast the ultimate loss ratios.

Annex: Trendline Calculations

The trendline $y = bx + c$ is to be fitted to the n data points $(x_i\ y_i)$. (See §B8 for theory.)

Formulae: $c = \bar{y}$ $\quad b = \Sigma x_i y_i \,/\, \Sigma x_i^2$

where: $\bar{y} = \Sigma y_i / n$ $\quad$ and x-axis chosen so that: $\bar{x} = \Sigma x_i \,/\, n = 0$.

d=1:

a	x_i	y_i	x_i^2	$x_i y_i$
1	–2	19.04	4	–38.08
2	–1	19.71	1	–19.71
3	0	20.56	0	0
4	1	20.99	1	20.99
5	2	20.52	4	41.04
Σ	0	100.82	10	4.24

Hence: $c = 100.82 / 5 = 20.164$

$b = 4.24 / 10 = .424$

Projection: $20.164 + 3 \times .424 = 21.44$

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d=2:

a	x_i	y_i	x_i^2	$x_i y_i$
1	–1.5	12.66	2.25	–18.99
2	–.5	13.35	.25	–6.68
3	.5	14.09	.25	7.05
4	1.5	15.28	2.25	22.92
Σ		55.38	5	4.30

Hence: $c = 55.38 / 4 = 13.845$

$b = 4.30/5 = .860$

Projections: $13.845 + (2.5, 3.5) \times .860 = (16.00, 16.86)$

d=3:

a	x_i	y_i	x_i^2	$x_i y_i$
1	–1	12.60	1	–12.60
2	0	12.90	0	0
3	1	13.10	1	13.10
Σ		38.60	2	.50

Hence: $c = 38.60/3 = 12.867$
$b = .50/2 = .250$

Projections: $12.867 + (2, 3, 4) \times .250 = (13.37, 13.62, 13.87)$

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[G10]
PAID LOSS RATIO & PAID CLAIMS PROJECTION

In this section, we shall examine the projection of loss ratio as an alternative for projecting the triangle of paid claims. The starting point is the triangle of paid loss ratios, as first derived in the previous section. The figures show the progress of the ratio for each accident year as development time *d* increases:

		d					
		0	1	2	3	4	5
	1	22.31	41.35	54.01	66.61	74.34	77.64%
	2	22.15	41.86	55.21	68.11	76.51	
a	3	22.27	42.83	56.92	70.02		
	4	22.61	43.60	58.88			
	5	23.06	43.58				
	6	22.22					

The triangle is, of course, closely related to that of the original data on paid claims. It differs in having each row scaled against the earned premium data for the accident years. But it is the same as the paid claims data in that it is a development pattern in triangular form showing the past progress of the business in question. It can, therefore, be worked through by any of the grossing up or link ratio methods of §E. The assumption required, as before, is just that the paid claims run to a stable development pattern over the accident years.

What happens if we take up the suggestion, and do a projection by one of the familiar methods of §E using the paid loss ratio figures? An example is worked through overleaf. It uses the Arabic version of grossing up, with simple averaging of the factors down the columns. The result obtained is very much in accord with the earlier projection of the paid claims itself. Mathematically this is no surprise, since the loss ratio projection as here shown is really a repetition of the paid claims development in a light disguise. *But the difference becomes more important if* **"premiums"** *as well as claims show a development pattern. Such is generally the case with London Market business and reinsurance as opposed to direct business.*

Referring to the calculations themselves, the table is slightly confusing in that *all* the figures in it are proportions of one kind or another. But they are of quite different kinds. In each cell of the table, the upper of the two figures is just the paid loss ratio copied from the data triangle above. The lower figure is then the appropriate grossing up factor, calculated by the usual procedure (see §E3 for full details). The result of the procedure is to generate the ultimate loss ratios in the far right hand column. The topmost of these, however, must be determined by other means — we take it here to be 83%, as previously in §G.

a \ d	0	1	2	3	4	5	*ult*
1	22.31	41.35	54.01	66.61	74.34	77.64	83.00
	26.88	49.82	65.07	80.25	89.57	93.54%	
2	22.15	41.86	55.21	68.11	76.51		85.42
	25.93	49.00	64.63	79.74	89.57%		
3	22.27	42.83	56.92	70.02			87.54
	25.44	48.93	65.02	79.99%			
4	22.61	43.60	58.88				90.71
	24.93	48.07	64.91%				
5	23.06	43.58					89.03
	25.90	48.95%					
6	22.22						86.06
	25.82%						

Applying the projected loss ratios which appear in the last column to the earned premiums, we can derive the loss and reserve estimates as follows:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	86.06	89.03	90.71	87.54	85.42	83.0%
$\hat{}L\text{-}ult$	7317	6661	5978	4972	4292	3723

Overall Values:	$\Sigma L\text{-}ult$	32,943
	ΣpC^*	20,334
	Reserve	12,609

The comparable result from the series of paid claim projections in §E is that from §E3, Variation 3:

a	6	5	4	3	2	1	
^*L-ult*	7293	6628	5951	4947	4271	3705	(from E3.2)

Overall Values:	Σ*L-ult*	32,795
	Σ*pC**	20,334
	Reserve	12,461

The results are so close that it would scarcely seem worthwhile to do both. Nevertheless, the paid loss ratio projection does provide something new — it shows the movement in the estimated final loss ratio down the accident years. The ratio first increases steadily from 83.0% to 90.7% and then falls back quickly to 86.1%. The pattern differs from that brought out by the projection of the previous section, §G9. But if the present forecast is at all sound, a fascinating thought naturally arises. It is that here we could have real evidence of the underwriting cycle at work. We must, therefore, ask carefully whether the observed effect is a genuine one, or a spurious result of some quirk in the data or method.

A first test is to look back at the loss ratios brought out by the original paid claims projection. The calculations are:

a	6	5	4	3	2	1	
^*L-ult*	7293	6628	5951	4947	4271		
aP	8502	7482	6590	5680	5024	3705	(from E3.2)
λ	85.8	88.6	90.3	87.1	85.0	4486	
						82.6%	

As expected, the pattern is almost identical, except that it is shifted down by about 0.4%. That is purely the effect of starting from a lower initial loss ratio of 82.6% as opposed to 83% in the paid loss ratio projection. It helps to verify the mathematical equivalence of the two methods when there is a development pattern of premiums.

A better test is to look back critically at the workings above on the triangle of paid loss ratios. It is particularly instructive to compare the progressions in accident years a=4, 5, 6. (These data are repeated below, for convenience.) Here it can be seen that the paid loss ratios in periods d=0 and d=1 are very stable. But the ultimate loss ratios fall away appreciably, as the result of the influence of values higher in the triangle.

		d 0	1	2	3	4	5	*ult*
	4	22.61	43.60	58.88				90.71
		24.93	48.07	64.91%				
a	5	23.06	43.58					89.03
		25.90	48.95%					
	6	22.22						86.06
		25.82%						

From the point of view of projecting the claims for year a=6, this is very unsatisfactory. Following the strong progression of the loss ratio to 90.7% in year 4, there is really no clear evidence at all to support the later fall. The triangle of data, on its own, is insufficient. What the result is pinpointing is not, after all, evidence of the underwriting cycle at work — rather, it is the inherent instability of the projections in the last 2 or 3 accident years.

A final test on the loss ratio progression would be to consult with underwriters for the given class of business. They might be able to confirm that the figures accorded with their own experience of the market in the recent past. Alternatively, they might say there was no evidence as yet to show loss ratios had reached their peak. The upward trend appeared to be continuing, and was not likely to be arrested until such time as a major shakeout in the market occurred. In this event, taking a cautious view, it would be proper to *extend* the loss ratio trend into the accident years 5 and 6. The trend is very close to +2.4% p.a. in years a=1 to 4, so the projection yields:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	95.90	93.50	91.10	87.81	85.42	83.0%
^*L-ult*	8153	6996	6003	4988	4292	3723

Overall Values:	Σ*L-ult*	34,155
	Σ*pC**	20,334
	Reserve	13,821

This is a salutary result. We have introduced very little that is new mathematically, or in computational procedure, but have been led to revise the reserve estimate upward by an appreciable amount. The conclusion is that it is well worthwhile examining the paid loss ratio and its progression for the new clues which the data may provide. Failing this, the results of any paid loss projection should at least be evaluated by means of the loss ratio. Any such projection will implicitly yield a variation of the ultimate loss ratio by accident year, and this pattern should be tested for its reasonableness. <>

[G11]
INCURRED LOSS RATIO & INCURRED CLAIMS PROJECTION

Having defined the loss ratio for paid claims, and looked into its development, it is a short step to do the same for the incurred claims. This section therefore provides a parallel treatment to that of §G10. The quantity to examine is the incurred loss ratio:

$$i\lambda(d) = iC\,(d)/aP$$

Again, we simply divide the incurred claims for a given accident year by the earned premium, and study the development of the ratio with time. The given data from our main example from F7.1 yield the following:

aP	*a*	*d* 0	1	2	3	4	5
4486	1	2866	3334	3503	3624	3719	3717
		63.89	74.32	78.09	80.78	82.90	82.86%
5024	2	3359	3889	4033	4231	4319	
		66.86	77.41	80.27	84.22	85.97%	
5680	3	3848	4503	4779	4946		
		67.75	79.28	84.14	87.08%		
6590	4	4673	5422	5676			
		70.91	82.28	86.13%			
7482	5	5369	6142				
		71.76	82.09%				
8502	6	5818					
		68.43%					

Here, the divisions run along each row. Thus 2866/4486 = 63.89%, 3334/4486 = 74.32% . . 3717/4486 = 82.86% and so on, down to the last row where 5818/8502 = 68.43%.

The patterns of the development can be seen more easily if the loss ratio figures are separated out (as in the following table). Here, as with the paid loss ratios, there are strong increases down the columns. The periods *d*=2, 3, 4 are particularly suggestive, and on the evidence of these one might expect an upward trend of up to 3% p.a. in the ultimate loss ratio.

	d	0	1	2	3	4	5
	1	63.89	74.32	78.09	80.78	82.90	82.86%
	2	66.86	77.41	80.27	84.22	85.97	
a	3	67.75	79.28	84.14	87.08		
	4	70.91	82.28	86.13			
	5	71.76	82.09				
	6	68.43					

We shall proceed with a full projection of the triangle, to test out the strength of the loss ratio trend. A grossing up technique was used in §G10 for the paid loss ratios, so we shall here try a link ratio variation for contrast.

	d	0	1	2	3	4	5	*ult*
		1.163	1.051	1.034	1.026	.999	1.002	
	1	63.89	74.32	78.09	80.78	82.90	82.86	83.00
							1.002	
		1.158	1.037	1.049	1.021			
	2	66.86	77.41	80.27	84.22	85.97		86.06
						1.001		
		1.170	1.061	1.035				
	3	67.75	79.28	84.14	87.08			89.26
a					*1.025*			
		1.160	1.047					
	4	70.91	82.28	86.13				91.73
				1.065				
		1.144						
	5	71.76	82.09					91.69
			1.117					
	6	68.43						88.62
		1.295						
r		1.159	1.049	1.039	1.024	.999	1.002	
f		1.295	1.117	1.065	1.025	1.001	1.002	
aP		8502	7482	6590	5680	5024	4486	

INCURRED LOSS RATIO & INCURRED CLAIMS PROJECTION

λ	88.62	91.69	91.73	89.26	86.06	83.00%
$\hat{L}$*-ult*	7534	6860	6045	5070	4324	3723

Overall Values:	Σ*L-ult*	33,556
	Σ*pC**	20,334
	Reserve	13,222

The strong trends in the main data effectively disappear in the sets of link ratios. Hence simple averaging down the columns is used to determine the *r*-factors. But the upward trend reappears clearly in the ultimate loss ratios obtained. The increase is steady from years *a*=1 to 4, this time at close to 3% p.a., and reaching a high of nearly 92%. The only awkward feature is the falling away to approximately 89% in the year *a*=6.

Again, it is useful to compare the result with that which would have been obtained from the projection of the incurred claims themselves. The ultimate loss ratios implicit in this earlier projection can be set out as follows:

a	6	5	4	3	2	1
^*L-ult*	7505	6842	6028	5055	4315	3717
aP	8502	7482	6590	5680	5024	4486
λ	88.3	91.4	91.5	89.0	85.9	82.9%

The set of values is very similar indeed, helping to confirm the mathematical identity which underlies the two projections. Again, a step-by-step projection, carried out in the manner for paid claims (§G9), will bring out a very similar set of ratios. To settle the estimate in this case, we may perhaps use a set of rounded loss ratios, increasing at 3% p.a. until year *a*=4, then steadying at the value of 92%. This yields:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	92.0	92.0	92.0	89.0	86.0	83.0%
^*L-ult*	7822	6883	6063	5055	4321	3723

Overall Values:	Σ*L-ult*	33,867
	Σ*pC**	20,334

Reserve	13,533

Finally, for a pessimistic estimate assuming the worst, we can allow the +3% trend to continue right up to the most recent accident year. The figures become:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
λ	98.0	95.0	92.0	89.0	86.0	83.0%
^*L-ult*	8332	7108	6063	5055	4321	3723

Overall Values:	Σ*L-ult*	34,602
	Σ*pC**	20,334
	Reserve	14,268

If confirmatory evidence were to be found for the truth of this scenario, clearly it is time that more stringent underwriting criteria and/or financial controls were placed on the class of business in question.

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[G12]
COMPARISON OF RESULTS (*pC*/*iC*/*p*λ/*i*λ)

It is time to return to the question of our main estimate for the claims reserve. We shall bring together the main results from the paid and incurred loss ratio projections in §G10 and §G11 with the earlier comparison of paid and incurred claims projections themselves (see table in §F7).

		Value of Reserve	
		Best Estimate	Conservative
Paid Claims	*pC*	12,461	12,931
Paid Loss Ratio	*p*λ	13,096	13,821
Incurred Claims	*iC*	13,151	13,645
Incurred Loss Ratio	*i*λ	13,533	14,268

The evidence from the *p*λ/*i*λ projections is undoubtedly strong. Unless we have firm evidence that the ultimate loss ratio is retreating from the high levels forecast for year a=4, we must conclude that the original paid claims projections are giving too low a figure for the reserve. But the incurred claims figures do not seem at all inflated, and are much in agreement with the paid loss ratio evidence. Finally, there are the incurred loss ratio values at the high end of the spectrum. Here, the "conservative" figure is in fact based on a very pessimistic assumption. Unless there is confirmatory evidence, it will be as well to disregard it.

The final determination must rest on the reserver's judgment. In this case, it will be reasonable to set a reserve at say £13,125 for the best estimate, and £13,750 for the conservative value.

Summing Up on Loss Ratios

A brief summing up on the use of loss ratios in reserving may be useful at this stage. The starting point is inauspicious, since the naive loss ratio appears to be of little help and to prejudge the required answers. But when the more subtle methods such as Bornhuetter-Ferguson and Loss Ratio projection are brought into play, it is as if an anchor were provided for the work. The assumptions underlying the claims development methods (both *pC* and *iC)* can easily go adrift for the most recent accident years, and B/F or the loss ratio projections can help provide the needed stabilisation.

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Section H
METHODS BASED ON CLAIM NUMBERS & AVERAGE COST PER CLAIM

Preamble

To this point in the Manual, we have used three main sources of data for the claims projections — paid claims, case reserves and earned or written premiums. These data items, all monetary amounts, lead on naturally to the paid and incurred claim projections and the loss ratio methods. But a further dimension can be provided by a fourth main data item, not itself a monetary unit, which is the *number* of claims. Such data are frequently available in direct insurance work, but seldom in the reinsurance field.

When the claim amounts paid or incurred are divided by the relevant number of claims, an average cost per claim results. This average cost can be projected, just as were the claim amounts themselves. Then, combined with a separate projection for the number of claims, it will yield the new estimate for the ultimate loss. The reserver can also examine the movement of the claim numbers and average costs as the accident years develop, and look for significant trends or discontinuities. A fuller view of the business can thus be obtained, perhaps leading to adjustment of the reserving figures, or showing where further investigation is needed.

One point about average cost per claim methods is that many variations are possible. The reserver should ask, what quantities go into making the average, what is the basis of projection, and what claim numbers are used for the eventual multiplier of the projected average? It is vital to be clear as to exactly what definitions are being used — the term "Average Cost per Claim Method" on its own is rather inadequate. The present section describes some of the average claim methods available, but is far from being exhaustive of the genre.

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H2. Number Settled & Number Reported
H3. Incurred Average Claims Projection
H4. Risk Exposure & Claim Frequency
H5. Correspondence of Claim Numbers & Claim Amounts

[H1]
PAID AVERAGE CLAIMS PROJECTION

Work on claim numbers and average costs per claim methods can begin quite easily from the starting point of paid claim amounts. We will use the data first introduced in §E1.

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880				
	5	1725	3261				**[*pC*]**	
	6	1889						

The additional data needed are the corresponding claim numbers. Since we are talking about paid claims, the appropriate data are the numbers of claims *settled*, for which we shall use the symbol *nS*. In average cost methods, there are always questions to be answered about the exact definition of claim amounts (for example whether they include partial payments made on claims that are still outstanding) and claim numbers and their correspondence one with another. But we will return to these in §H5, and for the moment press on with describing the projection method itself. Let us suppose that the following data become available for the claim numbers settled:

		d						
		0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	498
	2	303	411	463	500	522		
a	3	328	446	503	544			
	4	343	462	530				
	5	350	469				**[*nS*]**	
	6	355						

In this table, the numbers refer to the cumulative number of claims settled for each accident year as development time *d* progresses. The final number in the table, 498 in the *ult* column, is not fully objective, since we take it that year a=1 has not yet developed beyond d=5. The number will have been estimated, presumably, from a study of earlier years' data in which the development to ultimate is complete.

The first step in the working is very simple. The elements in the nS-triangle are divided into those of the pC-triangle to give the average costs per claim at each stage. The results are as follows, using the symbol pA to denote the average cost, which in this case may be called the "paid average" for short. (As with the original data on paid claims, the figures are in £1,000's. They are not supposed to be related to any particular class or grouping of business.)

		d 0	1	2	3	4	5	*ult*
	1	3.588	4.894	5.674	6.454	6.919	7.137	7.440
	2	3.673	5.117	5.991	6.844	7.364		
a	3	3.857	5.455	6.427	7.311			
	4	4.344	6.219	7.321				
	5	4.929	6.953				**[*pA*]**	
	6	5.321						

The table of average costs is interesting in itself. There is a marked increase in cost both along the rows and down the columns. The increase results from two major causes. Both rows and columns reflect the general tendency for claim sizes to inflate with the passing of the years. The row increase also reflects the fact that, for any given accident year, the more serious claims tend to take longer to settle — a factor which will be present in non-inflationary conditions.

We now want to project the paid average costs, to estimate the ultimate that will be reached for each accident year. Since we have a triangle of data no different in form from that tackled in earlier chapters, the standard methods can apply. Perhaps the simplest technique is to use a grossing-up procedure, with averaging of factors down the columns. The result is as follows (see ξE3 for full details of method of working):

		d 0	1	2	3	4	5	*ult*
	1	3.588	4.894	5.674	6.454	6.919	7.137	7.440
		48.2	65.8	76.3	86.7	93.0	<u>95.9%</u>	
	2	3.673	5.117	5.991	6.844	7.364		7.918
		46.4	64.6	75.7	86.4	<u>93.0%</u>		
a	3	3.857	5.455	6.427	7.311			8.442
		45.7	64.6	76.1	<u>86.6%</u>			
	4	4.344	6.219	7.321				9.633
		45.1	64.6	<u>76.0%</u>				
	5	4.929	6.953			**[*pA*]**		10.713

		46.0	64.9%			[*g*]		
	6	5.321						11.492
		46.3%						

We now have our estimate for the ultimate average claim cost for each accident year. It is only necessary to multiply these figures by the number of expected claims in each accident year to give the final estimate of the loss. The expected claim numbers can, of course, be found by applying a grossing up procedure to the *nS*-table above, and this is done below.

		d 0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	498
		56.0	76.1	85.7	93.0	96.8	98.0%	
	2	303	411	463	500	522		539
		56.2	76.3	85.9	92.8	96.8%		
a	3	328	446	503	544			586
		56.0	76.1	85.8	92.9%			
	4	343	462	530				618
		55.5	74.8	85.8%				
	5	350	469			[***nS***]		619
		56.6	75.8%			[***g***]		
	6	355						634
		56.0%						
								3494

The final multiplication of average costs and claim numbers can now be done:

a	*^A-ult*	*^n-ult*	*^L-ult*
1	7.440	498	3705
2	7.918	539	4268
3	8.442	586	4947
4	9.633	618	5953
5	10.713	619	6631
6	11.492	634	7286

Overall Values: S^*L-ult* 32,790
S*pC** 20,334

Reserve	12,456

The final value for the reserve is very close indeed to the best estimate from the grossing up of paid claim amounts. (Value £12,461, as given on E4.1.) Since it is much simpler just to use paid claims, is there any real advantage in the more complicated paid average method? In fact, the answer is yes, but not with the version just described. To gain the benefit, a variation must be brought in which concerns the way the claim numbers are handled. This variation arises from looking at the claim settlement pattern, and is the subject of the next section.

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[H2]
NUMBER SETTLED & NUMBER REPORTED

Beginning from paid claim amounts, we were led naturally to consider the numbers of claims settled, and to project the ultimate number of claims from these data. But if we had begun from the incurred claim position, then the corresponding numbers would have been of claims *reported* instead. Of the claims reported at any stage, clearly a subset will be the claims already settled. The remainder will be open claims, i.e. those claims to which (in most classes of business) the data for case reserves will relate.

To continue the example begun in §H1, let us use the symbol nR for the number of claims reported, and suppose that the following data have been given:

					d			
		0	1	2	3	4	5	*ult*
	1	414	460	482	488	492	494	494
	2	453	506	526	536	539		
a	3	494	548	572	582			
	4	530	588	615				
	5	545	605				**[*nR*]**	
	6	557						

Again, the numbers refer to the cumulative number of claims reported for each accident year as development time progresses. The final number for year a=l, 494, is derived on the assumption that all claims have been reported by time d=5.

Looking at the table, it is clear that the data could be projected just as was done for the numbers settled. Hence we have an alternative route for estimating the ultimate numbers of claims, and a check on the earlier projection. Before carrying this out, it is useful to examine the direct relationship between the numbers settled and reported. For convenience, the data on numbers settled are repeated here:

					d			
		0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	498
	2	303	411	463	500	522		
a	3	328	446	503	544			
	4	343	462	530				

					d			
		0	1	2	3	4	5	*ult*
	5	350	469				**[*nS*]**	
	6	355						

The obvious route for the comparison is to calculate the proportion which the number settled bears to the number reported at each stage. This is done in the table below:

					d			
		0	1	2	3	4	5	*ult*
	1	67.4	82.4	88.6	94.9	98.0	98.8	100.8
	2	66.9	81.2	88.0	93.3	96.8		
a	3	66.4	81.4	87.9	93.5			
	4	64.7	78.6	86.2				
	5	64.2	77.5				**[*nS*/*nR*]**	
	6	63.7						

The pattern which emerges is that the number of claims settled has in recent years been a decreasing proportion of the claims reported. If the pattern of the reported claim numbers is a stable one, then we have strong evidence here that the settlement rate for the class of business is tending to slow down. Enquiries should be made to see whether the point can be corroborated by the experience of the claims department staff. If it can, the conclusion will be that the claim number projection on the basis of numbers settled is likely to be at fault.

Indeed, even if the confirmatory evidence is not to hand, the projection of the numbers reported is still likely to be the more reliable. That is simply because, at any point in the development, the numbers reported must of necessity be further advanced towards the ultimate than the numbers settled. It is often found that the numbers reported yield one of the most stable patterns in the claims development scene. In general, numbers of claims tend to be easier to handle and predict than claim amounts or average costs.

Let us now carry out the claim number projection from the given data on claims reported. A grossing up method with averaging of the factors will be used, as for claims settled in §H1.

		d						
		0	1	2	3	4	5	*ult*
	1	414	460	482	488	492		494
		83.8	93.1	97.6	98.8	99.6	494	
							100.0%	
	2	453	506	526	536	539		541
		83.7	93.5	97.2	99.1	99.6%		
a	3	494	548	572	582			588
		84.0	93.2	97.3	99.(			
	4	530	588	615				631
		84.0	93.2	97.:				
	5	545	605			**[*nR*]**		648
		84.1	93.:			**[*g*]**		
	6	557						664
		83.9%						
								3566

The comparison of the projected claim numbers in the two cases is as follows:

a	6	5	4	3	2	1
^*n-ult* (*nS*-base)	634	619	618	586	539	498
^*n-ult* (*nR*-base)	664	648	631	588	541	494
Δ	+4.7%	+4.7%	+2.1%	+0.3%	+0.4%	-0.8%

The difference is mainly shown in the most recent three accident years.

