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THE EXECUTIVE COMMITTEE OF THE CONTINUOUS MORTALITY INVESTIGATION BUREAU

as on 17th May 1990

INTRODUCTION

THE Executive Committee of the Continuous Mortality Investigation Bureau of the Institute of Actuaries and the Faculty of Actuaries has pleasure in presenting this, the tenth number of its Reports.

The only item is a report describing the Committee's new 'Standard Tables of Mortality Based on the 1979–82 Experiences'. This is the natural sequel to the report in *C.M.I.R.* **9** on 'The Graduation of the 1979–82 Mortality Experiences'. Since the publication of that Report, the Committee has obtained the views of members of the actuarial profession and of contributing offices on the standard tables that are required, and has taken account of these views in its preparation of the new standard tables.

On no previous occasion has the Committee published a complete range of standard tables of mortality on a single occasion. On this occasion new standard tables for male and female assured lives, pensioners and immediate annuitants are presented, and the range is extended by the publication for the first time of standard tables for temporary assurances and for widows.

The report contains complete tables of the values of q_x (select and ultimate as appropriate) for the assured lives tables, and the base tables for pensioners, immediate annuitants and widows, together with the formula for projection for double entry tables for pensioners, annuitants and widows. This is the minimum information necessary for the calculation of any desired function on the basis of the new standard tables, so this report can be taken as the formal publication of the new tables.

However, for the convenience of users, the Committee proposes to publish a printed volume, which will repeat the values of the basic mortality rates, and include values such as those of l_x, μ_x, etc. and values of *specimen* monetary functions at selected rates of interest, and also to make available a computer package for use on the popular types of personal computer, by means of which a wide range of functions can be calculated at any desired rate of interest. The Committee believes that this method of making the tables available will be preferred to the previous system of publishing fat volumes with extensive monetary functions, though it recognises that this may disappoint those traditional actuaries who have enjoyed displaying a row of large volumes with variously coloured dust jackets on their bookshelves.

The Executive Committee wishes to thank all those who contributed to the production of this number of *C.M.I.R.*, in particular Tony Leandro of the Secretariat of the Bureau, who acted as Secretary to the Standard Tables Working Party, to Mary McCamley of R. Watson & Sons who typed the manuscript and to Brian Winchester of Alden Press who oversaw the printing process. Readers may be interested to know that the text of the manuscript was typed on a word processor, the numerical values were calculated by computer,

and the whole was transmitted through a series of computer processes into the form of printed proofs. This did not avoid the necessity for careful proof reading, nor for rearrangement of the layout, formulae and table headings, as well as corrections when the Committee wished to revise its original draft, but it did make the process considerably easier.

I personally should like to thank also the other members of the Standard Tables Working Party, Colin Berman, Chris Daykin, David Forfar, John McCutcheon and Rodney Barnett for the very large amount of work that went into constructing these standard tables, which was approached with enthusiasm, inspiration and attention to detail. The profession has reason to be grateful to them.

May, 1990

A. D. Wilkie
Chairman of the Executive Committee

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STANDARD TABLES OF MORTALITY BASED ON THE 1979-82 EXPERIENCES

1. INTRODUCTION

1.1 One of the objectives for which the C.M.I. Bureau was set up was the production and publication of standard tables of mortality. The Bureau has published a series of tables in the past – A1924-29, A1949-52, *a*(55), A1967-70, *a*(90), PA(90) and FA1975-78. On no previous occasion, however, has the Committee prepared a large number of standard tables at the same time based on all its main investigations. The Committee has decided to do just this in respect of the experience for 1979-82. This report describes the new standard tables.

The simultaneous publication of a complete range of standard tables is not the only novel feature of this report. The standard tables were graduated using new methods, which gave formulae for μ_x, rather than for q_x as on previous occasions; the graduated rates have then been adjusted using certain new methods; a new formula for projecting mortality rates has been used; and the Committee has decided that extensive tables of monetary functions should not be published, but that a computer package should be made available suitable for use on the now widespread personal computers.

It has often been the case that proposed new standard tables have been presented to the profession and discussed at sessional meetings of the Institute and the Faculty. On this occasion, since the graduated rates received extensive discussion at meetings of the Faculty and the Institute during 1988, it is not proposed that special sessional meetings be held to discuss this report.

1.2 The usual quadrennial report on the mortality of the assured lives', pensioners' and annuitants' experiences for 1979-82 was published in *C.M.I.R.*, **8**, in 1986. The mortality experience shown for 1979-82 was sufficiently different from that of the period 1967-70, on which the current standard tables are based, for the Committee to decide that it was appropriate to consider graduating the data with a view to the preparation of new standard tables.

The Committee presented its report 'The Graduation of the 1979-82 Mortality Experiences' in *C.M.I.R.*, **9**, in 1988. The methodology used for the graduation was fully described in the paper 'On Graduation by Mathematical Formula', by D O Forfar, J J McCutcheon and A D Wilkie (*J.I.A.*, **115**, 1 and *T.F.A.*, **41**, 97). This paper and the graduation report were discussed together at a sessional meeting of the Faculty on 15 February 1988 and at a special meeting of the Institute on 28 June 1988. The graduation report and the paper were also circulated to contributing offices, and submissions were invited.

The Committee has taken into account the views expressed at the meetings at the Institute and the Faculty, and the written submissions from contributing offices and from members of the profession. In July 1988 a Working Party was set up by the Committee to consider the preparation of standard tables of mortality. Its members were: A D Wilkie (Chairman), C Berman, C D Daykin, D O Forfar and J J McCutcheon, assisted by H A R Barnett (Secretary of the Committee) and P A Leandro of the Secretariat of the Bureau. The Committee is grateful to the members of this Working Party for preparing this report, for which the whole Committee takes responsibility.

The experience for 1979-82 had been used as the basis for the graduated rates described in the graduation report (*C.M.I.R.*, **9**) and these graduated rates have been used as the basis for the standard tables described in this report. Recently the experience for the various investigations for 1983-86 has become available. A comparison of the mortality experience in these investigations with that expected by the new standard tables will form the subject of a separate report.

2. THE NEW STANDARD TABLES

2.1. INTRODUCTION

In the report on the graduation of the 1979-82 mortality experiences the Committee presented graduated tables for the following experiences:

Males. United Kingdom.
Permanent Assurances (two year and five year select periods)
Temporary Assurances (level and decreasing)
Linked Assurances

Females. United Kingdom.
Permanent Assurances
Linked Assurances

Males. Republic of Ireland.
Permanent Assurances

Males and Females. United Kingdom.
Immediate Annuitants
Retirement Annuitants
Life Office Pensioners

Females. United Kingdom.
Widows of Life Office Pensioners

The Committee recommended that no standard table be prepared on the basis of the experience of linked assurances, either for males or for females, in the United Kingdom. There was no dissension from this view at the Faculty and Institute meetings.

The Committee would have been willing to prepare a new standard table for permanent assurances in the Republic of Ireland, but the general view of the meetings seemed to be that such a table was not required.

The Committee would also have been prepared to produce a standard table for retirement annuitants, but raised the question as to whether the change in legislation introducing the new style of personal pension possibly made a standard table based on an experience of retirement annuitants unnecessary. The general view of the meetings seemed to agree that it did.

No standard tables are therefore proposed for linked assurances, the Republic of Ireland or retirement annuities.

This leaves the following investigations, all based on the experience in the United Kingdom:

Permanent Assurances (males and females)
Temporary Assurances (males only, level and decreasing combined)
Immediate Annuitants (males and females, based on Lives)
Life Office Pensioners (males and females, Lives and Amounts)
Widows of Life Office Pensioners (females only, Lives and Amounts)

2.2 SOME GENERAL OBSERVATIONS

Although the graduated rates provide a satisfactory fit to the data over the age ranges where the number of deaths is sufficiently large, the formulae used for graduation did not necessarily provide satisfactory values of the mortality rates outside this range, in particular at the extreme ends of the age range. In some cases the graduated rates for successive select durations did not lie in a rational relationship to each other, or the rates for one table appeared inconsistent with another. It was therefore felt necessary to adjust the graduated rates to make them more satisfactory for use in standard tables.

The methods used for adjustment have been somewhat *ad hoc*. An alternative approach considered by the Working Party, but not pursued, would have been to refit mathematical curves to the experience data, subject to particular constraints, for example constraining μ_x at one or two selected ages to have specific values. However, the mathematical complexity of this approach seemed too great to be solved satisfactorily within the desired time-scale, and there was no certainty that the resulting rates would not require yet further adjustment. The mathematical problem remains an interesting one for academic study in future.

Another approach, suitable for dealing with experiences investigated by select duration, would be to fit a series of mathematical curves to all durations simultaneously, constraining them to be consistent. This too would have been an interesting mathematical exercise, but one that the Working Party did not wish to undertake in the time available.

The Committee has therefore fallen back on the *ad hoc* methods described in this report. Although the resulting mortality tables do not exhibit ideal smoothness in the sense used by Barnett (1985), the Committee does not expect that this will cause practical difficulties when the tables are used.

In each case the adjusted rates have been tested against the original experience, to see whether they fit that experience as well as, or better or worse than, the unadjusted graduated rates. This is discussed in Section 3.

The adjustments made to each of the tables are described in Section 2.3, but it will be useful first to make some general observations about the methods used.

2.2.1 *Quadratic adjustment*

We first explain what is meant by a 'quadratic adjustment'. All the experiences were graduated using what were denoted in the graduation report as GM(r,s)

formulae, restricted in fact to GM(0,2), GM(2,2) and GM(1,3) formulae, viz:

$$\text{GM}(0,2)\ \mu_x = \exp(b_0 + b_1 t)$$

$$\text{GM}(2,2)\ \mu_x = a_0 + a_1 t + \exp(b_0 + b_1 t)$$

$$\text{GM}(1,3)\ \mu_x = a_0 + \exp(b_0 + b_1 t + b_2(2t^2 - 1))$$

where in each case $t = (x - 70)/50$. The GM(0,2) formula is simply a Gompertz formula. The graph of $\log \mu_x$, plotted against x, is a straight line.

The GM(2,2) formula, which has been the most commonly used, is Makeham's second modification of Gompertz' original formula. At high ages the Gompertz part wholly dominates the relatively small first two terms. At these ages, the graph of $\log \mu_x$ is therefore almost a straight line.

In the case of the GM(1,3) formula the first (constant) term becomes relatively unimportant at high ages and the graph of $\log \mu_x$ at these ages is approximately a quadratic.

In a number of cases it was felt appropriate to adjust the graduated mortality rates at the highest ages by moving the graph of $\log \mu_x$ up or down in an appropriate way beyond a certain age. A convenient way of doing this without producing a discontinuity in the first derivative of μ_x was to add or subtract a suitable quadratic term to the polynomial inside the exponential term.

Formally, we choose an age x_0 above which the adjustment is to apply. Let $t_0 = (x_0 - 70)/50$. We wish the quadratic adjustment to be zero at age x_0, so the form $c(t - t_0)^2$ is appropriate. This formula makes the first derivative of the adjustment at age x_0 also zero, so that the first derivative of μ_x remains continuous over the relevant age ranges, in particular at x_0.

Suppose that the value of c is chosen to give an adjusted mortality rate at some higher age, denoted by x_1, which is approximately r times the graduated rate. Let $t_1 = (x_1 - 70)/50$. We require the exponential term to be multiplied by r at age x_1, so we put:

$$c(t_1 - t_0)^2 = \log r$$

The original polynomial and the quadratic adjustment are then combined to give new parameters:

$$b_0' = b_0 + ct_0^2 + \tfrac{1}{2}c$$

$$b_1' = b_1 - 2ct_0$$

$$b_2' = b_2 + \tfrac{1}{2}c \text{ (where } b_2 = 0 \text{ for a GM(2,2) formula)}$$

If the original formula was GM(2,2), the adjusted formula becomes GM(2,3), viz:

$$\mu_x = a_0 + a_1 t + \exp(b_0' + b_1' t + b_2'(2t^2 - 1))$$

If the original formula was GM(1,3), the adjusted formula remains GM(1,3) with altered parameters, viz:

$$\mu_x = a_0 + \exp(b_0' + b_1' t + b_2'(2t^2 - 1))$$

2.2.2 *Calculation of* q_x

The graduation formulae give values of μ_x for all x in the appropriate age range. In practice it is inappropriate to use any graduated values below age 17, and values of mortality rates at young ages have been taken from the population experience; these are discussed further in Section 2.4. The adjustments to μ_x have all been designed in such a way as to provide adjusted values of μ_x for all values of x. It is therefore possible to calculate accurate values of q_x for integral values of x through the formula

$$q_x = 1 - \exp\left(-\int_0^1 \mu_{x+t}\,dt\right).$$

It is possible to integrate, using elementary calculus and common functions, the function for μ_{x+t} in the above formula for the GM(0,2) and GM(2,2) formulae. However, it is not possible to do this for the GM(1,3) formula. It was therefore convenient for all the tables to use the same approximate method for the calculation of values of q_x from those of μ_x. The integral was calculated to a high degree of accuracy by repeated use of Simpson's rule, with progressively smaller subdivision, aided by the use of Romberg integration or 'accelerated convergence', a technique described by Waters and Wilkie in 'A Short Note on the Construction of Life Tables and Multiple Decrement Tables' (*J.I.A.*, **114**, 569), or in many text books on numerical analysis.

For each table values of q_x rounded to six decimal places are given in Appendix A and these values form the definitive new table.

2.2.3 *Graphical representation*

It is helpful to display rates of mortality graphically, using a vertical log scale. But on such a graph the rates for the different tables under consideration are quite close together, and differences between the rates are not readily apparent. A convenient alternative is to show the rates for each table as percentages of the rates for some standard table. For males the standard taken throughout is the table of q_x based on the adjusted values of μ_x for male permanent assurances duration 2+ (AM80). On such a graph the permanent assurances duration 2+ rates themselves appear as a horizontal line with a value of 100. For females the standard taken throughout is the corresponding table of q_x based on the adjusted values of μ_x for female permanent assurances duration 2+ (AF80).

An alternative standard would be the rates of mortality for English Life Tables No 14 Males and Females (ELT14M and ELT14F). The rates for ELT14 are greater than those for most of the new standard tables, and in particular are greater than the rates for permanent assurances duration 2+ for the corresponding sex, except for males at ages 96 to 104, as will be discussed below. The percentage ratios of ELT14M and ELT14F rates to those for the corresponding permanent assurances duration 2+ are generally also shown on the graphs.

2.2.4 *Naming of tables*

The new standard tables require convenient names by which they can be identified. The Committee, after consulting the profession through the pages of *Fiasco* (Berman, 1989, and Barnett and Berman, 1990), has devised a naming scheme for the tables which is now described.

The first letter of each name identifies the type of investigation:

A for (permanent) Assurances
T for Temporary (assurances)
P for Pensioners
I for Immediate (annuitants)
W for Widows

The second letter of the name identifies, where necessary, the sex:

M for Males
F for Females

This letter is not necessary for widows, there being no corresponding widowers' tables.

The next letter distinguishes, where necessary, between tables based on an investigation by Lives or by Amounts:

L for Lives
A for Amounts

This is the third letter in the name for the tables for pensioners, and the second letter in the name for the tables for widows.

The next part of the name is '80', representing 1980, one of the central years for the 1979-82 experience, on which the tables are based. As shown in Section 4.6, the values of q_x apply on average to a life reaching age x in the middle of 1980.

For permanent assurances, males, two tables with different select periods have been constructed, one with a two-year select period, the other with a five-year select period. The latter is distinguished by the symbol (5) after the name.

For permanent and temporary assurances it is not intended to construct any projected tables, so the names for the new standard tables for these are:

Permanent assurances, Males, two-year select	AM80
Permanent assurances, Males, five-year select	AM80(5)

Permanent assurances, Females	AF80
Temporary assurances, Males	TM80

In the case of pensioners, annuitants and widows the Committee has prepared projected tables to allow for possible future improvements in mortality. The method of projection results in full double-entry tables, indexed by calendar year or year of birth and by attained age. The double-entry tables will have no particular suffixes attached. The names for these tables will therefore be:

Pensioners, Males, Lives	PML80
Pensioners, Males, Amounts	PMA80
Pensioners, Females, Lives	PFL80
Pensioners, Females, Amounts	PFA80
Immediate annuitants, Males, one-year select	IM80
Immediate annuitants, Females, one-year select	IF80
Widows, Lives	WL80
Widows, Amounts	WA80

In order to construct projected tables it is necessary first to construct a base table for each investigation. The base tables are based, like the tables for assurances, on the graduated rates derived from the 1979-82 experiences. They are denoted by the addition of the word Base to the name shown above. For example, the base table for Pensioners, Males, Lives will be known as PML80Base. As is discussed in Section 4.6 there are three forms of single-entry standard table which can be derived from the double-entry tables. These are: a table for a single calendar year; a table for a single year of birth; and a table for monetary functions, based on the double-entry table, for a single 'year of use'. These will be denoted by a suffix to the names of the double-entry tables.

A single-entry table based on a specified calendar year will be denoted by the suffix (C = year), e.g. (C = 1990) or (C = 2010), which can be abbreviated as C90 or C10 for calendar years in the range 1980 to 2079 (which should cover most practical cases). Thus the table for Pensioners, Males, Lives, projected to calendar year 2010 can be known as PML80 (C = 2010) or more briefly as PML80C10. Each of the Base tables, applicable to calendar year 1980, could alternatively be described by the suffix (C = 1980) or just C80.

A single-entry table based on a specified year of birth will be denoted by the suffix (B = year), e.g. (B = 1935) for a table for a life born in 1935. For years of birth in the range 1900 to 1999 this can be abbreviated as e.g. B35. The table for female annuitants born in 1935 would therefore be known as IF80 (B = 1935) or IF80B35.

A table of monetary functions extracted from the full double-entry table, but applicable to a particular year of entry or year of use, will be denoted by the suffix (U = year). For example, functions for Pensioners, Females, Amounts for year of use 1990 would be denoted PFA80 (U = 1990), abbreviated to

PFA80U90 if the year of use is between 1980 and 2079, a range which will cover all practical cases.

It is hoped that these abbreviated names for the new standard tables will be both unambiguous and convenient to the user.

2.3 THE NEW STANDARD TABLES IN DETAIL

In the Sub-sections below the adjustments made to the graduated rates for each of the new standard tables are described. Values of q_x for the new tables (and $q_{[x]}$ etc for select tables) are shown in Appendix A, and details of the formulae used for calculation are shown in Appendix B.

2.3.1 *Permanent assurances, males - AM80 and AM80(5)*

In the graduation report the Committee indicated its intention to prepare standard tables based on the graduations of the experience for male lives assured in the United Kingdom for permanent assurances (whole-life and endowment assurances), for the medical and non-medical sections combined. The question was asked as to whether a two-year select period, as in A1967-70, or a five-year select period, as in A1967-70(5), should be produced. There was statistical justification for a five-year select period, but there are great practical advantages in preparing a table with a two-year select period.

The Committee decided to prepare tables on both bases, as was done for the A1967-70 tables. As with those tables, however, the Committee expects that the two-year select table will be more widely used and will be treated as the primary table, with the five-year select table being used rather less frequently. The tables will be denoted AM80 and AM80(5).

The graduated rates for durations two and over (2+) were based only on data up to age 90, because the experience beyond that age appeared erratic and unreliable, as had been found when graduating the 1967-70 experience. The rates produced by the graduation formula (for parameters of the formulae see Appendix B) extrapolated beyond age 90 appeared to be too high, particularly in comparison with English Life Table No 14 Males (ELT14M), which was based on the experience of the England and Wales male population for the years 1980 to 1982, a period almost exactly matching the period of the C.M.I. experience. Since the rates for duration 2+ were generally lower than those for ELT14M at lower ages, it seemed implausible that they should rise substantially above the latter rates at the highest ages. This was simply a consequence of the fact that the assured lives rates were a low proportion of those for the population at middling ages (about 60% in the 40s of age) but approached closer as age increased (about 80% in the 80s of age). The nearly straight line extrapolation implied by the use of a GM(2,2) formula necessarily meant that the graduated assured lives rates crossed the population rates at about age 96.

A quadratic adjustment, as described in § 2.2.1 above, was therefore used to reduce the duration 2+ rates above age 80. It seemed appropriate to reduce the graduated rates at age 110 by about 30%. This reduction was applied only to the exponential term so the actual reduction at age 110 is from $\mu_{110} = 1.445$ to 1.010.

The adjusted formula above age 80 becomes GM(2,3) with parameters:

$$a_0 = -0.00338415$$

$$a_1 = -0.00386512$$

$$b_0 = -3.887248$$

$$b_1 = 5.052347$$

$$b_2 = -0.495382$$

The graduated rates for duration 2+ were used without adjustment from ages 17 to 80.

The reduction in the values of μ_x above age 80 means that, for the 1979-82 experience, the expected deaths immediately above age 80 on the adjusted basis are slightly lower than those calculated on the unadjusted graduated rates, but this brings them rather closer to the actual numbers of deaths for ages 85 to 90. The question of how well the adjusted rates fit the 1979-82 experiences is discussed for all investigations in Section 3 below.

The graduated rates for duration 0 appeared to require no adjustment throughout the entire age range that was considered relevant. There was almost no exposure in the 1979-82 experience above age 75. The published A1967-70 tables gave select rates for $q_{[x]}$ up to age 80. It was felt that there might possibly be a use for select rates up to age 90, but that the values of select rates beyond that age were of little practical relevance, so this was adopted as the limiting age for these rates.

It had already been noted in the graduation report that the graduated rates for duration 1 for lower ages were higher than those for duration 2+, and that this feature would be eliminated in the standard tables. Although the high rates were justified to some extent by the experience (actual deaths for ages 18 to 20 being 97 compared with expected on the graduated basis of 82.6) the irregularity in the run of values of the mortality rates as duration increased was felt to be uncomfortable.

The rates for duration 1 were therefore adjusted below age 28 so that they held a constant proportionate distance between the rates for duration 0 and duration 2+, i.e., in an obvious notation:

$$\mu_x^{D1} = (1 - k)\mu_x^{D0} + k.\mu_x^{D2+} \qquad x < 28$$

where

$$k = \frac{\mu_{28}^{D1} - \mu_{28}^{D0}}{\mu_{28}^{D2+} - \mu_{28}^{D0}} = 0.60083712$$

The resulting values of μ_x at duration 1 are no longer produced by a single GM(r,s) formula, but instead are the weighted average of two such formulae.

No adjustments were needed to the rates for duration 1 at higher ages.

Values of q_x were then calculated for integral ages from 17 to 90 for select duration 0, from 17 to 91 inclusive for select duration 1, and from 17 to 119 inclusive for duration 2+ (the 'ultimate' rates). The derivation of values of q_x for ages under 17 is discussed in Section 2.4 below. The resulting table, the AM80 table, is shown in full in Table A1 in Appendix A. Details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B1 in Appendix B.

For the table with five years selection it was necessary to consider only the rates for durations 2 to 4 combined, since the graduation report had shown that the rates at these durations were not sufficiently different to need to be treated separately, and the rates for duration 5 and over (5+); the rates for durations 0 and 1 could be taken as the adjusted rates for the two-year select table.

The graduated rates for duration 5+ were very similar to those for duration 2+ over most of the age range, rising somewhat above the rates for duration 2+ only in the 20s of age. Although the 1979-82 experience gave some justification for this feature, it was considered that the tables were elsewhere so similar that it was appropriate to use the duration 2+ adjusted rates for the 5+ rates in the five-year select table. The ultimate rates in the two tables are therefore identical.

The only difference between the tables is therefore in the rates of μ_x for durations 2 to 4. These lie comfortably between the adjusted rates for duration 1 and the adjusted rates for duration 5+ (i.e. the adjusted 2+ rates), so there was no need to adjust these rates at all.

Values of q_x for integral ages from 0 to 90 for select duration 0, from 1 to 91 for select duration 1, from 2 to 94 for select durations 2 to 4, and from 0 to 119 for the ultimate rates for the proposed five-year select table, AM80(5), are also shown in Table A1 in Appendix A, and details of the formulae and parameters are shown in Section B1 in Appendix B.

Figure 1 shows a graph for the AM80 and AM80(5) tables, wherein the lines representing the rates (as percentages of the ultimate) for durations 0, 1 and 2-4 lie conformably below the horizontal line representing the ultimate rates (2+ and 5+). The line representing the percentage ratios of the ELT14M rates to the AM80 ultimate rates is also shown. It lies considerably above the 100% line, except for ages 96 to 104, where the quadratic adjustment for the duration 2+ rates was not quite enough to pull them down below those for ELT14M.

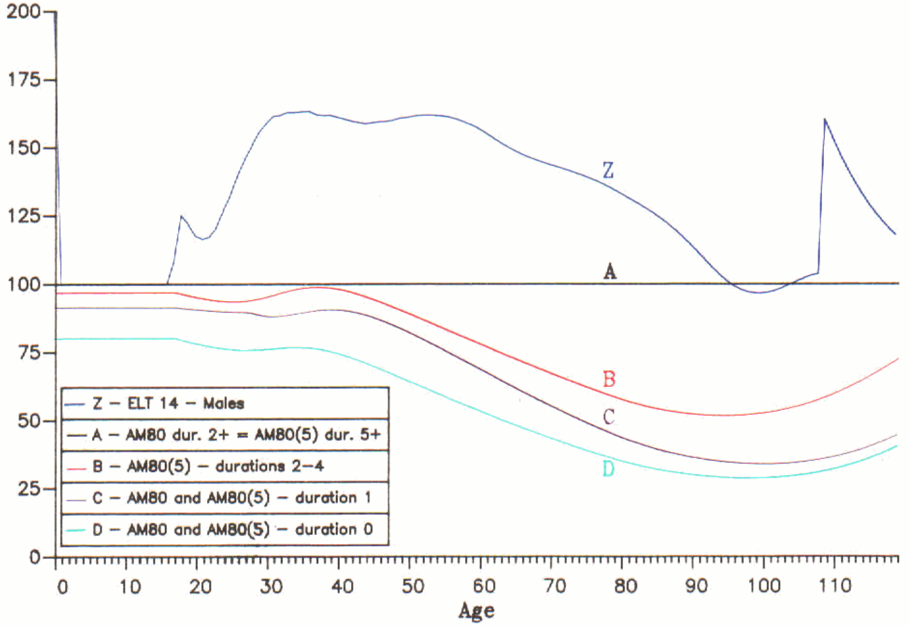

Figure 1. Permanent Assurances Males
AM80 and AM80(5) q_x as percent of q_x for AM80 Duration 2+

2.3.2 Permanent assurances, females - AF80

It was shown in the graduation report that there was good statistical justification for grouping durations 1, 2, 3 and 4 of the experience for female permanent assurances in the United Kingdom, to give a table with a five-year select period: duration 0, durations 1-4 and durations 5 and over. However, the general feeling at the meetings at the Faculty and the Institute seemed to be that a table for females with a two-year select period, as in the FA1975-78 table, would be more convenient for practical use. The Committee has therefore produced such a table.

When the graduated rates for the different durations were compared with one another and with the rates for male lives, it was apparent that the graduated rates for females for duration 1 required no adjustment, but that the rates for duration 0 were too low at the highest ages, and the rates for duration 2+ were too low at the youngest ages.

The graduated rates for duration 2+ below age 28 were therefore replaced by rates which bore a constant proportion to the rates for duration 1. Formally:

$$\mu_x^{D2+} = k.\mu_x^{D1} \qquad x < 28$$

where

$$k = \frac{\mu_{28}^{D2+}}{\mu_{28}^{D1}} = 1.08156196$$

The graduated rates for duration 0 above age 75 were replaced by adjusted rates which bore a constant ratio to the graduated rates for duration 1. Formally:

$$\mu_x^{D0} = k.\mu_x^{D1} \qquad x > 75$$

where

$$k = \frac{\mu_{75}^{D0}}{\mu_{75}^{D1}} = 0.46345460$$

Values for q_x were then calculated for integral ages from 17 to 90 for select duration 0, from 17 to 91 for select duration 1, and from 17 to 119 for duration 2+ (the 'ultimate' rates). Values of q_x for ages under 17 were derived as will be discussed in Section 2.4 below. The resulting table, the AF80 table, is shown in

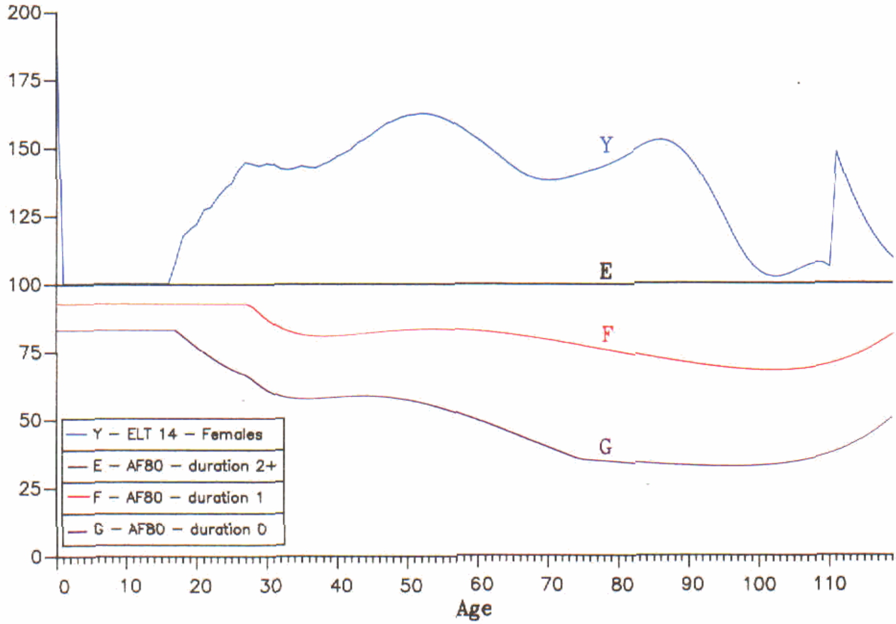

Figure 2. Permanent Assurances Females — AF80 q_x as percent of q_x for AF80 Duration 2+

full in Table A2 in Appendix A, and details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B2 in Appendix B.

Figure 2 shows the percentage ratios of the values of q_x, based on the adjusted values of μ_x for duration 0, duration 1 and duration 2+, to the value of q_x for duration 2+; the latter ratio is everywhere 100%. Also shown are the values of the ratios of English Life Table No 14 Females (ELT14F), which everywhere lie above the rates for duration 2+.

2.3.3 *Temporary assurances, males - TM80*

Separate investigations into the mortality experience of male lives assured for level temporary assurances and for decreasing temporary assurances have been carried out by the Bureau. In the graduation paper, however, the Committee showed that the levels of mortality in these two investigations in 1979-82 were sufficiently similar to justify combining them for the purposes of graduation. (The Committee subsequently decided to amalgamate the investigations as from the beginning of 1988.)

The mortality experience of the temporary assurances was shown to be rather lower than that of permanent assurances, and it seemed worth preparing a standard table based on this experience. The Committee suggested that a table with a five-year select period would be appropriate, with durations 1 to 4 combined. Three separately graduated tables therefore needed to be considered, duration 0, durations 1-4 and duration 5+.

Although the rates for duration 5+ were generally lower than those for the male permanent assurances duration 2+, the graduation formula caused them to rise well above the latter rates at younger ages. The rates for temporary assurances duration 5+ were therefore held down below age 32 to a constant ratio compared with the rates for permanent assurances duration 2+. Formally:

$$\mu_x^{D5+} = k.\mu_x^{D2+(AM)} \qquad x < 32$$

where

μ_x^{D5+} is the adjusted value of μ_x for male temporary assurances, duration 5+,

$\mu_x^{D2+(AM)}$ is the adjusted value of μ_x for male permanent assurances, duration 2+,

and

$$k = \frac{\mu_{32}^{D5+}}{\mu_{32}^{D2+(AM)}} = 0.98018301$$

The rates for durations 1-4 were generally below those for duration 5+, but rose above them slightly between ages 35 and 42. It was felt that, for a practical table, this feature should be eliminated, so the rates for durations 1-4 were held to a constant proportion of those for duration 5+ between ages 34 and a little over 44. Formally:

$$\mu_x^{D1\text{-}4} = k.\mu_x^{D5+} \qquad 34 < x < 44 + e$$

where

$$k = \frac{\mu_{34}^{D1\text{-}4}}{\mu_{34}^{D5+}} = 0.95021746$$

and e is such that

$$\frac{\mu_{44+e}^{D1\text{-}4}}{\mu_{44+e}^{D5+}} = k \quad \text{so that } e \text{ is approximately } 0.26$$

The graduated rates for duration 0 appeared to be too high at the youngest ages and rather too low at the highest ages. They were therefore adjusted to be a constant proportion of the rates for durations 1-4 at ages below 31 and above 65. Formally:

$$\mu_x^{D0} = k.\mu_x^{D1\text{-}4} \qquad x < 31$$

where

$$k = \frac{\mu_{31}^{D0}}{\mu_{31}^{D1\text{-}4}} = 0.80124215$$

and

$$\mu_x^{D0} = k.\mu_x^{D1\text{-}4} \qquad x > 65$$

where

$$k = \frac{\mu_{65}^{D0}}{\mu_{65}^{D1\text{-}4}} = 0.60410293$$

Values for q_x were then calculated for integral ages from 17 to 90 for select duration 0, from 17 to 94 for select durations 1-4, and from 17 to 119 for duration 5+ (the 'ultimate' rates). Values of q_x for ages under 17 were derived as will be discussed in Section 2.4 below. The resulting table, the TM80 table, is shown in full in Table A3 in Appendix A. Details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B3 in Appendix B.

Figure 3a shows percentage ratios of the values of q_x for duration 0, durations 1-4 and duration 5+, and also ELT14M, to the values of permanent assurances 2+ (AM80). Figure 3b shows the ratios for permanent assurances (durations 0, 2-4 and 5+) and for temporary assurances (durations 0, 1-4 and 5+), which allows these rates to be compared.

