ISSN 0954-2388
CMIR, **12** (1991)

Continuous Mortality Investigation Reports

Number 12

Institute of Actuaries

Faculty of Actuaries

Published by the Institute of Actuaries
and the Faculty of Actuaries
1991

THE EXECUTIVE COMMITTEE OF THE CONTINUOUS MORTALITY INVESTIGATION BUREAU

as on 30th June 1991

Institute Representatives

H. A. R. Barnett
P. H. Bayliss
B. J. Brindley
C. D. Daykin
T. S. Leigh
R. H. Plumb
A. D. Wilkie (Chairman)

Faculty Representatives

A. Neill (President)
C. Berman
D. O. Forfar
C. G. Kirkwood (Deputy Chairman)
I. C. Lumsden
J. J. McCutcheon

THE MORTALITY SUB-COMMITTEE

C. G. Kirkwood (Chairman)
H. A. R. Barnett
C. Berman
C. D. Daykin
D. O. Forfar
I. C. Lumsden
J. J. McCutcheon
A. D. Wilkie

THE IMPAIRED LIVES SUB-COMMITTEE

T. S. Leigh (Chairman)
H. A. R. Barnett
C. G. Kirkwood
S. F. Margutti
P. J. Savill
A. D. Wilkie

THE PERMANENT HEALTH INSURANCE SUB-COMMITTEE

R. H. Plumb (Chairman)
P. H. Bayliss
R. J. L. Blackwood
R. Garden
E. A. Hertzman
G. J. Hockings
A. E. J. Jefferies
A. R. Marshall
G. C. Orros
H. R. Waters

INTRODUCTION

THE Executive Committee of the Continuous Mortality Investigation Bureau of the Institute of Actuaries and the Faculty of Actuaries has pleasure in presenting this, the twelfth number of its Reports. This number contains what can hardly be described as a paper, but rather a book, consisting of several 'Chapters', described as 'Parts', some of which are attributed to named authors.

The paper—or book—is entitled 'The Analysis of Permanent Health Insurance Data', and it describes the results of many years study by the members of the PHI Sub-Committee. In it a radical new approach to the analysis of PHI data is presented, and it will be seen that it reconciles the two previous approaches, the 'Manchester Unity' sickness rates method and the 'American' incidence rate and disability annuity method, which were previously thought to conflict.

Credit is due to the members of the PHI Sub-Committee for preparing this massive work, and in particular to Philip Bayliss and Howard Waters, whose names appear on the Parts for which they are particularly responsible. Thanks are also due to the staff of Pensions and Insurance Computing Services, who provide the basic computing for the investigation, and also to the offices who have contributed PHI data over many years, receiving more promises of results than production of them. The Committee hopes that the offices will feel that their forbearance is rewarded, though it is appreciated that the new method will not be understood without some effort.

I should also like to take the opportunity to pay tribute to the work of Hugh Jarvis, who resigned from the Executive Committee on his retirement in April. He had served on the Committee since 1980, latterly being Chairman of the Impaired Lives Sub-Committee. His place on the main Committee and as Chairman of that Sub-Committee is taken by Spencer Leigh.

I should also like to welcome Peter Savill to the Impaired Lives Sub-Committee and Roger Blackwood and Graham Hockings to the PHI Sub-Committee, to which they have recently been appointed.

A. D. Wilkie
Chairman, Executive Committee

June, 1991

CONTENTS

LIST OF FIGURES

LIST OF TABLES

Tables E14 to E19 are on the 1 week deferred period basis, conditional on starting at age 30 with initial status healthy.

Tables E20a to E25 are on the 1 week deferred period basis.

Tables E26 to E29 are on the 4 week deferred period basis.

Tables E30 to E33 are on the 13 week deferred period basis.

Tables E34 to E37 are on the 26 week deferred period basis.

Tables F1 and F2 are on the 1-week deferred period basis, conditional on starting at age 30 with initial status healthy: rate of interest 6%.

Tables F3 to F5 are on the 1-week deferred period basis, rate of interest 6%.

CMIR **12** (1991) 1–263

THE ANALYSIS OF PERMANENT HEALTH INSURANCE DATA

INTRODUCTION

It has always been envisaged by the PHI Sub-Committee, from the first Report of the Advisory Sub-Committee for the investigation of sickness statistics in September 1971 (reprinted in 'Investigation of Sickness Statistics', *C.M.I.R.* **2** (1976)), that an investigation on a 'Disability Annuity' basis would be carried out. However, it was noted in that first Report that "disability annuities have to be derived from 'select' data with a very long period of selection (15 years was used in the United States of America) and a number of years' experience must be amalgamated to produce results which are statistically reliable". Records for claims were therefore gathered in a form which would allow rates of termination of claim 'whether by recovery or death' to be investigated. In this Report the Sub-Committee presents the results of its investigations on these lines for the first time.

Rates of claim inception, i.e. 'rates of starting a claim at age x' have been calculated and published from the very first investigation. Graduations of the male claim inception rates for 1972–75 were published in 'Sickness Experience 1972–75 for Individual Policies', *C.M.I.R.* **4** (1979) and graduations of the male standard experience for 1975–78 were published in 'Sickness Experience 1975–78 for Individual PHI Policies', *C.M.I.R.* **7** (1984).

In this Report the Sub-Committee is at last able to present its investigations of sickness claims on a 'disability annuity' basis for consideration by the actuarial profession and by PHI offices. The Report is long, and it has taken a long time to produce. As the investigation progressed it became apparent that a new and clearly stated model of sickness was required. Such a model had been proposed by Dr H. R. Waters (1984). In 1986 Dr Waters was invited to become a member of the Sub-Committee. His contribution to the development of the model is readily apparent from the fact that three of the six Parts into which this Report is divided carry his name as the author.

The use of this new multiple state model resulted in complexities that had not initially been suspected. It was felt that the use of the full model, although theoretically justifiable, resulted in what were probably unacceptably heavy computational requirements. It was necessary therefore to search for a way of simplifying the results in order to facilitate practical calculations. This too took a substantial amount of time, as did the search for satisfactory bivariate formulae to represent rates of recovery and death which varied both by age and by duration of sickness. This latter task fell to Mr P. H. Bayliss, who had been responsible for the graduation work in the earlier reports on sickness statistics. Part B of this Report is recognised as his work.

A major advantage of the multiple state model used as the basis of this Report is that it allows the two different approaches, the Manchester Unity Sickness Rate approach and the Claim Inception Rate and Disability Annuity approach, to be seen as alternative representations of the same underlying model, providing alternative ways of calculating the same functions. The apparent conflict between the approaches is seen to be groundless, and it is shown in Part F of the Report that each approach has its merits for calculating the values of different types of benefit. The choice is not one of principle, but of computational convenience. It has to be noted, however, that a single Sickness Rate table, as in the original Manchester Unity tables, does not provide a satisfactory approximation; rather a table dependent on age at entry is required.

In its Report in *C.M.I.R.* **7** (1984) the Sub-Committee discussed the problems of interpreting figures for sickness rates gathered on an 'aggregate' basis, i.e. not sub-divided by duration since the commencement of the policy. The investigations in Part E about the construction of select tables of sickness rates demonstrates why an 'aggregate' rather than a 'select' investigation is unsatisfactory, at least for the sickness period denoted as '104/all'.

Many techniques new to the actuarial profession have had to be developed in the course of this investigation. These include:

graduation of bivariate data, and corresponding graduation tests;
numerical solution of the simultaneous differential equations that define the multiple state model;
criteria for condensing complete tables of rates, sub-divided by age at entry and attained age, into tables with a select period for a limited number of years;
investigation into the relative accuracies of different methods of approximation.

Many of these are interesting problems in their own right.

The Report is sub-divided into six Parts. In Part A the mathematical basis of the multiple state model is described. In Parts B and C the rates of recovery and death among the sick and rates of falling sick among the healthy are analysed, and graduation formulae that satisfactorily fit the data are developed. In Part D the numerical methods required to solve the differential equations of the model are described, ready for Parts E and F, in which the calculation of probabilities and the calculation of monetary functions are described, and many numerical examples are given.

A glossary of the notation for those functions that appear in more than one Part is included as an Appendix.

It should be noted that the data used throughout this investigation is that for the Male Standard Experience for individual PHI policies for 1975–78. Comparison of the experiences for females, for group and unit cost policies and for all investigations for 1979–82 will follow in subsequent Reports.

The size of the task that has been undertaken by the members of the Sub-Committee and in particular by members of the Task Force responsible for the work—R. H. Plumb (Chairman), P. H. Bayliss, E. A. Hertzman, G. C. Orros,

H. R. Waters and A. D. Wilkie—has been daunting. The length of this Report may be just as daunting to many readers. The Task Force in particular feels perhaps that, to adapt Horace:

Parturiunt mures, nascetur immensus mons;

but it hopes that some others may attempt to scale the mountain with them.

Those who feel that they need to revise their knowledge of the practical aspects of PHI business may like to read or reread the papers by Bond (1963), Sansom (1978) and Sanders and Silby (1988). Those who wish to review the earlier investigations of the PHI Sub-Committee are referred in particular to reports in *C.M.I.R.* **2** (1976), *C.M.I.R.* **4** (1979) and *C.M.I.R.* **7** (1984).

CMIR **12** (1991) 5–20

PART A: A MULTIPLE STATE MODEL FOR PERMANENT HEALTH INSURANCE

BY H. R. WATERS

SUMMARY

The mathematical model for PHI which has been investigated by the C.M.I. Bureau, and which is the subject of this Report, is introduced in this Part. This Part also provides a brief and somewhat general introduction to topics which will be discussed more fully in other Parts of this Report; in particular, parameter estimation is discussed in sections 3 and 4 of this Part, and in much more detail in Parts B and C. In section 5 of this Part we discuss the derivation of formulae for probabilities; the numerical evaluation of these probabilities is discussed in detail in Part D.

1. INTRODUCTION

1.1 The purpose of this Part is to describe a mathematical model which can be used as a model for PHI business. This model can perform two functions:

(i) it can form the basis for the statistical analysis of PHI data and
(ii) it can be used to derive formulae, for example in terms of claim inception rates and disability annuities or Manchester Unity type sickness rates, which could conveniently be used for valuing and setting premiums for PHI business.

1.2 The need for a new model for PHI is apparent on reading the most recent report of the PHI Sub-Committee of the Continuous Mortality Investigation Bureau, *Continuous Mortality Investigation Reports* 7 (1984), subsequently referred to as *C.M.I.R.* 7. Part 4 of *C.M.I.R.* 7 is a detailed explanation of the reasons why Manchester Unity type functions are considered unsuitable for valuing and setting premiums for modern PHI business and throughout the report it is clear that the Sub-Committee have experienced difficulties as a result of estimating a quantity, $z_x^{a/b}$, which is very complicated mathematically, using estimates whose statistical properties are unknown.

1.3 The requirements of a model for PHI are stated below:

(i) It should be sufficiently realistic. Any model incorporates some simplifications but these should not be so severe as to make it difficult to accept as a model for the purposes being considered. For example, a model for PHI which has recovery rates depending only on the policyholder's attained age and not on the duration of his sickness may be considered too unrealistic to be of any use.

(ii) It should be possible to use the data which is available, or which could easily be made available, to estimate the parameters which determine the model, and, more importantly, to estimate them in such a way that the statistical properties of the estimators are known.

(iii) It should be possible to derive from the model numerical values of some functions which can conveniently be used to set premium rates for, and carry out valuations of, PHI business.

Broadly speaking, this Part discusses (i) and (ii) above for the model being proposed; (iii) is discussed in Part D of this Report.

1.4 The model for PHI discussed in this Part is a multiple state model with three states, Healthy, Sick and Dead. It is very similar to a standard illness/death model which first appeared in the actuarial literature early in this century, see Du Pasquier (1912, 1913), and which was used for illustrative purposes by Waters (1984) in a general discussion of multiple state models. The essential difference between the earlier model and the one discussed in this Part is that, whereas in the earlier model all probabilities depended only on the policyholder's attained age, in the proposed model, although all probabilities still depend on the policyholder's attained age, the probabilities of either recovering or dying from a state of sickness depend also on the duration of the sickness. The model is described in detail in §2 of this Part. Hoem (1972) discusses some mathematical aspects of models of this type and indicates that it was proposed as long ago as 1924 as a model for health insurance.

1.5 The important quantities for the proposed model are the transition intensities, which are analogous to the force of mortality for a life table, and in §3 we discuss how these could be estimated in such a way that the statistical properties of the estimators are known, at least asymptotically.

The discussion in §3 assumes there are no problems caused by having incomplete data. In practice this is not the case and in §4 we discuss the practical problems of parameter estimation resulting from using only that data which is currently available to the PHI offices and hence to the C.M.I. Bureau.

1.6 In §5 we show how, given the transition intensities, we can derive formulae for some of the important probabilities for the model. Some of these probabilities can be easily evaluated but in some cases the resulting formulae are integro-differential equations. The numerical solution of these equations will be discussed in Part D of this Report.

2. THE MODEL

2.1 The model proposed as a basis for the analysis of PHI data can be very simply described, in intuitive terms, with the help of Figure A1. On effecting his policy the policyholder enters state H (since we assume he is not sick at that time). From state H he may transfer at any future time either to state S, i.e. become sick, or to state D, i.e. die. (Note that entering state S is not equivalent to making a claim since to make a claim the policyholder must remain in state S for at least the deferred period of his policy.) The transition intensities, or forces of decrement, for these two transitions are denoted σ_x and μ_x respectively and depend only on x, the policyholder's attained age. Once in state S the policyholder may transfer back to state H, i.e. recover, or transfer to state D, i.e. die. The transition intensities for these transitions are denoted $\rho_{x,z}$ and $\nu_{x,z}$ respectively and depend on x, the policyholder's attained age, and z, the duration of his current sickness. Note that all the probabilities in this model depend only on the policyholder's attained age and, in some cases, on the duration of his current sickness. These probabilities take no account of any other information; for example, they do not take account of the number of, or durations of, or time since, any previous periods of sickness.

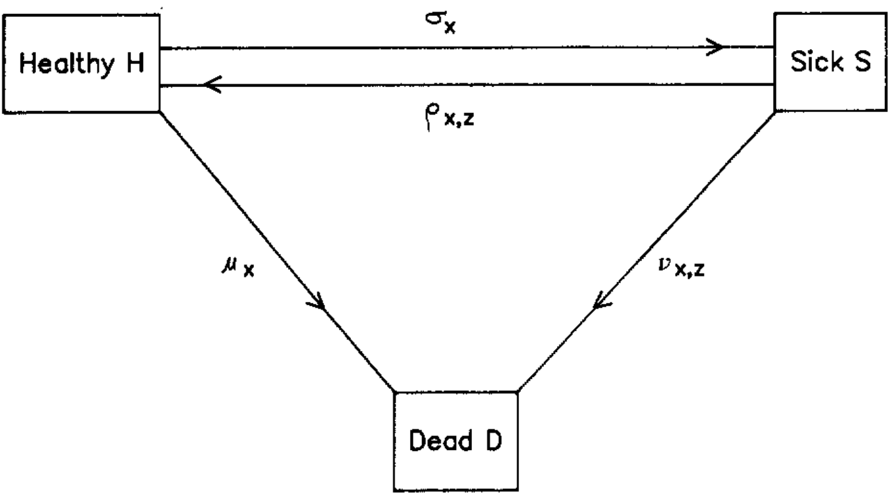

Figure A1. A diagrammatic representation of the model for sickness.

2.2 The model can be described more formally in terms of a pair of continuous time stochastic processes

$$\{Y(x), Z(x)\}\ x \geqslant 0 \tag{1}$$

$Y(x)$ can take any of the three values H, S or D and we interpret the event

$$\{Y(x) = H\},$$

for example, to mean 'the policyholder is healthy at age x'.

$Z(x)$ takes values in $[0,\infty]$ and is defined as follows

$Z(x) = \max\{t : t \leqslant x$ and $Y(x-h) = Y(x)$ for all h such that $0 \leqslant h \leqslant t\}$

so that $Z(x)$ denotes the duration, for a life now age x, of the sojourn so far in the current state, $Y(x)$. Hence the event

$$\{Y(x) = S, Z(x) = z\}$$

is the event that the policyholder is sick at age x and that the duration of his current sickness is z.

2.3 The joint process (1) is assumed to be a Markov process so that the future of the process after age x depends only on the values of $Y(x)$ and $Z(x)$ and not on any other events prior to age x. This means that, for example, if the policyholder has just fallen sick, the probability that he will remain sick for a long period takes no account of information such as that he has experienced many lengthy or short periods of sickness in the past.

2.4 We shall use the following notation

$${}_tp_{x,z}^{j,k} = P[Y(x+t) = k | Y(x) = j \text{ and } Z(x) = z]$$

where $j,k = H,S$ or D and $t,x,z \geqslant 0$. (Note that all the probabilities relating to the model are conditional on some information; unconditional probabilities have no meaning in this model.) We assume that if the current state is H, i.e. if the policyholder is healthy at age x, then the future of the process does not depend on the duration of the current period in state H, i.e. how long the policyholder has been healthy. Formally, we assume ${}_tp_{x,z}^{HH}$, ${}_tp_{x,z}^{HS}$ and ${}_tp_{x,z}^{HD}$ are all independent of the value of z and so we shall denote these probabilities ${}_tp_x^{HH}$, ${}_tp_x^{HS}$ and ${}_tp_x^{HD}$ respectively. We also assume, for obvious reasons,

that $${}_tp_{x,z}^{Dk} = 0 \qquad k = H \text{ or } S$$

and $${}_tp_{x,z}^{DD} = 1$$

The following notational definitions will be useful later in this Report:

$${}_tp_x^{\overline{HH}} = P[Y(x+t) = H \text{ and } Z(x+t) \geqslant t | Y(x) = H] \qquad (2)$$

$${}_tp_{x,z}^{\overline{SS}} = P[Y(x+t) = S \text{ and } Z(x+t) = z+t | Y(x) = S \text{ and } Z(x) = z] \qquad (3)$$

$${}_{w,t}p_x^{HS} = P[Y(x+t) = S \text{ and } Z(x+t) \leqslant w | Y(x) = H] \qquad (4)$$

In the special case where $z = 0$, we shall denote ${}_tp_{x,0}^{\overline{SS}}$ by ${}_tp_x^{\overline{SS}}$.

Note that ${}_tp_x^{\overline{HH}}$ is the probability of staying healthy from age x to age $(x + t)$, given that the individual was healthy at age x. (This is not the same as ${}_tp_x^{HH}$.) Note also that ${}_tp_{x,z}^{\overline{SS}}$ is the probability of staying sick from age x to age $(x + t)$, given that at age x the individual was sick with duration of sickness z.

We shall assume that all probabilities for our model are mathematically well behaved, in particular continuous, functions of w, t, x and z.

2.5 The transition intensities between the three states are denoted μ_x, σ_x, $\nu_{x,z}$ and $\rho_{x,z}$ and are defined as follows

$$\mu_x = \underset{t \to 0-}{\mathrm{Lim}}\, {}_t p_x^{HD}/t \tag{5}$$

$$\sigma_x = \underset{t \to 0+}{\mathrm{Lim}}\, {}_t p_x^{HS}/t \tag{6}$$

$$\nu_{x,z} = \underset{t \to 0+}{\mathrm{Lim}}\, {}_t p_{x,z}^{SD}/t \tag{7}$$

$$\rho_{x,z} = \underset{t \to 0+}{\mathrm{Lim}}\, {}_t p_{x,z}^{SH}/t \tag{8}$$

We shall assume that all the above limits exist and that the transition intensities are mathematically well behaved functions; in particular we assume

all the transition intensities are continuous functions of either x or (x,z). (9)

An important consequence of (9) is that

the transition intensities are bounded on any bounded set of values of (x,z). (10)

Using (9) and (10) it can be shown that for any time interval $(t,t+\tau)$

$$P[2 \text{ or more transitions in } (t,t+\tau)] = o(\tau) \tag{11}$$

$$P[Y(x+t+\tau) = S \mid Y(x+t) = H] = \tau \cdot \sigma_{x+t} + o(\tau) \tag{12}$$

$$P[Y(x+t+\tau) = D \mid Y(x+t) = H] = \tau \cdot \mu_{x+t} + o(\tau) \tag{13}$$

$$P[Y(x+t+\tau) = H \mid Y(x+t) = S \text{ and } Z(x+t) = z] = \tau \cdot \rho_{x+t,z} + o(\tau) \tag{14}$$

$$P[Y(x+t+\tau) = D \mid Y(x+t) = S \text{ and } Z(x+t) = z] = \tau \cdot \nu_{x+t,z} + o(\tau) \tag{15}$$

where $o(\tau)$ is any quantity such that

$$\underset{\tau \to 0-}{\mathrm{Lim}}\, o(\tau)/\tau = 0 \tag{16}$$

3. PARAMETER ESTIMATION

3.1 To be able to use the model described in the previous section we need to be able to estimate some parameters which determine the model. The choice of the parameters to be estimated depends on the form of the data available, but, in general, the most obvious choice for our model is the set of transition intensities. The reasons for this are discussed in Waters (1984) and in more detail in Hoem and Funck Jensen (1982).

3.2 There is a well established statistical theory concerning the estimation of transition intensities; useful references are Sverdrup (1965), Hoem (1969, 1976), Aalen and Hoem (1978) and Borgan (1984). In this section we shall discuss how we could estimate the transition intensities given data in the most convenient form. We shall not discuss the theory underlying the estimation procedure; the reader interested in a more detailed treatment should consult the references given above. In the next section we shall discuss the practical problems resulting from having available only that data which is currently available to the C.M.I. Bureau.

3.3 Suppose we can observe over a period of time a group of PHI policies as they move between the states of our model as described in §(2.1). We shall, in this section, assume

(a) the behaviour of any single policy is independent of the behaviour of the other policies and
(b) we can observe every transition made by a policy.

For the sake of example let us suppose we wish to estimate the value of $\rho_{x,z}$ for a given value of (x,z), say (x',z'). The first step is to choose intervals $x_1 \leqslant x' \leqslant x_2$ and $z_1 \leqslant z' \leqslant z_2$ which are sufficiently small for us to accept that $\rho_{x,z}$ is approximately constant over the rectangle $[x_1,x_2] \times [z_1,z_2]$ but not so small that the exposed to risk we shall calculate is small.

Having chosen this rectangle let us denote by ρ the assumed constant value of $\rho_{x,z}$ over the rectangle. The next step is to determine an observation period, perhaps a period of several calendar years, during which we shall record the movements of the policies. At the end of the observation period we can calculate two quantities, O and E, where

O is the observed number of recoveries by policyholders who, at the time of recovery, were aged between x_1 and x_2 and whose duration of sickness was between z_1 and z_2, and

E is the total time spent sick in the observation period by individual policies, counting only the time when the policyholders were aged between x_1 and x_2 and when the duration of their sickness was between z_1 and z_2.

It can then be shown that under reasonable assumptions the maximum likelihood estimator of ρ is $\hat{\rho}$ where

$$\hat{\rho} = O/E$$

3.4 For small sample sizes the statistical properties of the estimator $\hat{\rho}$ are not easily determined, but asymptotically, i.e. as the number of individuals who contribute to the exposure E becomes large, it can be shown that, under reasonable assumptions:

(i) the estimator is unbiased,
(ii) no other estimator has a smaller variance,
(iii) the distribution of the estimator is normal, and
(iv) the variance of the estimator can consistently be estimated by O/E^2.

Speaking very loosely we can assume for large sample sizes that

$$\hat{\rho} \sim N(\rho, O/E^2)$$

Investigations by Schou and Vaeth (1980) suggest this distributional assumption is reasonable if the expected number of recoveries exceeds 10.

3.5 By dividing the range of values of (x,z) into a number of non-overlapping rectangles, assuming that $\rho_{x,z}$ is approximately constant over each rectangle and estimating this constant value in the manner described above we can obtain a sequence of point estimates of $\rho_{x,z}$ with known asymptotic statistical properties. The same procedure can be used to obtain sequences of estimators of σ_x, μ_x and $v_{x,z}$ at selected points. It can be shown that each of the resulting estimators is independent of all the other estimators both for the same transition intensity and for the other three transition intensities.

3.6 With a set of point estimates of $\rho_{x,z}$ (or σ_x, μ_x or $v_{x,z}$) with known asymptotic distributions it would be possible to test for significant differences between recovery rates estimated from independent sets of data. There are some obvious questions of interest which could be investigated in this way:

(i) are recovery rates estimated from groups of policies with different deferred periods significantly different?
(ii) are recovery rates obtained from current data significantly different from those obtained from earlier data?
(iii) are recovery rates estimated from the experience of one group of offices significantly different from those estimated from the experience of another group of offices?

A simple example of hypothesis testing in this way is given in Hoem and Funck Jensen (1982, §4.1).

4. PRACTICAL DIFFICULTIES OF PARAMETER ESTIMATION

4.1 In our discussion of parameter estimation in §3 we made two important assumptions, (a) and (b) in §(3.3), which are unlikely to hold in practice. In this section we shall discuss how in practical terms we could estimate the transition intensities using only that data which is currently available to the C.M.I. Bureau. In particular we shall discuss in turn problems due to:

(i) duplicate policies,
(ii) observing transitions out of state S only when the duration of the sickness is greater than the deferred period of the policy,
(iii) observing transitions from state H to state S only when the duration of the ensuing sickness is greater than the deferred period of the policy,
(iv) not observing any transitions from state H to state D.

4.2 It is likely that any large group of PHI policies will contain some duplicates, i.e. several policies effected by the same life, and the presence of duplicates, for obvious reasons, makes assumption (a) in §(3.3) difficult to justify. Let us suppose we wish to estimate ρ as described in §§(3.3) and (3.4) but the data contain duplicates. Let O' and E' be the observed numbers of transitions and the observed exposure without eliminating duplicates. We assume we know that a proportion f_t of individuals contributing to E' have exactly t policies and we denote by m_i the i-th moment about zero of the distribution of policies, so that

$$m_i = \sum_{t=1}^{\infty} t^i \cdot f_t \qquad i = 1, 2, \ldots .$$

Under reasonable assumptions, it can be shown that if we define the estimator ρ' by

$$\rho' = O'/E'$$

then, loosely speaking,

$$\rho' \sim N(\rho,(O'/E'^2)(m_2/m_1))$$

(The derivation of this is given in the Appendix.) Hence, the only result of not eliminating duplicates has been to increase the (asymptotic) variance of our estimator for ρ by a factor (m_2/m_1). This is precisely the 'correction factor' for duplicates to be found in C.M.I.R. 7 (1984, Appendix F), whose history can be traced back via Daw (1951) to Beard and Perks (1949). This is not surprising since the argument used in the Appendix to derive it is the same as that used by these earlier authors, although the present setting is somewhat different.

4.3 When calculating probabilities for our model, as we shall see in the next section, we shall need to know, amongst other things, the values of $\rho_{x,z}$ and $\nu_{x,z}$ for values of the duration of sickness z from zero upwards. Suppose we are considering a group of policies with deferred period d. It is very unlikely that we shall be able to observe transitions out of state S, either recoveries or deaths, if the duration of the sickness is less than d and hence we cannot use the method of §3 to estimate $\rho_{x,z}$ or $\nu_{x,z}$ for a value of z less than d. One possible way of reducing the scale of this problem would be to test whether, for example, recovery rates were significantly different for policies with different deferred periods. If they were not, it would be possible to estimate $\rho_{x,z}$ for some values of z less than d by making use of data relating to policies with deferred periods less than d. Even if these recovery rates were significantly different, it might be reasonable to assume that $\rho_{x,z}$ followed a similar pattern for policies with different deferred periods. A practical solution to the problem, whether we can reduce its scale or not, is to extrapolate from graduated values of $\rho_{x,z}$ and $\nu_{x,z}$ for values of z for which we can estimate these parameters from data, to values of z for which we cannot.

4.4 The estimation of σ_x is likely to be more difficult than that of $\rho_{x,z}$ and $\nu_{x,z}$. The problem is that in practice a sickness of less than the deferred period of the policy is unlikely to be reported and so it is not possible to determine either the relevant number of transitions or the exposure necessary to estimate σ_x as outlined in §3. Hence we are forced to adopt a somewhat different approach. Let us suppose we wish to estimate σ_x at a point x' for policies with deferred period d. Recall from §2 that ${}_dp_x^{\overline{SS}}$ denotes the probability that a policyholder who falls sick at age x will remain sick for at least a period d. It will be shown in the next section that this probability is a function only of $\rho_{x,z}$ and $\nu_{x,z}$, whose estimation is independent of that of σ_x. Now choose an interval $x_1 \leqslant x' \leqslant x_2$ over which we may reasonably assume both σ_x and ${}_dp_x^{\overline{SS}}$ to be approximately constant and let σ and π respectively denote these assumed constant values. Intuitively, the product $\sigma \cdot \pi$ is the intensity of falling sick and staying sick for at least a period d for policyholders aged between x_1 and x_2. The population at risk, i.e. exposed to this intensity, is the set of policyholders who are in state H, i.e. who are not sick, and we must try to identify this population using only the data we assume to be available. Let $N(t)$ denote the number of policyholders who, at time t, are aged between x_1 and x_2 and whose policies have deferred period d. We have to subtract from this number those policyholders who are in state S at time t, and we do this in two stages. Let $Q_1(t)$ denote the proportion of policyholders in $N(t)$ who are sick at time t and whose sickness, at time t, has already lasted beyond the deferred period d and hence become a claim. Let $Q_2(t)$ denote the proportion of policyholders in $N(t)$ who are sick at time t and whose sickness, at time t, has not yet lasted, and may or may not last, beyond the deferred period. Now define

$$M(t) = N(t)\{1 - Q_1(t) - Q_2(t)\}$$

so that $M(t)$ represents the number of policyholders who, at time t, are aged between x_1 and x_2, are healthy and have policies with deferred period d. The appropriate exposure we wish to calculate is E, where

$$E = \int_{T_1}^{T_2} M(t)dt$$

T_1 and T_2 denote the start and the end of the observation period so that E represents the total time spent in state H during the observation period by policies with deferred period d, counting only that time when the policyholder is aged between x_1 and x_2. Let us assume for the moment that we can calculate E from the available data and also that the data does not contain any duplicate policies; we shall return to these points in the next paragraph.

Let I denote the number of sicknesses among policies with deferred period d which start in the observation period, for which the policyholder is aged between x_1 and x_2 at the start of the sickness and which last beyond the deferred period. (I is equivalent to the number of claims which start in $[T_1+d, T_2+d]$ and for which the policyholder is aged between (x_1+d) and (x_2+d) at the start of the claim.) As a result of the way in which we assume data to be collected, the exposure E is a random variable whose distribution depends on the parameter σ. However, if we regard E as in some way pre-determined then I has a Poisson distribution with parameter $\sigma \cdot \pi \cdot E$. (See Sverdrup (1965, §8) for an interesting discussion of this point.) Even if we do not assume that E is pre-determined, it can be shown that, asymptotically and under some reasonable assumptions, I has a Poisson distribution with the parameter given above. (See Hoem (1987, Appendix 2) for details.)

Hence we may regard $\hat{\sigma}$, where

$$\hat{\sigma} = I/(\pi \cdot E)$$

as a maximum likelihood estimator of σ which, asymptotically, has a normal distribution whose variance can be consistently estimated by $I/(\pi \cdot E)^2$. For the purposes of hypothesis testing we may assume that

$$\hat{\sigma} \sim N(\sigma, I/(\pi \cdot E)^2) \tag{17}$$

4.5 The estimation of σ in the previous paragraph assumed that duplicate policies had been eliminated from the data. Let I and E denote, as in the previous paragraph, the number of claims and the exposure, assuming duplicate policies have been eliminated and let I' and E' be the corresponding quantities assuming duplicate policies have not been eliminated. Let m_i denote, as in §(4.2), the i-th moment about zero of the distribution of policies per policyholder (for the relevant set of policyholders). Using the same argument as was used in §(4.2), we can show that we can estimate σ by σ', where

$$\sigma' = I'/(\pi \cdot E')$$

and that corresponding to (17), we have

$$\sigma' = N(\sigma,(I'/(\pi \cdot E')^2)\ \ (m_2/m_1))$$

The major difficulty with the estimation procedure for σ_x described in this section is that, although we should be able to determine, or estimate, $N(T)$,$Q_1(t)$ and I from the available data, it is unlikely, for obvious reasons, that we will be able to estimate $Q_2(t)$ directly. However, if we knew the values of all the transition intensities, including σ_x, we could calculate ${}_{d,x'-x}p_x^{HS}$ for various values of x ($< x'$), as we shall see in the next section, and from these values we could estimate $Q_2(t)$. Hence we could estimate σ_x using an iterative procedure as follows:

(i) start with a reasonable value for $Q_2(t)$ and estimate σ as described in §(4.4),
(ii) use the estimates from (i), and the values of the other transition intensities, to re-estimate $Q_2(t)$ by calculating ${}_{d,x'-x}p_x^{HS}$, the probability that a life who was healthy at some age x ($< x'$) is sick at age x' with duration of sickness less than d,
(iii) use the new estimate of $Q_2(t)$ to re-estimate σ_x,
(iv) continue this procedure until the estimates of $Q_2(t)$ and σ_x converge to limiting values.

4.6 The practical estimation of μ_x is likely to be even more difficult than that of σ_x since we cannot reasonably expect to have any information about transitions direct from state H to state D. One solution to this problem would be to assume some arbitrary values for μ_x; for example, μ_x equals the mortality of select assured lives at duration zero. Another solution could be to assume that policyholders who effect their policies at some conveniently early age will, overall, experience the mortality of a known table, say A1967–70 Ultimate. The overall force of mortality at any future age x' for this population can be expressed as a function of $\mu_{x'}$, and μ_x, σ_x, $\rho_{x,z}$ and $\nu_{x,z}$ for $x \leqslant x'$. By equating this overall force of mortality to that of the known table, $\mu_{x'}$ could be calculated. This will be discussed further in §5 and also in Part D.

5. FORMULAE FOR PROBABILITIES

5.1 In this section we shall indicate how we can derive formulae for some of the probabilities for our model. We shall assume throughout this section that the transition intensities are known functions of x or of (x,z). The formal derivations of formulae for all but the very simplest probabilities are extremely lengthy and for this reason we shall omit them. However, most of the formulae can be easily derived on an intuitive level and we shall give some of these intuitive derivations.

5.2 The probabilities we shall discuss in this section are ${}_tp_x^{\overline{HH}}$, ${}_tp_{x,z}^{\overline{SS}}$, ${}_tp_x^{HH}$, ${}_{w,t}p_x^{HS}$, and ${}_tp_x^{HD}$, which are all defined in §2. These probabilities satisfy the following (integro-) differential equations:

$$\frac{\partial}{\partial t}\,{}_tp_x^{\overline{HH}} = -\,{}_tp_x^{\overline{HH}}\,(\mu_{x+t} + \sigma_{x+t}) \tag{18}$$

$$\frac{\partial}{\partial t}\,{}_tp_{x,z}^{\overline{SS}} = -\,{}_tp_{x,z}^{\overline{SS}}\,(\nu_{x+t,z+t} + \rho_{x+t,z+t}) \tag{19}$$

$$\frac{\partial}{\partial t}\,{}_tp_x^{HH} = -\,{}_tp_x^{HH}(\mu_{x+t} + \sigma_{x+t}) + \int_{u=0}^{t} {}_up_x^{HH}\cdot\sigma_{x+u}\cdot{}_{t-u}p_{x+u}^{SS}\cdot\rho_{x+t,t-u}\,du \tag{20}$$

$$\begin{aligned}\frac{\partial}{\partial w}\,{}_{w,t}p_x^{HS} &= {}_{t-w}p_x^{HH}\,\sigma_{x+t-w}\cdot{}_wp_{x+t-w}^{SS} \qquad \text{for } 0 \leqslant w < t \\ &= 0 \qquad\qquad\qquad\qquad\qquad\qquad \text{for } w > t\end{aligned} \tag{21}$$

$$\frac{\partial}{\partial t}\,{}_tp_x^{HD} = {}_tp_x^{HH}\cdot\mu_{x+t} + \int_{u=0}^{t} {}_up_x^{HH}\cdot\sigma_{x+u}\cdot{}_{t-u}p_{x+u}^{\overline{SS}}\cdot\nu_{x+t,t-u}\,du \tag{22}$$

It should be noted that formulae (18) and (19) are just generalisations of the corresponding formula for ${}_tp_x$ in terms of the force of mortality for an ordinary life table. (See, for example, Neill (1977, formula (1.6.12)).) Note also that all the above formulae can be regarded as 'Kolmogorov forward equations' for our model. (See, for example, Cox and Miller (1965, §4.5).)

5.3 An intuitive derivation of (20) can be given very easily, especially if we replace the left hand side by

$$({}_{t+\tau}p_x^{HH} - {}_tp_x^{HH})/\tau + o(\tau)/\tau$$

and re-arrange, so that (20) becomes

$$
\begin{aligned}
{}_{t+\tau}p_x^{HH} &= {}_tp_x^{HH}\,(1-\tau(\mu_{x-t}+\sigma_{x+t})) \\
&+ \int_{u=0}^{t} {}_up_x^{HH}\cdot\sigma_{x+u}\cdot{}_tp_{x+u}^{\overline{SS}}\cdot\tau\cdot\rho_{x+t,t-u}\,du + o(\tau)
\end{aligned}
\tag{23}
$$

where $\tau > 0$. The intuitive derivation is as follows:

(i) the left hand side of (23) is the probability that the life is healthy at age $(x + t + \tau)$ given that the life was healthy at age (x), and this probability can be split into two parts according to whether the life was healthy or sick at age $(x + t)$,

(ii) the first term on the right hand side is the first of these two parts; it is the probability that the life was healthy at age x,multiplied by, using (12) and (13), the probability that a healthy life aged $(x+t)$ does not die or become sick before age $(x+t+\tau)$,

(iii) the remaining term on the right side of (23) (we are ignoring the term $o(\tau)$) is the probability that a life who was healthy at age x falls sick at some age $(x+u)$, $0 < u < t$, remains sick until age $(x+t)$ and then recovers before age $(x+t+\tau)$; it is calculated by integrating over u, the time at which the final sickness starts.

5.4 The intuitive derivation of (21) is as follows:

(i) first note that for $w > t$ we have,

$$_{w,t}p_x^{HS} = {}_{t,t}p_x^{HS} \qquad (= {}_tp_x^{HS})$$

and hence

$$\frac{\partial}{\partial w}\,{}_{w,t}p_x^{HS} = 0 \qquad \text{for } w > t$$

(ii) for $w < t$ consider the difference

$$_{w+dw,t}p_x^{HS} - {}_{w,t}p_x^{HS} \tag{24}$$

for $0 < dw \leqslant t - w$; this difference is the probability that a life who was healthy at age x will be sick at age $(x + t)$ with duration of sickness between w and $(w + dw)$; hence the life must have been healthy at age

$(x + t - w - dw)$, fallen sick between ages $(x + t - w - dw)$ and $(x + t - w)$ and then remained sick until age $(x + t)$; this probability can be written

$$ {}_{t-w-dw}p_x^{HH} \cdot \sigma_{x+t-w-dw} \cdot dw \cdot {}_{w}p_{x+t-w}^{\overline{SS}} \tag{25}$$

(iii) equating (24) and (25) and dividing by dw and letting dw decrease to zero, we obtain (21) for $0 \leqslant w < t$.

5.5 The intuitive derivation of (22) is very similar to that of (20) and is given briefly below:

(i) we consider the difference ${}_{t+\tau}p_x^{HD} - {}_{t}p_x^{HD}$ for $\tau > 0$, which is the probability that a healthy life aged x will die between ages $(x + t)$ and $(x + t + \tau)$,

(ii) this probability can be split into two parts according to whether the life was healthy or sick at age $(x + t)$ and these two parts correspond to the two terms on the right hand side of (22), in each case multiplied by τ,

(iii) note that for the second term on the right hand side of (22) we integrate over u, where $x + u$ is the age at which the final sickness starts.

5.6 Formulae (18), (19), (21) and (22) can be integrated to give the following formulae:

$$ {}_{t}p_x^{\overline{HH}} = \exp\left(-\int_0^t (\mu_{x+u} + \sigma_{x+u})\, du\right) \tag{26}$$

$$ {}_{t}p_{x,z}^{\overline{SS}} = \exp\left(-\int_0^t (\nu_{x+u,z+u} + \rho_{x+u,z+u})\, du\right) \tag{27}$$

$$ {}_{w,t}p_x^{HS} = \int_{\max[0,t-w]}^{t} {}_{u}p_x^{HH} \cdot \sigma_{x+u} \cdot {}_{t-u}p_{x+u}^{\overline{SS}}\, du \tag{28}$$

$$ {}_{t}p_x^{HD} = \int_0^t {}_{u}p_x^{HH} \cdot \mu_{x+u}\, du + $$

$$ + \int_{u=0}^{t} {}_{u}p_x^{HH} \cdot \sigma_{x+u} \int_{w=0}^{t-u} {}_{w}p_{x+u}^{\overline{SS}} \cdot \nu_{x+u+w,w}\, dw\, du \tag{29}$$

That the above formulae are the correct solutions to the corresponding formulae in paragraph 5.2 can be checked by differentiating and checking an appropriate boundary condition, for example ${}_{0}p_x^{HD} = 0$ in the case of (29). Formulae (26) and (27) correspond to the familiar formula for ${}_{t}p_x$ for an ordinary life table. Waters (1984, §4) gives a formal derivation of (26) (albeit in respect of a simpler multiple state model); the derivation of (27) is similar. It is possible to 'solve' (20) to give a formula for ${}_{t}p_x^{HH}$ along the lines of formula (29). However, the resulting formula is neither particularly useful nor intuitively appealing so we have not given it here.

5.7 Since we are assuming that the transition intensities are known, formulae (26) and (27) can be used to evaluate ${}_tp_x^{\overline{HH}}$ and ${}_tp_x^{\overline{SS}}$ by numerical integration. The numerical evaluation of the other probabilities can require a little care. Consider for example formula (20). In principle this is a standard form of integro-differential equation which can be solved by standard methods. However, while we can assume that the transition intensities have known functional forms and hence can be easily evaluated at any point, the term ${}_{t-u}p_{x+u}^{\overline{SS}}$ does not have a known functional form. This term can be evaluated numerically for any values of t, u and x using formula (27), but since in the numerical solution of (20) its value will be required at very many points, the computing time required to solve (20) by standard methods could be excessive. The numerical evaluation of these probabilities is one of the points discussed in Part D of this Report.

5.8 For an individual who was known to have been healthy at age x, the overall force of mortality at age $(x + t)$ is given by

$$(1 - {}_tp_x^{HD})^{-1} \frac{\partial}{\partial t} {}_tp_x^{HD} \tag{30}$$

Another of the points discussed in Part D is the numerical evaluation of μ_{x+t} assuming all the other transition intensities are known, together with the overall force of mortality given by (30).

APPENDIX A

PARAMETER ESTIMATION AND DUPLICATE POLICIES

In this Appendix we shall discuss the mathematical technicalities of parameter estimation in the presence of duplicate policies. In particular we shall discuss the estimation of the (assumed constant) parameter ρ as in §§(3.3) and (3.4).

First let us assume that duplicates have been eliminated from the data. Let N denote the number of individuals we observe and, as in §(3.3), let O and E denote respectively the observed number of transitions and the observed exposure for these N individuals. We assume there is a number A, intuitively the average exposure per individual, such that:

$$\lim_{N\to\infty} E/N = A$$

with probability one. With this assumption it can be shown that the asymptotic distribution as N goes to infinity of

$$N^{1/2}\,(O/N-\rho E/N)$$

is normal with mean zero and variance ρA. (See Sverdrup (1965,§5), Hoem (1976, §2), Aalen and Hoem (1978, §4.6) and Borgan (1984, §5).) This is the result from which the rather loose statement in §3.4 about the distribution of $\hat{\rho}$ is derived.

Now let us suppose that duplicates have not been eliminated from the data and that we know that a proportion f_t, $t = 1, 2, \ldots,$ of individuals have exactly t policies. Let N' denote the total number of policies we observe, so that:

$$N' = Nm_1,$$

where m_i ($i = 1, 2, \ldots,$) is the i-th moment about zero of the number of policies per individual, as in §(4.2).

Let O_t and E_t denote respectively the number of transitions and the exposure for individuals having exactly t policies, counting each individual only once, and let O' and E' be, as in §(4.2), the total observed number of transitions and the total observed exposure without eliminating duplicates, so that:

$$O = \sum_{t=1}^{\infty} O_t; \qquad O' = \sum_{t=1}^{\infty} tO_t$$

$$E = \sum_{t=1}^{\infty} E_t; \qquad E' = \sum_{t=1}^{\infty} tE_t$$

If we assume there is a number A, independent of t, such that

$$\lim_{N\to\infty} E_t/(f_tN) = A$$

with probability one, we can apply the result above to those individuals with t policies as follows:

$$(f_t N)^{1/2}\,(O_t/(f_t N) - \rho E_t/(f_t N)) \sim N(O,\rho A) \quad \text{(asymptotically)}$$

Hence

$$(Nm_1)^{1/2}(tO_t/(Nm_1) - \rho t E_t/(Nm_1) \sim N(O,\rho A t^2 f_t/m_1))$$

and, summing, we have:

$$N'^{1/2}(O'/N' - \rho E'/N') \sim N(O,\rho A m_2/m_1)$$

from which we can derive the statement in §4.2 about the distribution of ρ'.