This shows the importance of choosing the most appropriate claim number projection in an average cost per claim method. Unless there is evidence to the contrary, it will generally be right to prefer the projection of numbers reported, and this principle will be followed throughout the rest of §H.

We can now begin to answer the question posed at the end of §H1, i.e. as to the relevance of the paid average projection. The fact is that such an average cost per claim analysis, if properly applied, can be responsive to certain of the variations in the claim settlement pattern. It will be recalled from §E12 and §G8 above that such variations are a major point of difficulty with the straight projection of paid claim amounts. Indeed, the problem is of such importance that it should never be far from the reserver's mind. The information which comes from the claim numbers, and in particular from the comparison of numbers settled against numbers reported, is very useful in beginning to provide the needed evidence.

The other part of the evidence relates to the claim severities, and in particular to the relative costs of claims settled at different stages of the overall development.

To conclude the section, we summarise the movement observed in the claim settlement pattern by calculating the numbers settled as a proportion of the estimated ultimate values. (The latter, of course, come from the projection of numbers reported.)

		d						
		0	1	2	3	4	5	*ult*
	1	279	379	427	463	482	488	494
	2	303	411	463	500	522		541
a	3	328	446	503	544			588
	4	343	462	530				631
	5	350	469					648
	6	355					**[*nS*]**	664

	1	56.5	76.7	86.4	93.7	97.6	98.8%
	2	56.0	76.0	85.6	92.4	96.5	
a	3	55.8	75.9	85.5	92.5		
	4	54.4	73.2	84.0			
	5	54.0	72.4				
	6	53.5					**[*nS/n-ult*]**

The ability given to study patterns such as these shows the usefulness to the reserver of the data on claim numbers. The picture that can be built of the development pattern is fuller than can be obtained from using claim amounts alone.

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[H3]
INCURRED AVERAGE CLAIMS PROJECTION

The method already developed for paid claims and the projection of the average cost can also be applied using incurred claims. The mechanics are straightforward, and exactly parallel those set out in §H1. We begin with the original incurred claim data (first given in §F3), and the figures for numbers reported from the previous section:

					d			
		0	1	2	3	4	5	*ult*
	1	2777	3264	3452	3594	3719	3717	3717
	2	3252	3804	3973	4231	4319		
a	3	3725	4404	4779	4946			
	4	4521	5422	5676				
	5	5369	6142				**[*iC*]**	
	6	5818						

	1	414	460	482	488	492	494	494
	2	453	506	526	536	539		
a	3	494	548	572	582			
	4	530	588	615				
	5	545	605				**[*nR*]**	
	6	557						

Dividing the elements in the *iC* triangle by those of the *nR* triangle gives the average incurred cost per claim at each stage of development. This we shall refer to as the incurred average, symbol *iA*.

					d			
		0	1	2	3	4	5	*ult*
	1	6.708	7.096	7.162	7.365	7.559	7.524	7.524
	2	7.179	7.518	7.553	7.894	8.013		
a	3	7.540	8.036	8.355	8.498			
	4	8.530	9.221	9.229				
	5	9.851	10.152				**[*iA*]**	
	6	10.445						

The incurred average is projected to ultimate, using some standard method of the grossing up or link ratio type. Here, grossing-up is employed, working backward down the main diagonal and with averaging of factors in the columns:

					d			
		0	1	2	3	4	5	*ult*
	1	6.708	7.096	7.162	7.365	7.559	7.524	7.524
		89.2	94.3	95.2	97.9	100.5	100.0%	
	2	7.179	7.518	7.553	7.894	8.013		7.973
		90.0	94.3	94.7	99.0	100.5%		
a	3	7.540	8.036	8.355	8.498			8.627
		87.4	93.1	96.8	98.5%			
	4	8.530	9.221	9.229				9.654
		88.4	95.5	95.				
	5	9.851	10.152			**[*iA*]**		10.766
		91.5	94.3%			**[*g*]**		
	6	10.445						11.697
		89.						

It remains to bring in the relevant claim numbers. These are the numbers reported, which have already been projected in §H2 with the result:

a	6	5	4	3	2	1
^*n-ult (nR*-base)	644	648	631	588	541	494

Multiplying the projected average claims by the projected numbers then yields the loss estimate in the usual way:

a	^*A-ult*	^*n-ult*	^*L-ult*
1	7.524	494	3717
2	7.973	541	4313
3	8.627	588	5073
4	9.654	631	6092
5	10.766	648	6976
6	11.697	664	7767

Overall Values:	S^*L-ult*	33,938
	S*pC**	20,334
	Reserve	13,604

The final figure for the reserve is very close to that obtained by the incurred claims projection itself (§F3.2), which was £13,634. The fact is that the incurred average method will almost always produce such results. For projection purposes, the incurred average cannot be recommended as providing any real advantage over the incurred claims method itself. (It is included in the Manual for purposes of completeness and consistency in the exposition.)

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[H4]
RISK EXPOSURE & CLAIM FREQUENCY

Although the incurred average claims method has few advantages, the numbers reported themselves can be of further use. They can be made to bring out evidence on the claim frequency in the given class of business. What we need in addition are data on the risk exposure for the accident years in question. This exposure can be measured in a number of ways, but a common means will be via a standard exposure unit, which can be on an earned or written basis. The exact definition of the unit will vary with the class of business — it can be a vehicle-year in Motor, or a dwelling-year in domestic Fire, and so on. Let us suppose this information becomes available in the present case as follows (no particular specification of the unit-type is intended):

a	6	5	4	3	2	1
aX	20.59	19.82	19.21	18.94	18.44	18.03

Here, the figures give the 1,000s of exposure units for the years in question. aX is taken as the symbol for exposure X measured on the accident year, i.e. it is the *earned* exposure. (For the *written* exposure, corresponding to the underwriting year, we would write wX.)

The claim frequencies can now be calculated as numbers reported divided by the earned exposure for each accident year. (The symbol used here is Cq, where $Cq = nR/aX$.)

aX	*a*	*d* 0	1	2	3	4	5
18.03	1	414	460	482	488	492	494
		23.0	25.5	26.7	27.1	27.3	27.4
18.44	2	453	506	526	536	539	
		24.6	27.4	28.5	29.1	29.2	
18.94	3	494	548	572	582		
		26.1	28.9	30.2	30.7		
19.21	4	530	588	615			
		27.6	30.6	32.0			
19.82	5	545	605			**[*nR*]**	
		27.5	30.5			**[*Cq*]**	
20.59	6	557					
		27.1					

The picture shown here is that claim frequencies have been increasing over accident years a=1 to 4, but appear now to be stabilising. But the evidence for the latter point is very far from complete, and it will need further confirmation as the development proceeds. For a clearer view, it is worth setting out the frequencies on their own:

		d 0	1	2	3	4	5
	1	23.0	25.5	26.7	27.1	27.3	27.4
	2	24.6	27.4	28.5	29.1	29.2	
a	3	26.1	28.9	30.2	30.7		
	4	27.6	30.6	32.0			
	5	27.5	30.5			**[*Cq*]**	
	6	27.1					

A further useful item that can be discovered from this analysis (given that the exposure data are available) is the premium paid per unit exposure. Repeating the premium data from §G2 yields the figures:

a	6	5	4	3	2	1
aP	8502	7482	6590	5680	5024	4486
aX	20.59	19.82	19.21	18.94	18.44	18.03
aP/aX	412.9	377.5	343.1	299.9	272.5	

Pj	1.0938	1.1003	1.1440	1.1006	1.0953	248.8

The premium per unit exposure *aP/aX* has been increasing steadily during the period in question, and its inflation factors *Pj* are given in the bottom row of the table. (1.0938 is 412.9¸377.5, and so on.) Premium and claim inflation will not be coincident, of course, but a knowledge of their relationship is very important in the overall control of the business.

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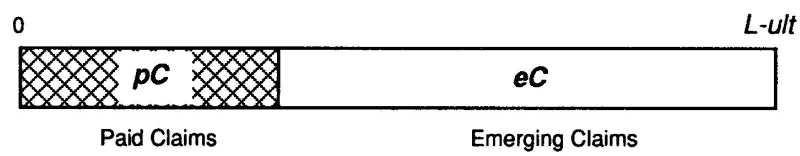
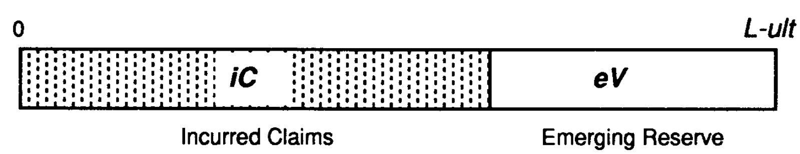
[H5]
CORRESPONDENCE OF CLAIM NUMBERS & CLAIM AMOUNTS

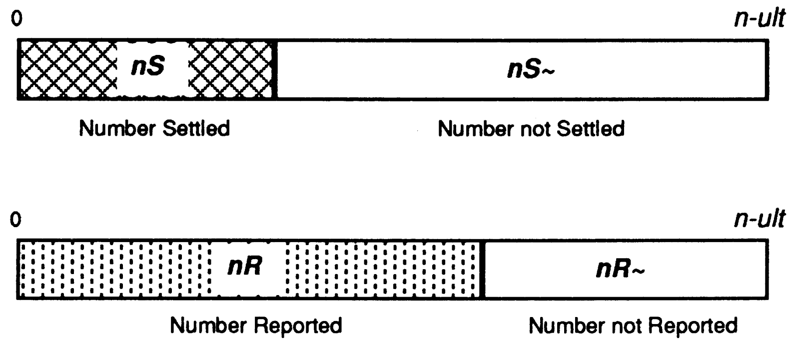
The work of this section of the Manual brings out a basic correspondence in the reserving data on claim amounts and claim numbers. The correspondence is a simple one, between paid claims and numbers settled on the one hand, and incurred claims and numbers reported on the other. It has its uses in establishing a theoretical framework for claims reserving, as will be shown more fully in §M. But its simplicity tends to disguise some very real difficulties which should not be neglected by the reserver, and one or two will be brought out in this section.

First, though, some diagrams to characterize the correspondence may be helpful. These depend on the idea of *completion.* If we are thinking about paid claims then the question is, what further claims are expected to emerge before the ultimate loss is reached? If we are on numbers reported the point is, how many further claims will come in before arriving at the final position? Diagrams can soon be drawn to reflect this idea, and appear as follows (cf those already given in §G3):

Claim Amounts







Claim Numbers

The diagrams are useful conceptually, and help to show the relationships of the various quantities. They lead naturally to the paid and incurred average claim definitions as used earlier in this section of the Manual.

Paid Average = Paid Claims/Number Settled
Incurred Average = Incurred Claims/Number Reported

But the diagrams conceal the fact that the definitions of the claim amounts and claim numbers are not always properly reconciled. The groups of claims concerned can be subtly different, thus leading to possible bias in the projections. The main point is to know exactly what definitions apply to the data in hand, so that any necessary adjustments can be made. Particular examples of this relate to partially paid claims and claims settled at zero, which can affect the paid average as outlined below.

Partially Paid Claims

Considering the paid claim data, the main element is the full payments on settled claims. If that were all, then numerator and denominator in the paid average would be in harmony with each other. But to the full payments will be added any partial payments made on claims still open at the reserving date. (Such payments arise typically where liability is admitted by the insurer, but where a long period is needed for the liability to be fully assessed.) Hence a discrepancy arises in the calculation of the paid average, which needs to be tackled. Three solutions seem possible.

a) If the partial payments are a small proportion of the whole, the distortion introduced by using pC/nS for pA can perhaps be ignored.

b) Provided the data are available, the paid amounts on settled claims alone can be used in the numerator. Alternatively, the partial payments can be subtracted from payments as a whole. The average is then $pA = pS/nS$.

c) If the general proportion of claim, say k, which is redeemed by a partial payment can be estimated, then the denominator can be adjusted. The addition to nS needed is just k times the number of partially paid claims.

Claims Settled at Zero

This is another point which relates to the paid average. Of the claims settled, a number will be at zero, perhaps through being disproved, or retracted by the insured. The question arises as to whether such claims should be included in the number settled — clearly they will make no difference to the paid amount itself. Often there will be no problem, and the number settled can be taken with or without the zero claims, as most convenient. The choice may, of course, be forced by the availability of the data. But where a choice exists, the main point is to ensure that a consistent definition is used throughout the working.

The problem arises where the proportion of claims settled at zero changes, perhaps as a result of a revised claims handling policy by the insurer, or an attempt to reduce the backlog of waiting claims. The general effect on projections of such changes is not easy to assess, however. It may be best to work with both bases, and assess the two results for their relative dependability.

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Section I
METHODS FOR IBNR

Preamble

IBNR means "Incurred but not reported". The term refers to claims not yet known to the insurer, but for which a liability is believed to exist at the reserving date. That is simple enough in itself, but the four letters contain a wealth of meaning, and of ambiguity. The wealth arises from the fact that IBNR acts as the remainder term in General Insurance reserving. Many things which cannot be dealt with explicitly elsewhere can be left to fall into the IBNR bag — and as a result, one sometimes cannot be sure what it contains, or even is supposed to contain.

Apart from this, there is the slight mystery of how to go about reserving for claims which have not yet come in, and are still in some sense a figment of the future. The only certainty is that such claims *will* come in, and that there is a duty to make provision for them. Of course, there are ways of dealing with the problem, and fortunately many of these can be described through a single basic principle. It is largely a question of finding a surrogate measure which will stand in place of the IBNR, of substituting the known for the unknown, and justifying one's case for so doing.

An important concept is that of the IBNR run-off. Although IBNR claims are at first unknown, at some time in the future they must become manifest. At this stage, values can be recorded for them — payments, numbers, case reserves, and incurred amounts, just as for any other group of claims. Hence data can be built up on their development patterns, and the data used in projections. A few of the methods are described here, but in the end the reserver need only be limited by his or her own ingenuity. There are some pitfalls to be avoided, however, particularly when it comes to combining IBNR estimates with those for reported claims. It is very possible to double count certain of the elements, or to leave out others completely.

Contents

[I1]
IBNR — DEFINITION & AMBIGUITIES

IBNR is an acronym standing for "Incurred but not reported". IBNR claims are thus that group which are incurred before the reserving date, but not reported until after it. They are the claims for which, in symbols:

$$a] < v] < r]$$

where $a]$ = accident date, $v]$ = reserving date, $r]$ = reporting date.

The picture is simple enough, but as always in General Insurance some complications enter. To begin with, the process of claims reporting is not instantaneous. Claims will normally come in through branch offices, brokers or agents for the insurer or reinsurer concerned. There will be an interval between such actual first report and the later time when the claim is notified to head office and/or formally recorded in the insurer's main data base. For reserving purposes the latter event will be the more convenient one to take. Hence a "reported" claim is normally one which has already been processed to the extent that a central record on it is held. The corollary is that, at any time, there are likely to be claims in the pipeline, already reported at branch level, but still counting for reserving purposes as IBNR.

A second complication is that the cut-off date for IBNR purposes may well be distinct from the reserving date itself. This arises because the extraction of information from the claims data base takes time, and because month and year ends do not fall regularly with respect to the working week.

Hence in examining IBNR data, the reserver must take precautions. Possible variations in the time taken between first report of a claim and its formal recording should be watched for carefully, as should the gap between the IBNR cut-off and reserving dates. Either of these factors can act as a source of bias in the IBNR projections.

Ambiguities

IBNR is a term much used in General Insurance reserving, but which has a good deal of ambiguity about it. This ambiguity has its uses, but can lead to imprecision in the discussion. Hence it is important to be aware of the variations, and to know what is being referred to at any given time. In the first place, IBNR can refer either to the claims themselves in this category, or to the reserve which needs to be established for them. This ambiguity is seldom troublesome, and it is usually clear from the context which sense is intended.

Of far greater import is the following distinction. It can best be seen by approaching IBNR from two different points of view, that of: a) the direct insurer, and b) the reinsurer. The direct insurer should have fairly full information on the pattern of claims, including numbers of claims in each category, and individual case reserves on claims still open at the reserving date. Particular claims can be identified at will, and it is natural to consider the liability as divided into that for the group of known claims on the one hand, and for IBNR claims on the other. The known claims can be estimated by taking the total of case reserves, plus some adjustment for future development, while the IBNR can be estimated by a statistical method (e.g. as described later in this section of the Manual). The point is that the two groups of claims are logically separate, and can be estimated as such.

Coming now to the reinsurer, the amount of information on the claims pattern is likely to be much less. The ceding company or agent will not generally provide information on numbers of claims or individual case reserves — the simple facts of the paid and incurred claims to date will have to suffice. Hence the reinsurer will consider the liability as divided into that which has been reported to it to date (i.e. the incurred claims less his or her payments on account), and that which remains to be reported in the future. The latter will naturally be termed by the reinsurer the "IBNR". It is thus the liability rather than the claim groups which generates the logical distinction. The difference from the direct insurer's view is that the two parts of the liability no longer correspond to fully identifiable groups of claims. (The position is summarised with a diagram, on the next page.)

To this point, we have stated the logical distinction. In practical terms, the main difference is that for the reinsurer the IBNR liability will include the subsequent development in case reserves for the known claims, i.e. in addition to the liability for genuine IBNR claims. There are other elements which can complicate the position further (e.g. reserves for settled claims later re-opened), but this is the central feature.

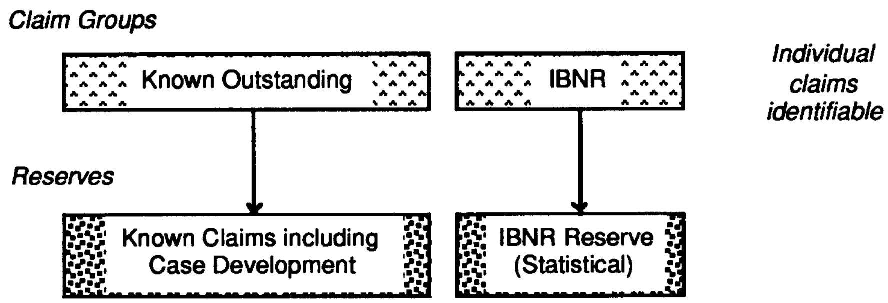
The discussion has been presented in terms of direct vs. reinsurance, but the definitions given above should not be thought of as in exclusive use in the two respective areas. In particular, direct insurers may also think of IBNR as including the subsequent development in case reserves. To help the distinction, the term "true IBNR" is sometimes used for the first version described above, with "IBNER" for the second. IBNER stands for "Incurred but not enough reserved". The algebraic relationship, in its simplest terms, is:

True IBNR + Development on Open Claims = IBNER

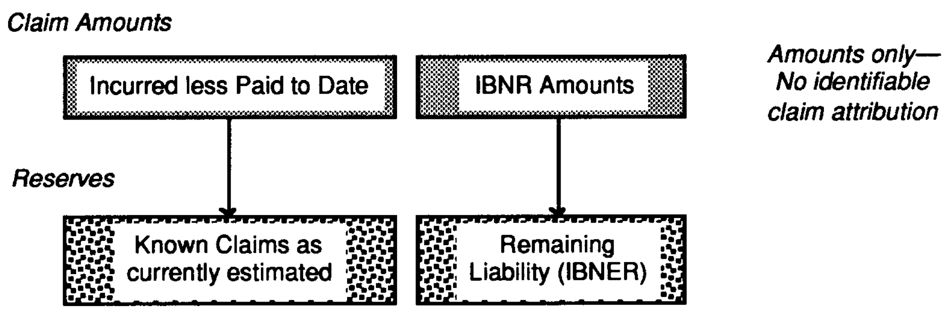
Finally the term IBNYR may also be encountered. This stands for "Incurred but not yet reserved", and has the same meaning as true IBNR.

Diagram — Clarification of the Ambiguity in IBNR

1) Direct Insurer



2) Reinsurer



[I2]
IBNR AS THE REMAINDER TERM

The majority of reserving methods described in the Manual thus far approach the estimation problem by a particular route. The main estimate that is made is of the full ultimate loss — by projecting some convenient quantity such as the paid or incurred claims. This done, subtraction of the paid claims to date then gives the required reserve. The general formula is:

$$CV = \hat{}L\text{-}ult - pC^*$$

As a matter of course, the reserve so determined includes provision for both the reported and the IBNR claims. Why then is there any need to find the IBNR reserve separately from the whole? A number of good reasons exist:

a) A breakdown of the overall claims reserve gives more information to decision takers, and helps with the control of business.

b) A separate statement of the IBNR liability is in any case needed for purposes of the returns to the supervisory authority.

c) It may be that a report year classification of data is being used in the main projections. These will then cover reported claims only, and a separate estimate of the IBNR must be made.

d) The chosen route for determining the overall reserve may be to build it up as the sum of parts, e.g. as case reserves plus IBNER element.

An example of d) is the Bornhuetter-Ferguson method on incurred claims, described in §G4 . But whatever the method, the first major analysis of the claims reserve will be into the reported and IBNR elements. In symbols:

$$CV = rV + ibV$$

Here, rV denotes the reserve for reported claims, while ibV means the complementary IBNR reserve.

The equation shows a simple algebraic relationship between three quantities. Any two of the quantities, if known, will determine the third. This leads directly to the first main method by which the IBNR reserve is determined. It is just the remainder, when the reported claims value is taken from the overall reserve, i.e:

$$\hat{}ibV = CV - rV$$

As was seen at the beginning of this section, many methods do develop an overall value for the claims reserve. Then, say, case reserves can be used for the estimate of *rV,* and the IBNR value is left as the residue. In this event, if case reserves are unadjusted for subsequent development, the IBNR value is correctly described as IBNER rather than true IBNR. If the case reserves *are* adjusted, then the value will be closer to true IBNR.

In the reinsurance world, it is very common to estimate IBNR by this route. As a result, what are often described as "IBNR methods" in this sphere are in fact general claims reserving methods. Thus a reinsurance paper on IBNR will often just describe the projection of paid or incurred claims, or of loss ratio, by the chain ladder and other familiar techniques. In the Manual, however, "IBNR methods" will be used to refer to techniques specifically aimed at estimating the IBNR component of the overall reserve.

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[I3]
IBNR ESTIMATES — THE UNDERLYING PRINCIPLE

Where IBNR is not being determined as a remainder, there is an underlying principle which applies quite generally. Almost by definition, direct statistical data on the current IBNR cannot be available. Hence it is necessary to find some other measure or base to which the IBNR value can be correlated. If this can be done, then the IBNR estimate will follow as some proportion of the alternative measure. In symbols, we are interested in the quotient:

$$ibV_y \,/\, M_y$$

where ibV denotes the IBNR value, M is the chosen measure, and y is a suffix to identify the current year (or other period).

Many different choices are possible for the alternative measure, and these are elaborated below. One simple example is quoted in Skurnick's 1973 paper, where a US Treasury formula at the time required an IBNR reserve for fidelity insurance of at least 10% of the premiums in force. For surety insurance the figure was 5% of the same quantity.

Once such a figure has been set, the IBNR estimation can follow automatically, e.g. by taking the reserve at the minimum required, or by adding a percentage margin for safety. But it is not necessary to assume that the ibV/M quotient is a constant. Indeed, there is bound to be variation in its true value over time. The simplest way of taking this variation into account is to assume that, for the current period, the value will be as for the immediately preceding period. Symbolically, this is:

$$ibV_y \,/\, M_y = ibV_{y\text{-}1} \,/\, M_{y\text{-}1}$$

Multiplying by M_y then gives:

$$\textit{Current IBNR Estimate} = \textit{Previous IBNR Value} \;{}^\prime\; \textit{Ratio}\ (M_y \,/\, M_{y\text{-}1})$$

The stated relationship is quite general. The ratio $M_y / M_{y\text{-}1}$ can be thought of as a kind of growth factor, linking successive IBNR values in the sequence of years. The means of application of the formula will depend, however, on the nature of the line of business, and how the data are structured. The simplest cases arise for the short-tail lines of business, particularly those where all or most of the IBNR claims are run off during the year following the year of exposure. The formula can then be applied directly to the data in hand. But where there is a medium or long tail to the business, tables of IBNR run-off for earlier accident years may need to be constructed, and the formula applied in a more roundabout way. (Examples of possible procedures are given in §§I7,I8.) <>

[I4]
ALTERNATIVE BASES FOR IBNR PROJECTION

In the previous section, it was seen that it is generally necessary to find some alternative measure or base against which to correlate the IBNR value. One of the skills in IBNR estimation is the choice of this alternative base. Many different examples have been used in practice, and many are quoted in the literature. Such measures are usually either related to premiums or to claims, and may be in either money or unit terms. The general classification is thus a 2-way one:

	Premium Related	**Claim Related**
Money Terms	Earned Premium Written Premium Premium in Force	Paid Claims Incurred Claims Case Reserves Late Reported Claims
Unit Terms	Earned Exposure Units Written Exposure Units No. of Policies in Force	No. of Reported Claims No. of Open Claims

In deciding upon an adequate and appropriate measure, there are two main elements to consider. In brief, these are a) risk exposure, and b) inflation of claims. Both will play their part in the IBNR value which ultimately results. For this reason the money related measures tend to be easier to use, since they contain elements relating both to risk exposure and to inflation. The unit related measures only include the risk exposure part, and hence must be supplemented by some allowance for claims inflation. (The alternatives are for the reserver either to gather further evidence on the position, or make an inflation assumption explicitly.)