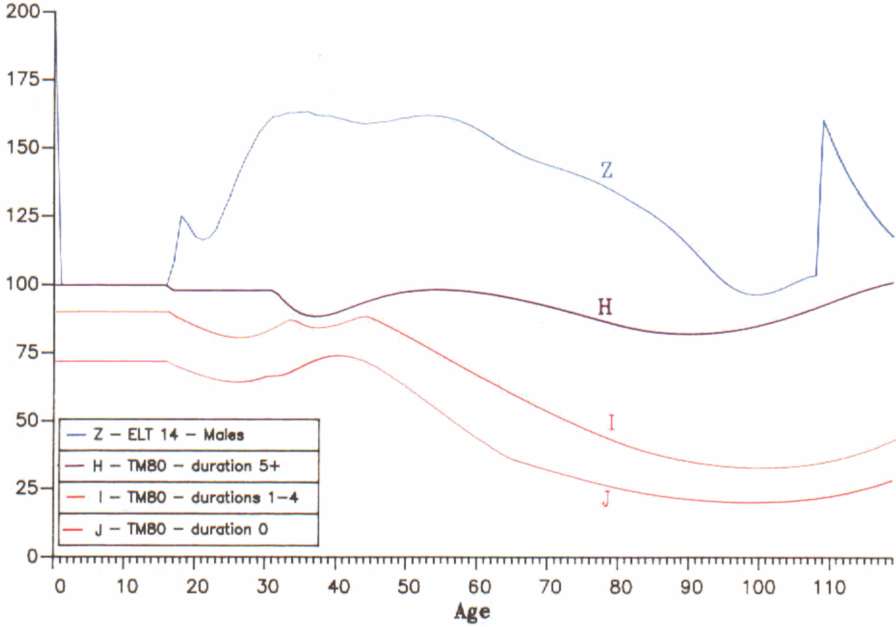

Figure 3a. Temporary Assurances Males — TM80 q_x as percent of q_x for AM80 Duration 2+

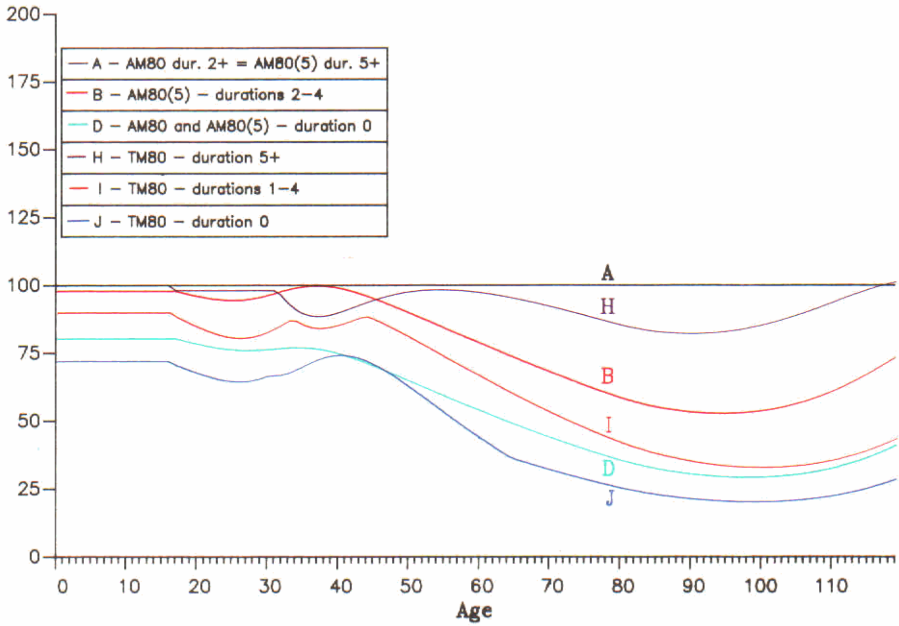

Figure 3b. Temporary and Permanent Assurances Males
TM80 and AM80 q_x as percent of q_x for AM80 Duration 2 +

2.3.4 *Pensioners*

The experience of life office pensioners retiring at or after their normal pension age was used as the basis of the Peg 1967-70 Experience Graduated Tables, on which the PL(90) and PA(90) projected tables were based. The investigation is carried out both on a Lives basis and on an Amounts basis, and these formed the basis respectively of the PL(90) and PA(90) tables. There is little experience for either sex below age 60, and the rates in the Peg 1967-70 tables below age 60 were based on those for assured lives, which it was thought might be similar to the rates for those at work, so that the resulting tables could, if desired, be used both before and after retirement.

The PL(90) and PA(90) tables are projected tables for calendar year 1990, based on projections from the base tables, the Peg 1967-70 graduated and adjusted tables, taking account of projected improvements in mortality.

Similar considerations applied on this occasion. In the graduation paper the Committee assumed that a new standard table, based on the pensioners experience and allowing for projected improvements in mortality, would be required by the profession. It should be based on the experience of those retiring at or after normal pension age, and not on the experience of early retirements. The rates at younger ages should be based on those for assured lives.

The first step was therefore to adjust the rates derived from the graduation formulae to provide satisfactory base tables for the years 1979-82. These will be called the Pxx80Base tables, with xx taking the values ML, MA, FL and FA for males Lives, males Amounts, females Lives and females Amounts respectively. The way in which projected tables have been derived is discussed in Section 4 below. The rates for males and for females are discussed in § 2.3.5 and 2.3.6 below.

2.3.5 *Male pensioners - PML80Base and PMA80Base*

The rates derived from the graduation formula for male pensioners Amounts were very close to those for male permanent assurances duration 2+ between ages 55 and 65, rising above the latter up to age 93, and then falling well below them at the highest ages. The rates for Lives derived from the graduation formula were generally higher than those for Amounts, but fell below them above age 92. A similar pattern of adjustments seemed to be needed in both cases.

The graduated rates for Amounts were therefore adjusted below age 55 to hold a constant ratio compared with the rates for permanent assurances (AM80) for duration 2+. Formally:

$$\mu_x^A = k.\mu_x^{D2+(AM)} \qquad x < 55$$

where

$$k = \frac{\mu_{55}^A}{\mu_{55}^{D2+(AM)}} = 1.00074607$$

The graduated rates for Amounts were then adjusted above age 93 by a quadratic adjustment, such that there was no change in the rate at age 93, and at age 110 the exponential term was multiplied by 1.25. The graduation formula used was GM(1,3), and the quadratic adjustment created a different GM(1,3) formula with parameters:

$$a_0 = 0.00200555$$

$$b_0 = -3.342787$$

$$b_1 = 4.056202$$

$$b_2 = -0.312522$$

The rates for Lives were adjusted below age 65 and above age 91 to hold constant ratios relative to the adjusted rates for Amounts. Formally:

$$\mu_x^L = k.\mu_x^A \qquad x < 65$$

where

$$k = \frac{\mu_{65}^L}{\mu_{65}^A} = 1.27871549$$

and

$$\mu_x^L = k.\mu_x^A \qquad x > 91$$

where

$$k = \frac{\mu_{91}^L}{\mu_{91}^A} = 1.01216060$$

Since the rates for Lives are based on the adjusted rates for Amounts, they are in practice based below age 55 on the rates for permanent assurances, and above age 93 on the rates for Amounts after the quadratic adjustment described above.

The PL(90) Tables and PA(90) Tables were extended down to age 20. This is well below what might be needed in practice for pensioners, but it was felt then that on occasion the tables might be used for calculating benefits for widows and widowers, or for active service mortality. The Committee is now publishing a separate table for widows, but has no data yet on which to base a table for widowers. The reasons for extending the table for males to younger ages therefore remain, and age 16 has been taken as the lowest age in the table.

The adjusted values of μ_x are appropriate only down to age 17. The derivation of the rate for age 16 is discussed in Section 2.4 below.

Values of q_x were calculated for integral ages from 17 to 119 inclusive. The resulting tables, the PML80Base and PMA80Base Tables, are shown in full in

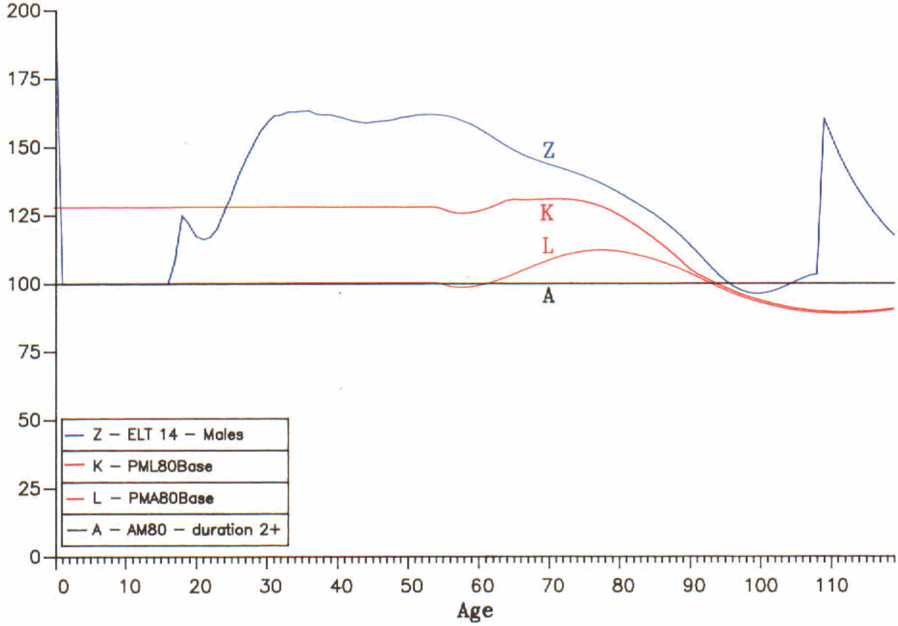

Figure 4. Male Pensioners — PM.80Base q_x as percent of q_x for AM80 Duration 2+

Table A4 in Appendix A, and details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B4 in Appendix B.

Figure 4 shows the percentage ratios of the adjusted values of q_x for Lives and for Amounts to those for AM80 duration 2+ and also those for ELT14M.

2.3.6 *Female pensioners - PFL80Base and PFA80Base*

The general considerations applying to pensioners tables have already been discussed in § 2.3.4. Similar considerations apply to the tables for female pensioners as to those for male pensioners and similar adjustments were also appropriate.

For ages below 67 the rates for female pensioners Amounts were taken as very close to those for female assured lives, duration 2+, after adjustment (AF80). Formally:

$$\mu_x^A = k.\mu_x^{D2+(AF)} \qquad x < 67$$

where

$\mu_x^{D2+(AF)}$ is the value of μ_x for female assured lives, duration 2+ (AF80)

and

$$k = \frac{\mu_{67}^{A}}{\mu_{67}^{D2+(AF)}} = 1.00769467$$

The rates for Amounts above age 95 were adjusted by a quadratic adjustment so that there was no change in the value of μ_x at age 95 and at age 110 the exponential term in the GM(1,3) formula was doubled. The resulting adjustment produced a different GM(1,3) formula with parameters:

$$a_0 = 0.00679085$$

$$b_1 = -2.138566$$

$$b_2 = 1.663488$$

$$b_3 = 0.492034$$

Although the graduated rates were based on a considerable body of experience for female pensioners between ages 60 and 67, the adjustments described above were not inconsistent with the data, as will be commented on further in Section 3 below.

The rates for Lives were adjusted at the lowest and highest ages in a similar way. Below age 67 the rates for Lives were taken as a constant ratio of the adjusted rates for Amounts, and hence were based on the adjusted rates for female permanent assurances duration 2+. Formally:

$$\mu_x^L = k.\mu_x^A \qquad x < 67$$

where

$$k = \frac{\mu_{67}^{L}}{\mu_{67}^{A}} = 1.17657278$$

Above age 95 the rates for Lives were also taken as a constant factor times the adjusted rates for Amounts, i.e. the rates after application of the quadratic adjustment. Formally:

$$\mu_x^L = k.\mu_x^A \qquad x > 95$$

where

$$k = \frac{\mu_{95}^{L}}{\mu_{95}^{A}} = 1.21543261$$

The adjusted values of μ_x are appropriate only down to the age 17. The derivation of the rate for age 16 is discussed in Section 2.4 below.

Values of q_x were calculated for integral ages from 17 to 119. The resulting tables, the PFL80Base and PFA80Base Tables, are shown in full in Table A4 in

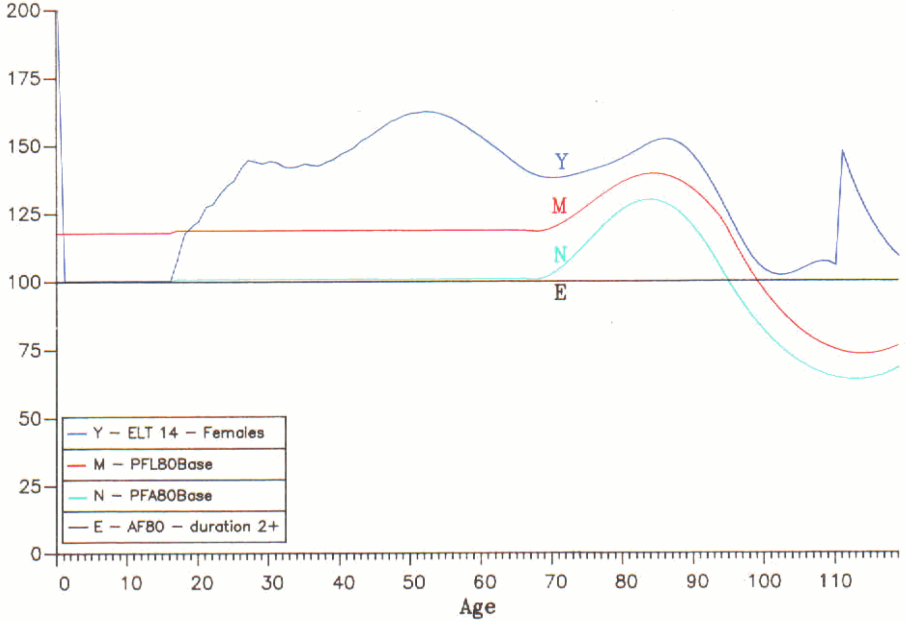

Figure 5. Female Pensioners — PF.80Base q_x as percent of q_x for AF80 Duration 2+

Appendix A, and details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B5 in Appendix B.

Figure 5 shows the percentage ratios of the adjusted values of q_x for Lives and for Amounts to those for AF80 duration 2+ and also those for ELT14F.

2.3.7 *Annuitants*

Previous tables for immediate annuitants, the *a*(55), *aeg* 1967-70 and *a*(90) Tables, for both males and females, were based only on the experience gathered by Lives, since at the time the experience by Amounts had not been collected. On this occasion the experience by both Lives and Amounts was available. However, in the graduation paper it was shown that the experience by Amounts was not very different from that for Lives, and the Committee therefore decided to construct tables, as before, only on the Lives basis.

Earlier tables for annuitants had been constructed using a one-year select period. In the graduation paper it was shown that there would be statistical justification for a five-year select period, with durations 0 to 4 combined and duration 5+ separate. The practical advantages of retaining a one-year select period seemed sufficiently great, however, for the earlier pattern to be retained.

The Committee has therefore constructed a table with only a one-year select period, with rates for duration 0 and duration 1+.

There is little experience for either sex below age 65, but in a table for practical use it may occasionally be found necessary to have rates for much younger ages. The tables have therefore been continued down to age 16, as was done for pensioners. The rates for younger ages for both sexes (males below age 65 and females below age 52) have been based on the rates for permanent assurances for the corresponding sex.

2.3.8 *Male annuitants - IM80Base*

The graduated rates for male annuitants duration 1+ were graduated with a simple GM(0,2) formula. This produced rates which were reasonably close to those for male permanent assurances duration 2+ after adjustment, being lower than the latter between ages 77 and 104 and higher below and above this age range. It seemed sufficient to adjust the rates below age 65 to maintain a constant ratio with those for permanent assurances duration 2+. Formally:

$$\mu_x^{D1+} = k.\mu_x^{D2+(AM)} \qquad x < 65$$

where

$\mu_x^{D2+(AM)}$ is the value of μ_x for male permanent assurances, duration 2+ (AM80)

and

$$k = \frac{\mu_{65}^{D1+}}{\mu_{65}^{D2+(AM)}} = 1.16879491$$

The rates for duration 0 were adjusted below age 65 so that they remained at a suitable distance below the rates for duration 1+, and also above age 80 so that they did not fall too far below the rates for duration 1+.

The rates below age 65 were held as a constant proportion of the adjusted rates for duration 1+. Formally:

$$\mu_x^{D0} = k.\mu_x^{D1+} \qquad x < 65$$

where

$$k = \frac{\mu_{65}^{D0}}{\mu_{65}^{D1+}} = 0.96155011$$

The rates above age 80 were also held as a constant proportion of the rates for duration 1+. Formally:

$$\mu_x^{D0} = k.\mu_x^{D1+} \qquad x > 80$$

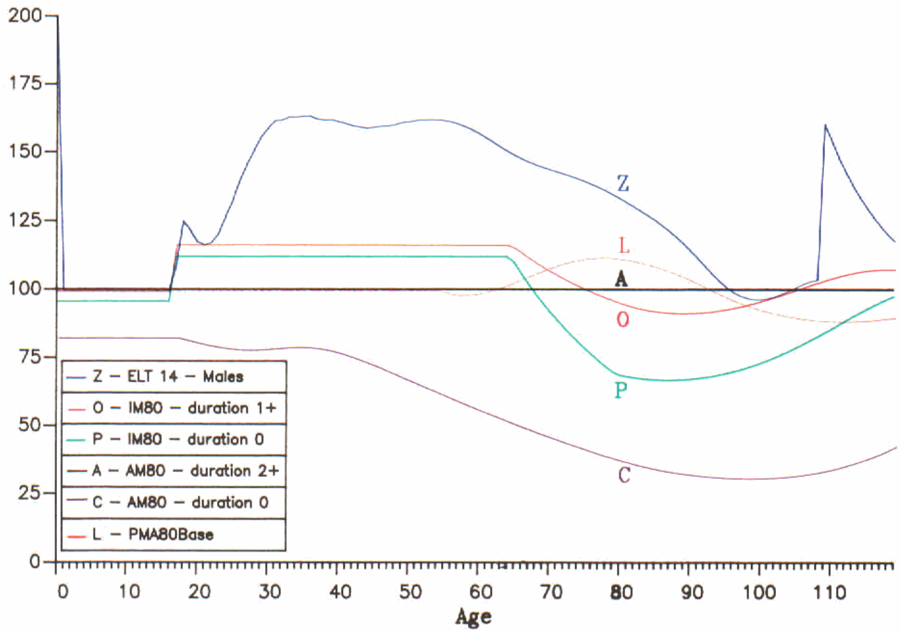

Figure 6. Male Annuitants — IM80Base q_x as percent of q_x for AM80 Duration 2+

where

$$k = \frac{\mu_{80}^{D0}}{\mu_{80}^{D1+}} = 0.71341513$$

Values of q_x were then calculated for integral ages from 17 to 100 for select duration 0 and 17 to 119 for duration 1+ (the 'ultimate' rates). Values of q_x for age 16 were derived as will be discussed in Section 2.4 below. The resulting table, the IM80Base table, is shown in full in Table A5 in Appendix A. Details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B6 in Appendix B.

Figure 6 shows percentage ratios of the values of q_x for duration 0 and duration 1+ to those for AM80 duration 2+, and also includes the ratios for AM80 duration 0, PMA80Base and ELT14M.

2.3.9 *Female annuitants - IF80Base*

The same general considerations apply to the table for female annuitants as to that for male annuitants. The bulk of the 1979-82 experience lies above age 65, and in order to extend the table down to younger ages it is desirable to take account of the rates for female permanent assurances. It is perhaps surprising to discover that the graduated rates for female annuitants duration 1+ lie above those for permanent assurances from age 52 upwards. By age 100 the graduated μ_x for annuitants duration 1+ is 30% higher than that for permanent assurances duration 2+. There is a sufficiently large experience in both investigations to support these observations.

The Committee felt, however, that it was necessary to reduce the graduated rates at the highest ages as well as at the lower ages. At the lower ages it was decided to base the rates for duration 1+ on the rates for permanent assurances for duration 2+, and to base the rates for duration 0 on the adjusted rates for duration 1+. Formally:

$$\mu_x^{D1+} = k.\mu_x^{D2+(AF)} \qquad x < 52$$

where

$$k = \frac{\mu_{52}^{D1+}}{\mu_{52}^{D2+(AF)}} = 1.00069170$$

and

$$\mu_x^{D0} = k.\mu_x^{D1+} \qquad x < 52$$

where

$$k = \frac{\mu_{52}^{D0}}{\mu_{52}^{D1+}} = 0.76087110$$

The rates for duration 1+ crossed those for female pensioners Amounts just below age 90, say at exact age e. The value of e is approximately 89.99. The following formula was used for ages above this point:

$$\mu_x^{D1+(Adj)} = \frac{(\mu_x^{D1+(Grad)} + \mu_x^{PFA})}{2} \qquad x > e$$

where

$\mu_x^{D1+(Adj)}$ is the adjusted value of μ_x for female annuitants duration 1+

$\mu_x^{D1+(Grad)}$ is the graduated value of μ_x for female annuitants duration 1+

and

μ_x^{PFA} is the adjusted value of μ_x for female pensioners Amounts (PFA80Base).

The rates for duration 0 at ages above 89 were based on the adjusted rates for duration 1+, so that

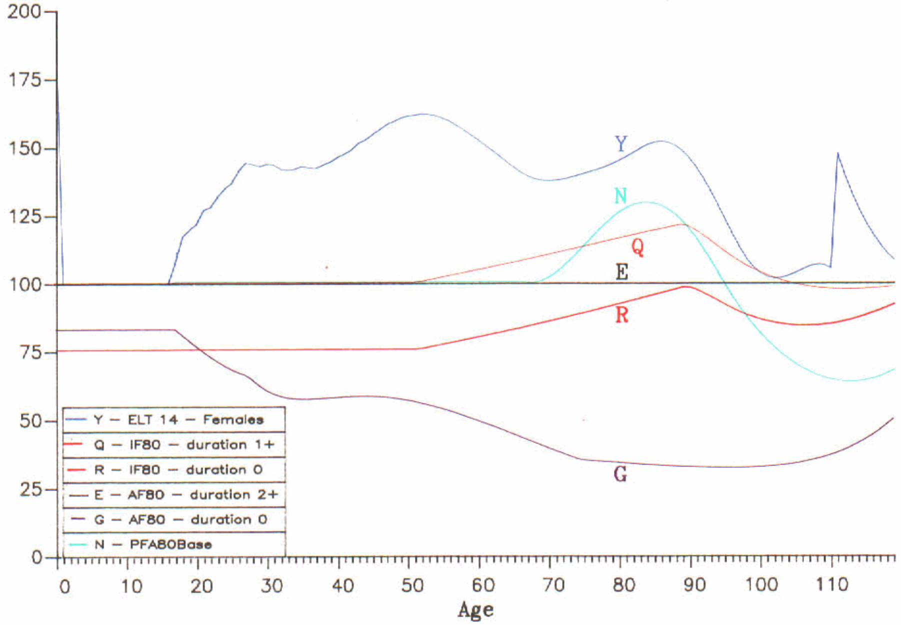

Figure 7. Female Annuitants — IF80Base q_x as percent of q_x for AF80 Duration 2+

$$\mu_x^{D0} = k.\mu_x^{D1+} \qquad x > 89$$

where

$$k = \frac{\mu_{89}^{D0}}{\mu_{89}^{D1+}} = 0.80284353$$

Values of q_x were then calculated for integral ages from 17 to 100 for select duration 0 and 17 to 119 for duration 1+ (the "ultimate" rates). Values of q_x for age 16 were derived as will be discussed in Section 2.4 below. The resulting table, the IF80Base table, is shown in full in Table A5 in Appendix A, and details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B7 in Appendix B. Figure 7 shows percentage ratios of the values of q_x for duration 0 and duration 1+ to those for AF80 duration 2+, and also includes the ratios for AF80 duration 0, PFA80Base and ELT14F.

2.3.10 *Widows - WL80Base and WA80Base*
This is the first occasion that the experience of widows of life office pensioners has been available to the Committee. The investigation has been carried out

both on a Lives and on an Amounts basis. In the graduation report, the Committee showed that a simple GM(0,2) formula would fit both the Lives data and the Amounts data satisfactorily. However, when extrapolated beyond the main age range, which was from about age 60 to age 90, the rates appeared unrealistically high, as compared with the adjusted rates for pensioners.

The rates for widows Amounts were therefore based on the adjusted rates for female pensioners Amounts below age 45 (and hence on the permanent assurances duration 2+ rates), and on the rates for female pensioners Amounts (after quadratic adjustment) above age 90. Formally:

$$\mu_x^A = k.\mu_x^{PFA} \qquad x < 45$$

where

$$k = \frac{\mu_{45}^A}{\mu_{45}^{PFA}} = 1.49458889$$

and

$$\mu_x^A = k.\mu_x^{PFA} \qquad x > 90$$

where

$$k = \frac{\mu_{90}^A}{\mu_{90}^{PFA}} = 1.11235210$$

The rates for widows Lives at younger ages were based on those for widows Amounts. Formally:

$$\mu_x^L = k.\mu_x^A \qquad x < 55$$

where

$$k = \frac{\mu_{55}^L}{\mu_{55}^A} = 1.43336009$$

Finally, the rates for widows Lives were based directly on those for female pensioners Lives at ages above 81. This keeps them above those for widows Amounts. Formally:

$$\mu_x^L = k.\mu_x^{PFL} \qquad x > 81$$

where

$$k = \frac{\mu_{81}^L}{\mu_{81}^{PFL}} = 1.03414340$$

It is possible, though rare, for widows to be as young as age 16, so the table has been extended down to this age. The derivation of the rate for age 16 is discussed in Section 2.4 below.

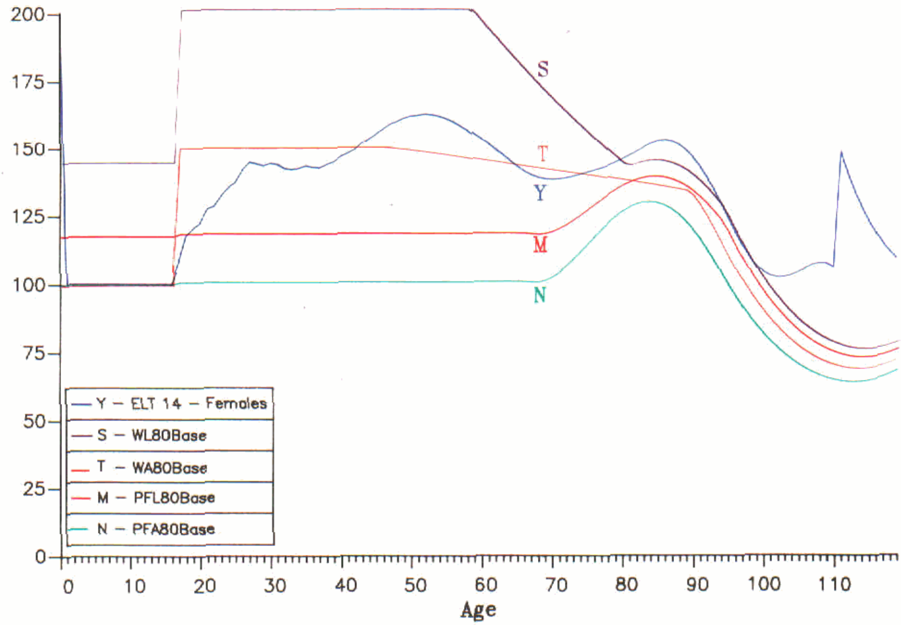

Figure 8. Widows — W80Base q_x as percent of q_x for AF80 Duration 2 +

Values of q_x were calculated for integral ages from 17 to 119 inclusive. The resulting tables, the WL80Base and WA80Base Tables, are shown in full in Table A6 in Appendix A. Details of the formulae and parameter values for μ_x for the adjusted rates are shown in Section B8 in Appendix B. Figure 8 shows percentage ratios of the adjusted values of q_x for Lives and for Amounts to those for AF80 duration 2+, and also those for female pensioners Lives and Amounts and for ELT14F.

2.4 EXTENSIONS TO YOUNG AGES

The published rates of mortality in the A1967–70 Tables for Assured Lives were based on the graduated rates from age 17 upwards. Below that age the experience was too sparse to provide any realistic rates for assured lives, and the published rates were based on those for male population mortality at that time. It was observed that this produced a discontinuity in the value of μ_x at age 17, which can be justified on the basis of population experience, and explained by the considerable increase in accidents, particularly road traffic accidents, at about this age. A similar feature is found in the population mortality rates for females, though the extent of the rise is considerably smaller.

It is convenient for some purposes to have mortality rates for assured lives available at childhood ages, and the Committee has therefore extended the tables for male and female assured lives and for male temporary assurances down to age 0, as has already been noted in Section 2.3. The Committee has also extended the tables for pensioners, annuitants and widows down to age 16, as has also been noted in Section 2.3.

The graduated rates based on the experience are only reasonable down to age 17. Below that age it is appropriate to have regard to population mortality. On investigation it was found that the graduated rates at ages 17 and above blended satisfactorily with the graduated rates based on England and Wales population mortality which have been published as ELT14M and ELT14F.

The values of q_x below age 17 were therefore derived from the rates in ELT14M and ELT14F. 'Basic' rates were defined as follows: at ages 1 to 16 the 'basic' rates were taken as the same as the ELT14M and ELT14F rates; at age 0 the 'basic' rates were derived from the population experience on which ELT14 was based, but omitting mortality in the first month of life. These basic rates are shown in Table 2.4 and how these basic rates were used for each table is described below.

Table 2.4 *Basic rates for young ages derived from ELT14*
Values of 10,000 q_x

Age	Males	Females
0	130	112
1	85	72
2	51	45
3	38	31
4	35	25
5	32	22
6	30	20
7	27	19
8	25	19
9	24	18
10	24	18
11	24	18
12	26	18
13	29	19
14	34	22
15	41	26
16	53	30

2.4.1 *Permanent assurances, males – AM80 and AM80(5)*

For these tables the ultimate rates (for durations 2+ and 5+) were taken as equal to the basic rates. The rates for duration 0, duration 1 and durations 2-4 were taken as constant proportions of the basic rates, the proportions being equal to the ratios of the values of q_{17} for duration 0, duration 1 and durations 2-4 to q_{17} for duration 2+. Formally:

$$q_x^{D0} = k_0 q_x^{\text{Basic}} \qquad x < 17$$

$$q_x^{D1} = k_1 q_x^{\text{Basic}}$$

and

$$q_x^{D2\text{-}4} = k_2 q_x^{\text{Basic}}$$

where

$$k_0 = \frac{q_{17}^{D0}}{q_{17}^{D2+}} = 0.805763$$

$$k_1 = \frac{q_{17}^{D1}}{q_{17}^{D2+}} = 0.923159$$

and

$$k_2 = \frac{q_{17}^{D2\text{-}4}}{q_{17}^{D2+}} = 0.977588$$

2.4.2. *Permanent assurances, females – AF80*

Rates for ages 0 to 16 for durations 0, 1 and 2+ were derived using the same methods as for males. The values of q_x for duration 2+ were set as equal to the basic rates for females, and the values of q_x for duration 0 and duration 1 were taken by multiplying the basic rates by the ratios of the values of q_{17} for the respective tables. The formulae are the same as shown in § 2.4.1 above, with different numerical values, viz:

$$k_0 = \frac{q_{17}^{D0}}{q_{17}^{D2+}} = 0.836601$$

and

$$k_1 = \frac{q_{17}^{D1}}{q_{17}^{D2+}} = 0.924837$$

2.4.3 *Temporary assurances, males - TM80*

The values of q_x for duration 5+ for ages 0 to 16 were set equal to the basic rates for males. The values of q_x for duration 0 and durations 1-4 were calculated by multiplying the basic rates by the ratios of the values of q_{17}, in the same way as for male permanent assurances, with different numerical values, viz:

$$k_0 = \frac{q_{17}^{D0}}{q_{17}^{D5+}} = 0.720348$$

$$k_1 = \frac{q_{17}^{D1\text{-}4}}{q_{17}^{D5+}} = 0.898803$$

2.4.4 *Pensioners - PML80Base, PMA80Base, PFL80Base and PFA80Base*
For each sex

$$q_{16}^{A} = q_{16}^{\text{Basic}}$$

$$q_{16}^{L} = \frac{q_{17}^{L}}{q_{17}^{A}} q_{16}^{\text{Basic}}$$

2.4.5 *Annuitants - IM80Base and IF80Base*
For each sex

$$q_{16}^{D1+} = q_{16}^{\text{Basic}}$$

$$q_{16}^{D0} = \frac{q_{17}^{D0}}{q_{17}^{D1+}} q_{16}^{\text{Basic}}$$

2.4.6 *Widows - WL80Base and WA80Base*
The basic rates for females were used, viz:

$$q_{16}^{A} = q_{16}^{\text{Basic}}$$

$$q_{16}^{L} = \frac{q_{17}^{L}}{q_{17}^{A}} q_{16}^{\text{Basic}}$$

2.5 THE LIMITING AGE OF THE TABLES

The new standard tables contain values of q_x up to age 119, as shown in the tables in Appendix A. For each table the value of q_{119} is given. It is possible to calculate, choosing a suitable radix, the values of l_x up to l_{119}, and by using the value of q_{119} the value of l_{120} can be derived.

For all tables, it is assumed that the value of q_{120} equals 1, so that $p_{120} = 0$, $l_{121} = 0$, $d_{120} = l_{120}$, $\ddot{a}_{120} = 1$, $a_{120} = 0$ and $A_{120} = v$.

Further:

$$D_{120} = N_{120} = S_{120} = l_{120} v^{120}$$

and

$$C_{120} = M_{120} = R_{120} = d_{120}v^{121}$$

2.6 RECALCULATION OF μ_x FROM ROUNDED VALUES OF q_x

2.6.1 In standard tables prepared by the Committee in the past, the values of q_x have been derived first, and values of μ_x have been calculated therefrom, generally by using simple finite difference methods. In this section we describe how values of $\mu_{[x-t]+t}$ or μ_x can be calculated from the published values for q_x. At first sight it may seem curious that one should wish to do this, since the values of q_x have already been derived from graduated values of μ_x. As will be explained below, the graduated values of μ_x for the select tables are not the appropriate ones for normal actuarial use, and for ultimate or aggregate tables there are certain conveniences in being able to recalculate the values of μ_x.