CMIR **12** (1991) 21–50

PART B: THE GRADUATION OF CLAIM RECOVERY AND MORTALITY INTENSITIES

BY P. H. BAYLISS

SUMMARY

The claims data available for investigation and how it was compiled and classified are described in §1. Various preliminary considerations governing the approach to the investigation and graduation of the data are described in §2.

The investigation of recovery intensities, reported in §3, revealed several notable patterns in the data, which are illustrated by accompanying graphs and from which the construction of a graduation formula evolved. Details of the graduation of recovery intensities are given in §4, the final graduation formula and the numerical values of its coefficients being stated respectively in §§4.4 and 4.6. A summary of the data with a comparison of the actual and expected recoveries based on the graduated rates is given in Table B4.

With relatively few deaths, investigation of the mortality experienced by claimants under policies was necessarily limited. The main features discerned are described in a short §5. The mortality graduation is dealt with in §6, the graduation formula and coefficient values being set out in §§6.2 and 6.3 respectively. Table B5 contains a summary of actual and expected deaths based on the graduated rates. Finally the derivation of a double decrement claim continuation table based on the graduations is explained in §7, specimen examples of such tables being given in Table B6.

1. INTRODUCTION

1.1 An investigation was made into the distributions of the duration of claims under PHI individual policies, as reported by contributing offices to the C.M.I. Bureau for the quadrennium 1975–78. The investigation was confined to the Standard male lives experience. An explanation of the categories of policy included in the Standard experience is given in *C.M.I.R.* **7** (1984). This is the first investigation of its kind made by the PHI Sub-Committee, although other features of the Male Standard Experience, 1975–78, including claim inception rates, were reported upon in *C.M.I.R.* **7**.

1.2 Claims suspected to be duplicate claims on the same life were eliminated. A suspected duplicate claims record is defined as one where a match is found on all of the following items between one claims record and another: sex; deferred period; age definition and month/year of birth; exact date of falling sick; investigation year.

1.3 The basic data for the investigation was compiled by combining records of individual claims into summary records, or data cells, classified by deferred period, by the sickness period, and by age at start of sickness, differentiating in this last respect between whether a 'nearest birthday' or 'next birthday' basis of stating age is used. For each such data cell, totals were recorded of the exposed to risk of claim termination (in units of life-days of exposure) and of the respective numbers of terminations by recovery or death, and of claim revivals. The exposed to risk was calculated as a central exposed to risk.

1.4 Deferred periods (or elimination periods) of 1, 4, 13, 26 and 52 weeks are denoted for convenience in this Report by the symbols D1, D4, D13, D26 and D52 respectively.

1.5 The following is a summary of the amount of data under investigation:

	D1	D4	D13	D26	D52	Total
Exposed to risk (days)	391,746	234,124	238,680	236,171	40,931	1,141,652
Number of recoveries	6,336	1,364	368	131	9	8,208
Number of deaths	84	49	48	46	5	232

Tables B4 and B5 show the data in more detail. The exposed to risk was calculated in units of days of exposure and it is convenient to state it in those units in these tables. However, as the recovery and mortality transition intensities derived in this Report are expressed as yearly rates, an exposed to risk (in days) should be converted to years by division by 365.25 before being multiplied by a transition intensity to calculate expected recoveries or deaths.

1.6 Data was classified by sickness period, in the first instance into single weeks of sickness duration for the first year of sickness, and into yearly intervals of sickness duration for sickness periods exceeding one year. The minute amount of data for sickness of duration exceeding 11 years was disregarded. In carrying out the investigations, the breakdown of the data into single weeks of sickness duration was found to be worthwhile only for the first 30 weeks of sickness, because the rapid change in termination rates with duration in the shorter periods did not extend further than this. This period of 30 weeks also enabled certain special features of the termination rates observed in the few weeks immediately following the end of the deferred period to be examined, as described later. For longer durations, where data was increasingly scanty, it was decided to use wider sickness bands. The following breakdown was used for the longer durations for most of the data analysis and graduation work:

Sickness period	Assumed centre of interval
30 weeks–39 weeks	34·5 weeks
39 weeks–1 year	45·6 weeks
1 year–2 years	1·5 years
2 years–3 years	2·5 years
3 years–4 years	3·5 years
4 years–5 years	4·5 years
5 years–11 years	7·35 years

The central point quoted above for each interval is simply its mid-point, except that, for the final interval of 5–11 years, the central point is the mean duration over the period, weighted by the exposed to risk in the individual years of the interval.

1.7 Where, in the course of the investigations and graduations, it was necessary to convert sickness duration measurements from one unit of time to another, this was done on the basis that one year equals 365·25 days or 52·18 weeks. The constant 52·18 is employed so frequently in the formulae given in this Part that it is convenient to denote it by the symbol w. Thus, taking z as the variable for sickness duration in years, then $w \cdot z$, or more simply wz, will represent the duration expressed in weeks.

1.8 Although the data was originally classified by individual years of age at falling sick, such a detailed breakdown, cross-classified with as many as 36 sickness periods, would have meant an inconveniently large number of data cells. It was therefore decided to group the data into nine quinquennial age groups: 20–24, 25–29, . . . , 60–64. The trivial amount of data for ages just below 20 and for age 65 was discarded.

Most of the data supplied by offices has age recorded on an 'age at nearest birthday' basis. A relatively small amount of data is submitted on the basis of 'age at next birthday', but is converted to an 'age at last birthday' basis in the course of data processing for this investigation. For the same nominal age, as reported on the two bases, there is thus a displacement, by approximately a half-year on average, between the true ages of the respective claimants. In grouping the data into quinquennial age groups, it was decided to amalgamate the age nearest birthday and age last birthday data sets. The data allocated to age group 20–24, for instance, comprises sicknesses starting at ages 20–24 nearest birthday for the former set, and at ages 20–24 last birthday for the latter, and similarly for other age groups. However, during this grouping, the exact mean age, y, for each group was calculated, taking into account the differences in age definition described and based on the exposed to risk at all sickness durations. The mean ages are as follows:

Age group	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64
Mean age y	23·3	27·3	32·2	37·0	42·3	47·1	52·1	57·1	61·4

Thereafter, during the investigations and graduations, the exact age at falling sick for all claims in a given age group was treated as independent of sickness duration and equal to the mean age for that group as quoted above. Because of the selective effect of variation in termination rates by age within a quinquennial group, the mean age at falling sick of survivors in successive sickness periods does not in fact remain constant. This is especially true of age group 60–64, due to automatic expiries at age 65. However, it was decided to employ the overall mean ages in the interests of simplicity.

1.9 The records include a few claim revivals. These have been treated as reversals of recorded recoveries, with their numbers being netted off against the numbers of recoveries.

2. PRELIMINARY CONSIDERATIONS

2.1 The lack of previous acquaintance with the kind of data now being investigated made it particularly important to carry out preliminary explorations aimed at recognising the principal features of the experience and at forming a view about possible types of graduation formulae. §§3 and 5 below describe the more interesting results of those data analyses, for recovery and mortality rates respectively.

2.2 As the data is confined to reported claims, no information is available about sickness inceptions (as distinct from claim inceptions), nor as to the persistency of sickness during the deferred period of a policy before a claim arises. For the purpose of the model to be employed, however, assumptions about sickness inception intensities and recovery and mortality intensities during the deferred period are required, and the derivation of suitable estimates for this purpose is dealt with in Part C. In this connection, a question of particular interest is whether the experiences under different deferred periods are significantly different from one another, so as to necessitate separate graduations of their recovery and mortality rates, or whether they exhibit sufficient common characteristics to be regarded, if not as identical, at least as a related family. If sufficient similarity exists, then it may be possible, from the short-duration experience of claims under policies with a short deferred period (especially D1) to make inferences about sickness termination rates before the start of claim of policies with longer deferred periods.

A practical problem in this area is that there are clear indications, discussed in §3, that the date of recovery as reported to an insurer may differ, under the influence of practical and maybe moral factors, from what may be considered the true, natural, recovery date. There appears to be a tendency, for example, for claims to be brought to a conclusion after a round number of weeks, leading to a bunching in the distribution of claim durations. When, as in the early days of claim under D1, the underlying natural recovery rates are high and changing quite rapidly, any artificial distortion in the reporting of recoveries adds to the difficulty of drawing reliable conclusions in this area. Another problem is the absence of data for the very first week of sickness, which could only be met by extrapolating, back to the start of sickness, from the known experience for sickness durations exceeding one week. That extrapolation is purely for the purpose of the model and should not be taken out of context.

2.3 The considerations discussed in §2.2 are relevant not only to the exploratory analysis of the data, but also to its graduation. The object is not simply to produce valid graduated rates for the range of ages and durations under actual observation, but also, as a secondary requirement, sensible rates, by extrapolation, for ages and durations which, though outside the observed ranges, are relevant for the full modelling described above. In practice this required graduation formulae with more parameters than would otherwise have been chosen.

2.4 It is considered desirable that the results of the graduations be expressed in mathematical formulae, enabling a potential user to calculate rates at the ages and sickness durations required for a particular application. Rates presented only as a set of numbers in a table with predetermined sickness durations, for example, would by contrast be inherently inflexible. Graduation by mathematical formula has other advantages which need not be mentioned here.

2.5 Although for some purposes it may be unnecessary to differentiate between recovery and death as the reason for claim termination, separate decremental rates are required for the purpose of the model as a whole. The calculation of the exposed to risk, as mentioned previously, was designed to enable the investigation to be conducted in terms of central rates of recovery and mortality. In this Report, the term 'rate' is used always to mean a central rate. The central rate calculated for a given data cell by dividing the number of decrements in that cell by its central exposed to risk is taken to be an estimate of the corresponding force of decrement (or transition intensity) at the central duration and mean age of that cell, as detailed in §1.6 and §1.8.

The number of decrements in each cell is assumed to have an expected value, equal to the exposed to risk multiplied by the true, underlying parametric value, of the transition intensity.

2.6 In fitting the graduation formulae, data cells corresponding to the sickness period and age bands quoted in §1.6 and §1.8 were used, in the belief that these bands were sufficiently narrow to ensure reasonable homogeneity within each cell. However, it is thought to be more convenient to the reader, in setting out the results of the graduations in Tables B4 and B5, to condense the tables by combining data cells to some extent, so as to provide enough decrements in each case for a reasonable comparison of actual with expected numbers.

3. RECOVERY INTENSITIES—INVESTIGATION

3.1 The claims data is heavily concentrated at short durations. Of all the claims terminating by recovery under D1, for instance, some 60% are for claims of no more than two weeks, and only 6% for claims exceeding 12 weeks. For longer deferred periods the numbers of claims are limited by the absence of data for sickness durations shorter than the deferred period. Of a total of 8,208 recoveries investigated, 6,336 are under D1, leaving 1,872 for the other deferred periods. For 4,456 of the claims under D1 the sickness duration did not exceed four weeks and was thus shorter than the deferred period under any of the other deferred periods. This heavy weighting in the total data of short-term claims under D1 is of some importance when it comes to graduating the experience. In contrast, for all deferred periods there are only 200 claims where sickness exceeded one year, and only a quarter of those continued for sickness periods of more than two years.

3.2 Initial examination of the recovery rates, for data grouped and cross-classified by age and duration, as described previously, suggested that, subject to some doubt about very short claim durations under D1, those two factors may be largely independent of each other and multiplicative in effect. Thus, the recovery intensity might be explained at least approximately by an expression of the general form

$$\rho_{y+z,z} = f_y \cdot g_z \tag{1}$$

where f_y, g_z are respectively functions of y, age at sickness inception, and z, sickness duration. The functions may differ between deferred periods. In the search for a pattern, an attempt to separate the age and duration effects is in any case desirable.

It is perhaps worth pointing out that, for the investigations described in this Part, the recovery and mortality intensities are regarded as functions of age at the date of falling sick, y, and of sickness duration, z, whereas in other Parts those intensities have been more conveniently treated, upon their incorporation into the overall model, as functions of attained age, x, and duration z. The notation $\rho_{y+z,z}$ is consistent with the form $\rho_{x,z}$ used elsewhere in the Report.

One might simply take the marginal rates, shown by the row and column totals of each table, as an indication of the variation by duration and age respectively. However, to avoid distortions due to the uneven distribution of exposed to risk amongst the cells, a slightly more elaborate approach is needed. For the data cell for mean age y and mean sickness duration z, let $E(y,z)$ be the exposed to risk and $O(y,z)$ be the observed number of recoveries. The ratio $O(y,z)/E(y,z)$ is taken as an estimate of the recovery intensity, $\rho_{y+z,z}$. Assuming the $O(y,z)$ to be mutually independent, the log-likelihood of their joint distribution may be shown to be

$$L = \sum_y \sum_z \{O(y,z) \cdot \log(\rho_{y+z,z}) - E(y,z) \cdot \rho_{y+z,z}\} \tag{2}$$

where the summations cover the 9 values of y ($y_0, y_1, \ldots, y_8$) and up to 36 values of $z(z_1, z_2, \ldots, z_{36})$ into which the data for each table was classified.

Now, substituting for $\rho_{y+z,z}$ from (1), and maximising L by setting its differential coefficient with respect to each of the coefficients to zero in turn, estimates of the coefficients are obtained as the values satisfying the set of equations

$$\left.\begin{aligned} \sum_z O(y,z) &= f_y \cdot \sum_z \{E(y,z) \cdot g_z\} \qquad \text{for } y = y_0, y_1, \ldots y_8 \\ \sum_y O(y,z) &= g_z \cdot \sum_y \{E(y,z) \cdot f_y\} \qquad \text{for } z = z_1, z_2, \ldots z_{36} \end{aligned}\right\} \tag{3}$$

The equations were solved by iteration. In a trivial sense, there is actually an infinite number of solutions, since, given any one solution, consisting of sets $\{f_y\}$ and $\{g_z\}$, another may be stated by multiplying all the f_y by a constant, and dividing all the g_z by the same constant. To facilitate comparisons of the coefficients obtained for the different deferred periods, the scales of the coefficients were adjusted so as to bring the age coefficients f_y onto a similar average level for each deferred period.

3.3 The durational factors g_z which were derived for each deferred period showed some clear and interesting relationships between the factors and the variable z. Empirical mathematical transformations of g_z and wz indicated in the case of D1 an apparently linear relationship between the logarithm of g_z and the square root of wz, for sickness durations up to one year. A similar linearity was found for D4, D13 and D26, apart from a 'run-in' period of roughly 4 weeks immediately after the end of the deferred period, which was particularly well-defined for D4. Figure B1, which shows $\log(g_z)$ plotted against the square root of wz for the four deferred periods separately, demonstrates these features.

It can also be seen that the slopes of the linear parts of the curves are similar, suggesting that, leaving aside the run-in periods, the same durational factors might be taken to apply to all policies regardless of deferred period. Prior statistical appraisal of the data had, in any case, not revealed significant differences between deferred periods, except that the termination rates for the

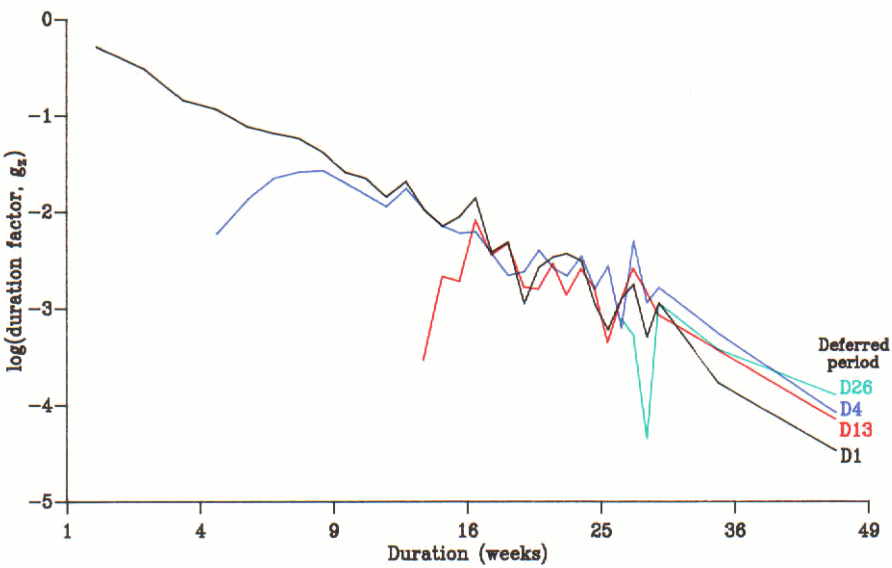

Figure B1. Recovery rates: duration related factors: deferred periods separate.

first few weeks of claim under D4 were quite clearly lower than the rates under D1 for corresponding sickness durations. This had led us to expect to pool the data for all deferred periods at longer claim durations. The graph in Figure B2 results from combining the data for all deferred periods (including, at this stage, D52), except for continuing to distinguish the factors for the first four weeks of claim under D4, D13 and D26.

The lower recovery rates during the run-in periods under D4, D13 and D26, compared with the general trend by duration, cannot be plausibly attributed to natural differences in the sickness characteristics of claimants with different deferred periods, but must presumably reflect differences in attitudes to claiming or in claims procedures. Two opposing explanations for the lower recovery rates soon after claims commence may be considered. The first is that some who are near to recovery at the end of their deferred period do not bother to submit a claim for what would be only a short period, and that their imminent recoveries are thereby excluded from the reported experience. The second is that virtually all those entitled to claim do so, and that recoveries tend to be protracted beyond their natural duration if a claim has only just started. If this second explanation were true, the initial shortfall in recoveries should be succeeded by a period of relative superfluity and there appears to be no evidence of this. (The two explanations are not, of course, mutually exclusive; different policyholders may

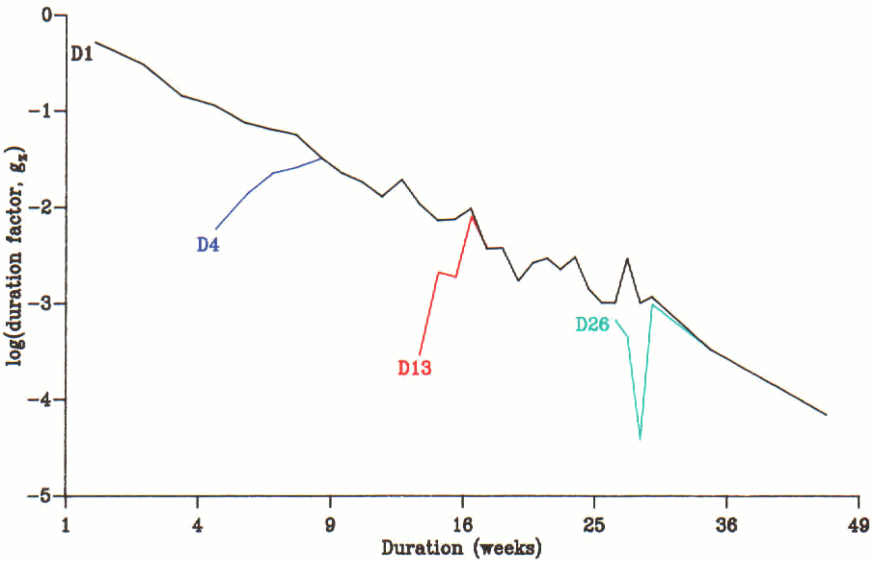

Figure B2. Recovery rates: duration related factors: all deferred periods combined except for run-in periods.

behave in different ways.) Comparable run-in periods of low termination rates were reported by Medin (1952) in a study of Swedish experience, with the comment that 'the low values for the first intervals may presumably be explained by supposing that a number of cases of sickness which lasted a little longer than the benefit waiting period were not reported to the insurance company.' The same explanation was given by Dillner (1969). The Sub-Committee came to the same view of the UK PHI experience.

The durational factors for the first few weeks of claim under D1 do not show a run-in pattern similar to those of the other deferred periods. There are, however, other distinctive features of this region of the experience (to be described later), and it may be that the conformity of the observed durational factors to the overall trend is somewhat fortuitous.

The linear trend exhibited by the transformed variables as shown in Figure B2 is not continued for sickness durations exceeding one year. When the graph is extended to include the longer durations, as in Figure B3, the curve veers off, reflecting higher recovery rates than would have been predicted by extrapolating the trend of the first year of claim. Conclusions should not be drawn too firmly, though, in view of the very small number of claims remaining after two years.

3.4 The factors f_y have been plotted for D1, D4, D13 and D26 separately in Figure B4, and for all deferred periods (including D52) combined in Figure B5,

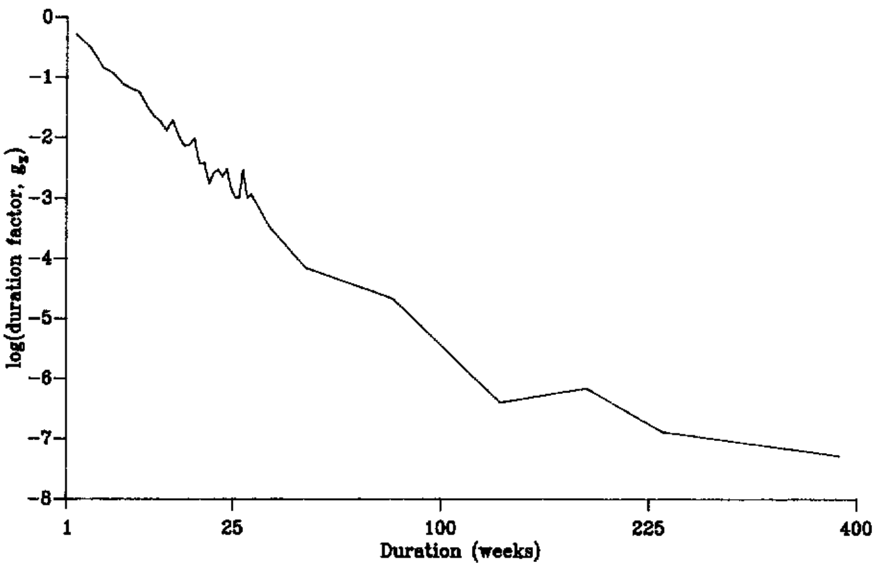

Figure B3. Recovery rates: duration related factors: extended to sickness durations greater than 1 year: all deferred periods combined excluding run-in periods.

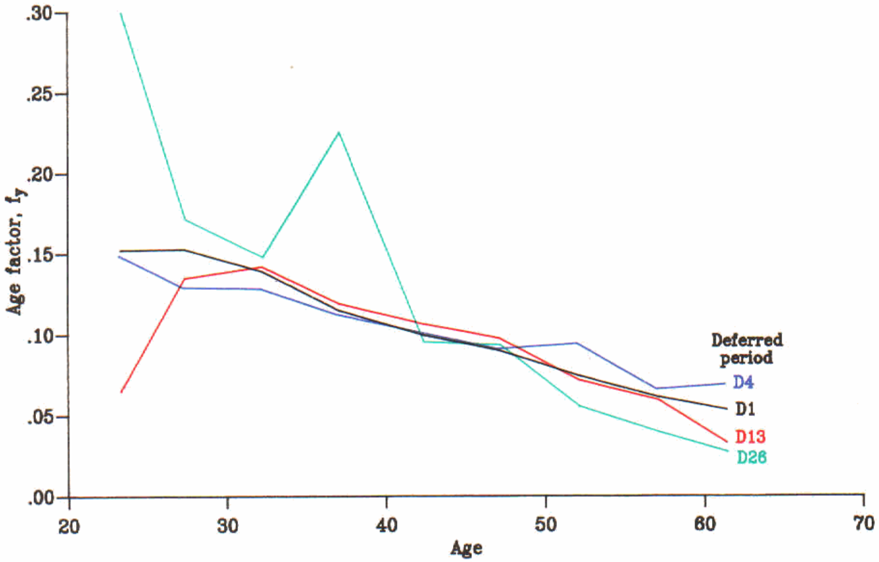

Figure B4. Recovery rates: age related factors: deferred periods separate.

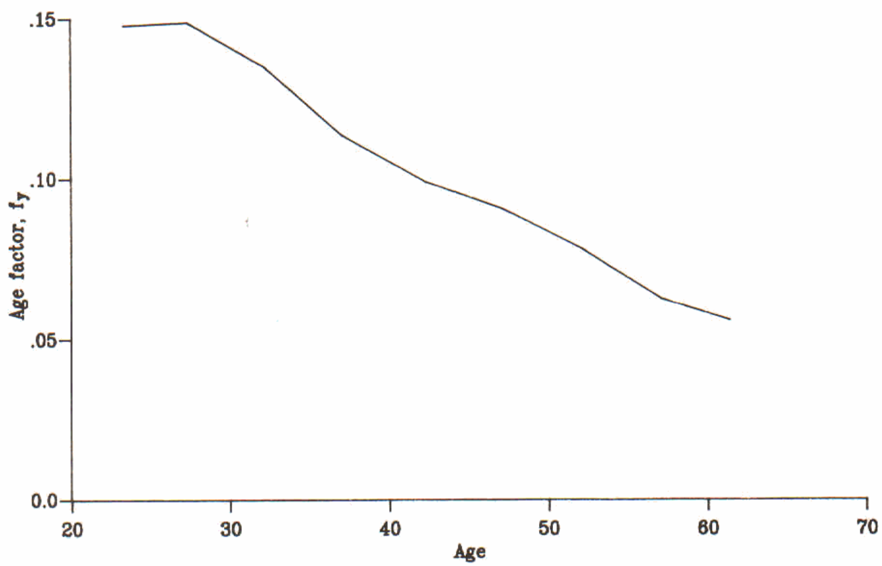

Figure B5. Recovery rates: age related factors: all deferred periods combined.

following an inspection which gave no reason to distinguish 'run-in' periods in the case of these factors. The variables have not been subjected to transformation in these graphs. The general appearance is of a broadly linear relationship of the factors with age, and no great disparity between the deferred periods. That observation may be thought dubious in respect of the youngest age group (ages 20–24). There is, however, relatively little data in that age group, which contains only approximately 0.5 per cent of the grand total exposed to risk, with all but 30 of the 152 recoveries in that age group occurring under D1. No simple transformation of the variables was found which would straighten the line further.

3.5 In the light of the foregoing investigations, it was thought that, apart from the need to recognise the peculiarities of the run-in periods under D4, D13 and D26, the experience for different deferred periods seemed insufficiently different to necessitate keeping them separate. There are, indeed, strong practical incentives to combine them: to gain the stability of larger numbers, and the simplicity of having one data set instead of four or five. It was therefore decided at this point to proceed with combined data, except for keeping the first four weeks of claim separate for each of D4, D13 and D26.

3.6 Although the investigations described above give an insight into the main age and durational effects, the provisional idea, put forward in §3.2, of a multiplicative model to represent recovery rates, requires more careful examination. Using values for the f_y and g_z factors recalculated for the data combined as described in §3.5, an estimate of $\rho_{y+z,z}$, and hence of the expected numbers of recoveries, was found for each data cell. Examination of the differences of actual from expected numbers reveals unfortunately a non-random distribution of signs of deviations, showing that the simple multiplicative model fails to fit the experience sufficiently well. A more detailed examination of the variation in observed recovery rates with age at sickness inception, while not invalidating the finding in §3.4 of a broadly linear trend with age, showed that the slope of the trend, instead of being constant, varies with sickness duration. After the first few weeks of sickness, during which the (negative) slope reduces, the slope then increases with sickness duration, and appears to be roughly proportional to the square root of duration. Consequential adjustments are therefore required to the multiplicative model if it is to form the basis of a graduation formula.

4. RECOVERY INTENSITIES—GRADUATION

4.1 The investigations described above suggested that a basic formula from which to start in seeking a suitable graduation formula would be

$$\rho_{x+z,z} = \{a + b(z) \cdot (y - \bar{y})\} \cdot e^{-c\sqrt{z}} \qquad (4)$$

where a and c are constants, but b is dependent on z, reflecting the association described in §3.6.

4.2 Attempts, during graduation trials, to find a single simple mathematical form for $b(z)$ to represent its variation over all durations were not fruitful, and for the purpose of this basic formula it was decided therefore to cope with the main range of medium to longer durations, by letting $b(z) = b \cdot \sqrt{z}$, (b being a constant), leaving the short durations of the first four weeks of sickness to be dealt with by a special adjustment.

It was convenient to substitute $\bar{y} = 50$, a round number near to the true mean. The basic formula may thus be restated:

$$\rho_{x+z,z} = \{a + b\sqrt{z} \cdot (y-50)\} \cdot e^{-c\sqrt{z}} \tag{5}$$

4.3 Whilst the latter formula may be suitable for graduating much of the data, modifications are needed to cope with the three special features identified earlier:

(i) The lowered recovery rates during the run-in periods of D4, D13 and D26,
(ii) The increased gradient of $\rho_{y+z,z}$ with respect to y for, say, $wz < 4$ weeks, and
(iii) The change in gradient of $\rho_{y+z,z}$ with respect to z for $z > 1$ year.

It was found that (i) could be covered reasonably well by assuming for each of D4, D13 and D26 that $\rho_{y+z,z}$ at the moment of claim commencement (when the value of wz is equal to the deferred period) is a proportion p of the value of $\rho_{y+z,z}$ calculated from the basic formula, and that the proportion increases at a uniform rate up to unity over the first 4 weeks of claim. The value of p is taken to be the same for all ages at sickness commencement and for all of these deferred periods. The reduction factor is therefore

$$r = p + (wz-d) \cdot (1-p)/4 \qquad \text{for } wz < d+4$$

$$\text{and } r = 1 \qquad \text{for } wz \geqslant d+4$$

where d is the deferred period expressed in weeks.

A similarly simple adjustment factor was introduced to deal with (ii), though, in this case, what is needed is a multiplier applying to the coefficient b, rather than to the whole formula. The chosen factor is $(1 + q \cdot [4 - wz])$, where q is a positive constant for $wz < 4$, and $q = 0$ for $wz \geqslant 4$.

It was thought desirable to limit the applicable period of this factor to a maximum of four weeks, so as not to overlap the start of D4 claims. Fortunately, a period of four weeks proved quite suitable.

As previously remarked, there is little data for sickness durations exceeding two years and it would perhaps be of little practical consequence and quite expedient if any adjustment in respect of (iii) were ignored. However, there are enough recoveries in the period 1–2 years to be statistically influential in a graduation, and it was felt unreasonable to ignore the departure of the recovery rates in that period from the trend of shorter durations. It was observed that the reduction in the rate at which recovery rates declined, after the first year of sickness, could be represented roughly as equivalent to a slowing down in the passage of time itself. Thus, in entering the basic formula for sickness durations of more than one year, an adjusted value of z should be used, $Z = 1 + s \cdot (z - 1)$, where s is a constant (approximately equal to 1/3), whose value is to be fitted in the graduation.

However, for convenience in the use of the graduated rates in the overall model, it was assumed that, after five years of sickness (i.e. for $z > 5$), recovery intensities depend on attained age only.

4.4 The complete formula used for the graduation may now be stated as follows:

$$\rho_{y+z,z} = r \cdot \{a + b \cdot (1 + q \cdot \max(4 - wz, 0)) \cdot \sqrt{Z} \cdot (Y - 50)\} \cdot e^{-c\sqrt{Z}} \quad (6)$$

where

(a) y is exact age (in years) at the date of falling sick, and

$$\begin{aligned} Y &= y && \text{for } z \leqslant 5, \\ Y &= y + z - 5 && \text{for } z > 5; \end{aligned}$$

z is duration of sickness (in years) and

$$\begin{aligned} Z &= z && \text{for } z \leqslant 1, \\ Z &= 1 + s \cdot (z - 1) && \text{for } 1 < z \leqslant 5, \\ Z &= 1 + 4 \cdot s && \text{for } z > 5; \end{aligned}$$

(b) for D4, D13 and D26 only,

$$r = \min\,(p + (wz - d)(1 - p)/4, 1) \text{ so that } r = 1 \text{ if } wz > d + 4$$

(c) a, b, c, p, q, s are constants.

4.5 The graduation formula is not intended for extrapolation to attained ages $(y+z)$ above age 65. If it were so used, then any negative values of $\rho_{y+z,z}$ appearing at ages over 65 should be set to zero.

4.6 The graduation formula was fitted by the method of maximum likelihood. A general explanation of the method is given by Forfar, McCutcheon and Wilkie (1988). The estimated standard errors of the parameter estimates quoted below were obtained by a method of simulating the experience and the parameter fitting process, and, although not claimed to be very accurate, were found to be quite consistent with estimates alternatively derived by approximate calculation of the theoretical values obtainable as an asymptotic result of the theory of maximum likelihood estimation. In comparison with the standard errors, it is clear that all six parameters are significantly different from zero or unity (as the case may be) and should therefore be retained in the graduation formula.

Table B1. *Graduated values of* $\rho_{y+z,z}$

Exact age y at falling sick	20	30	40	50	60	20	30	40	50	60
Exact duration of sickness										
Weeks										
0						51·0572	51·0572	51·0572	51·0572	51·0572
1						55·5765	45·6703	35·7641	25·8578	15·9516
2						42·6559	34·9398	27·2238	19·5077	11·7916
3						30·1077	25·3100	20·5121	15·7143	10·9165
	Run in period: D4									
4	3·8603	3·4689	3·0775	2·6861	2·2947	18·8203	16·9121	15·0039	13·0956	11·1874
5	6·7050	5·9713	5·2375	4·5038	3·7701	16·6034	14·7864	12·9695	11·1526	9·3357
6	8·9235	7·8863	6·8491	5·8119	4·7747	14·8094	13·0880	11·3667	9·6453	7·9240
7	10·6733	9·3697	8·0662	6·7626	5·4590	13·3203	11·6935	10·0666	8·4397	6·8129
8						12·0612	10·5253	8·9893	7·4534	5·9175
9						10·9811	9·5315	8·0819	6·6323	5·1826
10						10·0440	8·6756	7·3073	5·9390	4·5707
11						9·2231	7·9311	6·6390	5·3470	4·0549
12						8·4986	7·2779	6·0572	4·8365	3·6159
	Run in period: D13									
13	1·6111	1·3744	1·1377	0·9010	0·6643	7·8547	6·7007	5·5468	4·3928	3·2389
14	2·9395	2·4987	2·0579	1·6171	1·1762	7·2791	6·1875	5·0959	4·0043	2·9127
15	4·0745	3·4519	2·8292	2·2066	1·5839	6·7621	5·7287	4·6954	3·6621	2·6287
16	5·0445	4·2601	3·4758	2·6914	1·9071	6·2955	5·3167	4·3378	3·3589	2·3800
17						5·8729	4·9449	4·0170	3·0890	2·1611
18						5·4886	4·6083	3·7280	2·8478	1·9675
19						5·1380	4·3024	3·4668	2·6312	1·7956
20						4·8174	4·0236	3·2299	2·4361	1·6424
21						4·5232	3·7687	3·0143	2·2598	1·5053
22						4·2527	3·5351	2·8175	2·0999	1·3823
23						4·0035	3·3205	2·6375	1·9546	1·2716
24						3·7732	3·1229	2·4725	1·8221	1·1718
25						3·5602	2·9405	2·3208	1·7011	1·0814

Table B1 (*Continued*)

Exact age y at falling sick	20	30	40	50	60	20	30	40	50	60
Exact duration of sickness										
Weeks										
	Run in period: D26									
26	0·6897	0·5685	0·4474	0·3262	0·2050	3·3627	2·7719	2·1811	1·5903	0·9995
27	1·2839	1·0563	0·8287	0·6011	0·3736	3·1792	2·6157	2·0521	1·4886	0·9250
28	1·8129	1·4888	1·1647	0·8406	0·5165	3·0086	2·4708	1·9329	1·3951	0·8572
29	2·2834	1·8719	1·4604	1·0488	0·6373	2·8497	2·3361	1·8225	1·3090	0·7954
30						2·7014	2·2108	1·7201	1·2295	0·7389
40						1·6458	1·3275	1·0092	0·6908	0·3725
50						1·0582	0·8441	0·6299	0·4157	0·2016
Years										
1						0·9664	0·7692	0·5720	0·3748	0·1775
2						0·4683	0·3671	0·2659	0·1647	0·0635
3						0·2469	0·1914	0·1359	0·0803	0·0248
4						0·1382	0·1062	0·0741	0·0421	0·0101
5						0·0809	0·0617	0·0425	0·0233	0·0041

The fitted values of the parameters, and their estimated standard errors (given in parentheses) were as follows:

a	51·057202	(1·200)	p	0·205111	(0·024)
b	−2·687089	(0·124)	q	1·419428	(0·080)
c	4·914441	(0·038)	s	0·362456	(0·029)

4.7 A comparison of actual recoveries with those expected, based on the graduated recovery intensities, is given in Table B4. To facilitate appraisal, the results shown there have been grouped sufficiently by age and by sickness duration to ensure that there are generally not fewer than 5, and usually at least 10, expected recoveries in each cell. As a test of goodness of fit, a value for χ^2 was calculated for all the results as shown in Table B4(e), i.e. for Tables B4(a), B4(b), B4(c) and B4(d) together. The value found was 224.7. Having regard to the number of cells, 202, and the fitting of 6 parameters from the data, this is not statistically significant.

4.8 Table B1 gives values of the graduated $\rho_{y+z,z}$ for specimen exact values of y and z.

In view of the relatively low representation of lives under age 25 in the investigation, the recovery intensities shown in the table for age 20 at sickness inception should be treated with circumspection.

It may be noticed that there is a slight kink at durations 3 and 4 weeks in the run of rates for age 60. This no doubt is an anomaly arising from the rather crude adjustment in the graduation formula, by means of the parameter p, for the transition intensities in the first 4 weeks of sickness. It, was not felt sufficiently severe to compel a revision of the graduation.

4.9 For convenience of description later in this Report, the graduated rates $\rho_{y+z,z}$ (excluding those for the run-in periods) are referred to as deferred period 1 week rates.

5. Mortality Intensities–Investigation

5.1 The relatively small number of deaths precluded a detailed breakdown of the data by deferred period and sickness duration. The data for all deferred periods was therefore combined, there being no apparent distinguishing features which would necessitate their separation. The age and duration factors were examined by the same method as for recoveries, by fitting marginal coefficients f_y and g_z, on the assumption of simple multiplicative effects.

5.2 It appears from the graph of duration-related coefficients (Figure B6) that the force of mortality tends to rise from the start of sickness to a peak after about four months of sickness, after which it declines fairly rapidly. The graph indicates an eventual upwards turn in mortality at long durations, perhaps reflecting a reversion to a dependency primarily on attained age, once the short to medium term effects of sickness on the level of mortality have worn off.

5.3 There is a strongly rising trend by age at start of sickness, as shown by the age-related factors in Figure B7, but with only a small number of deaths below age 40 it is difficult to judge the precise shape of the underlying curve.

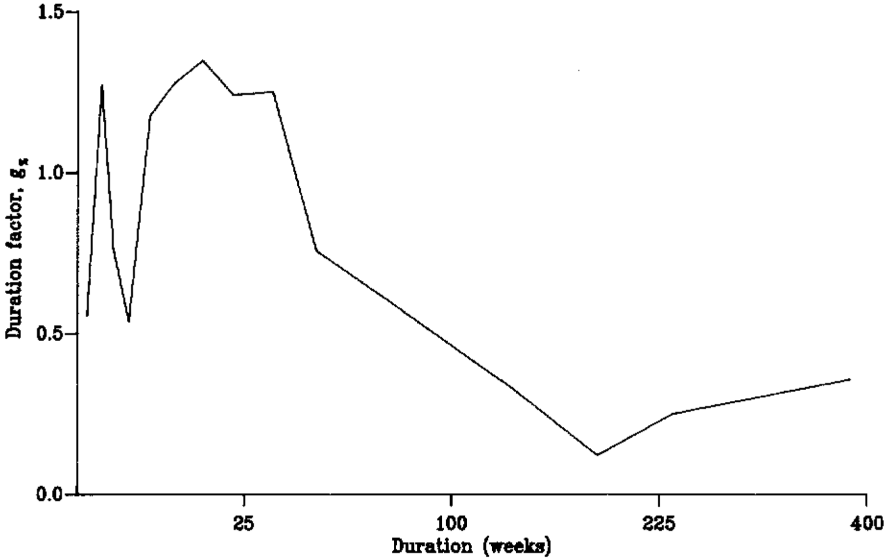

Figure B6. Mortality rates: duration related factors: all deferred periods combined.

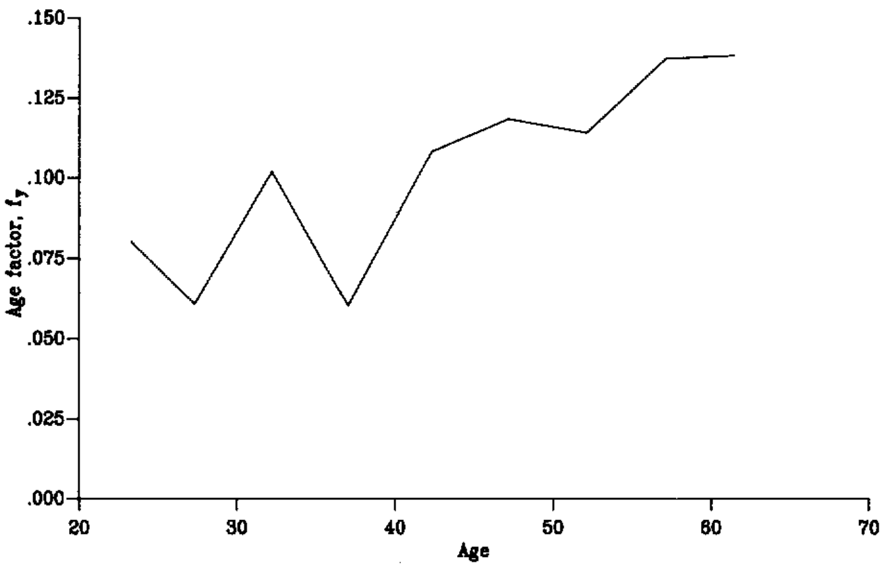

Figure B7. Mortality rates: age related factors: all deferred periods combined.

6. MORTALITY INTENSITIES—GRADUATION

6.1 A suitable formula for graduating the mortality rates of claimants must cater primarily for the hump-backed curve, related to sickness duration, which is the dominant feature of the experience. However, it was thought that the formula should also include a component representing an eventual trend towards mortality probabilities rising with attained age.

A constraint on the choice of formula for the hump-backed curve was the need for a function from which, by extrapolation, reasonable values of $v_{y+z,z}$ could be inferred to apply immediately upon commencement of sickness (i.e. for $z = 0$). A formula similar to that used by Starke (1980), to model sickness termination rates, was found to give a better fit than others which were tried. This postulates a distribution of the Weibull type with a negative parameter.

Expressed in life table terms, the probability of survival for a period z measured from sickness inception at exact age y, conditional on continuous sickness throughout that period, was assumed in the underlying independent single-decrement mortality table to be of the form:

$$-\log {}_z p_y = \frac{a \cdot \exp(-b/(z+c)^n)}{bn} \tag{7}$$

Hence the force of mortality with respect to claim duration is

$$v_{y+z,z} = -\frac{d}{dz} \cdot \log {}_z p_y = \frac{a \cdot \exp(-b/(z+c)^n)}{(z+c)^{n+1}} \tag{8}$$

To provide for the dependency of the force of mortality on age, the parameter a in the above formula was replaced by a quadratic expression in age, y. Although a quadratic was found to give slightly better graduation results than a linear function, it is questionable in the final outcome of the graduation whether the added refinement of a quadratic term is really justified.

It was decided to add a Gompertz term as the simplest formula for generating a customary pattern of long-term mortality probabilities depending on attained age. However with only a few deaths at long durations, there is very little support for the specific choice of a Gompertz term.

As in the case of recoveries, it was assumed for convenience that, after five years of sickness, mortality depends on attained age only.

6.2 The graduation formula is

$$v_{y+z,z} = (a_0 + a_1 \cdot Y + a_2 \cdot Y^2) \cdot \frac{\exp(-b/[Z+c]^n)}{(Z+c)^{n+1}} + r \cdot \exp(s \cdot [Y+Z]) \tag{9}$$

where y is age nearest birthday (in years) at the date of falling sick, and

$Y = y$ for $z \leqslant 5$,
$Y = y + z - 5$ for $z > 5$;

and z is duration of sickness (in years) and

$$Z = z \text{ for } z \leqslant 5,$$
$$Z = 5 \text{ for } z > 5.$$

6.3 Fitting by the method of maximum likelihood gave the following values for the parameters, with approximate estimates of their standard errors given in parentheses:

a_0	0·237884	(0·119)	b	0·874735	(0·252)	r	0·007221	(0·0063)
a_1	−0·00481923	(0·00169)	c	0·357384	(0·115)	s	0·024353	(0·0359)
a_2	0·0000958683	(0·0000353)	n	1·613917	(0·434)			

Table B2. *Graduated values of 10,000* $\nu_{y+z,z}$

	Exact age at falling sick				
	20	30	40	50	60
Exact duration of sickness					
Weeks					
0	383	415	484	593	745
1	453	485	562	686	860
2	526	558	642	782	979
3	599	630	722	877	1,098
4	669	701	800	970	1,213
5	735	767	873	1,057	1,321
6	797	828	941	1,138	1,421
7	852	884	1,002	1,211	1,512
8	901	933	1,057	1,276	1,593
9	944	976	1,104	1,332	1,662
10	981	1,012	1,144	1,380	1,722
11	1,011	1,042	1,177	1,419	1,771
12	1,035	1,066	1,204	1,452	1,811
13	1,054	1,085	1,225	1,476	1,842
14	1,068	1,099	1,241	1,495	1,865
15	1,077	1,108	1,251	1,507	1,880
20	1,071	1,103	1,245	1,500	1,871
25	1,012	1,043	1,179	1,422	1,774
30	930	962	1,089	1,314	1,641
40	762	794	904	1,093	1,366
Years					
1	595	627	719	874	1,094
2	277	311	370	458	578
3	193	228	280	351	444
4	165	201	250	316	401
5	154	190	239	303	386

Relative to their standard errors, the parameters r and s might be dismissed as non-significant. They have been retained, however, in view of the wider considerations referred to previously.