On the choice between a premium related and a claim related measure, the latter at first would seem to have the advantage, since it is well known that premium rates do not remain at a constant level of adequacy. However, the relationship between IBNR and reported claims is not constant either, and such events as a change in the claim reporting pattern can disturb the picture. A great deal could be said about each of the possible measures, but for now some brief notes on each of the four main categories in the table above may suffice.

Premium Related/Money Terms

The main problem here is that as the underwriting cycle takes its course, premium levels vary in relation to claims in general and to IBNR in particular. In addition, premium rates may react to influences which are not claim related at all, e.g. the general level of office expenses. One solution here is to use only the pure risk premium in the calculations.

On the choice of earned, written or in-force premium, it is largely a question of relating to the main data definition being used in the reserving projections. Thus, for accident year data the earned premium will be most appropriate, while for underwriting year the written premium is best.

Premium Related/Unit Terms

Exposure units are mainly of use in dealing with personal lines, such as motor and household insurance. In these lines, exposure units can be a good measure or base for IBNR claim numbers. This is in effect one half only of the determination, the other being the average amount of IBNR claim. No information is given about this latter element, or its progress with time, so that a separate estimate will be required.

As to the choice of earned, written or in-force units, similar considerations apply as given for premium in the paragraph above.

Claim Related/Money Terms

Paid claims can be useful for the short tail lines, but as the tail gets longer their relevance rapidly diminishes. For the medium and long tail lines, incurred claims will be more appropriate, and will give more information about the likely claims emergence. A problem that may arise here is the consistency over time of the case reserves which form part of the incurred value. Also, the extent to which such reserves include an allowance for inflation must be known, and adjusted for if necessary. Another problem is that variations in the IBNR cut-off date can produce distortions — if the date is later one year than another, say, then incurred claims will be increased by an amount equal to the reduction in IBNR. But the higher incurred value will lead to a *higher* projection for IBNR emergence.

Late reported claims can be a particularly useful measure, mainly for short tail lines. These claims are actual IBNR claims, emerging shortly after the valuation date, in a period in which information can still be brought into the accounting and financial statements. Since the period is likely to be short, it may frequently be the case that only claim numbers rather than the incurred values are available. If so, the measure is in unit terms only, and should strictly be re-classified in the paragraph below.

Claim Related/Unit Terms

Numbers of claims, either of the open or reported variety, can be used as a base for IBNR claim numbers. As with the use of exposure units, a separate estimate of the progress of IBNR claim amounts will have to be made. Of the two measures suggested here, number reported is probably the better choice, since variations in the claim payment pattern will have no disturbing influence.

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[I5]
TARBELL'S METHOD & IBNR RUN-OFF

Tarbell's method is included at this point, since it is a good practical illustration of the general principle set down in the previous section. It also happens to be a classic of the reserving literature, having first appeared in the *Proceedings of the Casualty Actuarial Society* as early as 1933. Finally, it introduces the useful concept of the IBNR run-off.

To explain IBNR run-off, consider the position for some given year of exposure, y. At the end of the year, the reported claims can be evaluated to their incurred value, iC_y, by adding paid claims to case reserves. Of the IBNR claims, denoted as ibC_y, nothing will yet be known. But during the following year, y+l, these claims will begin to emerge, moving effectively from IBNR to the reported group. However, their status of being IBNR at the end of year y will remain so that IBNR claims for the year y are those where the event took place in year y but which were not reported until after year y. As the emergence takes place, all the usual quantities of paid claims, incurred claims, case reserves, and so on can be applied to this IBNR group in its own right. Development tables can be constructed, just as has earlier been done for the overall claims run-off of given accident years.

It is useful to be able to represent the IBNR run-off in symbols. We have already used symbols such as *pC, iC, kV* for the main claims run-off. Coming to the IBNR, we will adapt these to *ibpC, ibiC, ibkV* and so on. The full symbol for incurred claims in the run-off will be:

$$ibiC_y(d)$$

where y is the year of exposure, and d is the development time following the end of year y. For example, d=l will denote the position at the end of year $y+l$.

Tarbell assumes that the main part of the IBNR run-off will occur during the 12 months following the end of any given year. He is therefore dealing essentially with short tail business. However, he allows that the 12 months may not be sufficient, in which case a tail factor must be introduced, which we will denote as $f(1)$. In general:

$$f(1) = ibL\text{-}ult \,/\, ibiC(1)$$

i.e. $f(1)$ is the ratio of the final IBNR loss to the incurred amount after 1 year. Based on experience of earlier IBNR run-offs, it should be possible to estimate this factor. There is the assumption, however, that f will show reasonable stability from year to year.

We come now to the main question. What is the equivalent measure or base which Tarbell uses in his IBNR estimations? Effectively, it is the claims incurred during the last three months of the year of exposure. However, the value is expressed not directly, but as the product of number of claims reported and average size of claim. Tarbell's reason for doing this is to allow more flexibility in the treatment of the various claim distributions experienced in different lines of business. In particular, he is concerned to exclude abnormally large claims which may distort the picture. The resulting formula for the IBNR estimate is as follows:

$$\hat{\ } ibV_y = ibiC_{y\text{-}1}(1) \times (nR_{y[10,12]} \times A_{y[10,12]}) \,/\, (nR_{y\text{-}1[10,12]} \times A_{y\text{-}1[10,12]})$$

The formula has a lot of detail in it, so it may be useful to re-define each term separately.

$\hat{\ } ibV_y$ – Estimated reserve for IBNR claims for the year y as required at the end of year y.

$ibiC_{y\text{-}1}(1)$ – Claims incurred by the end of year y in the run-off of the IBNR claims for year y-1.

$nR_{y[10,12]}$ – Number of claims reported in the last three months of year y.

$A_{y[10,12]}$ – Average size of claim for claims reported in the last three months of year y.

$nR_{y\text{-}1[10,12]}$ – As above, for year y-1.

$A_{y\text{-}1[10,12]}$ – As above, for the year y-1.

The expression of the formula here is essentially that given by Tarbell, but the notation has been adapted to the general form used throughout the Manual. Tarbell also restates the formula by combining the nR and A terms into their product, which is effectively the incurred claim value for the last three months of the respective year of exposure. The result is as follows (again in the Manual's notation):

$$\hat{\ } ibV_y = ibiC_{y\text{-}1}(1) \times iC_{y[10,12]} \,/\, iC_{y\text{-}1[10,12]}$$

Strictly speaking, both versions of the formula should have the factor $f(1)$ inserted, in order to provide for any further development in the $ibiC_{y\text{-}1}$ term beyond the first twelve months.

An important point, admitted by Tarbell, is that there is nothing sacrosanct about his choice of the last three months of the year over which to take the incurred claims measure. The point is to choose a period which is: a) recent enough to reflect current trends in claims numbers and claim sizes, and b) long enough to give dependable statistical data. Different length periods might therefore be selected for different lines of business, according to the balance between these two main requirements.

In conclusion, Tarbell cites his formula as being applicable to such lines as personal accident, motor and other miscellaneous property damage. These are essentially short tail, relatively high frequency lines. Medium and long tail lines are likely to need the more elaborate methods developed below in §§I7. I8.

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[I6]
AVERAGE COST PER IBNR CLAIM

An obvious way to analyse the IBNR position is to look separately at the numbers of such claims and at their average cost. If both can be projected, then their product will give the estimated liability. The main question will be to find suitable data from which to make the projections. The technique suggested here, in fact, is to relate the IBNR claims to the reported claims, both by numbers and average cost.

Number of IBNR Claims

Provided that data of previous years' IBNR run-offs are available, projecting the number of claims should be quite straightforward. The simplest method will be to assume a stable ratio as between the number of claims which are IBNR at the end of a given year (symbol ibn_y), and the number reported during the year (nR_y). Suppose first of all that the run-off for any given year of exposure is completed during the following 12 months. Then the previous year's data can be used to estimate the ratio, i.e. as:

$$ibn_{y\text{-}1}/nR_{y\text{-}1}$$

leading to the projected IBNR number of:

$$^\wedge ibn_y = nR_y \;\; ' \; (ibn_{y\text{-}1}/\; nR_{y\text{-}1})$$

It will be better, however, to gather in data from years earlier than y-1, so that variations in the *ibn/nR* ratio can be assessed. Thus, an average or a weighted average might be taken over the last 3–5 years say, or a trend might be observed and extrapolated.

What has been described so far leads to fairly rough and ready estimates. A more refined method is to use data on a monthly or quarterly basis, rather than just the annual figures. By this means, a development table for claim numbers reported can be built up, in the familiar triangular or parallelogram form. The axis on the left hand side of the triangle gives the accident month or quarter, while the axis along the top gives the equivalent development period. The table so constructed is no different in principle from the annual-period triangles shown in previous sections of the Manual. It can therefore be evaluated by the usual methods, and sets of grossing up factors or link ratios derived in the usual way. Then, applying the appropriate factor to the current year reported claims, the estimated final number of claims is given, and the IBNR claims can be derived by subtraction.

To this point, we have been assuming that the IBNR run-off for any given year of exposure is completed during the following 12 months. For many lines, this will not be the case. But once the concept of the development table is introduced, there is no problem in obtaining the projection for number of claims by the standard methods. Indeed, number of reported claims is one of the more stable quantities in the field of claims reserving, so that this part of the IBNR estimation can often be done with confidence.

Average Cost per IBNR Claim

The second part of the estimation is likely to be the more difficult, at least in terms of reliability. The approach which most readily suggests itself is to assume a stable relationship between the average cost of IBNR claims and reported claims for each period of exposure. Then observed trends in the average reported claims can be used to project the required average IBNR values. If only the previous year's data are being used, the formula will be:

$$^\wedge ibiA_y = iA_y \ ' \ (ibiA_{y-1} / iA_{y-1})$$

Here, $ibiA_y$ is used to denote the average cost per IBNR claim for the year y, while iA_y is the incurred average for the same year.

The same remarks as previously made for numbers of IBNR claims apply, i.e. it will be better to examine the data from several past years. In this way, the stability of the $ibiA/ iA$ relationship can be checked upon, and averages or trends calculated as appropriate. But it is likely that irregularities will be found in the ratio, particularly in times of varying inflation. The point is that IBNR claims will on average be settled at a significantly later date than will reported claims. Hence changes in inflation will soon disturb any otherwise steady relationship. Indeed, inflation is a factor which should specifically be taken into account in deriving the average cost for the IBNR claims.

An alternative to the above, where the $ibiA/iA$ ratio is not stable, is to look purely at the trends in $ibiA$ values themselves for recent years. Then taking account of current inflation rates and other known influences on the business, the trend can be carried forward to estimate the $ibiA$ value for the current year.

Where the IBNR run-off takes longer than a year or two, the $ibiA/iA$ ratio becomes more difficult to assess. This is because IBNR development for the most recent years will not be complete, and must itself be estimated on the way to making the current year's estimate. Of course, full development tables of the incurred claims or incurred average can be constructed and evaluated. But these will generate their own values for the ultimate loss, and hence for the IBNR by subtraction of the incurred claims to date. If that is done, the present method of evaluation is effectively made redundant, although the *ibiA/iA* ratios can be found as a by-product of the work.

The IBNR Estimate Itself

The final estimate of the IBNR liability follows simply as the product of the estimated number of claims and the average cost per claim. In symbols:

$$^\wedge ibV = {}^\wedge ibn \; ' \; {}^\wedge ibiA$$

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[I7]
IBNR BY ACCIDENT YEAR PROJECTION

To this point in §I6, we have been looking at methods suited to the case where the IBNR run-off is mainly completed during the year or two following the year of exposure. But for many lines nowadays, particularly in the liability class, the tail is very much longer. The aim then should be to construct full development tables showing the IBNR emergence by accident or underwriting year, (or possibly by calendar year).

The example given here uses accident year data, and sets the IBNR emergence against a base measure of earned premium. The same procedure could be followed using underwriting year data and written premium. To begin with, the data must be put into the appropriate triangular form, as shown immediately below. These particular data are an extension from the main example used in the text of §§E–H.

				d		
		1	2	3	4	5
	1	604	806	948	1018	1044
	2	718	952	1080	1160	
a	3	776	1004	1196		
	4	868	1114			
	5	962			**[*ibiC*]**	
	6					

It can be seen that the table follows the usual pattern already established. The accident years appear down the left hand side, while the development periods run across the top. The period d=1 is the year immediately following the accident year itself, and so on. d=0 would coincide with the accident year, but by definition there can be no IBNR emergence at that stage. Year a=6 appears at the foot of the table, but without any entries in its row. This is to show the point in time at which the table is constructed, i.e. the end of the year a=6.

A word is also necessary about the nature of the IBNR run-off data. Generally speaking, such data will be in *incurred* form. That is, they will include both actual payments made on the IBNR claims, plus the reserves needed for the open IBNR claims at the development date. The latter may well be derived as the sum of case reserves held on such claims at the time in question. Finally, the symbol used to denote IBNR run-off data is *ibiC.* Individual elements in the table can then be identified, if necessary, by expanding the symbol to $ibiC_a(d)$.

Once the data are in development table form, the first step is to construct the year-by-year figures. This is done overleaf.

		d				
		1	2	3	4	5
	1	604	202	142	70	26
	2	718	234	128	80	
a	3	776	228	192		
	4	868	246			
	5	962			**[Δ*ibiC*]**	
	6					

In the row a=l, 604 is just the value repeated from the first table above. Then 202 is found by subtracting 604 from 806, 142 is the value (948–806), and so on. (As usual, the symbol Δ is used to denote a step-by-step quantity, found by taking differences on the cumulative function.)

The second step in the work is to introduce the earned premium, *aP,* for each accident year. Then the step-by-step IBNR run-off figures are divided by the premium to which they relate. This is done in the table below, by working along each row in turn, using the earned premium figure from the left hand column. The results are expressed as percentages, appearing as the lower of the two figures in each cell of the table.

aP	*a*	*d*				
		1	2	3	4	5
4486	1	604	202	142	70	26
		13.46	4.50	3.17	1.56	.58%
5024	2	718	234	128	80	
		14.29	4.66	2.55	1.59%	
5680	3	776	228	192		
		13.66	4.01	3.38%		
6590	4	868	246			
		13.17	3.73%			
7482	5	962			**[*ibiC*]**	
		12.86%			**[*ibic*/*aP*]**	
8502	6					

Looking down the columns of the table, the percentage figures show a good degree of stability from year to year. That is, the IBNR emergence is closely correlated to the earned premium for each accident year. It is therefore reasonable to project the columns downwards, in this case by taking the average of the observed values. The projection is shown in the table below, which also gives the addition of the percentage figures along each row.

aP	*a*			*d*			*row*
		1	2	3	4	5	*sum*
4486	1	.	.	.	.	.	
5024	2	.	.	.	.	.58	.58%
5680	3	.	.	.	1.58	.58	2.16%
6590	4	.	.	3.03	1.58	.58	5.19%
7482	5	.	4.23	3.03	1.58	.58	9.42%
8502	6	13.49	4.23	3.03	1.58	.58	22.91%

The row sums show the amount of IBNR emergence still expected to occur for each accident year. Thus to make the full IBNR projection, it is only necessary now to multiply the added percentages by the respective earned premium figures. The calculations follow:

a	*row sum*	*aP*	*^ibV*	*kV**
1		4486		234
2	.58	5024	29	475
3	2.16	5680	123	969
4	5.19	6590	342	1796
5	9.42	7482	705	2881
6	22.91	8502	1948	3929

The table shows the current values of case reserves for each accident year, for comparison figures the IBNR figures. The overall value for the IBNR estimate is just the sum of the *^ibV* column, i.e. 3,108. This can, apparently, be added to the current case reserves to give the overall reserve required.

Overall Values:	S^*ibV*	3,147
	S*kV**	10,284
	Reserve	13,431

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[I8]
ADJUSTED PROJECTION USING ACCIDENT & REPORT YEAR

This section presents a modified version of the calculations of §I7 above.

The method depends on having data which are classified both by accident and report period. Thus, for accident year a=1, data are analysed according to the claims reported in year a+1, a+2, and so on. For each group, the incurred claim development is tracked, and a table drawn up to show the current position. As usual, accident years are shown down the LHS, while across the top is plotted the interval between the respective accident and report years. Thus, using r for year of report, (r-a) takes values 1, 2, 3, ... and the familiar triangular form results:

		(r-a) 1	2	3	4	5
	1	669	187	127	45	16
	2	788	214	108	50	
a	3	846	218	132		
	4	928	186			
	5	962			**[*iC*:a,r-a*]**	
	6					

The data in each cell can be characterised as $iC^*_{a,r-a}$, that is the incurred value to date for the given group of claims. E.g. 218 in cell a=3, r-a=2 is the incurred liability to date on those claims originating in year 3 and with report date in year 5, i.e. 2 years later. The actual time of evaluation of all the cells is the end of year 6.

The IBNR emergence can now be evaluated by comparing the values in the table with some suitable base measure. Using earned premium, as was previously done in §I7, the following percentages arise:

aP	a	(r-a) 1	2	3	4	5
4486	1	14.91	4.17	2.83	1.00	0.36%
5024	2	15.68	4.26	2.15	1.00%	
5680	3	14.89	3.84	2.32%		
6590	4	14.08	2.82%			
7482	5	12.86%		**[*iC*:a,r-a/aP*]**		

aP	*a*	(*r*-*a*)				
		1	2	3	4	5
8502	6					

The percentage values in each column are very stable, and can easily be projected downwards. This is done in the table below, taking the average of the most recent three values in each case. The resulting figures are then summed along the rows:

aP	*a*	(*r-a*) 1	2	3	4	5	*row sum*
4486	1						
5024	2					.36	0.36%
5680	3				1.0	.36	1.36%
6590	4			2.43	1.0	.36	3.79%
7482	5		3.64	2.43	1.0	.36	7.43%
8502	6	13.94	3.64	2.43	1.0	.36	21.37%

It remains to apply the row sums to the baseline figures for earned premium to produce the IBNR estimates:

a	*row sum*	*aP*	*^ibV*	*^VR*
1		4486		244
2	0.36	5024	18	507
3	1.36	5680	77	1023
4	3.79	6590	250	1855
5	7.43	7482	556	3002
6	21.37	8502	1817	4150

The overall IBNR value is now 2,718, by addition over the accident years. Data on reserves for reported claims, ^*VR*, are also shown for comparison. The full position is now:

Overall Values:	S^*ibV*	2,718
	^*VR*	10,781
	Reserve	13,499

One point remains to be made. The IBNR reserve, as constructed here, does not include any allowance for future development of the case reserves (i.e. the case reserves held at present on claims formerly in the IBNR category). This is because each cell in the development table is, as it were, an independent IBNR unit, and does not directly link with other cells in its row. Consequently, on projecting the figures down the columns, there is no "knock on" effect from previous columns — but which effect is present in the equivalent tables from §I7 above. Compared with §I7, the IBNR value provided here is an estimate of true IBNR, rather than a hybrid containing IBNER elements.

The result is the beneficial one that, when combining the IBNR estimate with case reserves to give the full liability, the case reserves can be treated consistently. In fact, the reserves *will* need to be adjusted for future development in this case. As an alternative, a projection of the main data on a report year basis can be used. This gives a direct estimate of the liability for reported claims, to which the IBNR reserve can be added.

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Section J
DEALING WITH INFLATION

Preamble

How to deal with inflation is a key question in General Insurance claims reserving. Past inflation of monetary values will affect the shape of the data, and the assumption made as to future rates will significantly affect the final value to be set on the reserve. It is not always necessary, however, to bring inflation explicitly into the calculations — many of the methods so far described will automatically project the level of past inflation into the future. Under stable economic conditions, therefore, the methods can work well on their own. But when inflation is unstable in its rate from year to year, it is necessary to bring it openly into the account.

An important point is that economic inflation is not the only force affecting the average cost per claim. Social influences also play their part, and among these such factors as court awards, attitudes in society towards compensation of accident victims, and legislation, including that of the EU, are of particular importance. Again, technical factors such as a change in the mix of business can produce inflation in the average claim size.

The present section of the Manual concentrates on a number of simple but practical ways of taking inflation into account. The methods are straightforward in use, and have the advantage of allowing the reserver to exercise judgement with regard to the future inflation levels assumed. But all are well based in a prior analysis of the past data which is to hand. This analysis is the homework which needs to be done, to provide the framework and the discipline that lend confidence to the future projections.

Contents

J4. The Separation Method

[J1]
INFLATION — GENERAL CONSIDERATIONS

Inflation is a factor that always needs to be taken into account when making reserving estimates. It is, perhaps, the most important source of uncertainty as regards the final liability still to be met on the claims incurred to date. But it is well to ask exactly what is meant by "inflation" in the context of claims reserving. Is one referring to general economic inflation, as measured for example by the decreasing purchasing power of the pound? Does one allow also for some element of what may be called social inflation, as reflected say in the increased level of damages awards by the courts? Or does one mean the inflation of claim amounts themselves, taking into account all possible influences which are operating at the time?

In general, the reserver will be interested in the rate of claims inflation resulting from all causes together. But economic and/or social inflation will be major elements in the package. If they can be assessed in relation to the data, then a more reliable base will be available from which to make projections.

Dealing with inflation for claims reserving purposes normally embraces two aspects:

a) To identify the inflation element implicit in the past data on claim amounts, average costs per claim, etc.

b) To set a suitable inflation assumption for the purpose of future projections of the data.

In many cases, the answer for b) will be to continue with the general level of rates found in a), or to make some simple extrapolation. But other influences can be taken into account, and if expectations are changing rapidly there is no need for future rates to be tied to the past. The forecasts of some economic schools, for example, might be thought just as relevant as the inflation revealed in the office's recent experience.

On the matter of inflation in past data, there are distinct approaches which can be taken. One is to compare the data against some suitable index of inflation. Of such indexes, the Retail Price Index (RPI) is the best known, but will often not be the most apt one for insurance reserving. It is better to seek an index with some more direct connection with the line of business in hand. Thus, motor claims for vehicle damage would be expected to relate more closely to the NAE (index of National Average Earnings), since labour charges are a large element in the cost of repair. An index, if available, on the price of motor spare parts would also be relevant. Similar considerations would apply to health care insurance, where indices on doctors' fees, drug prices and hospital charges will be the appropriate ones to look for.

Given that the relevant index is to hand, the data on claims can then be adjusted to take out the assumed inflationary element. The data should be in a year-by-year rather than a cumulative form. The job is then best done by scaling up payments in past years to make them comparable with payments in the most recent year. The new picture should show how far the variations in the data can be explained by the chosen inflation index.

If the index is satisfactory, the inflation adjusted data can then be projected by any of the normal methods. At this point comes the leading question — should future inflation be taken to reflect the recent values in the chosen index, or should some other assumption be made? There are no hard and fast answers to this question — the reserver must scrutinise the evidence in front of him, and come to his own considered opinion. Once the choice is made, the projected data can be adjusted to take account of the assumed inflation.

A useful step will be to make a number of such projections with different rates of inflation. This will show the sensitivity of the estimates to inflation — the longer the tail of the business, of course, the greater the sensitivity is likely to be.

In the above, it has been assumed that claims inflation is essentially related to the calendar period in which the claim is paid. But it is also possible for the inflation to correspond with the accident period itself. An example might be a business line giving compensation for loss of earnings. Claims paid out will not allow for earnings growth between the accident date and payment date, and hence their relative scale will relate to the accident year only. (In terms of the development triangle, this type of inflation affects the rows only. The normal type of claims inflation works essentially as an effect on the diagonals of the triangle.)

A final point of some importance refers back to the main methods so far described in the Manual. No mention was made of inflation during their description, which may be surprising in view of the subject's importance. But by dropping inflation until this point, the exposition has been simplified. Also, once the general techniques for dealing with inflation are understood, they can be applied fairly readily to any chosen method.

Lastly, the methods generally do make implicit allowance for inflation — they tend to project forward the inflation already present in the past data. The process only works properly, however, when inflation is fairly stable from year to year. Where it is rapidly varying, as say in the 1970s, the projections will tend to be distorted.

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[J2]
INFLATION ADJUSTED CLAIMS PROJECTION

This section presents a general technique for taking inflation explicitly into account in projections. The data are those of the standard paid claims example from earlier sections of the Manual, and the projection is the usual link ratio method. But the technique could equally be applied to many other combinations of data and projection method.