We first consider ultimate (duration 1+, 2+ or 5+) rates and aggregate (Lives or Amounts) rates. Values of q_x for ages 17 to 119 have been calculated accurately from the graduated and adjusted values of μ_x. These accurate values of q_x have been rounded to six decimal places as shown in the tables in Appendix A. There is a small practical convenience and some economy in being able to rederive the values of μ_x from those of q_x, by appropriate formulae. For this to be satisfactory the errors of approximation should not be too great. Experiments showed that the formulae described below reproduce the original values of μ_x in most cases exactly to six decimal places, with some ages at which the error was one unit in the sixth decimal place, and a very few ages where the error was greater than this. Further experiments show that about half these errors were caused by the rounding of q_x to six decimal places. When more decimal places were retained in the values of q_x the rederived values of μ_x were even closer to the original values.

For practical purposes, therefore, the formulae described below reproduce the original values of μ_x for ultimate and aggregate tables as accurately as is necessary.

As has been described in Section 2.4, the values of q_x for ages below 17 were derived from the values of q_x from ELT14. For these ages no 'source' values of μ_x exist. It was therefore necessary to derive the values of μ_x from those of q_x in the traditional way. The formulae used are also described below.

It was stated above that for select durations the graduated (and sometimes adjusted) values of μ_x were not what was required for normal actuarial use. The reason for this will now be explained. The experience described as 'duration 0' is in fact the experience for curtate duration 0, i.e. between exact durations 0 and 1, at exact duration $\frac{1}{2}$ on average. The graduated μ_x for 'duration 0' therefore represents $\mu_{[x-\frac{1}{2}]+\frac{1}{2}}$, and not $\mu_{[x]}$. Similar arguments apply to duration 1 and to higher select durations. The method of graduation assumed that the value of μ_x was constant over the appropriate duration, and from this a value of q_x for the appropriate duration was derived. For the calculation of monetary functions in which μ_x may be required a different assumption is made: it is assumed that $\mu_{[x-t]+t}$ varies continuously, both by age and by duration, until the 'ultimate' duration is reached. Thus the value of $\mu_{[x]}$ is the value of the force of mortality at the beginning of the year to which $q_{[x]}$ applies, and the value of $\mu_{[x-1]+1}$ is a value appropriate both to the end of the year to which $q_{[x-1]}$ applies and the beginning of the year to which $q_{[x-t]+t}$ applies. This continues throughout the select period, and then there is a possibly abrupt step as the ultimate duration is entered, the values of q_x having been derived simply from the ultimate experience, and without regard to how the select rates run in.

The formulae used for deriving values of μ_x from those of q_x are described below.

2.6.2 It is assumed that, over an appropriate age range, the force of mortality is a polynomial function of age of low degree. (See McCutcheon, 1983.) Over most of the age-range of each published table the value of μ_x (for each integer x) is determined on the basis that between exact age $(x-2)$ and exact age $(x+2)$ the force of mortality is a cubic function of age. An alternative method of calculation is, however, required at the extreme ends of each life table, in the late teens (where general population mortality is blended with each particular graduation), and for select values of the force of mortality. Recall that for each table $q_{120} = 1$, so that the limiting age ω is 121. The method of calculation is summarised below.

Let

$$\lambda_x = -\log(1 - q_x) = \operatorname{colog} p_x$$

For ultimate functions the relevant assumptions are as follows.

(a) For $2 \leqslant x \leqslant 15$ and for $19 \leqslant x \leqslant 118$

$$\mu_x = \frac{-\lambda_{x-2} + 7\lambda_{x-1} + 7\lambda_x - \lambda_{x+1}}{12} \tag{2.6.1}$$

(b) For special values of x

$$\mu_0 = \frac{11\lambda_0 - 7\lambda_1 + 2\lambda_2}{6}$$

$$\mu_1 = \frac{2\lambda_0 + 5\lambda_1 - \lambda_2}{6}$$

$$\mu_{16} = \frac{-\lambda_{14} + 5\lambda_{15} + 2\lambda_{16}}{6}$$

$$\mu_{17} = \frac{11\lambda_{17} - 7\lambda_{18} + 2\lambda_{19}}{6}$$

$$\mu_{18} = \frac{2\lambda_{17} + 5\lambda_{18} - \lambda_{19}}{6}$$

$$\mu_{119} = \frac{-\lambda_{117} + 5\lambda_{118} + 2\lambda_{119}}{6}$$

These last six equations have been derived on the assumption that, over an appropriate age-range, the force of mortality is a quadratic function of age. For the tables for pensioners and annuitants, where the lowest age for which q_x is published is 16, λ_{14} and λ_{15} are derived from 'hypothetical' values q_{14} and q_{15}, calculated in the same manner as q_{16}. (See § 2.4.4, § 2.4.5 and § 2.4.6 above.)

In relation to the select functions, where the select period is n years, it is convenient to define, for $d = 0, 1, \ldots, n$,

$$q_x^d = q_{[x-d]+d}$$

$$\mu_x^d = \mu_{[x-d]+d}$$

and

$$\lambda_x^d = -\log(1 - q_x^d)$$

Thus $q_x^n = q_x$, the ultimate rate of mortality at age x. Note that, if the select period is two years, q_x^2 denotes the ultimate rate of mortality but that, if the select period is five years, q_x^2 denotes the select rate at duration 2.

The select values are obtained on the assumption that over an appropriate age-range the force of mortality is a linear function of age, as follows.

(a) Duration 0.

(i) For $0 \leqslant x \leqslant 15$ and $x \geqslant 17$

$$\mu_x^0 = \frac{3\lambda_x^0 - \lambda_{x+1}^1}{2}$$

(ii) $$\mu_{16}^0 = \frac{3\lambda_{16}^0 - \lambda_{17}^{1*}}{2}$$

where

$$\lambda_{17}^{1*} = -\log(1 - q_{17}^{1*})$$

with q_{17}^{1*} obtained by linear extrapolation from q_{15}^1 and q_{16}^1 as

$$q_{17}^{1*} = 2q_{16}^1 - q_{15}^1$$

The special value λ_{17}^{1*} is required to maintain regularity.

(b) Durations 1–4.

(i) For $1 \leqslant x \leqslant 16$ and $x \geqslant 18$ and $d = 1,2,3,4$ (as appropriate)

$$\mu_x^d = \frac{\lambda_{x-1}^{d-1} + \lambda_x^d}{2}$$

(ii) $$\mu_0^d = \frac{3\lambda_0^d - \lambda_1^{d+1}}{2}$$

$$\mu_{17}^d = \frac{3\lambda_{17}^d - \lambda_{18}^{d+1}}{2}$$

except for two special values (to maintain regularity):

$$\mu_0^4 = \mu_0^3 \text{ (calculated as above)}$$

and

$$\mu_{17}^4 = \mu_{17}^3 \text{ (calculated as above).}$$

3. COMPARISON OF ADJUSTED RATES WITH EXPERIENCE

3.1 GENERAL CONSIDERATIONS

The graduated rates which have formed the basis of the new standard tables were based on the corresponding experiences for assured lives, pensioners, annuitants or widows for 1979-82, as described in the graduation paper. In each case the order of the GM(r,s) formula was chosen so that a satisfactory fit was obtained, and the values of the parameters were obtained by maximum likelihood estimation. These graduated rates have been adjusted as described in Section 2 in various ways, and it is reasonable to enquire whether the adjusted rates can also be considered to represent satisfactorily the corresponding 1979-82 experience, or whether they have been adjusted beyond the bounds of fidelity to the data in the interests of adherence to other criteria of reasonableness, i.e. compatibility with other experiences.

A number of criteria can be considered in this context. Some of these are shown in Table 3.1, which summarises statistics for all the experiences considered, and others are commented on in the sub-sections which follow, referring to each table.

First is the ratio of actual deaths to those expected according to the adjusted table. For the graduated rates the method used was such that when the parameters were estimated by the maximum likelihood method the expected number of deaths always equalled the actual number of deaths, so the ratio $100A/E$ always equalled 100. For the adjusted rates the ratios are shown in Table 3.1. They are all very close to 100. The highest value is 102.8, for temporary assurances, males, duration 0; and the lowest value is 96.5, for male annuitants, duration 0. Of the 21 separate sections shown in the Table, 13 have ratios in the range 99.0 to 101.0.

The ratio of actual to expected is not a test of significance. It is possible for the actual and expected deaths to be equal, and yet for a graduation to give a very poor fit; and conversely, when the numbers of deaths are small, it is possible for the actual to diverge to a fair extent from the expected, but for this not to be statistically significant. The next test used is that of the change in the value of the log likelihood function. This is described in Table 3.1 as 'Change in L'. A positive value means that the adjusted rates give a *higher* value of the log likelihood than the graduated rates, that is they give a better fit to the data. A negative value for the change in L shows a worse fit. Note that a change of 2 units in the value of L would not be considered a significant change.

Of the 21 experiences considered, nine show a positive change in L, though none of these changes is significant, and a further four show a negative change in L which is not of a significant size. Eight show reductions greater than 2.0, and these require further consideration. They are: permanent assurances, males, duration 5+; permanent assurances, females, duration 2+; temporary assur-

Table 3.1. *Statistics for comparisons of adjusted rates with experience*

	$100A/E$	Change in L	p(runs) adjusted	$p(\chi^2)$ graduated	$p(\chi^2)$ adjusted
Permanent Assurances, Males:					
Duration 0	100·3	+1·83	0·50	0·08	0·19
Duration 1	101·2	+1·32	0·27	0·42	0·44
Duration 2+	100·1	+1·73	0·13	0·0004	0·0018
Durations 2-4	100·0	+1·66	0·27	0·13	0·20
Duration 5+	101·4	−5·57	0·007	0·0007	0·0002
Permanent Assurances, Females:					
Duration 0	100·8	−0·53	0·44	0·11	0·16
Duration 1	100·2	+0·19	0·14	0·33	0·42
Duration 2+	99·6	−2·52	0·002	–	–
Temporary Assurances, Males:					
Duration 0	102·8	−6·24	0·79	0·54	0·31
Durations 1-4	101·4	−2·81	0·04	0·39	0·46
Duration 5+	100·2	+0·03	0·71	0·86	0·89
Pensioners:					
Males, Lives	100·0	−6·78	0·51	0·11	0·03
Males, Amounts	100·0	+0·13	0·91	–	–
Females, Lives	100·8	−1·26	0·98	0·008	0·004
Females, Amounts	102·0	−23·28	0·73	–	–
Annuitants:					
Males Duration 0	96·5	−1·59	0·87	0·14	0·32
Males Duration 1+	100·1	−2·34	0·68	–	–
Females Duration 0	101·4	−0·35	0·21	0·29	0·33
Females Duration 1+	101·9	−4·61	0·27	0·002	–
Widows:					
Lives	99·1	+1·36	0·18	0·50	0·83
Amounts	100·2	+0·42	0·84	–	–

ances, males, duration 0 and duration 1-4; male pensioners, Lives; female pensioners, Amounts; male annuitants, duration 1+; and female annuitants, duration 1+. It should be noted that four of these eight relate to 'ultimate' sections of their experiences, where there may be duplicate policies on one life to an extent greater than allowed for; and one relates to an Amounts experience, where again there may be more 'duplicate pounds' than have been allowed for.

The next test considered and summarised in Table 3.1 is the runs test. The value shown is that of p(runs), the probability of obtaining a number of runs less than or equal to the actual number, in the light of the number of positive and negative values of $(A - E)$ at each age (or group of ages where the numbers are small). Extreme values at either end of the range are significant, but a high value indicates that the differences tend to alternate between positive and negative, which is a sign of overgraduation rather than of an ill-fitting graduation. There are only two significantly low values in the table, those for permanent assurances, males, duration 5+ and permanent assurances, females, duration 2+. In both these cases the number of runs in the original graduation was significantly low, and the adjusted rates are no worse.

The classic actuarial test for graduation is the χ^2 test. For this to be valid, it is necessary to group ages with small numbers of cases so that the expected number of deaths is at least 5. Because the basis on which the expected numbers of deaths are calculated is different for the adjusted rates from the graduated rates, the grouping of ages is not necessarily the same. It is therefore convenient to compare by using $p(\chi^2)$, the probability of obtaining a value of χ^2 greater than the observed value.

In Table 3.1 the values of $p(\chi^2)$ for the original graduation and for the adjusted rates are shown. Where the value of $p(\chi^2)$ is increased it can be argued that the adjusted rates give a better fit to the data than the original graduation. (There are, however, problems in deciding the appropriate number of degrees of freedom.) In 11 of the 21 cases the value of $p(\chi^2)$ is increased, and these generally correspond with those experiences where the change in L has been positive. It is not necessarily appropriate to calculate χ^2 for Amounts data, and in a few cases the value of χ^2 is so large that the probability of obtaining such a value by chance is extremely small. In only one case, that of male pensioners Lives, is the value of $p(\chi^2)$ reduced from within a 5% significance level to beyond that level, from 0.11 to 0.03. In all other cases where the change in L is significantly negative, either the value of χ^2 is comfortably within a 5% level (as for temporary assurances, males, duration 0 and durations 1-4) or is very far outside it (as for several of the 'ultimate' experiences).

The tests described so far apply to the whole experience; yet in most cases the original graduated rates have been adjusted only over a small range of ages. It is reasonable therefore to look also at the numbers of actual and expected deaths over these age ranges, and to see whether any of these show an exceptionally large difference. The results of these tests are shown in Table 3.2 and are discussed in Section 3.2 for each experience separately.

3.2 COMPARISON OF EACH EXPERIENCE

In Table 3.2 we show certain statistics that provide a comparison between the original graduations and the adjusted rates for the specific age ranges where the rates have been adjusted. For each section of each experience the rates have been altered at least below age 17. For some experiences there is no exposure at these ages, but in every such case the rates at the low end of the age range, but above age 17, have been adjusted, and there is at least some exposure, even if small, at the relevant ages. In many cases the adjusted rates lie at the upper end of the age range of exposures. In only one case (temporary assurances, durations 1-4) have the rates been adjusted in the middle of the age range.

Table 3.2. *Comparisons of adjusted and graduated rates for specific age ranges*

	Age Range	Actual Deaths	Graduated Basis 100*A*/*E*	Graduated Basis χ^2	Graduated Basis df	Adjusted Basis 100*A*/*E*	Adjusted Basis χ^2	Adjusted Basis df
1 *Permanent assurances, males*								
Duration 0	10–17	18	64·9	3·8	2	80·1	0·9	1
Duration 1	10–27	332	98·6	12·1	11	107·1	14·6	11
Duration 2+	10–18	18φ	99·6	1·2	2	129·5	1·2	1
	81–90	4,091φ	99·3	17·6	10	100·6	13·1	10
Durations 2–4	10–18	13	88·3	1·3	2	114·8	0·2	1
Duration 5+	10–90	52,380φ	100·0	109·0	66**	101·4	118·0	70**
2 *Permanent assurances, females*								
Duration 0	10–18	12	144·7	1·7	1	152·9	2·2	1
	75–93	2	30·6	3·1	1	29·0	3·5	1
Duration 1	10–21	17	88·6	1·1	3	89·5	0·5	1
Duration 2+	10–27	129·3φ	115·2	14·0	9	98·6	16·2	9
3 *Temporary assurances, males*								
Duration 0	10–30	72	97·9	8·7	10	122·0	15·0	8
	64–90	11	89·7	0·1	2	84·6	0·3	2
Durations 1–4	10–22	13	165·8	3·4	1	167·4	3·5	1
	35–44	577	98·3	3·3	10	103·6	4·5	10
Duration 5+	10–31	88	101·5	3·2	6	112·5	3·6	5
4 *Male pensioners*								
Lives	19–64	315	114·9	19·4	9*	120·8	27·4	8**
	92–108	1,107	103·1	10·5	11	98·3	13·2	12
Amounts	19–55	5·2φ	62·9	1·1	1	67·8	0·8	1
	94–108	283·1φ	96·1	31·6	9@	95·1	26·6	9@
5 *Female pensioners*								
Lives	28–66	1,224	101·9	18·3	9*	109·5	24·2	8**
	96–104	43	83·7	17·5	5**	86·8	17·9	5**
Amounts	28–66	1,403·7φ	99·0	390·5	13@	110·6	307·1	10@
	96–108	24·7φ	71·0	12·1	4@	68·6	12·7	4@
6 *Male annuitants*								
Duration 0	21–66	14	87·2	2·9	2	92·0	0·7	2
	89–99	13	97·6	0·1	2	76·7	1·0	2
Duration 1+	22–64	54	148·0	17·0	5**	158·5	16·8	4**
7 *Female annuitants*								
Duration 0	15–67	8	144·3	1·1	1	144·3	1·1	1
	89–101	41	108·8	1·0	4	115·0	1·1	4
Duration 1+	16–58	6	116·9	0·1	1	116·3	0·1	1
	90–107	2,761	98·8	33·1	15**	105·8	46·7	14**
8 *Widows*								
Lives	17–55	29	100·5	4·4	4	106·0	2·2	4
	81–108	151	105·2	17·4	12	98·7	12·4	12
Amounts	17–46	2·0φ	36·5	2·2	1	37·1	2·1	1
	89–108	12·3φ	114·7	0·7	2	124·6	0·6	1

φ Actual deaths have been multiplied by variance ratios (see graduation paper), or, for Amounts data, divided by an average £ amount.
* 1% $< p(\chi^2) <$ 5%, i.e. χ^2 significantly high.
** $p(\chi^2) <$ 1%, i.e. χ^2 very significantly high.
@ $p(\chi^2) <$ 5%, but Amounts data, therefore not reliable.

In Table 3.2 are shown for each adjusted age range for each experience: the age range involved, the number of actual deaths, or equivalent; and the values on the graduated basis and on the adjusted basis of $100A/E$, χ^2 and the number of degrees of freedom, taken as the number of ages or age groups within the relevant range.

For the permanent assurances ultimate experiences, for males duration 2+ and 5+ and for females duration 2+, the numbers of exposed to risk and of actual deaths at each age have been divided by the 'variance ratios' described in the graduation report. For all the Amounts experiences (male pensioners, female pensioners and widows) the exposed to risk and number of actual deaths (in £s) for each age have been divided by the average £ amount of annuity, as also described in the graduation report. This is what is meant by the reference to 'equivalent' actual deaths in the paragraph above. Since the use of variance ratios makes assumptions about the distribution of duplicate policies which are not necessarily realised in practice, and since the amount of annuity is certainly not the same for all lives, the calculated values of χ^2 need to be treated with some caution.

In order to calculate a satisfactory value of χ^2 it is necessary to group ages so as to provide more than five expected deaths in each group. For this reason the age range shown in some cases exceeds, by one or two ages, the age range where the adjusted rates are different from the graduated rates. The volume of experience in these cases is therefore small.

The correct number of degrees of freedom to use in a comparison of χ^2 is problematic. One starts with the number of different ages or age groups for which a value of z is calculated; if a mathematical formula using k parameters has been fitted to the data, it is then appropriate to deduct k from the number of age groups to give the number of degrees of freedom for a χ^2 test. But when only the 'tail' of the age range is being considered, it is perhaps reasonable to suggest that the experience in this age range has had only a small influence on the chosen values of the parameters, which in effect have been fitted to the main body of the data. It therefore seems appropriate not to make any deduction on account of the number of parameters fitted, even for the graduated rates. In the case of the adjusted rates, scant attention has been paid to the data for that experience, and it is appropriate to make no deductions. In the columns headed 'df', i.e. the number of degrees of freedom, the number of ages or age groups is given without deduction, with one exception: that for permanent assurances, males, duration 5+ where the full age range is considered.

Comments on the individual experiences follow.

3.2.1 *Permanent assurances, males*

For duration 0 and durations 2-4 the graduated rates did not need to be adjusted, except for ages below 17. It can be seen from Table 3.2 that the number of actual deaths in the relevant age ranges in each case is small, that the values of $100A/E$ are fairly far from 100, but that the rates for the adjusted basis for duration 0 are considerably closer to the actual than the graduated were, and for durations 2-4 are about as far away on the other side of 100. In all cases the value of χ^2 is conspicuously small, and for both durations it is smaller on the adjusted than on the graduated basis.

For duration 1 the rates have been adjusted below age 28. For ages 10 to 27 inclusive the actual number of deaths was 332. The number expected on the graduated basis was 336.8, giving $100A/E = 98.6$. The value of χ^2 was 12.1, with 11 degrees of freedom, clearly not a significantly high value. On the adjusted basis the expected number of deaths was 309.9, giving $100A/E$ of 107.1, and a value of χ^2 of 14.6, again with 11 degrees of freedom. The adjusted rates are rather further away from the actual than the graduated ones were, but by no means significantly so.

For duration 2+ the graduated rates were based only on ages up to 90, the data beyond that age being considered unreliable. The adjusted rates differ from the graduated rates above age 80. For ages 81 to 90 inclusive the actual number of deaths (after division by the variance ratios described in the graduation report) was 4,091. On the graduated basis the value of $100A/E$ was 99.3, whereas for the adjusted rates its value was 100.6. The adjusted rates are therefore about as far below the actual deaths as the graduated rates were above. The value of χ^2 for the graduated basis was 17.6, with 10 degrees of freedom, and on the adjusted basis 13.1, again with 10 degrees of freedom. It can be argued that the adjusted rates fit the experience slightly better, but in both cases the actual deaths are not significantly different from those expected.

For low ages for duration 2+ (ages 10 to 18) the differences are recorded in the table, and are negligible.

For duration 5+ the entire set of graduated rates was replaced by the rates for duration 2+. The value of $100A/E$ changes from 100.0 to 101.4. The value of χ^2 goes up from 109.0, with 66 degrees of freedom, to 118.0 with 70 degrees of freedom. Both of these figures are highly significant. But there is little evidence that the adjusted rates fit the data substantially worse than the original graduated rates. It was concluded in the graduation report that the graduated rates for durations 2+ and 5+ were not significantly different.

3.2.2 *Permanent assurances, females*

The adjustments at low ages for durations 0 and 1, and at ages from 75 upwards for duration 0 are in areas where the volume of data is negligible.

For duration 2+ the rates were changed below age 27. There were 129.3 actual deaths (after adjustment by variance ratios). On the graduated basis actual deaths were 115.2% of the expected, while on an adjusted basis they were 98.6%. This would indicate that the adjusted rates are closer to the actual data. The value of χ^2, however, is increased from 14.0 to 16.2, in each case with 9 degrees of freedom, so it is not clear that the apparent improvement is significant.

3.2.3 *Temporary assurances, males*

The adjustments at high ages for duration 0 and at the lowest ages for durations 1-4 are negligible. The change in rates for durations 1-4 from age 35 to age 44 affects a reasonable volume of data, 577 actual deaths, but the change is quite small and nothing significant emerges.

For duration 0 up to age 30 and duration 5+ up to age 31 the rates have been reduced, so that the actual deaths are now distinctly higher than the expected; in both cases, however, the value of χ^2 is not significantly high.

3.2.4 *Male pensioners*

For the Lives data the rates were adjusted below age 64 and above age 92. There is a substantial number of deaths in both ranges. For the higher ages the changes are not significant, but for the lower ages the actual deaths are 120.8% of those expected on the adjusted basis, and the value of χ^2 goes up from a significant 19.4, with 9 degrees of freedom, to a very significant 27.4, with 8 degrees of freedom. However, the experience of those retiring below age 65 at what is described as a 'normal or late retirement age' appears to be higher than might be expected from a typically healthy group of employees, and it seems likely that, for example, where an employee has the option of retiring between 60 and 65, with any age in the range being described as normal, those who are in poorer health choose to go earlier than those who are particularly fit. The experience of those who are described as retiring before normal retirement age (the 'early' retirements), who must be expected to include many genuine ill-health retirements, is conspicuously high, as can be seen from the graduation report.

The appropriate level of mortality rates for a graduated pensioners table depends on the uses to which that table is to be put. If it is intended to describe the experience of those in a pension scheme with a fixed retirement age below age 65, then it seems appropriate that it should allow for an average level of good health. Where a scheme offers the member the option of retiring over a range of possible ages, it may be appropriate to allow for the possibility of those who choose to retire at earlier ages having worse than average mortality, with those retiring at later ages having relatively good mortality. *The Committee stresses that it is up to each actuary who makes use of any published standard tables to ensure that they are appropriate for the circumstances in which they are to be used.*

The Committee, therefore, considers that the decision to relate the mortality rates of male pensioners below age 65 to those of male assured lives of a comparable age is appropriate.

For the Amounts experience the changes at the younger age range, from 19 to 55, are negligible. The rates for Amounts were also adjusted above age 94. The change in the number of expected deaths is not large, and the value of χ^2 is somewhat reduced. The values of χ^2, however, are apparently significantly high, both on the graduated basis and on the adjusted basis, but it should be noted that the values of χ^2 for an Amounts investigation are unreliable.

3.2.5 *Female Pensioners*

For the Lives data the rates below age 66 have been reduced noticeably, so that the value of $100A/E$ goes up from 101.9 to 109.5. The value of χ^2 is also increased from 18.3, with 9 degrees of freedom (significant at a 5% level) to 24.2, with 8 degrees of freedom (significant at a 1% level). The same arguments apply to female pensioners as to male pensioners, though it must be noted that the rates have been adjusted both below the typical normal retirement age of 60 and for the 6 years above that age. The adjustments over the latter range are, however, small.

For the female Amounts data the rates were adjusted for ages up to 66 and from 96 upwards. For the lower age range the value of $100A/E$ is increased from 99.0 on the graduated basis to 110.6 on the adjusted basis. Of interest is the value of χ^2, which is 390.5, with 13 degrees of freedom on the graduated basis, reducing to 307.1 with 10 degrees on the adjusted basis. Most of this very large value comes from one age range: at age 54 there was one death with 50.6 units of pension. On the adjusted basis the expected number of units of pension was 0.68. The age-group including this age contributed 348.3 to the value of χ^2 on the graduated basis and 281.5 on the adjusted basis. This shows how the results on an Amounts basis can be affected by only a few pensioners with annuities very much larger than the average.

3.2.6 *Male annuitants*

The exposure for duration 0 is not large in aggregate, and for the ages where the rates have been adjusted, up to 66 and from 89 onwards, the experience is negligible. For duration 1+ there were 54 actual deaths in the age range 22-64, which is 148.0% of those expected on the graduated basis and 158.5% of those expected on the adjusted basis. In both cases the value of χ^2 is very significantly high. It has already been noted that the experience of those who purchase immediate annuities, or for whom immediate annuities are purchased, is rather poor. The Committee again is faced with the problem of whether to reflect this conspicuously poor experience in standard tables for annuitants, or whether to pull the rates more into line with what might be expected among reasonably healthy lives at younger ages. The Committee has chosen the latter course, but observes that it would not be unreasonable to assume rather heavier mortality at younger ages, say below 65, for male annuitants, if there is evidence that the annuity is being purchased in circumstances where the prospective longevity of the annuitant is not a prime consideration. The same remark applies, with perhaps less force, to younger female annuitants.

3.2.7 *Female annuitants*

The adjustments for duration 0 and duration 1+ at the younger ages are negligible. For ages from 89 upwards the duration 0 rates have been reduced, producing an increase in $100A/E$ from 108.8 to 115.0, though the value of χ^2 is not significant.

The rates for duration 1+ have been altered from age 90 upwards. The number of actual deaths, 2,761, is substantial. $100A/E$ increases from 98.8 to 105.8. The value of χ^2 increases from a very significant 33.1 with 15 degrees of freedom to an even more significant 46.7 with 14 degrees of freedom. In this case it looks as if the adjusted rates have been pushed substantially further from the experience than can be justified on statistical grounds. The Committee nevertheless feels that the reasons given in § 2.3.9 for making these adjustments to the female annuitants mortality rates are valid. It would be inappropriate to proffer a mortality table for annuitants which was as much higher than those for female pensioners and female assured lives as the original graduations showed.

3.2.8 *Widows*

The mortality rates for widows Lives were adjusted for ages up to 55 and from 81 upwards. Although the volume of data in each age range is not trivial, it is fairly small, and the changes are not significant.

For the Amounts rates the changes apply up to age 46 and from age 89 upwards. In both cases the changes are negligible.

4. TABLES ALLOWING FOR PROJECTED IMPROVEMENTS

4.1 INTRODUCTION

Previous standard tables produced by the Committee for pensioners and annuitants have all allowed in some way for projected improvements in mortality. The PA(90) and a(90) tables were constructed by allowing for a uniform reduction in the mortality rates equivalent to deducting one year of age for each period of 20 calendar years for which the rates were projected forwards. Each of these tables was based on the experience for 1967-70 projected forwards to calendar year 1990.

The a(55) tables were constructed in a different way, first by projecting the basic rates forwards, and then by constructing a table of mortality rates where the rate for each age was the projected rate applicable to the calendar year in which that age on average would be attained by entrants in 1955 (C.M.I., 1952).

In Section 4.2 the improvements in mortality experienced in the past are considered. This updates the similar discussion in 'Proposed Standard Tables for Life Office Pensioners and Annuitants', *C.M.I.R.*, **3**, 1 (1978), in which the method of projection for the PA(90) and a(90) tables was explained. In Section 4.3 the method of projection for the new standard tables is explained. In Section 4.4 alternative types of projected tables are described, and in Section 4.5 there is a note on the calculation of the force of mortality, μ_x, for projected tables.

4.2 OBSERVED IMPROVEMENTS IN MORTALITY

4.2.1. The new standard tables for pensioners are based on graduations of both the Lives and the Amounts data. In Table 4.2.1a we give '20-year standardised' reduction factors for male pensioners (on the basis of both lives and amounts) implied by the experiences of (i) the five most recent quadrennia (1967-70, 1971-74, 1975-78, 1979-82, 1983-86) and (ii) the last four of these periods.

It is perhaps helpful to describe briefly how these 20-year standardised reduction factors have been calculated. Consider, for example, the Lives experience for the age-group 66-70. For this group the crude rate of mortality in the quadrennium 1983-86 was 75% of the corresponding rate for the quadrennium 1967-70 (sixteen years earlier) and 78% of the corresponding rate for the quadrennium 1971-74 (twelve years earlier). In order to compare these factors (at least approximately) we standardise using a geometric basis.

If $r^{16} = 0{\cdot}75$, then $r^{20} = 0{\cdot}70$; if $r^{12} = 0{\cdot}78$, then $r^{20} = 0{\cdot}66$. We therefore define the '20-year standardised' reduction factor for the age-group 66-70 for the lives data to be 0·70 on basis (i) and 0·66 on basis (ii). These numbers are to be found in the Table 4.2.1a. The other factors have been calculated in a similar manner.

Table 4.2.1a. *Male Pensioners. 20-year standardised mortality reduction factors*

Age group	Lives Data (i)	Lives Data (ii)	Amounts Data (i)	Amounts Data (ii)
66–70	·70	·66	·64	·58
71–75	·74	·69	·67	·61
76–80	·88	·79	·82	·73
81–85	·95	·84	·90	·82
86–90	·99	·93	·98	·95

The above factors are calculated from the observed improvement in mortality (i) over the period 1967-70 to 1983-86 and (ii) over the period 1971-74 to 1983-86. At each age, for both lives and amounts, the figures in column (ii) of the above table are less than the figures in column (i). This reflects the more rapid rate of improvement in mortality in recent years. The improvements in mortality rates are greater on the amounts basis than on the lives basis. Note also that the figures in the above table increase steadily with age. This implies that the rate of improvement in mortality is a decreasing function of age.

For female pensioners the 20-year standardised reduction factors arising from the observed experience form a less clear pattern than for males. This is illustrated in Table 4.2.1b (corresponding to Table 4.2.1a), which, as before, reflects the improvement (i) from 1967-70 to 1983-86 and (ii) from 1971-74 to 1983-86.

Table 4.2.1b. *Female Pensioners. 20-year standardised mortality reduction factors*

Age group	Lives Data (i)	Lives Data (ii)	Amounts Data (i)	Amounts Data (ii)
61–65	·90	·89	·93	·76
66–70	·76	·81	·65	·66
71–75	·73	·73	·79	·57
76–80	·84	·72	·70	·65
81–85	·88	·82	·67	·54
86+	·80	·85	·85	·93

The dependence of these factors on age is less obvious than in Table 4.2.1a. Moreover the trend for the lives experience differs somewhat from that for the amounts experience. The values in the final column are in general relatively low.

4.2.2 The graduated Pensioners' experience for the quadrennium 1967-70 was published in *C.M.I.R.* **2**. Four graduated tables were published, for males and females and for lives and amounts. These were denoted by Peg 1967–70 ML, Peg 1967–70 MA etc.

Table 4.2.2a. *Male Pensioners*

Age (1)	(2)	(3)	(4)	r^{20} (5)	r^{20} (6)
Amounts	PMA80Base	Peg 1967-70MA	PA(90)M		
60	·011611	·017768	·016127	·49	·92
65	·020410	·027534	·025015	·61	·92
70	·035405	·042437	·038608	·74	·92
75	·059320	·064868	·059139	·86	·92
80	·094850	·097942	·089572	·95	·92
85	·143746	·145259	·133444	·98	·93
90	·205732	·210112	·194221	·97	·93
95	·278903	·293962	·273928	·92	·94
Lives	PML80Base	Peg 1967-70ML	PL(90)M		
60	·014823	·021569	·019696	·54	·92
65	·025872	·032506	·029713	·68	·92
70	·042799	·048713	·044592	·81	·92
75	·069099	·072397	·066413	·93	·92
80	·106334	·106309	·097817	1·00	·93
85	·154070	·153478	·141815	1·01	·93
90	·209121	·216504	·201191	·94	·94
95	·281765	·296352	·277390	·92	·94

Notes: col (5) = $\{\text{col (2) / col (3)}\}^{20/12}$
col (6) = $\{\text{col (4) / col (3)}\}^{20/22}$

Table 4.2.2b. *Female Pensioners*

Age (1)	(2)	(3)	(4)	r^{20} (5)	r^{20} (6)
Amounts	PFA80Base	Peg 1967-70FA	PA(90)F		
60	·006505	·007839	·006961	·73	·90
65	·010795	·013435	·011937	·69	·90
70	·018165	·022932	·020396	·68	·90
75	·033373	·038877	·034641	·78	·90
80	·060719	·065170	·058243	·89	·90
85	·101686	·107261	·096321	·91	·91
90	·151883	·171548	·155192	·82	·91
95	·200821	·263015	·240469	·64	·92
Lives	PFL80Base	Peg 1967-70FL	PL(90)F		
60	·007649	·008875	·007916	·78	·90
65	·012689	·014905	·013302	·76	·90
70	·021133	·024927	·022271	·76	·90
75	·037246	·041404	·037059	·84	·90
80	·065465	·068013	·061054	·94	·91
85	·109334	·109766	·098988	·99	·91
90	·169280	·172409	·156562	·97	·92
95	·238499	·260346	·238748	·86	·92

Notes: col (5) = $\{\text{col (2) / col (3)}\}^{20/12}$
col (6) = $\{\text{col (4) / col (3)}\}^{20/22}$

The graduated experiences for Pensioners during the quadrennium 1979-82, PMA80Base etc, were compared with the graduated rates for the quadrennium 1967-70 (twelve years earlier) and 20-year reduction factors derived. The results for the male pensioners are shown in Table 4.2.2a and for female pensioners in Table 4.2.2b. The tables show the 20-year reduction factors derived from a comparison of the P.80Base and Peg 1967-70 experiences. The 20-year reduction factors implicit in the derivation of the PA(90) and PL(90) Tables are also shown.