6.4 A comparison of actual deaths with those expected, based on the graduated mortality intensities, is given in Table B5. The results shown there have been grouped by age and duration into broad classes to provide cell totals large enough for meaningful comparisons to be made. The value of χ^2 calculated on the 23 cells given in Table B5 is 13.3, which, taking into account that 8 parameters were fitted from the data, is not significant.

6.5 Graduated values of v_{y+z} for a range of exact values of y and z are given in Table B2. After sickness duration of 5 years, the graduated force of mortality is a function only of attained age, and Table B3, which gives such values, may be of interest. It will be seen that the values are generally far higher than those customarily shown by a standard mortality table applying to a population base of healthy and sick lives combined. There is no reason, indeed, why the mortality of the long-term disabled should revert to a level comparable to that of such a standard conventional table. Since, nevertheless, the possibility of such a comparison may be of some interest, it may be observed that over the range of ages at which sickness insurance is normally provided, the graduated rates for lives suffering disability of more than 5 years' duration broadly correspond to a constant addition of 0.02 to the force of mortality under the A1967–70 ultimate mortality table. It is emphasised that no attempt has been made to discover a precise relationship, and the comparison given in Table B3 is intended to be purely illustrative. It does however suggest that an office which calculates disabled lives' reserves for claims in force, by assuming mortality according to a standard assured lives mortality table, may have an implicit margin of strength in its reserving basis.

Table B3. *Graduated values of 10,000* $v_{y+z,z}$ *for durations over 5 years*

Attained exact age $(y+z)$	25	30	35	40	45	50	55	60	65
10,000 $v_{y+z,z}$ (for $z>5$)	154	171	190	213	239	269	303	342	386
10,000 $(\mu_{y+z}+0{\cdot}02)$ (A1967–70 ult.)	207	206	208	214	225	245	280	338	431

6.6 From an experience containing relatively few deaths, very reliable graduated mortality rates cannot be expected, but it is thought that those obtained are acceptable for use in the context of the disability annuity model as a whole.

7. DISABILITY ANNUITIES—CONTINUATION TABLES

7.1 From the graduated recovery and mortality transition intensities, the graduated double decrement continuation tables contained in Table B6, for D1, D4, D13 and D26, for three specimen ages were calculated. In the tables, $l(y,z)$

represents the number of claims remaining in force at exact sickness duration z out of 100,000 claims commencing at the end of the specified deferred period, for lives aged exactly y at the date of falling sick. The decrements by recovery and death in the succeeding interval of duration are designated by $r(y,z)$ and $d(y,z)$ respectively. As the tables are given for illustrative purposes only, they have for convenience been truncated after 10 years (or on attainment of age 65 if sooner).

7.2 Standard actuarial methods for constructing multiple decrement tables were used to calculate the tables in Table B6.

Consider a life aged y at the date of falling sick who has been continuously sick for duration z, and let the independent rates of recovery and death in the ensuing short interval of time h be ${}_hq^r_{y+z,z}$ and ${}_hq^d_{y+z,z}$. The transition intensities at duration $z+h/2$ may be taken as approximations to their respective mean values over the interval from z to $z + h$. Better approximations may be obtained if required by reducing the size of h. Hence, for recoveries,

$$ {}_hq^r{}_{y+z,z} \simeq 1-\exp(-h \cdot \rho_{y+z-h/2,\,z+h/2}) \tag{10} $$

and similarly for deaths.

Independent decremental rates were calculated by (10), and the corresponding dependent rates for the interval from z to $z + h$ were then obtained by the approximations:

$$ \begin{aligned} {}_h(aq)^r &\simeq {}_hq^r \cdot (1 - \tfrac{1}{2} \cdot {}_hq^d) \\ {}_h(aq)^d &\simeq {}_hq^d \cdot (1 - \tfrac{1}{2} \cdot {}_hq^r) \end{aligned} \tag{11} $$

In the calculations, h was set equal to 1/25th of a week for sickness durations up to one year, and equal to 1/100th of a year for longer durations. These values were judged to be sufficiently small to give reasonably accurate results. The decrements shown in the tables for a particular week or year of sickness were the accumulated totals obtained by working step by step through that period, calculating the numbers of recoveries and deaths at each step.

7.3 It should be pointed out that for the tables in Table B6, a week is given its natural meaning of 7 days, with a year comprising 52·18 weeks. In effect, the last week of the year is allotted a length of 1·18 normal weeks. This irregularity would be unmanageable for the more complex numerical procedures described in Parts D and E. Because of the necessity for completely uniform intervals, it is more convenient in those later calculations to work in units related to a nominal week equal to exactly 1/52th of a year. This difference should be borne in mind in any close comparisons between the tables in this Part and those included in Part E.

Tables B4. *Exposed to risk and comparison of actual recoveries with those expected according to the graduated rates*

Table B4(a). *All deferred periods, but excluding the run in periods of D4, D13 and D26*

Age group	20–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	All ages
Sickness period 1–2 weeks									
Exposed to risk (days)	3,899	3,833	3,757	4,212	5,003	6,240	4,905	4,699	36,548
Actual recoveries	495	422	342	309	327	313	215	168	2,591
Expected recoveries	461·0	401·6	348·3	336·1	339·6	347·4	212·5	153·3	2,599·9
Standardised deviation	1·58	1·02	−0·34	−1·48	0·68	−1·85	0·17	1·19	
Sickness period 2–3 weeks									
Exposed to risk (days)	1,820	1,860	2,128	2,535	3,239	4,303	3,626	3,655	23,166
Actual recoveries	157	149	150	168	166	203	127	114	1,234
Expected recoveries	159·6	145·7	148·8	154·5	170·5	189·9	128·8	102·6	1,200·4
Standardised deviation	−0·21	0·27	0·10	1·09	−0·34	0·95	−0·16	1·13	
Sickness period 3–4 weeks									
Exposed to risk (days)	1,038	1,161	1,383	1,716	2,382	3,292	2,935	3,007	16,914
Actual recoveries	57	71	65	73	97	107	83	75	628
Expected recoveries	62·8	64·4	70·5	79·3	99·5	122·7	96·0	86·4	681·4
Standardised deviation	−0·73	0·82	−0·66	−0·71	−0·25	−1·42	−1·33	−1·23	
Sickness period 4–5 weeks									
Exposed to risk (days)	695	762	995	1,259	1,807	2,611	2,339	2,537	13,005
Actual recoveries	37	38	43	41	65	70	77	57	428
Expected recoveries	31·3	32·1	39·4	46·5	62·3	83·5	68·8	69·0	432·8
Standardised deviation	1·02	1·04	0·57	−0·81	0·34	−1·48	0·99	−1·44	
Sickness period 5–6 weeks									
Exposed to risk (days)	504	525	783	1,003	1,429	2,186	1,965	2,219	10,614
Actual recoveries	22	26	19	33	52	51	42	41	286
Expected recoveries	20·0	19·4	27·1	32·2	42·5	59·8	49·0	50·6	300·6
Standardised deviation	0·45	1·50	−1·56	0·14	1·46	−1·14	−1·00	−1·35	
Sickness period 6–7 weeks									
Exposed to risk (days)	389	399	645	811	1,136	1,813	1,667	1,937	8,797
Actual recoveries	12	14	26	22	28	47	30	39	218
Expected recoveries	13·8	13·1	19·7	22·9	29·5	43·0	35·7	37·7	215·4
Standardised deviation	−0·48	0·25	1·42	−0·19	−0·28	0·61	−0·95	0·21	
Sickness period 7–8 weeks									
Exposed to risk (days)	313	300	459	703	964	1,555	1,449	1,688	7,431
Actual recoveries	9	9	19	16	27	35	31	26	172
Expected recoveries	10·0	8·8	12·5	17·6	22·1	32·3	27·0	28·3	158·6
Standardised deviation	−0·32	0·07	1·84	−0·38	1·04	0·48	0·77	−0·43	
Sickness period 8–9 weeks									
Exposed to risk (days)	701	891	1,154	1,406	1,882	2,422	2,114	2,002	12,572
Actual recoveries	16	27	22	31	35	50	38	32	251
Expected recoveries	20·2	23·6	28·3	31·5	38·4	44·5	34·6	29·2	250·3
Standardised deviation	−0·93	0·70	−1·18	−0·09	−0·55	0·82	0·58	0·52	
Sickness period 9–10 weeks									
Exposed to risk (days)	582	715	1,003	1,264	1,615	2,080	1,909	1,809	10,977
Actual recoveries	18	23	12	19	34	36	23	19	184
Expected recoveries	15·3	17·2	22·2	25·5	29·5	34·1	27·6	23·1	194·4
Standardised deviation	0·69	1·40	−2·16	−1·29	0·83	0·33	−0·88	−0·85	
Sickness period 10–11 weeks									
Exposed to risk (days)	505	599	881	1,078	1,435	1,885	1,734	1,676	9,793
Actual recoveries	11	13	18	28	12	27	22	17	148
Expected recoveries	12·1	13·1	17·7	19·7	23·6	27·7	22·3	18·9	155·0
Standardised deviation	−0·32	−0·03	0·07	1·87	−2·39	−0·13	−0·06	−0·44	
Sickness period 11–13 weeks									
Exposed to risk (days)	839	951	1,494	1,801	2,536	3,186	3,052	2,968	16,827
Actual recoveries	12	21	21	30	35	48	36	33	236
Expected recoveries	17·6	19·1	26·3	28·6	36·2	40·2	33·4	28·1	228·8
Standardised deviation	−1·33	0·43	−1·03	0·26	−0·20	1·23	0·45	0·92	

Table B4(a) *(Continued)*

Age group	20–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	All ages
Sickness period 13–15 weeks									
Exposed to risk (days)	672	739	1,234	1,389	2,116	2,531	2,633	2,585	13,899
Actual recoveries	11	5	12	16	26	39	21	22	152
Expected recoveries	12·0	12·1	18·3	18·5	25·1	26·3	23·4	19·6	155·9
Standardised deviation	−0·29	−2·02	−1·47	−0·58	0·18	2·48	−0·50	0·54	
Sickness period 15–18 weeks									
Exposed to risk (days)	915	1,181	1,820	2,165	3,025	3,570	3,873	3,571	20,120
Actual recoveries	11	12	22	19	41	36	23	24	188
Expected recoveries	13·6	15·7	22·1	23·2	28·8	29·4	26·9	20·8	180·5
Standardised deviation	−0·71	−0·93	−0·02	−0·87	2·27	1·22	−0·75	0·70	
Sickness period 18–22 weeks									
Exposed to risk (days)	1,310	1,790	2,643	3,319	4,480	5,056	5,591	5,033	29,222
Actual recoveries	12	12	23	28	33	43	26	14	191
Expected recoveries	15·4	19·0	25·3	28·0	32·9	31·7	29·0	21·3	202·7
Standardised deviation	−0·87	−1·61	−0·46	0·00	0·02	2·01	−0·56	−1·58	
Sickness period 22–26 weeks									
Exposed to risk (days)	941	1,395	2,089	2,590	3,894	4,151	4,749	4,653	24,462
Actual recoveries	14	12	18	17	18	28	15	12	134
Expected recoveries	8·7	11·5	15·4	16·6	21·5	19·3	17·9	13·9	124·7
Standardised deviation	1·80	0·15	0·66	0·10	−0·75	1·98	−0·69	−0·59	
Sickness period 26–30 weeks									
Exposed to risk (days)	728	1,106	1,750	2,104	3,351	3,525	4,164	4,233	20,961
Actual recoveries	4	11	11	13	19	14	15	14	101
Expected recoveries	5·2	7·2	10·1	10·4	14·2	12·4	11·6	9·0	80·6
Standardised deviation	−0·53	1·42	0·28	0·81	1·27	0·45	1·00	1·67	
Sickness period 30–39 weeks									
Exposed to risk (days)	2,062	2,594	4,023	5,965	8,286	10,341	11,843	10,934	56,048
Actual recoveries	14	15	18	20	25	21	17	12	142
Expected recoveries	10·6	11·8	16·1	20·4	23·9	24·2	21·2	14·4	142·6
Standardised deviation	1·04	0·93	0·47	0·09	0·23	0·65	0·91	0·63	
Sickness period 39–52 weeks									
Exposed to risk (days)	2,230	2,814	4,630	7,763	8,987	12,872	16,098	13,969	69,363
Actual recoveries	3	9	17	10	17	15	9	7	87
Expected recoveries	6·8	7·5	10·7	15·1	14·5	16·4	14·8	8·7	94·5
Standardised deviation	−1·46	0·55	1·93	−1·31	0·66	−0·35	−1·51	−0·58	
Sickness period 1–2 years									
Exposed to risk (days)	5,012	6,518	11,016	21,781	25,294	38,879	58,823	40,671	207,994
Actual recoveries	16	13	24	18	28	25	12	15	151
Expected recoveries	7·9	8·8	12·8	21·0	19·7	23·0	23·5	9·5	126·2
Standardised deviation	2·88	1·42	3·13	−0·65	1·87	0·42	−2·37	1·78	

Age group	20–34	35–44	45–49	50–54	55–59	60–64	All ages
Sickness period over 2 years							
Exposed to risk (days)	19,117	25,051	65,216	69,165	120,962	178,390	477,901
Actual recoveries	8	3	7	12	8	11	49
Expected recoveries	7·6	6·4	13·6	10·7	13·8	13·5	65·6
Standardised deviation	0·15	−1·34	−1·79	0·40	−1·56	−0·68	

Table B4(b). *Run in period of D4 (deferred 4 weeks)*

Age group	20–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	All ages
Sickness period 4–5 weeks									
Exposed to risk (days)	720	1,039	1,323	1,440	1,709	1,606	1,261	772	9,870
Actual recoveries	6	20	13	11	20	15	13	6	104
Expected recoveries	9·9	13·3	16·0	16·2	17·9	15·6	11·3	6·4	106·6
Standardised deviation	−1·24	1·84	−0·75	−1·29	0·50	−0·15	0·51	−0·16	
Sickness period 5–6 weeks									
Exposed to risk (days)	666	912	1,207	1,332	1,590	1,506	1,194	718	9,125
Actual recoveries	15	21	23	22	15	17	14	9	136
Expected recoveries	13·3	17·0	21·0	21·5	23·8	20·7	15·0	8·2	140·6
Standardised deviation	0·47	0·97	0·44	0·11	−1·80	−0·81	−0·26	0·28	
Sickness period 6–7 weeks									
Exposed to risk (days)	566	801	1,030	1,122	1,451	1,378	1,070	665	8,083
Actual recoveries	13	13	25	33	26	22	12	6	150
Expected recoveries	14·1	18·5	22·1	22·2	26·5	22·9	16·1	9·1	151·5
Standardised deviation	−0·29	−1·28	0·62	2·29	−0·10	−0·19	−1·02	−1·03	
Sickness period 7–8 weeks									
Exposed to risk (days)	483	723	900	990	1,264	1,224	971	587	7,142
Actual recoveries	12	10	21	20	25	24	15	14	141
Expected recoveries	13·8	19·2	22·1	22·3	26·1	22·9	16·3	8·9	151·7
Standardised deviation	−0·48	−2·10	−0·23	−0·49	−0·22	0·23	−0·32	1·71	

Table B4(c). *Run in period of D13 (deferred 13 weeks)*

Age group	20–34	35–44	45–54	55–64	All ages
Sickness period 13–15 weeks					
Exposed to risk (days)	840	1,906	2,347	1,654	6,747
Actual recoveries	6	12	8	4	30
Expected recoveries	5·7	10·6	10·4	5·5	32·2
Standardised deviation	0·13	0·43	−0·74	−0·64	
Sickness period 15–17 weeks					
Exposed to risk (days)	758	1,738	2,235	1,514	6,245
Actual recoveries	7	20	18	8	53
Expected recoveries	8·8	16·4	16·6	8·3	50·0
Standardised deviation	−0·61	0·89	0·34	−0·10	

Table B4(d). *Run in period of D26 (deferred 26 weeks)*

Age group	20–34	35–44	45–54	55–64	All ages
Sickness period 26–30 weeks					
Exposed to risk (days)	979	1,493	2,487	2,867	7,826
Actual recoveries	4	6	6	7	23
Expected recoveries	4·0	4·5	5·6	4·2	18·1
Standardised deviation	0·00	0·71	0·17	1·37	

Table B4(e). *Totals of tables B4(a)–B4(d)*

Age group	20–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	All ages
All sickness periods									
Exposed to risk (days)	36,540	46,352	75,394	138,105	161,142	243,141	279,522	161,456	1,141,652
Actual recoveries	987	981	982	1,027	1,200	1,309	935	787	8,208
Expected recoveries	965	935	993	1,061	1,196	1,320	985	786	8,241

Table B5. *Exposed to risk and comparison of actual deaths with those expected according to the graduated rates, for all deferred periods combined*

Age group	20–39	40–49	50–54	55–59	60–64	All ages
Sickness period 1–13 weeks						
Exposed to risk (years)	132·3	142·7	102·1	88·1	84·7	549·9
Actual deaths	6	11	12	14	14	57
Expected deaths	10·0	13·7	11·6	11·3	11·9	58·5
Standardised deviation	−1·26	−0·73	0·12	0·80	0·61	
Sickness period 13–26 weeks						
Exposed to risk (years)	54·5	74·8	48·2	51·4	46·7	275·7
Actual deaths	6	11	6	10	8	41
Expected deaths	6·1	10·0	7·5	8·8	9·2	41·5
Standardised deviation	−0·04	0·32	−0·55	0·40	−0·40	
Sickness period 26 weeks–1 year						
Exposed to risk (years)	63·9	105·0	77·8	92·3	83·2	422·2
Actual deaths	4	19	9	9	10	51
Expected deaths	5·0	11·2	9·3	12·3	12·3	50·1
Standardised deviation	−0·45	2·33	−0·10	−0·94	−0·66	

Age group	20–49	50–54	55–59	60–64	All ages
Sickness period 1–2 years					
Exposed to risk (years)	190·6	106·4	161·0	111·4	569·5
Actual deaths	12	9	11	7	39
Expected deaths	9·6	6·6	11·2	8·6	35·9
Standardised deviation	0·77	0·93	−0·06	−0·55	
Sickness period over 2 years					
Exposed to risk (years)	488·8	331·2	372·4	116·0	1308·4
Actual deaths	14	9	16	5	44
Expected deaths	13·7	11·7	15·1	5·5	46·0
Standardised deviation	0·08	−0·79	0·23	−0·21	

Age group	20–39	40–49	50–54	55–59	60–64	All ages
All sickness periods						
Exposed to risk (years)	433·4	819·3	665·7	765·3	442·0	3125·7
Actual deaths	24	59	45	60	44	232
Expected deaths	27·6	52·5	46·4	58·5	47·0	232·1

Tables B6. *Graduated double decrement tables of claim terminations*

Table B6(a). *Deferred period 1 week*

Sickness duration	Age x exact at date of falling sick: 20			40			60		
Weeks	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$
1	100,000	61,016	59	100,000	45,085	85	100,000	22,643	154
2	38,925	19,500	29	54,830	20,037	57	77,203	14,910	138
3	19,396	7,229	18	34,736	9,973	42	62,156	11,797	124
4	12,148	3,487	14	24,720	5,792	35	50,235	8,923	110
5	8,646	2,242	11	18,893	3,918	29	41,202	6,253	99
6	6,393·2	1,507·4	8·8	14,945	2,767	25	34,849	4,571	91
7	4,876·9	1,050·9	7·3	12,153	2,023	22	30,187	3,454	84
8	3,818·8	755·2	6·0	10,108	1,522	19	26,648	2,680	79
9	3,057·6	556·9	5·1	8,567	1,172	17	23,890	2,125	74
10	2,495·5	420·0	4·3	7,378	921	15	21,692	1,715	70
11	2,071·2	322·9	3·7	6,441	736	14	19,907	1,406	66
12	1,744·6	252·6	3·2	5,691	598	13	18,435	1,168	62
13	1,488·7	200·6	2·8	5,081	492	11	17,204	982	59
14	1,285·3	161·5	2·5	4,578	409	10	16,163	833	56
15	1,121·2	131·7	2·2	4,158·2	344·2	9·6	15,273	714	54
16	987·4	108·5	1·9	3,804·5	292·2	8·8	14,505	616	51
17	877·0	90·3	1·7	3,503·5	250·2	8·1	13,838	535	49
18	784·9	75·9	1·5	3,245·3	215·8	7·5	13,254	468	47
19	707·5	64·3	1·4	3,021·9	187·5	7·0	12,739	412	45
20	641·8	54·9	1·3	2,827·4	163·9	6·5	12,282	364	43
21	585·7	47·2	1·1	2,657·0	144·2	6·1	11,875	323	41
22	537·4	40·8	1·0	2,506·7	127·5	5·7	11,510	288	40
23	495·5	35·5	0·9	2,373·6	113·2	5·3	11,182	258	38
24	459·0	31·1	0·9	2,255·0	101·1	5·0	10,886	232	37
25	427·1	27·4	0·8	2,148·9	90·6	4·7	10,617	209	35
26	398·9	24·2	0·7	2,053·6	81·5	4·4	10,372	189	34
27	374·0	21·5	0·7	1,967·6	73·6	4·2	10,149	171	33
28	351·8	19·2	0·6	1,889·8	66·7	4·0	9,945	156	32
29	332·0	17·2	0·6	1,819·2	60·6	3·8	9,757	142	31
30	314·2	15·4	0·5	1,754·8	55·3	3·6	9,584	130	30
31	298·2	13·9	0·5	1,695·9	50·5	3·4	9,425	119	29
32	283·8	12·6	0·5	1,642·0	46·3	3·2	9,277	109	28
33	270·7	11·4	0·4	1,592·4	42·6	3·1	9,141	100	27
34	258·9	10·4	0·4	1,546·8	39·2	2·9	9,014	92	26
35	248·0	9·5	0·4	1,504·6	36·2	2·8	8,895	85	25
36	238·1	8·7	0·4	1,465·7	33·5	2·7	8,785	78	24
37	229·1	8·0	0·3	1,429·5	31·0	2·6	8,682	73	24
38	220·7	7·3	0·3	1,395·9	28·8	2·5	8,586	67	23
39	213·1	6·8	0·3	1,364·7	26·8	2·4	8,496	62	22
40	206·0	6·2	0·3	1,335·5	24·9	2·3	8,411	58	22
41	199·5	5·8	0·3	1,308·3	23·3	2·2	8,332	54	21
42	193·4	5·4	0·3	1,282·9	21·7	2·1	8,257	50	21
43	187·8	5·0	0·3	1,259·0	20·3	2·0	8,186	47	20
44	182·5	4·6	0·2	1,236·7	19·1	1·9	8,119	43	19
45	177·7	4·3	0·2	1,215·7	17·9	1·9	8,056	41	19
46	173·1	4·0	0·2	1,195·9	16·8	1·8	7,997	38	18
47	168·9	3·8	0·2	1,177·3	15·8	1·8	7,940	35	18
48	164·9	3·5	0·2	1,159·7	14·9	1·7	7,887	33	18
49	161·2	3·3	0·2	1,143·2	14·0	1·6	7,836	31	17
50	157·7	3·1	0·2	1,127·5	13·2	1·6	7,788	29	17
51	154·4	2·9	0·2	1,112·7	12·5	1·5	7,742	27	16

Table B6a *(Continued)*

Sickness duration	20			40			60		
Weeks	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$
Years									
1	151·3	73·7	4·3	1,098·7	352·3	44·7	7,698	777	544
2	73·3	21·1	1·4	701·6	121·1	19·8	6,377	250	303
3	50·8	8·6	0·8	560·8	53·5	13·8	5,823	93	237
4	41·4	4·2	0·6	493·6	26·9	11·5	5,494	36	210
5	36·6	2·2	0·5	455·2	14·7	10·7	5,248		
6	33·8	1·3	0·5	429·8	8·4	10·4			
7	32·0	0·8	0·5	410·9	5·0	10·3			
8	30·7	0·5	0·5	395·6	3·1	10·2			
9	29·8	0·3	0·5	382·3	2·0	10·1			
10	29·0			370·2					

Table B6(b). *Deferred period 4 weeks*

Age x exact at date of falling sick

Sickness duration	20			40			60		
Weeks	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$
4	100,000	9,727	128	100,000	7,748	154	100,000	5,725	236
5	90,145	12,595	123	92,097	10,123	151	94,039	7,436	237
6	77,427	13,288	112	81,823	10,925	142	86,366	8,089	231
7	64,028	12,550	97	70,755	10,682	129	78,045	8,070	220
8	51,381	10,161	81	59,944	9,025	114	69,755	7,014	206
9	41,139	7,493	69	50,805	6,950	102	62,535	5,561	193
10	33,577	5,651	58	43,753	5,462	91	56,781	4,490	182
11	27,868	4,345	50	38,200	4,367	82	52,109	3,681	172
12	23,473	3,399	43	33,752	3,545	74	48,255	3,059	163
13	20,030	2,700	38	30,133	2,916	68	45,033	2,570	155
14	17,293	2,174	33	27,149	2,427	62	42,308	2,182	148
15	15,086	1,772	29	24,661	2,041	57	39,979	1,868	141
16	13,285	1,460	26	22,563	1,733	52	37,970	1,612	134
17	11,799	1,215	23	20,778	1,484	48	36,223	1,401	128
18	10,561	1,021	21	19,246	1,280	45	34,693	1,225	123
19	9,519	865	19	17,922	1,112	41	33,345	1,078	118
20	8,636	738	17	16,768	972	39	32,149	953	113
21	7,880	635	15	15,757	855	36	31,084	846	108
22	7,230	549	14	14,866	756	34	30,129	755	104
23	6,667	478	13	14,077	672	32	29,270	676	100
24	6,176	419	12	13,373	599	30	28,494	607	96
25	5,746	368	11	12,744	537	28	27,791	547	93
26	5,367·1	325·7	9·8	12,179	483	26	27,151	495	89
27	5,031·6	289·3	9·1	11,669	437	25	26,566	449	86
28	4,733·2	258·1	8·4	11,208	396	24	26,031	408	83
29	4,466·7	231·1	7·8	10,789	360	22	25,540	372	80
30	4,227·8	207·7	7·3	10,407	328	21	25,088	340	78
31	4,012·7	187·4	6·8	10,058	300	20	24,671	311	75
32	3,818·6	169·6	6·3	9,738	275	19	24,284	285	73
33	3,642·7	153·9	5·9	9,444	252	18	23,927	262	70
34	3,482·8	140·1	5·6	9,173	232	17	23,594	241	68
35	3,337·1	127·9	5·2	8,923	215	17	23,285	223	66
36	3,204·0	117·1	4·9	8,692	198	16	22,996	205	64
37	3,082·0	107·4	4·7	8,478	184	15	22,727	190	62
38	2,969·9	98·8	4·4	8,279	171	15	22,475	176	60
39	2,866·8	91·0	4·2	8,093	159	14	22,239	163	58

Table B6(b) *(Continued)*

Sickness duration	Age x exact at date of falling sick 20			40			60		
Weeks	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$
40	2,771·6	84·1	3·9	7,921	148	13	22,017	152	57
41	2,683·6	77·8	3·7	7,759	138	13	21,809	141	55
42	2,602·1	72·1	3·6	7,608	129	12	21,613	131	54
43	2,526·4	67·0	3·4	7,467	121	12	21,428	122	52
44	2,456·0	62·3	3·2	7,334	113	12	21,253	114	51
45	2,390·5	58·1	3·1	7,210	106	11	21,089	106	50
46	2,329·3	54·2	2·9	7,092	100	11	20,933	99	48
47	2,272·2	50·7	2·8	6,982	94	10	20,785	93	47
48	2,218·7	47·4	2·7	6,878	88	10	20,645	87	46
49	2,168·7	44·4	2·6	6,779·6	83·1	9·7	20,512	81	45
50	2,121·6	41·7	2·5	6,686·7	78·4	9·4	20,386	76	44
51	2,077·5	39·2	2·4	6,598·9	74·1	9·2	20,266	72	43
Years									
1	2,035·9	991·9	58·2	6,515·6	2,089·5	265·0	20,151	2,034	1,425
2	985·9	284·0	18·6	4,161·1	718·2	117·2	16,692	656	793
3	683·3	115·4	10·9	3,325·8	317·1	81·6	15,243	243	620
4	557·0	55·9	8·3	2,927·2	159·5	68·3	14,381	94	551
5	492·8	30·1	7·4	2,699·4	86·9	63·5	13,736		
6	455·3	17·3	7·1	2,549·0	49·9	61·9			
7	431·0	10·4	6·9	2,437·1	29·8	61·0			
8	413·7	6·5	6·8	2,346·3	18·4	60·4			
9	400·4	4·2	6·8	2,267·5	11·6	59·9			
10	389·4			2,196·0					

Table B6(c). *Deferred period 13 weeks*

Age x exact at date of falling sick

Sickness duration	20			40			60		
Weeks	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$
13	100,000	4,294	199	100,000	3,037	233	100,000	1,763	352
14	95,507	6,227	190	96,730	4,440	226	97,885	2,565	346
15	89,090	7,466	177	92,064	5,407	215	94,974	3,130	337
16	81,446	8,097	161	86,443	5,994	201	91,507	3,501	325
17	73,189	7,538	144	80,248	5,730	186	87,682	3,392	311
18	65,507	6,333	129	74,332	4,943	172	83,979	2,966	298
19	59,045	5,365	116	69,216	4,295	160	80,715	2,609	285
20	53,565	4,580	105	64,761	3,755	149	77,821	2,306	273
21	48,881	3,937	95	60,857	3,302	139	75,241	2,048	262
22	44,849	3,407	86	57,416	2,919	130	72,931	1,827	252
23	41,356	2,966	79	54,366	2,594	122	70,852	1,636	242
24	38,311	2,596	72	51,650	2,315	115	68,974	1,470	233
25	35,643	2,285	66	49,220	2,075	108	67,271	1,325	224
26	33,291	2,020	61	47,037	1,867	102	65,721	1,198	216
27	31,210	1,795	56	45,068	1,686	96	64,307	1,087	209
28	29,359	1,601	52	43,286	1,528	91	63,012	988	201
29	27,706	1,434	48	41,667	1,389	86	61,823	900	194
30	26,224	1,289	45	40,192	1,266	82	60,728	822	188
31	24,890	1,162	42	38,844	1,158	78	59,718	753	182
32	23,686	1,052	39	37,609	1,061	74	58,783	691	176
33	22,595	955	37	36,474	975	70	57,917	635	170
34	21,603	869	35	35,429	898	67	57,113	584	165
35	20,700	793	32	34,463	829	64	56,364	539	160
36	19,874	726	31	33,570	767	61	55,666	497	155
37	19,117	666	29	32,743	710	59	55,013	460	150
38	18,422	613	27	31,973	659	56	54,403	426	146
39	17,782	565	26	31,258	613	54	53,832	395	142
40	17,192	521	24	30,590	571	52	53,295	367	138
41	16,646	483	23	29,967	533	50	52,791	341	134
42	16,140	447	22	29,384	498	48	52,316	317	130
43	15,671	415	21	28,838	466	46	51,868	296	127
44	15,234	387	20	28,326	437	45	51,446	276	123
45	14,828	360	19	27,845	410	43	51,047	257	120
46	14,448	336	18	27,392	385	42	50,670	240	117
47	14,094	314	17	26,966	362	40	50,312	225	114
48	13,763	294	17	26,563	341	39	49,973	210	111
49	13,452	276	16	26,184	321	38	49,652	197	109
50	13,160	259	15	25,825	303	36	49,346	185	106
51	12,886	243	15	25,486	286	35	49,055	173	104
Years									
1	12,628	6,152	361	25,164	8,070	1,023	48,778	4,924	3,449
2	6,115	1,762	115	16,071	2,774	452	40,405	1,587	1,919
3	4,238	716	67	12,845	1,225	315	36,898	587	1,500
4	3,455	347	51	11,305	616	264	34,811	228	1,334
5	3,057	187	46	10,425	336	245	33,249		
6	2,824	107	44	9,845	193	239			
7	2,673	65	43	9,412	115	236			
8	2,566	40	42	9,062	71	233			
9	2,483	26	42	8,758	45	231			
10	2,415			8,482					

Tables B6 *(Continued)*

Table B6(d). *Deferred period 26 weeks*

Age x exact at date of falling sick

Sickness duration	20			40			60		
Weeks	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$	$l(y,z)$	$r(y,z)$	$d(y,z)$
26	100,000	1,882	188	100,000	1,221	220	100,000	556	331
27	97,930	2,870	180	98,559	1,869	212	99,112	844	323
28	94,880	3,657	170	96,477	2,400	204	97,945	1,078	314
29	91,053	4,250	160	93,874	2,820	195	96,553	1,266	304
30	86,643	4,257	149	90,860	2,862	185	94,984	1,286	294
31	82,237	3,840	139	87,812	2,617	176	93,404	1,178	284
32	78,258	3,475	130	85,020	2,399	167	91,943	1,080	275
33	74,653	3,154	122	82,454	2,204	159	90,588	993	266
34	71,377	2,872	114	80,091	2,030	152	89,329	914	258
35	68,391	2,621	107	77,909	1,874	145	88,158	842	250
36	65,662	2,399	101	75,890	1,733	139	87,066	778	242
37	63,162	2,201	95	74,019	1,606	133	86,046	719	235
38	60,865	2,024	90	72,280	1,491	127	85,092	666	228
39	58,751	1,865	85	70,662	1,386	122	84,197	618	221
40	56,801	1,723	81	69,153	1,291	117	83,358	574	215
41	54,997	1,594	77	67,744	1,205	113	82,569	533	209
42	53,326	1,478	73	66,426	1,126	109	81,827	496	204
43	51,775	1,373	69	65,192	1,053	105	81,127	462	198
44	50,334	1,277	66	64,034	987	101	80,467	431	193
45	48,990	1,190	63	62,946	926	97	79,843	402	188
46	47,737	1,111	60	61,923	870	94	79,252	376	183
47	46,567	1,038	58	60,959	818	91	78,693	351	179
48	45,471	972	55	60,050	770	88	78,163	329	174
49	44,444	911	53	59,192	726	85	77,660	308	170
50	43,481	855	51	58,381	685	82	77,182	289	166
51	42,575	803	49	57,614	647	80	76,727	271	162
Years									
1	41,724	20,327	1,192	56,887	18,243	2,313	76,294	7,702	5,395
2	20,204	5,820	381	36,330	6,270	1,023	63,197	2,482	3,002
3	14,003	2,364	223	29,037	2,768	712	57,712	918	2,346
4	11,416	1,147	170	25,557	1,392	597	54,448	356	2,086
5	10,100	617	151	23,568	759	554	52,006		
6	9,332	355	145	22,255	436	541			
7	8,832	214	141	21,278	260	533			
8	8,477	133	140	20,485	160	527			
9	8,205	85	139	19,797	101	523			
10	7,981			19,173					

CMIR **12** (1991) 51–78

PART C: THE GRADUATION OF SICKNESS INCEPTION INTENSITIES

BY H. R. WATERS

SUMMARY

In this Part we discuss the graduation of the sickness inception intensity σ_x for D1, D4, D13 and D26. The final graduations of σ_x are given in §9 and are different for each of the deferred periods; in general, the longer the deferred period, the lower the value of σ_x.

Throughout this Part we assume the intensity of recovery for each of the four deferred periods is the graduated value of $\rho_{x,z}$ for D1 given in Part B. In §1 we describe the statistical model used to estimate and graduate σ_x; this has already been described in §§3 and 4 of Part A. In §2 we describe the data we have used, most of which can be found in *C.M.I.R.* 7 (1984). In §§3 and 4 we describe the way in which we have calculated the number of claim inceptions and the exposed to risk. In §§5 and 6 we describe the method of graduation and the results of the preliminary graduations. For D1, D13 and D26 these preliminary graduations proved to be satisfactory and hence were regarded as final graduations. However, further investigations of the preliminary graduation of σ_x for D4, detailed in §7, showed it to be unsatisfactory and a re-graduation, detailed in §8, was carried out in this case.

1. THE STATISTICAL MODEL

1.1 A feature of the model described in Part A is that it requires the specification, and hence estimation and graduation, of a sickness inception intensity, denoted by σ_x. Intuitively, $\sigma_x \cdot dt$ is the probability that a healthy individual aged x falls sick within a small interval of time dt. (Note that for this sickness to result in a claim it must last for at least the deferred period of the policy, which it may or may not do. This is the essential difference between a sickness inception intensity and a claim inception intensity or rate.)

The model for PHI assumes that σ_x is a continuously varying function of the individual's age, x, but for the purposes of estimation it is assumed that σ_x is piece-wise constant, i.e. that σ_x is constant for a range of values of x, usually of length 1 year. The basic statistical model for the estimation of σ_x has been described in Part A §§3 and 4. We give a brief summary below.

1.2 For a given deferred period, d, a given observation period and an age interval x_1 to x_2 over which σ_x may be assumed to be reasonably constant, let

$I(x_1,x_2)$ denote the number of sicknesses which start in the observation period, with age at start of sickness between x_1 and x_2, and which last beyond the deferred period of the policy,

$E(x_1,x_2)$ denote the total time spent in the observation period by individuals who are healthy and aged between x_1 and x_2,

$\pi_{x,d}$ denote the probability that an individual who falls sick at age x, will remain sick for at least the deferred period, d, of the policy; it is assumed that $\pi_{x,d}$ is reasonably constant over the age interval x_1 to x_2. ($\pi_{x,d}$ is equivalent to ${}_dp_x^{\overline{SS}}$ in the notation of Part A),

V denote the factor by which the variance of the estimator for σ is inflated by the presence of duplicate policies in the data.

For conciseness we shall denote $I(x_1,x_2)$ and $E(x_1,x_2)$ by I and E respectively. Let σ and π denote the assumed constant values of σ_x and $\pi_{x,d}$ over the age interval x_1 to x_2.

Then it is assumed that

$$I \sim N(\sigma \cdot \pi \cdot E, \sigma \cdot \pi \cdot E \cdot V) \tag{1}$$

which leads to

$$I/(\pi \cdot E) \sim N(\sigma, \sigma \cdot V/(\pi \cdot E)) \tag{2}$$

An assumption underlying (1) and (2), in addition to the assumption of piecewise constancy of both σ and π, is that the expected value of I is sufficiently large for the normal distribution to be a reasonable approximation to the Poisson distribution. In practice we have taken this to mean that the expected value of I should be at least 10 for (1) and (2) to hold.

1.3 It should be noted that throughout this Part a unit of a 'week', when used in connection with a deferred period, is always assumed to be 1/52 of a year, and not an exact seven days.

2. THE AVAILABLE DATA

2.1 Formula (2) was used to produce graduated values of σ_x for D1, D4, D13 and D26 using the individual PHI data, males, 1975–78, Standard experience. Most of the data relevant to the graduation of σ_x can be found in *C.M.I.R.* **7** (1984). We give below a brief summary of the relevant available data.

(i) For each of the four deferred periods, *C.M.I.R.* **7**, Table K13, gives the number of observed claim inceptions in 1975–78 by single ages. For all deferred periods claim inceptions are classified by age nearest birthday at the 1st January immediately preceding the date when claim payments commenced (which is broadly equivalent to age last birthday when claim payments commenced). For D1, which has some unique features, it was assumed that claim payments commenced at the start of the sickness rather than at the end of the deferred period. For all other deferred periods it was assumed that claim payments commenced at the end of the relevant deferred period.

(ii) For each of the four deferred periods and for sickness durations 1–4 weeks, 4–13 weeks, 13–26 weeks, 26–52 weeks, 52–104 weeks and over 104 weeks, *C.M.I.R.* 7, Tables K1–K4, gives exposures for 1975–78 by single ages. (For each deferred period an exposure is given only for those sickness periods extending beyond the deferred period.) The exposure for sickness period 1–4 weeks is the total time spent during the years 1975–78 as healthy, or sick and claiming, or sick but not (yet) claiming by individuals aged x nearest birthday at the preceding 1st January (which is broadly equivalent to being aged x last birthday). For other sickness periods an adjustment has been made to this exposure so that new entrants do not contribute to an exposure until they have had their policies for at least the lower limit of the sickness period. The adjustment is a deduction from the exposure for sickness period 1–4 weeks. For all but the final sickness period this deduction can be expressed as:

$$f_1 \cdot \alpha_x + f_2 \cdot \beta_x$$

where α_x is the sum over each of the four years of the number of policies at the start of the year for which the policyholders were either aged x nearest birthday and entered in the previous calendar year or aged $(x+1)$ nearest birthday and entered between 1 and 2 years ago,

β_x is the sum over each of the four years of the number of policies at the start of each year for which the policyholders were aged $(x+1)$ nearest birthday and who entered in the previous year,

f_1 and f_2 are factors depending on the sickness period and are given below:

Sickness period	f_1	f_2
4–9 weeks	0·0034	0·0899
13–26 weeks	0·0312	0·2188
26–52 weeks	0·1250	0·3750
52–104 weeks	0·5000	0·5000

(iii) For each deferred period, *C.M.I.R.* 7, Tables K1–K4, gives for single ages the number of weeks in the observation period spent claiming at age x nearest birthday at the preceding 1st January.

(iv) *C.M.I.R.* 7, Appendix F, summarises the results of an investigation into the extent of duplicates among claims in the 1975–78 (Aggregate experience) data. In particular, it gives for each deferred period an estimate of the average number of policies per life and also an estimate of V, the factor by which the variance of the estimator for σ_x is increased due to the presence of duplicate policies. The estimate of the average number of policies per life for D4 given in *C.M.I.R.* 7 is 1·128 and this, together with the implied variance inflation factor, was the estimate used for the preliminary graduation of σ_x. Further investigations, see §7, indicated the possibility that this estimate should have been much higher and a figure of 1.34, together with the implied variance inflation factor, was used for the final graduation of σ_x for D4. See §8.

(v) As will be seen later in this section, it was necessary to estimate the rate of growth of the number of policies in force for all deferred periods, except D1, preferably split by age. No precise data was available for this and we were able to estimate only a rate of growth for each of these deferred periods for all ages combined. The estimated annual rates of growth are as follows:

Deferred period	Rate of growth p.a. (%)
4 weeks	6·2
13 weeks	6·8
26 weeks	3·4

(vi) For each of the deferred periods greater than 1 week we had available in 5-year age groups the number of recoveries, deaths and expiries in each of the four weeks following the end of the deferred period. This data was drawn from a data file from which duplicate policies could be, and had been, virtually eliminated. This is in contrast to items (i), (ii), (iii) and (iv) above, which were drawn from data files from which duplicate policies could not be, and had not been, eliminated.

3. THE NUMBER OF CLAIM INCEPTIONS

3.1 For D1, the number of claim inceptions, denoted I in (1), at age x last birthday was taken to be the figure given in *C.M.I.R.* **7**, Table K13, without any adjustment. For the other deferred periods the situation was not so straightforward.

3.2 The estimation and graduation of σ_x requires the value of $\pi_{x,d}$, the probability that a sickness starting at age x lasts for at least the deferred period, d, to be known. The value of $\pi_{x,d}$ can be calculated from formula (27) in Part A. It can be seen from this formula that to calculate $\pi_{x,d}$ we need to know the values of the intensities of recovery, ρ, and mortality from sick, ν, during the deferred period. The graduation of the recovery intensity revealed the following general features:

(i) For D1, ρ was, broadly speaking, a decreasing function of both age and duration of sickness.

(ii) For deferred periods greater than 1 week, ρ increased in each of the four weeks immediately following the end of the deferred period and was then a decreasing function of both age and duration of sickness not significantly different from the experience for D1.

Feature (i) above is very much in line with our expectations for a recovery rate, but (ii) is difficult to explain in physiological terms. (A different level for ρ for different deferred periods could be due to different classes of life and different underwriting standards between offices, but these factors would not explain a different shape for ρ.) The PHI Sub-Committee took the view that the low observed values for ρ immediately following the end of the deferred period were due to the non-reporting of some sicknesses which lasted beyond the deferred period but ended (with recovery) within the following four weeks. (See §3.3 of Part B where it is noted that a similar feature was observed in Swedish data.) It is assumed throughout this Part that the intensity of recovery, i.e. the values of ρ, for all deferred periods is given by the graduated values for D1 given in Part B. Hence, for deferred periods greater than 1 week the value of I to be used in (2) can be written:

$$I = IR + IN$$

where:

IR (= Inceptions Reported) is the number of claim inceptions given in *C.M.I.R.* 7, Table K13.