The main steps in the procedure are as follows:

a) If the data are in cumulative form, then the year-by-year values are obtained by subtraction along the rows of the development table.

b) An inflation index relevant for the business class in hand is selected, or a suitable assumption made about claims inflation in recent years.

c) The year-by-year values from previous years are brought into line with the current year by inflating them according to the index.

d) The resulting values are added up along the rows of the table, to produce the adjusted data in the cumulative form.

e) The adjusted data are projected by the reserver's chosen method, in such a way that the development triangle is completed into a rectangle.

f) Year-by-year values are once again obtained, this time relating to the future years' projection (i.e. in the area to the lower right of the development table).

g) An assumption on the level of future inflation is made. This may come from a projection forward of the index in b) above, or be made on independent grounds.

h) The projected year-by-year values are inflated according to the future rate(s) selected in g).

i) Adding the projected inflated year-by-year values along the rows leads to the figure for the estimated reserve on the business.

The procedure is now illustrated by a numerical example. The starting point is the familiar table of paid claims:

	d						
	0	1	2	3	4	5	*ult*
1	1001	1855	2423	2988	3335	3483	3705
2	1113	2103	2774	3422	3844		
3	1265	2433	3233	3977			
a 4	1490	2873	3880				
5	1725	3261				**[*pC*]**	
6	1889						

Step (a)

The year-by-year values are found straight away by subtraction along the rows:

	d						
	0	1	2	3	4	5	*ult*
1	1001	854	568	565	347	148	222
2	1113	990	671	648	422		
3	1265	1168	800	744			
a 4	1490	1383	1007				
5	1725	1536				**[Δ*pC*]**	
6	1889						

Step (b)

Suppose that an inflation index relevant to the business class in hand is available. It is based at 100 in year a=4, and the yearly values it shows are:

a	1	2	3	4	5	6
Index	78	82	89	100	111	120

If the index is a monthly one, the value chosen for each year should be that as at 30 June. If the index is annual or quarterly, the 30 June value can be found by simple interpolation. The assumption here is that claims are fairly evenly spread through the year, probably reasonable for most classes of business — to insist on 100% precision in the timing of the index would be out of place. But if there is a bias in the claims, such as might be found with storm damage, then an adjustment may be in order.

From the index, the effective annual inflation in the past can be found. Symbol j will be used for inflation, so that the multiplier for adjustments will be $(1 + j)$:

a	1	2	3	4	5	6
Index	78	82	89	100	111	120
$1+j$	1.051	1.085	1.124	1.110	1.081	
$\pi(1+j)$	1.538	1.463	1.348	1.200	1.081	1.000

In the table, 1.051 is 82/78, 1.085 is 89/82 and so on. The annual inflation rates brought out vary from 5.1% to 12.4%, as can be seen. The lowest row is put in to show the cumulative effect — it gives the relation between the earlier years and the current year a=6. In fact, a 53.8% increase has occurred overall since the year a=1.

(Note: the symbol π is used to denote a product of factors. In this case the working runs backwards along the row of annual values $(1+y)$. Thus 1.200 is 1.081 ´ 1.110, 1.348 is 1.081 ´ 1.110 ´ 1.124, and so on. Alternatively, the values can be calculated as 120/111, 120/100, 120/89, 120/82 and 120/78).

Step (c)

We now bring the year-by-year paid claims into line with the current year by inflating them according to the index. Looking at the development triangle, it is the diagonals which correspond to the years of payment. For example, the cells (a=3, d=0), (a=2, d=1), (a=1, d=2) all represent payments made in year 3. For this year, the inflation factor to the present time is 1.348, from the last row in the above table. It follows that a table of inflation adjustment factors will simply show these $\pi(1+y)$ values arranged on consecutive diagonals of the triangle:

		d						
		0	1	2	3	4	5	*ult*
	1	1.538	1.463	1.348	1.200	1.081	1.000	0.870
	2	1.463	1.348	1.200	1.081	1.000		
	3	1.348	1.200	1.081	1.000			
a	4	1.200	1.081	1.000				
	5	1.081	1.000				**[$\pi(1+j)$]**	
	6	1.000						

There is one slight problem in this table — how to deal with the value under (a=1, d=*ult*). The paid claims value for this cell, 222, is an estimate of a future payment, based on run-off data from earlier years. Exactly what inflation adjustment should be applied to it is therefore a moot point. But if the average time of payment in the run-off comes, say, at 1.5 years on, and 10% is reckoned as the typical rate from the earlier run-offs, then a 15% adjustment will be in order. This leads to the right-hand value in the above table, since 100/115 = .870.

To carry out the adjustment, it remains to multiply the year-by-year claim values by those in the above triangle. The claim values are repeated here for convenience:

	a \ d	0	1	2	3	4	5	*ult*
	1	1001	854	568	565	347	148	222
	2	1113	990	671	648	422		
	3	1265	1168	800	744			
a	4	1490	1383	1007				
	5	1725	1536				**[Δ*pC*]**	
	6	1889						

The multiplication proceeds on a cell-by-cell basis, and gives the following result:

	a \ d	0	1	2	3	4	5	*ult*
	1	1540	1249	766	678	375	148	193
	2	1628	1335	805	700	422		
	3	1705	1402	865	744			
a	4	1788	1495	1007				
	5	1865	1536				**[*Adj.*Δ*pC*]**	
	6	1889						

Step (d)

The next step is the straightforward one of regenerating the cumulative values for the paid claims, now in their adjusted form. This is done by adding the values along the rows of the year-by-year claims triangle:

	a \ d	0	1	2	3	4	5	*ult*
	1	1540	2789	3555	4233	4608	4756	4949
	2	1628	2963	3768	4468	4890		
	3	1705	3107	3972	4716			
a	4	1788	3283	4290				
	5	1865	3401				**[*Adj.pC*]**	
	6	1889						

Step (e)

The adjusted data can now be projected by a standard method. The grossing up procedure, which goes direct to the ultimate values, is not well suited in this case. But the link ratio, which enables the intermediate values for future claim years to be generated, works well. This method is employed here. The ratios at the foot of the table are found as the average of the values in the columns above them.

		d 0	1	2	3	4	5	*ult*
	1	1.811	1.275	1.191	1.089	1.032	1.041	
		1540	2789	3555	4233	4608	4756	4949
	2	1.820	1.272	1.186	1.094			
		1628	2963	3768	4468	4890		
	3	1.822	1.278	1.187				
a		1705	3107	3972	4716			
	4	1.836	1.307					
		1788	3283	4290				
	5	1.824					**[*r*]**	
		1865	3401				**[*Adj.pC*]**	
	6	1889						
	r	1.823	1.283	1.188	1.092	1.032	1.041	

The projection is now completed by applying the ratios successively to the diagonal elements in the claims triangle. This generates the cumulative claims values for future years, so completing the previously unfilled part of the claims triangle:

		d 0	1	2	3	4	5	*ult*
	1							4949
	2		**[*Adj.pC*]**				5046	5253
	3					5150	5315	5533
a	4				5097	5566	5744	5980
	5			4363	5183	5660	5841	6080
	6		3444	4419	5250	5733	5916	6159

Along the bottom row, 3444 = 1889 ´ 1.823, 4419 = 3444 ´ 1.283, etc. On row *a*=5, 4363 = 3401 ´ 1.283, 5183 = 4363 ´ 1.188, and so on through the triangle.

Step (f)

Having projected the cumulative figures with inflation removed, we now wish to put back the future inflation. Before this can be done, the claims must once more be put into their year-by-year form. Again, the procedure is to subtract values along the rows:

		d						
		0	1	2	3	4	5	*ult*
	1							193
	2		**[Adj.Δ*pC*]**				156	207
	3					434	165	218
a	4				807	469	178	236
	5			962	820	477	181	239
	6		1555	975	831	483	183	243

Step (g)

We come now to the critical question. What rate or rates are to be assumed for future inflation? The evidence from the index used in Step (b) above shows that inflation has been in the range 8.0–12.5% during the last three years, but that it has been declining towards the lower end of the range. In the circumstances, an assumption of future inflation of 10% p.a. seems reasonably cautious. If this is taken up, a further triangle of inflation factors can be constructed:

		d						
		0	1	2	3	4	5	*ult*
	1							1.150
	2		**[π(1+*j*)]**				1.100	1.265
	3					1.100	1.210	1.392
a	4				1.100	1.210	1.331	1.531
	5			1.100	1.210	1.331	1.464	1.684
	6		1.100	1.210	1.331	1.464	1.610	1.852

The table shows an increase of 10% on the first diagonal, which represents the year following the current one. Then the increase is 21%, 33.1% and so on in sequence, each year having a 10% increase over the one before it, except in the *ult* column where provision is made for one and a half years' increase.

Step (h)

These factors can be applied immediately to the adjusted year-by-year claim values. Again, the multiplication takes place on a cell by cell basis, with the following result:

					d			*row*
		1	2	3	4	5	*ult*	*sum*
	1						222	222
	2		**[^Δ*pC*]**			172	262	434
	3				477	200	303	980
a	4			888	567	237	361	2053
	5		1058	992	635	265	402	3352
	6	1711	1180	1106	707	295	450	5449

Step (i)

The last step is to add the projected claim values along the rows. This gives the estimated liability for each accident year. Finally, the accident years are totalled to give the required reserve and the estimate of ultimate loss:

Overall Values:	Reserve	12,490
	S*pC**	20,334
	S^*L-ult*	32,824

The value obtained is very close to those previously given by the unadjusted data (see §E8.2). The example data are a well behaved set in any case, but the main reason is to do with the rate chosen for future inflation. This was set to be consistent with the values experienced in the past. Whenever this is done, adjusted and unadjusted methods will tend to give similar answers, provided the inflation rates concerned are reasonably stable from year to year.

Sensitivity Calculations

It is worth re-evaluating the above example for different future inflation assumptions. This has been done in the table below, to give an idea of the variation in the result. But the sensitivity will depend very much on the length of the tail of the business under evaluation. A long-tail liability line will show a good deal more variation than the present example.

Inflation	5%	8%	9%	10%	11%	12%	15%
^Reserve	11,228	11,966	12,229	12,490	12,758	13,027	13,885

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[J3]
BENNETT & TAYLOR — METHOD A

Bennett & Taylor's method, entitled "Method A" in their 1979 paper, is essentially a projection of the payments pattern on a per claim basis. It was devised in the context of motor reserving, but can be of quite general application. The authors use data structured on a report year basis, i.e. claims are classified and allowed to develop according to the year in which they are notified to the insurer.

The method begins by taking the year by year figures for the paid claims, and adjusting these to present day values by means of some suitable index. This procedure is identical to Steps 1–3 in the general technique for inflation adjustment described in §J2.

The Crucial Step

The crucial step is then taken, which is to divide the claims figures for each year by the number of claims reported in that year. The result is, or should be, to put the payments data on to a normalised basis. This is because variations in the data from year to year must be attributable to a) average claim size, and b) the number of claims occurring. In principle, the inflation adjustment should deal with much of the type a) variation, so that number of claims is the chief remaining factor.

With the main sources of variation removed, the payments per claim table should show a good degree of regularity in its columns. If the data were perfect then each column would be constant, with a single value repeated down its length. Such a situation will never pertain in practice, but the reserver should soon be able to assess the stability of the data from the calculated table. If all is well, an average can be taken on each column, and used to project the values in the empty cells of the table.

Finally, it is a simple matter to generate the required reserve from the projected payments per claim. There are three main steps:

a) Multiply the figures in each row by the respective number of claims for the report year.

b) Project the inflation index used in the first part of the work, or make some suitable assumption about the level of future inflation.

c) Inflate the future claim payment figures for each report year, and add them to give the estimated reserve.

Other Points

A rider is that since Bennett & Taylor's method uses report year data, it will be necessary to make a separate estimation of the IBNR liability. This must be added in later to give the full reserve.

A second point is to do with the nature and rationale of the method. Bennett & Taylor call it an "average payments" method, which is a fair description. But it should be clearly distinguished from the average cost per claim techniques of §H. The latter allow for claim number development, and do not use a single divisor for each row of the table. Also, they concentrate on whole claims, whether paid, incurred or emerging, at their different stages of development. But Bennett & Taylor's method focuses on the pattern of payment, and in effect distributes the cost for any given claim over all the years of development.

As usual, we illustrate the procedure with a worked example. The data that follow are paid claim information, but is a new set of figures based on a report year definition. It is not intended to be representative of any particular type or class of business. Below are shown the cumulative data, followed by the corresponding year by year claims figures.

	d							
		0	1	2	3	4	5	*ult*
	1	500	737	915	1036	1107	1163	1245
	2	732	1065	1296	1476	1606		
	3	854	1263	1556	1800			
a	4	980	1493	1890				
	5	1101	1688				**[*pC*:*r*]**	
	6	1189						

	d							
		0	1	2	3	4	5	*ult*
	1	500	237	178	121	71	56	82
	2	732	333	231	180	130		
	3	854	409	293	244			
a	4	980	513	397				
	5	1101	587				**[Δ*pC*]**	
	6	1189						

An inflation index is needed, or some assumption on the past inflation contained in the claims figures. Here, the following index is taken to be available and relevant to the business in hand:

r	1	2	3	4	5	6
Index	97	100	107	118	126	136
$1+j$	1.031	1.070	1.103	1.068	1.079	
$\pi(1+j)$	1.402	1.360	1.271	1.153	1.079	1.000

The $(1+j)$ line shows inflation varying between 3.1% and 10.3% for the period in question. Overall, there has been a 40% rise in values or costs between the first and last years concerned. The index yields the following full set of factors for inflating the past data:

	d							
		0	1	2	3	4	5	*ult*
	1	1.402	1.360	1.271	1.153	1.079	1.000	0.870
	2	1.360	1.271	1.153	1.079	1.000		
	3	1.271	1.153	1.079	1.000			
a	4	1.153	1.079	1.000				
	5	1.079	1.000				**[$\pi(1+j)$]**	
	6	1.000						

The adjustment itself follows by straight multiplication of the ΔpC table:

	d							
		0	1	2	3	4	5	*ult*
	1	701	322	226	140	77	56	71
	2	996	423	266	194	130		
	3	1085	472	316	244			
a	4	1130	554	397				
	5	1188	587				**[*Adj.*ΔpC]**	
	6	1189						

Now comes the second adjustment, scaling the data according to the number of claims for each of the report years. This produces the pattern of payments per claim, which is the focus of interest. (The single symbol n is used here for number of claims. By definition, the number must be a constant for each of the report years.)

n	r	d 0	1	2	3	4	5	ult
128	1	5.477	2.516	1.766	1.094	.602	.438	.555
167	2	5.964	2.533	1.593	1.162	.778		
190	3	5.711	2.484	1.663	1.284			
203	4	5.567	2.729	1.956				
214	5	5.551	2.743				**.Δ*pC*/*n*]**	
220	6	5.405						
Col Avge		5.613	2.601	1.745	1.180	.690	.438	.555

The data in this case show good regularity, and it is reasonable to project forward by taking the average of the values in each column. The projection table simply shows the values repeated in the lower right part of the triangle, representing the future years of development:

	r	d 0	1	2	3	4	5	ult
	1							.555
	2		**j.Δ*pC*/*n*]**				.438	.555
	3					.690	.438	.555
r	4				1.180	.690	.438	.555
	5			1.745	1.180	.690	.438	.555
	6		2.601	1.745	1.180	.690	.438	.555

Now the reverse adjustments are made to the projected data. First, the scaling factor of number of claims is brought back in, by straight multiplication along the rows:

n	r	d 0	1	2	3	4	5	ult
128	1							71
167	2		**Adj.Δ*pC*]**				73	93
190	3					131	83	105
203	4				240	140	89	113
214	5			373	253	148	94	119
220	6		572	384	260	152	96	122

Next, future inflation factors are generated. Here, a uniform rate of 10% is chosen, as an extrapolation of the highest rate shown in the inflation index in the last five years.

		d 0	1	2	3	4	5	*ult*
	1							1.150
	2		**[^(1+*j*)]**				1.100	1.265
	3					1.100	1.210	1.392
a	4				1.100	1.210	1.331	1.531
	5			1.100	1.210	1.331	1.464	1.684
	6		1.100	1.210	1.331	1.464	1.610	1.852

A final multiplication yields the future paid claim estimates with inflation allowed for at the assumed rate. The value for the reserve then follows by addition of parts.

		1	2	3	*d* 4	5	*ult*	*row sum*
	1						82	82
	2		**[^Δ*pC*]**			80	118	198
	3				144	100	146	390
a	4			264	169	118	173	724
	5		410	306	197	138	200	1251
	6	629	465	346	223	155	226	2044

The total of the final column is 4,689. Since report year data have been used, this is the estimated reserve for reported claims. To it should be added the IBNR estimate to give the full reserve.

Sensitivity Calculations

As in §J2, the example can be evaluated for different future inflation assumptions, and this is done in the table below.

Inflation	5%	8%	9%	10%	11%	12%	15%
^Reserve	4,196	4,483	4,588	4,689	4,797	4,906	5,244

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[J4]
THE SEPARATION METHOD

The separation method is a mathematical method first devised by Verbeek in 1972. Verbeek applied the model in the reinsurance context to the projection of numbers of claims reported. Later, the method was developed to apply to average payments per claim by the Australian author Taylor. This is the version described here, although the main part of the working is identical with that of Verbeek.

The idea behind the separation method is to distinguish two patterns in the claims data from one another. These are: a) the development pattern for the accident year, and b) calendar year effects, of which inflation is usually the most important. The first pattern is the one which works across the columns of the development table, and which is elicited in the grossing up and link ratio methods. The second pattern is the one operating on the diagonals of the table, and which is the special subject of the present section of the Manual.

The basic assumption behind the method is that the two patterns are independent of one another. The assumption will never completely be satisfied, but in many cases it will serve as a working basis. A test can be applied during the work to test the truth of this assumption.

Using Verbeek's model, the patterns are generated without using any outside information. Thus, unlike the previous methods described in §J, no special inflation indices are required. The data are effectively analysed to reveal their own intrinsic inflation. Once the analysis has been done, it is as if the internal structure of the data was revealed to the light. This structure can then be extended so as to generate data values for future years which are in keeping with the existing data. Estimation of the required reserve then follows straight- forwardly.

The style of the earlier parts of the Manual is to present reserving methods by means of arithmetical example. Since the separation method can be applied without going into the full mathematics, the style will be retained here. The method requires, however, that the reserver follow through a detailed set of operations, called here the Separation Algorithm.

Work begins with the standard paid claims data used in the Manual examples:

		d						
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
	3	1265	2433	3233	3977			
a	4	1490	2873	3880				
	5	1725	3261				**[*pC*]**	
	6	1889						

		d						
		0	1	2	3	4	5	*ult*
	1	1001	854	568	565	347	148	222
	2	1113	990	671	648	422		
	3	1265	1168	800	744			
a	4	1490	1383	1007				
	5	1725	1536				**[Δ*pC*]**	
	6	1889						

The first step is to convert the year by year values into the average payment per claim figures. This is done simply by dividing through each row by the related number of claims. With report year data, as in the Bennett & Taylor method, there is no problem since the claim numbers for each year are known with certainty. But with accident year data, claim numbers are not fully developed, and some estimate must be used. There are at least three different possibilities:

a) Use the development to date of claim numbers reported to estimate the ultimate numbers for each year.

b) Estimate the ultimate numbers for each year separately, as at the end of the accident year itself.

c) Substitute for ultimate number of claims the number made in the accident year itself.

Of these, a) is not thought fully satisfactory, since more information is available about the earlier accident years. This means that a bias may be introduced into the working, which should be avoided. As far as b) is concerned, the estimate is likely to depend to a large extent on the number of claims actually made in the accident year. Hence option c) has much to recommend it, apart from being the simplest to use in any case. The fact that claim numbers in c) are not the ultimate ones does not matter, provided the proportionality is constant across the accident years. The claims numbers in c) are being used merely as a standardising factor.

Option c) is used in the table below. The numbers of claims reported in each accident year, $nR(0)$, are used to divide through the year by year claims figures. The result is a version of the average payments per claim table:

$nR(0)$	a	d = 0	1	2	3	4	5
414	1	2.418	2.063	1.372	1.365	.838	.357
453	2	2.457	2.185	1.481	1.430	.932	
494	3	2.561	2.364	1.619	1.506		
530	4	2.811	2.609	1.900			
545	5	3.165	2.818				
557	6	3.391					

It will be seen that the cell $a = 1$, $d = ult$ has been dropped from this table. That is because an exact triangle is needed for the main separation calculations — the cell will be brought back into account towards the end of the working. We now begin the separation method proper by calculating the sum of each column and each diagonal in the table:

$a + d$	*diagonal sums*	a	d = 0	1	2	3	4	5
		1	2.418	2.063	1.372	1.365	.838	.357
1	2.418	2	2.457	2.185	1.481	1.430	.932	
2	4.520	3	2.561	2.364	1.619	1.506		
3	6.118	4	2.811	2.609	1.900			
4	8.021	5	3.165	2.818			**[$\Delta pC/n$]**	
5	9.661	6	3.391					
6	10.904							
	col sum		16.803	12.039	6.372	4.301	1.770	.357

With these totals, we can apply the separation algorithm. It is set out in the following table, by example. There are eight rows in the table, and one column for each of the columns in the claim payment table, i.e. six in this case. The work proceeds down the columns, beginning from top left of the table.

	{1}	*diag sum*	10.904	9.661	8.021	6.118	4.520	2.418
D/gen	{2}	{1}/{8}	10.904	9.988	9.088	8.278	7.895	7.757
	{3}	*col sum*	.357	1.770	4.301	6.372	12.039	16.803
	{4}	Σ{2}	10.904	20.892	29.980	38.258	46.153	53.910
C/gen	{5}	{3}/{4}	.0327	.0847	.1435	.1666	.2608	.3117
	{6}	Σ{5}	.0327	.1174	.2609	.4275	.6883	1.0000
	{7}	1-{6}	.9673	.8826	.7391	.5725	.3117	
	{8}	{7} shifted	1	.9673	.8826	.7391	.5725	.3117

To explain in more detail:

Row {1} contains the diagonal sums, in reverse order, starting with the sum of the main diagonal ($a + d = 6$) in the triangle.
Row {3} contains the column sums, in reverse order, from the triangle.
Row {8} starts with the value 1 (the other values being produced progressively by the steps described below).
Row {2} = Row {1} element divided by the corresponding Row {8} element.
Row {4} is the cumulative sum of the Row {2} elements.
Row {5} is Row {3} divided by the Row {4} element.
Row {6} is the cumulative sum of the Row {5} elements.
Row {7} is the value 1 minus the Row {6} element.
This becomes the value of Row {8} in the next column.

The procedure should continue automatically until Row {6} in the final column is reached. The value 1 should be obtained, exactly. This is a good check on the working — if the answer is not unity, then a mistake has occurred somewhere along the line.

In the algorithm table, Row {2} is marked D/gen (= diagonal generator), and Row {5} as C/gen (= column generator). These are the output values from the table, and can be used to remodel the original data. This is done by setting out the Row {5} values in reverse order across the top of a new table, and the Row {2} values down the left hand side, again in reverse order. The element in the table for row a, column d is obtained by multiplying the value of D/gen for $a + d$ by the value of C/gen for column d.

	C/gen		.3117	.2608	.1666	.1435	.0847	.0327
		a				*d*		
a + *d*	*D/gen*		0	1	2	3	4	5
		1	2.418	2.059	1.379	1.304	.846	.357
1	7.757	2	2.461	2.159	1.514	1.433	.924	
2	7.895	3	2.580	2.370	1.664	1.565		
3	8.278	4	2.833	2.605	1.817			
4	9.088	5	3.113	2.844			**ı.Δ*pC/n*]**	
5	9.988	6	3.399					
6	10.904							

For example, on the first row of the table, 2.418 = 7.757 ´ .3117. Then 2.059 = .2608 ´ 7.895, and 1.379 = .1666 ´ 8.278, and so on. On the second row, 2.461 = .3117 ´ 7.895, and 2.159 = .2608 ´ 8.278, and so on.

The success or otherwise of the remodelling process can be checked by comparing the generated table of average payments with the original version.

This is repeated below for convenience:

		d					
		0	1	2	3	4	5
	1	2.418	2.063	1.372	1.365	.838	.357
	2	2.457	2.185	1.481	1.430	.932	
	3	2.561	2.364	1.619	1.506		
a	4	2.811	2.609	1.900			
	5	3.165	2.818			**[Δ*pC/n*]**	
	6	3.391					

The agreement between the two sets of figures is very good in this example, with the possible exception of the last element in column d = 2 and the first and last in d = 3. In practice, one would seldom obtain such a good fit. However, the fit may often be reasonable enough to justify the use of the method.

Further information about the data is afforded by the D/gen values obtained in the analysis. They are a key to the inflation intrinsic in the data on the separation principle. In fact, the D/gen values supply the missing inflation index for the past calendar years, which was brought in externally by the other methods. In the present case, the values are:

a	1	2	3	4	5	6
Index	7.757	7.895	8.278	9.088	9.988	10.904
$1+j$		1.018	1.049	1.098	1.099	1.092
$\pi(1+j)$	1.408	1.383	1.318	1.200	1.092	1.000

The year 2-on-1 inflation looks suspiciously low. However, it should be remembered that the method estimates not only the inflation for past years, but also any other calendar year effects on the data (e.g. the introduction of a new claims administration system). The low index may therefore be explicable by some other special feature of the business in hand.