For males, both by lives and by amounts, the 20-year reduction factor shows an increasing trend with age, suggesting that the rate of improvement in mortality slows down with age.

For females the trend in the rate of improvement in mortality with age is not quite so clear as for males although there is a monotonic increase in the reduction factor between.ages 68 and 84 for amounts and between 68 and 88 for lives. It is within these age limits that the bulk of the data lies and the graduation is at its most accurate.

It will be seen from column (6) of Tables 4.2.2a and 4.2.2b that an almost uniform 20-year reduction factor varying between 0.92 and 0.94 for males and between 0.90 and 0.92 for females applies to the derivation of both the PA(90) and PL(90) tables. The P.80Base rates are already lighter than the PA(90) rates at and below a certain age (age of cross-over) depending on the table, and heavier than the PA(90) tables above that age. The cross-over age is given in Table 4.2.2c.

Table 4.2.2c.

Table	Age of cross-over
Male Amounts	74
Male Lives	72
Female Amounts	77
Female Lives	74

The above shows that the reduction factor used to derive the PA(90) and PL(90) Tables underestimated improvements in mortality at relatively younger pensioner ages (since the 1979-82 mortality is already below that deemed appropriate to the year 1990) and may well have over-estimated the improvements at the older ages, although a true comparison cannot be made until the actual mortality rates for the year 1990 are known. Certainly the uniform reduction factor of about 0.92 has proved to be inadequate up to about age 80.

The above suggests that a reduction factor varying with age would be more appropriate, certainly for males.

4.2.3 Table 4.2.3 shows the ultimate mortality rates for assured lives taken from the standard tables A1949-52 and A1967-70 and the graduated experience for the quadrennium 1979-82. The 20-year reduction factors derived from these mortality experiences are shown.

Table 4.2.3. *Ultimate Rates of Mortality*

Age (1)	A1949–52 (2)	A1967–70 (3)	AM80 (4)	r^{20} (5)	r^{20} (6)
60	·01720	·01443	·01174	·82	·71
65	·02810	·02403	·01980	·84	·72
70	·04543	·03911	·03272	·85	·74
75	·07257	·06229	·05317	·84	·77
80	·11369	·09703	·08505	·84	·80
85	·17282	·14727	·13233	·84	·84
90	·25168	·21651	·19841	·86	·86
95	·34683	·30593	·28592	·87	·89

Notes: col (5) $= \{q(\text{A1967–70 ult}) \;/\; q(\text{A1949–52 ult})\}^{20/18}$
col (6) $= \{q(\text{AM80 ult}) \;/\; q(\text{A1967–70 ult})\}^{20/12}$

Table 4.2.4. *20-year reduction factors: England and Wales Population*

Age (1)	1951 to 1981 (2)	1961 to 1981 (3)	1951 to 1961 (4)	1961 to 1971 (5)	1971 to 1981 (6)
Males					
60	·85	·81	·93	·82	·79
65	·86	·81	·98	·90	·73
70	·88	·84	·97	·99	·72
75	·90	·88	·93	·96	·80
80	·88	·89	·87	·89	·89
85	·86	·89	·81	·86	·92
90	·84	·89	·77	·88	·89
95	·84	·89	·74	·98	·82
Females					
60	·84	·91	·73	·89	·93
65	·82	·85	·76	·82	·87
70	·78	·79	·77	·80	·77
75	·76	·77	·76	·79	·74
80	·76	·77	·76	·77	·76
85	·80	·81	·78	·77	·85
90	·84	·83	·84	·80	·87
95	·86	·82	·94	·90	·75

Notes:
The factors are: col (2) $= \{q(\text{ELT14}) \;/\; q(\text{ELT11})\}^{2/3}$
col (3) $= \{q(\text{ELT14}) \;/\; q(\text{ELT12})\}$
col (4) $= \{q(\text{ELT12}) \;/\; q(\text{ELT11})\}^{2}$
col (5) $= \{q(\text{ELT13}) \;/\; q(\text{ELT12})\}^{2}$
col (6) $= \{q(\text{ELT14}) \;/\; q(\text{ELT13})\}^{2}$

A reduction in the rates of improvement in mortality with increasing age is evident for assured lives, particularly over the twelve year period between the quadrennia 1967-70 and 1979-82 i.e. r^{20} is an increasing function of age.

4.2.4 *Population data*

Table 4.2.4 shows the 20-year reduction factors for population mortality derived from English Life Tables (ELT).

There is little evidence from Table 4.2.4 of a reduction in the rates of improvement of population mortality with increasing age in the range 60-95.

4.2.5 *Long-term trends in mortality*

Table 4.2.5 is derived from the work done by Forfar & Smith (1987). They fitted a smooth mathematical curve to each of the twenty-eight ELT tables published to date. Table 4.2.5 shows the 20-year reduction factors for the mortality rates derived from the fitted mathematical curves. For example the 20-year reduction factors shown for the year 1841 are:

$$(q_x^2/q_x^1)^{20/140}$$

where each q_x^1 is derived from the mathematical formula fitted to ELT1 which relates to the year 1841 and each q_x^2 is derived from the mathematical formula fitted to ELT14 which relates to the year 1981 (i.e. 140 years later).

Table 4.2.5. *20-year reduction factors (derived from work of Forfar and Smith)*

Age	1841 to 1981	1846 to 1981	1886 to 1981	1906 to 1981	1921 to 1981	1951 to 1981	1971 to 1981
Males							
60	·92	·91	·85	·84	·86	·83	·75
65	·94	·93	·88	·87	·89	·85	·76
70	·95	·95	·91	·90	·91	·86	·78
75	·96	·96	·93	·93	·93	·89	·81
80	·98	·98	·95	·96	·95	·91	·84
85	·98	·99	·97	·98	·97	·93	·87
90	·99	1·00	·98	1·00	·99	·95	·90
95	1·00	1·00	·99	1·01	1·00	·97	·93
Females							
60	·86	·85	·79	·77	·79	·81	·77
65	·88	·87	·81	·80	·81	·80	·78
70	·89	·88	·83	·82	·82	·79	·79
75	·90	·90	·85	·84	·83	·78	·81
80	·91	·91	·87	·85	·84	·78	·83
85	·92	·92	·88	·87	·85	·78	·85
90	·93	·94	·90	·89	·87	·78	·87
95	·94	·95	·92	·91	·89	·80	·89

The 20-year reduction factors all relate to the improvements between the average year to which the ELT table relates and 1981, the average year to which ELT14 relates.

It will be seen that this approach shows a reduction in the rate of improvement in mortality with increasing age. However, as there is a limit to which a single mathematical curve can accurately fit the mortality data over the whole range of ages (particularly at the higher ages) care should be taken with the interpretation of this result.

4.3 ALLOWANCE FOR IMPROVING MORTALITY

In the light of the past experience of improvements in mortality the Committee considers that it would be imprudent not to incorporate into the new tables for pensioners, annuitants and widows an allowance for projected improvements in mortality with the passage of time. In this section we describe the model adopted to allow for future improvement in mortality.

For each experience let $q_{x,t}$ denote the rate of mortality (i.e.'q_x') for an ultimate life attaining exact age x at time t (measured in years from an appropriate origin). For the annuitants let $q_{[x],t}$ be the select rate of mortality ('$q_{[x]}$') for a life newly selected at exact age x at time t. The (adjusted) graduated rates of mortality at age x apply on average to lives attaining exact age x in 1980 and thus provide the values of $q_{x,0}$ and, for annuitants, $q_{[x],0}$, where the time origin is taken in the middle of 1980. For practical purposes, therefore, $q_{x,t}$ may be considered to be the rate of mortality for a life attaining exact age x in calendar year $(1980+t)$. The problem is to estimate the values of $q_{x,t}$ (and, for annuitants, $q_{[x],t}$) for $t>0$. In order to model mortality rates which change with time, it is convenient to define RF(x,t), the 'time t reduction factor at age x', to be

$$\mathrm{RF}(x,t) = \frac{q_{x,t}}{q_{x,0}} \tag{4.3.1}$$

so that

$$q_{x,t} = q_{x,0} \cdot \mathrm{RF}(x,t) \tag{4.3.2}$$

The reduction factor function is the key to further progress. In the past some simple form of 'geometric' decrease in mortality rates with time has been used to produce 'projected' tables. This corresponds to assuming that RF(x,t) equals $(r_x)^t$, or even simply r^t (independent of x). (For the $a(90)$ and PA(90) tables a slightly different model was adopted, which—in conjunction with the particular graduation formulae used—enabled improving mortality to be expressed as an equivalent reduction in age with the passage of time.) Consideration of recent trends in mortality for various groups of lives (see Section 4.2 above) suggests, however, that rates of improvement do depend on age.

A simple geometric model (i.e. RF(x,t) = $(r_x)^t$) implies that at each age the rate of mortality tends to zero with the passage of time. In the view of the Committee it is more realistic to assume that at each age the limiting rate of mortality is non-zero and that the rate of mortality decreases to its limiting value by exponential decay with the passage of time. Such a model for improving mortality is characterised by taking

$$\mathrm{RF}(x,t) = \alpha(x) + [1 - \alpha(x)]\,\mathrm{e}^{-\beta_x \cdot t}, \qquad (4.3.3)$$

where, for all x, $0 < \alpha(x) \leqslant 1$ and $\beta_x > 0$. Note that (since $\beta_x > 0$)

$$\lim_{t\to\infty} \mathrm{RF}(x,t) = \alpha(x)$$

and

$$q_{x,\infty} = \lim_{t\to\infty} q_{x,t} = \alpha(x) \cdot q_{x,0}.$$

Thus at age x the limiting value of the rate of mortality is $\alpha(x)$ times the value at time 0 (i.e. the 'current' value). In equation (4.3.3) the parameter β_x determines the speed with which the mortality rate decreases to its limiting value. (A greater value of β_x implies a more rapid convergence to the limit.) If, relative to the limiting value, this speed of convergence does not depend on age (and for practical purposes this may be a reasonable hypothesis, even if the limit itself varies with age), then $\beta_x = \beta$ (constant) and

$$\mathrm{RF}(x,t) = \alpha(x) + [1 - \alpha(x)]e^{-\beta t}. \qquad (4.3.4)$$

An alternative way of expressing this last equation can be obtained by specifying the fraction of the total future fall in q_x which will occur (at all ages) by some given future time n. If we denote this fraction by f_n, then *by definition*

$$q_{x,0} - q_{x,n} = f_n[q_{x,0} - q_{x,\infty}]$$

Dividing both sides of this last equation by $q_{x,0}$, we have (using equation (4.3.2)

$$1 - \mathrm{RF}(x,n) = f_n[1 - \mathrm{RF}(x,\infty)]$$

so that

$$\begin{aligned} f_n &= [1 - \mathrm{RF}(x,n)]/[1 - \mathrm{RF}(x,\infty)] \\ &= [1 - \alpha(x)][1 - e^{-n\beta}]/[1 - \alpha(x)] \qquad \text{(by 4.3.4)} \\ &= 1 - e^{-n\beta} \end{aligned}$$

Hence

$$e^{-\beta} = [1 - f_n]^{1/n}$$

and, substituting this last expression in equation (4.3.4), we obtain

$$\mathrm{RF}(x,t) = \alpha(x) + [1 - \alpha(x)][1 - f_n]^{t/n} \qquad (4.3.5)$$

This last equation defines the reduction factor $\mathrm{RF}(x,t)$ in terms of the limiting ratio, $\alpha(x)$, and the 'time n fraction', f_n.

Having considered a variety of relevant factors (in particular the detailed discussion in Section 4.2 above) and made an assessment of likely changes in future rates of improvement, the Committee is of the opinion that for practical purposes future mortality for male pensioners should be modelled by equations 4.3.2 and 4.3.5 above with $n = 20$, $f_{20} = 0{\cdot}6$, and $\alpha(x)$ a linear function of x between ages 60 and 110, so that

$$\mathrm{RF}(x,t) = \alpha(x) + [1 - \alpha(x)](0{\cdot}4)^{t/20} \qquad (4.3.6)$$

and

$$\alpha(x) = \begin{cases} 0{\cdot}5 & x < 60 \\ \dfrac{x-10}{100} & 60 \leqslant x \leqslant 110 \\ 1 & x > 110 \end{cases} \qquad (4.3.7)$$

It follows from these last equations that

$$\mathrm{RF}(x,20) = 0{\cdot}4 + 0{\cdot}6\alpha(x)$$

and thus

$$\mathrm{RF}(x,20) = \begin{cases} 0{\cdot}7 & x < 60 \\ \dfrac{0{\cdot}6x + 34}{100} & 60 \leqslant x \leqslant 110 \\ 1 & x > 110 \end{cases} \qquad (4.3.8)$$

It is of interest to compare the values of $\mathrm{RF}(x,20)$, evaluated at the central age of each age group, with those in Table 4.2.1a. The relevant values are given in Table 4.3.2.

The Committee is of the opinion that it would be excessively cautious to incorporate into the projected tables for female pensioners the rapid rates of improvement in mortality reflected in the final column of Table 4.2.1b. Moreover, to do so would be to widen the current differences between male and female mortality in a manner which might be difficult to justify—particularly as the latest U.K. population projections assume a narrowing of the sex differential in mortality, on the basis of a consideration of trends in mortality from specific causes.

There is considerably less data for annuitants than for pensioners. However, analysis of the available data indicates that the mortality of annuitants has improved somewhat less in recent years than that of pensioners. Having con-

Table 4.3.2. *20-year reduction factors (per cent) (from equation 4.3.8)*

Age x	100RF(x, 20)
68	74·8
73	77·8
78	80·8
83	83·8
88	86·8

sidered several possible alternative mortality improvement bases, the Committee concludes that for practical purposes it is reasonable to adopt the same reduction factors for each projected table, including the annuitants' select table. (This implies that at each age the ratio of the select rate of mortality to the ultimate rate does not change with time.)

The projected mortality tables for pensioners and annuitants (male and female) and for widows will incorporate in each case the projected rates of mortality defined (from the graduated 'base' rates) by equations 4.3.2, 4.3.6, and 4.3.7 above.

4.4. ALTERNATIVE TYPES OF PROJECTED TABLES

The projection basis having been determined, it is necessary to decide the type of mortality tables to be produced. In principle, for each experience the two-dimensional array $\{q_{x,t}\}$ could be published in full. This, however, would require considerable space in any printed volume, as the range of values of x and t would necessarily be extensive. On the other hand, since it is intended that the projected mortality rates be issued on a diskette, it would be relatively simple to include the complete arrays, i.e. to provide full tables for each generation. For practical purposes, however, there is merit in having a conventional one-dimensional table, dependent only on age. Two alternatives spring immediately to mind as the basis for published tables. A third basis, although of a somewhat artificial nature, is also possible. These bases are described below.

(a) *A specified calendar year*
We may choose a specific time, say $t = T$, and construct a mortality table based on the rates of mortality which are estimated to apply at that time. In such a table the rates of mortality are given by the one-dimensional array $\{q_{x,T}\}$ (for fixed T). For example, if we were to let $T = 20$, we would produce a table in which the rates of mortality were those projected for each age in the calendar year 2000. (Recall that the base rates apply to the year 1980.) The rates of mortality in such a table do not apply to any one individual over his lifetime, but may nevertheless provide an acceptable basis for practical calculations. (The

$a(90)$ and PA(90) Tables were constructed on this basis for the calendar year 1990.)

(b) *A specified year of birth*
Each generation ('year of birth cohort') has it own forecast mortality rates and it is possible to construct a mortality table based on the anticipated experience of one particular generation. For example, in relation to male pensioners we might adopt the mortality rates of the generation which will attain age 65 in the year 2000—i.e. the generation born in 1935. If in future mortality rates follow the projections, then the annuity values at age 65 from the resulting table will contain a safety margin for lives attaining that age before the year 2000 and will understate the true value for lives attaining age 65 in later years.

Suppose that we wish to adopt the estimated mortality rates for the generation born in calendar year b. A life in this generation will attain age x in calendar year $(b + x)$. The mortality rates in a table based on this generation are thus given by the array $\{q_{x,x+b}\}$ (where, for example, $b = -45$ for the generation born in 1935).
There is one problem in the construction of such a table: if it is desired to extend the table to very young ages, then $(x + b)$ may well be negative for certain ages, and the question arises as to whether or not the improvement factors should be used to extrapolate 'backwards' to estimate past mortality rates. For example, the rate of mortality at age 25 for the 1935 year of birth cohort is the rate applying in 1960, some twenty years prior to the period from which the base rates were derived. In certain situations such backward extrapolation might well be inappropriate.

(c) *A table derived from annuity values for a specified year of entry*
A third possible mortality table is one which, *for one particular year of entry and specified rate of interest*, produces at each age a single-life whole-life annuity value (or other appropriate function) equal to the true value on the basis of projected mortality. This method was discussed by the Committee in the construction of the $a(55)$ Tables (C.M.I., 1952).

More precisely, let the year of entry be calendar year $1980+e$. (For example, if we consider the year 2000, then $e = 20$.) For a life attaining age x in this year, the whole-life annuity value at any rate of interest depends on the mortality rates $\{q_{x+r,e+r} : r \geqslant 0\}$. With an interest rate of j per annum the value of an annuity of 1 p.a. payable annually in arrear to a life attaining age x in year $1980+e$ is simply

$$a_x^{e,j} = \sum_{r=0}^{\infty} \{(1 + j)^{-(r+1)} \cdot \prod_{s=0}^{r} (1 - q_{x+s,e+s})\} \qquad (4.4.1)$$

Having chosen particular values for e and j, we may determine the resulting set of annuity values $\{a_x^{e,j}\}$. It is then a simple matter to find the corresponding set of mortality rates—$\{q_x^{e,j}\}$, say—which *at rate of interest j* will reproduce these annuity values. The mortality table incorporating these rates may be described as the table 'derived from annuity values for year of entry 1980 + e with interest rate j'.

The mortality rates determined in this way are, of course, somewhat artificial, having been chosen for one specific purpose. They depend on both the year of entry, 1980 + e, and the specified rate of interest, j, (although in practice they may not be unduly sensitive to changes in this rate), and also on the particular financial function reproduced (in the above example the value of a whole-life annuity).

The Committee does not propose to construct projected tables on a year of entry basis, although individual actuaries may wish to do so for their own purposes.

4.5 CALCULATION OF THE FORCE OF MORTALITY FOR PROJECTED TABLES

For each projected table ('calendar year' or 'year of birth' basis) for pensioners, annuitants and widows values of the force of mortality are derived from the array of values $\{q_x\}$ *for that table* (and, where appropriate for annuitants, from the corresponding select rates). The formulae used are analogous to those described in § 2.6.2.

It is perhaps worth pointing out that this procedure leads to small inconsistencies between different tables. Consider, for example, the value of μ_{65}. If this is calculated from the 'calendar year 2000' rates of mortality, the resulting value will differ from that calculated from the 'year of birth 1935' rates of mortality —even although a member of this birth cohort will attain age 65 in the calendar year 2000. The difference arises from the fact that the calendar year table uses a sequence $\{q_x\}$ which comprises a column in the full two-dimensional array of mortality rates as a function of age and time, while the 'year of birth' table uses a different sequence $\{q_x\}$, which comprises a diagonal in the two-dimensional array. Thus the tables for calendar year 2000 and for year of birth 1935 will have different values of μ_{65} (although they will have the same value of q_{65}). For practical purposes, however, this difference is of no significance.

To be precise, consider the calculation in general of $\mu^C_{x,T}$, the force of mortality for age x in the calendar year table for year T. Using the general formula for ultimate tables given in § 2.6.2, we put $\lambda_{x,T} = -\log(1 - q_{x,T})$, and then:

$$\mu^C_{x,T} = \frac{-\lambda_{x-2,T} + 7\lambda_{x-1,T} + 7\lambda_{x,T} - \lambda_{x+1,T}}{12}$$

Now consider the calculation of $\mu^B_{x,x+b}$, the force of mortality for age x in the year of birth table for a life born in year b. This is derived from values of $q_{x,x+b}$:

$$\mu^B_{x,x+b} = \frac{-\lambda_{x-2,x-2+b} + 7\lambda_{x-1,x-1+b} + 7\lambda_{x,x+b} - \lambda_{x+1,x+1+b}}{12}$$

If $x+b = T$ we get

$$\mu^B_{x,T} = \frac{-\lambda_{x-2,T-2} + 7\lambda_{x-1,T-1} + 7\lambda_{x,T} - \lambda_{x+1,T+1}}{12}$$

This rate applies 'on average' to year $T-\frac{1}{2}$. It is found that $\mu^B_{x,x+b}$, where $x+b = T$, is approximately equal to $\frac{1}{2}(\mu^C_{x,T-1} + \mu^C_{x,T})$, and the approximation is very close. A rationale for this result can also be given. The mortality rates $\{q_{x,T}\}$ are assumed to apply throughout calendar year T, and values of $\{\mu^C_{x,T}\}$ also apply throughout calendar year T. The value of $\mu^B_{x,x+b}$, however, applies at the beginning of year $x+b$. By the end of the year the force $\mu^B_{x,x+b+1}$ applies. Thus, if $x+b = T$, $\mu^B_{x,x+b}$ is roughly half way between $\mu^C_{x,T-1}$ and $\mu^C_{x,T}$, and $\mu^C_{x,T}$ is roughly half way between $\mu^B_{x,x+b}$ and $\mu^B_{x,x+b+1}$. (Note that both these statements cannot be exact unless $\mu^C_{x,T}$ and $\mu^B_{x,T}$ are both exactly linear in T, which is not the case.)

4.6 THE PROJECTED TABLES

The Committee has decided to apply the method of projection described in Section 4.3 to the basic tables for pensioners, annuitants and widows, in order to provide projected double-entry tables for each of these three classes of life. The base values of q_x for each of these classes (and of $q_{[x]}$ for annuitants) are those from the corresponding 1980Base tables, which have been described in Section 2. The value of q_x in such a table applies on average to a life attaining age x in the middle of 1980, and gives the probability of death before he (or she) reaches age $x+1$ in the middle of 1981. The values of $q_{x,t}$ in the projected tables should be interpreted similarly, as applying from mid-year to mid-year on average.

The projected double-entry tables are listed below.

Pensioners: PML80, PMA80, PFL80 and PFA80, for Male Lives, Male Amounts, Female Lives and Female Amounts respectively;

Annuitants: IM80 and IF80 for Males and Females respectively, in both cases with a one-year select period;

Widows: WL80 and WA80 for Lives and Amounts respectively.

Tables for a specific calendar year extracted from the full double-entry projected tables, as described in Section 4.4(a), are denoted by the suffix (C = year) where 'year' is the specified calendar year. For example, a projected table for calendar year 2010 is denoted by (C = 2010), or abbreviated to e.g. C10, for years from 1980 to 2079. Tables A7, A8 and A9 in Appendix A show the values of q_x for pensioners, annuitants and widows respectively (and of $q_{[x]}$ for annuitants) for calendar year 2010. The first column in Table A7, for example, is denoted by PML80 (C = 2010) or PML80C10, i.e. the Pensioners projected table, using the base year 1980, for Male Lives, projected to calendar year 2010.

The second type of single-entry table that can be extracted from a double-entry table is that for a specified year of birth, as described in Section 4.4(b). Such a table is denoted by the suffix (B = year), where 'year' is the specified year of birth. For example, a table for year of birth 1935 is denoted by (B = 1935), or abbreviated to e.g. B35, for years from 1900 to 1999. Tables A10, A11 and A12 in Appendix A show the values of q_x for year of birth 1935 for pensioners, annuitants and widows respectively. The last column in Table A10, for example, is denoted by PFA80 (B = 1935), or PFA80B35 i.e. the Pensioners projected table, using the base year 1980, for Female Amounts, for year of birth 1935. A life born in 1935 attained age 45 in 1980, and will attain age 75 in 2010. The value of q_{45} in each of Tables A10, A11 and A12 is therefore the same as the corresponding value in Tables A4, A5 and A6, the base tables, which apply to the year 1980; the values of q_{75} in each of Tables A10, A11 and A12 correspond to the values for q_{75} in each of Tables A7, A8 and A9, the tables for calendar year 2010.

Tables A13, A14 and A15 in Appendix A show the values of q_x for pensioners, annuitants and widows respectively for a life born in 1964 (B = 1964), who attained age 16 in 1980. This is the first year of birth for which a complete table

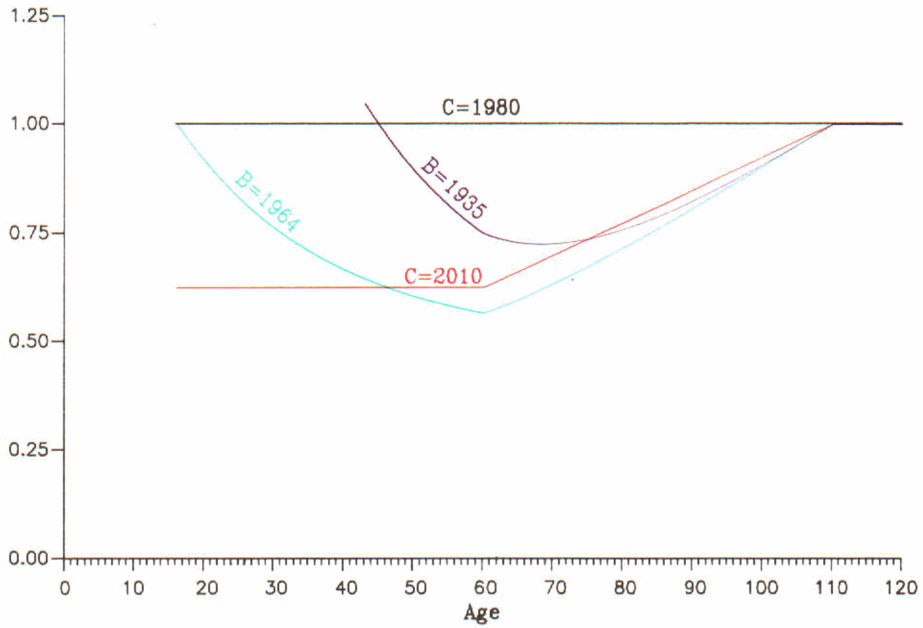

Figure 9. Reduction Factors for C = 1980, C = 2010, B = 1935, B = 1964

from age 16 is available. Tables for earlier years of birth have not been projected backwards, and so are truncated appropriately. (The Committee makes one exception to its principle of not projecting backwards: where values of μ_x for a year of birth table are to be calculated, it is convenient to project backwards for the necessary couple of years in order that smooth values of μ_x can be derived.)

Graphs of the reduction factors applicable to the tables for pensioners, annuitants and widows given in Appendix A are shown in Figure 9. The reduction factors are the same for each table. For the base table the reduction factors are uniformly equal to unity. For calendar year 2010 the reduction factors show a substantial reduction in mortality rates at young ages, and no reduction at ages above 110. For year of birth 1935 there is no reduction at age 45, attained in 1980, and the reduction factors then decrease to a minimum at age 68 in 2003, increasing again to show no reduction at ages above 110, reached in 2045. For year of birth 1964 the reduction factors start at age 16 in 1980, showing no reduction, and have the same general shape as the reduction factors for year of birth 1935, with a minimum at age 60 in 2024, and no reduction beyond age 110, reached in 2074.

The Committee does not propose to present any particular calendar year or year of birth table as a 'standard', in contrast to what it did for the PA(90) and *a*(90) tables, where the tables projected to calendar year 1990 were proposed as a standard. The Committee takes the view that actuaries should choose a particular calendar year table, a particular year of birth table or the full double-entry table as is most appropriate to the circumstances.

Where the full double-entry tables are used, it may be desirable for certain functions appropriate to a specified 'year of entry' or 'year of use' to be calculated and displayed. Such a set of functions, for example annuity values, appropriate to each age in 1990 might be denoted by the suffix (U = 1990), abbreviated to U90. Each annuity value would be calculated from a different set of projected values of q_x, each for a distinct year of birth.

5. SPECIMEN MONETARY VALUES AND COMPARISONS WITH OTHER TABLES

5.1 The tables which follow show specimen monetary functions derived from the new tables and compare these with the corresponding functions from earlier standard tables. The layouts of the tables resemble the layouts of Table 12 of 'Considerations Affecting the Preparation of Standard Tables of Mortality', *J.I.A.*, **101**, on page 179 and *T.F.A.*, **34**, on page 187 and of Tables 20 and 21 of 'Proposed Standard Tables for Life Office Pensioners and Annuitants', *C.M.I.R.*, **3**, on pages 29 and 30.

5.1.1 *Assurances*

Tables 5.1 to 5.4 are relatively straightforward and require little explanation. As is to be expected the greatest effect of changing mortality tables is to be seen in the monetary functions for term and whole life assurances.

Table 5.3 compares monetary functions derived from the new table for female assured lives with those from FA1975–78, thus showing the effect of mortality changes over two successive quadrennia. A comparison is also made with monetary functions derived from A1967–70 with the popular 4-year rating down in age for females; this comparison shows how inappropriate such an adjustment to A1967–70 has become. Table 5.4 shows that for short duration term assurances at young ages based on the new table for term assurances, the policy reserves can be negative. This is a function of the shape of the underlying values of q_x. Actuaries basing their reserves on this table will no doubt wish to adjust for this feature if it is considered to be material.