IN (= Inceptions Not Reported) is the number of recoveries within four weeks after the end of the deferred period which were not reported as claims or recoveries.

3.3 The value of IN at each age was calculated, or rather estimated, as follows. For a given deferred period (greater than 1 week) and one of the five-year age groups 20–24, 25–29, . . . , 60–64, let E_i and R_i, $i = 1, 2, 3, 4$, denote the reported exposure (in weeks) and recoveries in each of the four weeks following the end of the deferred period. Let XR_i denote the number of non-reported recoveries in each of these four weeks. The observed recovery intensity is

$$\hat{\rho}_i = R_i/E_i \qquad i = 1, 2, 3, 4$$

but should have been, if the extra recoveries had been reported,

$$\hat{\rho}_4^* = (R_4 + XR_4)/(E_4 + XR_4/2)$$

$$\hat{\rho}_3^* = (R_3 + XR_3)/(E_3 + XR_4 + XR_3/2)$$

$$\hat{\rho}_2^* = (R_2 + XR_2)/(E_2 + XR_4 + XR_3 + XR_2/2)$$

$$\hat{\rho}_1^* = (R_1 + XR_1)/(E_1 + XR_4 + XR_3 + XR_2 + XR_1/2)$$

assuming each of the extra recoveries recovered on average half way through the relevant week. The XR_i's can now be calculated by assuming the ratio $\hat{\rho}_i/\hat{\rho}_i^*$ is the same as the ratio of the graduated recovery intensities for the particular deferred period and for deferred period 1 week at the mid-point of the relevant week, i.e. by assuming:

$$\hat{\rho}_4/\hat{\rho}_4^* = 0{\cdot}9006$$

$$\hat{\rho}_3/\hat{\rho}_3^* = 0{\cdot}7019$$

$$\hat{\rho}_2/\hat{\rho}_2^* = 0{\cdot}5032$$

$$\hat{\rho}_1/\hat{\rho}_1^* = 0{\cdot}3045$$

(See Part B.) (Note that the above ratios are independent of age.) The value of *IN* at a single age was calculated by apportioning the extra recoveries, $XR_1 + XR_2 + XR_3 + XR_4$, for the relevant five year age group to individual ages in proportion to the exposure at these ages and then multiplying by the average number of policies per life for the relevant deferred period. (This last adjustment was included to make *IN* consistent with *IR*, from which duplicates had not been excluded.)

3.4 Table C1 shows for each of the three deferred periods involved, the values of *IN* and *IR* in five year age groups. Note that the values of *IN* in Table C1 are those used in the preliminary graduations of σ_x. In particular, the average number of policies per individual for D4 has been taken to be 1.128. This estimate, and the estimated values of *IN*, were later revised. See §§7 and 8 and Table C11.

Table C1. *Values of IR and IN used in the preliminary graduations of* $\sigma_{i,x}$

	Deferred Period					
	4 weeks		13 weeks		26 weeks	
Age group	*IR*	*IN*	*IR*	*IN*	*IR*	*IN*
20–24	25·0	13·0	4·5	0	3·0	0
25–29	107·0	28·7	25·5	5·5	19·5	0·3
30–34	192·5	87·2	44·0	5·9	18·5	0·3
35–39	247·0	77·2	73·0	11·5	26·5	5·3
40–44	270·0	74·1	91·5	11·9	37·5	3·5
45–49	305·0	86·7	105·5	9·2	43·0	3·7
50–54	294·0	73·3	100·5	6·6	69·5	5·4
55–59	210·5	58·5	78·0	4·5	75·0	4·6
60–64	125·5	31·4	60·0	3·7	60·5	3·8
Totals	1,776·5	530·1	582·5	58·8	353·0	26·9

4. THE CALCULATION OF THE EXPOSURE

4.1 For a given integer age x and a given deferred period, let TE_x denote the total time spent in the observation period 1975–78 at age x last birthday. For D1,

TE_x is given in *C.M.I.R.* **7**, Table K1, as the exposure for sickness period 1–4 weeks. For D4 and D13, TE_x can be calculated by 'unadjusting' the exposures for sickness periods 4–13, 13–26 and 26–52 weeks and 13–26, 26–52 and 52–104 weeks respectively. (This is nothing more than solving three linear simultaneous equations with three unknowns.) For D26 weeks we have only two equations with three unknowns; however, we can solve these equations by making the extra, and not unreasonable, assumption that α_x equals β_x at each age, in the notation of §2. Let CL_x denote the total time spent claiming at age x last birthday in the observation period 1975–78. For D1, the values of CL_x are given in *C.M.I.R.* **7**, Table K1. For deferred periods greater than 1 week we need to add to the figures given in *C.M.I.R.* **7**, Tables K2–K4, the (very small) number of weeks when the unreported sicknesses, as calculated in §3 above, could have been claiming. Then $TE_x - CL_x/52.18$ represents the time spent in the period 1975–78, at age x last birthday, as either healthy or sick but not yet claiming. (Note that the exposures in *C.M.I.R.* **7** are measured in years whereas the time spent claiming is measured in weeks.) Let p_x denote the proportion of TE_x which is spent as sick but not yet claiming. Then EH_x, where

$$EH_x = TE_x \cdot (1-p_x) - CL_x/52{\cdot}18 \tag{3}$$

represents the time spent in 1975–78 as healthy by policyholders aged x last birthday. We do not know the value of p_x but, using the numerical algorithms outlined in Part D, and provided we know the values of all the transition intensities, including σ_x, we can calculate for any age x_0 the probability that a policyholder who was healthy at age x_0 and who is alive at exact age y ($> x_0$), is sick but not yet claiming at age y. (Note that we need the extra conditioning at age x_0 to make this a well-defined probability in terms of our model.) We denote this probability $p(y;x_0)$. Table C2 shows some values of $p(y;x_0)$ for selected ages y and x_0 and each of the deferred periods. These values have been calculated using the final graduations of σ_x for each of the deferred periods. It can be seen from Table C2 that $p(y;x_0)$ is not very sensitive to the value of x_0 and hence that, for any reasonable x_0, $p(x+\frac{1}{2};x_0)$ should be a reasonable approximation to p_x.

4.2 To calculate the values in Table C2 using the algorithms described in Part D we have used a step size of 1/104 of a year (see Part D §2.1) and the following function for μ_x:

$$\mu_x = a + bx' + \exp(c + dx')$$

$$x' = (x-70)/50$$

$$\begin{aligned} a &= -\ 0{\cdot}0250148 \\ b &= -\ 0{\cdot}016037 \\ c &= -\ 3{\cdot}32957 \\ d &= 1{\cdot}30571 \end{aligned}$$

Table C2. *Values of 100,000 $p(y;x_0)$*

		Deferred Period			
y	x_0	1 week	4 weeks	13 weeks	26 weeks
31	20	397	499	505	318
31	30	397	499	505	318
34	20	387	513	512	328
34	30	387	513	512	328
41	30	365	526	533	386
41	40	366	527	534	387
44	30	358	533	552	432
44	40	359	534	553	432
51	40	359	591	657	613
51	50	364	597	662	617
54	40	369	650	745	741
54	50	373	654	749	744

This formula for μ_x is based on a graduation of male Assured Lives, 1975–78, select mortality at duration zero.

4.3 We would expect the value of $p(y;x_0)$, for given y and x_0, to increase with the deferred period, but Table C2 shows that this is not always the case. For example, the value of $p(31;x_0)$ for x_0 equal to 20 or 30, is 397×10^{-5} for D1 and 318×10^{-5} for D26. The practical interpretation of these figures is as follows: for a group of D1 policyholders all aged 31, on average a proportion 397×10^{-5} will be sick with a duration of current sickness less than 1 week; for a group of D26 policyholders all aged 31, on average a proportion 318×10^{-5} will be sick with duration of current sickness less than 26 weeks. An important distinction between these figures is that they have been calculated using different functions for σ_x (see §6); σ_{31} is 0.3218 for D1 and 0.1145 for D26. A helpful way of assessing the effect of this difference is to note that if we used the values of σ_x appropriate to D1, the proportion of individuals aged 31 whom we would expect to be sick with duration of current sickness less than 26 weeks would be just under 900×10^{-5}.

4.4 For D1 the exposure E at age x in (2) was taken to be EH_x as given by (3) without any further adjustment. For the other deferred periods two further adjustments were made. Let d, measured in years, be one of these deferred periods. The claim inceptions are recorded as age x last birthday at the start of the claim payments, which is on average at exact age $(x+\frac{1}{2}-d)$ at the start of the sickness which resulted in the claim. Hence we require the corresponding exposure between ages $(x-d)$ and $(x+1-d)$. We can approximate this by

$$d \times EH_{x-1} + (1-d) \times EH_x$$

Finally, the sicknesses which result in claims in the period 1975–78 are those which start in the period $(1975-d)$ – $(1978-d)$ and again we require the corresponding exposure. This can be approximated by

$$(1-g)^{-d} \times (d \times EH_{x-1} + (1-d) \times EH_x) \qquad (4)$$

where g is the annual rate of growth of the number of policies in force for the relevant deferred period, and is given in §2.

4.5 In summary, the exposure 'at age x', E, to be used in (2) is given by (3) for D1, and by (4) for the other deferred periods. Note that the calculation of these exposures requires the calculation of the proportions p_x, or, more precisely, of the probabilities $p(x+\frac{1}{2};x_0)$, which in turn require σ_x to be known. This is a difficulty since our aim in the section is to estimate σ_x! We shall return to this problem in the next section.

4.6 Table C3 gives for each deferred period and selected ages the exposure 'at age x', E, used in (2) for the final graduation of σ_x. By comparing the figures in Table C3 and those given for the exposures in *C.M.I.R.* 7, the combined effect of all the adjustments outlined in this section can be assessed.

Table C3. *Exposures (in years) at age* x

	Deferred Period			
Age	1 week	4 weeks	13 weeks	26 weeks
30	2,847	3,254	4,716	8,787
35	2,073	3,006	5,153	8,382
40	2,087	3,130	5,188	7,625
45	2,206	3,987	4,648	6,721
50	2,418	2,430	3,517	5,247
55	1,919	1,478	2,052	3,602
60	1,255	559	920	1,749

5. THE GRADUATION PROCESS

5.1 The next stage of the graduation process, after the calculation of the inceptions, I, and exposures, E, at single ages, was the grouping of ages, if necessary, to ensure an expected number of inceptions of at least ten in each group. This produced the following age groupings:

Deferred period 1 week: single ages from 23 to 64 inclusive,
Deferred period 4 weeks: single ages from 25 to 64 inclusive,
Deferred period 13 weeks: ages 25 to 28 as one group and then single ages from 29 to 63 inclusive,
Deferred period 26 weeks: three year age groups from 31 to 39 inclusive,
two year age groups from 40 to 49 inclusive
and then single ages from 50 to 63 inclusive.

(Initially, an extra age group, covering ages 28 to 30 inclusive, was included for D26 but this gave a very high observed value for σ_x which distorted the graduation, particularly when the graduation of σ_x was extrapolated back to age 20. For this reason this age group was not used for the graduation of σ_x for D26.)

5.2 The values of the probabilities $\pi_{x,d}$ were then calculated for each age group and an appropriate value of x. (For example, for age 50, $\pi_{x,d}$ was calculated at $x = 50.5$ for D1 and at $x = 50$ for D26). The values of $I/\pi_{x,d}E$ were regarded as point estimates of σ_x, with x taken to be the mid-point of the age interval (less the deferred period where this is greater than 1 week.) (For D13, age group 25–28, the estimate of σ_x was taken to be a point estimate at exact age $27\frac{1}{4}$ rather than exact age $26\frac{3}{4}$, with $\pi_{x,d}$ calculated accordingly; this adjustment was made to take account of the uneven incidence of exposure over this particular age group.)

5.3 After preliminary investigations it was decided that σ_x should be graduated by functions of the form:

$$\sigma_x = \exp\{f(x)\} \tag{5}$$

where $f(x)$ is a polynomial, usually of degree 3. The combination of the chosen functional form for σ_x, (5), and the statistical model, (2), is ideally suited to the statistical computing package GLIM. However, σ_x needs to be known since it is required for the calculation of the exposures, E, and since it appears in (2) in the variance of the estimator.

To overcome these difficulties, the following iterative procedure was used:

(i) The factors p_x used in the calculation of the exposure E were set at zero. The variance of the estimate in (2) was set at $I.V/(\pi_{x,d}E)^2$.

(ii) GLIM was then used to fit, by maximum likelihood, the coefficients of the polynomial $f(x)$.

(iii) The 'graduated' values of σ_x calculated in (ii) were used in the variance term in (2) and GLIM was used again to re-graduate σ_x.

(iv) Step (iii) was repeated until the 'graduations' of σ_x converged to a limit. This usually took about four GLIM runs. (In practice, σ_x was deemed to have converged to a limit when each of the relative differences between corresponding coefficients in $f(x)$ in successive 'graduations' was less than 10^{-4}.)

(v) The 'graduation' of σ_x resulting from (iv) was used to calculate the p_x factors and hence to update the values of the exposures E.

(vi) Steps (iii), (iv) and (v) were then repeated until the 'graduations' of σ_x converged to a limit. This usually took about three or four calculations of the p_x factors. (In practice, σ_x was deemed to have converged when a new cycle of the process resulted in the same coefficients for $f(x)$ to four significant figures.) This graduation was regarded as the final graduation of σ_x.

6. RESULTS OF THE PRELIMINARY GRADUATIONS

6.1 The results of the graduation process described in §5 are presented and discussed in this section. Throughout this section these results are referred to as the 'preliminary graduations'. For D1, D13 and D26 the preliminary graduations were found to be satisfactory and were adopted as the final graduations. For D4 the preliminary graduation discussed in this section was found to be unsatisfactory in one respect (see §7) and an adjustment was made to produce a final graduation. See §8.

6.2 The formula for the polynomial $f(x)$ in (5) was chosen to be a cubic for all graduations except D26, for which $f(x)$ was chosen to be a quadratic, i.e.

$$f(x) = a + bx + cx^2 + dx^3 \tag{6}$$

with $d = 0$ for D26. The preliminary estimates of a, b, c and d, together with the standard errors of these estimates, are shown in Table C4. (Note that the parameterisation used by GLIM makes the standard error of the parameter a irrelevant.) Tables C5, C6, C7, and C8 show for each of the four preliminary graduations and each age (or age group) used for the graduation,

(i) the estimate of σ_x at the mid-point of the age interval used for the graduation (less the deferred period where this is greater than 1 week),
(ii) the corresponding graduated value of σ_x,
(iii) I, the adjusted observed number of claim inceptions for this age (or age group) including an estimate of the number of unreported inceptions as explained in §3 above,
(iv) $E\pi\sigma$, the expected number of inceptions for this age (or age group),
(v) $100I/E\pi\sigma$ and
(vi) $(I - E\pi\sigma)/(VE\pi\sigma)^{\frac{1}{2}}$, which, from (1), can be regarded as a standardised residual coming from a $N(0,1)$ distribution.

In each case the totals of actual and expected claim inceptions are equal, apart from rounding errors. This can be shown to be a mathematical consequence of the statistical model and the graduating function specified by (1), (5) and (6). Similar results are presented in Forfar, McCutcheon and Wilkie (1988, Appendix 2).

6.3 Table C9 summarises the results of some standard tests of the preliminary graduations. This table shows a value of a χ^2 goodness of fit statistic, with the appropriate number of degrees of freedom (this is the scaled deviance computed

Table C4. *Parameter estimates for the preliminary graduations*

	Deferred Period			
Parameter	1 week	4 weeks	13 weeks	26 weeks
a	$-1{\cdot}796$	$-4{\cdot}168$	$-2{\cdot}722$	$-4{\cdot}819 \times 10^{-1}$
b	$8{\cdot}083 \times 10^{-2}$	$2{\cdot}290 \times 10^{-1}$	$1{\cdot}290 \times 10^{-1}$	$-8{\cdot}434 \times 10^{-2}$
Standard error of b	$6{\cdot}93 \times 10^{-2}$	$1{\cdot}39 \times 10^{-1}$	$2{\cdot}963 \times 10^{-1}$	$7{\cdot}44 \times 10^{-2}$
c	$-2{\cdot}686 \times 10^{-3}$	$-6{\cdot}258 \times 10^{-3}$	$-4{\cdot}240 \times 10^{-3}$	$9{\cdot}749 \times 10^{-4}$
Standard error of c	$1{\cdot}60 \times 10^{-3}$	$3{\cdot}15 \times 10^{-3}$	$6{\cdot}57 \times 10^{-3}$	$7{\cdot}58 \times 10^{-4}$
d	$2{\cdot}498 \times 10^{-5}$	$5{\cdot}299 \times 10^{-5}$	$3{\cdot}888 \times 10^{-5}$	0
Standard error of d	$1{\cdot}19 \times 10^{-5}$	$2{\cdot}32 \times 10^{-5}$	$4{\cdot}80 \times 10^{-5}$	—

Table C5. *Preliminary graduation of σ_x for deferred period 1 week: estimated and graduated values of σ_x and actual and expected numbers of claim inceptions*

Age x	Estimate of σ_x	Graduated σ_x	Claim Inceptions Observed I	Expected $E\pi_{x,d}\sigma_x$	$\frac{100I}{E\pi_{x,d}\sigma_x}$	$\frac{(I-E\pi_{x,d}\sigma_x)}{(VE\pi_{x,d}\sigma_x)^{\frac{1}{2}}}$
23	0·2767	0·3480	33·0	41·5	79·5	−0·87
24	0·4341	0·3463	124·5	99·3	125·4	1·67
25	0·3120	0·3440	146·5	161·5	90·7	−0·78
26	0·3702	0·3412	231·0	212·9	108·5	0·82
27	0·3251	0·3379	247·5	257·3	96·2	−0·40
28	0·3700	0·3343	341·0	308·2	110·6	1·23
29	0·3418	0·3303	350·5	338·9	103·4	0·42
30	0·2644	0·3261	291·5	359·7	81·0	−2·38
31	0·3282	0·3217	369·0	361·8	102·0	0·25
32	0·2970	0·3171	311·0	332·1	93·6	−0·77
33	0·3078	0·3124	304·5	309·2	98·5	−0·18
34	0·2790	0·3077	263·5	290·8	90·6	−1·06
35	0·3342	0·3030	288·0	261·2	110·3	1·10
36	0·3184	0·2984	272·0	255·0	106·7	0·70
37	0·3211	0·2938	282·0	258·2	109·2	0·98
38	0·2647	0·2894	244·5	267·6	91·4	−0·93
39	0·3006	0·2852	283·0	268·7	105·3	0·58
40	0·2681	0·2812	249·5	261·9	95·3	−0·51
41	0·3017	0·2775	279·0	256·8	108·6	0·92
42	0·2479	0·2740	223·0	246·7	90·4	−1·00
43	0·2869	0·2708	275·0	259·8	105·9	0·62
44	0·2600	0·2680	263·0	271·3	96·9	−0·33
45	0·2677	0·2656	282·5	280·5	100·7	0·08
46	0·2393	0·2635	265·5	292·7	90·7	−1·05
47	0·2628	0·2619	298·0	297·4	100·2	0·02
48	0·3024	0·2608	349·0	301·2	115·9	1·82
49	0·2631	0·2601	314·0	310·8	101·0	0·12
50	0·2736	0·2600	339·5	323·1	105·1	0·60
51	0·2885	0·2605	362·5	327·8	110·6	1·27
52	0·2294	0·2616	279·5	319·1	87·6	−1·46
53	0·2606	0·2633	314·0	317·8	98·8	−0·14
54	0·2831	0·2658	314·0	295·2	106·4	0·72
55	0·2532	0·2690	267·5	284·7	94·0	−0·67
56	0·2868	0·2731	275·5	262·8	104·8	0·52
57	0·2151	0·2781	186·0	240·8	77·2	−2·33
58	0·2658	0·2841	229·0	245·2	93·4	−0·68
59	0·2753	0·2912	223·0	236·3	94·4	−0·57
60	0·3488	0·2995	258·5	222·4	116·2	1·60
61	0·3185	0·3093	230·5	224·2	102·8	0·28
62	0·2982	0·3206	208·5	224·6	92·8	−0·71
63	0·3191	0·3336	207·5	217·4	95·4	−0·44
64	0·4085	0·3486	191·0	163·4	116·9	1·43
Totals			11,068·0	11,067·8		

Table C6. *Preliminary graduation of σ_x for deferred period 4 weeks: estimated and graduated values of σ_x and actual and expected numbers of claim inceptions*

Age x	Estimate of σ_x	Graduated σ_x	Claim Inceptions Adjusted observed I	Expected $E\pi_{x,d}\sigma_x$	$\frac{100I}{E\pi_{x,d}\sigma_x}$	$\frac{(I-E\pi_{x,d}\sigma_x)}{(VE\pi_{x,d}\sigma_x)^{\frac{1}{2}}}$
25	0·2731	0·2187	21·2	17·0	124·7	0·91
26	0·2974	0·2212	30·6	22·8	134·2	1·46
27	0·1479	0·2228	19·8	29·8	66·4	−1·63
28	0·1327	0·2236	22·4	37·8	59·3	−2·23
29	0·2113	0·2236	41·7	44·1	94·6	−0·32
30	0·2428	0·2229	53·6	49·3	108·7	0·55
31	0·2625	0·2215	64·7	54·6	118·5	1·22
32	0·2384	0·2197	60·1	55·5	108·3	0·55
33	0·1943	0·2173	51·0	57·1	89·3	−0·72
34	0·1902	0·2146	50·3	56·8	88·6	−0·77
35	0·2549	0·2116	66·6	55·4	120·2	1·34
36	0·2606	0·2083	70·5	56·4	125·0	1·67
37	0·1978	0·2049	57·3	59·5	96·3	−0·25
38	0·1984	0·2015	62·3	63·4	98·3	−0·12
39	0·2029	0·1980	67·4	65·9	102·3	0·16
40	0·1478	0·1946	51·5	67·9	75·8	−1·77
41	0·2211	0·1913	79·9	69·3	115·3	1·13
42	0·1738	0·1882	66·4	72·1	92·1	−0·60
43	0·1636	0·1854	64·8	73·5	88·2	−0·90
44	0·1989	0·1828	81·6	75·1	108·7	0·67
45	0·1758	0·1805	74·9	77·1	97·1	−0·22
46	0·1850	0·1787	81·0	78·4	103·3	0·26
47	0·1520	0·1772	67·4	78·7	85·6	−1·13
48	0·1903	0·1762	85·9	79·7	107·8	0·62
49	0·1831	0·1757	82·5	79·3	104·0	0·32
50	0·1746	0·1758	77·5	78·2	99·1	−0·07
51	0·1417	0·1764	61·2	76·4	80·1	−1·55
52	0·1693	0·1778	69·1	72·7	95·0	−0·38
53	0·2017	0·1799	77·6	69·4	111·8	0·88
54	0·2218	0·1829	82·0	67·7	121·1	1·55
55	0·2089	0·1868	72·1	64·7	111·4	0·82
56	0·1939	0·1918	60·4	59·9	100·8	0·06
57	0·2030	0·1979	54·4	53·2	102·3	0·15
58	0·1870	0·2055	43·5	47·9	90·8	−0·57
59	0·1973	0·2146	38·7	42·2	91·7	−0·48
60	0·2705	0·2256	45·3	37·9	119·5	1·07
61	0·1983	0·2387	29·6	35·7	82·9	−0·91
62	0·1748	0·2544	22·8	33·3	68·5	−1·62
63	0·2434	0·2730	27·7	31·1	89·1	−0·54
64	0·4266	0·2954	31·2	21·6	144·4	1·84
Totals			2,268·5	2,268·4		

Table C7. *Preliminary graduation of σ_x for deferred period 13 weeks: estimated and graduated values of σ_x and actual and expected numbers of claim inceptions*

Age x	Estimate of σ_x	Graduated σ_x	Claim Inceptions: Adjusted observed I	Claim Inceptions: Expected $E\pi_{x,d}\sigma_x$	$\frac{100I}{E\pi_{x,d}\sigma_x}$	$\frac{(I-E\pi_{x,d}\sigma_x)}{(VE\pi_{x,d}\sigma_x)^{\frac{1}{2}}}$
25–28	0·2232	0·2086	17·3	16·2	106·8	0·25
29	0·2426	0·2012	13·7	11·4	120·2	0·63
30	0·2293	0·1972	11·6	10·0	116·0	0·47
31	0·1768	0·1930	10·7	11·7	91·5	−0·27
32	0·2037	0·1887	13·7	12·7	107·9	0·26
33	0·1118	0·1843	8·2	13·6	60·3	−1·35
34	0·0733	0·1799	5·7	14·0	40·7	−2·05
35	0·1828	0·1755	14·8	14·2	104·2	0·15
36	0·2508	0·1711	21·2	14·5	146·2	1·63
37	0·1296	0·1669	11·7	15·1	77·5	−0·81
38	0·1638	0·1628	16·3	16·2	100·6	0·02
39	0·1857	0·1589	20·4	17·4	117·2	0·66
40	0·1099	0·1551	13·0	18·4	70·7	−1·16
41	0·1749	0·1516	21·9	19·0	115·3	0·62
42	0·1468	0·1484	19·4	19·6	99·0	−0·04
43	0·1530	0·1454	21·3	20·3	104·9	0·21
44	0·1890	0·1427	27·8	21·0	132·4	1·37
45	0·1557	0·1403	24·0	21·7	110·6	0·46
46	0·1463	0·1382	23·4	22·2	105·4	0·24
47	0·1210	0·1365	19·8	22·4	88·4	−0·51
48	0·1569	0·1351	26·3	22·7	115·9	0·70
49	0·1244	0·1342	21·1	22·9	92·1	−0·35
50	0·1093	0·1336	18·6	22·7	81·9	−0·80
51	0·1421	0·1335	24·5	23·0	106·5	0·29
52	0·1256	0·1338	20·8	22·2	93·7	−0·27
53	0·0859	0·1347	13·7	21·5	63·7	−1·56
54	0·1916	0·1361	29·6	21·0	141·0	1·73
55	0·1017	0·1380	14·7	20·0	73·5	−1·10
56	0·1389	0·1406	18·5	18·8	98·4	−0·06
57	0·1596	0·1440	19·3	17·5	110·3	0·40
58	0·1831	0·1481	20·7	16·8	123·2	0·88
59	0·0873	0·1531	9·1	16·1	56·5	−1·61
60	0·1812	0·1592	17·0	15·0	113·3	0·48
61	0·1481	0·1664	12·9	14·5	89·0	−0·39
62	0·2393	0·1749	18·8	13·7	137·2	1·27
63	0·1632	0·1851	11·1	12·6	88·1	−0·39
Totals			632·4	632·6		

Table C8. *Preliminary graduation of σ_x for deferred period 26 weeks: estimated and graduated values of σ_x and actual and expected numbers of claim inceptions*

			Claim Inceptions			
Age x	Estimate of σ_x	Graduated σ_x	Adjusted observed I	Expected $E\pi_{x,d}\sigma_x$	$\dfrac{100I}{E\pi_{x,d}\sigma_x}$	$\dfrac{(I-E\pi_{x,d}\sigma_x)}{(VE\pi_{x,d}\sigma_x)^{\frac{1}{2}}}$
31–33	0·1053	0·1128	12·6	13·5	93·3	−0·22
34–36	0·0895	0·1065	12·7	15·1	84·1	−0·55
37–39	0·1306	0·1024	22·6	17·8	127·0	1·01
40–41	0·1362	0·1004	19·4	14·3	135·7	1·20
42–43	0·0692	0·0997	11·4	16·4	69·5	−1·10
44–45	0·1318	0·0998	25·0	18·9	132·3	1·25
46–47	0·0443	0·1007	9·5	21·7	43·8	−2·33
48–49	0·0954	0·1024	22·4	24·0	93·3	−0·29
50	0·1319	0·1042	16·2	12·8	126·6	0·85
51	0·1078	0·1057	13·7	13·4	102·2	0·07
52	0·1045	0·1074	13·6	14·0	97·1	−0·10
53	0·1226	0·1093	16·0	14·3	111·9	0·40
54	0·1166	0·1115	15·4	14·8	104·1	0·14
55	0·1118	0·1140	15·2	15·5	98·1	−0·07
56	0·1222	0·1168	16·6	15·8	105·1	0·18
57	0·0951	0·1198	12·4	15·6	79·5	−0·72
58	0·1363	0·1232	16·8	15·2	110·5	0·37
59	0·1628	0·1269	18·7	14·5	129·0	0·98
60	0·1369	0·1310	14·5	13·9	104·3	0·14
61	0·1071	0·1355	10·9	13·8	79·0	−0·70
62	0·1498	0·1404	14·8	13·9	106·5	0·22
63	0·1462	0·1457	13·7	13·6	100·7	0·02
64	0·1346	0·1516	10·5	11·8	89·0	−0·34
Totals			354·6	354·6		

by GLIM, which, for our particular model, can be interpreted as a χ^2 value), the numbers of +'s and −'s, where a + indicates that the estimate of σ_x is greater than the graduated value (or, equivalently, that the observed number of claim inceptions is greater than the expected number) and a - indicates the reverse, and the number of runs of +'s and −'s. None of the results in Table C9 indicates an unsatisfactory graduation.

6.4 A noticeable feature of Table C4 is that for D13 the estimates of b, c and d are not significantly different from zero. This suggests that one or more of these parameters could be set equal to zero. Putting d equal to zero has the effect of making the revised estimates of b and c significantly different from zero, and increasing the χ^2 statistic by only 0.6 with an increase of 1 in the number of degrees of freedom. These are arguments in favour of setting d equal to zero. The argument against, and the reason why d was not set equal to zero, is that while its

Table C9. *Tests of the graduations*

	Deferred Period			
	1 week	4 weeks	13 weeks	26 weeks
χ^2	42·0	41·7	28·4	14·3
Degrees of freedom	38	36	32	20
No. of +'s	22	20	20	13
No. of −'s	20	20	16	10
No. of runs	28	21	24	15

inclusion does not significantly improve the fit over the range of the data, when the function for σ_x is extrapolated back to age 20, it gives a shape for σ_x much more in line with the graduations of σ_x for D1 and D4 , for which there is more data in this range of values of x than there is for D13. For D26, the estimates of b and c are not significantly different from zero; in fact, setting both b and c equal to zero, and hence fitting a constant function to σ_x, results in a χ^2 statistic of only 18.2 with 22 degrees of freedom! This is partly a result of the relatively scanty data for D26. The parameters b and c have been included for this graduation so that the resulting function for σ_x has broadly the same shape as for the other deferred periods. The inclusion of parameter d for D26 not only did not improve the fit over the range of the data but also did not change significantly the extrapolated values of σ_x in the age range 20 to 30; for this reason d was set equal to zero in this case.

7. FURTHER INVESTIGATIONS

7.1 An important assumption in our graduation of σ_x is that the intensities ρ and ν are the same for all deferred periods. A consequence of this assumption, taking account of the low observed values of ρ in the four weeks following the end of the deferred period for all deferred periods except 1 week, is that some sicknesses which last beyond the deferred period and then recover within four weeks are not reported. We have estimated these 'unreported inceptions' as explained in §3 and then graduated σ_x using the total of reported and unreported inceptions. An alternative approach, and a useful way of checking our assumption, would be to deduct from the reported inceptions those claims which were reported to have terminated within four weeks of the end of the deferred period and regard the resulting figure as the number of claim inceptions with an effective deferred period four weeks longer than the original deferred period.

7.2 To clarify this check, consider D4; the same check was applied to D13 and D26. We know the number of reported claim inceptions at each age, including duplicates, since these are given in *C.M.I.R.* 7, Table K13. This is *IR* in the notation of §3. We also know the number of reported claims which terminated for any reason within four weeks of the end of the deferred period, excluding duplicates. By multiplying this last figure by the average number of policies per individual and then deducting it from IR we obtain an estimate of the number of sicknesses at each age which lasted beyond eight weeks. For a given age let us denote this figure by IR^*. We can calculate the expected number of sicknesses for this age which last beyond eight weeks, denoted IE^*, as

$$IE^* = \sigma \cdot \pi' \cdot E$$

where E is the same exposure as calculated in §4, σ is the graduated value of σ_x at the mid-point of the interval less four weeks and π' is the probability that a sickness which starts at this age lasts for at least eight weeks. If all our assumptions are valid, we have

$$IR^* \sim N(IE^*, V \cdot IE^*) \tag{7}$$

where V is the variance inflation factor to take account of the effect of including duplicate policies in both IR^* and IE^*. In particular, we can calculate

(i) the number of $+$'s, i.e. ages where IR^* is greater than $< IE^*$, and $-$'s,
(ii) the total over all ages of both IR^* and IE^*,
(iii) the sum of $(IR^* - IE^*)^2/(V \cdot IE^*)$, which, according to (7), has a χ^2 distribution with number of degrees of freedom equal to the number of ages less the number of fitted parameters in the formula for σ_x.

7.3 The results of these checks for each of the deferred periods are summarised in Table C10. The results in Table C10 for D13 and D26 are encouraging but the results for D4 are disappointing. A possible cause of the disappointing results for D4 is that the estimate of the average number of policies per individual for this class of business is too low. (The estimate we have used is 1.128 and is taken from *C.M.I.R.* 7, Appendix F.) The effects of a higher estimate of the average number of policies per individual would be

(i) to increase IN, in the notation of §3, and hence
(ii) to increase the graduated values of σ_x, and hence
(iii) to increase IE^*,
(iv) to decrease IR^* by increasing the number of terminations to be deducted from IN.

Hence, the overall effect of an increase in this estimate would be to reduce IR^* and increase IE^*. A rough calculation, based only on totals, shows that if the average number of policies per individual were 1.34 rather than 1.128, then IR^* would be approximately equal to IE^*. (It is interesting to note in *C.M.I.R.* 7 Appendix F that an under-estimate of this factor could be an explanation of the difficulties experienced by the PHI Sub-Committee in the graduation of claim inception rates for D4.)

7.4 Although the evidence is by no means conclusive, the PHI Sub-Committee decided it was prudent to graduate σ_x for D4 on the assumption that the average number of policies per individual was 1.34 and not 1.128 as given in *C.M.I.R.* 7 Appendix F. The details of this re-graduation are given in the next section.

Table C10. *The effect of eliminating terminations within 4 weeks of the end of the deferred period*

	Deferred Period		
	4 weeks	13 weeks	26 weeks
*IR**	1,155	475	298
*IE**	998	459	317
χ^2	115·7	38·0	16·8
Degrees of freedom	36	32	20
No. of +'s	28	22	10
No. of −'s	12	14	13
No. of runs	17	23	16

8. THE REGRADUATION OF σ_x FOR DEFERRED PERIOD 4 WEEKS

8.1 As explained in the previous section, the PHI Sub-Committee decided it would be prudent to re-graduate σ_x for D4 on the assumption that the average number of policies per individual was 1.34. Using the distributional assumptions outlined in *C.M.I.R.* **7** this implies that the variance inflation factor V in (1) should be 1.68 rather than 1.26.

8.2 A consequence of these new assumptions is that the estimates of *IN*, the number of sicknesses assumed to be not reported, have to be revised (by multiplying by a factor 1.34/1.128). The revised figures are given in Table C11. The figures in Table C11 should be compared with those in Table C1.

Table C11. *Reported and non-reported claim inceptions for deferred period 4 weeks*

Age group	*IR*	*IN*
20–24	25	15·4
25–29	107	34·1
30–34	192·5	103·6
35–39	247	91·7
40–44	270	88·0
45–49	305	103·0
50–54	294	87·1
55–59	210·5	69·5
60–64	125·5	37·3
Totals	1,776·5	629·7

8.3 The function used to re-graduate σ_x was, as for the preliminary graduation, of the form

$$\sigma_x = \exp(a + bx + cx^2 + dx^3) \tag{8}$$

The final estimates of the parameters together with their standard errors are given in Table C12. The figures in Table C12 should be compared with the corresponding results for the preliminary graduation, which are given in Table C4.

The results of standard tests of the graduation together with actual and expected values corresponding to Tables C9 and C6 are given in Tables C13 and C14 respectively.

8.4 The results of the tests of the graduation summarised in Table C13 are satisfactory. The graduation was further tested, as in §7, by eliminating terminations within four weeks of the end of the deferred period. The results of this test are given in Table C15 using the same notation as in §7.

8.5 The outstanding feature of Table C15 is the very high χ^2 value. (Note that the number of degrees of freedom is not very clear since not only have four parameters been fitted from the data but IE^* has been constrained to be approximately equal to IR^*. We have taken account of this by reducing the number of degrees of freedom by one.) A large contribution to this χ^2 value comes from a few ages, notably 26, 54 and 64 where the number of claim inceptions is very much higher than expected, and a few ages, notably 40 and 62, where it is very much lower. This feature can be observed in the number of reported claim inceptions as given in *C.M.I.R.* 7 Table K13. This feature did not give a large χ^2 value for the re-graduation of σ_x (see Table C13), or for the

Table C12. *Parameter estimates and standard errors for the re-graduation of σ_x for deferred period 4 weeks*

Parameter	Estimate	Standard Error
a	$-4{\cdot}256$	
b	$2{\cdot}392\times10^{-1}$	$1{\cdot}568\times10^{-1}$
c	$-6{\cdot}498\times10^{-3}$	$3{\cdot}561\times10^{-3}$
d	$5{\cdot}476\times10^{-5}$	$2{\cdot}623\times10^{-5}$

Table C13. *Tests of the re-graduation of σ_x for deferred period 4 weeks*

χ^2	30·6
Degrees of freedom	36
No. of +'s	20
No. of −'s	20
No. of runs	21

Table C14. *Re-graduation of σ_x for deferred period 4 weeks: estimated and graduated values of σ_x and actual and expected numbers of claim inceptions*

			Claim Inceptions			
Age x	Estimate of σ_x	Graduated σ_x	Adjusted observed I	Expected $E\pi_{x,d}\sigma_x$	$\frac{100I}{E\pi_{x,d}\sigma_x}$	$\frac{(I-E\pi_{x,d}\sigma_x)}{(VE\pi_{x,d}\sigma_x)^{\frac{1}{2}}}$
25	0·2822	0·2288	21·9	17·8	123·0	0·75
26	0·3059	0·2317	31·5	23·8	132·4	1·22
27	0·1560	0·2337	20·8	31·2	66·7	−1·44
28	0·1404	0·2347	23·7	39·5	60·0	−1·94
29	0·2187	0·2349	43·1	46·3	93·1	−0·36
30	0·2574	0·2343	56·9	51·7	110·1	0·56
31	0·2765	0·2330	68·1	57·3	118·8	1·10
32	0·2516	0·2311	63·4	58·2	108·9	0·53
33	0·2068	0·2287	54·3	59·9	90·7	−0·56
34	0·2022	0·2258	53·5	59·6	89·8	−0·61
35	0·2659	0·2226	69·5	58·1	119·6	1·15
36	0·2711	0·2191	73·3	59·2	123·8	1·41
37	0·2078	0·2155	60·2	62·3	96·6	−0·21
38	0·2079	0·2118	65·3	66·4	98·3	−0·10
39	0·2119	0·2081	70·4	69·0	102·0	0·13
40	0·1559	0·2044	54·3	71·1	76·4	−1·54
41	0·2289	0·2009	82·7	72·4	114·2	0·93
42	0·1811	0·1975	69·2	75·3	91·9	−0·54
43	0·1707	0·1944	67·6	76·8	88·0	−0·81
44	0·2056	0·1915	84·3	78·4	107·5	0·51
45	0·1839	0·1890	78·3	80·4	97·4	−0·18
46	0·1928	0·1870	84·4	81·7	103·3	0·23
47	0·1594	0·1853	70·6	81·9	86·2	−0·96
48	0·1974	0·1841	89·1	82·9	107·5	0·53
49	0·1898	0·1835	85·5	82·5	103·6	0·25
50	0·1820	0·1835	80·8	81·2	99·5	−0·03
51	0·1488	0·1841	64·3	79·4	81·0	−1·31
52	0·1760	0·1855	71·8	75·4	95·2	−0·32
53	0·2082	0·1876	80·0	71·9	111·3	0·74
54	0·2279	0·1907	84·2	70·3	119·8	1·28
55	0·2175	0·1947	75·2	67·1	112·1	0·76
56	0·2023	0·1999	63·0	62·0	101·6	0·10
57	0·2110	0·2063	56·5	55·1	102·5	0·15
58	0·1948	0·2142	45·3	49·6	91·3	−0·47
59	0·2048	0·2238	40·1	43·7	91·8	−0·42
60	0·2805	0·2354	47·0	39·3	119·6	0·95
61	0·2079	0·2492	31·0	37·0	83·8	−0·76
62	0·1840	0·2658	24·0	34·6	69·4	−1·39
63	0·2521	0·2856	28·7	32·3	88·9	−0·49
64	0·4350	0·3093	31·8	22·5	141·3	1·51
Totals			2,365·6	2,365·1		

Table C15. *The effect of eliminating terminations within 4 weeks of the end of the deferred period*

IR^*	1,043
IE^*	1,044
χ^2	67·7
Degrees of freedom	35
No. of +'s	20
No. of −'s	20
No. of runs	23

preliminary graduation (see Table C9), because by smoothing the values of *IN* over five year age groups and then adding them to *IR*, any unevenness in the individual values of *IR* becomes relatively less important. The reverse is true when smoothed values are deducted from *IR* to obtain IR^*. This seems to be the explanation for the large χ^2 value in Table C15.

In the circumstances the PHI Sub-Committee regarded the re-graduation of σ_x for D4 as a satisfactory final graduation.

9. CONCLUSIONS AND FURTHER COMMENTS

9.1 The PHI Sub-Committee regard the preliminary graduations of σ_x for D1, D13 and D26, as given in §6, and the re-graduation of σ_x for D4, as given in §8, as satisfactory final graduations.

In summary, the final graduations of σ_x for the four different deferred periods are:

Deferred period 1 week

$$\sigma_x = \exp\{-1{\cdot}796 + 8{\cdot}083 \times 10^{-2} \times x - 2{\cdot}686 \times 10^{-3} \times x^2 + 2{\cdot}498 \times 10^{-5} \times x^3\} \qquad (9)$$

Deferred period 4 weeks

$$\sigma_x = \exp\{-4{\cdot}256 + 2{\cdot}392 \times 10^{-1} \times x - 6{\cdot}498 \times 10^{-3} \times x^2 + 5{\cdot}476 \times 10^{-5} \times x^3\} \qquad (10)$$

Deferred period 13 weeks

$$\sigma_x = \exp\{-2{\cdot}722 + 1{\cdot}290 \times 10^{-1} \times x - 4{\cdot}240 \times 10^{-3} \times x^2 + 3{\cdot}888 \times 10^{-5} \times x^3\} \qquad (11)$$

Deferred period 26 weeks

$$\sigma_x = \exp\{-4{\cdot}819 \times 10^{-1} - 8{\cdot}434 \times 10^{-2} \times x + 9{\cdot}749 \times 10^{-4} \times x^2\} \qquad (12)$$

Graphical summaries of the final graduations for D1, D4, D13 and D26 are shown in Figures C1, C2, C3 and C4, respectively. In particular, these figures

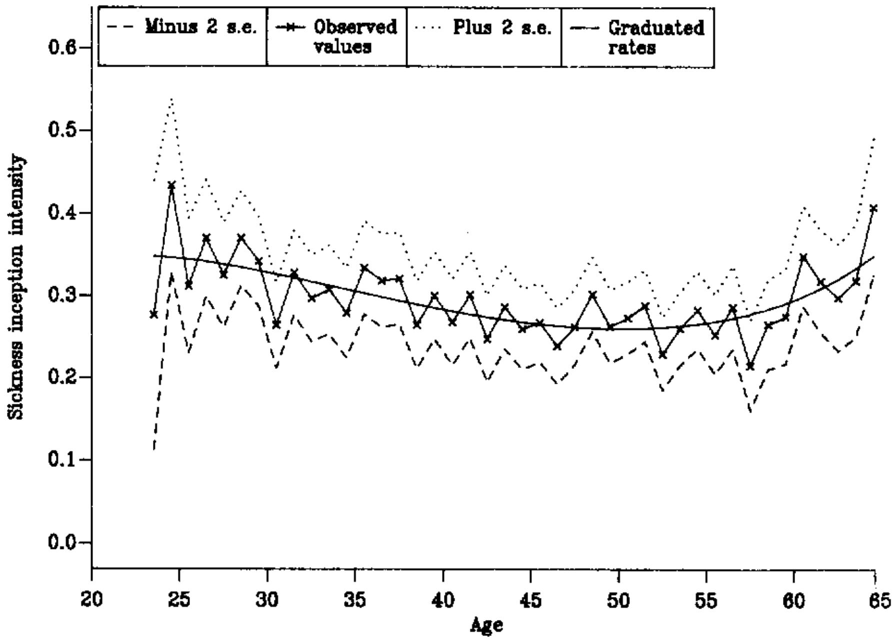

Figure C1. Sickness inception intensities: deferred period 1 week.