The last three year's figures are very consistent, and suggest continuing inflation at around the 10% mark. This assumption simplifies the situation, and can be used immediately in projection forward the data.

The projection, then, is to take the latest value in the D/gen set, and uprate it by 10%p.a. compound. This yields:

$a + d$	6	7	8	9	10	11
D/gen projection	10.904	11.994	13.193	14.512	15.963	17.559

We now return to the special table for generation of data. This time, the new values of D/gen are added down the right hand side of the table. Again, the element in the table for row a, column d is obtained by multiplying the value of D/gen for $a + d$ by the value of C/gen for column d.

C/gen		.3117	.2608	.1666	.1435	.0847	.0327		
	a				*d*				*a + d*
D/gen		0	1	2	3	4	5	*D/gen*	
	1							11.994	7
7.757	2			**Δ*pC*/*n*]**			.392	13.193	8
7.895	3					1.016	.431	14.512	9
8.278	4				1.721	1.117	.475	15.963	10
9.088	5			1.998	1.893	1.229	.522	17.559	11
9.988	6		3.128	2.198	2.082	1.352	.574		
10.904									

By way of example, in the bottom row 3.128 = .2608 ′ 11.994, and 2.198 = .1666 ′ 13.193, and so on. In the next row up, 1.998 = .1666 ′ 11.994, and 1.893 = .1435 ′ 13.193, and so on.

We now have the future pattern of average payments per claim, with inflation included. It remains to multiply through each row by the related number, in order to reinstate the full paid claim amounts. The reader is reminded that the $nR(0)$ are being used solely as standardising factors. This is done below:

					d			*row*
nR(0)	*a*	1	2	3	4	5	*ult*	*sum*
414	1						222	222
453	2			**n.Δ*pC*]**		178	267	445
494	3				502	213	320	1035
530	4			912	592	252	378	2134
545	5		1089	1032	670	284	426	3501
557	6	1742	1224	1160	753	320	480	5679

Overall Values:	Reserve	13,016
	S*pC**	20,334
	S^*L-ult*	33,350

One final point needs explanation here, relating to the *ult* column in the table. Earlier on, the element for $a = 1$ in the *ult* position had to be dropped, and with it the tail of the claims development from $d = 5$ onwards. Now some means has to be found for bringing the tail back into the reckoning. The simplest way is to take the ratio of the ΔpC elements at d=5 and d=*ult*, and use it as a scaling factor. The elements are 148 and 222 respectively, giving a ratio of 1.500. This ratio is applied in the above table to generate the *ult* column, as 1.5 times the values in the $d = 5$ column alongside. The procedure is not fully satisfactory, but perhaps the best that can be devised in the circumstances.

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Section K
MISCELLANEOUS TOPICS

Preamble

The section's purpose is to catch those reserving topics which have fallen through the net of the main classification system used in the Manual. At present, three main items are dealt with, but the section has room for extension as other worthwhile topics come to light.

The first area taken is that of average cost reserving systems. Such systems are very useful for dealing with claims which are small but numerous, and with recently reported claims where not enough evidence is yet available to support a proper case estimate.

The second area is that of re-opened claims. For a variety of reasons, claims already settled may sometimes have to be re-evaluated, and further payments made to the insured. Depending on the reserving method used, and on the relative importance of the re-opens, it may be necessary to set up a separate reserve for such claims. There is an interesting question as to how the re-opens should be treated: as extensions of the original claims, or as new claims in their own right? In the latter case, re-opens have much in common with IBNR claims, and indeed can be evaluated as such for reserving purposes.

The third area is claims expense, of both the direct and indirect type. Indirect expense is not an area in which great precision can be achieved, and relatively crude methods will suffice. The problem is to find the overall reserve needed, i.e. for the business as a whole, and then to allocate it between the different classes. Coming to direct expense, this may often be treated simply as a component of claims themselves. If it is evaluated separately, then many of the methods already described for claims can be used. A good way of proceeding, however, is to look at the ratio which the expense bears to claims, and to project this ratio directly. It is commonly found that the ratio increases markedly with the development life of the claim.

Contents

[K1]
AVERAGE COST RESERVING SYSTEMS

Average cost reserving systems are, in effect, used as an alternative to the setting of individual case reserves. They may be used either for replacing case reserves where insufficient information is available, or as a means for reducing paperwork and administrative cost.

Situations where such systems can be appropriate are as follows:

a) For lines of business where claims tend to be small and quickly settled.
b) For lines with large numbers of claims and a relatively stable pattern of claim sizes.
c) Generally, for recently reported claims where the facts provided are insufficient for setting a proper case reserve.

General Principle

The general principle behind the average cost systems is to put the claims into a standard pool, but with time and value cut-off points beyond which the claims must be individually estimated. Thus, when first reported, all claims are reserved at an average value previously set for the line of business in hand. Then, for those claims settled by the cut-off date, no further estimating need be done. But for claims persisting beyond the cut-off, a case reserve will be established and used in preference to the average value.

The value limit, if set, works in a similar way to the time limit. Any claim exceeding the set value is immediately withdrawn from the pool and given an individual reserve.

This would apply where further facts may be obtained at an early date enabling a case reserve to be properly set. The claim can then immediately be estimated as such, rather than waiting for the time cut-off to operate.

Average cost reserving systems introduce some interesting problems for the reserver. These relate to the time and value cut-offs, and the average cost figure itself. The setting of the time cut-off will normally depend on how long is needed to gain the fuller facts about a given case. For most lines the period will be fairly short, say from one to six months at most following the date of report. The value cut-off needs to be selected for practical reasons, balancing cost savings against the need to give individual attention to the larger claims. It will be helpful if the reserver can obtain data on the general pattern of claim sizes which pertains to the line of business in question.

The Average Cost Figure

Coming to the average cost figure, one basis for it would be to estimate the ultimate average cost for claims in the given line of business. To this end, the methods of §H and §J could be used. The objection is that the ultimate average covers claims which are IBNR as well as reported claims. The IBNR claims are likely to differ in size from the generality — hence the ultimate average will show bias if used to estimate reported claims only. The distortion may not necessarily be serious, but if it is, one solution would be to make the average estimate using report year rather than accident year data.

The reserver should also monitor the data for other sources of distortion. For example, the most severe claims may be rapidly taken from the average cost pool and reserved on an individual basis. But the average used for valuing claims in the pool may be that for reported claims as a whole, i.e. including the severe cases. In this event the average will tend to be overstated.

The opposite effect can occur where the majority of claims are small and quickly settled. In this event, the larger claims remaining longer in the pool can be underestimated by the reported claims average.

As a final point, care is needed whenever the time cut-off or the money limits for inclusion in the pool are changed. The group of claims that will qualify will be different, and hence the average cost figure is likely to need adjustment. It should be remembered that the statistical results (used, for example, for rating purposes) could be distorted.

By-passing the Main Claims System

For lines of business where claims tend to be small and quickly settled, i.e. situation a) above, special systems can be put into use. Various names are used for these: either a "first and final" payments system, or in the USA a "fast track" or a "one-shot" system.

The idea is that authority can be given for claims below a set limit to be settled directly at branch or agent level. The first report of any claim at head office will then be of the settlement itself and the amount paid. Such claims, although reported at branch level, are not coded into the main claims recording system and effectively by-pass it. But, as always, reserves must be established. It is most conveniently done by taking a periodic count of the number of claims in the category, and multiplying by an average cost figure.

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[K2]
RESERVES FOR RE-OPENED CLAIMS

It is possible for claims which have been finalised, and on which the files have been closed, to be re-opened at a later stage. There is a number of reasons why this might happen. A good example would be in employers' liability, where an injury or disease sustained in course of employment may sometimes recur or produce fresh symptoms after a lapse of time. In such case it may be necessary to increase the amount paid out. Another example might be in property damage, where new evidence on the causal events comes to light, and an earlier settlement is challenged through a court action. Finally, an office's practice on old claims which have been inactive for an appreciable time may be to close them automatically. But such claims can still be re-opened by the insured at a later date.

How should such possibilities be treated by the reserver? For most lines of business, the reserve needed for re-opens will be small in comparison to the overall liability. Even where it is not, many of the standard methods for claims reserving will automatically cover the re-opens along with the first-time claims. Take for example the projection of paid claims on an accident year basis. For each accident year, running along the row in the development table, the amounts paid out on re-opens will be included as a matter of course. The accident date will be the same as that for the original claim, and so the paid amounts will be recorded as if the claim had never been closed in the first place.

Classification of Re-opened Claims

Although accident year projections of overall liability lead to few problems with re-opened claims, other methods may need more careful consideration. Again, where the liability has to be split into that for reported claims and for IBNR separately, problems can arise. The essential question is whether to count the re-opens as new claims, or as just further development on the existing ones. Either alternative can be used, so long as it is used consistently, though the choice may have to depend on the office's practice in the recording of such claims.

To give an example of the dilemmas which can arise, consider the case where reported claims are estimated from case reserves and the IBNR are found separately. There are two main possibilities:

a) The case reserves are adjusted for future development, and combined with an estimate of true IBNR.

b) Case reserves are not adjusted, but are combined with IBNR as the remainder term, or IBNER.

If a) holds, and case reserves are adjusted, the re-opens can be taken as part of the development on existing claims, and incorporated in the adjustment. They will then form a part of the reported claims estimate. But in case b), the re-opens cannot be put in with case reserves, and instead must be defined as new claims in their own right. As such they effectively take on IBNR status, and will be estimated as part of the remainder liability.

Where report year data are used for projections, the question of claim classification becomes an important one. Thus, if re-opens are counted as extensions of old claims, then the amounts paid out on them will appear at a later stage in the original row of the development table. Hence the projection will include the necessary allowance for the re-opens. But if the re-opens are treated as new claims in their own right, then the amounts paid on them will only appear in the table as new report year rows are added. The procedure is equivalent to that for emerging IBNR claims, and the re-opens effectively take on the IBNR status. They are thus no longer covered by the projection, and must either be brought in to the IBNR reserve or estimated as a third, independent category in their own right.

To sum up, re-opened claims can usually be treated as part of the reported or IBNR claim groups without further trouble. The problems mainly arise where re-opens form an appreciable element in the overall picture, or where there is a marked variation in their occurrence. In these cases, a separate analysis may be called for. The parallel between re-opens and IBNR claims is useful here, and many of the IBNR methods can be adapted for the purpose.

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[K3]
RE-OPENED CLAIMS — BALCAREK'S METHOD

This is a method based on number and average cost for re-opened claims. The number of re-opens is estimated from the number of claims settled in recent years, using observed experience. The average cost for the re-opens is then found, again from experience, as a multiple of the average cost for settled claims. Multiplication of the two leads to a simple formula for the re-opened claims reserve.

Taking the number of re-opens first, function $r(t)$ is defined as the probability that a claim first settled t years previously re-opens during the current year. The probabilities are taken to be stable, and are derived from the past experience. (Balcarek fits an exponential function to his data for this purpose.)

Let nS_y be the number of claims settled in a given year y. Then, taking a stance at the end of year y, the claim numbers settled in recent years are:

$$nS_y,\ nS_{y-1},\ \ldots\ nS_{y-k+1}$$

where k is some appropriate limit, i.e. the time-span beyond which re-opens are rarely encountered. The required number of re-opens from the end of year y, i.e. commencing in the year $y+1$, is then:

$$nS_y \,.\, r(1) + nS_{y-1} \,.\, r(2) \ \ldots\ \ldots + nS_{y-k+1} \,.\, r(k)$$

or:

$$\text{S}\ nS_{y-t+1} \,.\, r(t)$$

where summation is from $t = 1$ to k.

It remains to determine the average cost for a re-opened claim. Balcarek relates this to the average cost for a settled claim, where settlement is taken to occur in the original year to which the re-open relates. Denoting this average cost for year y by sA_y, the formula for the re-opened claims reserve becomes:

$$r(1).\ nS_y.\ z_1.\ sA_y + r(2).\ nS_{y-1}.\ z_2.\ sA_{y-1} + \text{---}$$

$$= \text{S}\ r(t) \,.\, nS_{y-t+1} \,.\, z_t \,.sA_{y-t+1}$$

where summation is again from t=1 to k, and z is the ratio between the average re-open and the average settled claim. Balcarek found, for his own data, that a constant ratio of 4.5 could be taken for the value of $_z$. Using this property leads to the simplified formula:

$$z \ . \ \mathrm{S} \ r(t).pS_{y-t+1}$$

where pS_y denotes the amount paid out on claims settled during the year y.

A point to note is that Balcarek's paper dates from 1961, and that he was using data from workmen's compensation business in the USA. The formula given above should therefore not be applied indiscriminately when a reserve for re-opened claims is required. The stability of the required assumptions on re-open probabilities and average cost ratios should be tested in the light of the data available.

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[K4]
CLAIMS EXPENSE — INDIRECT TYPE

In dealing with the expenses related to the settling of claims, it is usual to recognize two main categories — direct and indirect expense.

Direct Expenses are those which can be related to the settlement of particular claims. Examples are lawyers' and loss adjusters' fees, medical and court expenses, costs of special investigations and so on.

Indirect Expenses are those which cannot be allocated to the settlement of particular claims. They are, typically, claim department salaries and national insurance, office costs, data processing costs, and so on.

Claims expenses are usually known in the USA as loss adjustment expenses (LAE), and the direct and indirect kind are known as allocated LAE and unallocated LAE respectively.

Reserving for Indirect Expenses

The simpler methods are available for indirect expense, so this type will be treated first. Since the expenses are of a general nature, as incurred in the overall running of the claim department, they must first be considered in relation to the business at large. The prime problem, then, is to determine the reserve as a whole — which may be sufficient in itself. But in some cases, it may also be desired to allocate the amount between the different classes of business.

The assumption needed for valuing the reserve is just that indirect claims expense will tend to vary over time very much as claims themselves do. The simplest method then is to find the paid expense/paid claims ratio for the year or period just past. Applying this ratio to the end-year claims reserve will then generate a first estimate for the expense reserve. The formula for year y will be:

$$^\wedge EV_y = (pE_y \,/\, pC_y) \;'\; ^\wedge CV_y$$

where pE_y denotes the paid expense for year y, and $^\wedge EV_y$ is the required expense reserve.

This first estimate, however, is likely to be an overstatement of the actual need. The argument runs as follows: During the year, expense will be incurred as claims are opened, investigated, paid out on and closed. For any given claim, the expense will have some pattern of incidence while the claim is extant. But it would hardly be worthwhile to determine the pattern in full detail — a crude assumption will be enough. Perhaps the easiest to make is that 50% of the expense is incurred when the claim is first reported, and the other 50% when it is settled.

With this assumption, consider the claims outstanding and IBNR at the end of the year y. For the first group, 50% of the expense will already have been incurred, during the year itself or earlier. But for the IBNR group, it is clear that no expense can yet have been incurred. Hence, in applying the formula above, it will be correct to reduce the overall claims reserve by 50% of the reported claims portion. The modified estimate is then:

$$\hat{}EV_y = (pE_y / pC_y)\ ´\ (.5\ ´\ \hat{}VR_y + \hat{}ibV_y)$$

The estimate may, perhaps, be improved by looking at the paid expense to paid claims ratio in years prior to the current year. An average can then be taken for the ratio, or a trend followed through, to produce a more reliable figure. For years affected by one or more catastrophes, the paid expense to paid claims ratio may need to be adjusted to reflect the fact that claims resulting from a castrophe receive less individual consideration than claims in normal circumstances and hence the expense may be a lower proportion of the claim cost.

Other methods for determining the indirect expense reserve will depend on a more detailed analysis of the incidence of expense. For example, claim payments might be identified as well as openings and closings as significant events in attracting costs. If such payments were more concentrated towards the closing date of a claim, then the 50% multiplier in the above formula would need to be increased by an apt amount.

On the question of allocating the indirect expense reserve among the different classes of business, this is a matter of finding a suitable weighting factor. Premium income might be suitable, but not necessarily. Reported and IBNR reserves could be used, as in the above formula. Alternatively, for private rather than commercial lines, a weighting based on numbers of claims reported and settled might be appropriate.

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[K5]
CLAIMS EXPENSE — DIRECT TYPE

The first question with direct expenses is as to whether they require separate recognition, or can be included along with the claims themselves. For most lines of business in the UK, the latter alternative is probably to be preferred, since it reduces the amount of work to be done. The principles are simply that all expense payments are counted as claim payments, and that an element for future expense is included when case reserves are set up.

Separate recognition of expenses is necessary only where the expenses are large in relation to claims, or where the expense pattern is changing in a markedly different way from the claims pattern. Such effects are more likely to occur in the long-tail lines, particularly liability business, where the legal costs of a long drawn out case can be heavy. This feature, of course, is exacerbated where North American business is concerned, and separate evaluation of claims expense may be vital.

General Methods

Given that data are available, it is possible to analyse expenses for their development by accident year, just as has been done for claims themselves. Thus completely separate projections can be made, using the normal claims methods already described. But a more favoured principle is to work on the relationship of expenses to claims, and to project the ratios found in past data. The ratios, of course, will not be constant, but provided they are reasonably stable for each development period the method will be valid.

One warning must be given. The expense/claims ratio will not be reliable in projections for direct expense if it is calculated on the calendar year basis only (i.e. as for indirect expenses, in §K4). The problem is that the characteristics of the group of claims settled during the year are very different from those of the group outstanding or IBNR at the end of the year. The settled group will contain a large proportion of early settling claims, and for these the direct expense/claims ratio is likely to be comparatively low. But the outstanding group at the year-end will have many more by proportion of late settling claims. For these the direct expense/claims ratio will be a good deal higher, thus completely invalidating the forecast.

Accident Year Projection

The safe method is therefore to use an accident year projection, and this is now illustrated by a worked example. To begin with, we give on the next page the usual paid claims data, together with a ready-worked out projection to the ultimate values. (The projection is taken from the result of the link-ratio method, as applied in §E6.)

					d			
		0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		4271
a	3	1265	2433	3233	3977			4947
	4	1490	2873	3880				5948
	5	1725	3261			**[*pC*]**		6626
	6	1889						7284

Next we require the expense data, in the same accident/development year format. The symbol *pE* denotes that the figures are for *paid* expense, while the value 320 in the *ult* column is an estimate, based if possible on data from earlier years.

					d			
		0	1	2	3	4	5	*ult*
	1	30	74	144	210	260	294	320
	2	36	88	165	250	309		
a	3	42	100	201	289			
	4	51	124	250				
	5	54	136			**[*pE*]**		
	6	63						

From the two tables, it is easy to calculate the expense/claims ratio at each point in the development. Thus, in the following table, 3.00 is 30/1001, 3.99 is 74/1855, ... each ratio being given as a percentage.

					d			
		0	1	2	3	4	5	*ult*
	1	3.00	3.99	5.94	7.03	7.80	8.44	8.64
	2	3.23	4.18	5.95	7.31	8.04		
a	3	3.32	4.11	6.22	7.27			
	4	3.42	4.32	6.44				
	5	3.13	4.17			**[*pE/pC*]**		
	6	3.34						

The ratios show reasonable stability down the columns, and the table can be projected by the link ratio method of §E. This is done below.

a \ *d*	0	1	2	3	4	5	*ult*
	1.330	1.489	1.184	1.110	1.082	1.024	
1	3.00	3.99	5.94	7.03	7.80	8.44	8.64
	1.294	1.423	1.229	1.100			
2	3.23	4.18	5.95	7.31	8.04		8.91
	1.238	1.513	1.169				
3	3.32	4.11	6.22	7.27			8.90
	1.263	1.491					
4	3.42	4.32	6.44				9.41
	1.332				**[*r*]**		
5	3.13	4.17			**[*pE/pC*]**		9.01
6	3.34						9.32
r	1.291	1.479	1.194	1.105	1.082	1.024	
f	2.790	2.161	1.461	1.224	1.108	1.024	

The result of the projection is to give an ultimate value for the expense/claims ratio for each succeeding accident year. It remains to apply these ratios to the projected claims figures, given in the first table above. The calculations are:

a	6	5	4	3	2	1
^*C-ult*	7284	6626	5948	4947	4271	3705
^*pE/pC*	9.32	9.01	9.41	8.90	8.91	8.64
^*E-ult*	679	597	560	440	381	320

Here, 679 is 9.32% of 7284 and so on. The final result for the expense projection is now given, by addition of the accident year figures. Symbol *pE** refers to the current values for paid expense, i.e. the data in the main diagonal of the *pE* triangle above.

Overall Values:	S^*E-ult*	2,977
	S*pE**	1,341

	Reserve	1,636	(55.0% of ^*E-ult*)
Comparison:	S^*C-ult*	32,781	
	S*pC**	20,334	
	Reserve	12,447	(38.0% of ^*C-ult*)

Comparison of the overall expense and claims figures is instructive. The proportion of the ultimate amount which needs to be held as a reserve is markedly higher in this example for expenses than for claims. This results simply from the fact that the expenses escalate more rapidly than the claims as development time increases.

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Section L
ACTUARIAL CONSIDERATIONS

Preamble

Actuarial methods have for a long time been at the heart of life assurance, providing the essential discipline and long-term financial control. But in the last three decades, it has been increasingly realised in the UK that actuarial methods have an important part to play in general insurance as well. Although the time span is for the most part shorter, the business is likewise concerned with the measurement and financial control of risk. The problems of risk assessment and reserving, of solvency and the release of surplus are present in equal measure as with the life side.

The more sophisticated aspects of actuarial practice in claims reserving are to be found in Volume 2 of the Manual. The present section takes as its main subject the financial question of the discounting of reserves. There is a short discussion of some pros and cons of discounting, but the aim is to raise pertinent questions rather than provide conclusive answers. Some practical examples are given, showing how to put discounting into practice, if and when it is required.

The other actuarial topic dealt with at this stage is the monitoring of the claims estimates. Every method which is used for the estimating job should be regularly checked for the stability and reliability it shows in practice. Such procedure is a necessity if claims reserving is to be properly and professionally done.

Contents

[L1]
THE ACTUARIAL APPROACH

What is the contribution which actuaries are able to make to claims reserving? It is partly a question of the attitude of mind, of the determination to take as impartial and scientific a view as is permitted by the available data. But it is also the bringing into play of the techniques and principles which are required for a numerate and disciplined approach. From his or her training, such techniques and principles will be adopted almost as second nature by the actuary. Four of the most prominent will be mentioned here.

a) Variance of the claims estimate
b) Use of explicit mathematical models
c) Discounting of future cash flows
d) Tracking the performance of the estimates

a) Variance of the Claims Estimate

It is apparent, even from the simplest projection methods, that a variety of values for the claims estimate can easily be produced. In any case, since future events are involved, there can be no final certainty of what the exact figure should be. It may, however, be possible to give a best estimate, and a range (or variance) about that estimate within which the true value is likely to fall. Formal statistical techniques may throw some light on the confidence limits of the range of values.

b) Explicit Mathematical Models

The methods of §§E-K are set out in terms of strictly numerical development. This is the natural way to start in the work of claims reserving. But the methods can generally be restated in mathematical terms, introducing formal assumptions, equations and parameters of various kinds. The numerical work then resolves itself into: i) estimating the parameters, and ii) making the projection using the estimates. The advantage of the mathematical statement is that it brings into the open the mechanics of any particular method. The method's assumptions can then more easily be questioned, and the circumstances in which it is valid can be more precisely defined.

The use of statistical method and explicit mathematical models puts claims reserving on to a formal and fully reasoned basis. It is an area in which actuaries are particularly well fitted to contribute, and indeed provides the main thrust for Volume 2 of the Manual.

c) Discounting of Future Cash Flows

In life assurance work, it is customary for actuaries to discount the values of future cash flows of the business by some appropriate rate of interest. The main point of so doing is to take account of the fact that investment income is earned by the life office, and is a very material factor in the financial operation. In general insurance, investment income is not so dominant as on the life side, but it is still extremely important in contributing to profitability especially for insurers writing long-tail business. The implication at first sight would be that claims reserves, which relate very much to future cash flows, should allow for discounting. In the UK, however, that is not the practice, and almost all general insurance reserves are stated at the undiscounted value.

The actuarial view helps to clarify the issue of the possible discounting of claims reserves, so that advantages and disadvantages can be seen in the fullest light. Actuarial methods also provide the means for calculating discounted claims reserves, whenever and wherever these are required.

d) Tracking the Performance of the Estimates

How well do the claims reserving methods in common use perform? It is not always enough to make a claims estimate one year and then forget about it the next. One should ask, how good was the estimate in the light of subsequent events? If it was an accurate figure, that tends to show that the method used was sound in the circumstances, and may be expected to continue to give reasonable results, other things being equal. But if the estimate was off-beam, then the position needs to be re-evaluated. Perhaps a different method should be used, or the old one should be retained, but adjusted in a suitable way.