Table 5.1. *Comparison of monetary values on AM*80, *A*1967–70 *and A*1949–52
Rate of interest 4%

*Sample AM*80 *monetary functions*

Premium rates per £1,000 sum assured

x	$P_{[x]}$	$P_{[x]:\overline{15}\rvert}$	$P_{[x]:\overline{25}\rvert}$
15	4·51	48·39	23·50
20	5·45	48·36	23·49
25	6·67	48·33	23·54
30	8·28	48·39	23·75
35	10·41	48·62	24·22
40	13·21	49·10	25·11
45	16·89	49·96	26·64
50	21·75	51·44	29·19
55	28·18	53·88	33·28
60	36·74	57·84	39·73
65	48·17	64·18	49·55

Policy values

t	${}_tV_{25}$	${}_tV_{45}$
5	0·0345	0·0817
10	0·0766	0·1722
15	0·1268	0·2697
20	0·1853	0·3710
25	0·2518	0·4724
30	0·3256	0·5694
35	0·4050	0·6575
40	0·4875	0·7327
45	0·5701	
50	0·6492	
55	0·7210	
60	0·7822	

t	${}_tV_{25:\overline{25}\rvert}$
5	0·1298
10	0·2880
15	0·4804
20	0·7142

t	${}_tV_{45:\overline{25}\rvert}$
5	0·1371
10	0·2972
15	0·4850
20	0·7108

*AM*80 *monetary functions as a percentage of the equivalent A*1967–70 *functions*

x	$P_{[x]}$	$P_{[x]:\overline{15}\rvert}$	$P_{[x]:\overline{25}\rvert}$
15	92	100	100
20	92	100	100
25	92	100	100
30	92	100	99
35	92	100	99
40	92	99	98
45	91	99	97
50	91	99	96
55	91	98	94
60	91	97	93
65	92	96	92

t	${}_tV_{25}$	${}_tV_{45}$
5	93	94
10	93	94
15	93	95
20	93	96
25	94	97
30	94	97
35	95	98
40	96	99
45	97	
50	97	
55	98	
60	99	

t	${}_tV_{25:\overline{25}\rvert}$
5	100
10	100
15	100
20	100

t	${}_tV_{45:\overline{25}\rvert}$
5	99
10	99
15	100
20	100

*AM*80 *monetary functions as a percentage of the equivalent A*1949–52 *functions*

x	$P_{[x]}$	$P_{[x]:\overline{15}\rvert}$	$P_{[x]:\overline{25}\rvert}$
15	83	100	99
20	83	100	99
25	84	99	98
30	84	99	98
35	84	99	97
40	84	99	96
45	84	98	94
50	84	97	91
55	83	95	89
60	83	93	86
65	83	91	84

t	${}_tV_{25}$	${}_tV_{45}$
5	88	88
10	88	89
15	88	90
20	89	92
25	89	93
30	90	94
35	91	95
40	92	96
45	93	
50	94	
55	96	
60	97	

t	${}_tV_{25:\overline{25}\rvert}$
5	100
10	100
15	100
20	100

t	${}_tV_{45:\overline{25}\rvert}$
5	98
10	99
15	100
20	100

Table 5.2. *Comparison of monetary values on AM*80(5), *A*1967–70(5) *and A*1949–52
Rate of interest 4%

*Sample AM*80(5) *monetary functions*

Premium rates per £1,000 sum assured				Policy values				
x	$P_{[x]}$	$P_{[x]:\overline{15}\vert}$	$P_{[x]:\overline{25}\vert}$	t	$_tV_{25}$	$_tV_{45}$	t	$_tV_{25:\overline{25}\vert}$
15	4·50	48·38	23·49	5	0·0345	0·0817	5	0·1298
20	5·45	48·36	23·49	10	0·0766	0·1722	10	0·2880
25	6·66	48·32	23·53	15	0·1268	0·2697	15	0·4804
30	8·28	48·39	23·75	20	0·1853	0·3710	20	0·7142
35	10·41	48·62	24·22	25	0·2518	0·4724		
40	13·20	49·09	25·10	30	0·3256	0·5694		
45	16·86	49·92	26·61	35	0·4050	0·6575	t	$_tV_{45:\overline{25}\vert}$
50	21·65	51·31	29·07	40	0·4875	0·7327		
55	27·91	53·55	33·00	45	0·5701		5	0·1371
60	36·09	57·10	39·07	50	0·6492		10	0·2972
65	46·73	62·62	48·10	55	0·7210		15	0·4850
				60	0·7822		20	0·7108

*AM*80(5) *monetary functions as a percentage of the equivalent A*1967–70(5) *functions*

x	$P_{[x]}$	$P_{[x]:\overline{15}\vert}$	$P_{[x]:\overline{25}\vert}$	t	$_tV_{25}$	$_tV_{45}$	t	$_tV_{25:\overline{25}\vert}$
15	91	100	100	5	92	93	5	100
20	92	100	100	10	92	94	10	100
25	92	100	100	15	93	95	15	100
30	91	100	99	20	93	96	20	100
35	91	100	99	25	93	97		
40	91	99	98	30	94	97		
45	91	99	97	35	95	98	t	$_tV_{45:\overline{25}\vert}$
50	91	99	96	40	96	99		
55	91	98	94	45	96		5	99
60	92	97	93	50	97		10	99
65	92	97	93	55	98		15	100
				60	99		20	100

*AM*80(5) *monetary functions as a percentage of the equivalent A*1949–52 *functions*

x	$P_{[x]}$	$P_{[x]:\overline{15}\vert}$	$P_{[x]:\overline{25}\vert}$	t	$_tV_{25}$	$_tV_{45}$	t	$_tV_{25:\overline{25}\vert}$
15	83	100	99	5	88	88	5	100
20	83	100	99	10	88	89	10	100
25	84	99	98	15	88	90	15	100
30	84	99	98	20	89	92	20	100
35	84	99	97	25	89	93		
40	84	99	96	30	90	94		
45	84	98	94	35	91	95	t	$_tV_{45:\overline{25}\vert}$
50	83	97	91	40	92	96		
55	83	95	88	45	93		5	98
60	82	92	85	50	94		10	99
65	81	89	82	55	96		15	100
				60	97		20	100

Table 5.3. *Comparison of monetary values on AF*80, *A*1967–70 *less* 4 *years and FA*1975–78
Rate of interest 4%

*Sample AF*80 *monetary functions*

Premium rates per £1,000 sum assured — Policy values

Age	$P_{[x]}$	$P^1_{[x]:\overline{10}\rvert}$	$P^1_{[x]:\overline{20}\rvert}$	$P_{[x]:\overline{25}\rvert}$	t	$_tV_{25}$	$_tV_{45}$	t	$_tV_{25:\overline{25}\rvert}$	$_tV^1_{25:\overline{25}\rvert}$
15	3·51	0·27	0·30	23·28	5	0·0281	0·0636	5	0·1303	0·0022
20	4·28	0·27	0·37	23·31	10	0·0618	0·1362	10	0·2885	0·0041
25	5·27	0·33	0·52	23·39	15	0·1016	0·2174	15	0·4807	0·0053
30	6·53	0·49	0·81	23·55	20	0·1484	0·3059	20	0·7145	0·0046
35	8·14	0·76	1·32	23·84	25	0·2026	0·3996			
40	10·20	1·25	2·17	24·33	30	0·2644	0·4951			
45	12·86	2·06	3·58	25·14	35	0·3335	0·5885	t	$_tV_{45:\overline{25}\rvert}$	$_tV^1_{45:\overline{25}\rvert}$
50	16·32	3·41	5·89	26·49	40	0·4089	0·6753			
55	20·86	5·64	9·62	28·71	45	0·4887		5	0·1333	0·0173
60	26·89	9·28	15·51	32·33	50	0·5701		10	0·2920	0·0325
65	34·99	15·18	24·54	38·17	55	0·6496		15	0·4819	0·0413
					60	0·7235		20	0·7120	0·0357

*AF*80 *monetary functions as a percentage of the equivalent A*1967–70, *less* 4 *years, functions*

Age	$P_{[x]}$	$P^1_{[x]:\overline{10}\rvert}$	$P^1_{[x]:\overline{20}\rvert}$	$P_{[x]:\overline{25}\rvert}$	t	$_tV_{25}$	$_tV_{45}$	t	$_tV_{25:\overline{25}\rvert}$	$_tV^1_{25:\overline{25}\rvert}$
15	84	43	46	99	5	92	85	5	101	198
20	84	34	49	99	10	90	86	10	101	137
25	86	51	69	99	15	89	87	15	100	109
30	87	77	83	100	20	88	88	20	100	93
35	86	89	82	99	25	88	90			
40	86	85	77	99	30	88	92			
45	85	78	72	97	35	89	94	t	$_tV_{45:\overline{25}\rvert}$	$_tV^1_{45:\overline{25}\rvert}$
50	84	72	70	95	40	90	96			
55	83	69	69	92	45	91		5	98	66
60	83	68	70	88	50	93		10	99	66
65	82	69	73	86	55	94		15	99	65
					60	96		20	100	65

*AF*80 *monetary functions as a percentage of the equivalent FA*1975–78 *functions*

Age	$P_{[x]}$	$P^1_{[x]:\overline{10}\rvert}$	$P^1_{[x]:\overline{20}\rvert}$	$P_{[x]:\overline{25}\rvert}$	t	$_tV_{25}$	$_tV_{45}$	t	$_tV_{25:\overline{25}\rvert}$	$_tV^1_{25:\overline{25}\rvert}$
15	95	106	90	100	5	96	97	5	100	84
20	95	85	81	100	10	96	97	10	100	86
25	95	78	80	100	15	97	97	15	100	87
30	95	79	82	100	20	97	97	20	100	88
35	95	82	85	99	25	97	97			
40	95	86	88	99	30	97	96			
45	95	89	91	99	35	97	96	t	$_tV_{45:\overline{25}\rvert}$	$_tV^1_{45:\overline{25}\rvert}$
50	96	92	93	99	40	97	96			
55	96	94	95	99	45	97		5	100	95
60	96	96	95	98	50	97		10	100	95
65	95	96	94	98	55	97		15	100	96
					60	96		20	100	96

Table 5.4. *Comparison of monetary values on TM*80, *and A*1967–70(5)
Rate of interest 6%

*Sample TM*80 *monetary functions*

Premium rates per £1,000 sum assured					Policy values					
x	$P_{[x]}$	$P^{1}_{[x]:\overline{10}\vert}$	$P^{1}_{[x]:\overline{25}\vert}$	$P^{1}_{[x]:\overline{n}\vert}$ $x+n=65$	t	${}_tV_{25}$	${}_tV^{1}_{30:\overline{35}\vert}$	t	${}_tV^{1}_{25:\overline{25}\vert}$	${}_tV^{1}_{45:\overline{25}\vert}$
15	2·34	0·61	0·59	1·24	5	0·0196	0·0107	5	0·0019	0·0277
20	2·94	0·53	0·63	1·45	10	0·0460	0·0240	10	0·0045	0·0534
25	3·78	0·47	0·81	1·77	15	0·0805	0·0386	15	0·0069	0·0696
30	5·00	0·56	1·33	2·29	20	0·1238	0·0518	20	0·0069	0·0616
35	6·71	0·90	2·33	3·04	25	0·1763	0·0580			
40	9·08	1·69	4·10	4·10	30	0·2374	0·0472			
45	12·24	3·07	6·88	5·46	35	0·3074		t	$1000\ {}_tV^{1}_{25:\overline{10}\vert}$	${}_tV^{1}_{45:\overline{10}\vert}$
50	16·42	5·24	11·02	7·06	40	0·3834				
55	21·85	8·58	16·93	8·58	45	0·4631		2	−0·0496	0·0028
60	28·85	13·53	24·92	8·07	50	0·5434		4	−0·0339	0·0047
65	37·75	20·70	35·19		55	0·6209		6	0·0057	0·0053
					60	0·6924		8	0·0237	0·0039

*TM*80 *monetary functions as a percentage of the equivalent A*1967–70(5) *functions*

x	$P_{[x]}$	$P^{1}_{[x]:\overline{10}\vert}$	$P^{1}_{[x]:\overline{25}\vert}$	$P^{1}_{[x]:\overline{n}\vert}$ $x+n=65$	t	${}_tV_{25}$	${}_tV^{1}_{30:\overline{35}\vert}$	t	${}_tV^{1}_{25:\overline{25}\vert}$	${}_tV^{1}_{45:\overline{25}\vert}$
15	84	75	75	77	5	86	77	5	68	79
20	85	79	75	77	10	86	78	10	69	79
25	85	77	74	77	15	87	79	15	73	78
30	85	73	75	77	20	87	79	20	75	77
35	85	72	76	77	25	88	79			
40	85	75	78	78	30	89	78			
45	85	77	78	78	35	89		t	${}_tV^{1}_{25:\overline{10}\vert}$	${}_tV^{1}_{45:\overline{10}\vert}$
50	85	78	79	78	40	90				
55	85	77	79	77	45	91		2	119	80
60	84	77	80	76	50	92		4	−982	80
65	84	76	81		55	93		6	7	79
					60	94		8	21	79

5.1.2 *Pensioners*

Tables 5.5 to 5.8 show annuity values for the new pensioner tables and compare these with the corresponding values from existing standard tables. These annuity values are based on mortality tables projected on each of the three bases described in Section 4.6. The first column, for the year of projection 1980, is derived from the graduated and adjusted values of q_x, i.e. P..80Base. The 1990 column is for the same projection year as underlies the PL(90) and PA(90) tables and corresponding values are shown for calendar years of projection 2000 and 2010 as well.

The next section follows cohorts for different years of birth. These years were chosen so that age 65 is attained in each successive year of projection as detailed in the previous columns. Thus a life born in 1925 will attain age 65 in 1990.

The section headed Year of Use gives specimen annuity values for cohorts reaching the designated ages in 1990 and 2000. These values can be picked out from the Year of Birth section where the values appropriate for the year 2000 are highlighted.

5.1.3 *Annuitants*

Tables 5.9 to 5.12 show specimen annuity values for the new annuitants tables. The format is similar to that used for pensioners.

5.1.4 *Widows*

Tables 5.13 and 5.14 show specimen annuity values for the new widows tables. Again the format is similar to that used for pensioners.

Tables 5.5 to 5.14 follow on pages 66 to 85 inclusive

Table 5.5. *Pensioners Males Lives, PML*80

Sample annuity values using different types of projected table

Rate of interest %	Age	Calendar year 1980	1990	2000	2010	Year of birth 1915	1925	1935	1945	Year of use 1990	2000
0	55	20·683	22·009	22·950	23·592		22·360	23·208	**23·771**	23·208	**23·771**
	65	13·161	14·182	14·908	15·403	14·044	14·826	**15·354**	15·703	14·826	**15·354**
	75	7·589	8·241	8·702	9·016	8·504	**8·886**	9·140	9·305	8·504	**8·886**
	85	4·149	4·489	4·725	4·883	**4·780**	4·920	5·011	5·070	4·570	**4·780**
5	55	11·884	12·359	12·685	12·904		12·391	12·713	**12·924**	12·713	**12·924**
	65	8·742	9·236	9·578	9·808	9·106	9·496	**9·756**	9·926	9·496	**9·756**
	75	5·699	6·098	6·375	6·561	6·236	**6·470**	6·625	6·725	6·236	**6·470**
	85	3·411	3·658	3·827	3·939	**3·862**	3·963	4·028	4·070	3·709	**3·862**
10	55	7·922	8·132	8·272	8·365		8·109	8·259	**8·357**	8·259	**8·357**
	65	6·359	6·632	6·818	6·941	6·531	6·753	**6·900**	6·995	6·753	**6·900**
	75	4·505	4·769	4·950	5·071	4·848	**5·004**	5·107	5·173	4·848	**5·004**
	85	2·883	3·069	3·196	3·280	**3·220**	3·296	3·345	3·377	3·104	**3·220**
15	55	5·818	5·928	6·000	6·047		5·900	5·983	**6·036**	5·983	**6·036**
	65	4·927	5·096	5·208	5·282	5·018	5·158	**5·250**	5·310	5·158	**5·250**
	75	3·699	3·885	4·011	4·095	3·934	**4·044**	4·117	4·163	3·934	**4·044**
	85	2·489	2·635	2·734	2·799	**2·751**	2·810	2·849	2·873	2·660	**2·751**

Sample annuity values expressed as a percentage of

		PL(90) males						a(90) ultimate males		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	110	115	112	116	116	119	101	103	105
	65	107	113	112	116	112	116	99	102	102
	75	103	109	111	115	107	111	95	100	97
	85	102	108	112	114	104	109	94	100	95
5	55	108	110	108	111	111	113	102	102	104
	65	106	110	109	112	109	112	100	102	102
	75	103	107	109	112	105	109	96	100	97
	85	102	107	111	112	103	108	95	100	96
10	55	106	108	106	107	107	109	102	102	103
	65	105	108	107	109	107	109	100	102	102
	75	102	106	108	110	104	108	97	100	98
	85	102	106	109	111	103	107	95	100	96
15	55	105	106	104	106	106	107	101	101	102
	65	104	107	106	108	106	108	101	101	101
	75	102	106	106	108	103	106	97	100	98
	85	101	105	108	110	102	106	96	100	96

Table 5.6. *Pensioners Males Amounts, PMA80*

Sample annuity values using different types of projected table

Rate of interest %	Age	Calendar year 1980	1990	2000	2010	Year of birth 1915	1925	1935	1945	Year of use 1990	2000
0	55	22·165	23·430	24·319	24·922		23·845	24·616	**25·126**	24·616	**25.126**
	65	14·252	15·247	15·949	16·426	15·168	15·907	**16·403**	16·729	15·907	**16·403**
	75	8·174	8·824	9·281	9·591	9·099	**9·472**	9·719	9·880	9·099	**9·472**
	85	4·309	4·650	4·885	5·043	**4·943**	5·082	5·172	5·230	4·733	**4·943**
5	55	12·434	12·865	13·160	13·356		12·919	13·201	**13·385**	13·201	**13·385**
	65	9·292	9·757	10·077	10·291	9·658	10·015	**10·252**	10·407	10·015	**10·252**
	75	6·073	6·463	6·732	6·913	6·604	**6·830**	6·978	7·074	6·604	**6·830**
	85	3·533	3·779	3·947	4·058	**3·983**	4·083	4·147	4·189	3·832	**3·983**
10	55	8·170	8·353	8·475	8·555		8·341	8·469	**8·552**	8·469	**8·552**
	65	6·672	6·922	7·091	7·202	6·840	7·039	**7·169**	7·254	7·039	**7·169**
	75	4·761	5·015	5·188	5·303	5·095	**5·243**	5·339	5·402	5·095	**5·243**
	85	2·979	3·164	3·290	3·373	**3·314**	3·389	3·437	3·468	3·200	**3·314**
15	55	5·949	6·042	6·103	6·142		6·021	6·090	**6·134**	6·090	**6·134**
	65	5·124	5·274	5·374	5·440	5·211	5·334	**5·414**	5·466	5·334	**5·414**
	75	3·884	4·061	4·180	4·258	4·109	**4·213**	4·280	4·323	4·109	**4·213**
	85	2·568	2·712	2·809	2·873	**2·826**	2·884	2·922	2·946	2·737	**2·826**

Sample annuity values expressed as a percentage of

		PA(90) males						*a*(90) ultimate males		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	110	114	112	116	116	118	108	109	111
	65	108	113	113	116	113	116	106	109	109
	75	104	110	112	115	108	112	101	106	103
	85	102	107	112	114	104	108	97	103	99
5	55	107	110	108	110	110	112	106	106	107
	65	107	110	109	112	109	112	105	107	107
	75	104	108	110	112	106	110	101	105	103
	85	102	106	110	112	103	107	98	103	99
10	55	105	107	105	107	107	108	104	104	105
	65	106	108	107	109	107	109	104	106	106
	75	103	107	108	110	105	108	101	104	102
	85	101	105	109	110	103	106	98	102	99
15	55	104	105	104	105	105	106	103	103	104
	65	105	107	106	107	106	107	104	105	105
	75	103	106	107	109	104	107	101	104	102
	85	101	105	108	109	102	106	98	102	99

Table 5.7. *Pensioners Females Lives, PFL80*

Sample annuity values using different types of projected table

Rate of interest		Calendar year				Year of birth				Year of use	
%	Age	1980	1990	2000	2010	1915	1925	1935	1945	1990	2000
0	55	25·683	26·853	27·661	28·202		27·370	28·018	**28·442**	28·018	**28·442**
	65	17·298	18·243	18·897	19·337	18·299	18·945	**19·372**	19·650	18·945	**19·372**
	75	10·194	10·851	11·306	11·612	11·174	**11·527**	11·758	11·909	11·174	**11·527**
	85	5·263	5·627	5·877	6·044	**5·948**	6·091	6·184	6·244	5·730	**5·948**
5	55	13·546	13·903	14·143	14·301		13·980	14·197	**14·337**	14·197	**14·337**
	65	10·668	11·066	11·335	11·513	11·029	11·314	**11·501**	11·622	11·314	**11·501**
	75	7·269	7·634	7·884	8·049	7·789	**7·988**	8·118	8·202	7·789	**7·988**
	85	4·208	4·459	4·629	4·742	**4·672**	4·770	4·834	4·875	4·521	**4·672**
10	55	8·607	8·747	8·839	8·899		8·747	8·840	**8·900**	8·840	**8·900**
	65	7·390	7·587	7·718	7·804	7·542	7·690	**7·786**	7·849	7·690	**7·786**
	75	5·534	5·758	5·908	6·008	5·839	**5·963**	6·043	6·095	5·839	**5·963**
	85	3·481	3·664	3·787	3·868	**3·814**	3·885	3·932	3·961	3·704	**3·814**
15	55	6·156	6·223	6·267	6·295		6·211	6·259	**6·290**	6·259	**6·290**
	65	5·545	5·656	5·729	5·777	5·617	5·704	**5·761**	5·797	5·704	**5·761**
	75	4·418	4·566	4·665	4·730	4·613	**4·696**	4·750	4·785	4·613	**4·696**
	85	2·956	3·095	3·187	3·248	**3·206**	3·260	3·295	3·317	3·122	**3·206**

Sample annuity values expressed as a percentage of

		PL(90) females						a(90) ultimate females		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	104	107	106	109	109	110	100	101	103
	65	104	108	108	111	108	111	99	102	102
	75	103	107	109	111	106	109	96	99	97
	85	101	106	110	111	103	107	92	97	93
5	55	103	104	103	105	105	106	101	101	102
	65	103	106	106	107	106	107	100	102	102
	75	102	106	107	109	104	107	97	100	98
	85	101	104	108	109	102	105	93	97	94
10	55	102	103	102	103	103	104	101	101	101
	65	103	104	104	105	104	105	100	101	101
	75	102	105	106	107	103	106	98	100	99
	85	100	104	106	108	101	104	94	97	94
15	55	101	102	101	102	102	102	101	100	101
	65	102	104	103	104	103	104	101	101	101
	75	102	104	105	106	103	105	99	100	99
	85	100	103	105	107	101	104	94	97	95

Table 5.8. *Pensioners Females Amounts, PFA*80

Sample annuity values using different types of projected table

Rate of interest		Calendar year				Year of birth				Year of use	
%	Age	1980	1990	2000	2010	1915	1925	1935	1945	1990	2000
0	55	26·729	27·876	28·667	29·196		28·445	29·057	**29·456**	29·057	**29·456**
	65	18·163	19·107	19·760	20·198	19·214	19·841	**20·255**	20·524	19·841	**20·255**
	75	10·794	11·474	11·945	12·260	11·823	**12·182**	12·418	12·570	11·823	**12·182**
	85	5·782	6·174	6·442	6·620	**6·521**	6·673	6·771	6·834	6·290	**6·521**
5	55	13·854	14·187	14·411	14·558		14·275	14·472	**14·599**	14·472	**14·599**
	65	11·019	11·400	11·658	11·829	11·382	11·649	**11·824**	11·937	11·649	**11·824**
	75	7·575	7·942	8·191	8·356	8·103	**8·300**	8·428	8·511	8·103	**8·300**
	85	4·536	4·797	4·974	5·092	**5·021**	5·122	5·187	5·230	4·865	**5·021**
10	55	8·726	8·852	8·935	8·989		8·857	8·939	**8·992**	8·939	**8·992**
	65	7·563	7·746	7·868	7·948	7·710	7·846	**7·934**	7·990	7·846	**7·934**
	75	5·711	5·931	6·078	6·175	6·014	**6·134**	6·212	6·262	6·014	**6·134**
	85	3·703	3·889	4·014	4·096	**4·042**	4·115	4·161	4·191	3·931	**4·042**
15	55	6·213	6·272	6·311	6·336		6·263	6·305	**6·332**	6·305	**6·332**
	65	5·643	5·744	5·811	5·854	5·712	5·790	**5·841**	5·873	5·790	**5·841**
	75	4·532	4·675	4·770	4·832	4·722	**4·801**	4·852	4·885	4·722	**4·801**
	85	3·114	3·253	3·346	3·406	**3·365**	3·419	3·453	3·476	3·282	**3·365**

Sample annuity values expressed as a percentage of

		PA(90) females						a(90) ultimate females		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	106	109	108	110	110	112	104	105	107
	65	107	110	111	113	111	113	104	106	106
	75	106	111	113	115	109	113	101	105	103
	85	110	115	119	121	112	116	101	106	102
5	55	103	105	104	105	105	106	103	103	104
	65	105	107	107	108	107	108	103	104	104
	75	104	108	109	111	107	109	101	104	102
	85	107	111	115	116	109	112	100	104	101
10	55	102	103	102	103	103	104	102	102	102
	65	103	105	105	106	105	106	102	103	103
	75	103	106	107	108	105	107	101	103	102
	85	106	109	112	113	107	110	99	103	100
15	55	101	102	101	102	102	102	101	101	102
	65	103	104	103	104	103	104	102	103	103
	75	103	105	106	107	104	106	101	102	101
	85	104	107	110	111	105	108	99	102	99

Table 5.9. *Male Annuitants, IM80 Ultimate*

Sample annuity values using different types of projected table

Rate of interest %	Age	Calendar year 1980	1990	2000	2010	Year of birth 1915	1925	1935	1945	Year of use 1990	2000
0	55	22·310	23·672	24·626	25·273		24·061	24·906	**25·464**	24·906	**25·464**
	65	14·805	15·850	16·583	17·079	15·756	16·531	**17·050**	17·390	16·531	**17·050**
	75	8·985	9·651	10·117	10·430	9·944	**10·318**	10·565	10·725	9·944	**10·318**
	85	4·827	5·172	5·410	5·568	**5·472**	5·609	5·699	5·757	5·263	**5·472**
5	55	12·368	12·834	13·151	13·361		12·871	13·180	**13·383**	13·180	**13·383**
	65	9·473	9·954	10·283	10·502	9·844	10·214	**10·459**	10·619	10·214	**10·459**
	75	6·543	6·932	7·199	7·377	7·079	**7·299**	7·444	7·537	7·079	**7·299**
	85	3·914	4·159	4·325	4·436	**4·364**	4·461	4·525	4·565	4·216	**4·364**
10	55	8·099	8·298	8·431	8·519		8·277	8·419	**8·511**	8·419	**8·511**
	65	6·731	6·988	7·160	7·274	6·899	7·103	**7·238**	7·325	7·103	**7·238**
	75	5·057	5·305	5·473	5·584	5·385	**5·527**	5·620	5·679	5·385	**5·527**
	85	3·270	3·452	3·575	3·655	**3·600**	3·672	3·719	3·749	3·490	**3·600**
15	55	5·896	5·999	6·066	6·110		5·972	6·049	**6·099**	6·049	**6·099**
	65	5·139	5·293	5·396	5·463	5·225	5·352	**5·434**	5·488	5·352	**5·434**
	75	4·082	4·252	4·366	4·441	4·300	**4·398**	4·462	4·503	4·300	**4·398**
	85	2·797	2·937	3·031	3·092	**3·048**	3·104	3·140	3·163	2·963	**3·048**

Sample annuity values expressed as a percentage of

		a(90) ultimate males						a(55) ultimate males		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	105	109	106	110	110	113	115	117	119
	65	105	110	110	113	110	113	120	123	123
	75	105	110	112	115	108	112	128	134	131
	85	103	108	112	114	105	109	135	142	137
5	55	103	106	103	106	106	107	108	109	110
	65	104	107	106	109	106	109	113	115	115
	75	104	108	110	112	106	110	122	126	123
	85	103	107	110	112	104	108	130	136	131
10	55	102	104	102	103	103	105	105	105	106
	65	103	105	105	107	105	107	109	110	110
	75	104	107	108	110	105	108	117	120	118
	85	103	106	109	111	104	107	127	132	128
15	55	101	103	101	102	102	103	103	103	104
	65	102	104	103	105	103	105	107	108	108
	75	103	106	107	108	104	107	114	117	115
	85	103	106	108	110	104	107	124	128	125

Table 5.10. *Male Annuitants, IM80 Select*

Sample annuity values using different types of projected table

Rate of interest		Calendar year				Year of birth				Year of use	
%	Age	1980	1990	2000	2010	1915	1925	1935	1945	1990	2000
0	55	22·316	23·677	24·632	25·278		24·068	24·912	**25·470**	24·912	**25·470**
	65	14·821	15·865	16·596	17·092	15·774	16·546	**17·063**	17·403	16·546	**17·063**
	75	9·094	9·753	10·213	10·523	10·049	**10·416**	10·658	10·816	10·049	**10·416**
	85	5·012	5·350	5·582	5·737	**5·646**	5·779	5·866	5·922	5·443	**5·646**
5	55	12·372	12·837	13·153	13·364		12·875	13·183	**13·385**	13·183	**13·385**
	65	9·484	9·963	10·291	10·510	9·855	10·223	**10·468**	10·627	10·223	**10·468**
	75	6·622	7·005	7·267	7·443	7·153	**7·368**	7·509	7·601	7·153	**7·368**
	85	4·064	4·301	4·463	4·570	**4·503**	4·597	4·657	4·696	4·360	**4·503**
10	55	8·101	8·300	8·433	8·520		8·280	8·421	**8·513**	8·421	**8·513**
	65	6·739	6·994	7·166	7·279	6·906	7·110	**7·244**	7·330	7·110	**7·244**
	75	5·118	5·361	5·525	5·633	5·442	**5·580**	5·670	5·728	5·442	**5·580**
	85	3·395	3·570	3·688	3·766	**3·714**	3·783	3·828	3·856	3·609	**3·714**
15	55	5·898	6·000	6·067	6·111		5·974	6·051	**6·101**	6·051	**6·101**
	65	5·145	5·298	5·400	5·467	5·231	5·357	**5·439**	5·492	5·357	**5·439**
	75	4·132	4·297	4·407	4·480	4·345	**4·440**	4·501	4·541	4·345	**4·440**
	85	2·904	3·038	3·127	3·186	**3·145**	3·198	3·232	3·253	3·064	**3·145**

Sample annuity values expressed as a percentage of

		a(90) select males						a(55) select males		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	104	109	106	110	110	112	115	116	119
	65	105	109	109	112	109	112	119	122	122
	75	104	109	111	114	108	111	126	132	129
	85	102	106	110	112	104	107	132	138	133
5	55	103	105	103	106	106	107	108	108	110
	65	103	107	106	108	106	108	112	114	114
	75	104	107	109	111	106	109	120	124	121
	85	102	106	109	110	103	106	127	132	128
10	55	102	103	102	103	103	104	105	105	106
	65	102	105	104	106	104	106	108	109	109
	75	103	106	107	109	104	107	115	118	117
	85	102	105	108	109	103	106	123	128	124
15	55	101	102	101	102	102	103	103	103	104
	65	102	104	103	104	103	104	106	107	107
	75	102	105	106	107	104	106	112	115	113
	85	101	104	107	108	102	105	121	125	121

Table 5.11. *Female Annuitants, IF80 Ultimate*

Sample annuity values using different types of projected table

Rate of interest %	Age	Calendar year 1980	1990	2000	2010	Year of birth 1915	1925	1935	1945	Year of use 1990	2000
0	55	26·730	27·884	28·676	29·205		28·424	29·046	**29·451**	29·046	**29·451**
	65	18·220	19·163	19·813	20·247	19·239	19·872	**20·289**	20·560	19·872	**20·289**
	75	11·037	11·692	12·143	12·445	12·025	**12·369**	12·594	12·739	12·025	**12·369**
	85	5·755	6·118	6·365	6·530	**6·438**	6·578	6·668	6·726	6·225	**6·438**
5	55	13·833	14·173	14·401	14·551		14·256	14·458	**14·589**	14·458	**14·589**
	65	11·022	11·409	11·670	11·842	11·379	11·653	**11·832**	11·948	11·653	**11·832**
	75	7·732	8·088	8·329	8·488	8·243	**8·433**	8·557	8·637	8·243	**8·433**
	85	4·561	4·808	4·974	5·084	**5·018**	5·113	5·174	5·214	4·872	**5·018**
10	55	8·710	8·839	8·925	8·980		8·843	8·928	**8·982**	8·928	**8·982**
	65	7·549	7·737	7·862	7·943	7·696	7·836	**7·927**	7·985	7·836	**7·927**
	75	5·811	6·025	6·168	6·262	6·105	**6·222**	6·297	6·345	6·105	**6·222**
	85	3·745	3·922	4·041	4·119	**4·068**	4·137	4·181	4·209	3·963	**4·068**
15	55	6·203	6·264	6·304	6·329		6·254	6·298	**6·326**	6·298	**6·326**
	65	5·626	5·731	5·800	5·844	5·695	5·777	**5·830**	5·864	5·777	**5·830**
	75	4·597	4·737	4·829	4·889	4·782	**4·859**	4·909	4·941	4·782	**4·859**
	85	3·160	3·293	3·381	3·439	**3·400**	3·451	3·484	3·505	3·320	**3·400**

Sample annuity values expressed as a percentage of

		a(90) ultimate females						a(55) ultimate females		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	101	104	103	105	105	107	113	115	116
	65	100	104	104	106	104	106	116	119	119
	75	99	103	105	106	102	105	119	123	121
	85	96	100	103	104	97	101	120	126	122
5	55	101	103	101	103	103	104	108	108	109
	65	101	103	103	104	103	104	111	112	112
	75	99	102	104	105	101	104	115	118	116
	85	96	100	102	104	98	101	117	122	118
10	55	101	102	101	102	102	102	105	105	105
	65	101	102	102	103	102	103	107	108	108
	75	100	102	103	104	101	103	112	114	112
	85	97	100	102	103	98	101	115	119	116
15	55	101	101	100	101	101	102	103	103	103
	65	101	102	101	102	101	102	105	106	106
	75	100	102	103	104	101	103	109	111	110
	85	97	100	102	103	98	100	113	117	114

Table 5.12. *Female Annuitants, IF80 Select*

Sample annuity values using different types of projected table

Rate of interest		Calendar year				Year of birth				Year of use	
%	Age	1980	1990	2000	2010	1915	1925	1935	1945	1990	2000
0	55	26·755	27·905	28·695	29·222		28·450	29·068	**29·470**	29·068	**29·470**
	65	18·267	19·205	19·850	20·281	19·289	19·915	**20·328**	20·596	19·915	**20·328**
	75	11·117	11·765	12·212	12·510	12·100	**12·438**	12·660	12·803	12·100	**12·438**
	85	5·870	6·228	6·471	6·633	**6·545**	6·682	6·771	6·828	6·337	**6·545**
5	55	13·846	14·184	14·410	14·559		14·269	14·469	**14·598**	14·469	**14·598**
	65	11·051	11·433	11·692	11·862	11·408	11·678	**11·855**	11·968	11·678	**11·855**
	75	7·788	8·138	8·376	8·533	8·294	**8·481**	8·602	8·680	8·294	**8·481**
	85	4·652	4·894	5·057	5·165	**5·102**	5·194	5·254	5·293	4·960	**5·102**
10	55	8·718	8·846	8·930	8·985		8·851	8·934	**8·988**	8·934	**8·988**
	65	7·568	7·754	7·876	7·957	7·716	7·853	**7·942**	7·999	7·853	**7·942**
	75	5·853	6·063	6·203	6·295	6·144	**6·257**	6·330	6·377	6·144	**6·257**
	85	3·820	3·992	4·108	4·184	**4·136**	4·203	4·245	4·273	4·034	**4·136**
15	55	6·209	6·269	6·308	6·333		6·260	6·302	**6·330**	6·302	**6·330**
	65	5·641	5·743	5·811	5·854	5·710	5·790	**5·841**	5·874	5·790	**5·841**
	75	4·630	4·766	4·856	4·915	4·812	**4·886**	4·935	4·965	4·812	**4·886**
	85	3·223	3·352	3·438	3·494	**3·456**	3·506	3·538	3·558	3·380	**3·456**

Sample annuity values expressed as a percentage of

%	Age	a(90) select females: Calendar year 1990	a(90) select females: Calendar year 2000	a(90) select females: Year of birth 1925	a(90) select females: Year of birth 1935	a(90) select females: Year of use 1990	a(90) select females: Year of use 2000	a(55) select females: Calendar year 2000	a(55) select females: Year of birth 1935	a(55) select females: Year of use 2000
0	55	101	104	103	105	105	106	113	115	116
	65	100	103	104	106	104	106	116	118	118
	75	98	102	104	106	101	104	118	122	120
	85	95	99	102	103	97	100	118	123	119
5	55	101	102	101	103	103	104	107	108	109
	65	100	103	103	104	103	104	110	112	112
	75	99	102	103	105	101	103	113	116	115
	85	96	99	102	103	97	100	115	119	116
10	55	101	102	101	102	102	102	104	104	105
	65	100	102	102	103	102	103	107	108	108
	75	99	102	102	104	101	102	110	113	111
	85	96	99	101	103	97	100	113	116	113
15	55	100	101	100	101	101	101	103	103	103
	65	100	102	101	102	101	102	105	106	106
	75	99	101	102	103	100	102	108	110	109
	85	97	99	101	102	98	100	111	114	111