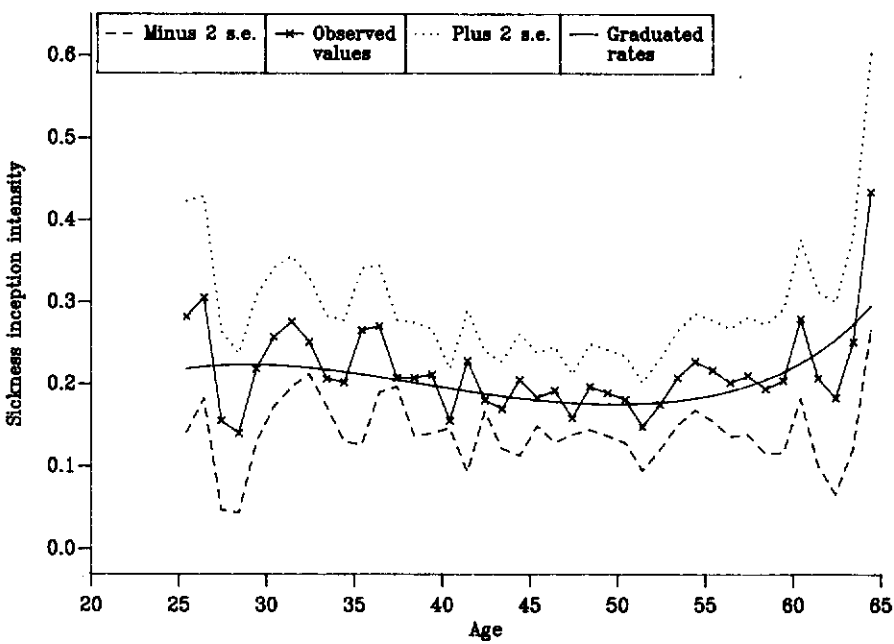

Figure C2. Sickness inception intensities: deferred period 4 weeks.

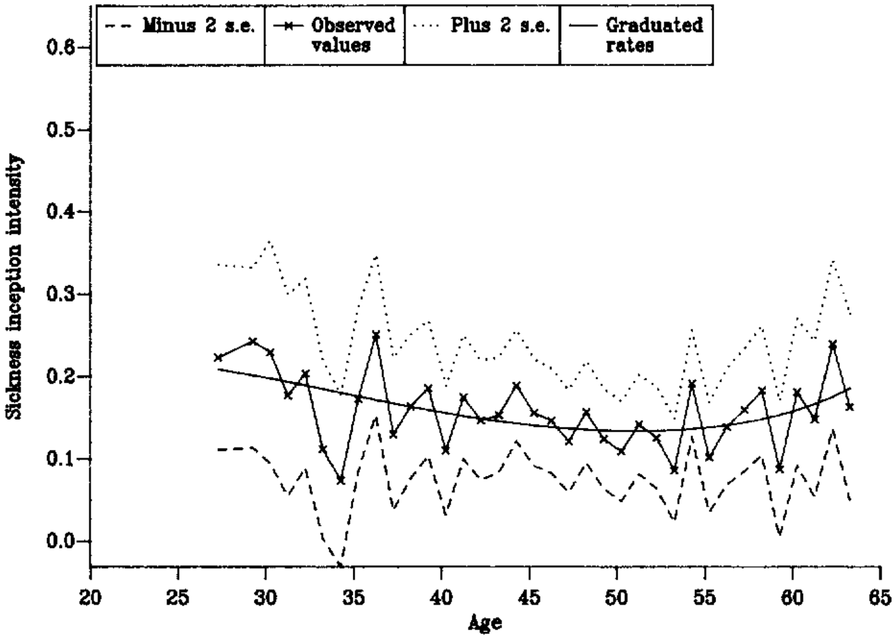

Figure C3. Sickness inception intensities: deferred period 13 weeks.

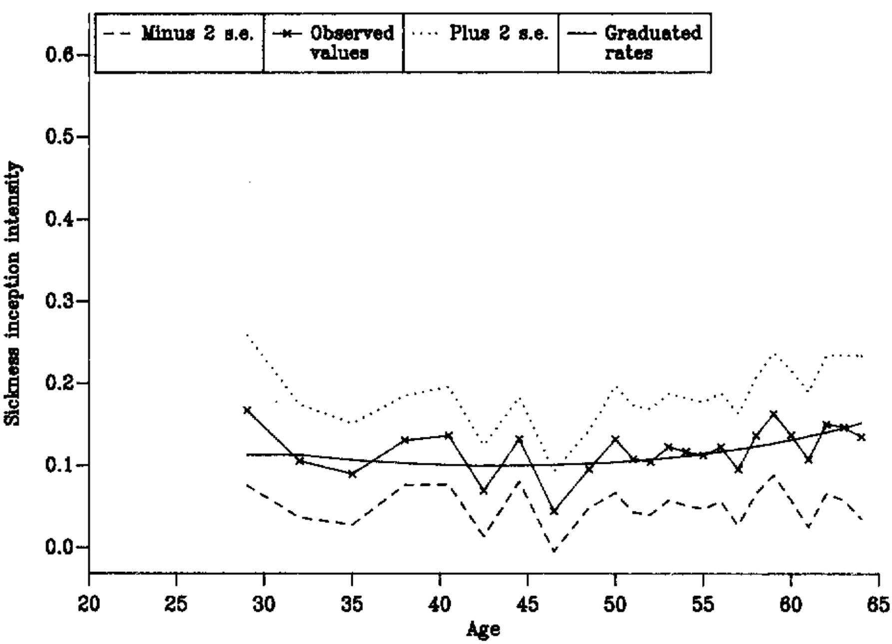

Figure C4. Sickness inception intensities: deferred period 26 weeks.

show the final graduation, the point estimates of σ_x and the point estimate $\pm$ 2 standard errors.

9.2 Table C16 gives graduated values of σ_x at selected exact ages x for each of the deferred periods. It may be seen from these figures that, apart from some slight aberrations between ages 20 and 30, where the data is relatively scanty, the functions have the same general shape, with a minimum at about age 50. It is tempting to speculate on why a sickness inception intensity should have such a shape, or even whether such a shape is reasonable. However, care should be exercised in such speculation. The estimates of σ_x given in Tables C5, C7, C8, and C14, and hence the final graduations of σ_x, depend very much on the assumed values for the intensities ρ and ν for durations of sickness within the deferred period, values which are the result of a (not unreasonable) extrapolation exercise. To put it another way, changes in the values of ρ and ν for durations of sickness within the deferred period which resulted in the probabilities π increasing by a constant factor would, broadly speaking, result in the estimates and graduated values of σ_x decreasing by the same factor, without any change to the expected number of claim inceptions. A more reasonable object of speculation is the product $\sigma_x \cdot \pi_{x,d}$. Intuitively, $\sigma_x \cdot \pi_{x,d}dt$ is the probability that a healthy individual aged x falls sick in a small interval of time dt and remains sick for at least the deferred period of his policy. Hence $\sigma_x \cdot \pi_{x,d}$ represents a claim inception intensity to be applied to healthy individuals. Table C17 gives values of $\sigma_x \cdot \pi_{x,d}$ for selected exact ages x and each deferred period.

Table C16. *Graduated values of* σ_x

	Deferred Period			
Age x	1 week	4 weeks	13 weeks	26 weeks
20	0·3486	0·1953	0·2172	0·1689
25	0·3452	0·2273	0·2145	0·1379
30	0·3282	0·2346	0·1982	0·1183
35	0·3054	0·2240	0·1766	0·1065
40	0·2832	0·2059	0·1560	0·1007
45	0·2667	0·1901	0·1408	0·1000
50	0·2600	0·1834	0·1337	0·1042
55	0·2673	0·1929	0·1375	0·1140
60	0·2952	0·2302	0·1576	0·1310
65	0·3569	0·3251	0·2073	0·1580

9.3 The values of $\sigma_x \cdot \pi_{x,d}$ given in Table C17 can reasonably be compared with the graduated claim inception rates given in *C.M.I.R.* **7**, Table H5. The former are, in general, a little higher for two reasons:

(i) the exposure appropriate to $\sigma_x \cdot \pi_{x,d}$ is time spent as healthy at age x, which is less than the exposure to which the graduated inception rates in *C.M.I.R.* 7 are designed to be applied,

(ii) for all deferred periods other than 1 week, the values of σ_x have been based

Table C17. *Values of 10,000* $\sigma_x \cdot \pi_{x,d}$

	Deferred Period			
Age x	1 week	4 weeks	13 weeks	26 weeks
20	1,164	79·7	10·75	2·246
25	1,237	117·9	15·49	2·965
30	1,262	155·9	20·87	4·105
35	1,260	190·6	27·13	5·975
40	1,254	224·6	34·91	9·113
45	1,267	265·4	45·89	14·58
50	1,325	328·0	63·43	24·46
55	1,461	441·7	94·89	43·00
60	1,732	675·0	158·2	79·32
65	2,246	1,220·0	302·3	153·4

on claim inceptions, both reported and unreported; the claim inception rates in *C.M.I.R.* 7 were based only on the reported inceptions.

9.4 A feature of the graduated values of σ_x, as shown in Table C16, is that, although σ_x has the same basic shape for each of the deferred periods, the values of σ_x decrease as the deferred period increases. (This is not strictly true for D4 and D13 at the youngest ages, but this is an area where there is little or no reliable data.) It is of interest to check whether the differences between the sickness inception intensities for different deferred periods are significant. The answer to this question is already reasonably clear, particularly for the shorter deferred periods, and can be seen by comparing the point estimates of σ_x given in Tables C5, C7, C8 and C14. Comparing the estimates in Tables C5 and C14 we can see that the estimate of σ_x for D4 is less than the corresponding estimate for D1 at every age except age 64. More formally we can check whether the data for one deferred period is consistent with the graduated values of σ_x for a different deferred period. If not, then we have been justified in graduating separately for these different periods. To clarify this, consider D1 and D4. At each age we have an observed number of claim inceptions, denoted I, for D1. Using the graduated values of σ_x for D4 we can calculate the expected numbers of claim inceptions for D1 at each age, denoted $I\hat{E}$. We can calculate $I\hat{E}$ using the formula

$$I\hat{E} = \sigma \cdot \pi \cdot \hat{E}$$

where σ is the appropriate value of σ_x using the graduation for D4, π is unchanged and the exposure $\hat{E}$ is calculated as in §4 with the exception that the factors p_x are calculated using the graduated values of σ_x for D4. If the data for D1 is consistent with the graduation of σ_x for D4, we would have for each age

$$I \sim N(I\hat{E}, V \cdot I\hat{E}) \qquad (13)$$

9.5 Table C18 shows the results of testing the data for D1 and D13 against the graduation of σ_x for D4. The results shown in Table C18 are

(i) the totals of I and $I\hat{E}$ over all ages included in the graduation of σ_x for D1 or D4.
(ii) the number of +'s, ages where I is greater than $I\hat{E}$, and −'s,
(iii) the total over all ages of $(I-\hat{I}E)^2/(V\cdot\hat{I}E)$, which, if (13) holds, has a χ^2 distribution with number of degrees of freedom equal to the number of ages.

Table C18. *Testing the data for deferred periods 1 week and 13 weeks against the graduated values of σ_x for deferred period 4 weeks*

	Deferred Period	
	1 week	13 weeks
I	11,068	633
$I\hat{E}$	8,070	851
χ^2	562·7	70·4
Degrees of freedom	42	36
Number of +'s	42	3
Number of −'s	0	33
Number of runs	1	7

9.6 Table C19 shows the results of testing the data for D4 and D26 against the graduated values of σ_x for D13. These two tables show very clearly that there are significant differences, in terms of the values of σ_x, between the data for different deferred periods and, in particular, that assuming all our other assumptions are valid, notably that the intensities ρ and ν do not depend on the deferred period, the sickness intensity decreases as the deferred period increases.

9.7 The magnitude of the differences between the graduated values of σ_x for different deferred periods, as shown in Table C16, may, at first sight, seem a little surprising. At ages 30 to 40 the ratio of the values of σ_x for D1 and D26 is about

Table C19. *Testing the data for deferred periods 4 weeks and 26 weeks against the graduated values of σ_x for deferred period 13 weeks*

	Deferred Period	
	4 weeks	26 weeks
I	2,365	340
$I\hat{E}$	1,782	367
χ^2	162·3	36·0
Degrees of freedom	40	24
Number of +'s	38	1
Number of −'s	2	23
Number of runs	3	3

3:1. However, the evidence for a substantial difference between D1 and D26 in the 1975–78 data can be seen in *C.M.I.R.* 7. From Tables K1 and K4 it can be seen that the total number of weeks of sickness of duration 26/all divided by the exposure for 26/26 is 0.5686 for D1 and 0.1893 for D26. The ratio of these two figures is almost exactly 3:1.

9.8 Having established that this difference exists, at least in the 1975-78 data, it is then necessary to explain, in terms of the model, how it arises. Broadly speaking, there are two possible explanations for this difference:

(a) the sickness inception intensities for D1 and D26 are the same, but the intensities of sickness termination (i.e. of recovery plus mortality from sick) in the first 26 weeks of sickness are different,
(b) the intensities of claim termination in the first 26 weeks of sickness are the same for D1 and D26 but the sickness inception intensities are different.

9.9 To investigate explanation (a) above a little more closely, let ${\sigma_x}^{(1)}$ and ${\sigma_x}^{(26)}$ denote the sickness inception intensities at age x for D1 and D26 respectively and let $(\rho_{x,z}^{(1)} + \nu_{x,z}^{(1)})$ and $(\rho_{x,z}^{(26)} + \nu_{x,z}^{(26)})$ denote the intensities of claim termination for D1 and D26 respectively. Let $\pi_x^{(26)}$ denote the probability that a sickness starting at age x lasts, for a policyholder having a deferred 26 weeks policy, for at least 26 weeks, i.e.

$$\pi_x^{(26)} = \exp\left(\int_0^{1/2} (\rho_{x+z,z}^{(26)} + \nu_{x+z,z}^{(26)})dz\right)$$

Let us assume that $\sigma_x^{(1)} = \sigma_x^{(26)}$ for all x and also that the common value is as for D1 in Table C16. Let us assume also that $\sigma_x^{(26)} \cdot \pi_x^{(26)}$ is as shown for D26 in Table C17. In other words, we are assuming that the sickness inception intensities for D1 and D26 are the same (and equal to the graduated values for D1) and that the 'claim intensity' for D26, the product $\sigma_x \cdot \pi_{x,d}$ is as in Table C17 for D26. These assumptions are not consistent with the intensities of claim termination being the same for D1 and D26 in the first 26 weeks of sickness. It is clear that $(\rho_{x,z}^{(26)} + \nu_{x,z}^{(26)})$ must be higher than $(\rho_{x,z}^{(1)} + \nu_{x,z}^{(1)})$ but how much higher? In an attempt to quantify this let us assume that there is a function $k(y)$ such that

$$(\rho_{x+z,z}^{(26)} + \nu_{x+z,z}^{(26)}) = k(x-z) \cdot (\rho_{x+z,z}^{(1)} + \nu_{x+z,z}^{(1)})$$

i.e. for sickness starting at age y there is a constant ratio, $k(y)$, between the sickness termination intensities for D1 and D26. We can calculate $k(y)$ from the figures given in Tables C16 and C17 and the assumption that $\sigma_x^{(26)} \cdot \pi_{x,d}^{(26)}$ is as given in Table C17. These calculations show that the values of $k(y)$ increase from 1.11 at age 20 to 1.29 at age 60, i.e. broadly speaking, the sickness termination intensity for D26 must be between 10% and 30% higher than for D1, in the first 26 weeks of sickness, if the sickness inception rates for these two deferred periods are to be the same.

The difficulty with adopting this explanation for the difference in experience between D1 and D26 is that there is insufficient data to reveal significant differences in the sickness termination intensities after 26 weeks of sickness between D1 and D26 other than the run-in period. For this reason the PHI Sub-Committee decided to assume that the sickness termination intensities for D1 and D26 (and also D4 and D13) were the same both before and after 26 weeks of sickness. In other words, the PHI Sub-Committee decided to explain the whole of the difference between D1 and D26 in terms of the sickness inception intensity, σ_x. The result is the set of values of σ_x shown in Table C16.

CMIR **12** (1991) 79–96

PART D: COMPUTATIONAL PROCEDURES FOR THE MODEL

BY H. R. WATERS

SUMMARY

Formulae for basic probabilities for our model are given in Part A. In this Part we present numerical algorithms which can be used to evaluate these probabilities. These algorithms are given in §2, with the assumption that the transition intensities, σ, μ, ρ and ν are known. In §3 we discuss the necessary amendments to these algorithms if μ is not known but the overall force of mortality is known. In §4 we discuss the construction of an increment–decrement table, which is the natural extension to our model of a simple life table. In §5 we discuss the definition, and calculation, of claim inception rates. In §6 we discuss Manchester Unity-type sickness rates, and, finally, in §7 we discuss the calculation of annuity values. Throughout this Part the emphasis is on practical formulae which can be used to evaluate the quantities being considered. Numerical examples of these calculations are given in Parts E and F.

1. INTRODUCTION

1.1 Part A of this Report discusses the model proposed for the analysis of PHI data, with the emphasis on the statistical estimation of the transition intensities but also mentioning formulae for basic probabilities. For this model to be of practical use it is important that it can be used to calculate premiums and reserves for PHI business, and to do this it must be possible to calculate basic probabilities, such as ${}_tp_x^{HH}$ and ${}_{w,t}p_x^{HS}$ in the notation of Part A, in an efficient manner. Sections 2 and 3 of this Part give a set of numerical algorithms which can be used to evaluate these basic probabilities and §4 shows how an increment–decrement table can be constructed from the basic probabilities; the final sections show how these basic probabilities can be used to calculate other quantities, which themselves are useful in the calculation of premiums and reserves.

1.2 Throughout most of this Part we shall assume that the transition intensities, μ_x, σ_x, $\rho_{x,z}$ and $\nu_{x,z}$, are known functions of x, or of x and z. (In §3 we shall discuss briefly how the algorithms should be amended in a situation where μ_x is not known but the overall force of mortality is known.) The choice of mathematical functions for σ_x, $\rho_{x,z}$ and $\nu_{x,z}$, i.e. the graduation of these transition intensities, is discussed elsewhere in this Report. Numerical examples of the application of the algorithms in this Part to the calculation of, for example, premiums for PHI business are presented in Parts E and F.

1.3 In terms of the simple life table, the problems addressed in this Part are the numerical evaluation of basic probabilities, such as ${}_tp_x$, and subsequently the evaluation of annuities and assurances, given the force of mortality, μ_x. However, while there is a very simple formula for ${}_tp_x$ in terms of μ_x, viz.

$${}_tp_x = \exp\{-\int_0^t \mu_{x+s}\, ds\}$$

the formulae for the basic probabilities for the PHI model described in Part A, such as ${}_tp_x^{HH}$ and ${}_{w,t}p_x^{HS}$, can be regarded as simultaneous integro-differential equations. The numerical solution of these equations requires a little care. The general method of solution of these equations, as described in this Part, has been described briefly, and exemplified in relation to the simple life table, by Waters and Wilkie (1987).

2. NUMERICAL CALCULATION OF PROBABILITIES

2.1 In this section we show how basic probabilities, such as ${}_tp_x^{HH}$ and ${}_{w,t}p_x^{HS}$, can be evaluated numerically when the transition intensities μ_x, σ_x, $\rho_{x,z}$ and $\nu_{x,z}$ are known functions of x, or of x and z. More precisely we consider for an individual who is healthy at some initial age, x_0 say, the probabilities that at some later age, x, he is healthy, ${}_{x-x_0}p_{x_0}^{HH}$, or that he is sick with duration of sickness between $(m-1)h$ and mh,

$${}_{mh,x-x_0}p_{x_0}^{HS} - {}_{(m-1)h,x-x_0}p_{x_0}^{HS}$$

where $m = 1, 2 \ldots$ and h is the 'step size' for the numerical algorithms. It is convenient to choose a value for h such that h^{-1} is an integer multiple of 52, and in the applications later in this Report we shall usually take h to be 1/156. In general, the smaller the value of h the more accurate the numerical procedures, and also the longer the computer time necessary to perform them; halving the value of h roughly quadruples the computer time necessary.

2.2 At this stage it is convenient to introduce a notation different from that used in Part A. We define $pH(x)$, $pS(x)$, $pD(x)$ and $pS(x,m)$ as follows:

$$pH(x) = {}_{x-x_0}p_{x_0}^{HH} \tag{1}$$

$$pS(x) = {}_{x-x_0,x-x_0}p_{x_0}^{HS} \tag{2}$$

$$pD(x) = {}_{x-x_0}p_{x_0}^{HD} \tag{3}$$

$$pS(x,m) = {}_{mh,x-x_0}p_{x_0}^{HS} - {}_{(m-1)h,x-x_0}p_{x_0}^{HS} \tag{4}$$

for any $x \geqslant x_0$ and $m = 1, 2, \ldots$, so that

- $pH(x)$ is the probability of being healthy at age x,
- $pS(x)$ is the probability of being sick at age x,
- $pD(x)$ is the probability of being dead at age x,
- $pS(x,m)$ is the probability of being sick at age x with duration of sickness between $(m-1)h$ and mh,

with all probabilities conditional on being healthy at the initial age x_0. Note that

$$pS(x) = \sum_m pS(x,m)$$

It is convenient also to put:

$pDH(x)$ is the probability of being dead at age x, having died as healthy,
$pDS(x)$ is the probability of being dead at age x, having died as sick, with $pD(x) = pDH(x) + pDS(x)$.

It will also be convenient to have a different notation for the transition intensities and so we define functions $s(x)$, $m(x)$, $r(x,m)$ and $n(x,m)$ as follows:

$$s(x) = \sigma_x \tag{5}$$

$$m(x) = \mu_x \tag{6}$$

$$r(x,m) = \rho_{x,(m-\frac{1}{2})h} \tag{7}$$

$$n(x,m) = \nu_{x,(m-\frac{1}{2})h} \tag{8}$$

for any x and $m = 1, 2, \ldots\ldots,$

2.3 We shall start by discussing the following approximate formula, which is valid for $m = 1, 2, \ldots.$

$$\begin{aligned} pS(x+h,m+1) \simeq pS(x,m) - \tfrac{1}{2}h\{&pS(x,m)[r(x,m)+n(x,m)] \\ &+ pS(x+h,m+1)[r(x+h,\, m+1)+n(x+h,m+1)]\} \end{aligned} \tag{9}$$

The derivation of this approximation is rather messy. To avoid distracting attention from the more important points being made in this Part, these details have been included as Appendix D1. Note that (9) can be given an intuitive explanation. In moving from $pS(x,m)$ to $pS(x+h, m+1)$ we are increasing both the attained age and the duration of sickness simultaneously by h. The difference between the two is the probability that the sickness terminates, either by recovery or death, in this interval of length h, and this is the second term on the right hand side of (9). In principle, provided that we can calculate $pS(x,1)$ for each x, we can use (9) to calculate $pS(x,m)$ recursively for each x and each $m > 1$. In practice this involves recording for each x the value of $pS(x,m)$ for a large number of values of m. We shall assume for the remainder of this Part that the intensities of recovery and death from sickness depend only on the individual's attained age after a sufficiently long duration of sickness. This assumption is not only computationally convenient, but is also not contradicted by the data used earlier in this Report to graduate $\rho_{x,z}$ and $\nu_{x,z}$. (See Part B.) More precisely, we shall assume that there is an integer N such that for each age x we have

$$r(x,m) = r(x,N) \tag{10}$$

$$n(x,m) = n(x,N) \tag{11}$$

whenever $m > N$.

2.4 We define $pS(x,N^+)$ for each x as follows:

$$pS(x,N^+) = \sum_{k=1}^{\infty} pS(x,N+k) \tag{12}$$

so that $pS(x,N^+)$ is the probability of being sick at age x with duration greater than Nh, conditional on being healthy at age x_0.

By rearranging (9), summing as appropriate and using (10), (11) and (12) we obtain our first two useful recurrence relations:

$$pS(x+h,m+1) \simeq \frac{pS(x,m)\{1-\frac{1}{2}h\,[r(x,m)+n(x,m)]\}}{\{1+\frac{1}{2}h[r(x+h,m+1)+n(x+h,m+1)]\}} \tag{13}$$

for $x \geqslant x_0$ and $m = 1, 2, \ldots, N-1$, and

$$pS(x+h,N^+) \simeq \frac{\{pS(x,N)+pS(x,N^+)\}\cdot\{1-\frac{1}{2}h\,[r(x,N)+n(x,N)]\}}{\{1+\frac{1}{2}h\,[r(x+h,N)+n(x+h,N)]\}} \tag{14}$$

2.5 Our next useful recurrence relation is

$$\begin{aligned} pH(x+h) \simeq pH(x) - \tfrac{1}{2}h\,[pH(x)\cdot\{s(x)+m(x)\} \\ + pH(x+h)\{s(x+h)+m(x+h)\}] \\ + \tfrac{1}{2}h \sum_{m=1}^{N} \{pS(x,m)\cdot r(x,m) + pS(x+h,m)\cdot r(x+h,m)\} \\ + \tfrac{1}{2}h\{pS(x,N^+)\cdot r(x,N) + pS(x+h,N^+)\cdot r(x+h,N)\} \end{aligned} \tag{15}$$

The intuitive interpretation of (15) is that to calculate the difference between $pH(x)$ and $pH(x+h)$ we must first subtract the probability of ceasing to be healthy between ages x and $x+h$, which is the second term on the right hand side of (15), and then add the probability of recovering from being sick between ages x and $x+h$, taking account of the duration of the sickness; this gives the final two terms on the right hand side of (15). More formally, (15) follows from (20) in Part A in the same way as (13) and (14) follow from (21) in Part A. See Appendix D.1 for details.

2.6 We need one more recurrence relation and this is as follows:

$$\begin{aligned} \tfrac{1}{2}h\{pH(x)\cdot s(x) + pH(x+h)\cdot s(x+h)\} \simeq pS(x+h,1)\cdot \\ \{1+\tfrac{1}{2}h\,[r(x+h,1)+n(x+h,1)]\} \end{aligned} \tag{16}$$

Intuitively, the left hand side of (16) is the probability of falling sick between ages x and $(x+h)$ and the right hand side is the probability of being sick at age $(x+h)$ with duration less than h plus the probability of recovering or dying within duration of sickness h. It is possible to derive (16) formally from (21) in Part A.

2.7 Now let us assume that $pH(x)$, $pS(x,m)$ and $pS(x,N^+)$ are known for some

x and $m = 1, 2, \ldots, N$. Using (13) and (14) we can calculate $pS(x+h,m)$ for $m = 2, \ldots, n$ and $pS(x+h,N^+)$. (15) and (16) are then two linear equations with two unknowns, $pS(x+h,1)$ and $pH(x+h)$, which can then be solved. From the initial conditions:

$$pH(x_0) = 1$$

$$pS(x_0,m) = 0 = pS(x_0,N^+)$$

we can then calculate recursively $pH(x)$, $pS(x,m)$ and $pS(x,N^+)$, and hence $pS(x)$, for any $x > x_0$, or at least for any x which is an integer multiple of h greater than x_0. (Note that we do not have to start with the initial distribution given above; any initial distribution at age x_0 can be specified. The same recursive procedure then applies.)

2.8 If we know $pH(x)$ and $pS(x)$ for some x, we can calculate $pD(x)$ from the obvious relationship

$$pD(x) = 1 - pH(x) - pS(x) \tag{17}$$

However, we can derive recurrence relations for $pDH(x)$ and $pDS(x)$ directly from (22) in Part A following the method of Appendix D.1. The recurrence relations are

$$pDH(x+h) \simeq pDH(x) + \tfrac{1}{2}h\{pH(x)\cdot m(x) + pH(x+h)\cdot m(x+h)\} \tag{18}$$

and

$$\begin{aligned} pDS(x+h) \simeq pDS(x) + \tfrac{1}{2}h \sum_{m=1}^{N} \{&pS(x,m)\cdot n(x,m) \\ &+ pS(x+h,m)\cdot n(x+h,m)\} \\ + \tfrac{1}{2}h\{pS(x,N^+)\cdot n(x,N) &+ pS(x+h,N^+)\cdot n(x+h,N)\} \end{aligned} \tag{19}$$

Intuitively, the probability of dying between x and $(x+h)$ is the sum of the probability of dying from being healthy, given by the second term on the right hand side of (18), and the probability of dying from being sick, taking account of the duration of sickness, given by the second and third terms on the right hand side of (19). It can be checked that the approximations used to derive (13), (14), (15), (16), (18) and (19) are consistent in the sense that if these relations are used to calculate $pH(x)$, $pS(x)$ and $pD(x)$, then (17) is satisfied. This is a useful computational check.

3. THE CALCULATION OF μ_x

3.1 The previous section provided numerical algorithms for the calculation of basic probabilities, conditional on an initial distribution at some initial age x_0, when the transition intensities σ_x, μ_x, $\rho_{x,z}$ and $\nu_{x,z}$ are known. In this section we show how these algorithms should be amended if μ_x is not known but the overall force of mortality is known.

3.2 We shall assume throughout this section that σ_x, $\rho_{x,z}$ and $\nu_{x,z}$ are known and that the individual was healthy at the initial age x_0. This individual's overall, or total, force of mortality at age x_0+t, denoted $\mu L_{[x_0]+t}$ is defined by

$$\mu L_{[x_0]+t} = -\left({}_tp_{x_0}^{HH} + {}_tp_{x_0}^{HS}\right)^{-1}\frac{d}{dt}\left({}_tp_{x_0}^{HH} + {}_tp_{x_0}^{HS}\right) \tag{20}$$

This can be rewritten

$$\mu L_{[x_0]+t} = \left[\mu_{x_0+t} \cdot {}_tp_{x_0}^{HH} + \int_0^\infty \nu_{x_0+t,z} \cdot \frac{d}{dz}\left({}_{z,t}p_{x_0}^{HS}\right)dz\right] \cdot \left[{}_tp_{x_0}^{HH} + {}_tp_{x_0}^{HS}\right]^{-1} \tag{21}$$

which shows that the overall force of mortality is a weighted average of $\mu_{x_0+t,z}$ and $\nu_{x_0+t,z}$ with weights equal to the probabilities of being healthy or being sick with duration z at age x_0+t.

3.3 In the spirit of the notation introduced in §2 for the basic transition intensities, we make the following notational definitions:

$$mL(x) = \mu L_{[x_0]+x-x_0} \tag{22}$$

$$mS(x) = \left[\int_0^\infty \nu_{x_0+t,z} \cdot \frac{d}{dz}\left({}_{z,t}p_{x_0}^{HS}\right)dz\right] / {}_tp_{x_0}^{HS} \tag{23}$$

So that $mS(x)$ represents the average force of mortality among the sick, weighted by duration. Note that both $mL(x)$ and $mS(x)$ are conditional on the individual being healthy at age x_0. The following approximate formulae can then be derived:

$$mS(x) = \sum_{m=1}^{N} n(x,m) \cdot pS(x,m)/pS(x) + n(x,N) \cdot pS(x,N^+)/pS(x) \tag{24}$$

$$mL(x) = [m(x) \cdot pH(x) + mS(x) \cdot pS(x)] / [pH(x) + pS(x)] \tag{25}$$

3.4 It is convenient to define $pL(x)$ to be the probability that the individual is alive at age x, whether healthy or sick, given that he was alive and healthy at the initial age x_0. Hence,

$$pL(x) = pH(x) + pS(x) \tag{26}$$

Next we note that the following approximate relationship holds

$$pL(x+h) \simeq pL(x)[1 - \tfrac{1}{2}h \cdot mL(x)] / [1 + \tfrac{1}{2}h \cdot mL(x+h)] \tag{27}$$

(See Waters and Wilkie (1987, formula (10A)).)

Let us suppose now that for some age x_0+t, where t is an integer multiple of the step size h, we know the values of

$$pH(x_0+t), pS(x_0+t,m), m = 1, 2, \ldots N,$$

$pS(x_0+t,N^+)$ and $m(x_0+t)$, as well as the values of the other transition intensities. Then we can calculate

$$pS(x_0 + t + h,m)\ m = 2, 3, \ldots, N, \text{ and } pS(x_0 + t + h,N^+)$$

using (13) and (14). We can calculate $pL\,(x_0 + t + h)$ from (27). Adding formulae (18) and (19), and noting that

$$pDH(x_0 + t + h) + pDS(x_0 + t + h) = 1 - pL\,(x_0 + t + h) \qquad (28)$$

gives us, together with formulae (15) and (16), three linear equations in three unknowns, $pH(x_0 + t + h)$, $pS(x_0 + t + h,1)$ and $m(x + t + h)$, which can then be solved. This recursive procedure starts at age x_0, where

$$m(x_0) = mL\,(x_0) \qquad (29)$$

Hence we can calculate recursively $m(x_0 + h)$, $m(x_0 + 2h)$, etc. It should be noted that the value of $m(x)$ calculated in this way might not be positive; for $m(x)$ to be positive we must assume an overall force of mortality which is consistent with the graduated values of σ_x, $\rho_{x,z}$ and $\nu_{x,z}$.

4. THE CONSTRUCTION OF AN INCREMENT–DECREMENT TABLE

4.1 The basic probabilities for a simple life table can be conveniently summarized, usually at integral ages, by columns of l_x, the number alive at age x out of a given number (the radix) alive at some initial age, and of d_x, the number dying between ages x and $x+1$. A corresponding summary can be constructed for the model for PHI discussed in this Report, although the resulting table, an increment-decrement table, is somewhat more complex than the usual life table.

4.2 We start with a radix, R, at some initial age x_0. R represents a (large) number of lives aged x_0 with some specified initial distribution of healthy lives and sick lives, with specific durations of sickness. In all our illustrations in Part E we shall assume that all lives are healthy at age x_0, although this assumption is not necessary. We make the following definitions for age $x > x_0$:

$lH(x)$ the number healthy at age x $(= R \cdot pH(x))$
$lS(x)$ the number sick at age x, at all durations combined $(= R \cdot pS(x))$
$lDH(x)$ the number dead at age x, having died as healthy $(= R \cdot pDH(x))$
$lDS(x)$ the number dead at age x, having died as sick $(= R \cdot pDS(x))$,
$lL(x)$ the number alive at age x $(= lH(x) + lS(x))$
$lD(x)$ the number dead at age x $(= lDH(x) + lDS(x))$

(Note that for convenience we refer to $lH(x)$, for example, as the number healthy at age x rather than as the expected number healthy at age x. Thus in this section we could be said to be taking a deterministic, rather than stochastic, view of the model. Note also that to emphasise the analogy with a simple life table, we shall sometimes denote $lL(x)$ by l_x in the remainder of this Report.)

4.3 The quantities defined below correspond to d_x for a simple life table.

$dHS(x)$ the number of transitions from healthy to sick between ages x and $x+1$
$dSH(x)$ the number of transitions from sick to healthy between ages x and $x+1$
$dHD(x)$ the number of transitions from healthy to dead between ages x and $x+1$
$dSD(x)$ the number of transitions from sick to dead between ages x and $x+1$.

4.4 The quantities defined in the previous paragraph can be calculated as follows:

$$dHS(x) \simeq R\sum_{j=0}^{h^{-1}-1} \tfrac{1}{2}h\,[pH(x+jh)\cdot s(x+jh)+pH(x+(j+1)h)\cdot s(x+(j+1)h)] \qquad (30)$$

$$\begin{aligned} dSH(x) \simeq{} & R\sum_{j=0}^{h^{-1}-1} \tfrac{1}{2}h \sum_{m=1}^{N} \{pS(x+jh,m)\cdot r(x+jh,m) \\ & + pS(x+(j+1)h,m)\cdot r(x+(j+1)h,m)\} \\ & + R\sum_{j=0}^{h^{-1}-1} \tfrac{1}{2}h\{pS((x+jh),N^{+})\cdot r(x+jh,N) \\ & + pS(x+(j+1)h,N^{+})\cdot r(x+(j+1)h,N)\} \end{aligned} \qquad (31)$$

$$\begin{aligned} dHD(x) \simeq{} & R\sum_{j=0}^{h^{-1}-1} \tfrac{1}{2}h\{pH(x+jh)\cdot m(x+jh) \\ & + pH(x+(j+1)h)\cdot m(x+(j+1)h\} \end{aligned} \qquad (32)$$

$$\begin{aligned} dSD(x) \simeq{} & R\sum_{j=0}^{h^{-1}-1} \tfrac{1}{2}h \sum_{m=1}^{N} \{pS(x+jh,m)\cdot n(x+jh,m) \\ & + pS(x+(j+1)h,m)\cdot n(x+(j+1)h,m)\} \\ & + R\sum_{j=0}^{h^{-1}-1} \tfrac{1}{2}h\{pS(x+jh,N^{+})+n(x+jh,N) \\ & + pS(x+(j+1)h,N^{+})\cdot n(x+(j+1)h,N)\} \end{aligned} \qquad (33)$$

Note that (30) and (31) represent the sum over the steps of age $x\rightarrow x+h$, $x+h\rightarrow x+2h,\ldots, x+1-h\rightarrow x+1$ of the number of relevant transitions (multiplied by

R) as given by the right hand side of (15). Formulae (32) and (33) can be derived similarly using (18) and (19) respectively.

5. CLAIM INCEPTION RATES

5.1 In this section we show how we can calculate a claim inception rate. The reader should note that claim inception rates can be defined in a variety of similar, but not identical, ways. In this section we discuss only two of these ways. Throughout this section we assume that we start with R individuals at age x_0, all of whom are then healthy.

5.2 We define the following function, L_x, which is analogous to the corresponding function for a simple life table.

$$L_x = \int_x^{x+1} l_y \, dy \qquad x \geqslant x_0 \tag{34}$$

so that L_x represents the (expected) number of years lived between ages x and $x+1$ by the R individuals who were healthy at age x_0. We could also interpret L_x as the average number of individuals alive between ages x and $x+1$. We can calculate L_x as follows:

$$L_x = \tfrac{1}{2}h \sum_0^{h^{-1}-1} \{l_{x+jh} + l_{x+(j+1)h}\} \tag{35}$$

5.3 Next we define a function $ca(x,d)$, which represents the (expected) number of claim inceptions between ages x and $x+1$ for a given deferred period d. For any step size h we can write

$$ca(x,d) = \sum_0^{h^{-1}-1} E\,[\text{number of claim inceptions between ages } (x+jh) \text{ and } (x+(j+1)h)\,|\,R \text{ individuals were healthy at age } x_0] \tag{36}$$

For a value of h less than the deferred period d, an individual can have at most one claim inception between ages $(x+jh)$ and $(x+(j+1)h)$. Hence (36) can be written

$$ca(x,d) = R\sum_0^{h^{-1}-1} P\,[\text{a claim inception between ages } (x+jh) \text{ and } (x+(j+1)h)\,|\,\text{the individual was healthy at age } x_0] \tag{37}$$

We can calculate $ca(x,d)$ from formula (37), at least approximately, as follows

$$ca(x,d) = R\sum_{0}^{h^{-1}-1} pS\left(x + (j+1)h, \frac{d}{h} + 1\right)\left\{1 + \frac{1}{2}h \cdot n\left(x + (j+1)h, \frac{d}{h} + 1\right)\right.$$

$$\left. + \tfrac{1}{2}h \cdot r\left(x + (j+1)h, \frac{d}{h} + 1\right)\right\} \quad (38)$$

In the special case where the deferred period d is zero (which is useful for illustrative if not for practical purposes) we can still regard (38) as a reasonable approximation to $ca(x,0)$ provided the step size h is sufficiently small for the probability of two or more sicknesses starting within a time interval of length h to be negligible. In this case it would also be possible to use (30) to approximate $ca(x,0)$ since $ca(x,0)$ is equivalent to $dHS(x)$. Note that (38) is (R times) the probability that an individual is sick at exact age $(x+(j+1)h)$ with duration of sickness between d and $d+h$ plus an approximation to the probability of dying or recovering between ages $(x+jh)$ and $(x+(j+1)h)$ and between durations d and $d+h$. (See formula (15).)

5.4 We denote by $ia(x,d)$ the claim inception rate at age x for deferred period d defined as follows

$$ia(x,d) = ca(x,d)/L_x \quad (39)$$

Note that $ia(x,d)$ is, in probabilistic terms, the ratio of two expected values: the expected number of claim inceptions between ages x and $x+1$ for deferred period d and the expected number of years lived between these ages, in both cases out of R individuals who were healthy at some initial age x_0. Note that the value of the radix R affects the numerical values of $ca(x,d)$ and L_x, but not of $ia(x,d)$, and that the initial condition that all individuals were healthy at age x_0 affects the values of all three functions. The dependence of $ia(x,d)$ on the initial condition that all individuals were healthy at age x_0 is not explicit in the notation. For some purposes it is convenient to make this dependence explicit in the notation and for this reason we define $ia(x,d,t)$ to be the same as $ia(x,d)$ with t denoting the duration since the lives were 'select', i.e. $t = x - x_0$.

5.5 We shall refer to $ia(x,d)$ as a 'type (a) inception rate'. We can define a 'type (b) inception rate' as follows. For any age x and deferred period d we define $cb(x,d)$ by

$$cb(x,d) = ca(x+d,d) \quad (40)$$

so that $cb(x,d)$ represents the (expected) number of claim inceptions between ages $(x+d)$ and $(x+d+1)$ out of R individuals who were healthy at age x_0. We can interpret $cb(x,d)$ as the (expected) number of sickness inceptions between ages x and $(x+1)$ which will eventually give rise to claims (between ages $(x+d)$ and $(x+1+d)$) out of R individuals who were healthy at age x_0. We can calculate

$cb(x,d)$ using formula (38) with $(x+d)$ replacing x. We can also approximate $cb(x,d)$ by

$$cb(x,d) \simeq (1-d) \cdot ca(x,d) + d \cdot ca(x+1,d) \tag{41}$$

We define our second claim inception rate, $ib(x,d)$, as follows

$$ib(x,d) = cb(x,d)/L_x \tag{42}$$

Note that $ib(x,d)$ is not equal to $ia(x+d,d)$ but, using (40), and (41), we have the following approximation

$$ib(x,d) \simeq [(1-d) \cdot L_x \cdot ia(x,d) + d \cdot L_{x+1} \cdot ia(x+1,d)]/L_x \tag{43}$$

6. MANCHESTER-UNITY-TYPE SICKNESS RATES

6.1 In this section we show how we can define and calculate Manchester-Unity-type sickness rates using the basic probabilities for the model. We start by introducing some notation which will be very convenient in this section. For $x > x_0$ and numbers a and b, which must be non-negative integer multiples of the step size h, we define

$$pS\,(x,a/b) = \sum_{a/h+1}^{(a+b)/h} pS(x,m) \tag{44}$$

so that $pS(x,a/b)$ is the probability of being sick at exact age x with duration of sickness between a and $(a+b)$, conditional as always on being healthy at age x_0. Next we introduce the function $\zeta(x,a/b)$ which is defined by

$$\zeta(x,a/b) = 52 \cdot pS\,(x,a/b)\,/pL\,(x) \tag{45}$$

so that $\zeta(x,a/b)$ represents (52 times) the probability that an individual is sick at age x with duration of sickness between a and $(a+b)$, given that the individual is alive at age x and was healthy at some earlier age x_0.

6.2 Finally in this section we define the function $z(x,a/b)$ as follows

$$z\,(x,a/b) = \left[52 \int_x^{x+1} pS\,(y,a/b)\,dy \right] \Bigg/ \left[\int_x^{x+1} pL\,(y)\,dy \right] \tag{46}$$

which, using (34), can be written

$$z(x,a/b) = 52 \cdot R \int_x^{x+1} pS\,(y,a/b)\,dy/L_x \tag{47}$$

The numerator of (47) can be calculated using numerical integration as follows

$$52R \int_{x}^{x+1} pS\,(y,a/b)\,dy \simeq$$

$$52R\left[\tfrac{1}{2}h \sum_{j=0}^{h^{-1}-1} \left(pS\,(x+jh,a/b) + pS\,(x+(j+1)\,h,a/b)\right)\right] \quad (48)$$

The denominator of (47) can be calculated using (35). The numerator of (47) can be interpreted as the expected number of weeks of sickness between durations a and $a+b$ between ages x and $x+1$ for a group of R individuals who were healthy at age x_0; the denominator represents the expected man-years lived between ages x and $x+1$ by these individuals. It can be seen that $z(x,a/b)$ is very similar in definition to the central rate of sickness $z_x^{a/b}$, the important difference being that both the numerator and denominator of $z(x,a/b)$ are conditional on an initial status at some earlier age x_0.

7. THE EXACT CALCULATION OF MONETARY FUNCTIONS

7.1 In this section we consider the evaluation of monetary functions which are useful for the calculation of premiums and reserves for PHI business. In particular we consider the evaluation of annuities payable while the individual has a given status; for example, sick, with duration between a and $(a+b)$. The formulae given in this section for the evaluation of these annuities are based on the probabilities whose evaluation is outlined in §2 of this Part. Numerical illustrations of these formulae will be given in Part F. Also given in Part F will be alternative, approximate, formulae for the evaluation of these annuities.