The aim should be to analyse the errors which occur in the past estimates, and make adjustments in future according to the experience gained. The process can be called "tracking the performance", and can be seen as a form of adaptive control.

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[L2]

THE DISCOUNTING OF CLAIMS RESERVES

In the insurance industry, premiums are normally collected in advance of the period of risk for which cover is given. In addition, it takes time for claims to be reported, processed and settled. It follows that an appreciable period is likely to pass between the receipt of premium and payment of the corresponding claims. During this time, the insurance office will make use of the moneys received by investing them in various securities. Investment income therefore enters the picture, and indeed plays an integral role in the overall balance of insurance accounts, affecting the office's solvency, profitability and competitiveness in the market.

This fact of commercial life raises interesting questions when it comes to the setting of claims reserves. Income will be produced in the time until the settlement of outstanding claims, and perhaps should be recognised in advance of its receipt. If this is to be done, the appropriate means is to discount the claims reserves by the principles of compound interest. The reserves will then appear at a lower figure in the balance sheet than they would otherwise have done on a flat, or undiscounted, basis. The practice is not an alien one, since it is followed almost universally in life assurance and pensions work.

However, against this, standard practice for general insurance in the UK is to publish undiscounted reserves. Discounting is rare in published and statutory accounts, and appears to have been largely limited to a few cases in the reinsurance market. There is the point that while life insurance and pensions have very long liabilities to consider, much business in general insurance is short tail, say of duration two years or less. For such business, investment income is not a large factor, and it may seem scarcely worthwhile to introduce it into the reserving equation.

Discounting for the purpose of the published accounts and the returns to the supervisory authority is constrained by legislation introduced in the UK to implement the provisions of the 1991 EC Council Directive on the accounts of insurance undertakings. It is not permitted except for categories of claim where the average expected period from the accounting date to final settlement, weighted on the basis of expected gross claims, is at least four years, and even for such categories there are other conditions that must be satisfied. Under the legislation, implicit discounting is prohibited.

Some additional disclosure is required in the accounts and supervisory returns where discounting has been applied. This disclosure should cover the effect of discounting on the overall provisions, the categories of claims to which discounting has been applied, the assumed mean term of the discounted claim payments (and the method used to assess the pattern of those claim payments) and the rate of interest used in the discounting.

Points of View on Discounting

When reserves are discounted, their values will almost always be reduced. The insurance office which consistently uses discounting will operate on a less conservative basis, and its emergence of profit will be accelerated. The desirability of this depends on the point of view being taken.

If solvency is in question, then discounting in effect removes one of the safety margins in the business. Although an explicit solvency margin is required by law, undiscounted reserves will provide a further cushion against adverse experience in the future. But at the other end of the scale, when premium rates are at issue, and we are not considering the reserves for the published accounts, discounting, implicit or explicit, is required if competitive rates are to result. For general management purposes, say in assessing the relative profitability of different lines, discounting will as a rule give the truer and fairer view.

The ultimate questions relate to the quantum of capital which is needed for underwriting new risks, and to how far profit should be held up in deference to the needs of solvency. There are no absolute answers to such questions — the application of sound professional judgment is the only practical way forward.

Three Stages in Discounting

For practical purposes, three main steps have to be taken in order to produce discounted reserves. These are as follows:

a) Assess the flat, or undiscounted, value for the outstanding liability.
b) Choose an appropriate payment pattern over the future years of settlement.
c) Select a suitable discounting rate, and apply it to the payment pattern.

We look at these three steps in turn.

a) Assess Undiscounted Value of the Liability

The first step is to determine the flat, or undiscounted value for the claims reserve. This may be found in many different ways, a fair number of which have been described in the earlier sections of the Manual. The undiscounted value is likely to include allowance for claims inflation, but the point should be checked. Questions may arise, for example, over case estimates. These can be valued to include future inflation, or alternatively for immediate settlement excluding inflation. In the latter event, an explicit allowance for inflation should be brought in during the course of the calculations.

b) Choose Payment Pattern over Future Years of Settlement

The payment pattern may often arise naturally from the derivation of the undiscounted estimate, but this is not always the case. Projections of paid claims or of paid loss ratio easily yield the payment pattern, but those of incurred claims do not. Case estimates are another example where information on the pattern is not normally provided. In such cases, the solution may be to use a standard pattern, perhaps gained from a study of industry data relating to the line of business in question. Alternatively, if data are available, a separate projection of paid claims may be made.

Standard patterns are in fact prescribed in the USA for discounting purposes. But they are open to the objection that they encapsulate features of the past which may not be repeated in future. This is particularly the case with levels of inflation, which can have a strong secular variation. A further point is that standard patterns are the most difficult to derive for the longer tail lines, precisely those for which discounting has the greatest effect.

c) Select Interest Rate for Discounting and apply to Estimated Claims

The actual application of the interest rate for discounting to the payment pattern is straightforward, and need not detain us here — the method is shown in the example of the next section. However, the choice of the appropriate discounting rate raises some difficult questions.

The starting point should normally be the expected yield on investments over the term of the outstanding claims. Since a cautious view is likely to be taken, this yield will provide an upper limit for the discounting rate. By taking a lesser value, the reserver will build in an implicit margin of safety for the office.

The next point is on the identity of the investments themselves. The obvious choice would be the actual assets held by the insurance office, but these may not be suitable (it is a matter of debate how non-interest bearing assets should be treated). The foundation is to select such investments as would by their income and maturity values exactly match the claim liabilities both in amount and timing. The most likely choice is therefore a portfolio of gilts, with term chosen to match that of the outstanding claims. The yield on such a portfolio should be relatively easy to determine. It should be remembered, that, because exact matching is not possible, reinvestment problems with different yields will arise.

Finally, there is the matter of tax. We shall illustrate the discounting process by using an interest rate of 5% per annum. The main reason is that investment income used to meet claims or strengthen reserves is deductible for tax purposes. But the insurance office may lose tax benefits where it is in a loss-making position, and in this case a yield less than the full gross yield should be used for discounting.

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[L3]
PRACTICAL EXAMPLE OF DISCOUNTING

The preceding section gave the three main stages for obtaining discounted reserves. These were:

a) Assess the flat, or undiscounted, value for the outstanding liability.
b) Choose an apt payment pattern over the future years of settlement.
c) Choose a suitable interest rate for discounting, and apply it to the payment pattern.

We will now follow through these stages in a simple practical example.

a) Assess the Undiscounted Value for the Liability

We begin from the usual data on the paid claims, and carry out a link ratio projection (see §E5). The projection in this case must provide values for intermediate years as well as the ultimate position. The work therefore employs the one-step ratios r throughout rather than the final ones f.

		d 0	1	2	3	4	5	*ult*
	1	1001	1855	2423	2988	3335	3483	3705
	2	1113	2103	2774	3422	3844		
a	3	1265	2433	3233	3977			
	4	1490	2873	3880				
	5	1725	3261				**[*pC*]**	
	6	1889						

		0	1	2	3	4	5	*ult*
		1.853	1.306	1.233	1.116	1.044	1.064	
	1	1001	1855	2423	2988	3335	3483	3705
a		1.889	1.319	1.234	1.123			
	2	1113	2103	2774	3422	3844		
		1.923	1.329	1.230				
	3	1265	2433	3233	3977			
		1.928	1.351					
	4	1490	2873	3880				
		1.890					**[*r*]**	
	5	1725	3261				**[*pC*]**	
	6	1889						
	r	1.897	1.326	1.232	1.120	1.044	1.064	

The one-step ratios here are just the average of the values in the column above them. The ratios are now applied to generate the expected payments in future years, which appear in the table below. Explanation of the figures:

In the bottom row:

3583 = 1889 × 1.897; 4751 = 3583 × 1.326; 5853 = 4751× 1.232; etc.

In the next row up:

4324 = 3261 × 1.326; 5327 = 4324 × 1.232; etc through the table.

						d		
		0	1	2	3	4	5	*ult*
	1							3705
	2		**[*pC*]**				4013	4270
a	3					4454	4650	4948
	4				4780	5354	5590	5948
	5			4324	5327	5966	6229	6628
	6		3583	4751	5853	6555	6843	7281

Addition of the last column gives the estimated final loss, which is £32,780.

Paid claims to date are (as usual) £20,334. If these are deducted from the loss, we arrive at the undiscounted liability of £12,446.

b) Choose a Payment Pattern over Future Years of Settlement

In the present case, the step is simple, since the link ratio method on paid claims produces its own payment pattern. The figures in the table above are the cumulative values for paid claims as the years progress. Subtraction along the rows gives the year-by-year figures:

		0	1	2	3	4	5	*ult*
					d			
	1							222
	2		**[Δ*pC*]**				169	257
a	3					477	196	298
	4				900	574	236	358
	5			1063	1003	639	263	399
	6		1694	1168	1102	702	288	438

The payment pattern as a whole can now be obtained by adding the values along the diagonals. The sum of the top diagonal gives the amount to be paid in the year following the accounting date, the diagonal below that gives the year next following, and so on. We designate these years as t=1, t=2 ... ... The pattern is then:

t	1	2	3	4	5	6	7
^pC(t)	4525	3198	2275	1323	687	438	0
%	36.4	25.7	18.3	10.6	5.5	3.5	0.0

Overall Values: Σ *^pC(t)* 12446

There is a point of detail in that the payments in the *ult* column do not quite follow the main pattern, since they represent the tail of the run-off. We will suppose as before (in §J2) that these payments occur on average 18 months later than those for year d=5. The practical effect is that, for example, the first *ult* payment of 222 can be seen as divided equally between the years t=l and t=2. The second *ult* payment of 257 can be divided between the years t=2 and t=3, and so on. This requires only a simple adjustment to the figures, effected in the next table:

t	1	2	3	4	5	6	7
^pC(t)	4414	3180	2255	1293	667	418	219
%	35.4	25.5	18.1	10.4	5.4	3.4	1.8

Overall Value: Σ *^pC(t)* 12446

c) Choose Interest Rate for Discounting, and Apply to Payment Pattern

The third step can now be taken. We shall illustrate the discounting process by using an interest rate of 5% per annum. As seen in the preceding section, however, the choice of the rate is not an easy one, and should never be taken for granted. The reserver must weigh up the factors which bear on the situation, not least of which will be the purpose for which the reserve estimations are required.

To calculate the discounting factors themselves, and apply them, is a straightforward matter. An assumption has first to be made on how the claim payments will fall in each future year. Usually, it is adequate to assume an even spread, so that payments can be taken on average as falling at the midpoint of each year. With a 5% rate, the discounting factor for year t=1 will then be 1.025, since only half the year's earnings will be received by the time the average payment is due. Further payment points then follow at yearly intervals, so that succeeding factors are given by multiplying 1.025 by 1.05 the requisite number of times:

t	1	2	3	4	5	6	7
$\pi(1+h)$	1.025	1.076	1.130	1.187	1.246	1.308	1.373

The factors are labelled as $\pi(1+h)$. Here, π is used to denote a product, while h stands for the hypothetical rate of earnings on the fund.

The main discounting calculation can now follow. For each future year, the estimated payment $pC(t)$ is divided by the discounting factor $\pi(1+h)$. The resulting values are the amounts which should be set by as discounted reserves. According to the assumptions, if these reserves are invested at the accounting date and if they yield the required investment income, then estimated claims can be met exactly on their due dates.

t	1	2	3	4	5	6	7
$\hat{}pC(t)$	4414	3180	2255	1293	667	418	219
$\pi(1+h)$	1.025	1.076	1.130	1.187	1.246	1.308	1.373
Disct^{d}V	4306	2955	1996	1089	535	320	160

Overall Values:	Outstanding Claims	12,446
	Discounted Reserve	11,361

The 5% rate has here produced an appreciable reduction in the reserve required. It is useful to look at the sensitivity of the result, by evaluating it for a number of different rates of discount. This is done below:

Discount Rate	0%	2.5%	5%	7.5%	10%
Reserve	12,446	11.873	11,361	10,880	10,455

In selecting an appropriate rate for discounting the starting-point or benchmark could be a position where assets in risk-free investments are exactly matched to the run-off of liabilities. Gilts could be regarded as a first approximation but they do not cover currency and/or inflation risks; and in practice it is unlikely that they would match exactly by the term of run-off. Any deviation from that position leads to the consideration of a margin in the expected yield and capital growth; and a margin for mismatching of the timing. Whether such margins are introduced will depend upon the purpose of the discounting. The important point is that discounting has little meaning without reference to the actual or hypothecated assets. In §L4 the link between claims inflation and the discount rate (or investment return) is considered.

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[L4]
DISCOUNTING COMBINED WITH INFLATION

When using discounted reserves, it is prudent to check the relationship between the discounting and any claims inflation in the projection. As seen in §J inflation will often be implicit in the data, and projected forward at its past rate. Alternatively, the reserver may have put in an explicit inflation assumption. In either case, the assumptions should be examined for their consistency, since times of high inflation often coincide with those of high earnings on invested funds, and vice versa. It is difficult to generalise over this point, however, and the reserver should keep aware of the prevailing influences on both claims inflation and investment yields.

The remainder of this section deals with the mechanics of combining discounting and inflation allowances in the same projection, by means of a practical example. The data used are the usual paid claims data with inflation adjustment built in, and are taken directly from §J2.4.

		d						
		0	1	2	3	4	5	*ult*
	1	1540	2789	3555	4233	4608	4756	4949
	2	1628	2963	3768	4468	4890		
a	3	1705	3107	3972	4716			
	4	1788	3283	4290			**Adj.*pC***	
	5	1865	3401					
	6	1889						

These data are first projected using the link ratio method, averaging the factors down the columns:

	d	0	1	2	3	4	5	*ult*
		1.81115	1.275	1.191	1.089	1.032	1.041	
	1	40	2789	3555	4233	4608	4756	4949
		1.820	1.272	1.186	1.094			
	2	1628	2963	3768	4468	4890		
		1.822	1.278	1.187				
	3	1705	3107	3972	4716			
		1.836	1.307					
	4	1788	3283	4290				
a		1.824					**[*r*]**	
	5	1865	3401				**[Adj.*pC*]**	
	6	1889						
	r	1.823	1.283	1.188	1.092	1.032	1.041	

The ratios are used to project the claims for the various accident years, working along each row in turn:

	d	0	1	2	3	4	5	*ult*
	1							4949
	2		[Adj.*pC*]				5046	5253
a	3					5150	5315	5533
	4				5097	5566	5744	5980
	5			4363	5183	5660	5841	6080
	6		3444	4419	5250	5733	5916	6159

Addition of the *ult* column gives the estimate of the final loss, adjusted to the value of the present year's currency. This loss is £33,954. The paid claims to date, again in the same currency, are £23,942. (This value comes from adding the figures on the leading diagonal of the first data triangle above.) Subtraction of paid claims to date from the final loss gives the required reserve, which is £10,012.

This is the value which we now wish to adjust for i) inflation and ii) discounting of investment income. The first step in the adjustment is to find the year-by-year values for the future paid claims. As usual, this is done by subtracting along the rows of the above table:

		d						
		0	1	2	3	4	5	*ult*
	1							193
	2	**ıdj.Δ*pC*]**					156	207
a	3					434	165	218
	4				807	469	178	236
	5			962	820	477	181	239
	6		1555	975	831	483	183	243

Labelling future years beyond the accounting point as t=l, t=2, ...we can now build up the claim payments pattern. This is by adding the values along the diagonals in the above table (t=1 is the top diagonal, t=2 the second one, and so on):

t	1	2	3	4	5	6	7
^Adj.*pC(t)*	4107	2636	1704	900	422	243	0
%	41.1	26.3	17.0	9.0	4.2	2.4	0.0

Overall Value: Σ^Adj.*pC*(*t*) 10,012
(100%)

As in §L3, there is an adjustment to make for the payments estimated in the *ult* column. These payments are taken to occur 18 months later than those for year d=5. As a result, they need to be divided, for example so that half of the amount 193 falls in t=l and half in t=2. When this is done, the adjusted pattern is:

t	1	2	3	4	5	6	7
^Adj.*pC(t)*	4011	2629	1698	891	420	241	122
%	40.0	26.3	17.0	8.9	4.2	2.4	1.2

Overall Value: Σ^Adj.*pC(t)* 10,012
(100%)

Now we are ready to put inflation back into the payments. The future rate assumed will be 10%. This generates the following set of inflation multipliers:

t	1	2	3	4	5	6	7
$\pi(1+f)$	1.100	1.210	1.331	1.464	1.610	1.771	1.948

The multipliers, when applied to the payments pattern, give:

t	1	2	3	4	5	6	7
^Adj.$pC(t)$	4011	2629	1698	891	420	241	122
$\pi(1+j)$	1.100	1.210	1.331	1.464	1.610	1.771	1.948
^$pC(t)$	4412	3181	2260	1304	676	427	238

Overall Value: Σ^$pC(t)$ 12,498

The inflated payments now have to be discounted back by an appropriate rate, to allow for investment income. If 5% is chosen, as in §L3, the same set of discounting factors can be used. The result of the operation is:

t	1	2	3	4	5	6	7
^$pC(t)$	4412	3181	2260	1304	676	427	238
$\pi(1+h)$	1.025	1.076	1.130	1.187	1.246	1.308	1.373
Disct^{d}V	4304	2956	2000	1099	543	326	173

Overall Values: Outstanding Claims 12,498
Discounted Reserve 11,401

The result of discounting, as is often the case, is a substantial reduction in the required reserve. The assumptions in the projection are conservative, nonethe-less, and inflation has been allowed a more powerful influence than the invest-ment yield. At times, however, the reserver may wish to allow for a positive return on investments, net of any inflation of claims. If, for example, the above figures were reworked with inflation at 5% and investment yield at 7.5%, the reserve estimate would be reduced to £9,845.

It is well worth repeating the calculations for a number of different inflation and investment assumptions, and this has been done below.

		Discounting Rate (=Hypothetical Investment Yield)				
		0%	2.5%	5%	7.5%	10%
	0%	10,012	9,569	9,220	8,867	8,550
Inflation	5%	11,192	10,700	10,258	9,845	9,476
	10%	12,498	11,920	11,401	10,915	10,487
	15%	13,959	13,280	12,672	12,107	11,603

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[L5]
TRACKING THE PERFORMANCE

This section is a brief introduction to a large subject. The fact is that a claims estimate, once made, cannot be regarded as in any way sacrosanct. Future events are more than likely to prove it lacking in some respect. But that does not mean that the past estimates made in previous years should be forgotten. It is all too easy to consign them to insignificance in the archives when it comes to making the new set of reserves for the ending of the current accounting period. But much can be learned from the errors and inconsistencies in past estimates when compared with the latest set of data available to the reserver. In particular, the analysis of such errors can tell us more about the data in hand, and about the relative reliability of the different reserving methods under different conditions. In brief, the aim should be to track the performance of the past estimates, and take account of the information gained in setting the current reserves.

What follows in this section is a simple numerical example by way of a first illustration. The interpretation of the results, however, will not always be as straightforward as in the case shown here. It is an aspect of reserving which greatly needs a more formal, mathematical approach, although comparatively little seems to have been done on this to date.

Worked Example

The example uses figures from §L3, where paid claims were projected by the link ratio method. (The aim in §L3 was to illustrate the effects of discounting the claims estimates, but that aspect is not relevant here.) The result of the projection, discounting apart, was to produce the following figures for the ultimate losses (see §L3.2):

a	6	5	4	3	2	1
^L-ult	7281	6628	5948	4948	4270	3705

Overall Values:	Σ*^L-ult*	32,780
	Σ*pC**	20,334
	Reserve	12,446

A further result was to provide year-by-year figures for the projected paid claims, as in the table overleaf:

		d						
		0	1	2	3	4	5	*ult*
	1							222
	2		**[Δ*pC*]**				169	257
a	3					477	196	298
	4				900	574	236	358
	5			1063	1003	639	263	399
	6		1694	1168	1102	702	288	438

Now let us examine the position a year later. We suppose that the following paid claims are actually recorded during the year, the breakdown being given by accident year as usual:

a	7	6	5	4	3	2	1
Δ*pC*	2023	1850	1146	810	428	163	105

These values enable the new totals of paid claims to date to be found (time 0 in *pC**(0) denotes the original position, time 1 the new, current postion):

a	7	6	5	4	3	2	1
*pC**(0)	-	1889	3261	3880	3977	3844	3483
Δ*pC*	2023	1850	1146	810	428	163	105
*pC**(1)	2023	3739	4407	4690	4405	4007	3588

Also, we may compare the estimates that were made under the link ratio method with the actual, emerging figures:

a	7	6	5	4	3	2	1
Estimate	-	1694	1063	900	477	169	111
Δ*pC*	2023	1850	1146	810	428	163	105
Deviation	-	+9.2%	+7.8%	-10.0%	-10.3%	-3.6%	-5.4%

Sum for years *a*=1 to 6:	Estimate	4304
	Δ*pC*	4397
	Deviation	+2.2%

(Note: 111 appears as the estimate for year *a*=l, since only half the tail of 222 is expected to be realised during the year in question. The deviation figures show the % increase or decrease of the actual figures over the estimates).

What the table shows is very clear: the method of projection has underestimated the payments for the two recent accident years, *a*=5,6; and it has overestimated payments for the earlier years *a*=1,2,3,4. The evidence is that a change in the claim development pattern is taking place, with a particular shift between years *a*=4 and

5. However, the evidence is not conclusive, since there are still comparatively few items of data on the years from a=5 onwards. Fortunately, for the projection as a whole, the deviations tend to be self-cancelling. But the warning sign is there, and must be heeded by the reserver — it suggests that an adjustment to the projection may be needed.

To develop the analysis further, it is useful to recast the whole projection for the current date. The new projection will be made for the years a=2 to 7, ignoring any small variation which may be indicated in the tail values for a=1. First, we shall set up the paid claims triangle, by reference to the original one (given on § L3.1). The new values required appear in the table on the previous page as the pC^*(l) row. They provide the new leading diagonal for the triangle, whose updated version is:

		d						
		0	1	2	3	4	5	*ult*
	2	1113	2103	2774	3422	3844	4007	4263
	3	1265	2433	3233	3977	4405		
a	4	1490	2873	3880	4690		[p*C*]	
	5	1725	3261	4407				
	6	1889	3739					
	7	2023						

Here, the value 4263 in the *ult* column has been estimated as bearing the same proportion to the d=5 value as 3705 did in the earlier triangle. We now apply the link ratio method to the triangle in the usual way:

		d 0	1	2	3	4	5	*ult*
		1.88911	1.319	1.234	1.123	1.042	1.064	
	1	13	2103	2774	3422	3844	4007	4263
		1.923	1.329	1.230	1.108			
	2	1265	2433	3233	3977	4405		4885
		1.928	1.351	1.209				
a	3	1490	2873	3880	4690			5806
		1.890	1.351					
	4	1725	3261	4407				6677
		1.97918				**[*r*]**		
	5	89	3739			**[*pC*]**		7579
	6	2023						7882
	r	1.922	1.338	1.224	1.116	1.042	1.064	
	f	3.896	2.027	1.515	1.238	1.109	1.064	

Overall Values:	Σ*L-ult*	37,092
	Σ*pC**	23,271
	Reserve	13,821

We can now compare the results of the original and the new projections of the ultimate loss, on an accident year basis:

a	7	6	5	4	3	2	1
^L-ult(0)	-	7281	6628	5948	4948	4270	3705
^L-ult(1)	7882	7579	6677	5806	4885	4263	-
Shift %	-	+4.1	+0.7	-2.4	-1.3	-0.2%	-

The figures confirm the earlier picture, of an underestimation for the later accident years, and an overestimation for the others. But the information becomes even more useful when the loss ratios for the two projections are calculated. This is done below, using the earned premium figures, *aP*.

Original Projection:

a	7	6	5	4	3	2	1
^*L-ult(0)*	-	7281	6628	5948	4948	4270	3705
aP	-	8502	7482	6590	5680	5024	4486
^λ	-	85.6	88.6	90.3	87.1	85.0	82.6%

New Projection:

a	7	6	5	4	3	2	1
^*L-ult(1)*	7882	7579	6677	5806	4885	4263	-
aP	9211	8502	7482	6590	5680	5024	-
^λ	85.6	89.1	89.2	88.1	86.0	84.9%	-

Comparison:

a	7	6	5	4	3	2	1
^λ*(0)*	-	85.6	8.6	90.3	87.1	85.0	82.6%
^λ*(1)*	85.6	89.1	89.2	88.1	86.0	84.9%	
Shift	-	+3.5	+0.6	-2.2	-1.1	-0.1%	-
							-

The message coming through in these figures is clear: for the older accident years, there is a slight but welcome reduction in the estimated loss ratio. But for the latest accident year, the method is appreciably undervaluing the liability, owing to a change in the claim settlement pattern. It will be prudent to increase the estimate for this year (*a*=7), to reflect an anticipated loss ratio of at least 89%.