Table 5.13. *Widows Lives, WL*80

Sample annuity values using different types of projected table

Rate of interest		Calendar year				Year of birth				Year of use	
%	Age	1980	1990	2000	2010	1915	1925	1935	1945	1990	2000
0	55	23·547	24·895	25·834	26·466		25·304	26·125	**26·664**	26·125	**26·664**
	65	15·994	17·020	17·736	18·217	16·981	17·720	**18·212**	18·533	17·720	**18·212**
	75	9·783	10·449	10·912	11·223	10·763	**11·127**	11·366	11·521	10·763	**11·127**
	85	5·123	5·483	5·731	5·896	**5·799**	5·942	6·034	6·094	5·583	**5·799**
5	55	12·744	13·191	13·493	13·693		13·228	13·522	**13·714**	13·522	**13·714**
	65	10·015	10·469	10·778	10·984	10·384	10·726	**10·951**	11·097	10·726	**10·951**
	75	7·017	7·394	7·651	7·822	7·545	**7·754**	7·890	7·978	7·545	**7·754**
	85	4·113	4·363	4·533	4·645	**4·574**	4·673	4·737	4·778	4·424	**4·574**
10	55	8·237	8·426	8·551	8·633		8·404	8·538	**8·625**	8·538	**8·625**
	65	7·017	7·252	7·410	7·513	7·178	7·362	**7·483**	7·561	7·362	**7·483**
	75	5·364	5·599	5·757	5·861	5·680	**5·811**	5·897	5·952	5·680	**5·811**
	85	3·413	3·596	3·719	3·800	**3·745**	3·817	3·864	3·894	3·635	**3·745**
15	55	5·956	6·052	6·116	6·157		6·026	6·099	**6·146**	6·099	**6·146**
	65	5·308	5·447	5·538	5·598	5·388	5·501	**5·574**	5·621	5·501	**5·574**
	75	4·296	4·453	4·558	4·627	4·500	**4·590**	4·648	4·685	4·500	**4·590**
	85	2·905	3·044	3·137	3·198	**3·155**	3·210	3·245	3·268	3·071	**3·155**

Sample annuity values expressed as a percentage of

		PL(90) females						$a(90)$ ultimate females		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	96	100	98	101	101	103	94	95	97
	65	97	101	101	104	101	104	93	95	95
	75	99	103	105	107	102	105	92	96	94
	85	99	103	107	109	100	104	90	94	91
5	55	97	100	98	100	100	101	96	96	98
	65	98	101	100	102	100	102	95	97	97
	75	99	102	104	106	101	104	94	97	95
	85	98	102	105	107	100	103	91	95	92
10	55	98	100	98	99	99	100	97	97	98
	65	98	100	100	101	100	101	96	97	97
	75	99	102	103	104	101	103	95	98	96
	85	98	102	105	106	100	103	92	95	93
15	55	98	99	98	99	99	100	98	98	99
	65	98	100	99	101	99	101	97	98	98
	75	99	102	102	104	100	102	96	98	97
	85	98	101	104	105	99	102	93	96	93

Table 5.14. *Widows Amounts, WA*80

Sample annuity values using different types of projected table

Rate of interest		Calendar year				Year of birth				Year of use	
%	Age	1980	1990	2000	2010	1915	1925	1935	1945	1990	2000
0	55	24·862	26·124	26·999	27·588		26·618	27·344	**27·819**	27·344	**27·819**
	65	16·788	17·790	18·486	18·954	17·807	18·508	**18·973**	19·276	18·508	**18·973**
	75	10·146	10·822	11·292	11·607	11·151	**11·517**	11·757	11·912	11·151	**11·517**
	85	5·405	5·780	6·037	6·209	**6·111**	6·258	6·353	6·414	5·888	**6·111**
5	55	13·229	13·624	13·891	14·067		13·690	13·938	**14·100**	13·938	**14·100**
	65	10·389	10·817	11·108	11·301	10·758	11·073	**11·279**	11·413	11·073	**11·279**
	75	7·213	7·589	7·845	8·015	7·745	**7·950**	8·085	8·172	7·745	**7·950**
	85	4·296	4·551	4·724	4·839	**4·768**	4·868	4·933	4·974	4·616	**4·768**
10	55	8·462	8·621	8·727	8·795		8·615	8·724	**8·794**	8·724	**8·794**
	65	7·224	7·440	7·584	7·679	7·381	7·547	**7·655**	7·725	7·547	**7·655**
	75	5·484	5·715	5·870	5·972	5·797	**5·925**	6·008	6·062	5·797	**5·925**
	85	3·540	3·724	3·848	3·929	**3·875**	3·948	3·994	4·024	3·765	**3·875**
15	55	6·080	6·158	6·209	6·242		6·141	6·199	**6·236**	6·199	**6·236**
	65	5·438	5·562	5·644	5·697	5·514	5·613	**5·678**	5·719	5·613	**5·678**
	75	4·376	4·529	4·631	4·698	4·577	**4·663**	4·719	4·755	4·577	**4·663**
	85	2·998	3·136	3·229	3·290	**3·248**	3·302	3·337	3·360	3·164	**3·248**

Sample annuity values expressed as a percentage of

		PA(90) females						a(90) ultimate females		
		Calendar year		Year of birth		Year of use		Calendar year	Year of birth	Year of use
%	Age	1990	2000	1925	1935	1990	2000	2000	1935	2000
0	55	99	102	101	104	104	105	98	99	101
	65	99	103	103	106	103	106	97	99	99
	75	100	105	107	109	103	107	95	99	97
	85	103	108	112	113	105	109	94	99	96
5	55	99	101	100	101	101	103	99	99	100
	65	99	102	102	103	102	103	98	100	100
	75	100	103	105	106	102	105	96	99	98
	85	102	106	109	110	103	107	95	99	96
10	55	99	101	99	101	101	101	99	99	100
	65	99	101	101	102	101	102	99	100	100
	75	100	102	103	105	101	103	97	100	98
	85	101	104	107	108	102	105	95	99	96
15	55	99	100	99	100	100	101	100	100	100
	65	99	101	100	101	100	101	99	100	100
	75	100	102	103	104	101	103	98	100	98
	85	101	104	106	107	102	104	95	99	96

6. PUBLICATION OF THE NEW TABLES

6.1 The Committee has given considerable attention to the question of the format in which the new standard tables should be made available to the profession. The basic question is whether nowadays, with the very widespread use of computers, there is any need to publish printed volumes of commutation and monetary functions over a wide range of interest rates, in addition to mortality functions.

This question is not new; it was addressed more than ten years ago in the discussions preceding the publication of the *a*(90) and PA(90) tables (see the discussions on C.M.I. (1978) in *J.I.A.* **105**, 117 and *T.F.A.* **36**, 180). At that time there still appeared to be a demand for the traditional printed volumes of commutation and monetary functions at a 'full' range of interest rates. Accordingly such a set of volumes for those tables was published (at considerable expense).

The Committee is now of the view, supported by comments received from members of the profession, that the publication of printed volumes of commutation and monetary functions is no longer necessary. The widespread use of personal computers enables such functions to be generated easily, at any required rates of interest, from the basic mortality functions and this is already the procedure in many offices. In any case, the production of a full set of printed volumes for each of the new standard tables described in Section 2.1 would be inordinately cumbersome and expensive, compared with the more effective and less expensive route outlined below.

6.2 The Committee has made arrangements to publish and make available for purchase:

(a) a single printed volume which will contain the basic mortality functions (q, μ and l) for each of the new standard tables for assurances and the base tables for pensioners, annuitants and widows, together with corresponding values of q for selected projected tables, as well as examples of monetary functions at only one or two rates of interest for each of the tables; and

(b) a computer package comprising a diskette (which will be compatible with many of the popular makes of personal computers) and a set of instructions; this package will enable the user to generate the basic mortality functions and a wide range of commutation and monetary functions, at any reasonable rate of interest, for any of the new standard tables or, in the case of joint-life functions, a combination of any two of the new standard tables.

The computer package will be as 'user friendly' as possible. Essentially the user will be invited to specify the mortality table(s), interest rate and the required functions; these will be displayed on the screen with the option to print out all or part of the display. The user will also be able to specify an adjustment to the mortality basis as a rating in age, as a constant addition, as a percentage adjustment, or any combination of these, i.e. $aq_{x-b} + c$, where a, b and c can be specified by the user.

In the case of the tables for pensioners, annuitants and widows, projected improvements in mortality can be allowed for as discussed in Section 4.6 above. The user will specify whether a 'calendar year', 'year of birth' or 'year of use' basis is required and also the desired year or years.

The intended basis of publication, i.e. a single volume of mortality functions plus the computer package, is not only cheaper to produce and more convenient to store, but also enables the production of a much wider range of functions, over a much wider range of interest rates (including negative and fractional rates) than could possibly be produced in printed form. The purpose of including examples of monetary functions in the printed volume is twofold. First, they will indicate the different formats in which these functions will be displayed on the screen by the computer program, i.e. which monetary functions are grouped together in a single display. Secondly, for those who write their own programs for generating monetary functions from the basic mortality rates, the printed functions may be used to verify the accuracy of their programs.

6.3 The computer program will be designed to generate the functions listed below.

6.3.1 *Mortality functions*

(a) For tables with a 5-year select period, a display showing

$$q_{[x]},\ q_{[x]+1},\ q_{[x]+2},\ q_{[x]+3},\ q_{[x]+4},\ q_{x+5}$$

and likewise for μ, l and d.

(b) For tables with a 2-year select period, as above with obvious modification.

(c) For all tables, a display showing

$$q_x,\ \mu_x,\ l_x$$

and, if the table has a select period, the same for each select duration, i.e.

$$q_{[x]},\ \mu_{[x]},\ l_{[x]}$$

$$q_{[x]+1},\ \mu_{[x]+1},\ l_{[x]+1},\ \text{etc.}$$

6.3.2 *Commutation Functions*

(a) As for 6.3.1 (a) and (b) above for

$$D,\ N,\ S,\ C,\ M \text{ and } R.$$

(b) For all tables

$$D_x,\ N_x,\ S_x \text{ in a single display}$$

and

C_x, M_x, R_x in a single display

or possibly all six functions in one display.

If the table has a select period, the same for each select duration.

6.3.3 *Single-life monetary functions*

$$\ddot{a}_x \quad A_x \quad P_x$$
$$\ddot{a}_{x:\overline{n}|} \quad A_{x:\overline{n}|} \quad P_{x:\overline{n}|}$$
$${}_nA_x \quad {}_nE_x \quad {}_nP_x$$
$$a_{x:\overline{n}|} \quad a_{\overline{x:\overline{n}|}} \quad \ddot{a}_{\overline{x:\overline{n}|}}$$

Each of the above four lines constitutes a separate display. For the three displays involving both x and n the user may specify either a fixed value of n or a fixed value of $x+n$.

Each of the nine functions involving both x and n will also be available separately in a large-spread format for 'all x and all n'.

If the table has a select period, each of the four displays will be available at each select duration, e.g.

$$\ddot{a}_{[x]} \quad A_{[x]} \quad P_{[x]}$$
$$\ddot{a}_{[x]+1} \quad A_{[x]+1} \quad P_{[x]+1} \text{ etc}$$

and the 'all x and all n' format for functions involving x and n will be available in select form at duration 0.

It will be noted that the intended range of single-life monetary functions does not include a_x, perhaps the most basic of all such functions. Isolated values of a_x can of course be derived by a simple adjustment to $\ddot{a}_x$. Alternatively, if the user wishes to generate a screen display or a print-out of a full column of a_x, this can be done by requesting $a_{x:\overline{n}|}$ for $x+n = 120$.

6.3.4 *Joint-life monetary functions*

Each of the joint-life functions noted below will be displayed in a large-spread format and the user will be able to choose between the 'x:y' format, which was used in the $a(55)$ tables, or the 'x:$x \pm t$' format used in the PA(90) tables.

Where the function also involves n the user will specify a fixed value of n. It will be possible alternatively to specify a fixed value of $x+n$, or a fixed value of $x-y$.

Joint-life functions can be produced using any combination of two of the new standard tables; furthermore they can be produced with both lives select (at any select duration), both lives ultimate, or one select and the other ultimate.

The following functions will be produced:

$$
\begin{array}{lllll}
a_{xy} & \ddot{a}_{xy} & a_{\overline{xy}} & \ddot{a}_{\overline{xy}} & a_{x|y} \\
A_{xy} & A_{\overline{xy}} & A^{1}_{xy} & & \\
a_{xy:\overline{n}|} & \ddot{a}_{xy:\overline{n}|} & a_{\overline{xy:\overline{n}|}} & \ddot{a}_{\overline{xy:\overline{n}|}} & \\
a_{\overline{xy}:\overline{n}|} & \ddot{a}_{\overline{xy}:\overline{n}|} & & & \\
A_{xy:\overline{n}|} & A_{\overline{xy}:\overline{n}|} & {}_{n}A_{xy} & {}_{n}A_{\overline{xy}} & {}_{n}A^{1}_{xy}
\end{array}
$$

6.4 There are a number of functions for which the demand is likely to be limited or which involve complexities of production which are probably not justified by the demand. For example, it is not intended to include

joint-life mortality functions,
joint-life commutation functions,
continuous functions (i.e. $\bar{a}$ and $\bar{A}$),
increasing annuities and assurances,
deferred annuities and assurances.

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APPENDIX A

VALUES OF MORTALITY RATES FOR THE NEW STANDARD TABLES

Basic tables for 1979–82 experience, and projected tables for calendar year 2010 and years of birth 1935 and 1964

Table A1. *Permanent Assurances, Males – AM80 and AM80(5)*
Two and five years select. Values of $q_{[x-t]+t}$

Age x	Duration 0 / Duration 0	Duration 1 / Duration 1	Durations 2–4	Durations 2+ / Durations 5+
0	0·001047			0·001300
1	0·000685	0·000784		0·000850
2	0·000411	0·000470	0·000498	0·000510
3	0·000306	0·000351	0·000371	0·000380
4	0·000282	0·000323	0·000342	0·000350
5	0·000258	0·000295	0·000313	0·000320
6	0·000242	0·000277	0·000293	0·000300
7	0·000217	0·000249	0·000264	0·000270
8	0·000201	0·000231	0·000244	0·000250
9	0·000193	0·000221	0·000235	0·000240
10	0·000193	0·000221	0·000235	0·000240
11	0·000193	0·000221	0·000235	0·000240
12	0·000209	0·000240	0·000254	0·000260
13	0·000234	0·000267	0·000283	0·000290
14	0·000274	0·000314	0·000332	0·000340
15	0·000330	0·000378	0·000401	0·000410
16	0·000427	0·000489	0·000518	0·000530
17	0·000755	0·000865	0·000916	0·000937
18	0·000708	0·000815	0·000860	0·000886
19	0·000663	0·000768	0·000808	0·000837
20	0·000622	0·000723	0·000759	0·000791
21	0·000584	0·000682	0·000713	0·000747
22	0·000549	0·000644	0·000672	0·000708
23	0·000518	0·000610	0·000635	0·000671
24	0·000490	0·000580	0·000603	0·000639
25	0·000467	0·000554	0·000576	0·000611
26	0·000448	0·000532	0·000555	0·000588
27	0·000434	0·000516	0·000539	0·000570
28	0·000425	0·000503	0·000530	0·000558
29	0·000422	0·000494	0·000527	0·000553
30	0·000424	0·000493	0·000532	0·000554
31	0·000432	0·000500	0·000545	0·000563
32	0·000447	0·000517	0·000566	0·000580
33	0·000468	0·000543	0·000596	0·000607
34	0·000497	0·000578	0·000637	0·000644
35	0·000534	0·000625	0·000687	0·000692
36	0·000579	0·000683	0·000749	0·000752
37	0·000634	0·000753	0·000823	0·000826
38	0·000697	0·000835	0·000911	0·000914
39	0·000771	0·000932	0·001012	0·001019

Table A1. *(Continued)*
Permanent Assurances, Males – AM80 and AM80(5)
Two and five years select. Values of $q_{[x-t]+t}$

Age	Duration 0	Duration 1		Durations 2+
x	Duration 0	Duration 1	Durations 2–4	Durations 5+
40	0·000855	0·001042	0·001129	0·001141
41	0·000952	0·001168	0·001261	0·001283
42	0·001060	0·001311	0·001412	0·001446
43	0·001181	0·001470	0·001581	0·001632
44	0·001317	0·001648	0·001771	0·001844
45	0·001467	0·001846	0·001982	0·002084
46	0·001632	0·002064	0·002217	0·002355
47	0·001815	0·002304	0·002477	0·002660
48	0·002016	0·002567	0·002764	0·003003
49	0·002236	0·002856	0·003080	0·003386
50	0·002476	0·003170	0·003428	0·003813
51	0·002737	0·003513	0·003809	0·004290
52	0·003022	0·003885	0·004227	0·004821
53	0·003332	0·004288	0·004683	0·005410
54	0·003667	0·004725	0·005181	0·006065
55	0·004031	0·005198	0·005724	0·006790
56	0·004424	0·005708	0·006315	0·007592
57	0·004848	0·006258	0·006958	0·008480
58	0·005307	0·006850	0·007656	0·009461
59	0·005801	0·007488	0·008415	0·010544
60	0·006333	0·008173	0·009237	0·011739
61	0·006906	0·008909	0·010129	0·013056
62	0·007523	0·009698	0·011094	0·014507
63	0·008185	0·010544	0·012139	0·016105
64	0·008896	0·011451	0·013270	0·017863
65	0·009659	0·012422	0·014491	0·019796
66	0·010478	0·013460	0·015811	0·021921
67	0·011356	0·014570	0·017236	0·024256
68	0·012296	0·015757	0·018775	0·026819
69	0·013304	0·017025	0·020434	0·029632
70	0·014382	0·018378	0·022223	0·032718
71	0·015536	0·019821	0·024151	0·036100
72	0·016770	0·021361	0·026228	0·039807
73	0·018089	0·023003	0·028464	0·043866
74	0·019499	0·024752	0·030872	0·048308
75	0·021005	0·026615	0·033462	0·053168
76	0·022614	0·028599	0·036248	0·058480
77	0·024331	0·030710	0·039242	0·064283
78	0·026164	0·032956	0·042461	0·070619
79	0·028119	0·035345	0·045918	0·077531

Table A1. *(Continued)*
Permanent Assurances, Males – AM80 and AM80(5)
Two and five years select. Values of $q_{[x-t]+t}$

Age x	Duration 0 Duration 0	Duration 1 Duration 1	 Durations 2–4	Durations 2 + Durations 5 +
80	0·030205	0·037884	0·049629	0·085053
81	0·032428	0·040583	0·053613	0·093183
82	0·034798	0·043450	0·057886	0·101944
83	0·037323	0·046495	0·062469	0·111367
84	0·040013	0·049727	0·067380	0·121485
85	0·042876	0·053156	0·072642	0·132329
86	0·045925	0·056794	0·078275	0·143929
87	0·049169	0·060652	0·084303	0·156312
88	0·052620	0·064741	0·090750	0·169505
89	0·056289	0·069074	0·097642	0·183529
90	0·060190	0·073663	0·105003	0·198405
91		0·078521	0·112861	0·214148
92			0·121243	0·230769
93			0·130178	0·248272
94			0·139694	0·266656
95				0·285915
96				0·306032
97				0·326985
98				0·348742
99				0·371263
100				0·394496
101				0·418383
102				0·442854
103				0·467831
104				0·493226
105				0·518941
106				0·544873
107				0·570910
108				0·596936
109				0·622829
110				0·648466
111				0·673723
112				0·698476
113				0·722607
114				0·746000
115				0·768550
116				0·790158
117				0·810740
118				0·830220
119				0·848541

Table A2. *Permanent Assurances, Females – AF80*
Two years select
Values of $q_{[x-t]+t}$

Age x	Duration 0	Duration 1	Durations 2+
0	0·000938		0·001120
1	0·000603	0·000666	0·000720
2	0·000377	0·000416	0·000450
3	0·000259	0·000287	0·000310
4	0·000209	0·000231	0·000250
5	0·000184	0·000203	0·000220
6	0·000167	0·000185	0·000200
7	0·000159	0·000176	0·000190
8	0·000159	0·000176	0·000190
9	0·000151	0·000166	0·000180
10	0·000151	0·000166	0·000180
11	0·000151	0·000166	0·000180
12	0·000151	0·000166	0·000180
13	0·000159	0·000176	0·000190
14	0·000184	0·000203	0·000220
15	0·000218	0·000240	0·000260
16	0·000251	0·000277	0·000300
17	0·000256	0·000283	0·000306
18	0·000243	0·000276	0·000298
19	0·000232	0·000270	0·000292
20	0·000222	0·000265	0·000287
21	0·000214	0·000262	0·000283
22	0·000207	0·000260	0·000281
23	0·000202	0·000259	0·000280
24	0·000198	0·000260	0·000281
25	0·000196	0·000263	0·000285
26	0·000197	0·000268	0·000290
27	0·000199	0·000275	0·000298
28	0·000204	0·000285	0·000313
29	0·000212	0·000297	0·000335
30	0·000222	0·000312	0·000361
31	0·000234	0·000330	0·000390
32	0·000250	0·000352	0·000422
33	0·000269	0·000377	0·000459
34	0·000291	0·000407	0·000500
35	0·000317	0·000441	0·000545
36	0·000347	0·000481	0·000596
37	0·000381	0·000525	0·000653
38	0·000419	0·000576	0·000717
39	0·000462	0·000634	0·000788

Table A2. *(Continued)*
Permanent Assurances, Females – AF80
Two years select
Values of $q_{[x-t]+t}$

Age x	Duration 0	Duration 1	Durations 2+
40	0.000510	0.000699	0.000867
41	0.000564	0.000771	0.000955
42	0·000623	0·000853	0·001052
43	0·000688	0·000944	0·001161
44	0·000760	0·001045	0·001281
45	0·000838	0·001158	0·001415
46	0·000924	0·001284	0·001564
47	0·001018	0·001423	0·001729
48	0·001120	0·001578	0·001912
49	0·001231	0·001749	0·002115
50	0·001352	0·001938	0·002340
51	0·001483	0·002148	0·002589
52	0·001625	0·002379	0·002865
53	0·001778	0·002634	0·003171
54	0·001944	0·002916	0·003510
55	0·002123	0·003226	0·003885
56	0·002316	0·003568	0·004300
57	0·002524	0·003944	0·004760
58	0·002748	0·004359	0·005269
59	0·002989	0·004815	0·005832
60	0·003249	0·005316	0·006455
61	0·003527	0·005867	0·007144
62	0·003827	0·006473	0·007907
63	0·004149	0·007138	0·008750
64	0·004494	0·007869	0·009682
65	0·004864	0·008672	0·010713
66	0·005261	0·009553	0·011853
67	0·005687	0·010520	0·013112
68	0·006143	0·011580	0·014504
69	0·006632	0·012744	0·016042
70	0·007155	0·014020	0·017741
71	0·007715	0·015419	0·019617
72	0·008314	0·016953	0·021689
73	0·008955	0·018633	0·023976
74	0·009641	0·020474	0·026500
75	0·010487	0·022491	0·029285
76	0·011524	0·024699	0·032357
77	0·012660	0·027116	0·035744
78	0·013905	0·029762	0·039478
79	0·015269	0·032656	0·043591

Table A2. *(Continued)*
Permanent Assurances, Females – AF80
Two years select
Values of $q_{[x-t]+t}$

Age x	Duration 0	Duration 1	Durations 2+
80	0.016764	0.035821	0.048121
81	0.018401	0.039282	0.053107
82	0·020194	0·043064	0·058593
83	0·022157	0·047196	0·064625
84	0·024306	0·051708	0·071252
85	0·026657	0·056632	0·078529
86	0·029230	0·062004	0·086512
87	0·032044	0·067861	0·095262
88	0·035121	0·074243	0·104844
89	0·038485	0·081193	0·115326
90	0·042161	0·088755	0·126777
91		0·096978	0·139273
92			0·152886
93			0·167695
94			0·183775
95			0·201200
96			0·220043
97			0·240368
98			0·262235
99			0·285691
100			0·310770
101			0·337487
102			0·365835
103			0·395782
104			0·427264
105			0·460180
106			0·494392
107			0·529716
108			0·565921
109			0·602733
110			0·639827
111			0·676837
112			0·713363
113			0·748977
114			0·783240
115			0·815720
116			0·846009
117			0·873750
118			0·898653
119			0·920519

Table A3. *Temporary Assurances, Males – TM80*
Five years select
Values of $q_{[x-t]+t}$

Age x	Duration 0	Durations 1–4	Durations 5+
0	0.000936		0.001300
1	0.000612	0.000764	0.000850
2	0.000367	0.000458	0.000510
3	0·000274	0·000342	0·000380
4	0·000252	0·000315	0·000350
5	0·000231	0·000288	0·000320
6	0·000216	0·000270	0·000300
7	0·000194	0·000243	0·000270
8	0·000180	0·000225	0·000250
9	0·000173	0·000216	0·000240
10	0·000173	0·000216	0·000240
11	0·000173	0·000216	0·000240
12	0·000187	0·000234	0·000260
13	0·000209	0·000261	0·000290
14	0·000245	0·000306	0·000340
15	0·000295	0·000369	0·000410
16	0·000382	0·000476	0·000530
17	0·000662	0·000826	0·000919
18	0·000617	0·000770	0·000868
19	0·000574	0·000717	0·000820
20	0·000535	0·000668	0·000775
21	0·000500	0·000623	0·000733
22	0·000467	0·000583	0·000694
23	0·000439	0·000548	0·000658
24	0·000414	0·000517	0·000626
25	0·000394	0·000492	0·000599
26	0·000379	0·000473	0·000576
27	0·000368	0·000459	0·000559
28	0·000362	0·000452	0·000547
29	0·000362	0·000452	0·000542
30	0·000368	0·000459	0·000543
31	0·000375	0·000473	0·000552
32	0·000388	0·000496	0·000559
33	0·000410	0·000528	0·000567
34	0·000442	0·000558	0·000587
35	0·000484	0·000589	0·000620
36	0·000536	0·000634	0·000668
37	0·000598	0·000694	0·000730
38	0·000671	0·000769	0·000810
39	0·000754	0·000862	0·000907

Table A3. *(Continued)*
Temporary Assurances, Males – TM80
Five years select
Values of $q_{[x-t]+t}$

Age x	Duration 0	Durations 1–4	Durations 5+
40	0·000847	0·000973	0·001024
41	0·000952	0·001105	0·001163
42	0·001067	0·001258	0·001324
43	0·001193	0·001436	0·001511
44	0·001330	0·001629	0·001724
45	0·001479	0·001819	0·001967
46	0·001639	0·002028	0·002242
47	0·001811	0·002257	0·002552
48	0·001994	0·002507	0·002898
49	0·002189	0·002780	0·003285
50	0·002396	0·003078	0·003717
51	0·002615	0·003401	0·004195
52	0·002846	0·003752	0·004725
53	0·003090	0·004133	0·005311
54	0·003346	0·004544	0·005958
55	0·003615	0·004987	0·006669
56	0·003897	0·005466	0·007452
57	0·004192	0·005982	0·008311
58	0·004499	0·006537	0·009254
59	0·004821	0·007134	0·010286
60	0·005155	0·007776	0·011415
61	0·005504	0·008464	0·012650
62	0·005866	0·009202	0·013998
63	0·006242	0·009993	0·015469
64	0·006632	0·010841	0·017073
65	0·007113	0·011748	0·018821
66	0·007703	0·012718	0·020724
67	0·008333	0·013756	0·022795
68	0·009006	0·014864	0·025046
69	0·009726	0·016048	0·027493
70	0·010494	0·017312	0·030151
71	0·011315	0·018661	0·033036
72	0·012191	0·020100	0·036165
73	0·013125	0·021634	0·039558
74	0·014122	0·023268	0·043235
75	0·015184	0·025010	0·047218
76	0·016316	0·026865	0·051528
77	0·017523	0·028839	0·056192
78	0·018807	0·030940	0·061233
79	0·020175	0·033176	0·066681

Table A3. *(Continued)*
Temporary Assurances, Males – TM80
Five years select
Values of $q_{[x-t]+t}$

Age x	Duration 0	Durations 1–4	Durations 5+
80	0·021631	0·035553	0·072563
81	0·023181	0·038080	0·078911
82	0·024829	0·040766	0·085755
83	0·026583	0·043619	0·093130
84	0·028447	0·046649	0·101071
85	0·030429	0·049867	0·109614
86	0·032535	0·053281	0·118796
87	0·034773	0·056903	0·128656
88	0·037150	0·060745	0·139234
89	0·039675	0·064817	0·150569
90	0·042354	0·069133	0·162701
91		0·073706	0·175672
92		0·078548	0·189520
93		0·083673	0·204283
94		0·089095	0·219997
95			0·236696
96			0·254409
97			0·273161
98			0·292972
99			0·313853
100			0·335808
101			0·358830
102			0·382902
103			0·407992
104			0·434054
105			0·461028
106			0·488833
107			0·517373
108			0·546528
109			0·576162
110			0·606118
111			0·636218
112			0·666269
113			0·696061
114			0·725373
115			0·753976
116			0·781639
117			0·808136
118			0·833250
119			0·856784

Table A4. *Pensioners – PML80Base, PMA80Base, PFL80Base and PFA80Base. Values of* q_x

Age	Males		Females	
	Lives	Amounts	Lives	Amounts
x	PML80Base	PMA80Base	PFL80Base	PFA80Base
16	0·000678	0·000530	0·000353	0·000300
17	0·001200	0·000938	0·000363	0·000308
18	0·001134	0·000887	0·000354	0·000301
19	0·001071	0·000838	0·000346	0·000294
20	0·001012	0·000791	0·000340	0·000289
21	0·000956	0·000748	0·000336	0·000285
22	0·000905	0·000708	0·000333	0·000283
23	0·000859	0·000672	0·000332	0·000282
24	0·000818	0·000639	0·000334	0·000284
25	0·000782	0·000612	0·000337	0·000287
26	0·000752	0·000588	0·000344	0·000292
27	0·000730	0·000571	0·000353	0·000300
28	0·000714	0·000559	0·000371	0·000315
29	0·000707	0·000553	0·000398	0·000338
30	0·000709	0·000554	0·000428	0·000364
31	0·000720	0·000563	0·000462	0·000393
32	0·000743	0·000581	0·000501	0·000426
33	0·000777	0·000608	0·000544	0·000462
34	0·000824	0·000644	0·000592	0·000503
35	0·000885	0·000693	0·000646	0·000549
36	0·000962	0·000753	0·000707	0·000601
37	0·001057	0·000826	0·000775	0·000658
38	0·001170	0·000915	0·000850	0·000722
39	0·001303	0·001019	0·000934	0·000794

Table A4. *(Continued)*
Pensioners – PML80Base, PMA80Base, PFL80Base and PFA80Base. Values of q_x

Age	Males		Females	
	Lives	Amounts	Lives	Amounts
x	PML80Base	PMA80Base	PFL80Base	PFA80Base
40	0·001460	0·001142	0·001027	0·000873
41	0·001641	0·001283	0·001132	0·000962
42	0·001849	0·001447	0·001247	0·001060
43	0·002088	0·001633	0·001376	0·001170
44	0·002359	0·001845	0·001519	0·001291
45	0·002666	0·002086	0·001678	0·001426
46	0·003013	0·002357	0·001854	0·001576
47	0·003403	0·002662	0·002050	0·001742
48	0·003841	0·003005	0·002266	0·001927
49	0·004331	0·003388	0·002507	0·002131
50	0·004877	0·003816	0·002773	0·002358
51	0·005487	0·004293	0·003069	0·002609
52	0·006165	0·004824	0·003396	0·002887
53	0·006918	0·005414	0·003758	0·003195
54	0·007754	0·006069	0·004160	0·003537
55	0·008634	0·006759	0·004604	0·003915
56	0·009580	0·007500	0·005097	0·004333
57	0·010657	0·008344	0·005641	0·004797
58	0·011878	0·009301	0·006244	0·005309
59	0·013261	0·010386	0·006911	0·005877
60	0·014823	0·011611	0·007649	0·006505
61	0·016583	0·012992	0·008465	0·007199
62	0·018561	0·014545	0·009368	0·007967
63	0·020779	0·016287	0·010366	0·008817
64	0·023259	0·018236	0·011469	0·009756
65	0·025872	0·020410	0·012689	0·010795
66	0·028620	0·022831	0·014037	0·011943
67	0·031667	0·025519	0·015491	0·013180
68	0·035032	0·028496	0·017111	0·014578
69	0·038736	0·031784	0·018982	0·016228
70	0·042799	0·035405	0·021133	0·018165
71	0·047240	0·039383	0·023596	0·020422
72	0·052075	0·043741	0·026404	0·023036
73	0·057321	0·048502	0·029591	0·026042
74	0·062992	0·053687	0·033193	0·029477
75	0·069099	0·059320	0·037246	0·033373
76	0·075649	0·065420	0·041784	0·037763
77	0·082649	0·072007	0·046844	0·042674
78	0·090099	0·079098	0·052457	0·048128
79	0·097996	0·086708	0·058655	0·054140

Table A4. *(Continued)*
Pensioners – PML80Base, PMA80Base, PFL80Base and PFA80Base
Values of q_x