7.2 In this paragraph we consider continuous annuities payable in a particular status. All the annuities in this paragraph are the expected present values of annuities of 1 p.a. payable continuously for a maximum of n years to an individual who is currently healthy at age x_0. These annuities are payable only when the individual is in a given status. The notation for these annuities is as follows:

$\bar{a}^{HL}_{x_0:\overline{n|}}$ which is payable only if the individual is alive,
$\bar{a}^{HH}_{x_0:\overline{n|}}$ which is payable only if the individual is healthy,
$\bar{a}^{HS}_{x_0:\overline{n|}}$ which is payable only if the individual is sick,
$\bar{a}^{HS(a/b)}_{x_0:\overline{n|}}$ which is payable only if the individual is sick, with duration of sickness between a and $(a+b)$.

Formulae for these annuities are as follows:

$$\bar{a}^{HH}_{x_0:\overline{n|}} = \int_0^n v^t\,{}_tp^{HH}_{x_0}\,dt \quad (49)$$

$$\bar{a}^{HS}_{x_0:\overline{n}|} = \int_0^n v^t \,{}_t p^{HS}_{x_0}\, dt \tag{50}$$

$$\bar{a}^{HS(a/b)}_{x_0:\overline{n}|} = \int_0^n v^t \left({}_{(a+b),t} p^{HS}_{x_0} - {}_{a,t} p^{HS}_{x_0} \right) \tag{51}$$

$$\bar{a}^{HL}_{x_0:\overline{n}|} = \bar{a}^{HH}_{x_0:\overline{n}|} + \bar{a}^{HS}_{x_0:\overline{n}|} = \int_0^n v^t \left({}_t p^{HH}_{x_0} + {}_t p^{HS}_{x_0} \right) dt \tag{52}$$

where $v = (1+i)^{-1}$ and i is the rate of interest. We can evaluate these annuities, at least approximately, using the repeated trapezium rule with step size h, together with the algorithms in §2 above. The relevant formulae are:

$$\bar{a}^{HH}_{x_0:\overline{n}|} \simeq \sum_0^{(n/h)-1} \tfrac{1}{2} h \left[v^{jh} \cdot pH(x_0 + jh) + v^{(j+1)h} \cdot pH(x_0 + (j+1)h \right] \tag{53}$$

$$\bar{a}^{HS}_{x_0:\overline{n}|} \simeq \sum_0^{(n/h)-1} \tfrac{1}{2} h \left[v^{jh} \cdot pS(x_0 + jh) + v^{(j+1)h}\, pS(x_0 + (j+1)h) \right] \tag{54}$$

$$\bar{a}^{HS(a/b)}_{x_0:\overline{n}|} \simeq \sum_0^{(n/h)-1} \tfrac{1}{2} h \left[v^{jh}\, pS\,(x_0 + jh, a/b) + v^{(j+1)h} \cdot pS_0\,(x_0 + (j+1)\,h, a/b) \right] \tag{55}$$

$$\bar{a}^{HL}_{x_0:\overline{n}|} \simeq \sum_0^{(n/h)-1} \tfrac{1}{2} h \left[v^{jh} \cdot pL\,(x_0 + jh) + v^{(jh+1)h} \cdot pL\,(x_0 + (j+1)h) \right] \tag{56}$$

7.3 In this paragraph we consider a 'current claim' annuity, which we denote $\bar{a}^{\overline{SS}}_{x,z:\overline{n}|}$. This annuity is the expected present value of an annuity of 1 p.a. payable continuously for a maximum of n years to an individual who is currently aged x and sick, with duration of sickness z. The annuity ceases at age $(x+n)$, or on recovery or on death, whichever occurs soonest. The integral formula for this annuity is:

$$\bar{a}^{\overline{SS}}_{x,z:\overline{n}|} = \int_0^n v^t \cdot {}_t p^{\overline{SS}}_{x,z}\, dt \tag{57}$$

which can be evaluated using the formulae:

$$\bar{a}^{\overline{SS}}_{x,z:\overline{n}|} \simeq \sum_0^{(n/h)-1} \tfrac{1}{2} h \left[v^{jh} \cdot {}_{jh} p^{\overline{SS}}_{x,z} + v^{(j+1)h} \cdot {}_{(j+1)h} p^{\overline{SS}}_{x,z} \right] \tag{58}$$

$${}_{t+h} p^{\overline{SS}}_{x,z} \simeq {}_t p^{\overline{SS}}_{x,z} \exp\{ -\tfrac{1}{2} h \left(\rho_{x+t,z+t} + \nu_{x+t,z+t} + \rho_{x+t+h,z+t+h} + \nu_{x+t+h,z+t+h} \right) \} \tag{59}$$

where (59) follows from formula (27) in Part A.

7.4 Now consider an annuity similar to $\bar{a}^{\overline{SS}}_{x,z:\overline{n}|}$, but payable monthly in advance rather than continuously. We denote the value of this annuity $\ddot{a}^{(12)\overline{SS}}_{x,z:\overline{n}|}$. Hence, $\ddot{a}^{(12)\overline{SS}}_{x,z:\overline{n}|}$ denotes the value of an annuity of 1 p.a. payable monthly in advance (so that the first payment is payable immediately) for at most n years to an individual who is currently aged x and sick, with duration of sickness z. The annuity ceases at age $(x+n)$, or on recovery or on death, whichever occurs soonest. The formula for this annuity is

$$\ddot{a}^{(12)\overline{SS}}_{x,z:\overline{n}|} = \sum_{0}^{12n-1} \tfrac{1}{12} v^{t/12} \cdot {}_{t/12}p^{\overline{SS}}_{x,z} \tag{60}$$

which can be used to evaluate $\ddot{a}^{(12)\overline{SS}}_{x,z:\overline{n}|}$ with the help of (the approximate) formula (59).

7.5 In this paragraph we consider annuities payable monthly in advance corresponding to the continuous annuities discussed in §7.2. Formulae for these annuities are as follows:

$$\ddot{a}^{(12)HH}_{x_0:\overline{n}|} = \sum_{0}^{12n-1} \tfrac{1}{12} v^{t/12} \cdot {}_{t/12}p^{HH}_{x_0} \tag{61}$$

$$= \sum_{0}^{12n-1} \tfrac{1}{12} \cdot v^{t/12} \cdot pH(x_0 + t/12) \tag{62}$$

$$\ddot{a}^{(12)HS}_{x_0:\overline{n}|} = \sum_{0}^{12n-1} \tfrac{1}{12} v^{t/12} \cdot {}_{t/12}p^{HS}_{x_0} \tag{63}$$

$$= \sum_{0}^{12n-1} \tfrac{1}{12} \cdot v^{t/12} \cdot pS(x_0 + t/12) \tag{64}$$

$$\ddot{a}^{(12)HS(a/b)}_{x_0:\overline{n}|} = \sum_{0}^{12n-1} \tfrac{1}{12} v^{t/12} \cdot \left({}_{a+b,t/12}p^{HS}_{x_0} - {}_{a,t/12}p^{HS}_{x_0}\right) \tag{65}$$

$$= \sum_{0}^{12n-1} \tfrac{1}{12} \cdot v^{t/12} \cdot pS(x_0 + t/12, a/b) \tag{66}$$

$$\ddot{a}^{(12)HL}_{x_0:\overline{n}|} = \sum_{0}^{12n-1} \tfrac{1}{12} v^{t/12} \cdot \left({}_{t/12}p^{HH}_{x_0} + {}_{t/12}p^{HS}_{x_0}\right) \tag{67}$$

$$= \sum_{0}^{12n-1} \tfrac{1}{12} \cdot v^{t/12} \cdot pL\,(x_0 + t/12) \tag{68}$$

Note that in each case the payment of 1/12 is made at time $t/12$ if and only if the individual is in the specified status at that time.

7.6 Finally in this section we consider an annuity which is rather less straightforward than those considered so far. Consider the value at age x_0, for an individual who is healthy, of all future claims, payable continuously at the rate of 1 p.a., up to age (x_0+n), counting only those claims for which the sickness started between ages (x_0+m_1) and (x_0+m_2) where $m_1<m_2<n$. (In practice, particularly for group PHI, we may be interested in the case where $m_1 = 0$ and $m_2 = 1$.) The value of this annuity can be written:

$$\int_{y=x_0+m_1}^{x_0+m_2} v^{y-x_0} \cdot {}_{y-x_0}p_{x_0}^{HH} \cdot \sigma_y \cdot \int_{t=d}^{n-y} v^t \cdot {}_tp_y^{\overline{SS}}\, dt\, dy \tag{69}$$

or as:

$$\int_{y=x_0+m_1}^{x_0+m_2} v^{y-x_0} \cdot {}_{y-x_0}p_{x_0}^{HH} \cdot \sigma_y \cdot v^d \cdot \pi_{y,d} \cdot \bar{a}^{\overline{SS}}_{y+d,d:\overline{n-y-d}|}\, dy \tag{70}$$

(Recall that d denotes the deferred period for the policy.) Intuitively, (69) can be explained as follows: y is the age at which a sickness leading to a claim starts (the possible values of y are $x_0 + m_1 \leqslant y \leqslant (x_0 + m_2)$ and t is the time after age y when a payment is made (the possible values of t are $d \leqslant t \leqslant x_0 + n - y$). (Note that, as with the annuities in §§7.2 and 7.5, the annuity valued by (69) may include several disjoint periods, corresponding to different bouts of sickness, when the annuity is payable. This is not the case for the 'current claim' annuities considered in §§7.3 and 7.4.). The payments valued in formula (69) can be valued by a different method, separating those payments made between ages $(x_0 + m_1 + d)$ and $(x_0 + m_2 + d)$ from those made between ages $(x_0 + m_2 + d)$ and $(x_0 + n)$. Consider the following two values:

$$\int_{y=x_0+m_1+d}^{x_0+m_2+d} v^{y-x_0} \cdot \{{}_{y-x_0-m_1,y-x_0}p_{x_0}^{HS} - {}_{d,y-x_0}p_{x_0}^{HS}\}\, dy \tag{71}$$

and

$$\int_{y=x_0+m_1+d}^{x_0+n} v^{y-x_0} \cdot \{{}_{y-x_0-m_2,y-x_0}p_{x_0}^{HS} - {}_{y-x_0-m_1,y-x_0}p_{x_0}^{HS}\}\, dy \tag{72}$$

Formula (71) gives the value of a continuous annuity at the rate of 1 p.a. payable from ages $(x_0 + m_1 + d)$ to $(x_0 + m_2 + d)$ if an individual now aged x_0 and healthy is sick with duration of sickness between d and $(y - x_0 - m_1)$. Formula (72) gives the value of payments between ages $(x_0 + m_2 + d)$ and $(x_0 + n)$, where payments are made only if the duration of sickness is between $(y - x_0 - m_2)$ and $(y - x_0 - m_1)$. It can be seen intuitively that (69) equals the sum of (71) and (72). The algebraic proof of this is included as Appendix D2. These annuities are not illustrated numerically in Part F and hence we have not given formulae in terms of $pH(\ )$,$pS(\ )$ etc. corresponding to (69), (71) and (72).

APPENDIX D1

In this Appendix we give details of the derivation of (9); the derivations of the other recurrence relations in §2 are similar.

From the definition of $pS(x+h,m+1)$ and formula (28) in Part A, we have

$$pS(x+h,m+1)-pS(x,m) = \int_{(m-1)h}^{mh} {}_{x-x_0-s}p_{x_0}^{HH} \cdot \sigma_{x-s}\{{}_{s+h}p_{x-s}^{\overline{SS}} - {}_{s}p_{x-s}^{\overline{SS}}\}\, ds \quad (73)$$

provided $x-x_0>mh$. It is implicit in §2 that $(x-x_0)$ is an integer multiple of h and, if $x-x_0<mh$ there is nothing to prove since both $pS(x,m)$ and $pS(x+h,m+1)$ are zero. Using formula (27) in Part A, the right hand side of (73) can be written

$$\int_{(m-1)h}^{mh} {}_{x-x_0-t}p_{x_0}^{HH} \cdot \sigma_{x-t} \cdot {}_{t}p_{x-t}^{\overline{SS}}\{\exp[-\int_0^h (\rho_{x+u,t+u} + \nu_{x+u,t+u})du] - 1\}\, dt \quad (74)$$

and using the mean value theorem for integrals and formula (28) in Part A, (74) is equivalent to

$$[\exp\{-\int_0^h (\rho_{x+u,t+u} + \nu_{x+u,t+u})\, du\} - 1]\, pS(x,m) \quad (75)$$

for some t between $(m-1)h$ and mh. The mean value theorem for derivatives applied to the first term in (75) shows that (75) is equivalent to

$$-h\{\rho_{x+f,t+f} + \nu_{x+f,t+f}\} \exp\{-\int_0^f (\rho_{x+u,t+u} + \nu_{x+u,t+u})\, du\}\, pS(x,m) \quad (76)$$

for some f between 0 and h. Formula (76) is an exact expression for $(pS(x+h,m+1)-pS(x,m))$; to obtain an approximation we first take the average of (76) with $f=0$ and $f=h$ as follows:

$$pS(x+h,m+1)-pS(x,m) \simeq [-\tfrac{1}{2}h\{\rho_{x,t} + \nu_{x,t}\} - \tfrac{1}{2}h\{\rho_{x+h,t+h} + \nu_{x+h,t+h}\} \cdot \exp\{-\int_0^h(\rho_{x+u,t+u} + \nu_{x+u,t+u})\, du\}] \cdot pS(x,m) \quad (77)$$

Now note that from the definition of t

$$\exp\{-\int_0^h (\rho_{x+u,t+u} + \nu_{x+u,t+u})\, du\} \cdot pS(x,m) = pS(x+h,m+1) \quad (78)$$

Using (77) and (78) and then replacing the remaining t's by the average value $(m-\frac{1}{2})h$ we obtain

$$pS(x+h,m+1)-pS(x,m)$$
$$\simeq -\tfrac{1}{2}h\{\rho_{x+h,(m+\frac{1}{2})h}+\nu_{x+h,(m+\frac{1}{2}h)}\}\,pS(x+h,m+1)$$
$$-\tfrac{1}{2}h\{\rho_{x,(m-\frac{1}{2})h}+\nu_{x,(m-\frac{1}{2})h}\}\,pS(x,m) \quad (79)$$

which is equivalent to (9).

APPENDIX D2

In this Appendix we show that formula (69) is the sum of formulae (71) and (72). First we note the general result that for any (reasonably well behaved) function, $f(z,y)$,

$$\int_{x_0+m_1}^{x_0+m_2} \int_{z+d}^{x_0+n} f(z,y)\, dy\, dz =$$

$$\int_{x_0+m_1+d}^{x_0+m_2+d} \int_{x_0+m_1}^{y-d} f(y,z)\, dz\, dy + \int_{x_0+m_2+d}^{x_0+n} \int_{x_0+m_1}^{x_0+m_2} f(z,y)\, dz\, dy \tag{80}$$

This result can most easily be proved by drawing a picture of the (z,y)-plane and noting the area over which integration takes place in each of the three integrals. Next we note that for $w \leqslant t$

$$_{w,t}p_x^{HS} = \int_{x_0+t-w}^{x_0+t} {}_{z-x_0}p_{x_0}^{HH} \cdot \sigma_z \cdot {}_{x_0+t-z}p_z^{\overline{SS}} dz \tag{81}$$

This formula follows, by a change of variable, from formula (28) in Part A. Using (81), formulae (71) and (72) become

$$\int_{x_0+m_1+d}^{x_0+m_2+d} \int_{x_0+m_1}^{y-d} v^{y-x_0} \cdot {}_{z-x_0}p_{x_0}^{HH} \cdot \sigma_z \cdot {}_{y-z}p_z^{\overline{SS}} dz\, dy \tag{82}$$

and

$$\int_{x_0+m_2+d}^{x_0+n} \int_{x_0+m_1}^{x_0+m_2} v^{y-x_0} \cdot {}_{z-x_0}p_{x_0}^{HH} \cdot \sigma_z \cdot {}_{y-z}p_z^{\overline{SS}} dz\, dy \tag{83}$$

With the change of variable $z = t+y$ in the inner integral, (69) becomes

$$\int_{x_0+m_1}^{x_0+m_2} v^{y-x_0} \cdot {}_{y-x_0}p_{x_0}^{HH} \cdot \sigma_y \cdot \int_{y+d}^{x_0+n} v^{z-y} \cdot {}_{z-y}p_y^{\overline{SS}}\, dz\, dy \tag{84}$$

Changing the notation in (84), i.e. writing y in place of z and z in place of y, we can see that, using the general result (80), (84) is the sum of (82) and (83).

CMIR **12** (1991) 97–226

PART E: CALCULATION OF PROBABILITIES

SUMMARY

In this Part the rates (transition intensities) which have been derived in Parts B and C are applied to the model whose mathematical formulation was defined in Part A, using the numerical methods described in Part D.

A further element of the basis, the mortality rates of the healthy, is discussed in Section 1. In Section 2 a graphical representation of the sickness model is presented, which may be a useful aid to understanding the model. In Sections 3 to 6 probabilities and rates are derived on one basis, that for a one-week deferred period. The construction of 'select' tables for different rates is discussed in Sections 7 to 9. A further useful probability, that of survival while sick during the deferred period, is introduced in Section 10. Results using the bases for other deferred periods are considered in Section 11.

1. MORTALITY OF THE HEALTHY

1.1 A necessary element of the fundamental basis for the calculation of PHI values is the mortality of the healthy, the function μ_x as defined in Part A. The PHI Sub-Committee has no direct information about the mortality rates experienced under PHI policies among those who are not making a claim, which include both the healthy and those who are sick but with a duration of sickness less than the deferred period. It is doubtful whether offices have comprehensive information on this point; sometimes they may be notified about the death of a policyholder; on other occasions a policy may apparently lapse because of the death of the policyholder but without the office being notified of the death.

1.2 The mortality of those who are sick with a duration less than the deferred period, $\nu_{x,z}$, has been extrapolated from the graduated mortality rates for the sick about whom information is known, as described in Part B. It is, however, necessary to make a plausible assumption about the level of mortality that might be experienced by healthy lives. The mortality assumed for the calculations in this Part is that of Male Permanent Assurances 1979–82, duration 0, as graduated by the GM(2,2) formula shown in the report "The Graduation of the 1979–82 Experience" in *C.M.I.R.* **9**.

The formula used was:

$$\mu_x = a_0 + a_1 t + \exp(b_0 + b_1 t) \tag{1}$$

where

$$t = \frac{(x-70)}{50},$$
$$a_0 = -0{\cdot}00465192$$
$$a_1 = -0{\cdot}00452546$$
$$b_0 = -3{\cdot}985723$$
$$b_1 = +3{\cdot}185063$$

A justification for using the duration 0 rates is that they apply to a group of assured lives who had been recently selected, either by medical examination or by medical questionnaire, and who therefore may be expected to have been healthy, or at least not suffering from long-term disability though they might have been currently temporarily sick. These rates are therefore potentially appropriate to represent the mortality of the healthy in a PHI model.

1.3 The rates for the data described as being for 'duration 0' apply to policies in the first year of insurance, and are therefore not strictly those at exact duration zero. If the 'duration 0' graduated rates were treated as applying at exact duration $\frac{1}{2}$ and the 'duration 1' rates at exact duration $1\frac{1}{2}$, then a hypothetical exact duration 0 rate could be obtained by linear extrapolation: 1·5 (duration 0) − 0·5 (duration 1). Mortality rates so calculated for 1979–82 Male Permanent Assurances were used in one of the experiments carried out to investigate the sensitivity of the results to alternative assumptions about the mortality of healthy lives.

An extreme choice for μ_x was to assume that the mortality of healthy lives was zero throughout.

Investigations using different assumptions for μ_x showed that the calculated results were in general fairly insensitive to the assumptions made about the mortality of the healthy, so it was deemed most convenient to use the 'duration 0' rates described above.

2. GRAPHICAL REPRESENTATION

2.1 For clarification of some of the factors discussed below it may be convenient to use a graphical representation of the development of sickness. Figure E1 shows such a graphical representation of the model for sickness. Imagine that a group of healthy lives aged x_0 starts at point A in the diagram. So long as they remain healthy they travel down the heavy vertical line, their age increasing as they go.

Various alternative paths are depicted in the diagram. At point B, for example, one life might become sick. The duration of sickness is represented by the horizontal axis to the right of the heavy vertical. Lines representing 13 weeks and 26 weeks sickness are shown. As time passes both the age and the duration of sickness increase simultaneously. This is represented by assuming that our

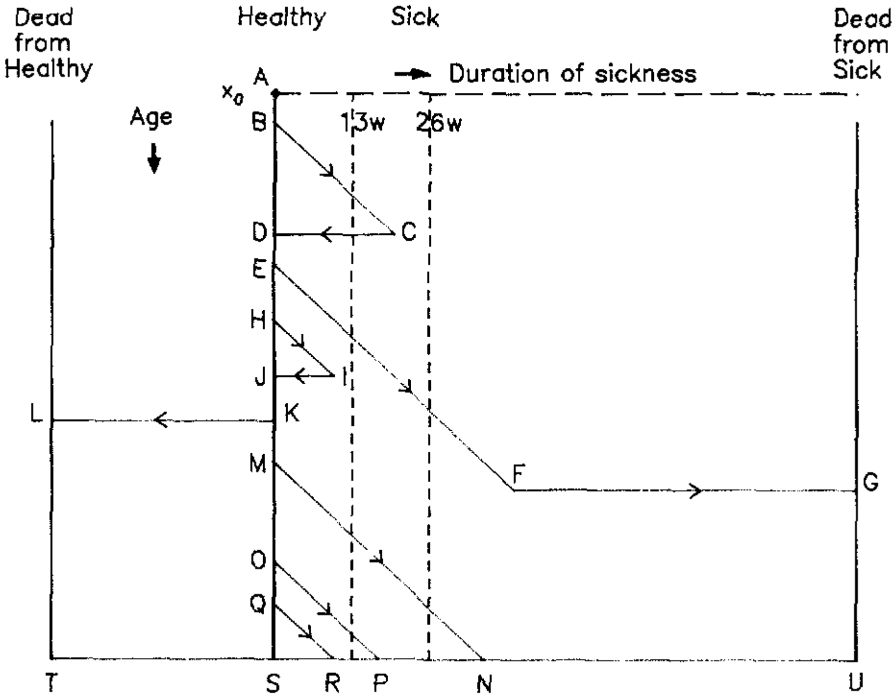

Figure E1. Model of sickness.

specimen life travels down the diagonal line from B towards C, atwhich point he recovers and moves immediately back to the healthy state at point D. It can be seen that on this occasion he was sick for rather longer than 13 weeks but less than 26 weeks, and would therefore have had a potential claim under a 13-week deferred policy, but not under a 26-week deferred one.

At point E another healthy life (or the same again) becomes sick. He continues down the diagonal line, beyond the 26 week line to point F, at which time he dies. This is represented by the line from F to G. The heavy vertical line to the right of the diagram represents the path of those who died while in the status sick.

Another example becomes sick at point H, continues to I and recovers to J. He is sick for less than 13 weeks, and could not claim under a 13-week deferred policy.

Another example dies while in the healthy state at point K, moving across to L in the vertical line to the left of the diagram, which represents those who die in the healthy state.

Yet other examples become sick at points M, O and Q. They are all still sick at the end of the period under consideration which is represented by the horizontal line from T to U. M has reached point N beyond the 26-week line, O has reached point P between 13 weeks and 26 weeks and Q has reached point R, sick for less than 13 weeks. At the end of the period the remaining healthy have reached point S, those who have died while healthy are at point T and those who have died while sick are gathered at U.

This diagram (analogous to a Lexis diagram) may be kept in mind while the numerical results are being described.

3. BASIC PROBABILITIES

3.1 In the following sections we give numerical examples which are all *based on the graduated rates for deferred period 1 week*. It will be seen that specimen results for all deferred periods can be derived on this basis. Results using the graduated sickness rates for other deferred periods will be discussed in Section 11. It should be noted that they also can be used to derive results for any deferred period. It is necessary therefore to remember that one can calculate, for example, 13-week claim inception rates on the 1-week deferred period basis or on the 13-week deferred period basis or on the 26-week deferred period basis. Similarly, one can calculate 1-week claim inception rates, 13-week claim inception rates and 26-week claim inception rates all on the 1-week deferred period basis.

It is also desirable to note that the results to be presented in this section and in Sections 4, 5 and 6 are all conditional on the starting status and the starting age, which in these examples are taken as *healthy at age 30*. The extent to which results for different starting ages can be combined is discussed in Sections 7 to 9.

Although in Parts A and D we have described functions mainly in terms of probabilities conditional on the starting age and status, it is convenient, by analogy with the ordinary life table, to speak in terms of 'a large number of persons' in this initial status, and 'the number of survivors' in a particular status or 'the number moving' from one status to another over any period. Although we present some results in 'numbers', calculated by multiplying the corresponding probabilities by a radix of 1,000,000, we use this terminology just as a figure of speech, and we intend throughout that the results should be interpreted as probabilities.

3.2 In our calculations we have assumed a step size h, of $\frac{1}{156}$ of a year. We assume throughout exactly 52 weeks in the year, so the step size is one-third of a week. A month consists of 13 steps. The number 156 was chosen because it is the lowest common multiple of 12 and 52.

Note therefore that in this Part (and in Part F) a 'week' is always $\frac{1}{52}$ of a year, and not an exact seven days.

The graduated recovery rates and mortality rates for the sick presented in Part B vary only by attained age and not by duration once the duration exceeds 5 years. We have therefore been able to amalgamate durations after 5 years (780 steps). Thus we assume that the integer N defined in Section 2.3 of Part D has the value 780.

3.3 The basis used has already been stated as that relating to deferred period 1 week. To be precise it is:

Mortality of the healthy, μ_x:
: Male Permanent Assurances 1979–82 duration 0 graduated rates, as described in Section 1 above.

Sickness inception rates, σ_x:
: the graduated rates for deferred period 1 week as given by formula (9) in Section 9.1 of Part C.

Recovery rates, $\rho_{x,z}$:
: the graduated rates for deferred period 1 week, as given by formula (6) in Section 4.4 of Part B.

Mortality rates of the sick, $\nu_{x,z}$:
: the graduated rates for all deferred periods as given by formula (9) in Section 6.2 of Part B.

3.4 The basic probabilities are given in Table E14, which is expressed in terms of 'numbers' based on a radix of 1,000,000 ($=R$) as described in Section 4.2 of Part D. The columns towards the left show the numbers in each status at each integral age from 30 to 65. These are analogous to the l_x column of a life table, and they have been denoted:

$lH(x)$ the number of healthy $= R \cdot pH(x)$,
$lS(x)$ the number sick (at all durations combined) $= R \cdot pS(x)$,
$lDH(x)$ the number dead, having died as healthy $= R \cdot pDH(x)$,
$lDS(x)$ the number who have died as sick $= R \cdot pDS(x)$,
$lL(x)$ the number living $= lH(x) + lS(x) = R \cdot pL(x)$,
$lD(x)$ the number dead $= lDH(x) + lDS(x) = R \cdot pD(x)$.

The number living plus the number dead in every case exhausts the statuses, and the sum of these columns is always equal to the radix R.

The columns towards the right of Table E14 show the numbers of transitions taking place between ages x and $x+1$. They are analogous to the number of deaths, d_x, in a life table and they have been denoted:

$dHS(x)$ the number of transitions from healthy to sick between ages x and $x+1$,
$dHD(x)$ the number of transitions from healthy to dead between ages x and $x+1$,
$dSH(x)$ the number of transitions from sick to healthy, i.e. recoveries, between ages x and $x+1$,
$dSD(x)$ the number of deaths among the sick between ages x and $x+1$.

It can be seen that certain accounting identities hold, for example:

$$lH(x+1) = lH(x) - dHS(x) - dHD(x) + dSH(x) \tag{2}$$

and

$$lS(x+1) = lS(x) + dHS(x) - dSH(x) - dSD(x), \tag{3}$$

and these are confirmed numerically (subject possibly to rounding errors, since we are strictly representing proportions not exact numbers).

The function $dHS(x)$ can be interpreted as R times the average number of transitions from healthy to sick between ages x and $x+1$ for a person who commenced healthy at age 30, and similar interpretations apply to $dHD(x)$, $dSH(x)$ and $dSD(x)$.

3.5 It can be seen that by age 65 about 69% of those starting healthy at age 30 are in the healthy state, and about 13% in the sick state. About 8.3% have died while in the status healthy and 9·6% have died while sick. On this basis rather more die each year while sick than while healthy, but this result is specific to this basis, as can be seen from Table E10 in which the results on different bases are compared.

3.6 It should be noted that the method of numerical calculation adopted requires the probabilities shown for integral ages in Table E14 to be calculated at steps of $\frac{1}{156}$ of a year; thus the table shows only every 156th entry in the full table available inside the computer.

3.7 Besides calculating probabilities at every $\frac{1}{156}$th of a year of age, it is necessary to calculate the proportion sick within each step of $\frac{1}{156}$th of a year of duration of sickness up to 780 steps (5 years) in all, with all sickness periods beyond that duration aggregated. It is convenient to print out only the proportion sick for selected sickness periods, as shown in Table E15. This shows the proportions sick at each age from 30 to 65 within the sickness periods shown, denoted in the usual 'a/b' notation. For example, '4/9 weeks' indicates those who have been sick for at least 4 weeks but not more than $4+9=13$ weeks. We denote these values $pS(x,a/b)$ and each is equal to the sum of appropriate terms $pS(x,m)$, as defined in Section 6.1 of Part D. In Table E15 the probabilities are shown as decimal fractions, not multiplied by the radix R. The last two columns show the 'Total' sick, equivalent to '0/All', and also '1/All'. The total figure, if multiplied by the radix of 1,000,000, is identical with the number sick, $lS(x)$ of Table E14.

3.8 In Table E16 the ζ functions are shown. These were defined in Section 6.1 of Part D as 52 times the proportion sick within a specific sickness period among the total living at age x (all conditional on the initial status at age x_0). The formula is therefore:

$$\zeta\,(x,a/b)=52\cdot\frac{pS(x,a/b)}{pL(x)} \tag{4}$$

where

$$pL(x)=pH(x)+pS(x).$$

The ζ rates will be referred to again when sickness rates are discussed in Section 6.

4. AGGREGATE MORTALITY RATES

4.1 In Section 3 of Part D it was shown how the overall force of mortality at age x, conditional on the initial status at age x_0, could be calculated. The total force of mortality of the living, $mL(x)$, can be considered as the weighted average of the mortality of the healthy and the mortality of the sick. The force of mortality of the healthy is simply $\mu_x(=m(x))$, and this applies at age x to the proportion healthy at age x, $pH(x)$. The average force of mortality of the sick, weighted by duration, is defined as $mS(x)$ and it can be calculated by the approximate formula (formula (24) of Part D)

$$mS(x) = \frac{\sum_{m=1}^{N-1} n(x,m) \cdot pS(x,m) + n(x,N) \cdot pS(x,N^+)}{pS(x)} \tag{5}$$

The total force of mortality of the living, $mL(x)$, is then calculated by the formula

$$mL(x) = \frac{m(x)pH(x) + mS(x)pS(x)}{pH(x) + pS(x)} \tag{6}$$

which is the same as formula (25) of Part D.

4.2 The values of μ_x, $mS(x)$ and $mL(x)$ are given in the first three columns of Table E17. It can be seen how, on the basis used, the value of $mS(x)$, which is undefined at the starting age of 30, is comparatively constant thereafter, rising only from about 0·061 at age 31 to 0·082 at age 65. The values of $mL(x)$, however, rise in line with the values of μ_x, and, on this basis, are generally rather more than twice the value of μ_x.

A comparison of the values of $mL(x)$ with the values of μ_x for durations 2 and over for Male Permanent Assurances 1979–82 (AM80 ultimate) is shown in Table E1. After one year the values of $mL(x)$ (on this basis) are substantially higher than the values of μ_x for permanent assurances, the ratio reducing with age. (For a comparison on the bases for other deferred periods see Section 11.)

It can be seen from Table E17 that the values of $mS(x)$ exceed the values of $mL(x)$ or of μ_x on AM80 ultimate by about 0·06 to 0·07.

4.3 The values previously shown for the total living, $lL(x)$, allow the calculation of a total life table, including both the healthy and the sick. Values of

Table E1. *Comparison of overall mortality rates*

Age	$mL(x)$	AM80 ultimate	Difference	Ratio %
31	0·000986	0·000557	0·000429	177
35	0·001216	0·000666	0·000550	182
40	0·001749	0·001077	0·000672	162
45	0·002720	0·001961	0·000759	139
50	0·004385	0·003598	0·000787	122
55	0·007214	0·006435	0·000779	112
60	0·012123	0·011184	0·000939	108
65	0·020975	0·018978	0·001997	111

l_x and q_x for this life table are shown in further columns of Table E17. The values of l_x are the same as $lL(x)$ in Table E14. The values of q_x are calculated in the usual way as

$$q_x = \frac{l_x - l_{x+1}}{l_x} \tag{7}$$

The values can equivalently be calculated by the formula

$$q_x = \frac{dHD(x) + dSD(x)}{lL(x)} \tag{8}$$

The average number of living between ages x and $x+1$ is given by the integral

$$L_x = \int_x^{x+1} l_y dy \tag{9}$$

and this has been calculated approximately by

$$L_x \doteqdot \frac{1}{156} \sum_{j=0}^{155} \tfrac{1}{2}\{l_{x+j/156} + l_{x+(j+1)/156}\} \tag{10}$$

as described in Section 5.2 of Part D.

Values of L_x are also shown in Table E17. Over the age range shown the simple approximation

$$L_x \doteqdot \frac{l_x + l_{x+1}}{2} \tag{11}$$

reproduces the more accurate approximation shown above to within two parts in 10,000, which is probably negligible for practical purposes.

5. CLAIM INCEPTION RATES

5.1 The calculation of claim inception rates on two different definitions was described in Section 5 of Part D. Values of both types of claim inception rates on the specimen basis are shown in Tables E18a and E18b. They are denoted $ia(x,d)$ and $ib(x,d)$, are conditional on the life being healthy at age x_0, and are defined as:

(*a*) the expected number of periods of sickness which pass through duration d between attained ages x and $x+1$ (sickness being commenced between ages $x-d$ and $x+1-d$), denoted by $ca(x,d)$, divided by the average number living between ages x and $x+1$, L_x; and

(*b*) the expected number of periods of sickness which pass through duration d

between attained ages $x+d$ and $x+d+1$ (sickness having commenced between ages x and $x+1$), denoted by $cb(x,d)$, also divided by L_x.

The values shown in Tables E18a and E18b are $10{,}000ia(x,d)$ and $10{,}000ib(x,d)$, and the value of $10{,}000\sigma_x$ at exact age x is shown for comparison.

It can be seen from Tables E18a and E18b that the claim inception rates for deferred period 0 weeks, which are the same in both tables, are smaller than the value of σ_x by an increasing amount as age increases. This is because the sickness inception intensity applies only to the healthy lives, whereas the claim inception rates that we have defined are applied both to the healthy and to the sick. However, the rates are of the same order of size, whereas the claim inception rates for higher durations (d greater than 0) reduce in value very rapidly as duration increases.

A healthy life aged 30 cannot reach duration of sickness 52 weeks within the first year, nor 104 weeks within the first two years. The claim inception rates of type (a), in the top right hand corner of Table E18a, for 52 weeks deferred at age 30 and 104 weeks deferred at ages 30 and 31 are therefore necessarily zero. This is not the case for the claim inception rates of type (b).

5.2 A graphical representation of the claim inception rates that we have defined is shown in Figure E2. A group of healthy lives commenced at age x_0 at point A. Those who are healthy at age x appear at point B, and those who are healthy at age $x+1$ are at point C. The line AEFG represents the course of a person who became sick immediately at age x_0 and has remained sick ever since. It forms the boundary of the area within which sickness can occur for this group of lives, so that no sickness can appear above and to the right of this line.

The average number living between ages x and $x+1$, L_x, is represented by the integral of the healthy over the line BC and the sick over the area BCFE. The end of deferred period d is represented by the vertical line through HIJK. The claim inceptions of type (a), $ca(x,d)$, are those which cross this line between H and J, having commenced sickness between ages $x-d$ and $x+1-d$. The claim inceptions of type (b) relate to cases which pass the end of deferred period d having commenced their period of sickness between ages x and $x+1$, represented by the cases that cross the d line between I and K.

5.3 The claim inception rates of type (a) derived for deferred period 1 week on the specimen basis (which is the basis for 1 week deferred business), can be compared with the graduated claim inception rates shown in Table H5 of *C.M.I.R.* 7, except for those in the first year. A summary comparison is shown in Table E2.

It can be seen that these rates are reasonably similar. A comparison of inception rates for other deferred periods is shown in Table E12.

5.4 Values of $ib(x,d)$ can be calculated by interpolation from values of $ia(x,d)$ using formulae of the type:

$$ib(x,d) = \{(1-d)ia(x,d)\cdot L_x + d\cdot ia(x+1,d)\cdot L_{x+1}\}/L_x \qquad (12)$$

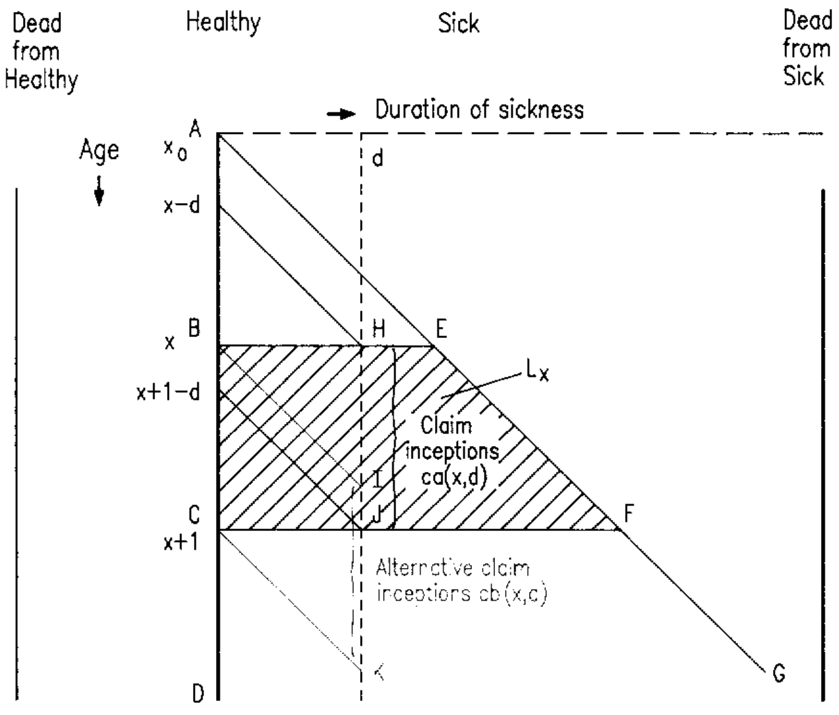

Figure E2. Claim inceptions.

with appropriate modifications for the early years, and where $d > 1$. Extensive experimental calculations showed that this formula is very accurate for years after the first and even in the first year is only inaccurate by about 1·5% at the maximum, for deferred period 26 weeks.

Table E2. *Claim inception rates of type* (a) *per 10,000*

Age	Calculated on D1 basis (age 30 at entry)	*C.M.I.R.* 7, Table H5 Dp 1 week
31	1,224	1,232
35	1,236	1,255
40	1,249	1,268
45	1,281	1,284
50	1,355	1,327
55	1,500	1,425
60	1,748	1,619
64	2,034	1,890

6. SICKNESS RATES

6.1 In Section 6 of Part D the formula for calculating Manchester-Unity-type sickness rates, $z_x^{a/b}$ or $z(x,a/b)$, was discussed.

Numerical values are shown in Table E19. The values of $z(x,a/b)$ are measured in weeks of sickness (assuming 52 weeks per year) for the usual sickness periods, together with the total for all periods '0/All' and '1/All'. It can be seen from Section 6 of Part D that the z rate is derived by integrating the numerator and the denominator of the ζ rates from ages x to $x+1$. Both integrals are approximated by summations using the trapezium rule, as shown above from the calculation of L_x, which forms the denominator of these rates.

It may be expected that there might be very little numerical difference between $z(x,a/b)$ and $\zeta(x+\frac{1}{2},a/b)$. It can be seen that, on the specimen basis shown, there is only a small numerical difference between $z(x,a/b)$ and the average of $\zeta(x,a/b)$ and $\zeta(x+1,a/b)$. Apart from the first year, for all periods of sickness, and for the first year or two for higher periods of sickness (what we shall call the 'run-in period'—not the same as the 'run-in period' of Part B) the difference is not greater than 3 parts in 1,000.

6.2 The values of $z(x,a/b)$ calculated on this specimen basis are compared in Table E3 with the graduated rates for deferred period 1 week business shown in Table H1 of *C.M.I.R.* 7.

After the run-in period the rates shown are at a similar level, except for sickness period 104/all. The rates there diverge as the age increases. It should be remembered that the rates shown in Table E19 apply only to those who are

Table E3. *Comparison of sickness rates: Table E19 and Table H1 of* C.M.I.R. 7

		Sickness periods					
Age	Table	1/3	4/9	13/13	26/26	52/52	104/all
30	E19	0·147	0·066	0·016	0·005	—	—
	H1	0·140	0·053	0·019	0·017	0·013	0·016
35	E19	0·168	0·098	0·035	0·028	0·029	0·044
	H1	0·162	0·078	0·032	0·030	0·019	0·021
40	E19	0·185	0·124	0·051	0·045	0·050	0·130
	H1	0·182	0·108	0·046	0·046	0·032	0·041
45	E19	0·207	0·161	0·074	0·073	0·087	0·279
	H1	0·203	0·145	0·063	0·068	0·057	0·101
50	E19	0·240	0·215	0·112	0·121	0·154	0·562
	H1	0·230	0·195	0·089	0·102	0·118	0·257
55	E19	0·292	0·300	0·177	0·209	0·284	1·132
	H1	0·272	0·269	0·146	0·171	0·250	0·614
60	E19	0·375	0·442	0·296	0·379	0·551	2·360
	H1	0·340	0·394	0·261	0·341	0·546	1·467

known to have been healthy at age 30, whereas the rates shown in *C.M.I.R.* 7 apply to a population who were healthy at a mixture of different entry ages.

7. CONSTRUCTION OF SELECT TABLES—MORTALITY RATES

7.1 In the preceding sections we have described the functions that can be calculated using the full multiple state model. The functions are, in every case, dependent upon the initial status, in our example healthy at age 30. Similar calculations can be carried out for any age at entry and for any chosen status, either healthy or sick with duration of sickness between j and $j+1$ steps. (It would be possible to consider a life with initial duration of sickness z exactly, but the consequent algorithms and computer programs would require many minor changes throughout to effect this, whereas the same program will work for healthy lives and for those sick within one duration step.) It would also be possible to start with any desired probability distribution of statuses as the initial position, corresponding perhaps to a mixed population with specified properties. It can readily be seen, however, that the volume of calculation and corresponding output of tables in order to present every possibility, or even every possible starting age for the healthy, would be substantial. It is appropriate, therefore, to investigate whether the tables can be summarised in some convenient form, in the same way as 'select' life tables are constructed.

In this section we consider the construction of a select total life table, and in Sections 8 and 9 we consider the construction of select tables for inception rates and for sickness rates. The 'selection' in this case results entirely from the model, in which it is assumed that the life enters in the state 'healthy'. It does not reflect any further medical selection, such as might occur in reality.

7.2 In order to investigate the construction of select tables we adopted the following procedure. We calculated tables similar to those shown in Tables E14 to E19 for each entry age of healthy lives from 16 to 64, though they are not shown in this Report. Commencing the calculations with an entry age of 16 allows suitable 'ultimate' rates to be derived, although great reliance should not be placed on the results for the earliest years of age.

We considered first the values of the aggregate 1-year mortality rates, q_x, which are shown for entry age 30 in Table E17. Putting together the results for all entry ages allowed a large triangular table to be constructed with entry ages from 16 to 64 and attained ages from 16 to 64, containing entries for $q(x,t)$ the value of q_x for attained age x for a life who entered at age $x-t$. It was soon seen, as might be expected, that the values of $q(x,t)$ for any one attained age were very similar after the first few years of duration. We describe the rates for age 16 at entry as 'ultimate' rates (although at attained age 16 the one rate is for entry age 16 duration 0). We calculated the ratio of each 'select' rate to the 'ultimate' rate for the same attained age, i.e.

$$\frac{q(x,t)}{q(x,x-16)}.$$

It could be seen that the select rates ran in very quickly to the ultimate rates. The maximum difference was at attained age 64. The ratios the select rates bear to the ultimate rate (whose value is 0·019647) are shown in Table E4a.

By comparison, at attained age 40 the comparable ratios are as shown in Table E4b. The value of the ultimate rate is 0·001831.

Table E4a. *Ratio of* q_{64} *for entry age shown to* q_{64} *for entry age 16*

Entry age	Duration	% Ratio
64	0	71·3
63	1	87·3
62	2	91·6
61	3	93·6
60	4	94·9
59	5	95·8
58	6	96·6
57	7	97·2
56	8	97·7
55	9	98·1

Table E4b. *Ratio of* q_{40} *for entry age shown to* q_{40} *for entry age 16*

Entry age	Duration	% Ratio
40	0	86·0
39	1	95·4
38	2	97·0
37	3	97·7
36	4	98·2
35	5	98·5
34	6	98·8
33	7	99·0
32	8	99·2
31	9	99·4

7.3 We construct a select table with T years selection as follows:

for select durations

$$q_{[x-t]+t} = q(x,t) \qquad \text{for } t = 0 \text{ to } T-1 \qquad \text{and } x = 16 \text{ to } 64; \qquad (13)$$

for the ultimate rates

$$q_x = q(x,x-16) \qquad \text{for } x = 16 + T \text{ to } 64. \qquad (14)$$

The question is what value to use for T. If the view were taken that it was sufficient to jump straight to the ultimate rate when the select rate approached to within 10% of the ultimate rate, then a table with 2 years selection would be sufficient. If it were felt that the select period should continue so long as the select rates were at least 5% different from the ultimate rate, then the select period would need to be 5 years.