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Section M
TOWARDS A FORMALISED APPROACH

Preamble

To this point, the Manual has not gone into a full statistical treatment of claims reserving, and mathematical models have not been explicitly used. But it will be apparent that the subject is a complex one, and that many different concepts and quantities have to be manipulated. The discussion, however, has been on a rather ad hoc basis, without any fuller systematisation. Yet such a systematisation would have its uses, and would help to tie together the many strands that make up general insurance reserving. In particular, it could help to show the relationships which the many methods bear to each other, and so assist in the choice of an appropriate method in particular circumstances.

For these reasons, the present section attempts a more formal approach to the subject, and puts forward some of the elements necessary for a systematised view.

There is a main foundation to the development. It is simply the basic analysis of all claims into three main states: settled, open and IBNR. As time progresses, of course, claims move between these states. But at any accounting or review point, the division can be made. It is of basic relevance to the reserving process — although it is not always possible to make the full division, particularly in reinsurance work. In such cases, substitute measures may be used, whose standing can be assessed through the basic analysis put forward here.

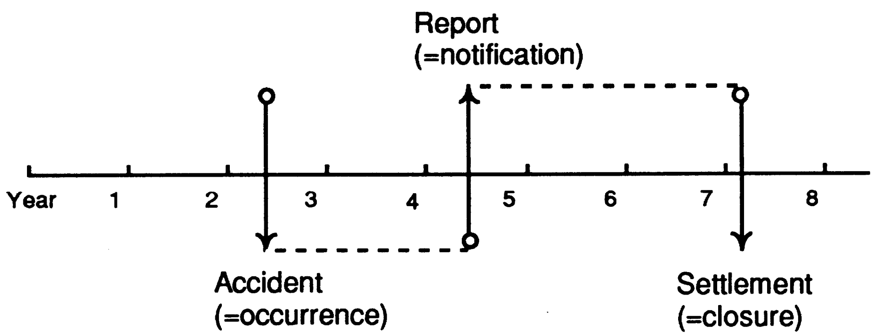
The section also has the function of systematising the notation which is used in Volume 1 of the Manual. The notation is not standard, and could not be, since there is no generally accepted standard notation in General Insurance. But there is a need to be able to express the quantities and ideas which come into reserving in a compact and precise way. The notation should therefore be seen as an attempt towards producing an acceptable algebra for claims reserving.

Contents

M12. Claim Development & Trend Analysis

[M1]
THE HISTORY OF A CLAIM

We begin by considering the simple history of an individual claim. This can be



shown diagrammatically along a time axis:

In the example shown, the accident or occurrence giving rise to the claim occurs in year 3. The claim is then reported to the insurer in year 5, and settled in year 8. (The alternative terms "notification" and "closure" may also be used for "reporting" and "settlement".) Although the scale is given in years, months or quarters or some other unit could equally be used.

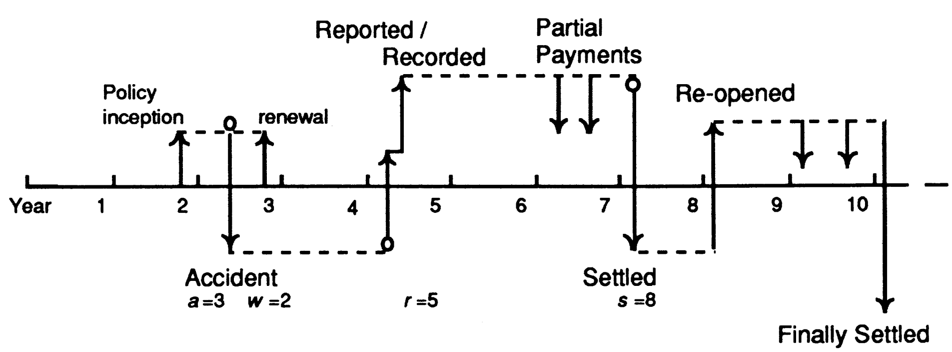
Using a to denote the accident year, r for report year and s for settlement year, the relation: $a \le r \le s$ can be seen to hold.

Over the time period in question, the *state* of the claim changes as follows:

Between the accident date and reporting dates, the claim is said to be "incurred but not reported", or IBNR for short. Once reported, the claim becomes an open one in the insurer's records, until it is fully settled with the claimant. In the settled state, the claim file will be kept for some further period, until finally archived or destroyed.

Detailed History

The above simple history gives the three basic states for claims, on which reserving analysis will depend. But there is a fuller story to be told (a good account is given in the first section of Ackman, Green & Young, 1985.) At each stage complications arise, which are shown diagrammatically below:



Accident/Occurrence

Each claim can be classified by underwriting year (i.e. the year in which the risk commences) as an alternative to accident year. If w denotes the underwriting year, then the relation with a) is:

$$w = a \quad \text{or} \quad w = a - 1$$

Reporting/Recording

We can distinguish:

a) Date of report to office by claimant, say r'
b) Date record reaches office's central files, say r''

For reserving purposes, general practice is to use the latter, r''. The term "reported" thus usually means "recorded on file", and is so taken in the Manual.

Settlement

We can distinguish:

a) Date claim is first considered settled, say s'
b) Date claim is finally settled, say s''

The problem here is that a settled claim can be re-opened by the claimant. In the Manual, "settled" normally indicates that at least a first-time settlement has taken place, i.e. it uses s' for the definition.

Thus, many potential complications exist. The important point, for claims reserving purposes, is that the fundamental classification of claims is still a 3-fold one:

{ IBNR Claims ^{3} Open Claims ^{3} Settled Claims }

Each of the groups will have its own requirement for reserves, since settled claims can be re-opened. According to the method chosen, the reserves can be estimated separately, or en bloc, or by some other combination. (See §§ M4, M5.)

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[M2]
THE CLAIMS COHORT

For reserving purposes, claims will first be divided according to class and subdivision of business, and geographical territory. Examples would be:

Motor/Non-comp/UK
Liability/Medical Malpractice/USA
Proportional Reinsurance/Aviation/Europe etc.

(The main classes of business are described briefly in §§ A2,A3 above.)

Within each grouping so obtained, it is often necessary and desirable to divide the claims according to their year of origin. The divisions are then known as cohorts, and for these symbol $\{C\}$ will be used:

$\{C\}$ = Cohort of Claims

The origin chosen for cohorts is commonly accident year a or underwriting year w, but report year r can also be used. The resulting cohorts can be distinguished by use of a suffix:

$\{C_a\}$ = Accident Year Cohort
$\{C_r\}$ = Report Year Cohort
$\{C_w\}$ = Underwriting Year Cohort

If report year is used, then the year of occurrence is disregarded in the classification. However, a 2-dimensional classification of report year within accident year can be used especially in analysing IBNR claims. This multiplies the work of reserving, but adds detail.

In the Manual, accident year has been taken as the norm, but there is reference to the other types as well. Report year cohorts in particular give rise to significantly different methods of reserving.

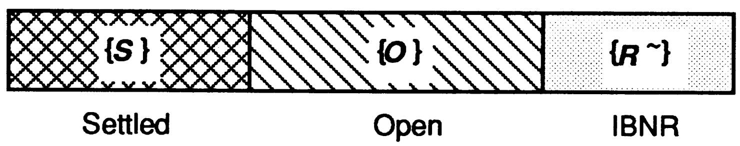
Primary Division of the Cohort

The claims in the cohort develop over time. At any point in time from the end of the year of origin onwards, each claim will be, by the analysis of §M1, either settled, open or IBNR. The claims in the cohort can thus be analysed into three main groups:

$$\{C\} = \{ S \mid O \mid R^{\sim} \}$$

where: $\{S\}$ denotes the subgroup of settled claims
$\{O\}$ ... ... open claims
$\{R\sim\}$ ... ... IBNR claims

This primary division is not static, and varies according to the moment at which it is made. In most cases, it will be made as at the end of a given accounting period — a year or a quarter. It can also be made at an interim review period, say a quarter or a month. A block diagram can be drawn to represent the analysis:



Development over time of individual claims will, in general, be from {R$^{\sim}$} to $\{O\}$ to $\{S\}$.

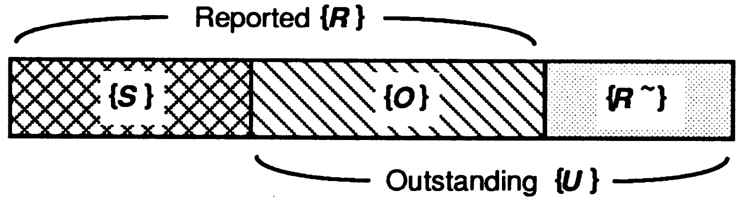
Apart from the primary subgroups of claims, it is also useful to be able to refer to combinations of these:

Reported Claims = Open claims + Settled claims
Outstanding Claims = IBNR claims + Open claims

Symbolically:

$$\{R\} = \{O\} + \{S\}$$
$$\{U\} = \{R^{\sim}\} + \{O\}$$

The full picture in the block diagram becomes:



There is an ambiguity in the term "outstanding claims". As well as being used for (open + IBNR) claims, as here, it is sometimes used as a synonym for open claims per se. This is perhaps a matter of taste. The main point is to be clear as to which set of claims is being referred to at any given time.

The term "IBNR" can also have different definitions. (See §I1.) The usage in this Manual is the most common one, i.e. IBNR claims are those for which liability has been incurred at the reserving date, but which have not *by that time* been reported (i.e. recorded on the insurer's central files).

Claims at Nil Cost

A further division of claims is that of the settled group, $\{S\}$. A claim may be settled at nil cost, i.e. eventually dismissed because it is shown to be valueless, wrongly made, or even fraudulent. Otherwise, it will be settled at some positive cost to the insurer. Symbolically:

$$\{S\} = \{S^{\circ}\} + \{S^{+}\}$$

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[M3]
CLAIM NUMBERS & CLAIM AMOUNTS

Given that the cohort $\{C\}$ of claims is defined, we shall be interested in the numbers of claims within the cohort, and the losses that are incurred on their account.

Numbers of Claims

At any review point or accounting date, the numbers of claims in the sets $\{S\}$ and $\{O\}$ can be decided from the office's main files (i.e. leaving aside any technical or data processing problems which may arise in interrogating the database). Symbol n will generally be used for claim numbers:

Number of claims in $\{S\}$ = nS "number settled"
Number of claims in $\{O\}$ = nO "number open"

In normal circumstances (at least, for a direct writing office), these values can be taken as known. But the number of claims which are IBNR by definition cannot be known and must be estimated:

$$\text{Estimated no. of claims in } \{\mathrm{R}^{\sim}\} = \hat{}n\mathrm{R}^{\sim}$$

If the true number is $nR^{\sim}$, the relation will hold that:

$$nS + nO + n\mathrm{R}^{\sim} = nC$$

where nC is the number in the whole cohort $\{C\}$. Ultimately, as time continues, all claims come to fruition, and the exact value of nC becomes known. It can thus be designated as the ultimate number, or *n-ult*.

In addition, there is the subsidiary relation for $\{S\}$:

$$nS = nS^{O} + nS^{+}$$

where nS^{O}, nS^{+} denote the numbers of claims settled without payment and with payment respectively.

Claims Amounts

In reserving, a general aim is to estimate the total claim amounts paid from the various claim cohorts, and hence from all cohorts together. For cohort $\{C\}$, we define the total of claims paid out, in past and future together, as the value *L-ult* (the ultimate loss).

At any accounting or review date, $\{C\}$ can be considered as broken into its three main elements:

$$\{C\} = \{S \mid O \mid R^{\sim}\}$$

Hence the value *L-ult* can equally be analysed into losses on settled, open and IBNR claims. Until the end of the development is reached, none of these values is known with absolute certainty (even settled claims can be re-opened). All may therefore have to be estimated. Symbolically:

Incurred claims on $\{S\}$ = $\hat{}iS$
Incurred claims on $\{O\}$ = $\hat{}iO$
Incurred claims on $\{R^{\sim}\}$ = $\hat{}iR^{\sim}$

Symbol i is used to denote the incurred claim values. The above formulae thus read as the amounts "incurred on settled claims", "incurred on open claims", and "incurred on IBNR claims".

True values for these 3 will yield:

$$iS + iO + iR^{\sim} = L\text{-}ult$$

i.e. the full losses on the cohort. These true values will be exactly known only at the end of the development, at which point $iS = L\text{-}ult$

As a first simple approach to the estimates for settled and open claims, it may be possible to use the following:

pC = paid losses to date on claims in $\{C\}$
kC = total of case estimates on $\{C\}$ at the reserving date

Here, pC will estimate iS, the incurred loss on settled claims, while kC will estimate iO, the loss on open claims. But the estimates will depart from the truth, since:

a) pC does not allow for re-opens on settled claims, but includes part payments on open claims.
b) kC does not allow for future development of the case estimates.

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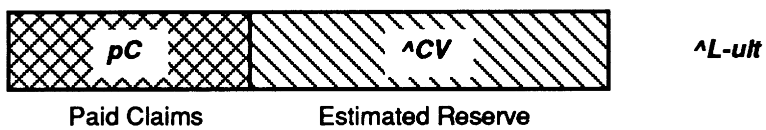
[M4]
OVERALL LOSS & THE CLAIMS RESERVE

The diagram shows the relationship between the overall loss *L-ult* and the claims reserve itself, *CV*:



Here, the reserve is shown as the estimated one, ^*CV*, and the overall loss ^*L-ult* is also the estimated value. The true values *CV* and *L-ult* could be substituted in the diagram.

The diagram can represent the overall picture, i.e. for all claims together, or the position for a given cohort {*C*}

Algebraically,

$$\hat{}L\text{-}ult \;=\; pC + \hat{}CV$$

This equality shows that there can be two distinct approaches to the overall question of the claims reserve:

a) Estimate the overall loss, *L-ult.* Then derive the reserve as:

$$\hat{}CV = \hat{}L\text{-}ult - pC^*$$

where pC^* denotes the paid claims to date.

b) Estimate the required reserve directly, and derive the overall loss from it:

$$\hat{}L\text{-}ult = \hat{}CV + pC^*$$

Either approach can be used, and the choice will often fall out automatically from the particular method used for reserving. It is always good, however, to be clear about the route which is being taken. The choice of routes implies an important distinction between two different approaches to a situation in which the claims paid progress at a higher level than previously anticipated. One approach is to assume that *L-ult* will remain the same and hence that the level of claims paid in the remainder of the cohort will be correspondingly less than expected. The alternative assumption is that the higher level of claims paid implies an increase in *L-ult*.

The overall loss/claims reserve identity can be further expanded:

$$\begin{aligned}\hat{}CV &= \hat{}L\text{-}ult - pC^* \\ &= \hat{}iS + \hat{}iO + \hat{}iR^{\sim} - pS^* - pO^*\end{aligned}$$

Here, iS, iO, $iR^{\sim}$ are the incurred amounts on the settled, open and IBNR claims. Then pS^* denotes the paid amounts on settled claims, and pO^* the partial payments on the open claims, both to the present date.

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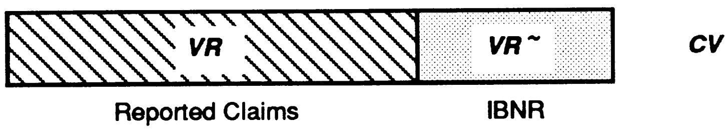
[M5]
PRIMARY DIVISION OF THE CLAIMS RESERVE

The primary division of the claims reserve *CV* is into the reserve for reported claims, and the IBNR reserve:



Algebraically,

$$CV = VR + \text{VR}^{\sim}$$

The equation can apply to claims as a whole, or to particular cohorts of claims. Further to the work of §M4, this analysis again shows that the reserving problem can be tackled from different perspectives:

a) Estimate the overall reserve and the IBNR separately, then find the reported claims as:

$$\hat{}VR = \hat{}CV - \hat{}ibV$$

b) Estimate the overall reserve and the reported claims separately, then find the IBNR reserve as:

$$\hat{}ibV = \hat{}CV - \hat{}VR$$

c) Estimate reported claims and IBNR separately, then find the overall reserve as:

$$\hat{\ } CV = \hat{\ } VR + \hat{\ } ibV$$

Note: In the above, the symbol ibV is used synonymously with $VR^{\sim}$ to denote the IBNR reserve.

The Accident Year vs. Report Year Comparison

As an example, consider the difference when using report year data as against accident year data.

Accident Year

1) A projection of the paid claims to the ultimate value *L-ult* will give an estimate for the overall loss (i.e. for all claims in the cohort).
2) The deduction of pC^* (paid claims to date) will give the required overall estimate for the claims reserve CV.
3) Further deduction of $\hat{}ibV$ (the estimated IBNR) for the cohort will then enable the reserve VR to be found.

Report Year

1. Projection of paid claims will give *L-ult* for the cohort. By definition, only claims already reported are members of $\{C_r\}$.
2. Deduction of pC^* will give the estimate of the reserve for reported claims, namely $\hat{}VR$.
3. There is no group of IBNR claims which can be identified with the specific cohort $\{C_r\}$. But there will be IBNR claims for the class of business as a whole. These claims can be estimated separately, after which the overall reserve for the class follows as $\hat{}VR + \hat{}ibV$.

Again, as with the relationship between overall loss and the claims reserve, it is useful to be clear which general approach is being used. Reserving methods will naturally proceed by one route or another to build up the full picture.

IBNR & IBNER

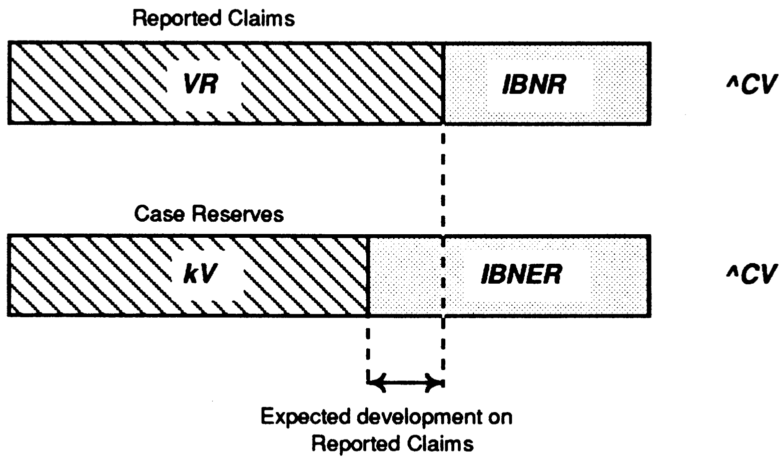
There is a further way of dividing the estimate of the overall claims reserve. This leads to the quantity IBNER, standing for: "Incurred but Not *Enough* Reserved". The term can be confusing, since the initials are so similar to IBNR itself. One derivation of IBNER, as used in the Manual is as follows. In dividing the overall claims reserve, case reserves can be used to estimate reported claims, and then the IBNR reserve found by deduction, i.e.

$$\hat{}VR = kV$$
$$\hat{}ibV = \hat{}CV - kV$$

Strictly speaking, this is satisfactory if case reserves can be shown to be a reliable estimate for reported claims. But usually further development on reported claims would be expected beyond the case value. To recognise this, the IBNR reserve found in the above way is given the name IBNER instead.

Thus:

IBNER = IBNR + Expected development on reported claims beyond the current value of case reserves



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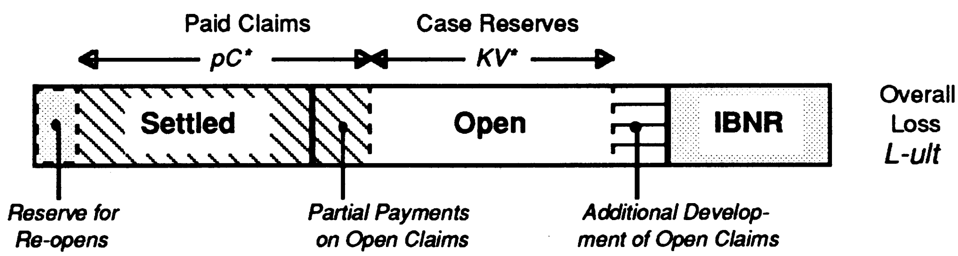
[M6]
THE FULL ANALYSIS OF LOSS

At this point, we summarise the relationships between all the main quantities which go to make up the overall loss. One new element to be introduced here is the reserve for re-opened claims (i.e. possible re-opens of claims which have been settled already). A diagram may be helpful:



To summarise the analysis:

$$L\text{-}ult = iS + iO + iR^{\sim}$$

or, in words, the overall loss is the sum of incurred amounts on the settled, open and IBNR claims.

At the further level of detail, this leads to:

$$\begin{aligned} iS &= pS + VS \\ iO &= pO + kV + \Delta O \\ iR^{\sim} &= iR^{\sim} \end{aligned}$$

or, in words:

Incurred amount on settled claims	= Amounts paid on such claims + Reserve for possible re-opens (*VS*)
Incurred amount on open claims	= Partial payments on such claims + Case reserves + Additional development

Incurred amount on IBNR claims = (No further breakdown)

(The symbol ΔO is used here to denote the additional development on open claims, i.e. beyond the current value of the case estimates.)

Alternative Breakdown

The full analysis in its above form is not often used in claims reserving. It is more common to concentrate on paid claims, incurred claims and the remainder (i.e. IBNER). These quantities can, however, be related to the full analysis:

$$
\begin{aligned}
pC &= pS + pO \\
iC &= pC + kV \\
IBNER &= iR^{\sim} + \Delta O + VS
\end{aligned}
$$

or, in words:

Paid claims = Amounts paid on settled claims + Partial payments on open claims
Incurred claims = Paid claims + Current value of case reserves
IBNER = IBNR + Additional development on open claims + Reserve for re-opens

A point to bear in mind through the above analysis is that we are considering the claims quantities as they refer to a given year of origin (accident, underwriting or report year). Where an accounting year is in question in the sense used by companies (as opposed to a Lloyd's "year of account") the analysis does not apply in the same way. For example, the incurred claims on the year are equal to paid claims plus the *increase* in total case reserves over the year.

(Refer to §F2 and §I1 for further discussion of the incurred claims function and the variations within IBNR.)

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[M7]
AVERAGE COST PER CLAIM

Data may not always be available on the number of claims, particularly in reinsurance work. But if they are, values for average cost per claim can be developed. These add another dimension to claims estimating, i.e. beyond the use of claim amounts alone, and can improve the picture which is obtained a great deal.

In general, at the end of the development, the average cost per claim will be:

$$A\text{-}ult = L\text{-}ult \,/\, n\text{-}ult$$

This might apply to a whole class of business, or to a cohort. But average claims can also be defined for the subgroups within the cohort. Thus:

Settled claims:	$AS = iS/nS$
Open claims:	$AO = iO/nO$
IBNR claims:	$AR^{\sim} = iR^{\sim}/nR^{\sim}$

The above averages should be considered as holding for the subgroups of the cohort as at a particular review date. The question of development in time will be considered later. (See §§M9,M10.)

If average claims can be estimated in any of the three subgroups of claims, then losses as a whole for that group can be estimated:

Settled	$\hat{}iS = nS \times \hat{}AS$
Open	$\hat{}iO = nO \times \hat{}AO$
IBNR	$\hat{}iR^{\sim} = \hat{}nR^{\sim} \times AR^{\sim}$

(Note that nS, nO should be known, but $nR^{\sim}$ must be estimated.)

Reported and Outstanding Averages

Averages can also be developed for reported claims as a whole, and for outstanding claims:

Reported claims $AR = iR / nR$
$= (iS + iO) / (nS + nO)$

Outstanding claims $AU = iU / nU$
$= (iO + iR^{\sim}) / (nO + nR^{\sim})$

Thus, if means can be found for estimating AR or AU, their values can be used towards the loss estimates $\hat{}iR$ and $\hat{}iU$.

Paid and Incurred Averages

Two important averages are the paid and incurred averages. These will be defined as follows:

Paid claims $pA = pC / nS$
Incurred claims $iA = iC / nR$

These are the most practical measures, for which data will frequently be avail-able. The problem is that they are not pure measures. The denominators show their relationship to the settled and reported groups of claims respectively. Pure measures would therefore be obtained (by reference to diagram in §M6) as:

Paid claims $pA = (pS + VS) / nS$
Incurred claims $iA = (iC + \Delta O + VS) / nR$

In practice, the reserve for re-opens (VS) will often be ignored, or taken as part of the IBNR liability. The development on open claims (ΔO) will be included in the incurred claims, if adjusted case reserves are used. Alternatively, it may be left to come out in the IBNR term. Perhaps the more serious objection is the use of the ratio pC/nS in the first paid average definition above. Since:

$$pC = pS + pO$$

the partial payments on open claims (pO) will distort the average. Where these partial payments are small, however, the definition will serve as a reasonable approximation to the true average.