	Males		Females	
Age x	Lives PML80Base	Amounts PMA80Base	Lives PFL80Base	Amounts PFA80Base
80	0·106334	0·094850	0·065465	0·060719
81	0·115102	0·103533	0·072912	0·067863
82	0·124282	0·112763	0·081013	0·075560
83	0·133855	0·122544	0·089781	0·083788
84	0·143795	0·132873	0·099222	0·092512
85	0·154070	0·143746	0·109334	0·101686
86	0·164647	0·155151	0·120107	0·111251
87	0·175485	0·167075	0·131521	0·121138
88	0·186542	0·179496	0·143548	0·131267
89	0·197771	0·192392	0·156150	0·141548
90	0·209121	0·205732	0·169280	0·151883
91	0·221830	0·219482	0·182882	0·162168
92	0·236079	0·233604	0·196890	0·172296
93	0·250713	0·248110	0·211231	0·182157
94	0·265925	0·263194	0·225825	0·191641
95	0·281765	0·278903	0·238499	0·200821
96	0·298227	0·295235	0·249526	0·210355
97	0·315304	0·312180	0·261161	0·220441
98	0·332980	0·329727	0·273430	0·231108
99	0·351239	0·347858	0·286361	0·242384
100	0·370059	0·366552	0·299979	0·254299
101	0·389412	0·385783	0·314310	0·266882
102	0·409267	0·405520	0·329379	0·280164
103	0·429587	0·425727	0·345208	0·294173
104	0·450330	0·446363	0·361816	0·308935
105	0·471447	0·467383	0·379220	0·324479
106	0·492888	0·488734	0·397432	0·340827
107	0·514596	0·510362	0·416459	0·358001
108	0·536508	0·532206	0·436302	0·376017
109	0·558559	0·554201	0·456956	0·394889
110	0·580680	0·576278	0·478405	0·414623
111	0·602798	0·598367	0·500625	0·435220
112	0·624837	0·620392	0·523582	0·456670
113	0·646722	0·642278	0·547228	0·478957
114	0·668373	0·663946	0·571502	0·502051
115	0·689713	0·685320	0·596327	0·525911
116	0·710664	0·706321	0·621612	0·550482
117	0·731151	0·726874	0·647247	0·575692
118	0·751099	0·746906	0·673106	0·601455
119	0·770441	0·766347	0·699046	0·627664

Table A5. *Annuitants – IM80Base and IF80Base*
One year select
Values of $q_{[x-t]+t}$

Age x	Males - IM80Base Duration 0	Males - IM80Base Durations 1+	Females - IF80Base Duration 0	Females - IF80Base Durations 1+
16	0·000510	0·000530	0·000228	0·000300
17	0·001054	0·001096	0·000233	0·000306
18	0·000996	0·001035	0·000227	0·000299
19	0·000941	0·000978	0·000222	0·000292
20	0·000889	0·000924	0·000218	0·000287
21	0·000840	0·000874	0·000216	0·000283
22	0·000795	0·000827	0·000214	0·000281
23	0·000754	0·000784	0·000213	0·000280
24	0·000718	0·000747	0·000214	0·000282
25	0·000687	0·000714	0·000217	0·000285
26	0·000661	0·000687	0·000221	0·000290
27	0·000641	0·000666	0·000227	0·000298
28	0·000627	0·000652	0·000238	0·000313
29	0·000621	0·000646	0·000255	0·000336
30	0·000623	0·000647	0·000275	0·000361
31	0·000633	0·000658	0·000297	0·000390
32	0·000652	0·000678	0·000322	0·000423
33	0·000682	0·000710	0·000349	0·000459
34	0·000724	0·000753	0·000380	0·000500
35	0·000778	0·000809	0·000415	0·000546
36	0·000845	0·000879	0·000454	0·000597
37	0·000928	0·000965	0·000497	0·000654
38	0·001027	0·001068	0·000546	0·000717
39	0·001145	0·001191	0·000600	0·000788

Table A5. *(Continued)*
Annuitants – IM80Base and IF80Base
One year select
Values of $q_{[x-t]+t}$

Age	Males - IM80Base		Females - IF80Base	
x	Duration 0	Durations 1 +	Duration 0	Durations 1 +
40	0·001282	0·001333	0·000660	0·000867
41	0·001441	0·001499	0·000727	0·000955
42	0·001624	0·001689	0·000801	0·001053
43	0·001834	0·001907	0·000884	0·001162
44	0·002072	0·002155	0·000976	0·001282
45	0·002342	0·002436	0·001078	0·001416
46	0·002647	0·002752	0·001191	0·001565
47	0·002989	0·003109	0·001317	0·001730
48	0·003374	0·003509	0·001456	0·001913
49	0·003804	0·003956	0·001611	0·002116
50	0·004285	0·004456	0·001782	0·002341
51	0·004820	0·005013	0·001972	0·002591
52	0·005416	0·005632	0·002190	0·002875
53	0·006078	0·006321	0·002440	0·003199
54	0·006813	0·007085	0·002719	0·003559
55	0·007627	0·007931	0·003030	0·003960
56	0·008528	0·008868	0·003376	0·004406
57	0·009525	0·009904	0·003762	0·004902
58	0·010626	0·011049	0·004192	0·005454
59	0·011842	0·012313	0·004670	0·006067
60	0·013183	0·013706	0·005204	0·006750
61	0·014661	0·015243	0·005798	0·007509
62	0·016289	0·016935	0·006459	0·008353
63	0·018081	0·018797	0·007196	0·009291
64	0·020053	0·020846	0·008016	0·010334
65	0·021813	0·022903	0·008930	0·011494
66	0·023299	0·024947	0·009947	0·012783
67	0·024885	0·027172	0·011080	0·014215
68	0·026577	0·029591	0·012340	0·015807
69	0·028382	0·032223	0·013743	0·017575
70	0·030309	0·035084	0·015304	0·019539
71	0·032363	0·038195	0·017041	0·021720
72	0·034555	0·041575	0·018973	0·024141
73	0·036892	0·045247	0·021122	0·026829
74	0·039384	0·049235	0·023511	0·029811
75	0·042040	0·053565	0·026167	0·033120
76	0·044872	0·058264	0·029119	0·036788
77	0·047889	0·063361	0·032398	0·040854
78	0·051104	0·068887	0·036039	0·045358
79	0·054528	0·074876	0·040081	0·050347

Table A5. *(Continued)*
Annuitants – IM80Base and IF80Base
One year select
Values of $q_{[x-t]+t}$

Age	Males - IM80Base		Females - IF80Base	
x	Duration 0	Durations 1+	Duration 0	Durations 1+
80	0·058746	0·081362	0·044565	0·055867
81	0·063884	0·088383	0·049539	0·061973
82	0·069454	0·095977	0·055051	0·068721
83	0·075490	0·104186	0·061156	0·076174
84	0·082027	0·113052	0·067914	0·084399
85	0·089103	0·122620	0·075388	0·093465
86	0·096756	0·132935	0·083647	0·103449
87	0·105027	0·144044	0·092765	0·114430
88	0·113960	0·155995	0·102820	0·126493
89	0·123599	0·168837	0·113815	0·139724
90	0·133990	0·182617	0·124852	0·153049
91	0·145180	0·197383	0·135711	0·166119
92	0·157216	0·213180	0·147152	0·179846
93	0·170148	0·230050	0·159184	0·194233
94	0·184023	0·248034	0·171819	0·209286
95	0·198888	0·267166	0·185145	0·225103
96	0·214789	0·287472	0·199497	0·242066
97	0·231767	0·308974	0·215014	0·260321
98	0·249862	0·331680	0·231765	0·279930
99	0·269108	0·355590	0·249819	0·300947
100	0·289531	0·380688	0·269239	0·323415
101		0·406942		0·347368
102		0·434302		0·372824
103		0·462700		0·399778
104		0·492042		0·428205
105		0·522214		0·458051
106		0·553075		0·489228
107		0·584458		0·521614
108		0·616173		0·555042
109		0·648002		0·589307
110		0·679708		0·624154
111		0·711035		0·659285
112		0·741713		0·694359
113		0·771464		0·729001
114		0·800013		0·762805
115		0·827094		0·795353
116		0·852462		0·826228
117		0·875900		0·855034
118		0·897235		0·881419
119		0·916341		0·905097

Table A6. *Widows – WL80Base and WA80Base*
Values of q_x

Age x	Lives WL80Base	Amounts WA80Base
16	0·000430	0·000300
17	0·000660	0·000461
18	0·000644	0·000449
19	0·000630	0·000440
20	0·000619	0·000432
21	0·000611	0·000426
22	0·000606	0·000423
23	0·000605	0·000422
24	0·000607	0·000424
25	0·000614	0·000429
26	0·000626	0·000437
27	0·000642	0·000448
28	0·000675	0·000471
29	0·000724	0·000505
30	0·000779	0·000544
31	0·000841	0·000587
32	0·000911	0·000636
33	0·000990	0·000691
34	0·001078	0·000752
35	0·001177	0·000821
36	0·001287	0·000898
37	0·001410	0·000984
38	0·001547	0·001079
39	0·001700	0·001186

Table A6. *(Continued)*
Widows – WL80Base and WA80Base
Values of q_x

Age x	Lives WL80Base	Amounts WA80Base
40	0·001870	0·001305
41	0·002059	0·001437
42	0·002270	0·001584
43	0·002504	0·001748
44	0·002764	0·001929
45	0·003052	0·002130
46	0·003370	0·002352
47	0·003721	0·002597
48	0·004108	0·002868
49	0·004536	0·003167
50	0·005008	0·003497
51	0·005529	0·003861
52	0·006104	0·004263
53	0·006739	0·004706
54	0·007440	0·005196
55	0·008159	0·005737
56	0·008892	0·006333
57	0·009690	0·006992
58	0·010559	0·007719
59	0·011505	0·008520
60	0·012536	0·009405
61	0·013659	0·010381
62	0·014881	0·011458
63	0·016212	0·012646
64	0·017661	0·013956
65	0·019238	0·015401
66	0·020954	0·016994
67	0·022822	0·018751
68	0·024853	0·020687
69	0·027064	0·022820
70	0·029468	0·025171
71	0·032082	0·027760
72	0·034924	0·030612
73	0·038012	0·033751
74	0·041367	0·037206
75	0·045012	0·041007
76	0·048970	0·045188
77	0·053266	0·049783
78	0·057927	0·054832
79	0·062982	0·060377

Table A6. *(Continued)*
Widows – WL80Base and WA80Base
Values of q_x

Age x	Lives WL80Base	Amounts WA80Base
80	0·068462	0·066462
81	0·075305	0·073137
82	0·083660	0·080452
83	0·092700	0·088464
84	0·102430	0·097230
85	0·112848	0·106813
86	0·123942	0·117278
87	0·135692	0·128692
88	0·148067	0·141126
89	0·161028	0·154651
90	0·174524	0·167435
91	0·188497	0·178659
92	0·202880	0·189696
93	0·217595	0·200427
94	0·232562	0·210734
95	0·245550	0·220697
96	0·256846	0·231032
97	0·268757	0·241950
98	0·281311	0·253478
99	0·294534	0·265646
100	0·308451	0·278482
101	0·323088	0·292012
102	0·338466	0·306265
103	0·354607	0·321266
104	0·371528	0·337039
105	0·389244	0·353604
106	0·407764	0·370981
107	0·427093	0·389184
108	0·447228	0·408220
109	0·468159	0·428095
110	0·489868	0·448803
111	0·512326	0·470333
112	0·535492	0·492662
113	0·559313	0·515756
114	0·583723	0·539570
115	0·608638	0·564044
116	0·633961	0·589103
117	0·659576	0·614655
118	0·685350	0·640590
119	0·711135	0·666782

Table A7. *Pensioners projected – PML80, PMA80, PFL80, PFA80 (C = 2010)*
Values of q_x for calendar year 2010

	Males		Females	
Age	Lives	Amounts	Lives	Amounts
x	PML80C10	PMA80C10	PFL80C10	PFA80C10
16	0·000425	0·000332	0·000221	0·000188
17	0·000752	0·000588	0·000227	0·000193
18	0·000710	0·000556	0·000222	0·000189
19	0·000671	0·000525	0·000217	0·000184
20	0·000634	0·000496	0·000213	0·000181
21	0·000599	0·000469	0·000211	0·000179
22	0·000567	0·000444	0·000209	0·000177
23	0·000538	0·000421	0·000208	0·000177
24	0·000512	0·000400	0·000209	0·000178
25	0·000490	0·000383	0·000211	0·000180
26	0·000471	0·000368	0·000216	0·000183
27	0·000457	0·000358	0·000221	0·000188
28	0·000447	0·000350	0·000232	0·000197
29	0·000443	0·000346	0·000249	0·000212
30	0·000444	0·000347	0·000268	0·000228
31	0·000451	0·000353	0·000289	0·000246
32	0·000465	0·000364	0·000314	0·000267
33	0·000487	0·000381	0·000341	0·000289
34	0·000516	0·000403	0·000371	0·000315
35	0·000554	0·000434	0·000405	0·000344
36	0·000603	0·000472	0·000443	0·000377
37	0·000662	0·000517	0·000486	0·000412
38	0·000733	0·000573	0·000533	0·000452
39	0·000816	0·000638	0·000585	0·000497

Table A7. *(Continued)*
Pensioners projected – PML80, PMA80, PFL80, PFA80 (C = 2010)
Values of q_x for calendar year 2010

	Males		Females	
Age	Lives	Amounts	Lives	Amounts
x	PML80C10	PMA80C10	PFL80C10	PFA80C10
40	0·000915	0·000715	0·000643	0·000547
41	0·001028	0·000804	0·000709	0·000603
42	0·001158	0·000907	0·000781	0·000664
43	0·001308	0·001023	0·000862	0·000733
44	0·001478	0·001156	0·000952	0·000809
45	0·001670	0·001307	0·001051	0·000893
46	0·001888	0·001477	0·001162	0·000987
47	0·002132	0·001668	0·001284	0·001091
48	0·002406	0·001883	0·001420	0·001207
49	0·002713	0·002123	0·001571	0·001335
50	0·003055	0·002391	0·001737	0·001477
51	0·003438	0·002690	0·001923	0·001635
52	0·003862	0·003022	0·002128	0·001809
53	0·004334	0·003392	0·002354	0·002002
54	0·004858	0·003802	0·002606	0·002216
55	0·005409	0·004234	0·002884	0·002453
56	0·006002	0·004699	0·003193	0·002715
57	0·006677	0·005227	0·003534	0·003005
58	0·007441	0·005827	0·003912	0·003326
59	0·008308	0·006507	0·004330	0·003682
60	0·009286	0·007274	0·004792	0·004075
61	0·010513	0·008236	0·005366	0·004564
62	0·011906	0·009330	0·006009	0·005110
63	0·013484	0·010569	0·006727	0·005721
64	0·015267	0·011970	0·007528	0·006404
65	0·017175	0·013549	0·008423	0·007166
66	0·019213	0·015327	0·009423	0·008017
67	0·021495	0·017322	0·010515	0·008946
68	0·024041	0·019555	0·011742	0·010004
69	0·026872	0·022049	0·013168	0·011258
70	0·030010	0·024826	0·014818	0·012737
71	0·033477	0·027909	0·016722	0·014472
72	0·037293	0·031324	0·018909	0·016497
73	0·041478	0·035096	0·021412	0·018844
74	0·046052	0·039249	0·024267	0·021550
75	0·051033	0·043810	0·027508	0·024647
76	0·056435	0·048804	0·031171	0·028172
77	0·062275	0·054256	0·035296	0·032154
78	0·068561	0·060190	0·039917	0·036623
79	0·075303	0·066629	0·045072	0·041603

Table A7. *(Continued)*
Pensioners projected – PML80, PMA80, PFL80, PFA80 (C = 2010
Values of q_x for calendar year 2010

	Males		Females	
Age	Lives	Amounts	Lives	Amounts
x	PML80C10	PMA80C10	PFL80C10	PFA80C10
80	0·082504	0·073594	0·050794	0·047112
81	0·090167	0·081104	0·057117	0·053161
82	0·098287	0·089177	0·064068	0·059755
83	0·106857	0·097828	0·071673	0·066888
84	0·115866	0·107066	0·079951	0·074544
85	0·125297	0·116901	0·088915	0·082696
86	0·135128	0·127335	0·098574	0·091305
87	0·145334	0·138369	0·108924	0·100325
88	0·155885	0·149997	0·119957	0·109694
89	0·166746	0·162211	0·131654	0·119343
90	0·177878	0·174995	0·143989	0·129191
91	0·190345	0·188330	0·156925	0·139151
92	0·204335	0·202193	0·170416	0·149129
93	0·218874	0·216602	0·184406	0·159024
94	0·234141	0·231736	0·198834	0·168736
95	0·250192	0·247651	0·211775	0·178318
96	0·267038	0·264359	0·223430	0·188356
97	0·284684	0·281863	0·235799	0·199033
98	0·303131	0·300170	0·248919	0·210391
99	0·322377	0·319274	0·262830	0·222467
100	0·342415	0·339170	0·277570	0·235302
101	0·363231	0·359846	0·293178	0·248939
102	0·384809	0·381286	0·309695	0·263421
103	0·407123	0·403465	0·327157	0·278790
104	0·430146	0·426357	0·345599	0·295088
105	0·453838	0·449926	0·365056	0·312359
106	0·478160	0·474130	0·385556	0·330643
107	0·503064	0·498925	0·407126	0·349978
108	0·528492	0·524255	0·429783	0·370399
109	0·554386	0·550061	0·453542	0·391939
110	0·580680	0·576278	0·478405	0·414623
111	0·602798	0·598367	0·500625	0·435220
112	0·624837	0·620392	0·523582	0·456670
113	0·646722	0·642278	0·547228	0·478957
114	0·668373	0·663946	0·571502	0·502051
115	0·689713	0·685320	0·596327	0·525911
116	0·710664	0·706321	0·621612	0·550482
117	0·731151	0·726874	0·647247	0·575692
118	0·751099	0·746906	0·673106	0·601455
119	0·770441	0·766347	0·699046	0·627664

Table A8. *Annuitants projected – IM80 and IF80 (C = 2010)*
One year select
Values of $q_{[x-t]+t}$ *for calendar year* 2010

Age	Males - IM80C10		Females - IF80C10	
x	Duration 0	Durations 1+	Duration 0	Durations 1+
16	0.000320	0.000332	0·000143	0·000188
17	0·000660	0·000687	0·000146	0·000192
18	0·000624	0·000648	0·000142	0·000187
19	0·000590	0·000613	0·000139	0·000183
20	0·000557	0·000579	0·000137	0·000180
21	0·000526	0·000548	0·000135	0·000177
22	0·000498	0·000518	0·000134	0·000176
23	0·000472	0·000491	0·000133	0·000175
24	0·000450	0·000468	0·000134	0·000177
25	0·000430	0·000447	0·000136	0·000179
26	0·000414	0·000430	0·000138	0·000182
27	0·000402	0·000417	0·000142	0·000187
28	0·000393	0·000408	0·000149	0·000196
29	0·000389	0·000405	0·000160	0·000211
30	0·000390	0·000405	0·000172	0·000226
31	0·000397	0·000412	0·000186	0·000244
32	0·000408	0·000425	0·000202	0·000265
33	0·000427	0·000445	0·000219	0·000288
34	0·000454	0·000472	0·000238	0·000313
35	0·000487	0·000507	0·000260	0·000342
36	0·000529	0·000551	0·000284	0·000374
37	0·000581	0·000605	0·000311	0·000410
38	0·000643	0·000669	0·000342	0·000449
39	0·000717	0·000746	0·000376	0·000494

Table A8. *(Continued)*
Annuitants projected – IM80 and IF80 (C = 2010)
One year select
Values of $q_{[x-t]+t}$ for calendar year 2010

Age x	Males - IM80C10 Duration 0	Males - IM80C10 Durations 1+	Females - IF80C10 Duration 0	Females - IF80C10 Durations 1+
40	0·000803	0·000835	0·000413	0·000543
41	0·000903	0·000939	0·000455	0·000598
42	0·001017	0·001058	0·000502	0·000660
43	0·001149	0·001195	0·000554	0·000728
44	0·001298	0·001350	0·000611	0·000803
45	0·001467	0·001526	0·000675	0·000887
46	0·001658	0·001724	0·000746	0·000980
47	0·001873	0·001948	0·000825	0·001084
48	0·002114	0·002198	0·000912	0·001198
49	0·002383	0·002478	0·001009	0·001326
50	0·002685	0·002792	0·001116	0·001467
51	0·003020	0·003141	0·001235	0·001623
52	0·003393	0·003528	0·001372	0·001801
53	0·003808	0·003960	0·001529	0·002004
54	0·004268	0·004439	0·001703	0·002230
55	0·004778	0·004969	0·001898	0·002481
56	0·005343	0·005556	0·002115	0·002760
57	0·005967	0·006205	0·002357	0·003071
58	0·006657	0·006922	0·002626	0·003417
59	0·007419	0·007714	0·002926	0·003801
60	0·008259	0·008587	0·003260	0·004229
61	0·009295	0·009663	0·003676	0·004760
62	0·010448	0·010863	0·004143	0·005358
63	0·011733	0·012197	0·004669	0·006029
64	0·013162	0·013683	0·005261	0·006783
65	0·014480	0·015204	0·005928	0·007630
66	0·015641	0·016747	0·006678	0·008581
67	0·016891	0·018444	0·007521	0·009649
68	0·018239	0·020307	0·008468	0·010848
69	0·019689	0·022354	0·009534	0·012192
70	0·021252	0·024601	0·010731	0·013701
71	0·022934	0·027067	0·012076	0·015392
72	0·024746	0·029773	0·013587	0·017288
73	0·026695	0·032741	0·015284	0·019414
74	0·028793	0·035994	0·017188	0·021794
75	0·031048	0·039560	0·019325	0·024461
76	0·033475	0·043466	0·021723	0·027444
77	0·036084	0·047742	0·024411	0·030783
78	0·038888	0·052420	0·027424	0·034515
79	0·041901	0·057537	0·030799	0·038688

Table A8. *(Continued)*
Annuitants projected – IM80 and IF80 (C = 2010)
One year select
Values of $q_{[x-t]+t}$ for calendar year 2010

Age x	Males - IM80C10 Duration 0	Males - IM80C10 Durations 1+	Females - IF80C10 Duration 0	Females - IF80C10 Durations 1+
80	0·045581	0·063128	0·034578	0·043347
81	0·050044	0·069236	0·038807	0·048547
82	0·054927	0·075902	0·043536	0·054347
83	0·060264	0·083172	0·048821	0·060810
84	0·066095	0·091095	0·054723	0·068007
85	0·072463	0·099720	0·061309	0·076010
86	0·079409	0·109102	0·068650	0·084902
87	0·086982	0·119295	0·076827	0·094769
88	0·095231	0·130358	0·085922	0·105705
89	0·104210	0·142351	0·095960	0·117805
90	0·113971	0·155333	0·106199	0·130183
91	0·124574	0·169368	0·116449	0·142541
92	0·136076	0·184515	0·127365	0·155663
93	0·148540	0·200835	0·138969	0·169567
94	0·162028	0·218388	0·151283	0·184272
95	0·176602	0·237229	0·164399	0·199880
96	0·192326	0·257407	0·178633	0·216750
97	0·209260	0·278969	0·194133	0·235041
98	0·227464	0·301947	0·210989	0·254836
99	0·246995	0·326370	0·229291	0·276218
100	0·267903	0·352250	0·249126	0·299255
101		0·379583		0·324014
102		0·408347		0·350544
103		0·438505		0·378873
104		0·469988		0·409012
105		0·502709		0·440942
106		0·536549		0·474610
107		0·571360		0·509924
108		0·606967		0·546749
109		0·643161		0·584905
110		0·679708		0·624154
111		0·711035		0·659285
112		0·741713		0·694359
113		0·771464		0·729001
114		0·800013		0·762805
115		0·827094		0·795353
116		0·852462		0·826228
117		0·875900		0·855034
118		0·897235		0·881419
119		0·916341		0·905097

Table A9. *Widows projected – WL80 and WA80 (C = 2010)*
Values of q_x for calendar year 2010

Age x	Lives WL80C10	Amounts WA80C10
16	0·000269	0·000188
17	0·000413	0·000289
18	0·000403	0·000281
19	0·000395	0·000276
20	0·000388	0·000271
21	0·000383	0·000267
22	0·000380	0·000265
23	0·000379	0·000264
24	0·000380	0·000266
25	0·000385	0·000269
26	0·000392	0·000274
27	0·000402	0·000281
28	0·000423	0·000295
29	0·000454	0·000316
30	0·000488	0·000341
31	0·000527	0·000368
32	0·000571	0·000398
33	0·000620	0·000433
34	0·000675	0·000471
35	0·000737	0·000514
36	0·000806	0·000563
37	0·000883	0·000616
38	0·000969	0·000676
39	0·001065	0·000743

Table A9. *(Continued)*
Widows projected – WL80 and WA80 (C = 2010)
Values of q_x for calendar year 2010

Age x	Lives WL80C10	Amounts WA80C10
40	0·001172	0·000818
41	0·001290	0·000900
42	0·001422	0·000992
43	0·001569	0·001095
44	0·001732	0·001209
45	0·001912	0·001334
46	0·002111	0·001474
47	0·002331	0·001627
48	0·002574	0·001797
49	0·002842	0·001984
50	0·003137	0·002191
51	0·003464	0·002419
52	0·003824	0·002671
53	0·004222	0·002948
54	0·004661	0·003255
55	0·005112	0·003594
56	0·005571	0·003968
57	0·006071	0·004380
58	0·006615	0·004836
59	0·007208	0·005338
60	0·007854	0·005892
61	0·008659	0·006581
62	0·009545	0·007350
63	0·010520	0·008206
64	0·011592	0·009160
65	0·012771	0·010224
66	0·014067	0·011408
67	0·015491	0·012728
68	0·017055	0·014197
69	0·018775	0·015831
70	0·020663	0·017650
71	0·022735	0·019672
72	0·025010	0·021922
73	0·027506	0·024422
74	0·030242	0·027200
75	0·033243	0·030285
76	0·036532	0·033711
77	0·040135	0·037511
78	0·044080	0·041725
79	0·048397	0·046395

Table A9. *(Continued)*
Widows projected – WL80 and WA80 (C = 2010)
Values of q_x for calendar year 2010

Age x	Lives WL80C10	Amounts WA80C10
80	0·053119	0·051568
81	0·058991	0·057293
82	0·066161	0·063624
83	0·074003	0·070621
84	0·082536	0·078346
85	0·091773	0·086865
86	0·101721	0·096252
87	0·112378	0·106581
88	0·123733	0·117933
89	0·135767	0·130390
90	0·148449	0·142420
91	0·161743	0·153301
92	0·175600	0·164189
93	0·189962	0·174974
94	0·204766	0·185546
95	0·218035	0·195967
96	0·229984	0·206870
97	0·242657	0·218454
98	0·256094	0·230756
99	0·270332	0·243817
100	0·285409	0·257679
101	0·301366	0·272380
102	0·318239	0·287962
103	0·336064	0·304467
104	0·354876	0·321933
105	0·374705	0·340397
106	0·395580	0·359896
107	0·417522	0·380462
108	0·440546	0·402121
109	0·464662	0·424897
110	0·489868	0·448803
111	0·512326	0·470333
112	0·535492	0·492662
113	0·559313	0·515756
114	0·583723	0·539570
115	0·608638	0·564044
116	0·633961	0·589103
117	0·659576	0·614655
118	0·685350	0·640590
119	0·711135	0·666782

Table A10. *Pensioners projected – PML80, PMA80, PFL80, PFA80 (B = 1935)*
Values of q_x for year of birth 1935

		Males		Females	
Age	in	Lives	Amounts	Lives	Amounts
x	year	PML80B35	PMA80B35	PFL80B35	PFA80B35
45	1980	0·002666	0·002086	0·001678	0·001426
46	1981	0·002946	0·002304	0·001812	0·001541
47	1982	0·003254	0·002545	0·001960	0·001666
48	1983	0·003594	0·002812	0·002121	0·001803
49	1984	0·003968	0·003104	0·002297	0·001953
50	1985	0·004378	0·003425	0·002489	0·002117
51	1986	0·004828	0·003777	0·002700	0·002295
52	1987	0·005319	0·004162	0·002930	0·002491
53	1988	0·005857	0·004583	0·003181	0·002705
54	1989	0·006444	0·005044	0·003457	0·002939
55	1990	0·007047	0·005517	0·003758	0·003196
56	1991	0·007684	0·006016	0·004088	0·003475
57	1992	0·008403	0·006580	0·004448	0·003783
58	1993	0·009213	0·007214	0·004843	0·004118
59	1994	0·010122	0·007927	0·005275	0·004486
60	1995	0·011139	0·008726	0·005748	0·004888
61	1996	0·012361	0·009685	0·006310	0·005366
62	1997	0·013740	0·010767	0·006935	0·005898
63	1998	0·015294	0·011988	0·007630	0·006490
64	1999	0·017040	0·013360	0·008402	0·007147
65	2000	0·018887	0·014899	0·009263	0·007880
66	2001	0·020839	0·016624	0·010221	0·008696
67	2002	0·023020	0·018551	0·011261	0·009581
68	2003	0·025448	0·020700	0·012430	0·010590
69	2004	0·028143	0·023092	0·013791	0·011790
70	2005	0·031125	0·025748	0·015369	0·013210
71	2006	0·034415	0·028691	0·017190	0·014878
72	2007	0·038030	0·031944	0·019283	0·016823
73	2008	0·041993	0·035532	0·021678	0·019078
74	2009	0·046321	0·039478	0·024408	0·021676
75	2010	0·051033	0·043810	0·027508	0·024647
76	2011	0·056144	0·048552	0·031011	0·028026
77	2012	0·061671	0·053730	0·034954	0·031842
78	2013	0·067625	0·059368	0·039372	0·036123
79	2014	0·074016	0·065490	0·044302	0·040892

Table A10. *(Continued)*
Pensioners projected – PML80, PMA80, PFL80, PFA80 (B = 1935)
Values of q_x for year of birth 1935

Age x	in year	Males Lives PML80B35	Males Amounts PMA80B35	Females Lives PFL80B35	Females Amounts PFA80B35
80	2015	0·080852	0·072120	0·049777	0·046168
81	2016	0·088137	0·079279	0·055831	0·051965
82	2017	0·095871	0·086985	0·062493	0·058287
83	2018	0·104052	0·095259	0·069791	0·065132
84	2019	0·112671	0·104113	0·077745	0·072488
85	2020	0·121715	0·113559	0·086374	0·080332
86	2021	0·131171	0·123606	0·095687	0·088631
87	2022	0·141016	0·134258	0·105687	0·097344
88	2023	0·151226	0·145514	0·116371	0·106415
89	2024	0·161771	0·157372	0·127727	0·115783
90	2025	0·172619	0·169821	0·139732	0·125372
91	2026	0·184805	0·182849	0·152358	0·135101
92	2027	0·198518	0·196437	0·165564	0·144883
93	2028	0·212819	0·210609	0·179304	0·154625
94	2029	0·227884	0·225544	0·193521	0·164227
95	2030	0·243777	0·241301	0·206344	0·173746
96	2031	0·260511	0·257897	0·217969	0·183752
97	2032	0·278099	0·275344	0·230345	0·194430
98	2033	0·296547	0·293650	0·243512	0·205821
99	2034	0·315858	0·312817	0·257515	0·217968
100	2035	0·336031	0·332847	0·272395	0·230916
101	2036	0·357059	0·353732	0·288197	0·244709
102	2037	0·378930	0·375461	0·304964	0·259397
103	2038	0·401625	0·398016	0·322738	0·275025
104	2039	0·425121	0·421376	0·341562	0·291641
105	2040	0·449383	0·445509	0·361473	0·309293
106	2041	0·474378	0·470380	0·382507	0·328027
107	2042	0·500060	0·495945	0·404695	0·347888
108	2043	0·526376	0·522156	0·428063	0·368916
109	2044	0·553271	0·548954	0·452630	0·391151
110	2045	0·580680	0·576278	0·478405	0·414623
111	2046	0·602798	0·598367	0·500625	0·435220
112	2047	0·624837	0·620392	0·523582	0·456670
113	2048	0·646722	0·642278	0·547228	0·478957
114	2049	0·668373	0·663946	0·571502	0·502051
115	2050	0·689713	0·685320	0·596327	0·525911
116	2051	0·710664	0·706321	0·621612	0·550482
117	2052	0·731151	0·726874	0·647247	0·575692
118	2053	0·751099	0·746906	0·673106	0·601455
119	2054	0·770441	0·766347	0·699046	0·627664

Table A11. *Annuitants projected – IM80 and IF80* (B = 1935)
One year select
Values of $q_{[x-t]+t}$ *for year of birth* 1935

Age	in	Males - IM80B35		Females - IF80B35	
x	year	Duration 0	Durations 1+	Duration 0	Durations 1+
45	1980	0·002342	0·002436	0·001078	0·001416
46	1981	0·002588	0·002690	0·001164	0·001530
47	1982	0·002858	0·002973	0·001259	0·001654
48	1983	0·003157	0·003284	0·001363	0·001790
49	1984	0·003486	0·003625	0·001476	0·001939
50	1985	0·003846	0·004000	0·001600	0·002101
51	1986	0·004241	0·004411	0·001735	0·002280
52	1987	0·004673	0·004859	0·001890	0·002481
53	1988	0·005145	0·005351	0·002066	0·002708
54	1989	0·005662	0·005888	0·002260	0·002958
55	1990	0·006225	0·006474	0·002473	0·003232
56	1991	0·006840	0·007113	0·002708	0·003534
57	1992	0·007511	0·007810	0·002966	0·003865
58	1993	0·008242	0·008570	0·003251	0·004230
59	1994	0·009039	0·009398	0·003565	0·004631
60	1995	0·009907	0·010300	0·003911	0·005073
61	1996	0·010929	0·011362	0·004322	0·005597
62	1997	0·012059	0·012537	0·004782	0·006184
63	1998	0·013308	0·013835	0·005297	0·006839
64	1999	0·014691	0·015272	0·005873	0·007571
65	2000	0·015923	0·016719	0·006519	0·008391
66	2001	0·016964	0·018164	0·007243	0·009308
67	2002	0·018090	0·019752	0·008054	0·010333
68	2003	0·019306	0·021496	0·008964	0·011483
69	2004	0·020621	0·023411	0·009985	0·012769
70	2005	0·022042	0·025515	0·011130	0·014210
71	2006	0·023577	0·027825	0·012414	0·015823
72	2007	0·025235	0·030362	0·013856	0·017630
73	2008	0·027027	0·033147	0·015474	0·019655
74	2009	0·028961	0·036205	0·017289	0·021921
75	2010	0·031048	0·039560	0·019325	0·024461
76	2011	0·033302	0·043241	0·021611	0·027303
77	2012	0·035734	0·047278	0·024175	0·030484
78	2013	0·038357	0·051704	0·027049	0·034044
79	2014	0·041185	0·056553	0·030273	0·038027

Table A11. *(Continued)*
Annuitants projected – IM80 and IF80 (B = 1935)
One year select
Values of $q_{[x-t]+t}$ for year of birth 1935