Another approach is to compare the values of ${}_{65-x}p_{[x]}$ on the select basis and on the exact basis for each age. The error is less than 3 parts in 1,000 for all ages for a 5-year select table (and for ages 16 and 60 to 64 at entry the error is zero) and is less than 5 parts in 1,000 for a 2-year select table. The maximum error for a 5-year select table is 0·23% for ${}_{11}p_{[54]}$ and for a 2-year select table is 0·43% for ${}_{8}p_{[57]}$.

Yet another way of assessing the select period needed is to consider the values of ${}_{t}q_{[x]} = 1 - {}_{t}p_{[x]}$, the probability of death within t years of a select life entering at age x. The proportionate error in ${}_{t}q_{[x]}$ is larger than that in ${}_{t}p_{[x]}$. Values of ${}_{t}q_{[x]}$ calculated using a limited-period select table are slightly higher than those calculated for each separate age at entry. The maximum excess for a 5-year select table is 1·77% for ${}_{9}q_{[56]}$, and for a 2-year select table reaches 4·66% for ${}_{5}q_{[60]}$.

If the select tables were to be used for calculating the value of benefits on death, then the error in the overestimation of deaths for a 2-year select table might be considered significant. However, since the life table in the context of PHI benefits is mainly used for the calculation of premium rates, the error in ${}_{t}p_{[x]}$ is the important one, and for this purpose a 2-year select life table may be satisfactory. However, because a 5-year select period proves to be preferable for inception rates and sickness rates, as is shown in Sections 8 and 9, we have used a 5-year select period in subsequent calculations.

7.4 Tables E20a and E20b show the values of $q_{[x-t]+t}$ for attained age x and select duration t, for a 2-year select period and for a 5-year select period respectively. The ultimate rates in each case are in the columns headed '2 and over' and '5 and over'. The rates for select durations 0 and 1 are the same in both tables.

Tables E21a and E21b show the values of $l_{[x-t]+t}$ for a 2-year and a 5-year select table respectively, in each case based on a radix of $l_{65} = 1{,}000{,}000$.

It should be noted that, unlike the select mortality tables sometimes displayed elsewhere, a life entering at select duration 0 runs diagonally down to the right as his attained age and duration since entry increases, until he reaches the ultimate column.

8. CONSTRUCTION OF SELECT TABLES—INCEPTION RATES

8.1 It is intuitively reasonable to assume that the claim inception rates described in Section 6 might, after some suitable run-in period, be similar at any one attained age for different starting ages. Tables comparable to Table E18a showing inception rates of type (a) for each entry age were calculated. Inspection of these showed that the assumption just described is indeed true. However, unlike values of $q(x,t)$, which for any one attained age increase by select duration, the values of $ia(x,d,t)$ (the values of $ia(x,d)$ for attained age x, deferred period d and duration t since entry (healthy) at age $x-t$), for any one attained age and deferred period, reduce as the select duration increases. This is because the inception rates are calculated by dividing the number of sickness periods that extend beyond the deferred period by the total number alive, in both cases between ages x and $x+1$. Comparing the extremes, a new entrant who is healthy at age 64 is more likely to reach 1 or 4 weeks sickness within his first year than is someone who was healthy at age 16 but by age 64 has a high chance of being sick with a long duration of sickness from which he is unlikely to recover in order to start another sickness period. The same is true for any ages x_2 and x_1, where $x_2 > x_1$.

For longer deferred periods there is a temporary reversal of this phenomenon. For example, at middling entry ages, when the proportion of long-term sick among the ultimate group has not risen greatly, the inception rates at select duration 0 for a 13-week deferred period are roughly three-quarters of those for any higher duration, simply because a life entering at age x while healthy cannot reach 13 weeks of sickness within 13 weeks of entry. A similar argument shows that the inception rate (of type a) at duration 0 for a 26-week deferred period is about one-half that for higher durations; that the inception rate for a 52-week deferred period is necessarily zero in the first year of selection, and for a 104-week deferred period is necessarily zero in the first two years of selection.

8.2 Apart from the 'run-in' feature noted in the previous paragraph, the inception rates at any one attained age for all ages at entry are very close to those for entry age 16 at all the middling ages. The select rates do not rise more than 5% higher than the rates for entry at age 16 until attained age 58 and above. The rates for attained age 64, as percentages of the rates for entry age 16, for select durations 0 to 4, for various deferred periods, are shown in Table E5a, along with the ultimate rate per 10,000 living.

The pattern is demonstrated visually in Figure E3, for deferred periods 0, 26 and 104 weeks.

Table E5a. *Inception rates of type (a) for attained age 64 as percentage of those for age 16 at entry*

Deferred period (weeks)	Select duration 0	1	2	3	4	Rate for age 16 at entry per 10,000
0	113·3	110·3	108·4	106·8	105·6	2,963
1	111·2	110·3	108·4	106·8	105·6	2,034
4	104·9	110·3	108·4	106·8	105·6	1,192
13	86·0	110·4	108·4	106·8	105·6	451
26	58·3	110·4	108·4	106·8	105·6	288
52	0·0	110·8	108·5	106·9	105·6	215
104	0·0	0·0	109·0	107·0	105·7	164

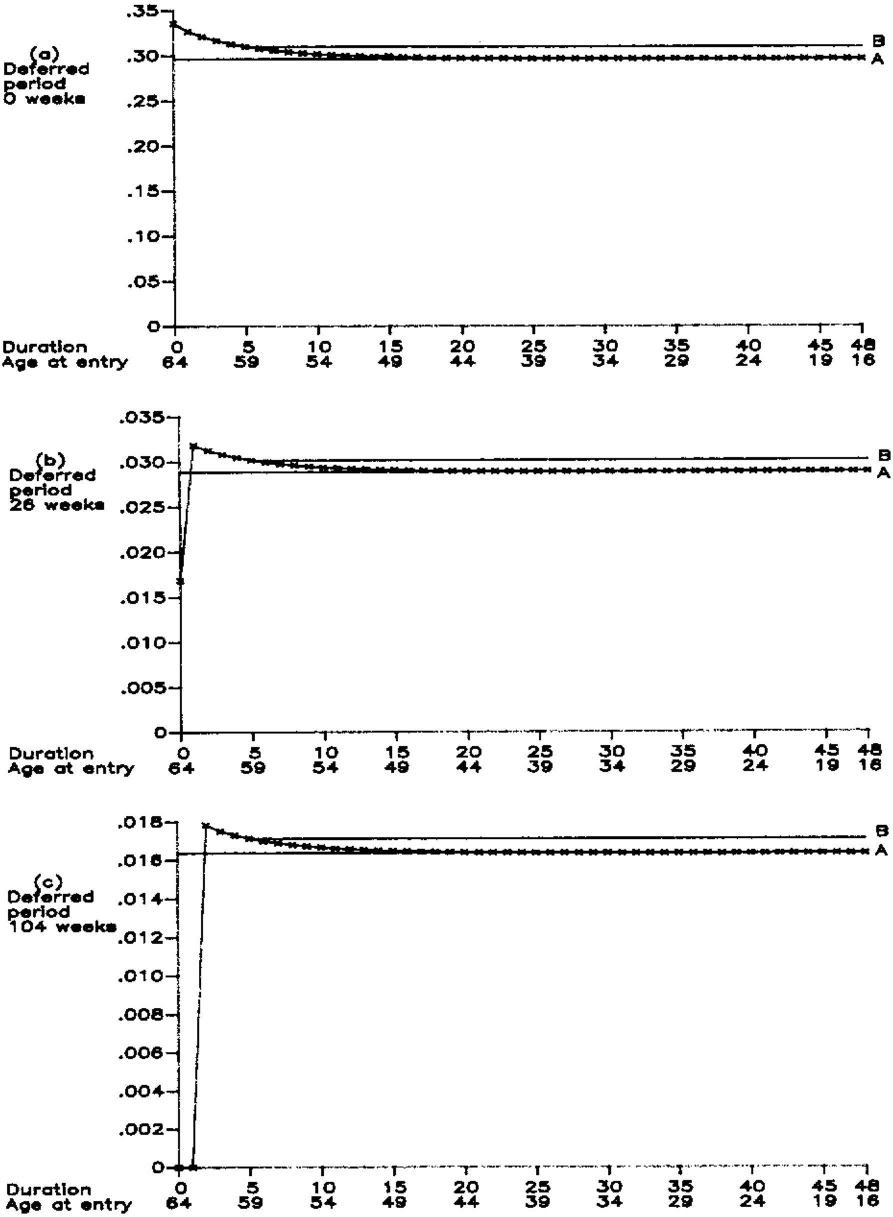

Figure E3. Inception rates of type (*a*) for attained age 64. A shows the inception rate for entry age 16. B shows the inception rate for duration 5 years.

8.3 One approach to constructing select tables for inception rates would be to use the rates for age 16 at entry as the ultimate rates, and to use a select period of, for example, 5 years. We describe this as Method A. However, it may be *important that the inception rates in any standard tables are not too low.* An alternative approach, which we describe as Method B, is to retain say the first five years of select rates, i.e. those for durations 0 to 4, and then to use the rates for the next select duration, i.e. 5, as if they were the ultimate rates, i.e. to use them for all subsequent durations. Rates for attained age 64 as percentages of the duration 5 rates are shown in Table E5b.

Table E5b. *Inception rates of type (a) for attained age 64 as percentage of those for duration 5*

Deferred period (weeks)	Select duration 0	1	2	3	4	Rate for duration 5 per 10,000
0	108·4	105·5	103·6	102·1	101·0	3,099
1	106·3	105·5	103·6	102·1	101·0	2,128
4	100·3	105·5	103·6	102·1	101·0	1,247
13	82·2	105·5	103·6	102·1	101·0	471
26	55·7	105·6	103·6	102·1	101·0	302
52	0·0	106·0	103·7	102·2	101·0	225
104	0·0	0·0	104·1	102·2	101·0	171

Method B slightly overestimates the inception rates, and the measure of the aggregate overestimation can be obtained by calculating the total 'number' of inceptions for a life aged x at entry (in the healthy state) calculated both on the complete table (treating each age at entry independently) and on the select table.

The pattern for type (b) rates is almost the same, except that the 'run-in' feature shown in the lower left hand corner of Tables E5a and E5b does not exist.

8.4 Formally, a select table with T years selection is defined as follows:

for the select durations (Methods A and B)

$$ia^d_{[x-t]+t} = ia(x,d,t) \qquad \text{for } t = 0 \text{ to } T-1 \qquad \text{and } x = 16 \text{ to } 64; \qquad (15)$$

for the ultimate rates (Method A)

$$ia^d_x = ia(x,d,x-16) \qquad (16)$$

for the ultimate rates (Method B)

$$ia^d_x = ia(x,d,T) \qquad (17)$$

Similar formulae would apply to the inception rates of type (b), which could be derived from the inception rates of type (a) by formula (12) of 5.4.

8.5 The inception rate of type (a), $ia(x,d)$, was defined in Section 5 above and in Section 5 of Part D by the formula

$$ia(x,d) = ca(x,d)/L_x. \qquad (18)$$

Given values for $ia(x,d)$ and L_x it is therefore possible to reconstruct $ca(x,d)$, the 'number' of claim inceptions starting between ages x and $x+1$, conditional on a life commencing as healthy at age x_0. To emphasise the fact that these rates are conditional on being healthy at age x_0 we use the notation:

$$ia(x,d,x_0-x) = ca(x,d,x_0-x)/L(x,x_0).$$

or

$$ia(x_0+t,d,t) = ca(x_0+t,d,t)/L(x_0+t,x_0) \tag{19}$$

The total number of claim inceptions for deferred period d, for a life who is healthy at age x_0, within n years can be defined as $Tca(x_0,d,n)$ and calculated as

$$\begin{aligned} Tca(x_0,d,n) &= \sum_{t=0}^{n-1} ca(x_0+t,d,t) \\ &= \sum_{t=0}^{n-1} ia(x_0+t,d,t)\cdot L(x_0+t,x_0) \end{aligned} \tag{20}$$

The equivalent function calculated using select tables can be defined as $Tca^*(x_0,d,n)$ which can be defined as

$$Tca^*(x_0,d,n) = \sum_{t=0}^{n-1} ia^d_{[x_0]+t}\cdot L_{[x_0]+t} \tag{21}$$

where ia and L are both taken from the appropriate select table, with the chosen numbers of years selection. (It is not essential that the same number of years selection be used for the life table and for the inception rate table.) Note that the subscript $[x_0]+t$ is taken as x_0+t for $t \geqslant T$, the select period for the table.

We can calculate L by

$$L_{[x_0]+t} \doteq \tfrac{1}{2}(l_{[x_0]+t} + l_{[x_0]+t+1}) \tag{22}$$

8.6 We have calculated $Tca(x_0,d,n)$ and $Tca^*(x_0,d,n)$ for both Method A and Method B for each entry age from 16 to 64, each termination age from 17 to 65, and for the deferred periods 0, 1, 4, 13, 26, 52 and 104 weeks, using a select period of 5 years. We have defined the percentage error as

$$100(Tca^*(x_0,d,n) - Tca(x_0,d,n))/Tca(x_0,d,n).$$

The maximum errors are as shown in Table E6.

It can be seen that the values of Tca^* calculated using Method A are at the worst slightly less than the values of Tca calculated using the full table for each separate entry age, but the maximum error never exceeds 2%. Using Method B the maximum error is in the the other direction, and for most deferred periods is slightly larger, reaching a maximum of more than 2%. Further, the maximum error using Method A occurs only for a rather high entry age, terminating at age 65, and for most ages and terms the percentage error is considerably smaller. By

Table E6. *Maximum error in total claims*

Deferred period (weeks)	Method A			Method B		
	Age	Term	%	Age	Term	%
0	56	9	−1·22	44	21	+1·05
1	56	9	−1·27	43	22	+1·18
4	56	9	−1·39	40	25	+1·47
13	56	9	−1·48	36	29	+1·71
26	57	8	−1·58	32	33	+1·91
52	57	8	−1·70	29	36	+2·09
104	57	8	−1·94	25	40	+2·24

contrast, using Method B the maximum error occurs at a more typical entry age, and the error over quite a large range of entry ages and terms is close to this maximum.

It is possible that practitioners would prefer a table that slightly overestimated inception rates to one which slightly underestimated them, even if the error were slightly larger in absolute terms. It is, however, anticipating the results of Part F to observe that annuity values calculated using Method A may be closer to the exact values than those calculated using Method B, so the choice is not clear-cut.

8.7 Tables E22a to E22g show the select inception rates of type (a), per 10,000 living, for the same series of deferred periods, with a 5-year select period, and with the ultimate rates calculated both on Method B and on Method A.

9. CONSTRUCTION OF SELECT TABLE—SICKNESS RATES

9.1 It might be hoped that the same technique as can be applied to the life table and to inception rates for construction of select tables might be used for the construction of select tables for sickness rates. We define $z(x, a/b, t)$ as the value of $z(x, a/b)$ for attained age x, sickness period a/b, and duration t since entry as healthy at age $x-t$, and consider tables of such rates for each sickness period a/b. For all lower sickness periods the same features are found as for inception rates. After an initial run-in period the rates at select durations are higher than those for entry age 16 (taken as ultimate rates), and the excess rises as age increases.

The run-in period for any period of sickness is intermediate between the run-in periods for the inception rates at the beginning and the end of the period of sickness. For example, for sickness between 52 and 104 weeks, denoted 52/52, there can be no sickness in the first year after entry and little more in the second year after entry than about 50% of the ultimate rate.

Sickness rates for attained age 64 as a percentage of those for age 16 at entry are shown in Table E7 for various sickness periods.

Table E7. *Sickness rates for attained age 64 as percentage of those for age 16 at entry*

Sickness period	Select duration 0	1	2	3	4	Sickness rate for age 16 at entry (weeks)
0/1	112·3	110·3	108·4	106·8	105·6	0·239
1/3	108·3	110·3	108·4	106·8	105·6	0·473
4/9	96·9	110·4	108·4	106·8	105·6	0·621
13/13	73·2	110·4	108·4	106·8	105·6	0·454
26/26	30·9	110·6	108·4	106·9	105·6	0·632
52/52	0·0	59·2	108·6	106·9	105·7	0·970
104/all	0·0	0·0	10·0	26·6	39·8	4·420

The pattern is demonstrated visually in Figure E4, for sickness periods 0/1, 26/26 and 104/all.

Comparison with the percentages shown above for inception rates at attained age 64 shown in Section 8 demonstrates the lengthened run-in period, represented by lower percentages at select duration 0 and in some cases select duration 1, followed by ratios which are very similar to those in Table E5a. Ratios as percentages of the sickness rates for duration 5 are shown in Table E8.

Table E8. *Sickness rates for attained age 64 as percentage of those for duration 5*

Sickness period	Select duration 0	1	2	3	4	Sickness rate for duration 5 (weeks)
0/1	107·4	105·5	103·6	102·1	101·0	0·251
1/3	103·5	105·5	103·6	102·1	101·0	0·495
4/9	92·7	105·5	103·6	102·1	101·0	0·649
13/13	70·0	105·5	103·6	102·1	101·0	0·475
26/26	26·6	105·7	103·7	102·1	101·0	0·662
52/52	0·0	56·6	103·8	102·2	101·0	1·015
104/all	not appropriate to calculate					2·236

9.2 Select tables for sickness periods up to 52/52 can therefore be constructed, on the same lines as for inception rates, both on Method A, using the rates for entry age 16 as the ultimate rates, and on Method B, using the rates for select duration 5 as if they were ultimate rates, for the same reasons as described under inception rates. Formally, we put:

for the select durations (Methods A and B)

$$z^{a/b}_{[x-t]+t} = z(x,a/b,t) \qquad \text{for } t = 0 \text{ to } T-1 \qquad \text{and } x = 16 \text{ to } 64;$$

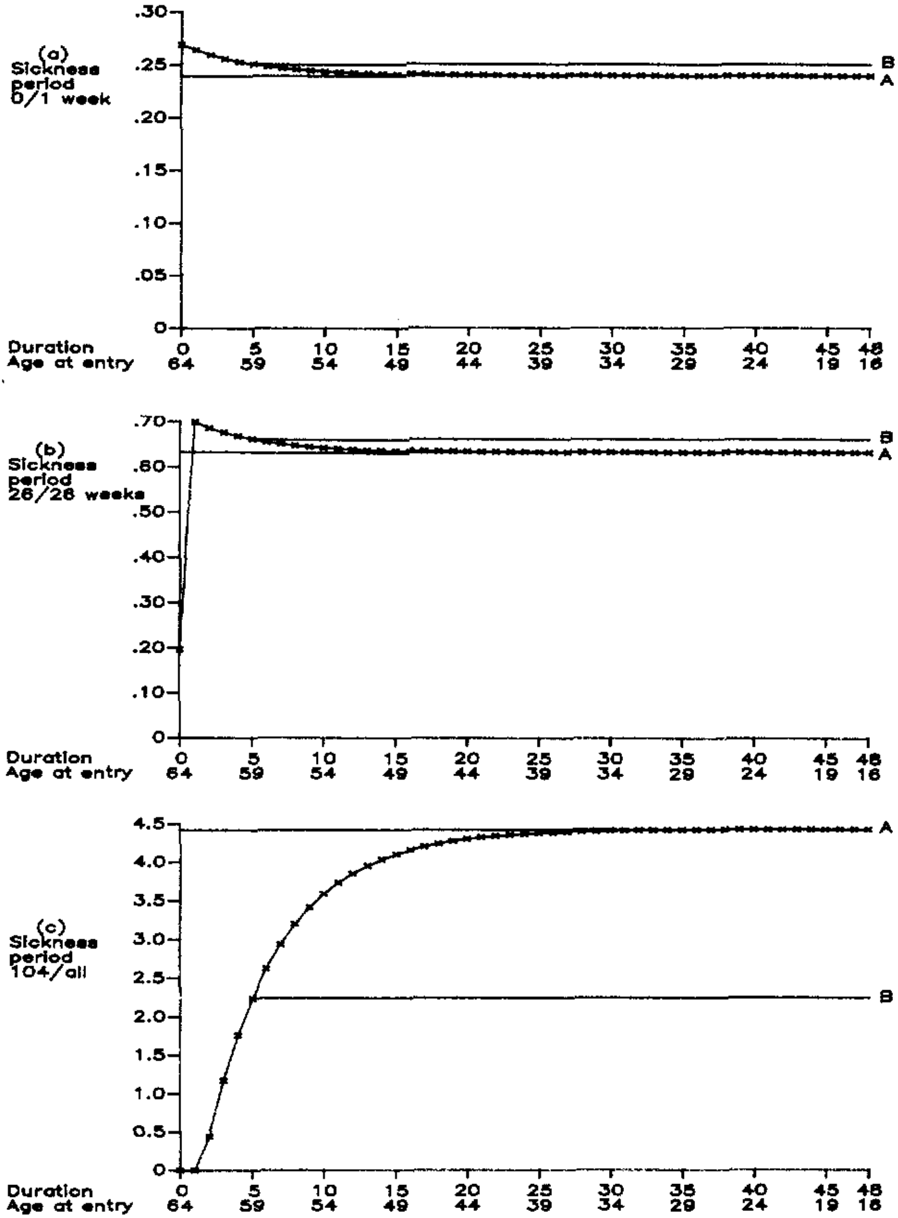

Figure E4. Sickness rates for attained age 64. A shows the sickness rate for entry age 16. B shows the sickness rate for duration 5 years.

for the ultimate rates (Method A)

$$z_x^{a/b} = z(x,a/b,x-16)$$

for the ultimate rates (Method B)

$$z_x^{a/b} = z(x,a/b,T)$$

9.3 It will, however, be seen from the bottom line in Table E7 or from the lowest diagram in Figure E4, that the select rates for sickness period 104/all do not run into the ultimate rates at all quickly. This is true not just for attained age 64, but for all attained ages. The run-in period lasts indefinitely. For example, no select rate approaches within 5% of the rate for entry age 16 within the first eight years from entry, and at select duration 8 it is only the rate for age 25 attained (age 17 at entry) that exceeds 95% of the rate for entry age 16.

It might be thought possible to avoid this problem by continuing the sickness periods. We could calculate, for example, a sickness rate for sickness period 104/52; this would have a run-in period that lasted three years; and we would be left with the same problem that the rates for 156/all would not converge on the rates for age 16 at entry. We could continue the process for further years, but on each occasion the run-in period would be extended by a year, and the residual balance would not provide a satisfactory select table with a limited number of select durations. Further, once we continued beyond five years of sickness, it would be necessary to subdivide sickness periods beyond the 780 steps that we have used. Although theoretically possible, we have found that this in practice considerably increases the calculation time necessary.

Another approach might be to jump straight to the ultimate rates after a chosen select period. This would, however, considerably exaggerate the rates of sickness for 104/all, which form an increasing fraction of total sickness as the duration from the initial entry age increases.

9.4 A measure of the inaccuracy of using select tables with a limited rather than an unlimited select period can be derived by calculating the total number of weeks sickness from entry age x up to a terminal age of say 65 using select tables constructed on both Methods.

Table E9 shows the maximum errors, for various periods of sickness. The errors are comparable with those for inception rates, except for 104/all and 1/all sickness periods; these two sickness periods are compared using both a 5-year and a 9-year select period.

9.5 Tables E23a to E23f show sickness rates using Methods A and B, with a 5-year select period, for sickness periods 0/1, 1/3, 4/9, 13/13, 26/26 and 52/52. Tables E24a to E24e show the full set of sickness rates for 104/all, for all entry ages. It is necessary to use this full table for calculations involving sickness rates for 104/all. There appear to be no short cuts for such calculations.

Table E9. *Maximum error in total weeks sickness*

Sickness period		Method A Age	Term	%	Method B Age	Term	%
0/1		56	9	−1·25	44	21	+1·10
1/3		56	9	−1·35	42	23	+1·33
4/9		56	9	−1·43	38	27	+1·57
13/13		57	8	−1·53	35	30	+1·81
26/26		57	8	−1·64	30	35	+2·01
52/52		57	8	−1·81	27	38	+2·17
104/all	(5)	58	7	+53·73	16	38	−49·46
	(9)	52	13	+13·40	16	39	−22·61
1/all	(5)	56	9	+15·14	33	32	−19·65
	(9)	51	14	+4·93	28	37	−8·75

(5) 5-year select period
(9) 9-year select period

10. PROBABILITIES OF SURVIVAL WHILE SICK

10.1 In Section 6 of Part F we show how it is possible to calculate the present value of future benefits using the inception rate method. For certain types of policy (mainly group policies) it is useful to count claim inceptions with deferred period d where the sickness commences between ages x and $x+1$, i.e. the potential claim commences between ages $x+d$ and $x+d+1$. We have already described these in Section 5 as claim inceptions of type (*b*). One way of approximating to the number of such inceptions is to count the number of periods of sickness that commence between ages x and $x+1$ and to multiply them by an average probability of survival while continuing sick from age $x+t$ $(0 \leqslant t \leqslant 1)$ to $x+t+d$. These probabilities, which are denoted in Section 2.4 of Part A as ${}_dp_x^{\overline{SS}}$ and in Section 1.2 of Part C as $\pi_{x,d}$, can be calculated directly by the formula

$$\pi_{x,d} = \exp(-\int_0^d (\rho_{x+t,t} + \nu_{x+t,t})\mathrm{dt} \tag{23}$$

which can be evaluated either numerically to as close a degree of approximation as desired, or, in the case of the formulae for ρ and ν derived in Part B and used in the calculations in this part, can be evaluated by direct integration.

Such a calculation gives us the value of $\pi_{x,d}$ for a single age. In order to calculate the value of $cb(x,d)$ from $cb(x,0)$ (which equals $ca(x,0)$ and $dHS(x)$) we need to multiply the latter by an 'average' value of $\pi_{y,d}$, taken over the year of age from x to $x+1$ and weighted by the number of sickness inceptions over this year of age. Rather than calculate such a weighted average, it is neater to derive the value of the average value of $\pi_{y,d}$, denoted $pi(x,d)$, over the year of age x to $x+1$ as

$$pi(x,d) = cb(x,d)/cb(x,0) \tag{24}$$

The values of $pi(x,d)$ so calculated are strictly dependent on the initial age x_0, but in practice they are almost insensitive to this age, and vary negligibly for almost all entry ages. The exception is that for entry ages above 55 the calculated values for $pi(x_0,d)$ fall below those for earlier entry ages, by up to 2 parts in 10,000 by age 64.

Table E25 shows values of these average probabilities $pi(x,d)$ for ages x from 16 to 64, and deferred periods d of 1, 4, 13, 26, 52 and 104 weeks, calculated from the claim inception rates for entry at age 16 (though as just noted, they are almost the same for all entry ages).

11. DEFERRED PERIODS 4, 13 AND 26 WEEKS

11.1 All the calculations in the earlier Sections of this Part are based on the graduated rates for deferred period 1 week. In this Section we consider calculations using the bases for deferred periods 4, 13 and 26 weeks. We first discuss the bases that we have used. There is no problem about two elements of the basis, the sickness intensities and the mortality rates of the sick.

The sickness intensities, σ_x, are those described in Section 9.1 of Part C, for the appropriate deferred period, defined by formulae (10) to (12). It can be noted that the values of σ_x, between the ages of 30 and 55, for each deferred period, are the following percentages of those for deferred period 1 week:

4 weeks	71–73%
13 weeks	51–60%
26 weeks	35–43%

These figures give an indication of the reduction in sickness, whether measured by sickness inceptions or sickness rates, to be expected from calculations using the sickness intensities for each deferred period, all other things being equal.

For the mortality rates of the sick, $\nu_{x,z}$, we have little choice but to use the graduated rates for all deferred periods as given by formula (9) in Section 6.2 of Part B, the same as are used for the deferred period 1 week basis.

11.2 In Section 4 of Part B it is shown how the recovery rates for deferred periods 4 weeks, 13 weeks and 26 weeks were different from those for 1 week, but only to the extent that the rates in the four weeks immediately following the deferred period were lower than those for deferred period 1 week. In the calculation of the sickness intensities in Part C it was assumed that these low recovery rates were accounted for by periods of sickness which lasted only a little longer than the deferred period and which were not reported to the insurance company, and the sickness intensity rates that were graduated were assumed to include these 'not reported' sicknesses. It is therefore appropriate in the calculations that follow to use the recovery rates for the deferred period 1 week basis, and this has been done. This procedure overestimates the number of inceptions reported to the extent shown in Table C1 of Part C, but it has very little effect on the number of weeks sickness, even for the first sickness period following the end of the deferred period, since the not-reported sickness periods last only for very few weeks (and in any case not more than 4 weeks) beyond the end of the deferred period.

The final element in the basis is the choice of the mortality rate of the healthy, μ_x. For the calculations for deferred period 1 week this was taken as the mortality for Male Permanent Assurances MA1979–82 duration 0. For other deferred periods there are two possible approaches: the first is to use the same mortality rates for the healthy; as will be seen, this results in lower aggregate mortality rates, mL_x, than for deferred period 1 week. The second is to use the aggregate rates for deferred period 1 week as the aggregate rates for the other deferred periods, using the method described in Section 3.4 of Part D; this would result in higher mortality rates for the healthy than was assumed for deferred period 1 week.

The first of these assumptions is consistent with the suggestion that the different experience of policies with different deferred periods reflects a genuinely different sickness experience among policyholders who take policies with different conditions. The second of these assumptions is consistent with the suggestion that the population of policyholders for all deferred periods is the same, and that the difference between the experiences of the different deferred periods results from different policy conditions or different propensities to claim.

For convenience we have followed the first of these assumptions, and have used the same mortality rates for the healthy for deferred periods 4 weeks, 13 weeks and 26 weeks as were used for deferred period 1 week. While the aggregate mortality level is affected by our choice, the effect on calculations relating to sickness benefits is small.

11.3 Calculations on the bases just described for 4 weeks, 13 weeks and 26 weeks were carried out, using the same methods as those described above for the deferred period 1 week basis. The necessary results are shown in Tables E26 to E37. Some general comments and comparisons, all based on the results for a life who is *healthy at age 30* follow.

First, in Table E10 are shown the values of pH_{65}, pS_{65}, pDH_{65} and pDS_{65}, on the four different bases, all conditional on a life who is healthy at age 30. These show the probabilities of being at age 65 in the states: healthy, sick, dead having died while healthy and dead having died while sick respectively. It can be seen how the lower sickness inception rates used in the higher deferred period bases result in a lower probability of reaching age 65 as sick, a higher probability of reaching age 65 as healthy, a higher probability of surviving to age 65 at all, a lower probability of having died while sick, and a slightly higher probability of having died while healthy.

Table E10. *Probabilities by age 65 conditional on being healthy at age 30*

Basis	pH_{65}	pS_{65}	pDH_{65}	pDS_{65}
1 week	0·689	0·132	0·083	0·096
4 weeks	0·727	0·113	0·085	0·075
13 weeks	0·780	0·079	0·087	0·054
26 weeks	0·804	0·064	0·088	0·043

The aggregate mortality on the four different bases for specimen ages is as shown in Table E11.

Table E11. *Aggregate mortality* mL(x) *conditional on being healthy at age 30*

	Deferred period basis:			
Age	1 week	4 weeks	13 weeks	26 weeks
31	0·000986	0·000831	0·000764	0·000628
35	0·001216	0·001030	0·000923	0·000761
40	0·001749	0·001496	0·001333	0·001147
45	0·002720	0·002345	0·002103	0·001890
50	0·004385	0·003801	0·003419	0·003170
55	0·007214	0·006292	0·005617	0·005295
60	0·012123	0·010760	0·009377	0·008827
65	0·020975	0·019633	0·016290	0·014845

Whereas the aggregate mortality rates on the deferred period 1 week basis exceeded those for AM80 ultimate at all ages shown, those for the higher deferred periods are lower than those for deferred period 1 week, and fall below those for AM80 ultimate above age 53 (4 weeks), 47 (13 weeks) and 43 (26 weeks), falling to as low as 96% (4 weeks, age 59), 83% (13 weeks, age 61) and 78% (26 weeks, age 63) of the AM80 ultimate rates.

11.4 Of more importance are the claim inception rates. Table E12 shows claim

Table E12. *Claim inception rates of type (a) per 10,000 conditional on being healthy at age 30*

Age	4 weeks 4 weeks basis	*C.M.I.R.* 7 Dp 4 weeks	13 weeks 13 weeks basis	*C.M.I.R.* 7 Dp 13 weeks	26 weeks 26 weeks basis	*C.M.I.R.* 7 Dp 26 weeks
31	161	114	22	18	4	5
35	190	140	27	25	6	6
40	227	171	35	33	9	9
45	274	204	47	44	15	13
50	344	250	67	60	25	23
55	470	334	101	87	45	41
60	716	509	169	143	83	78
64	1,095	812	276	240	139	135

inception rates of type (*a*) per 10,000, for a life who is healthy at age 30, showing the claim inception rates for 4 weeks deferred on the 4 weeks basis, 13 weeks deferred on the 13 weeks basis, and 26 weeks deferred on the 26 weeks basis, in each case compared with the graduated claim inception rates per 10,000 shown in table H5 of *C.M.I.R.* 7.

In each case the claim inception rates derived from the calculation basis are of the same order of size as those in *C.M.I.R.* 7, but rather larger. The excess is comparable with the excess of total inceptions ($IN+IR$) over reported inceptions (IR) shown in Table C1.

11.5 Although claim inceptions on the calculation basis are higher than those observed, for the reasons just explained, the sickness rates are closely comparable, with the exception of those for sickness period 104/all, as is discussed for the deferred period 1 week basis in Section 6. Tables E13a to E13c show comparisons of the sickness rates calculated on the 4 weeks basis (for sickness periods 4/9 upwards), 13 weeks basis (for sickness periods 13/13 upwards) and 26 weeks basis (for sickness periods 26/26 upwards) all compared with the graduated rates for the corresponding deferred period business shown in Tables H2, H3 and H4 of *C.M.I.R.* 7.

With a few exceptions the calculated sickness rates are similar to those in *C.M.I.R.* 7. The exceptions are: the 'run-in' period; sickness period 104/all where the calculated rates are considerably higher than the experience rates at ages up to 55; and by age 60 where the rates diverge somewhat erratically. A possible reason for this last feature is that the graduation formulae used in *C.M.I.R.* 7 are

Table E13a. *Comparison of sickness rates: 4 weeks basis and Table H2 of* C.M.I.R. 7

	Sickness periods				
Age	4/9	13/13	26/26	52/52	104/all
30	0·048	0·012	0·004	—	—
	0·055	0·025	0·017	0·011	0·014
35	0·072	0·026	0·021	0·022	0·026
	0·074	0·037	0·022	0·016	0·019
40	0·091	0·037	0·033	0·037	0·091
	0·094	0·048	0·032	0·027	0·036
45	0·115	0·053	0·052	0·062	0·200
	0·116	0·059	0·051	0·050	0·088
50	0·153	0·080	0·086	0·109	0·401
	0·150	0·077	0·087	0·099	0·226
55	0·220	0·130	0·152	0·206	0·806
	0·212	0·122	0·160	0·211	0·540
60	0·355	0·235	0·301	0·429	1·721
	0·346	0·257	0·317	0·461	1·289

Table E13b. *Comparison of sickness rates: 13 weeks basis and Table H3 of* C.M.I.R. 7

Age	Sickness periods 13/13	26/26	52/52	104/all
30	0·010	0·003	—	—
	0·013	0·014	0·010	0·014
35	0·020	0·017	0·017	0·026
	0·021	0·018	0·015	0·018
40	0·028	0·025	0·028	0·075
	0·030	0·027	0·025	0·035
45	0·040	0·039	0·047	0·155
	0·042	0·044	0·046	0·086
50	0·058	0·063	0·080	0·302
	0·060	0·077	0·092	0·220
55	0·093	0·110	0·148	0·594
	0·091	0·131	0·196	0·525
60	0·164	0·210	0·301	1·237
	0·153	0·214	0·429	1·254

Table E13c. *Comparison of sickness rates: 26 weeks basis and Table H4 of* C.M.I.R. 7

Age	Sickness periods 26/26	52/52	104/all
30	0·002	—	—
	0·008	0·007	0·010
35	0·010	0·010	0·016
	0·011	0·010	0·013
40	0·016	0·018	0·046
	0·015	0·016	0·026
45	0·028	0·033	0·100
	0·025	0·030	0·063
50	0·049	0·062	0·211
	0·045	0·061	0·162
55	0·091	0·123	0·451
	0·086	0·129	0·386
60	0·176	0·253	0·998
	0·179	0·283	0·923

based on a rather small experience at higher ages, and the values are therefore relatively unreliable.

11.6 Calculations were carried out for all entry ages from 16 to 64, and select tables of rates comparable to those calculated on the deferred period 1 week basis were calculated for each of the other deferred period bases. The results are shown in Tables E26 to E37, according to the following schedule.

Basis:	4 weeks	13 weeks	26 weeks
$q_{[x-t]+t}$ with 5 years selection	E26	E30	E34
$ia^{d}_{[x-t]+t}$ with 5 years selection, Methods A and B	E27 d = 4 weeks	E31 d = 13 weeks	E35a and b d = 26 and 52 weeks
$z^{a/b}_{[x-t]+t}$ with 5 years selection, Methods A and B	E28a to d a/b = 4/9, 13/13 26/26, 52/52	E32a to c a/b = 13/13 26/26, 52/52	E36a and b a/b = 26/26 52/52
$z(x,104/\text{all},t)$	E29a to e	E33a to e	E37a to e

Except for sickness rates for sickness period 104/all, for which rates are shown for all attained ages and entry ages, select tables using 5 years selection, and where appropriate using methods A and B are shown. Select mortality rates are shown for all bases. Select claim inception rates are shown only for the deferred period corresponding to the basis of calculation. Sickness rates are shown only for sickness periods after the deferred period corresponding to the basis of calculation.

11.7 The only remaining function values that were calculated and discussed above for the deferred period 1 week basis are the derived values of the average probabilities of survival while sick, $pi(x,d)$, discussed in Section 10 and shown in Table E25. Since the basis for recovery rates and mortality rates among the sick is the same on all four bases used, the values of $pi(x,d)$ for the other deferred bases are very close to those shown in Table E25. They have not therefore been repeated.