Caveat

As seen above, many different "average claims" can be defined and used in reserving. Indeed, in the literature, quite different methods for reserving can occur under the generic title of average claim method. Hence when speaking of the "average cost per claim" (or "average claim" for short), it is important to be clear to which set of claims precisely the average refers.

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[M8]
EXPOSURE MEASURES & LOSS RATIO

Referring to the claims cohort, there is a need for some base measure of it, or rather of the business from which it results. Actuaries customarily employ the expression "exposed-to-risk" to describe the base measure to which claims are related. The most obvious measure is the premium income. Others would be the total of sums assured on the written business, or the number of policy units.

The term "policy unit" may need further definition. It means a policy in force for a year, whether the policy year itself, the accident year or some other 12-month period. Some policies contributing to the claims cohort will have exposure only for a part year, and therefore be counted as a part unit only. The policy unit can also be replaced more graphically by a risk exposure such as the number of motor cars, or dwellings, or individuals covered under the business in question.

Whatever the choice, it will give the base measure X for the cohort. If, say, X is premium income (special symbol P) this will relate to the cohort definition itself:

$\{C_a\}$	Accident year cohort	X_a will be earned premium, aP
$\{C_w\}$	Underwriting year cohort	X_w will be written premium, wP
$\{C_r\}$	Report year cohort	No definition available

The use of report year data has the disadvantage of not allowing any meaningful exposure measure to be developed.

Loss Ratio

If the ultimate losses for a class of business, or a cohort, are known, then its loss ratio can be calculated:

$$\lambda = L\text{-}ult \,/\, P$$

If it can be shown that λ is stable for a given class of business over the years, then it can be used for estimating losses on that class for the current set of claims:

$$\hat{}L\text{-}ult = \lambda \times P$$

where λ is the loss ratio for the given set of claims.

If the loss ratio is not fully stable, but thought to fluctuate about a mean value with a moderate variance (or to be subject to an identifiable trend), then the product of the mean with the premium income can still be used as a first estimate of losses. This will give a target value, from which deviations can be checked as the years of development pass by. <>

[M9]
TIME AXES

The analysis so far has been relatively static i.e. as at a particular review date. The next step is to bring in the time dimension explicitly.

For a given cohort, there is a fixed reference year — whether accident, underwriting or report year. The chief time axis will then be the development time or period which has passed since the reference year itself. This is to be measured as:

$$d = 0, 1, 2, 3, \ldots\ldots \; ult$$

Here, $d = 1, 2, 3 \ldots$ will be taken to count the development years *subsequent* to the reference year. $d = 0$ then conveniently denotes the reference year itself. $d = u$ or *ult* gives the ultimate development period, i.e. that time after which it is known (or can be taken for practical purposes) that no further development of the cohort of claims will occur. True values for *L-ult, n-ult* and *A-ult* can be established at this time, but not with absolute certainty beforehand.

The whole period from inception of the reference year to end of the ultimate year has length $(u + 1)$.

Other Time Axes

Apart from development time, two other time axes are relevant:

a) Accident year (or other origin year for cohort)
b) Calendar year

Using the symbols *a* and *y* respectively, and measuring from a base year, the diagram shows the relationship:

In all cases:

$$a + d = y$$

or:	Accident Year	+	Development Year	=	Calendar Year
e.g.	$a = 5$		$d = 2$		$y = 7$
	$a = 3$		$d = 4$		$y = 7$

This is a useful relationship. It is gained by using the convention of starting the development year count at $d = 0$ rather than $d = 1$.

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[M10]
DEVELOPMENT OF CLAIMS

We now introduce a fundamental idea in the work. Very many of the claim reserving methods are based upon it. It is that:

> "There exists some recognisable pattern to claim amounts, claim numbers and/or average claims as development time progresses."

If the idea has some truth in it, then it will be worth examining the quantities C, *n, A,* considered essentially as functions of *d.*

$$C = C(d) n = n(d) A = A(d)$$

The pattern which is found (or perhaps disallowed) will depend on which subgroup of the cohort $\{C\}$ is examined, i.e.:

Settled claims $\{S\}$ Open claims $\{O\}$ Ibnr claims $\{R^{\sim}\}$
Reported claims $\{R\}$ Outstanding claims $\{U\}$

However, the actual functions chosen for analysis must depend on what data are to hand for reserving purposes.

Paid Claims Development

The most straightforward development to consider is that of paid claims:

$$pC = pC_a(d)$$

Paid claims is just the amount actually paid out to date on the claims overall in the cohort $\{C_a\}$. In effect, it is:

$$pC_a(d) = pS_a(d) + pO_a(d)$$

i.e. the sum of the amounts paid out on settled claims, plus the partial payments on open claims. There could be a case for using $pS_a(d)$ alone, but the extra effort involved may be more trouble than it is worth, especially if $pO_a(d)$ is relatively small in the sum. In any case, it is likely that the amount $pC_a(d)$ will relate mainly to the group $\{S\}$.

Incurred Claims Development

Again, this is a commonly used function. Different definitions are possible, but the usual one is:

$$iC_a(d) = pC_a(d) + kV_a(d)$$

Incurred Claims = Paid Claims + Case Reserves

Here, $kV_a(d)$ is used to signify the total of the individual case reserves at time d.

Incurred claims defined in this way gives a first estimate for the losses on the reported claims $\{R\}$, i.e. groups $\{S\} + \{O\}$. But it omits allowance for any development on the reported claims, and also for any losses to follow on the IBNR claims. (But in some definitions, amounts for reported claim development and/or IBNR can be brought into the iC function.)

A final and important point is that IBNR claims which are late reported will *at that later time* come into the iC value.

Both paid claims and incurred claims are functions that will develop over time. They have the useful property that both must home in to the ultimate loss for the cohort concerned:

$$\lim_{d \text{ becomes large}} pC_a(d) = \lim_{d \text{ becomes large}} iC_a(d) = L_a\text{-}ult$$

or

$$pC_a(u) = iC_a(u) = L_a\text{-}ult$$

For the report year cohort, $pC_r(d)$ and $iC_r(d)$ will also develop, although IBNR claims which are later reported will no longer enter as an element in iC. The development of iC_r will, in fact, directly show the correction that has to be made over time in the case reserves themselves.

Claim Number Developments

The available claim numbers are (or may be) those on $\{S\}$and $\{O\}$, the settled and open claims.

$$nS = nS_a(d)$$
$$nO = nO_a(d)$$

Also:

$$nR = nS_a(d) + nO_a(d)$$

Again, as development time increases, the n's home in to a limit:

$$nS_a(u) = nR_a(u) = n_a\text{-}ult$$

and

$$nO_a(u) = 0$$

But for report year cohorts, nR_r is fixed. In fact:

$$nR_r \;=\; n_r\text{-}ult \qquad \text{(by definition)}$$

Average Claim Developments

The idea of development extends naturally to the various types of average cost per claim function. Such functions can be defined for settled, reported or outstanding claims:

Settled claims	$AS_a(d)$	=	$iS_a(d)\ /\ nS_a(d)$
Reported claims	$AR_a(d)$	=	$iR_a(d)\ /\ nR_a(d)$
Outstanding claims	$AU_a(d)$	=	$iU_a(d)\ /\ nU_a(d)$

The settled average might be estimated as: $pC_a(d)\ /\ nS_a(d)$

and the reported average as: $iC_a(d)\ /\ nR_a(d)$

But there is no easy function available for the outstanding average.

Per Claim Payment Patterns

A significant group of methods uses the pattern of payments per claim incurred. The Bennett & Taylor method of §J3 is a good example. In this case, the developing payment values by accident year are divided through by the overall number of claims, or at least by an estimate of this number:

a) $pC_a(d)\ /\ \hat{}n_a\text{-}ult$
b) $pC_a(d)\ /\ \hat{}n_a\text{-}ult(0)$
c) $pC_a(d)\ /\ nR_a(0)$

In a), division is by the current best estimate of *n-ult.* In b), the estimate of *n-ult* is that made at the end of the accident year in question, and not later modified. In c), the number used is the number of claims actually reported in the accident year.

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[M11]
THE TRIANGULAR ARRAY

The basic analysis of claim cohorts by accident year (or other year of origin) and by development period leads naturally to a triangular format. The format is virtually identical whatever function is being considered:

		d					
		0	1	2	3	...	*ult*
	1	$el_1(0)$	$el_1(1)$	$el_1(2)$	$el_1(3)$	...	$el_1(u)$
	2	$el_2(0)$	$el_2(1)$	$el_2(2)$	$el_2(3)$		
a	3	$el_3(0)$	$el_3(1)$	$el_3(2)$	$el_3(3)$		
	4	$el_4(0)$	$el_4(1)$	$el_4(2)$		**[*fn*]**	
	...	...	...				
	c	$el_c(0)$					

Years of origin, say accident years *a*, are measured as from some base year, up to and including the most recent (or current) year *c*. Development years *d* are then measured from accident year as base, with the accident year itself appearing as d=0.

Conventionally, the rows of the triangle record the progress for given accident years, while the columns give the state of play for specific development periods. As a result, accident years are listed down the left hand side of the triangle, and development years across the top. Report or underwriting years may replace accident years to name the rows of the triangle.

The elements $el_a(d)$ in the triangle may be any one of a number of claims functions. Common examples are paid claims, incurred claims, number reported, paid average, incurred loss ratio, and so on.

The triangle as drawn above has equal breadth and depth, so that $c = u+1$. But equality of breadth and depth is not essential. In claims reserving many other shapes can be used. Some are shown in M11.3.

In any case, the development may not reach the *ult* value, even in the top row, but may fall short by several periods' length. This final part of the development is called the tail. If it has length *l*, then the last data value will fall in the cell (a=l, d= u-l).

Further points are that the time periods can be other than years, e.g. quarters or even months can be used. Also, there is no absolute need to employ the same time interval on the horizontal and vertical axes. E.g. the accident rows could be by years, and the development columns by quarters. But such triangles are rarely seen.

The Diagonals

In the basic triangle, the diagonals have the useful property of representing the calendar years:

		d					
		0	1	2	3	...	*ult*
	1	$el_1(0)$	$el_1(1)$	$el_1(2)$	$\boldsymbol{el_1(3)}$	...	$el_1(u)$
	2	$el_2(0)$	$el_2(1)$	$\boldsymbol{el_2(2)}$	$el_2(3)$		
a	3	$el_3(0)$	$\boldsymbol{el_3(1)}$	$el_3(2)$	$el_3(3)$		
	4	$\boldsymbol{el_4(0)}$	$el_4(1)$	$el_4(2)$		**[*fn*]**	
	...	...	...				
	c	$el_c(0)$					

The elements in boldface are those relating to calendar year 4. Thus:

$el_4(0)$	is element for	a=4 + d=0	sum = 4
$el_3(1)$	... .	a=3 + d=1	... ...
$el_2(2)$	... .	a=2 + d=2	... ...
$el_1(3)$	... .	a=1 + d=3	... ...

In each case, the sum of accident and development years is 4, i.e. the calendar year value. Thus the general element in the triangle, $el_a(d)$, will in all cases be the value occurring for the calendar year y, where:

$$y = a + d$$

In fact, the element *could* equally be indexed as $el_y(d)$, when a would be deduced as: $a = y\text{-}d$. But this convention does not seem to be used, in general.

Suffix Notation

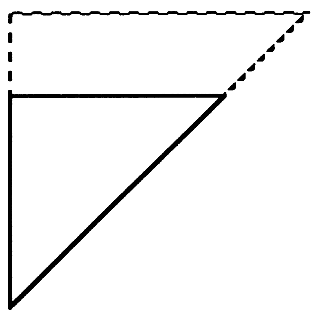
Having introduced the triangle, the general element $el_a(d)$ could equally be written in the double suffix form el_{ad}. This form is frequently used in the literature, though usually as el_{ij}, with i, j in place of a, d. The notation is a derivation from mathematical matrix theory, where i is commonly used as row index, and j as column index. The reason for the different approach of the Manual is that the form $el_a(d)$ emphasises strongly the inherent difference between the accident year classification *a)* and the development period progression *d*).

In General Insurance reserving, we are not dealing with matrices whose rows and columns are, for practical purposes, interchangeable. The mathematical notation tends to suggest a symmetry which is not in reality to be found. Quite apart from this, it is difficult to remember which of i, j refers to accident year or development period. There is no such problem when a, d are used instead.

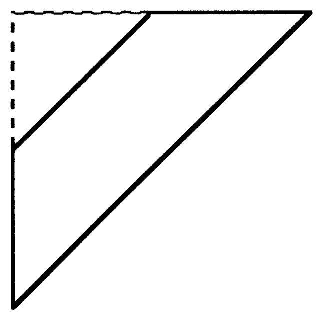
Variations in the Triangle Shape

The following variations can be met with in practice. The precise form will depend on the needs of the analysis, but more upon the data which is available.

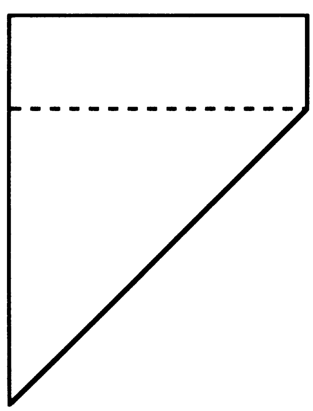
Reduced by Omission of Accident Years



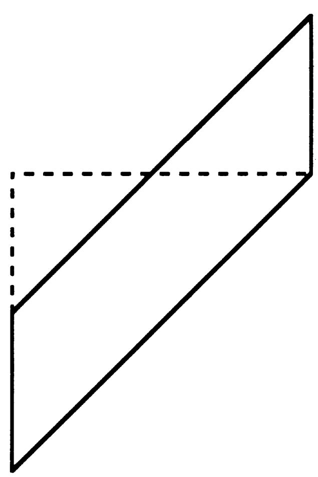
Reduced by Omission of Calendar Years



Extended by Earlier Accident Years



Extended by Accident Year & Reduced by Calendar Year



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[M12]
CLAIM DEVELOPMENT & TREND ANALYSIS

Once the development triangle has been set up, it is natural to begin looking at the ratio of one period's claim values to those of the preceding or succeeding period. Given that the triangle contains the general function $el_a(d)$ the development factor can be defined as:

$$r_a(d) = el_a(d+1) \,/\, el_a(d)$$

The general term "link ratio" is used in the Manual for such factors. The factor is here defined as a *forward* ratio from the period in question. (The backward ratio $el_a(d) \,/\, el_a(d\text{-}1)$ could also be used.)

Symbol r is used, since only a single step is being taken, i.e. from one period to the next. But it is also useful to define a ratio forward to the ultimate value, $el_a\text{-}ult$. This is the "final development" link ratio, symbol f:

$$f_a(d) = el_a\text{-}ult \,/\, el_a(d)$$

An example, using paid claims development, is:

$$r_a(d) = pC_a(d+1) \,/\, pC_a(d)$$
$$f_a(d) = L_a\text{-}ult \,/\, pC_a(d)$$

A grossing-up factor can also be defined, which, when divided into the current value of el, will yield the final value. In fact, g is just the inverse of f:

$$g_a(d) = el_a(d) \,/\, el_a\text{-}ult$$

hence: $el_a(d) \,/\, g_a(d) = el_a\text{-}ult$

Multiplying up the one-step link ratios leads to the final value:

$$f_a(d) = r_a(d) \times r_a(d+1) \times \ldots \ldots \times r_a(u\text{-}1)$$

Also, the relationship holds that:

$$f_a(d) = r_a(d) \times f_a(d+1)$$

In general, in the upper left-hand part of the triangle, i.e. for: $a + d < c$, the link ratios $r_a(d)$ may be found from the data. In the lower right-hand part of the triangle, i.e. for: $a + d \geq c$, the $r_a(d)$ must be estimated.

Once the one-step links have been estimated for the whole lower right triangle, the final ratios can be calculated by multiplication. The estimate for $L_a\text{-}ult$ can then be made:

$$L_a\text{-}ult \qquad = f_a(d) \times pC_a(d)$$

where the work runs through the values $a = 1, 2, \ldots c$, and d is chosen as $(c\text{-}a)$. That is, the paid claim values along the main diagonal are picked.

Trend Factors

Another variation using the basic triangular data pattern is to look at the *trend* in values from one accident year to another. The comparison with claim development analysis as just described is instructive:

Claim development looks at ratios along the *rows* of the triangle, e.g. the relative increase of paid claims over time for given accident years.

Trend analysis looks at ratios down the *columns* of the triangle, e.g. the trend in average cost per claim as experienced at given development periods.

Thus trend analysis constructs vertical rather than horizontal factors in the triangle. Normally, it would not be applied to a function such as paid claims direct, because claims arise in each accident year from a different exposure base. The data would first need to be reduced to an average, or a loss ratio, or a payments pattern. Then trending as applied to such quantities can be useful and valid.

Claim Development:	$r_a(d)$	=	$el_a(d+1) / el_a(d)$
Trend Analysis:	$h_a(d)$	=	$el_{a+1}(d) / el_a(d)$

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Section N
GLOSSARY OF NOTATION

Preamble

This section brings together for reference purposes the main symbols used in the system of notation used in describing the methods of Volume 1. Some special symbols which are specific to particular methods, and which fall outside the main structure of the system, have been omitted from the glossary; their use is usually apparent in the context of the method in question.

The notation does not extend to the methods considered in Volume 2 of the Manual. There each method calls for whichever mathematical/statistical notation is appropriate in the particular circumstances.

[N1]
GLOSSARY OF NOTATION

Symbol	*Meaning*
$\{C\}$	Cohort of claims, which may be identified as
	$\{C_a\}$ Accident-year cohort
	$\{C_r\}$ Report-year cohort
	$\{C_w\}$ Underwriting-year cohort
$\{S\}$	Subgroup of settled claims
$\{U\}$	.... outstanding claims
$\{O\}$	.... open claims
$\{R\}$	.... reported claims
$\{\tilde{R}\}$	.... IBNR claims

The following relationships hold —

$$\{U\} = \{O \cup \tilde{R}\}$$
$$\{R\} = \{S \cup O\}$$
$$\{C\} = \{S \cup O \cup \tilde{R}\}$$
$$= \{S \cup U\} \text{ or } \{R \cup \tilde{R}\}$$

$\{S^O\}$	Claims settled at nil cost
$\{S+\}$	Claims settled at some cost

$$\{S\} = \{S^O \cup S+\}$$

Symbol	*Meaning*
nC	Number of claims in cohort, which at a particular review date may be analysed into —

nS Number of settled claims

nU .... outstanding claims

nO .... open claims

nR .... reported claims

$nR^{\sim}$ .... IBNR claims

The following relationships hold —

$$nU = nO + nR^{\sim}$$

$$nR = nS + nO$$

$$nC = nS + nO + nR^{\sim}$$
$$= nS + nU \text{ or } nR + nR^{\sim}$$

nS^{O}	Number of claims settled at nil cost
$nS+$	Number of claims settled at some cost
	$nS = nS^{O} + nS+$

At the stage of ultimate development of the cohort $nC = nS$ as the other components become zero.

An alternative notation for the ultimate number of claims in the cohort is *n-ult.* In general *-ult* is used to denote the ultimate development value of the element involved.

Symbol	*Meaning*
d	Development time of a cohort. Conventionally $d = 0$ denotes the initial development period (usually year), so that d assumes the values 0, 1, 2, ... *ult* as the cohort runs off.

Any element, E, of a cohort may be identified by —

(i) the year of origin of the cohort, denoted by accident-year a or report-year r or underwriting-year w as the case may be,

and

(ii) the development-year d.

E_a(d)	The element in development year d in the cohort for accident-year a. The corresponding notation for report-year r and underwriting-year w would be $E_r(d)$ and $E_w(d)$ respectively.
^	Precedes an element to indicate that it is an <u>estimated</u> amount.

Symbol	*Meaning*

The following are examples of the "cohort development" notation applied to the element of "number of claims".

$nS_a(d)$	Number of claims originating in accident year which have been settled by the end of development year d.
$nO_a(d)$	Number of claims originating in accident year a which remain open at end of development year d.
$nR_a(d)$	Number of claims originating in accident year a which have been reported by the end of development year d $= nS_a(d) + nO_a(d)$
$\hat{}nR_a\sim(d)$	Estimated number of IBNR claims originating in accident year a, which remain unreported at end of development year d.
$\hat{}n_a\text{-}ult$	Estimated ultimate number of claims attributed to accident year a $= nS_a(d) + nO_a + \hat{}nR_a \sim (d)$

The same "cohort development" notation may be applied to other cohort elements defined on the following pages.

Symbol	*Meaning*
pC	Cumulative total claim <u>amounts paid</u> to end of development period.
pC^*	Cumulative claim amounts paid to end of most recent development period. These values lie on the "leading" diagonal of the claims triangle.
pS	Cumulative amounts paid on claims settled to end of development period.
pO	Cumulative amounts paid on claims that are still open at end of development period.
$\_pC$	Claim amounts paid in a specified development period (i.e. non-cumulative).

By way of example, the following relationships hold —

$pC_a(d)$	Cumulative claim amounts paid to end of development year d on claims originating in accident year a:

$$= \Delta pC_a(o) + \Delta pC_a(1) + ... + \Delta pCa(d)$$

alternatively

$$= pS_a(d) + pO_a(d)$$

Symbol	*Meaning*
iC	Cumulative total claim <u>amounts incurred</u> to end of development period.
	Corresponding <u>amounts incurred</u> on —
iS	Claims settled to end of development period.
iO	Claims still open at end of development period.
iR	Claims reported to end of development period.
$iR\sim$	Claims which are IBNR at end of development period.
iU	Claims which are outstanding at end of development period $= iO + iR\sim$

Symbol	*Meaning*
$kV_a(d)$	Case reserves on claims originating in accident year a which are open at end of development year d.
$\hat{}V_a(d)$	Estimated reserve at end of development year d on claims originating in accident year a.
$hV_a(d)$	Hypothecated reserve on claims outstanding at end of development d originating in accident year a.
$L_a\text{-}ult$	Ultimate liability on claims originating in accident year a.
$\hat{}L_a\text{-}ult$	Estimated ultimate liability on claims orginating in accident year a.

The following relationships hold —

$$
\begin{aligned}
\hat{}V_a(d) &= \hat{}L_a\text{-}ult - pC_a(d) \\
hV_a(d) &= \hat{}L_a\text{-}ult - pC_a(d) \\
iC_a(d) &= pC_a(d) + kV_a(d) \\
\hat{}L_a\text{-}ult &= iS_a(d) + iO_a(d) + iR_a \sim (d)
\end{aligned}
$$

VR	Reserve for reported claims.
$VR\sim$ (or $ib\ V$)	Reserve for IBNR claims.
VS	Reserve for re-opening of settled claims.

Symbol	*Meaning*
g	Grossing-up factor $= pC/L\text{-}ult$
$r_a(d)$	Link ratio $= pC_a(d+1)/pC_a(d)$
$f_a(d)$	Final link ratio from the current cumulative claims $pC_a(d)$ to the final ultimate value $L_a\text{-}ult$

$$f_a(d) = L_a\text{-}ult/pC_a(d)$$
$$= r_a(d) \times r_a(d+1) \times \ldots\ldots \times r_a(u-1)$$

Symbol	*Meaning*	
A-ult	Average cost per claim at end of cohort development = *L-ult/n-ult*	

Sub-groups of average costs —

AS	Settled claims	*iS/nS*
AO	Open claims	*iO/nO*
AR~	IBNR claims	*iR~/nR~*
AR	Reported claims = (*iS* + *iO*)/(*nS* + *nO*)	*iR/nR*
AU	Outstanding claims = (*iO* + *iR~*)/(*n*O + *nR~*)	*iU/nU*
pA	Paid average cost = *pC/nS*	
iA	Incurred average cost = *iC/nR*	

Symbol	*Meaning*
X	Base measure of risk exposure (units of exposed-to-risk)
X_a	Unit of exposure for accident-year cohort. (Example: Earned Premium aP)
X_w	Unit of exposure for underwriting-year cohort. (Example: Written Premium wP)
λ	Loss ratio = $L\text{-}ult/P$
$p\lambda_a(d)$	Paid loss ratio at end of development year d $= pC_a(d)/aP$
$i\lambda_a(d)$	Incurred loss ratio at end of development year d $= iC_a(d)/aP$

Symbol	*Meaning*
BF-pc	Bornhuetter-Ferguson Method applied to Paid claims.
BF-iC	Bornhuetter-Ferguson Method applied to Incurred claims.
B-ult	Benchmark Loss = $\lambda \times aP$
BF Proportion	$1 - {}^{1}/f$ where f is the final link ratio, or $1 - g$ where g is grossing-up factor
^eV	Emerging Liability $= (1 - {}^{1}/f) \times B\text{-}ult$
^eC	Emerging claims $= (1 - g) \times B\text{-}ult$
CV	Required Reserve $= \Sigma_a(\hat{\ }eC)$ i.e. the sum of Emerging Claims over all accident years.
^L-ult	Estimate Ultimate Loss $= pC + \hat{\ }eC$

[Section O]
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