Age	in	Males - IM80B35		Females - IF80B35	
x	year	Duration 0	Durations 1+	Duration 0	Durations 1+
80	2015	0·044668	0·061864	0·033885	0·042479
81	2016	0·048918	0·067678	0·037934	0·047455
82	2017	0·053577	0·074037	0·042466	0·053011
83	2018	0·058682	0·080989	0·047539	0·059214
84	2019	0·064272	0·088582	0·053214	0·066131
85	2020	0·070391	0·096870	0·059557	0·073837
86	2021	0·077084	0·105907	0·066640	0·082416
87	2022	0·084397	0·115751	0·074544	0·091953
88	2023	0·092385	0·126462	0·083354	0·102545
89	2024	0·101101	0·138104	0·093098	0·114291
90	2025	0·110602	0·150741	0·103059	0·126334
91	2026	0·120949	0·164439	0·113060	0·138393
92	2027	0·132203	0·179263	0·123740	0·151232
93	2028	0·144431	0·195279	0·135124	0·164875
94	2029	0·157698	0·212553	0·147240	0·179348
95	2030	0·172074	0·231146	0·160184	0·194754
96	2031	0·187625	0·251116	0·174267	0·211453
97	2032	0·204419	0·272516	0·189643	0·229604
98	2033	0·222523	0·295389	0·206406	0·249301
99	2034	0·242000	0·319770	0·224654	0·270632
100	2035	0·262908	0·345683	0·244482	0·293676
101	2036		0·373133		0·318508
102	2037		0·402109		0·345188
103	2038		0·432583		0·373756
104	2039		0·464497		0·404234
105	2040		0·497774		0·436614
106	2041		0·532304		0·470855
107	2042		0·567948		0·506879
108	2043		0·604537		0·544560
109	2044		0·641867		0·583728
110	2045		0·679708		0·624154
111	2046		0·711035		0·659285
112	2047		0·741713		0·694359
113	2048		0·771464		0·729001
114	2049		0·800013		0·762805
115	2050		0·827094		0·795353
116	2051		0·852462		0·826228
117	2052		0·875900		0·855034
118	2053		0·897235		0·881419
119	2054		0·916341		0·905097

Table A12. *Widows projected – WL80 and WA80 (B = 1935)*
Values of q_x for year of birth 1935

Age x	in year	Lives WL80B35	Amounts WA80B35
45	1980	0·003052	0·002130
46	1981	0·003295	0·002299
47	1982	0·003558	0·002483
48	1983	0·003844	0·002684
49	1984	0·004156	0·002902
50	1985	0·004495	0·003139
51	1986	0·004865	0·003397
52	1987	0·005267	0·003678
53	1988	0·005705	0·003984
54	1989	0·006183	0·004318
55	1990	0·006660	0·004683
56	1991	0·007132	0·005079
57	1992	0·007641	0·005513
58	1993	0·008190	0·005987
59	1994	0·008781	0·006503
60	1995	0·009421	0·007068
61	1996	0·010182	0·007738
62	1997	0·011016	0·008482
63	1998	0·011933	0·009308
64	1999	0·012939	0·010225
65	2000	0·014044	0·011243
66	2001	0·015257	0·012374
67	2002	0·016590	0·013631
68	2003	0·018054	0·015028
69	2004	0·019663	0·016580
70	2005	0·021430	0·018305
71	2006	0·023372	0·020223
72	2007	0·025505	0·022356
73	2008	0·027847	0·024725
74	2009	0·030419	0·027359
75	2010	0·033243	0·030285
76	2011	0·036344	0·033537
77	2012	0·039746	0·037147
78	2013	0·043478	0·041155
79	2014	0·047570	0·045602

Table A12. *(Continued)*
Widows projected – WL80 and WA80 (B = 1935)
Values of q_x for year of birth 1935

Age x	in year	Lives WL80B35	Amounts WA80B35
80	2015	0·052056	0·050535
81	2016	0·057663	0·056003
82	2017	0·064535	0·062061
83	2018	0·072060	0·068767
84	2019	0·080259	0·076185
85	2020	0·089150	0·084382
86	2021	0·098742	0·093433
87	2022	0·109039	0·103414
88	2023	0·120035	0·114408
89	2024	0·131717	0·126500
90	2025	0·144061	0·138209
91	2026	0·157036	0·148840
92	2027	0·170601	0·159515
93	2028	0·184706	0·170133
94	2029	0·199294	0·180588
95	2030	0·212445	0·190942
96	2031	0·224363	0·201814
97	2032	0·237045	0·213401
98	2033	0·250531	0·225743
99	2034	0·264865	0·238887
100	2035	0·280088	0·252875
101	2036	0·296245	0·267751
102	2037	0·313377	0·283563
103	2038	0·331526	0·300355
104	2039	0·350730	0·318172
105	2040	0·371027	0·337055
106	2041	0·392451	0·357049
107	2042	0·415028	0·378190
108	2043	0·438782	0·400511
109	2044	0·463727	0·424042
110	2045	0·489868	0·448803
111	2046	0·512326	0·470333
112	2047	0·535492	0·492662
113	2048	0·559313	0·515756
114	2049	0·583723	0·539570
115	2050	0·608638	0·564044
116	2051	0·633961	0·589103
117	2052	0·659576	0·614655
118	2053	0·685350	0·640590
119	2054	0·711135	0·666782

Table A13. *Pensioners projected – PML80, PMA80, PFL80, PFA80 (B = 1964)*
Values of q_x for year of birth 1964

		Males		Females	
Age x	in year	Lives PML80B64	Amounts PMA80B64	Lives PFL80B64	Amounts PFA80B64
16	1980	0·000678	0·000530	0·000353	0·000300
17	1981	0·001173	0·000917	0·000355	0·000301
18	1982	0·001084	0·000848	0·000339	0·000288
19	1983	0·001002	0·000784	0·000324	0·000275
20	1984	0·000927	0·000725	0·000312	0·000265
21	1985	0·000858	0·000671	0·000302	0·000256
22	1986	0·000796	0·000623	0·000293	0·000249
23	1987	0·000741	0·000580	0·000286	0·000243
24	1988	0·000692	0·000541	0·000283	0·000240
25	1989	0·000650	0·000509	0·000280	0·000239
26	1990	0·000614	0·000480	0·000281	0·000238
27	1991	0·000586	0·000458	0·000283	0·000241
28	1992	0·000563	0·000441	0·000293	0·000248
29	1993	0·000548	0·000429	0·000309	0·000262
30	1994	0·000541	0·000423	0·000327	0·000278
31	1995	0·000541	0·000423	0·000347	0·000295
32	1996	0·000550	0·000430	0·000371	0·000315
33	1997	0·000567	0·000444	0·000397	0·000337
34	1998	0·000593	0·000463	0·000426	0·000362
35	1999	0·000628	0·000492	0·000458	0·000389
36	2000	0·000673	0·000527	0·000495	0·000421
37	2001	0·000730	0·000571	0·000536	0·000455
38	2002	0·000799	0·000624	0·000580	0·000493
39	2003	0·000879	0·000687	0·000630	0·000535

Table A13. *(Continued)*
Pensioners projected – PML80, PMA80, PFL80, PFA80 (B = 1964)
Values of q_x for year of birth 1964

		Males		Females	
Age x	in year	Lives PML80B64	Amounts PMA80B64	Lives PFL80B64	Amounts PFA80B64
40	2004	0·000973	0·000761	0·000685	0·000582
41	2005	0·001082	0·000846	0·000746	0·000634
42	2006	0·001205	0·000943	0·000813	0·000691
43	2007	0·001347	0·001053	0·000888	0·000755
44	2008	0·001507	0·001178	0·000970	0·000824
45	2009	0·001686	0·001319	0·001061	0·000902
46	2010	0·001888	0·001477	0·001162	0·000987
47	2011	0·002113	0·001653	0·001273	0·001081
48	2012	0·002364	0·001849	0·001395	0·001186
49	2013	0·002643	0·002068	0·001530	0·001300
50	2014	0·002952	0·002310	0·001679	0·001427
51	2015	0·003295	0·002578	0·001843	0·001567
52	2016	0·003675	0·002876	0·002024	0·001721
53	2017	0·004094	0·003204	0·002224	0·001891
54	2018	0·004557	0·003567	0·002445	0·002079
55	2019	0·005040	0·003946	0·002688	0·002285
56	2020	0·005556	0·004350	0·002956	0·002513
57	2021	0·006143	0·004810	0·003252	0·002765
58	2022	0·006806	0·005329	0·003578	0·003042
59	2023	0·007555	0·005917	0·003937	0·003348
60	2024	0·008399	0·006579	0·004334	0·003686
61	2025	0·009491	0·007436	0·004845	0·004120
62	2026	0·010735	0·008412	0·005418	0·004608
63	2027	0·012147	0·009521	0·006060	0·005154
64	2028	0·013746	0·010778	0·006778	0·005766
65	2029	0·015463	0·012198	0·007584	0·006452
66	2030	0·017302	0·013802	0·008486	0·007220
67	2031	0·019366	0·015607	0·009474	0·008060
68	2032	0·021677	0·017633	0·010588	0·009021
69	2033	0·024255	0·019902	0·011886	0·010161
70	2034	0·027122	0·022436	0·013392	0·011511
71	2035	0·030299	0·025260	0·015134	0·013098
72	2036	0·033808	0·028397	0·017142	0·014955
73	2037	0·037670	0·031874	0·019446	0·017114
74	2038	0·041905	0·035715	0·022082	0·019610
75	2039	0·046535	0·039949	0·025083	0·022475
76	2040	0·051574	0·044601	0·028487	0·025745
77	2041	0·057042	0·049697	0·032331	0·029452
78	2042	0·062951	0·055265	0·036651	0·033626
79	2043	0·069312	0·061328	0·041486	0·038293

Table A13. *(Continued)*
Pensioners projected – PML80, PMA80, PFL80, PFA80 (B = 1964)
Values of q_x for year of birth 1964

		Males		Females	
Age	in	Lives	Amounts	Lives	Amounts
x	year	PML80B64	PMA80B64	PFL80B64	PFA80B64
80	2044	0·076134	0·067911	0·046872	0·043474
81	2045	0·083421	0·075037	0·052844	0·049184
82	2046	0·091175	0·082724	0·059432	0·055432
83	2047	0·099393	0·090994	0·066666	0·062216
84	2048	0·108067	0·099859	0·074569	0·069526
85	2049	0·117185	0·109332	0·083159	0·077342
86	2050	0·126731	0·119422	0·092448	0·085632
87	2051	0·136684	0·130134	0·102441	0·094354
88	2052	0·147018	0·141465	0·113134	0·103455
89	2053	0·157704	0·153415	0·124515	0·112872
90	2054	0·168706	0·165972	0·136565	0·122530
91	2055	0·181039	0·179123	0·149253	0·132348
92	2056	0·194891	0·192848	0·162540	0·142236
93	2057	0·209344	0·207170	0·176376	0·152100
94	2058	0·224571	0·222264	0·190707	0·161839
95	2059	0·240633	0·238189	0·203683	0·171505
96	2060	0·257544	0·254960	0·215487	0·181659
97	2061	0·275317	0·272589	0·228040	0·192484
98	2062	0·293956	0·291084	0·241385	0·204023
99	2063	0·313465	0·310447	0·255564	0·216317
100	2064	0·333842	0·330678	0·270620	0·229411
101	2065	0·355078	0·351769	0·286598	0·243352
102	2066	0·377162	0·373709	0·303541	0·258187
103	2067	0·400075	0·396480	0·321492	0·273963
104	2068	0·423790	0·420056	0·340492	0·290728
105	2069	0·448274	0·444410	0·360580	0·308530
106	2070	0·473492	0·469501	0·381792	0·327415
107	2071	0·499397	0·495288	0·404158	0·347427
108	2072	0·525936	0·521719	0·427705	0·368608
109	2073	0·553052	0·548737	0·452451	0·390996
110	2074	0·580680	0·576278	0·478405	0·414623
111	2075	0·602798	0·598367	0·500625	0·435220
112	2076	0·624837	0·620392	0·523582	0·456670
113	2077	0·646722	0·642278	0·547228	0·478957
114	2078	0·668373	0·663946	0·571502	0·502051
115	2079	0·689713	0·685320	0·596327	0·525911
116	2080	0·710664	0·706321	0·621612	0·550482
117	2081	0·731151	0·726874	0·647247	0·575692
118	2082	0·751099	0·746906	0·673106	0·601455
119	2083	0·770441	0·766347	0·699046	0·627664

Table A14. *Annuitants projected – IM80 and IF80* $(B = 1964)$
One year select
Values of $q_{[x-t]+t}$ *for year of birth* 1964

Age x	in year	Males - IM80B64 Duration 0	Males - IM80B64 Durations 1+	Females - IF80B64 Duration 0	Females - IF80B64 Durations 1+
16	1980	0·000510	0·000530	0·000228	0·000300
17	1981	0·001030	0·001071	0·000228	0·000299
18	1982	0·000952	0·000990	0·000217	0·000286
19	1983	0·000881	0·000915	0·000208	0·000273
20	1984	0·000815	0·000847	0·000200	0·000263
21	1985	0·000754	0·000785	0·000194	0·000254
22	1986	0·000699	0·000728	0·000188	0·000247
23	1987	0·000651	0·000676	0·000184	0·000242
24	1988	0·000608	0·000632	0·000181	0·000239
25	1989	0·000571	0·000593	0·000180	0·000237
26	1990	0·000540	0·000561	0·000180	0·000237
27	1991	0·000514	0·000534	0·000182	0·000239
28	1992	0·000494	0·000514	0·000188	0·000247
29	1993	0·000482	0·000501	0·000198	0·000261
30	1994	0·000476	0·000494	0·000210	0·000276
31	1995	0·000476	0·000494	0·000223	0·000293
32	1996	0·000483	0·000502	0·000238	0·000313
33	1997	0·000497	0·000518	0·000255	0·000335
34	1998	0·000521	0·000542	0·000273	0·000360
35	1999	0·000552	0·000574	0·000294	0·000387
36	2000	0·000591	0·000615	0·000318	0·000418
37	2001	0·000641	0·000667	0·000343	0·000452
38	2002	0·000701	0·000729	0·000373	0·000489
39	2003	0·000772	0·000803	0·000405	0·000531

Table A14. *(Continued)*
Annuitants projected – IM80 and IF80 (B = 1964)
One year select
Values of $q_{[x-t]+t}$ for year of birth 1964

Age x	in year	Males - IM80B64 Duration 0	Males - IM80B64 Durations 1+	Females - IF80B64 Duration 0	Females - IF80B64 Durations 1+
40	2004	0·000854	0·000888	0·000440	0·000578
41	2005	0·000950	0·000988	0·000479	0·000629
42	2006	0·001059	0·001101	0·000522	0·000686
43	2007	0·001183	0·001230	0·000570	0·000750
44	2008	0·001323	0·001376	0·000623	0·000819
45	2009	0·001481	0·001541	0·000682	0·000896
46	2010	0·001658	0·001724	0·000746	0·000980
47	2011	0·001856	0·001930	0·000818	0·001074
48	2012	0·002076	0·002159	0·000896	0·001177
49	2013	0·002321	0·002414	0·000983	0·001291
50	2014	0·002594	0·002697	0·001079	0·001417
51	2015	0·002895	0·003011	0·001184	0·001556
52	2016	0·003228	0·003357	0·001305	0·001714
53	2017	0·003597	0·003741	0·001444	0·001893
54	2018	0·004004	0·004164	0·001598	0·002092
55	2019	0·004452	0·004630	0·001769	0·002312
56	2020	0·004946	0·005143	0·001958	0·002555
57	2021	0·005490	0·005709	0·002168	0·002826
58	2022	0·006089	0·006331	0·002402	0·003125
59	2023	0·006747	0·007015	0·002661	0·003457
60	2024	0·007470	0·007766	0·002949	0·003825
61	2025	0·008391	0·008724	0·003318	0·004298
62	2026	0·009421	0·009794	0·003736	0·004831
63	2027	0·010570	0·010988	0·004207	0·005431
64	2028	0·011852	0·012320	0·004738	0·006108
65	2029	0·013037	0·013688	0·005337	0·006870
66	2030	0·014085	0·015081	0·006013	0·007728
67	2031	0·015219	0·016617	0·006776	0·008693
68	2032	0·016445	0·018310	0·007636	0·009781
69	2033	0·017772	0·020177	0·008605	0·011005
70	2034	0·019207	0·022233	0·009698	0·012382
71	2035	0·020757	0·024498	0·010930	0·013931
72	2036	0·022433	0·026991	0·012317	0·015673
73	2037	0·024244	0·029735	0·013881	0·017631
74	2038	0·026200	0·032754	0·015641	0·019832
75	2039	0·028312	0·036073	0·017622	0·022305
76	2040	0·030592	0·039722	0·019852	0·025081
77	2041	0·033052	0·043730	0·022360	0·028196
78	2042	0·035706	0·048130	0·025180	0·031691
79	2043	0·038567	0·052959	0·028349	0·035610

Table A14. *(Continued)*
Annuitants projected – IM80 and IF80 (B = 1964)
One year select
Values of $q_{[x-t]+t}$ for year of birth 1964

Age x	in year	Males - IM80B64 Duration 0	Durations 1+	Females - IF80B64 Duration 0	Durations 1+
80	2044	0·042061	0·058254	0·031908	0·040000
81	2045	0·046301	0·064056	0·035904	0·044916
82	2046	0·050952	0·070410	0·040386	0·050415
83	2047	0·056054	0·077362	0·045411	0·056562
84	2048	0·061646	0·084962	0·051040	0·063429
85	2049	0·067771	0·093264	0·057340	0·071089
86	2050	0·074474	0·102322	0·064384	0·079626
87	2051	0·081805	0·112195	0·072254	0·089129
88	2052	0·089815	0·122944	0·081035	0·099692
89	2053	0·098559	0·134632	0·090757	0·111417
90	2054	0·108095	0·147324	0·100723	0·123471
91	2055	0·118484	0·161087	0·110756	0·135572
92	2056	0·129787	0·175988	0·121479	0·148469
93	2057	0·142072	0·192090	0·132918	0·162183
94	2058	0·155405	0·209462	0·145099	0·176740
95	2059	0·169854	0·228165	0·158118	0·192242
96	2060	0·185488	0·248256	0·172282	0·209044
97	2061	0·202374	0·269790	0·187746	0·227307
98	2062	0·220579	0·292808	0·204603	0·247123
99	2063	0·240167	0·317348	0·222952	0·268581
100	2064	0·261195	0·343431	0·242889	0·291763
101	2065		0·371063		0·316741
102	2066		0·400234		0·343578
103	2067		0·430913		0·372313
104	2068		0·463043		0·402969
105	2069		0·496546		0·435537
106	2070		0·531310		0·469976
107	2071		0·567195		0·506208
108	2072		0·604032		0·544105
109	2073		0·641613		0·583497
110	2074		0·679708		0·624154
111	2075		0·711035		0·659285
112	2076		0·741713		0·694359
113	2077		0·771464		0·729001
114	2078		0·800013		0·762805
115	2079		0·827094		0·795353
116	2080		0·852462		0·826228
117	2081		0·875900		0·855034
118	2082		0·897235		0·881419
119	2083		0·916341		0·905097

Table A15. *Widows projected – WL80 and WA80 (B = 1964)*
Values of q_x for year of birth 1964

Age x	in year	Lives WL80B64	Amounts WA80B64
16	1980	0·000430	0·000300
17	1981	0·000645	0·000451
18	1982	0·000616	0·000429
19	1983	0·000590	0·000412
20	1984	0·000567	0·000396
21	1985	0·000548	0·000382
22	1986	0·000533	0·000372
23	1987	0·000522	0·000364
24	1988	0·000514	0·000359
25	1989	0·000510	0·000357
26	1990	0·000511	0·000357
27	1991	0·000515	0·000359
28	1992	0·000532	0·000371
29	1993	0·000562	0·000392
30	1994	0·000595	0·000415
31	1995	0·000632	0·000441
32	1996	0·000674	0·000471
33	1997	0·000722	0·000504
34	1998	0·000775	0·000541
35	1999	0·000835	0·000582
36	2000	0·000901	0·000629
37	2001	0·000974	0·000680
38	2002	0·001056	0·000736
39	2003	0·001146	0·000800

Table A15. *(Continued)*
Widows projected – WL80 and WA80 (B = 1964)
Values of q_x for year of birth 1964

Age x	in year	Lives WL80B64	Amounts WA80B64
40	2004	0·001246	0·000870
41	2005	0·001357	0·000947
42	2006	0·001480	0·001033
43	2007	0·001615	0·001128
44	2008	0·001765	0·001232
45	2009	0·001930	0·001347
46	2010	0·002111	0·001474
47	2011	0·002310	0·001612
48	2012	0·002528	0·001765
49	2013	0·002768	0·001933
50	2014	0·003031	0·002117
51	2015	0·003321	0·002319
52	2016	0·003639	0·002541
53	2017	0·003988	0·002785
54	2018	0·004372	0·003054
55	2019	0·004763	0·003349
56	2020	0·005157	0·003673
57	2021	0·005585	0·004030
58	2022	0·006050	0·004423
59	2023	0·006555	0·004854
60	2024	0·007103	0·005329
61	2025	0·007818	0·005942
62	2026	0·008606	0·006627
63	2027	0·009477	0·007392
64	2028	0·010438	0·008248
65	2029	0·011498	0·009205
66	2030	0·012667	0·010273
67	2031	0·013957	0·011467
68	2032	0·015379	0·012801
69	2033	0·016946	0·014289
70	2034	0·018674	0·015951
71	2035	0·020577	0·017805
72	2036	0·022673	0·019874
73	2037	0·024980	0·022180
74	2038	0·027519	0·024751
75	2039	0·030313	0·027616
76	2040	0·033386	0·030807
77	2041	0·036763	0·034359
78	2042	0·040473	0·038310
79	2043	0·044547	0·042704

Table A15. *(Continued)*
Widows projected – WL80 and WA80 (B = 1964)
Values of q_x for year of birth 1964

Age x	in year	Lives WL80B64	Amounts WA80B64
80	2044	0·049018	0·047586
81	2045	0·054578	0·053007
82	2046	0·061374	0·059021
83	2047	0·068833	0·065688
84	2048	0·076980	0·073072
85	2049	0·085831	0·081241
86	2050	0·095400	0·090271
87	2051	0·105690	0·100237
88	2052	0·116695	0·111225
89	2053	0·128405	0·123320
90	2054	0·140795	0·135076
91	2055	0·153835	0·145806
92	2056	0·167484	0·156601
93	2057	0·181690	0·167355
94	2058	0·196396	0·177963
95	2059	0·209705	0·188480
96	2060	0·221808	0·199516
97	2061	0·234673	0·211266
98	2062	0·248342	0·223771
99	2063	0·262858	0·237077
100	2064	0·278263	0·251227
101	2065	0·294602	0·266266
102	2066	0·311915	0·282240
103	2067	0·330246	0·299195
104	2068	0·349632	0·317175
105	2069	0·370112	0·336223
106	2070	0·391718	0·356382
107	2071	0·414478	0·377689
108	2072	0·438416	0·400176
109	2073	0·463543	0·423874
110	2074	0·489868	0·448803
111	2075	0·512326	0·470333
112	2076	0·535492	0·492662
113	2077	0·559313	0·515756
114	2078	0·583723	0·539570
115	2079	0·608638	0·564044
116	2080	0·633961	0·589103
117	2081	0·659576	0·614655
118	2082	0·685350	0·640590
119	2083	0·711135	0·666782

APPENDIX B

FORMULAE FOR THE NEW STANDARD TABLES

In this Appendix the formulae used for the calculation of the adjusted values of μ_x are described in detail. The formulae do not apply below age 17; the method of obtaining values of q_x at ages below 17 is described in Section 2.4

A number of basic (graduated) formulae have been used. They are referred to by formula number. Thus F11 means μ_x calculated using formula F11. Most of the formulae are the original graduation formulae, described in the report in *C.M.I.R.*, **9**. Three of them are formulae including 'quadratic adjustment' (explained in § 2.2.1.). Values of the parameters for the formulae are shown in Table B1. It should be noted that the parameter values for the basic formulae are shown to six decimal places, and these exact values have been used. They are the same as are shown in the Report in *C.M.I.R.*, **9** for the corresponding experience. For the formulae with quadratic adjustment, the adjusted parameters (b_0, b_1 and b_2) are shown to six decimal places, though a higher number of decimal places has been retained in the calculations.

It should be noted that the formulae described in this Appendix are used to calculate the values of μ_x for the various experiences into which the data are classified. These values of μ_x are then used to calculate the values of $q_{[x]}$, $q_{[x]+1}$, q_x, etc shown in Appendix A. The values of $q_{[x]}$, $q_{[x]+1}$, q_x, etc are then used to calculate values of $\mu_{[x]}$, $\mu_{[x]+1}$, μ_x, etc as described in Section 2.6. The recalculated values of μ_x for ultimate or aggregate tables agree closely with the original graduated and adjusted values of μ_x, but the values of $\mu_{[x]}$, etc, for select tables do *not* correspond closely with the values of μ_x for 'Duration 0', etc.

Table B1. *Graduated formulae and their parameter values*

Formula No		Parameters GM(r,s)	$100a_0$	$100a_1$	b_0	b_1	b_2
Permanent assurances, males							
F11	Duration 0	GM(2,2)	−0·465192	−0·452546	−3·985723	3·185063	-
F12	Duration 1	GM(2,2)	−0·713368	−0·676049	−3·689744	3·027036	-
F13	Duration 2+	GM(2,2)	−0·338415	−0·386512	−3·352236	4·656042	-
F14	Durations 2–4	GM(2,2)	−0·487122	−0·496613	−3·634119	3·647534	-
F15	Duration 2+*	GM(2,3)	−0·338415	−0·386512	−3·887248	5·052347	−0·495382
		(*F13 with quadratic adjustment, see B1.3)					
Permanent assurances, females							
F21	Duration 0	GM(2,2)	−0·169572	−0·158677	−4·755196	3·243579	-
F22	Duration 1	GM(2,2)	−0·076446	−0·089013	−4·253324	4·608019	-
F23	Duration 2+	GM(2,2)	−0·015700	−0·025164	−4·064944	5·044225	-
Temporary assurances, males							
F31	Duration 0	GM(2,2)	−8·935660	−3·775563	−2·318362	0·628717	-
F32	Durations 1-4	GM(2,2)	−0·616370	−0·576586	−3·773543	3·048161	-
F33	Duration 5+	GM(2,2)	−0·620970	−0·676328	−3·340552	4·051857	-
Male Pensioners							
F41	Lives	GM(1,3)	0·557291	-	−4·993529	5·882482	−1·668855
F42	Amounts	GM(1,3)	0·200555	-	−4·716394	5·832085	−1·277676
F43	Amounts*	GM(1,3)	0·200555	-	−3·342787	4·056202	−0·312522
		(*F42 with quadratic adjustment, see B4.2)					
Female Pensioners							
F51	Lives	GM(1,3)	0·662810	-	−6·473887	8·069982	−2·174915
F52	Amounts	GM(1,3)	0·679085	-	−7·914792	9·365123	−3·358784
F53	Amounts*	GM(1,3)	0·679085	-	−2·138566	1·663488	0·492034
		(*F52 with quadratic adjustment, see B5.2)					
Male Annuitants							
F61	Duration 0	GM(0,2)	-	-	−3·514486	3·332467	-
F62	Duration 1+	GM(0,2)	-	-	−3·375783	4·327411	-
Female Annuitants							
F71	Duration 0	GM(0,2)	-	-	−4·226617	5·419216	-
F72	Duration 1+	GM(0,2)	-	-	−3·979448	5·346654	-
Widows							
F81	Lives	GM(0,2)	-	-	−3·553013	4·316579	-
F82	Amounts	GM(0,2)	-	-	−3·719382	4·962087	-

B1 Permanent assurances, males – AM80 and AM80(5)

B1.1 Duration 0	all ages ($x > 17$)	F11
B1.2 Duration 1	$x < 28$	(1–k11)F11 + k11.F13
		where k11 = (F12–F11)/(F13–F11) at $x = 28$
		= 0·60083712
	$x \geqslant 28$	F12
B1.3 Duration 2+	$x \leqslant 80$	F13
	$x > 80$	F15

F15 is derived from F13 by quadratic adjustment above age 80 such that F13 = F15 at $x = 80$ and $r = 0{\cdot}7$ at $x = 110$

B1.4 Durations 2–4	all ages ($x > 17$)	F14
B1.5 Duration 5+	as Duration 2+ throughout	

B2 Permanent assurances, females – AF80

B2.1 Duration 0	$x \leqslant 75$	F21
	$x > 75$	k21.F22
		where k21 = F21/F22 at $x = 75$
		= 0·46345460
B2.2 Duration 1	all ages ($x > 17$)	F22
B2.3 Duration 2+	$x < 28$	k22.F22
		where k22 = F23/F22 at $x = 28$
		= 1·08156196
	$x \geqslant 28$	F23

B3 *Temporary assurances, males – TM80*

B3.1 Duration 0	$x < 31$	k31.F32
		where k31 = F31/F32 at $x = 31$
		= 0·80124215
	$31 \leqslant x \leqslant 65$	F31
	$x > 65$	k32.F32
		where k32 = F31/F32 at $x = 65$
		= 0·60410293
B3.2 Durations 1–4	$x \leqslant 34$	F32
	$34 < x \leqslant 44$	k33.F33
		where k33 = F32/F33 at $x = 34$
		= 0·95021746
	$44 < x < 45$	min(F32, k33.F33)
	$x \geqslant 45$	F32
B3.3 Duration 5+	$x < 32$	k34.F13
		where k34 = F33/F13 at $x = 32$
		= 0·98018301
	$x \geqslant 32$	F33

B4 Male pensioners – PML80Base and PMA80Base

B4.1 Lives	$x < 55$	$k41.k42.F13$
		where $k41 = F41/F42$ at $x = 65$
		$= 1{\cdot}27871549$
		$k42 = F42/F13$ at $x = 55$
		$= 1{\cdot}00074607$
	$55 \leqslant x < 65$	$k41.F42$
	$65 \leqslant x \leqslant 91$	F41
	$91 < x \leqslant 93$	$k43.F42$
		where $k43 = F41/F42$ at $x = 91$
		$= 1{\cdot}01216060$
	$x > 93$	$k43.F43$
B4.2 Amounts	$x < 55$	$k42.F13$
	$55 \leqslant x \leqslant 93$	F42
	$x > 93$	F43

F43 is derived from F42 by quadratic adjustment above age 93 such that F43 = F42 at $x = 93$ and $r = 1{\cdot}25$ at $x = 110$

B5 Female pensioners – PFL80Base and PFA80Base

B5.1 Lives	$x < 28$	$k52.k51.k22.F22$
		where $k51 = F52/F23$ at $x = 67$
		$= 1{\cdot}00769467$
		and $k52 = F51/F52$ at $x = 67$
		$= 1{\cdot}17657278$
	$28 \leqslant x < 67$	$k52.k51.F23$
	$67 \leqslant x \leqslant 95$	F51
	$x > 95$	$k53.F53$
		where $k53 = F51/F52$ at $x = 95$
		$= 1{\cdot}21543261$
B5.2 Amounts	$x < 28$	$k51.k22.F22$
	$28 \leqslant x < 67$	$k51.F23$
	$67 \leqslant x \leqslant 95$	F52
	$x > 95$	F53

F53 is derived from F52 by quadratic adjustment above age 95 such that F53 = F52 at $x = 95$ and $r = 2{\cdot}0$ at $x = 110$

B6 Male annuitants – IM80Base

B6.1 Duration 0	$x < 65$	$k61.k62.F13$
		where $k61 = F61/F62$ at $x = 65$
		$= 0{\cdot}96155011$
		$k62 = F62/F13$ at $x = 65$
		$= 1{\cdot}16879491$
	$65 \leqslant x \leqslant 80$	F61
	$x > 80$	$k63.F62$
		where $k63 = F61/F62$ at $x = 80$
		$= 0{\cdot}71341513$
B6.2 Duration 1+	$x < 65$	$k62.F13$
	$x \geqslant 65$	F62

B7 Female annuitants – IF80Base

B7.1 Duration 0	$x < 28$	$k72.k71.k22.$F22 where $k71$ = F72/F23 at $x = 52$ = 1·00069170 and $k72$ = F71/F72 at $x = 52$ = 0·76087110
	$28 \leqslant x < 52$	$k72.k71.$F23
	$52 \leqslant x \leqslant 89$	F71
	$89 < x \leqslant 90$	$k73.$ min(F72, $\frac{1}{2}$(F72+F52)) where $k73$ = F71/F72 at $x = 89$ = 0·80284353
	$90 < x \leqslant 95$	$k73.$ $\frac{1}{2}$(F72+F52)
	$x > 95$	$k73.$ $\frac{1}{2}$(F72+F53)
B7.2 Duration 1+	$x < 28$	$k71.k22.$F22
	$28 \leqslant x < 52$	$k71.$F23
	$52 \leqslant x \leqslant 89$	F72
	$89 < x \leqslant 90$	min(F72, $\frac{1}{2}$(F72+F52))
	$90 < x \leqslant 95$	$\frac{1}{2}$(F72+F52)
	$x > 95$	$\frac{1}{2}$(F72+F53)

B8 Widows – WL80Base and WA80Base

B8.1 Lives	$x < 28$	$k82.k81.k22.$F22 where $k81$ = F82/F23 at $x = 45$ = 1·50608926 $k82$ = F81/F82 at $x = 55$ = 1·43336009
	$28 \leqslant x < 45$	$k82.k81.$F23
	$45 \leqslant x < 55$	$k82.$F82
	$55 \leqslant x \leqslant 81$	F81
	$81 < x \leqslant 95$	$k83.$F51 where $k83$ = F81/F51 at $x = 81$ = 1·03414340
	$x > 95$	$k83.k53.$F53
B8.2 Amounts	$x < 28$	$k81.k22.$F22
	$28 \leqslant x < 45$	$k81.$F23
	$45 \leqslant x \leqslant 90$	F82
	$90 < x \leqslant 95$	$k84.$F52 where $k84$ = F82/F52 at $x = 90$ = 1·11235210
	$x > 95$	$k84.$F53

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