Table E14. *Survivors* (l) *and transitions* (d) *at each age based on a radix of 1,000,000: conditional on starting at age 30 with initial status healthy. One-week deferred period basis*

Age x	Healthy $lH(x)$	Sick $lS(x)$	Dead/H $lDH(x)$	Dead/S $lDS(x)$	Living $lL(x)$	Dead $lD(x)$	H to S $dHS(x)$	H to D $dHD(x)$	S to H $dSH(x)$	S to D $dSD(x)$
30	1,000,000	0	0	0	1,000,000	0	323,192	420	313,451	489
31	989,839	9,252	420	489	999,091	909	318,130	427	316,994	584
32	988,276	9,804	848	1,073	998,080	1,920	313,119	441	312,031	618
33	986,746	10,274	1,289	1,691	997,020	2,980	308,028	462	306,913	651
34	985,169	10,738	1,751	2,342	995,907	4,093	302,892	490	301,722	685
35	983,509	11,223	2,241	3,027	994,732	5,268	297,752	525	296,511	722
36	981,743	11,741	2,766	3,750	993,485	6,515	292,648	568	291,326	762
37	979,852	12,301	3,334	4,512	992,154	7,846	287,620	620	286,205	806
38	977,817	12,910	3,955	5,319	990,727	9,273	282,703	681	281,183	855
39	975,616	13,575	4,636	6,173	989,191	10,809	277,928	752	276,290	908
40	973,226	14,305	5,387	7,082	987,531	12,469	273,326	832	271,555	967
41	970,623	15,109	6,219	8,049	985,732	14,268	268,923	923	267,001	1,033
42	967,778	15,998	7,142	9,082	983,777	16,223	264,743	1,025	262,651	1,105
43	964,661	16,985	8,166	10,187	981,647	18,353	260,808	1,138	258,523	1,186
44	961,239	18,083	9,304	11,373	979,322	20,678	257,136	1,264	254,634	1,277
45	957,473	19,308	10,568	12,650	976,781	23,219	253,745	1,402	250,998	1,378
46	953,324	20,677	11,971	14,029	974,001	25,999	250,650	1,554	247,625	1,492
47	948,745	22,210	13,524	15,521	970,955	29,045	247,865	1,719	244,524	1,620
48	943,686	23,930	15,243	17,140	967,616	32,384	245,400	1,899	241,702	1,764
49	938,089	25,864	17,142	18,904	963,954	36,046	243,265	2,093	239,160	1,927

Table E14 *(Continued)*

Age x	Healthy $lH(x)$	Sick $lS(x)$	Dead/H $lDH(x)$	Dead/S $lDS(x)$	Living $lL(x)$	Dead $lD(x)$	H to S $dHS(x)$	H to D $dHD(x)$	S to H $dSH(x)$	S to D $dSD(x)$
50	931,892	28,042	19,235	20,831	959,934	40,066	241,468	2,301	236,900	2,111
51	925,022	30,499	21,536	22,943	955,521	44,479	240,015	2,525	234,919	2,321
52	917,401	33,274	24,061	25,263	950,676	49,324	238,907	2,764	233,209	2,558
53	908,939	36,415	26,825	27,822	945,353	54,647	238,147	3,018	231,759	2,829
54	899,534	39,973	29,843	30,651	939,506	60,494	237,729	3,286	230,554	3,138
55	889,073	44,009	33,128	33,789	933,082	66,917	237,647	3,567	229,570	3,491
56	877,429	48,595	36,695	37,281	926,024	73,976	237,885	3,861	228,775	3,895
57	864,458	53,809	40,557	41,176	918,267	81,733	238,424	4,166	228,130	4,358
58	849,998	59,745	44,723	45,534	909,743	90,257	239,231	4,480	227,582	4,889
59	833,868	66,506	49,203	50,423	900,374	99,626	240,263	4,799	227,061	5,497
60	815,867	74,210	54,002	55,920	890,078	109,922	241,459	5,120	226,483	6,196
61	795,771	82,991	59,122	62,116	878,762	121,238	242,738	5,437	225,738	6,997
62	773,333	92,994	64,560	69,112	866,328	133,672	243,990	5,745	224,690	7,914
63	748,288	104,380	70,305	77,027	852,668	147,332	245,074	6,035	223,173	8,963
64	720,352	117,318	76,341	85,989	837,670	162,330	245,804	6,298	220,981	10,157
65	689,230	131,984	82,639	96,147	821,214	178,786				

Table E15. *Subdivision of sick in each given sickness period at each age: conditional on starting at age 30 with initial status healthy. One-week deferred period basis*

Age	$pS(x,a/b)$ for sickness periods a/b shown								
x	0/1 wks	1/3 wks	4/9 wks	13/13 wks	26/26 wks	52/52 wks	104/all	0/all	1/all
30	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000	0·000000
31	0·003936	0·002963	0·001508	0·000486	0·000358	0·000000	0·000000	0·009252	0·005316
32	0·003901	0·003019	0·001583	0·000523	0·000393	0·000384	0·000000	0·009804	0·005903
33	0·003864	0·003075	0·001661	0·000563	0·000431	0·000429	0·000251	0·010274	0·006410
34	0·003826	0·003130	0·001741	0·000605	0·000473	0·000477	0·000485	0·010738	0·006912
35	0·003787	0·003186	0·001825	0·000651	0·000519	0·000532	0·000724	0·011223	0·007436
36	0·003747	0·003242	0·001912	0·000699	0·000569	0·000592	0·000980	0·011741	0·007994
37	0·003708	0·003298	0·002004	0·000751	0·000624	0·000658	0·001258	0·012301	0·008594
38	0·003669	0·003357	0·002099	0·000807	0·000683	0·000732	0·001562	0·012910	0·009241
39	0·003631	0·003417	0·002200	0·000867	0·000749	0·000814	0·001897	0·013575	0·009944
40	0·003595	0·003479	0·002306	0·000932	0·000820	0·000905	0·002267	0·014305	0·010710
41	0·003560	0·003545	0·002418	0·001002	0·000899	0·001007	0·002678	0·015109	0·011549
42	0·003528	0·003613	0·002537	0·001077	0·000986	0·001120	0·003137	0·015998	0·012470
43	0·003498	0·003686	0·002663	0·001159	0·001082	0·001246	0·003650	0·016985	0·013487
44	0·003472	0·003764	0·002798	0·001248	0·001187	0·001387	0·004227	0·018083	0·014612
45	0·003448	0·003846	0·002942	0·001345	0·001304	0·001545	0·004877	0·019308	0·015860
46	0·003428	0·003935	0·003097	0·001451	0·001434	0·001721	0·005610	0·020677	0·017248
47	0·003412	0·004030	0·003263	0·001567	0·001577	0·001920	0·006440	0·022210	0·018798
48	0·003400	0·004133	0·003442	0·001693	0·001737	0·002144	0·007381	0·023930	0·020530
49	0·003392	0·004244	0·003635	0·001832	0·001915	0·002395	0·008450	0·025864	0·022472

Table E15 *(Continued)*

Age x	0/1 wks	1/3 wks	4/9 wks	13/13 wks	26/26 wks	52/52 wks	104/all	0/all	1/all
	$pS(x,a/b)$ for sickness periods a/b shown								
50	0·003389	0·004364	0·003844	0·001985	0·002114	0·002680	0·009666	0·028042	0·024654
51	0·003390	0·004494	0·004071	0·002154	0·002337	0·003001	0·011053	0·030499	0·027109
52	0·003396	0·004635	0·004317	0·002340	0·002586	0·003365	0·012636	0·033274	0·029878
53	0·003407	0·004787	0·004585	0·002545	0·002866	0·003778	0·014447	0·036415	0·033008
54	0·003423	0·004952	0·004876	0·002773	0·003180	0·004246	0·016522	0·039973	0·036549
55	0·003444	0·005131	0·005193	0·003025	0·003534	0·004780	0·018903	0·044009	0·040565
56	0·003470	0·005324	0·005539	0·003305	0·003932	0·005387	0·021638	0·048595	0·045124
57	0·003502	0·005532	0·005916	0·003615	0·004380	0·006079	0·024785	0·053809	0·050308
58	0·003537	0·005755	0·006326	0·003960	0·004886	0·006869	0·028411	0·059745	0·056208
59	0·003577	0·005995	0·006773	0·004341	0·005456	0·007770	0·032594	0·066506	0·062929
60	0·003620	0·006250	0·007257	0·004764	0·006098	0·008798	0·037423	0·074210	0·070590
61	0·003666	0·006520	0·007780	0·005231	0·006820	0·009969	0·043004	0·082991	0·079325
62	0·003713	0·006804	0·008342	0·005746	0·007631	0·011303	0·049456	0·092994	0·089281
63	0·003759	0·007098	0·008943	0·006309	0·008537	0·012817	0·056917	0·104380	0·100621
64	0·003802	0·007398	0·009578	0·006922	0·009544	0·014530	0·065543	0·117318	0·113516
65	0·003838	0·007697	0·010241	0·007583	0·010656	0·016459	0·075510	0·131984	0·128146

Table E16. *ζ rates: 52 times proportions sick among the living at each age for each sickness period: conditional on starting at age 30 with initial status healthy. One-week deferred period basis*

Age x	$\zeta(x,a/b) = 52 \cdot pS(x,a/b)/pL(x)$ for sickness periods a/b shown								
	0/1 wks	1/3 wks	4/9 wks	13/13 wks	26/26 wks	52/52 wks	104/all	0/all	1/all
30	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000	0·0000
31	0·2049	0·1542	0·0785	0·0253	0·0186	0·0000	0·0000	0·4815	0·2767
32	0·2032	0·1573	0·0825	0·0273	0·0205	0·0200	0·0000	0·5108	0·3075
33	0·2015	0·1604	0·0866	0·0294	0·0225	0·0223	0·0131	0·5358	0·3343
34	0·1998	0·1634	0·0909	0·0316	0·0247	0·0249	0·0253	0·5607	0·3609
35	0·1980	0·1665	0·0954	0·0340	0·0271	0·0278	0·0378	0·5867	0·3887
36	0·1961	0·1697	0·1001	0·0366	0·0298	0·0310	0·0513	0·6146	0·4184
37	0·1943	0·1729	0·1050	0·0394	0·0327	0·0345	0·0659	0·6447	0·4504
38	0·1926	0·1762	0·1102	0·0424	0·0359	0·0384	0·0820	0·6776	0·4850
39	0·1909	0·1796	0·1157	0·0456	0·0394	0·0428	0·0997	0·7136	0·5227
40	0·1893	0·1832	0·1214	0·0491	0·0432	0·0477	0·1194	0·7532	0·5639
41	0·1878	0·1870	0·1276	0·0528	0·0474	0·0531	0·1413	0·7970	0·6092
42	0·1865	0·1910	0·1341	0·0569	0·0521	0·0592	0·1658	0·8456	0·6592
43	0·1853	0·1953	0·1411	0·0614	0·0573	0·0660	0·1934	0·8998	0·7144
44	0·1843	0·1998	0·1486	0·0663	0·0630	0·0736	0·2245	0·9602	0·7758
45	0·1836	0·2048	0·1566	0·0716	0·0694	0·0822	0·2596	1·0279	0·8443
46	0·1830	0·2101	0·1653	0·0775	0·0765	0·0919	0·2995	1·1039	0·9209
47	0·1827	0·2158	0·1748	0·0839	0·0845	0·1028	0·3449	1·1895	1·0067
48	0·1827	0·2221	0·1850	0·0910	0·0934	0·1152	0·3967	1·2860	1·1033
49	0·1830	0·2289	0·1961	0·0988	0·1033	0·1292	0·4558	1·3952	1·2123

Table E16 *(Continued)*

Age x	$\zeta(x,a/b)=52\cdot pS(x,a/b)/pL(x)$ for sickness periods a/b shown 0/1 wks	1/3 wks	4/9 wks	13/13 wks	26/26 wks	52/52 wks	104/all	0/all	1/all
50	0·1836	0·2364	0·2082	0·1075	0·1145	0·1452	0·5236	1·5191	1·3355
51	0·1845	0·2446	0·2215	0·1172	0·1272	0·1633	0·6015	1·6598	1·4753
52	0·1857	0·2535	0·2361	0·1280	0·1415	0·1840	0·6912	1·8200	1·6343
53	0·1874	0·2633	0·2522	0·1400	0·1576	0·2078	0·7947	2·0030	1·8156
54	0·1895	0·2741	0·2699	0·1535	0·1760	0·2350	0·9145	2·2124	2·0229
55	0·1919	0·2859	0·2894	0·1686	0·1969	0·2664	1·0534	2·4526	2·2607
56	0·1949	0·2989	0·3110	0·1856	0·2208	0·3025	1·2151	2·7288	2·5339
57	0·1983	0·3132	0·3350	0·2047	0·2480	0·3443	1·4036	3·0471	2·8489
58	0·2022	0·3290	0·3616	0·2263	0·2793	0·3926	1·6240	3·4150	3·2128
59	0·2066	0·3462	0·3911	0·2507	0·3151	0·4487	1·8824	3·8410	3·6344
60	0·2115	0·3651	0·4239	0·2783	0·3563	0·5140	2·1863	4·3355	4·1240
61	0·2169	0·3858	0·4604	0·3096	0·4036	0·5899	2·5447	4·9109	4·6940
62	0·2229	0·4084	0·5007	0·3449	0·4580	0·6784	2·9685	5·5818	5·3590
63	0·2293	0·4329	0·5454	0·3848	0·5206	0·7817	3·4711	6·3656	6·1364
64	0·2360	0·4592	0·5946	0·4297	0·5925	0·9020	4·0687	7·2828	7·0467
65	0·2430	0·4874	0·6485	0·4802	0·6748	1·0422	4·7814	8·3573	8·1143

Table E17. *Decrement rates and life table: conditional on starting at age 30 with initial status healthy. One-week deferred period basis*

Age	*H* to *D*	*S* to *D*	*L* to *D*		Life table	
x	$m(x)$	$mS(x)$	$mL(x)$	$l(x)$	$q(x)$	$L(x)$
30	0·000422	0·000000	0·000422	1,000,000	0·000909	999,567
31	0·000427	0·060814	0·000986	999,091	0·001012	998,589
32	0·000438	0·061409	0·001037	998,080	0·001062	997,554
33	0·000457	0·061769	0·001088	997,020	0·001116	996,468
34	0·000482	0·062190	0·001147	995,907	0·001180	995,325
35	0·000515	0·062661	0·001216	994,732	0·001254	994,115
36	0·000556	0·063170	0·001296	993,485	0·001340	992,827
37	0·000605	0·063713	0·001388	992,154	0·001438	991,449
38	0·000664	0·064285	0·001493	990,727	0·001550	989,969
39	0·000733	0·064878	0·001613	989,191	0·001678	988,372
40	0·000812	0·065488	0·001749	987,531	0·001822	986,644
41	0·000902	0·066111	0·001901	985,732	0·001984	984,768
42	0·001004	0·066741	0·002073	983,777	0·002165	982,727
43	0·001119	0·067377	0·002265	981,647	0·002368	980,501
44	0·001247	0·068017	0·002480	979,322	0·002594	978,071
45	0·001390	0·068657	0·002720	976,781	0·002847	975,412
46	0·001548	0·069299	0·002986	974,001	0·003127	972,501
47	0·001722	0·069940	0·003283	970,955	0·003439	969,311
48	0·001914	0·070582	0·003612	967,616	0·003785	965,813
49	0·002125	0·071223	0·003979	963,954	0·004170	961,975
50	0·002355	0·071867	0·004385	959,934	0·004597	957,762
51	0·002606	0·072512	0·004837	955,521	0·005071	953,136
52	0·002880	0·073162	0·005340	950,676	0·005599	948,056
53	0·003178	0·073815	0·005899	945,353	0·006185	942,476
54	0·003501	0·074475	0·006521	939,506	0·006837	936,345
55	0·003851	0·075141	0·007214	933,082	0·007565	929,609
56	0·004231	0·075814	0·007987	926,024	0·008376	922,207
57	0·004641	0·076495	0·008852	918,267	0·009283	914,072
58	0·005085	0·077182	0·009819	909,743	0·010298	905,132
59	0·005563	0·077874	0·010904	900,374	0·011436	895,307
60	0·006079	0·078569	0·012123	890,078	0·012713	884,509
61	0·006635	0·079262	0·013494	878,762	0·014150	872,642
62	0·007233	0·079949	0·015039	866,328	0·015767	859,605
63	0·007877	0·080621	0·016782	852,668	0·017590	845,286
64	0·008569	0·081270	0·018751	837,670	0·019645	829,569
65	0·009312	0·081881	0·020975	821,214		

Table E18a. *Claim inception rates of type (a) per 10,000 living at given deferred periods at each age: conditional on starting at age 30 with initial status healthy. One-week deferred period basis.*

Age	H to S	Claim inception rates $ia(x,d)$ per 10,000 living for deferred period d shown						
x	$10{,}000\sigma(x)$	0 weeks	1 week	4 weeks	13 weeks	26 weeks	52 weeks	104 weeks
30	3282·30	3233·32	1196·77	196·90	25·53	5·60	0·00	0·00
31	3238·95	3185·79	1223·91	221·46	36·03	11·94	5·22	0·00
32	3193·85	3138·87	1227·59	230·26	38·44	13·00	5·79	2·94
33	3147·56	3091·19	1230·68	239·28	41·00	14·16	6·41	3·31
34	3100·62	3043·14	1233·31	248·56	43·71	15·41	7·10	3·71
35	3053·54	2995·14	1235·63	258·12	46·59	16·76	7·86	4·17
36	3006·80	2947·63	1237·82	268·02	49·64	18·23	8·69	4·68
37	2960·84	2901·01	1240·05	278·29	52·89	19·82	9·61	5·24
38	2916·09	2855·67	1242·50	289·00	56·36	21·56	10·63	5·88
39	2872·95	2811·98	1245·35	300·21	60·07	23·45	11·76	6·59
40	2831·78	2770·26	1248·77	311·99	64·06	25·52	13·01	7·39
41	2792·94	2730·83	1252·95	324·42	68·34	27·78	14·40	8·29
42	2756·77	2693·96	1258·05	337·57	72·96	30·26	15·95	9·30
43	2723·58	2659·94	1264·27	351·56	77·95	32·98	17·67	10·44
44	2693·67	2629·01	1271·79	366·48	83·37	35·98	19·60	11·72
45	2667·36	2601·41	1280·80	382·47	89·25	39·30	21·75	13·18
46	2644·92	2577·38	1291·50	399·64	95·67	42·97	24·17	14·83
47	2626·68	2557·12	1304·08	418·14	102·68	47·04	26·89	16·70
48	2612·92	2540·86	1318·75	438·16	110·36	51·57	29·95	18·83
49	2603·98	2528·81	1335·74	459·86	118·81	56·63	33·41	21·26
50	2600·19	2521·17	1355·28	483·46	128·11	62·28	37·33	24·03
51	2601·92	2518·16	1377·60	509·18	138·40	68·61	41·78	27·21
52	2609·58	2519·97	1402·95	537·29	149·78	75·73	46·83	30·87
53	2623·60	2526·82	1431·62	568·07	162·42	83·74	52·60	35·07
54	2644·49	2538·91	1463·86	601·83	176·48	92·78	59·20	39·93
55	2672·82	2556·42	1499·96	638·93	192·15	103·02	66·75	45·54
56	2709·21	2579·52	1540·21	679·75	209·66	114·62	75·42	52·06
57	2754·41	2608·37	1584·88	724·70	229·24	127·79	85·41	59·63
58	2809·25	2643·05	1634·25	774·23	251·18	142·78	96·91	68·45
59	2874·72	2683·58	1688·55	828·81	275·77	159·86	110·20	78·74
60	2951·95	2729·87	1747·92	888·91	303·36	179·32	125·56	90·78
61	3042·24	2781·64	1812·44	954·99	334·28	201·52	143·34	104·88
62	3147·14	2838·40	1882·00	1027·45	368·91	226·82	163·93	121·43
63	3268·45	2899·30	1956·24	1106·58	407·60	255·63	187·75	140·85
64	3408·27	2963·03	2034·48	1192·46	450·64	288·33	215·27	163·64

Table E18b. *Claim inception rates of type (b) per 10,000 living at given deferred periods at each age: conditional on starting at age 30 with initial status healthy. One-week deferred period basis*

Age	H to S	Claim inception rates $ib(x,d)$ per 10,000 living for deferred period d shown						
x	$10{,}000\sigma(x)$	0 weeks	1 week	4 weeks	13 weeks	26 weeks	52 weeks	104 weeks
30	3282·30	3233·32	1220·25	213·62	34·31	11·43	5·22	2·94
31	3238·95	3185·79	1223·97	222·12	36·61	12·45	5·78	3·30
32	3193·85	3138·87	1227·63	230·93	39·06	13·56	6·40	3·70
33	3147·56	3091·19	1230·71	239·97	41·65	14·76	7·09	4·16
34	3100·62	3043·14	1233·32	249·26	44·40	16·06	7·85	4·66
35	3053·54	2995·14	1235·64	258·85	47·32	17·47	8·68	5·23
36	3006·80	2947·63	1237·83	268·76	50·42	19·00	9·60	5·86
37	2960·84	2901·01	1240·06	279·07	53·72	20·66	10·62	6·57
38	2916·09	2855·67	1242·52	289·81	57·24	22·47	11·74	7·37
39	2872·95	2811·98	1245·37	301·06	61·02	24·44	12·99	8·26
40	2831·78	2770·26	1248·80	312·88	65·07	26·60	14·37	9·26
41	2792·94	2730·83	1252·99	325·35	69·43	28·96	15·92	10·39
42	2756·77	2693·96	1258·11	338·56	74·13	31·55	17·63	11·67
43	2723·58	2659·94	1264·35	352·61	79·22	34·40	19·55	13·11
44	2693·67	2629·01	1271·89	367·60	84·73	37·55	21·69	14·74
45	2667·36	2601·41	1280·92	383·66	90·74	41·03	24·10	16·59
46	2644·92	2577·38	1291·64	400·91	97·28	44·88	26·80	18·70
47	2626·68	2557·12	1304·25	419·52	104·44	49·16	29·84	21·10
48	2612·92	2540·86	1318·96	439·64	112·29	53·93	33·28	23·83
49	2603·98	2528·81	1335·98	461·46	120·92	59·25	37·17	26·96
50	2600·19	2521·17	1355·56	485·19	130·44	65·20	41·57	30·55
51	2601·92	2518·16	1377·92	511·06	140·95	71·88	46·58	34·68
52	2609·58	2519·97	1403·32	539·33	152·60	79·38	52·29	39·44
53	2623·60	2526·82	1432·03	570·28	165·53	87·84	58·81	44·92
54	2644·49	2538·91	1464·32	604·24	179·92	97·40	66·27	51·27
55	2672·82	2556·42	1500·47	641·55	195·97	108·21	74·82	58·63
56	2709·21	2579·52	1540·77	682·61	213·89	120·47	84·65	67·18
57	2754·41	2608·37	1585·50	727·82	233·94	134·40	95·96	77·12
58	2809·25	2643·05	1634·93	777·62	256·39	150·24	109·00	88·71
59	2874·72	2683·58	1689·26	832·49	281·56	168·27	124·05	102·23
60	2951·95	2729·87	1748·68	892·89	309·76	188·81	141·42	118·01
61	3042·24	2781·64	1813·23	959·28	341·34	212·20	161·48	136·43
62	3147·14	2838·40	1882·80	1032·03	376·67	238·80	184·62	157·93
63	3268·45	2899·30	1957·04	1111·42	416·06	269·00	211·27	182·97
64	3408·27	2963·03	2035·24	1197·51	459·79	303·16	241·86	212·05

Table E19. *Sickness rates for each sickness period at each age: conditional on starting at age 30 with initial status healthy One-week deferred period basis*

Age	$z(x,a/b)$ in weeks of sickness for sickness periods a/b shown								
x	0/1 wks	1/3 wks	4/9 wks	13/13 wks	26/26 wks	52/52 wks	104/all	0/all	1/all
30	0·2040	0·1468	0·0663	0·0160	0·0052	0·0000	0·0000	0·4383	0·2343
31	0·2041	0·1558	0·0805	0·0263	0·0195	0·0107	0·0000	0·4969	0·2928
32	0·2024	0·1588	0·0845	0·0283	0·0215	0·0212	0·0067	0·5234	0·3210
33	0·2007	0·1619	0·0887	0·0305	0·0236	0·0236	0·0192	0·5482	0·3475
34	0·1989	0·1650	0·0931	0·0328	0·0259	0·0263	0·0315	0·5735	0·3747
35	0·1970	0·1681	0·0977	0·0353	0·0284	0·0294	0·0445	0·6004	0·4034
36	0·1952	0·1713	0·1025	0·0380	0·0312	0·0327	0·0585	0·6294	0·4342
37	0·1934	0·1745	0·1076	0·0409	0·0343	0·0364	0·0738	0·6609	0·4675
38	0·1917	0·1779	0·1129	0·0440	0·0376	0·0406	0·0907	0·6953	0·5036
39	0·1901	0·1814	0·1185	0·0473	0·0412	0·0452	0·1094	0·7331	0·5430
40	0·1885	0·1851	0·1245	0·0509	0·0453	0·0503	0·1301	0·7748	0·5862
41	0·1871	0·1890	0·1308	0·0549	0·0497	0·0561	0·1533	0·8209	0·6338
42	0·1859	0·1931	0·1376	0·0591	0·0547	0·0625	0·1793	0·8722	0·6863
43	0·1848	0·1975	0·1448	0·0638	0·0601	0·0697	0·2086	0·9294	0·7446
44	0·1839	0·2023	0·1526	0·0689	0·0662	0·0778	0·2417	0·9934	0·8094
45	0·1833	0·2074	0·1609	0·0745	0·0729	0·0870	0·2791	1·0651	0·8818
46	0·1829	0·2129	0·1700	0·0806	0·0804	0·0973	0·3217	1·1458	0·9629
47	0·1827	0·2189	0·1798	0·0874	0·0888	0·1089	0·3702	1·2367	1·0540
48	0·1828	0·2255	0·1904	0·0949	0·0982	0·1221	0·4256	1·3395	1·1566
49	0·1832	0·2326	0·2021	0·1031	0·1088	0·1370	0·4889	1·4558	1·2726
50	0·1840	0·2404	0·2148	0·1123	0·1207	0·1540	0·5616	1·5878	1·4039
51	0·1851	0·2490	0·2287	0·1225	0·1342	0·1734	0·6452	1·7381	1·5530
52	0·1865	0·2583	0·2440	0·1339	0·1494	0·1956	0·7416	1·9094	1·7229
53	0·1884	0·2686	0·2609	0·1466	0·1666	0·2211	0·8530	2·1052	1·9168
54	0·1907	0·2799	0·2795	0·1609	0·1862	0·2503	0·9821	2·3296	2·1389
55	0·1934	0·2923	0·3000	0·1769	0·2086	0·2840	1·1321	2·5873	2·3939
56	0·1965	0·3060	0·3228	0·1950	0·2341	0·3228	1·3067	2·8839	2·6874
57	0·2002	0·3210	0·3481	0·2153	0·2633	0·3678	1·5107	3·2263	3·0261
58	0·2043	0·3374	0·3761	0·2383	0·2968	0·4199	1·7495	3·6223	3·4180
59	0·2090	0·3555	0·4072	0·2642	0·3352	0·4805	2·0300	4·0816	3·8726
60	0·2142	0·3753	0·4418	0·2936	0·3793	0·5509	2·3602	4·6153	4·4011
61	0·2199	0·3969	0·4802	0·3268	0·4301	0·6329	2·7501	5·2369	5·0171
62	0·2260	0·4204	0·5226	0·3644	0·4885	0·7286	3·2119	5·9625	5·7365
63	0·2326	0·4458	0·5695	0·4067	0·5556	0·8401	3·7603	6·8108	6·5782
64	0·2395	0·4731	0·6210	0·4544	0·6326	0·9701	4·4133	7·8040	7·5645

Table E20a. *Select table of* $q_{[x]+t}$ *with 2 years selection. One-week deferred period basis*

Attained age x	Select durations (t) 0	1	2 and over
16	0·001107		
17	0·001069	0·001093	
18	0·001033	0·001060	0·001061
19	0·001000	0·001029	0·001032
20	0·000971	0·001002	0·001006
21	0·000945	0·000979	0·000983
22	0·000922	0·000959	0·000964
23	0·000903	0·000944	0·000950
24	0·000889	0·000933	0·000940
25	0·000879	0·000927	0·000935
26	0·000874	0·000925	0·000936
27	0·000874	0·000930	0·000942
28	0·000879	0·000940	0·000954
29	0·000891	0·000957	0·000974
30	0·000909	0·000981	0·001000
31	0·000934	0·001012	0·001035
32	0·000967	0·001052	0·001078
33	0·001007	0·001100	0·001130
34	0·001057	0·001157	0·001192
35	0·001115	0·001225	0·001266
36	0·001184	0·001304	0·001351
37	0·001263	0·001394	0·001449
38	0·001354	0·001497	0·001560
39	0·001458	0·001615	0·001687
40	0·001574	0·001747	0·001831
41	0·001706	0·001895	0·001992
42	0·001853	0·002062	0·002173
43	0·002017	0·002247	0·002376
44	0·002199	0·002454	0·002602
45	0·002401	0·002683	0·002853
46	0·002624	0·002937	0·003134
47	0·002870	0·003218	0·003445
48	0·003142	0·003530	0·003791
49	0·003441	0·003874	0·004175
50	0·003770	0·004255	0·004602
51	0·004131	0·004676	0·005077
52	0·004528	0·005141	0·005603
53	0·004964	0·005656	0·006190
54	0·005443	0·006226	0·006842

Table E20a. *(Continued)*

Select table of $q_{[x]+t}$ *with 2 years selection. One-week deferred period basis*

Attained age x	Select durations (t) 0	1	2 and over
55	0·005970	0·006857	0·007569
56	0·006549	0·007559	0·008380
57	0·007187	0·008338	0·009287
58	0·007889	0·009207	0·010302
59	0·008664	0·010176	0·011439
60	0·009520	0·011262	0·012717
61	0·010469	0·012480	0·014153
62	0·011522	0·013851	0·015770
63	0·012694	0·015400	0·017592
64	0·014002	0·017155	0·019647

Table E20b. *Select table of* $q_{[x]+t}$ *with 5 years selection. One-week deferred period basis*

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
16	0·001107					
17	0·001069	0·001093				
18	0·001033	0·001060	0·001061			
19	0·001000	0·001029	0·001031	0·001032		
20	0·000971	0·001002	0·001005	0·001005	0·001006	
21	0·000945	0·000979	0·000981	0·000982	0·000983	0·000983
22	0·000922	0·000959	0·000962	0·000963	0·000964	0·000964
23	0·000903	0·000944	0·000947	0·000948	0·000949	0·000950
24	0·000889	0·000933	0·000936	0·000938	0·000938	0·000940
25	0·000879	0·000927	0·000931	0·000932	0·000933	0·000935
26	0·000874	0·000925	0·000930	0·000932	0·000933	0·000936
27	0·000874	0·000930	0·000935	0·000937	0·000938	0·000942
28	0·000879	0·000940	0·000946	0·000949	0·000950	0·000954
29	0·000891	0·000957	0·000964	0·000967	0·000969	0·000974
30	0·000909	0·000981	0·000989	0·000992	0·000994	0·001000
31	0·000934	0·001012	0·001021	0·001025	0·001027	0·001035
32	0·000967	0·001052	0·001062	0·001066	0·001069	0·001078
33	0·001007	0·001100	0·001111	0·001116	0·001119	0·001130
34	0·001057	0·001157	0·001170	0·001176	0·001180	0·001192
35	0·001115	0·001225	0·001240	0·001247	0·001251	0·001266
36	0·001184	0·001304	0·001321	0·001328	0·001333	0·001351
37	0·001263	0·001394	0·001413	0·001422	0·001428	0·001449
38	0·001354	0·001497	0·001519	0·001530	0·001536	0·001560
39	0·001458	0·001615	0·001640	0·001651	0·001659	0·001687
40	0·001574	0·001747	0·001775	0·001789	0·001797	0·001831
41	0·001706	0·001895	0·001928	0·001943	0·001953	0·001992
42	0·001853	0·002062	0·002098	0·002116	0·002127	0·002173
43	0·002017	0·002247	0·002289	0·002309	0·002322	0·002376
44	0·002199	0·002454	0·002501	0·002524	0·002540	0·002602
45	0·002401	0·002683	0·002737	0·002764	0·002781	0·002853
46	0·002624	0·002937	0·002999	0·003030	0·003050	0·003134
47	0·002870	0·003218	0·003289	0·003325	0·003348	0·003445
48	0·003142	0·003530	0·003611	0·003651	0·003678	0·003791
49	0·003441	0·003874	0·003967	0·004014	0·004045	0·004175
50	0·003770	0·004255	0·004362	0·004415	0·004451	0·004602
51	0·004131	0·004676	0·004798	0·004860	0·004901	0·005077
52	0·004528	0·005141	0·005282	0·005353	0·005400	0·005603
53	0·004964	0·005656	0·005818	0·005900	0·005955	0·006190
54	0·005443	0·006226	0·006413	0·006507	0·006571	0·006842

Table E20b. *(Continued)*

Select table of $q_{[x]+t}$ *with 5 years selection. One-week deferred period basis*

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
55	0·005970	0·006857	0·007073	0·007183	0·007256	0·007569
56	0·006549	0·007559	0·007808	0·007935	0·008019	0·008380
57	0·007187	0·008338	0·008628	0·008774	0·008871	0·009287
58	0·007889	0·009207	0·009543	0·009712	0·009824	0·010302
59	0·008664	0·010176	0·010567	0·010763	0·010892	0·011439
60	0·009520	0·011262	0·011716	0·011943	0·012091	0·012717
61	0·010469	0·012480	0·013009	0·013271	0·013442	0·014153
62	0·011522	0·013851	0·014467	0·014770	0·014965	0·015770
63	0·012694	0·015400	0·016118	0·016466	0·016689	0·017592
64	0·014002	0·017155	0·017992	0·018391	0·018643	0·019647

Table E21a. *Select table of* $l_{[x]+t}$ *with 2 years selection. One-week deferred period basis*

Attained age x	Select durations (t) 0	1	2 and over
16	1,235,152		
17	1,233,752	1,233,784	
18	1,232,398	1,232,434	1,232,436
19	1,231,085	1,231,125	1,231,128
20	1,229,810	1,229,854	1,229,857
21	1,228,568	1,228,616	1,228,621
22	1,227,354	1,227,407	1,227,413
23	1,226,164	1,226,222	1,226,229
24	1,224,992	1,225,056	1,225,065
25	1,223,832	1,223,903	1,223,913
26	1,222,678	1,222,756	1,222,769
27	1,221,524	1,221,610	1,221,625
28	1,220,363	1,220,457	1,220,474
29	1,219,185	1,219,289	1,219,310
30	1,217,984	1,218,099	1,218,122
31	1,216,750	1,216,877	1,216,904
32	1,215,473	1,215,613	1,215,645
33	1,214,143	1,214,298	1,214,335
34	1,212,748	1,212,920	1,212,963
35	1,211,277	1,211,467	1,211,516
36	1,209,715	1,209,926	1,209,983
37	1,208,048	1,208,283	1,208,349
38	1,206,261	1,206,522	1,206,598
39	1,204,337	1,204,628	1,204,715
40	1,202,257	1,202,582	1,202,683
41	1,200,002	1,200,364	1,200,481
42	1,197,551	1,197,955	1,198,089
43	1,194,878	1,195,332	1,195,486
44	1,191,960	1,192,469	1,192,646
45	1,188,769	1,189,339	1,189,543
46	1,185,273	1,185,915	1,186,149
47	1,181,440	1,182,163	1,182,432
48	1,177,235	1,178,049	1,178,358
49	1,172,616	1,173,536	1,173,891
50	1,167,542	1,168,581	1,168,989
51	1,161,964	1,163,141	1,163,609
52	1,155,831	1,157,164	1,157,702
53	1,149,084	1,150,597	1,151,215
54	1,141,662	1,143,380	1,144,089

Table E21a *(Continued)*
Select table of $l_{[x]+t}$ *with 2 years selection. One-week deferred period basis*

Attained age x	Select durations (t) 0	1	2 and over
55	1,133,494	1,135,447	1,136,262
56	1,124,506	1,126,727	1,127,661
57	1,114,612	1,117,141	1,118,211
58	1,103,722	1,106,602	1,107,826
59	1,091,735	1,095,015	1,096,414
60	1,078,544	1,082,277	1,083,871
61	1,064,030	1,068,276	1,070,088
62	1,048,071	1,052,891	1,054,943
63	1,030,536	1,035,995	1,038,307
64	1,014,201	1,017,454	1,020,041
65	1,000,000	1,000,000	1,000,000

Table E21b. *Select table of* $l_{[x]+t}$ *with 5 years selection. One-week deferred period basis*

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
16	1,235,152					
17	1,233,751	1,233,784				
18	1,232,395	1,232,432	1,232,436			
19	1,231,080	1,231,122	1,231,126	1,231,128		
20	1,229,803	1,229,849	1,229,854	1,229,857	1,229,857	
21	1,228,559	1,228,609	1,228,616	1,228,619	1,228,620	1,228,621
22	1,227,343	1,227,398	1,227,406	1,227,410	1,227,412	1,227,412
23	1,226,150	1,226,211	1,226,221	1,226,225	1,226,228	1,226,229
24	1,224,974	1,225,042	1,225,053	1,225,059	1,225,063	1,225,065
25	1,223,811	1,223,885	1,223,899	1,223,906	1,223,911	1,223,913
26	1,222,653	1,222,735	1,222,751	1,222,760	1,222,765	1,222,769
27	1,221,494	1,221,584	1,221,604	1,221,614	1,221,621	1,221,625
28	1,220,326	1,220,426	1,220,449	1,220,461	1,220,469	1,220,474
29	1,219,142	1,219,253	1,219,279	1,219,294	1,219,303	1,219,310
30	1,217,933	1,218,055	1,218,086	1,218,103	1,218,115	1,218,122
31	1,216,689	1,216,825	1,216,861	1,216,881	1,216,895	1,216,904
32	1,215,402	1,215,552	1,215,594	1,215,618	1,215,634	1,215,645
33	1,214,059	1,214,227	1,214,274	1,214,303	1,214,322	1,214,335
34	1,212,650	1,212,836	1,212,891	1,212,925	1,212,947	1,212,963
35	1,211,162	1,211,369	1,211,433	1,211,472	1,211,498	1,211,516
36	1,209,581	1,209,811	1,209,885	1,209,931	1,209,962	1,209,983
37	1,207,892	1,208,149	1,208,234	1,208,287	1,208,323	1,208,349
38	1,206,080	1,206,366	1,206,465	1,206,526	1,206,569	1,206,598
39	1,204,126	1,204,446	1,204,560	1,204,631	1,204,681	1,204,715
40	1,202,013	1,202,371	1,202,502	1,202,585	1,202,642	1,202,683
41	1,199,719	1,200,120	1,200,271	1,200,367	1,200,434	1,200,481
42	1,197,222	1,197,672	1,197,845	1,197,957	1,198,034	1,198,089
43	1,194,498	1,195,004	1,195,203	1,195,332	1,195,422	1,195,486
44	1,191,520	1,192,089	1,192,318	1,192,467	1,192,572	1,192,646
45	1,188,260	1,188,900	1,189,164	1,189,336	1,189,457	1,189,543
46	1,184,686	1,185,407	1,185,711	1,185,909	1,186,049	1,186,149
47	1,180,763	1,181,577	1,181,926	1,182,155	1,182,316	1,182,432
48	1,176,453	1,177,373	1,177,774	1,178,038	1,178,225	1,178,358
49	1,171,716	1,172,757	1,173,217	1,173,521	1,173,736	1,173,891
50	1,166,505	1,167,684	1,168,213	1,168,563	1,168,811	1,168,989
51	1,160,772	1,162,108	1,162,716	1,163,118	1,163,404	1,163,609
52	1,154,461	1,155,977	1,156,674	1,157,137	1,157,466	1,157,702
53	1,147,513	1,149,234	1,150,034	1,150,565	1,150,943	1,151,215
54	1,139,863	1,141,817	1,142,734	1,143,343	1,143,777	1,144,089

Table E21b. *(Continued)*
Select table of $l_{[x]+t}$ with 5 years selection. One-week deferred period basis

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
55	1,131,440	1,133,659	1,134,708	1,135,406	1,135,903	1,136,262
56	1,122,165	1,124,685	1,125,885	1,126,682	1,127,251	1,127,661
57	1,111,954	1,114,815	1,116,184	1,117,093	1,117,742	1,118,211
58	1,100,718	1,103,963	1,105,520	1,106,554	1,107,292	1,107,826
59	1,088,362	1,092,035	1,093,799	1,094,970	1,095,807	1,096,414
60	1,074,784	1,078,932	1,080,922	1,082,241	1,083,185	1,083,871
61	1,061,077	1,064,551	1,066,781	1,068,258	1,069,317	1,070,088
62	1,046,304	1,049,968	1,051,266	1,052,904	1,054,082	1,054,943
63	1,030,536	1,034,249	1,035,425	1,036,057	1,037,353	1,038,307
64	1,014,201	1,017,454	1,018,321	1,018,736	1,018,997	1,020,041
65	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000

Table E22a. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *0 weeks. One-week deferred period basis*

Age x	Duration t: 0	1	2	3	4	5 and over B	5 and over A
16	0·336979						
17	0·340697	0·340581					
18	0·343512	0·343386	0·343368				
19	0·345455	0·345317	0·345297	0·345285			
20	0·346562	0·346412	0·346389	0·346375	0·346366		
21	0·346878	0·346715	0·346688	0·346672	0·346661	0·346653	0·346653
22	0·346450	0·346274	0·346243	0·346225	0·346213	0·346204	0·346196
23	0·345335	0·345145	0·345110	0·345089	0·345075	0·345064	0·345049
24	0·343590	0·343385	0·343346	0·343322	0·343305	0·343293	0·343269
25	0·341274	0·341055	0·341010	0·340983	0·340964	0·340950	0·340917
26	0·338451	0·338217	0·338166	0·338136	0·338114	0·338097	0·338055
27	0·335183	0·334933	0·334876	0·334842	0·334817	0·334798	0·334744
28	0·331533	0·331266	0·331203	0·331164	0·331136	0·331114	0·331048
29	0·327563	0·327278	0·327208	0·327164	0·327132	0·327107	0·327028
30	0·323332	0·323030	0·322951	0·322902	0·322866	0·322838	0·322744
31	0·318901	0·318579	0·318492	0·318437	0·318396	0·318365	0·318254
32	0·314325	0·313983	0·313887	0·313825	0·313779	0·313743	0·313615
33	0·309658	0·309295	0·309189	0·309119	0·309068	0·309028	0·308879
34	0·304952	0·304566	0·304449	0·304372	0·304314	0·304269	0·304097
35	0·300254	0·299845	0·299715	0·299629	0·299565	0·299514	0·299317
36	0·295612	0·295176	0·295033	0·294938	0·294866	0·294809	0·294583
37	0·291066	0·290603	0·290445	0·290339	0·290259	0·290195	0·289937
38	0·286658	0·286165	0·285991	0·285873	0·285783	0·285712	0·285418
39	0·282425	0·281899	0·281707	0·281576	0·281476	0·281396	0·281061
40	0·278403	0·277841	0·277628	0·277482	0·277371	0·277281	0·276901
41	0·274623	0·274022	0·273787	0·273625	0·273500	0·273399	0·272968
42	0·271119	0·270474	0·270214	0·270033	0·269893	0·269780	0·269290
43	0·267919	0·267226	0·266938	0·266736	0·266579	0·266453	0·265896
44	0·265054	0·264306	0·263987	0·263761	0·263585	0·263443	0·262810
45	0·262551	0·261742	0·261387	0·261133	0·260936	0·260776	0·260057
46	0·260438	0·259561	0·259165	0·258880	0·258658	0·258477	0·257659
47	0·258745	0·257790	0·257346	0·257026	0·256775	0·256571	0·255639
48	0·257499	0·256455	0·255958	0·255597	0·255313	0·255082	0·254018
49	0·256733	0·255587	0·255028	0·254619	0·254296	0·254033	0·252816
50	0·256477	0·255214	0·254583	0·254118	0·253751	0·253451	0·252056
51	0·256766	0·255369	0·254653	0·254123	0·253704	0·253361	0·251758
52	0·257640	0·256085	0·255270	0·254664	0·254183	0·253789	0·251943
53	0·259138	0·257400	0·256468	0·255771	0·255218	0·254764	0·252630
54	0·261309	0·259355	0·258284	0·257480	0·256840	0·256316	0·253841

Table E22a. *(Continued)*
Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *0 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·264204	0·261995	0·260759	0·259826	0·259082	0·258473	0·255594
56	0·267882	0·265370	0·263936	0·262848	0·261980	0·261269	0·257906
57	0·272410	0·269535	0·267862	0·266588	0·265569	0·264736	0·260793
58	0·277863	0·274553	0·272590	0·271088	0·269887	0·268904	0·264262
59	0·284328	0·280491	0·278173	0·276395	0·274970	0·273806	0·268316
60	0·291902	0·287424	0·284672	0·282552	0·280853	0·279466	0·272945
61	0·300699	0·295435	0·292146	0·289604	0·287566	0·285905	0·278123
62	0·310846	0·304613	0·300657	0·297589	0·295130	0·293128	0·283800
63	0·322488	0·315053	0·310262	0·306537	0·303552	0·301126	0·289890
64	0·335791	0·326855	0·321012	0·316460	0·312815	0·309858	0·296263

Table E22b. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *1 week. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·096935						
17	0·099800	0·101696					
18	0·102465	0·104411	0·104406				
19	0·104928	0·106919	0·106913	0·106910			
20	0·107185	0·109218	0·109211	0·109206	0·109203		
21	0·109237	0·111308	0·111299	0·111294	0·111291	0·111288	0·111288
22	0·111088	0·113192	0·113182	0·113176	0·113172	0·113169	0·113166
23	0·112743	0·114876	0·114864	0·114857	0·114852	0·114849	0·114844
24	0·114210	0·116367	0·116354	0·116346	0·116340	0·116336	0·116328
25	0·115498	0·117675	0·117660	0·117650	0·117644	0·117639	0·117627
26	0·116617	0·118811	0·118793	0·118783	0·118775	0·118769	0·118754
27	0·117580	0·119788	0·119767	0·119755	0·119746	0·119739	0·119720
28	0·118401	0·120619	0·120596	0·120581	0·120571	0·120563	0·120539
29	0·119095	0·121319	0·121293	0·121276	0·121265	0·121255	0·121226
30	0·119677	0·121904	0·121875	0·121856	0·121842	0·121832	0·121796
31	0·120162	0·122391	0·122358	0·122336	0·122321	0·122309	0·122266
32	0·120569	0·122797	0·122759	0·122735	0·122717	0·122703	0·122652
33	0·120914	0·123138	0·123095	0·123068	0·123047	0·123031	0·122972
34	0·121214	0·123433	0·123385	0·123354	0·123331	0·123312	0·123242
35	0·121488	0·123700	0·123646	0·123610	0·123584	0·123563	0·123481
36	0·121753	0·123956	0·123896	0·123855	0·123825	0·123801	0·123706
37	0·122027	0·124220	0·124152	0·124107	0·124073	0·124045	0·123935
38	0·122329	0·124511	0·124435	0·124383	0·124344	0·124313	0·124185
39	0·122676	0·124846	0·124761	0·124702	0·124658	0·124623	0·124474
40	0·123088	0·125245	0·125149	0·125083	0·125033	0·124992	0·124821
41	0·123582	0·125727	0·125618	0·125544	0·125486	0·125440	0·125242
42	0·124180	0·126310	0·126188	0·126103	0·126037	0·125985	0·125756
43	0·124899	0·127014	0·126877	0·126780	0·126706	0·126645	0·126381
44	0·125762	0·127860	0·127705	0·127596	0·127510	0·127442	0·127136
45	0·126789	0·128870	0·128694	0·128569	0·128472	0·128393	0·128039
46	0·128003	0·130065	0·129866	0·129723	0·129611	0·129521	0·129110
47	0·129428	0·131469	0·131242	0·131078	0·130950	0·130846	0·130370
48	0·131090	0·133107	0·132848	0·132660	0·132512	0·132392	0·131840
49	0·133016	0·135006	0·134709	0·134493	0·134322	0·134183	0·133540
50	0·135237	0·137195	0·136854	0·136604	0·136406	0·136245	0·135495
51	0·137787	0·139706	0·139313	0·139023	0·138793	0·138606	0·137728
52	0·140701	0·142574	0·142119	0·141781	0·141513	0·141294	0·140265
53	0·144021	0·145838	0·145308	0·144913	0·144599	0·144342	0·143132
54	0·147794	0·149540	0·148921	0·148456	0·148087	0·147784	0·146357

Table E22b. *(Continued)*

Select table of $\mathrm{ia}^{\mathrm{d}}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *1 week. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·152070	0·153727	0·153000	0·152452	0·152015	0·151658	0·149968
56	0·156908	0·158453	0·157595	0·156945	0·156426	0·156002	0·153993
57	0·162375	0·163778	0·162759	0·161984	0·161364	0·160858	0·158461
58	0·168546	0·169766	0·168550	0·167621	0·166878	0·166270	0·163399
59	0·175508	0·176494	0·175033	0·173913	0·173016	0·172283	0·168828
60	0·183358	0·184042	0·182277	0·180918	0·179830	0·178942	0·174765
61	0·192211	0·192503	0·190357	0·188700	0·187371	0·186288	0·181217
62	0·202197	0·201979	0·199353	0·197318	0·195687	0·194359	0·188173
63	0·213463	0·212582	0·209345	0·206831	0·204816	0·203179	0·195597
64	0·226183	0·224431	0·220416	0·217290	0·214786	0·212756	0·203420

Table E22c. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *4 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·009611						
17	0·010262	0·011086					
18	0·010926	0·011805	0·011804				
19	0·011602	0·012536	0·012535	0·012535			
20	0·012290	0·013279	0·013279	0·013278	0·013278		
21	0·012987	0·014034	0·014033	0·014032	0·014032	0·014031	0·014031
22	0·013694	0·014799	0·014797	0·014797	0·014796	0·014796	0·014795
23	0·014410	0·015573	0·015571	0·015571	0·015570	0·015569	0·015569
24	0·015135	0·016357	0·016355	0·016354	0·016353	0·016352	0·016351
25	0·015869	0·017150	0·017148	0·017146	0·017145	0·017145	0·017143
26	0·016612	0·017953	0·017950	0·017948	0·017947	0·017946	0·017944
27	0·017364	0·018766	0·018763	0·018761	0·018759	0·018758	0·018755
28	0·018127	0·019590	0·019586	0·019584	0·019582	0·019581	0·019577
29	0·018902	0·020427	0·020423	0·020420	0·020418	0·020416	0·020411
30	0·019690	0·021279	0·021273	0·021270	0·021268	0·021266	0·021260
31	0·020494	0·022146	0·022140	0·022136	0·022133	0·022131	0·022124
32	0·021316	0·023033	0·023026	0·023021	0·023018	0·023015	0·023006
33	0·022159	0·023942	0·023934	0·023928	0·023924	0·023921	0·023910
34	0·023025	0·024877	0·024867	0·024861	0·024856	0·024852	0·024838
35	0·023920	0·025841	0·025830	0·025822	0·025817	0·025812	0·025795
36	0·024847	0·026840	0·026826	0·026818	0·026811	0·026806	0·026785
37	0·025811	0·027878	0·027862	0·027852	0·027844	0·027838	0·027813
38	0·026818	0·028961	0·028943	0·028931	0·028922	0·028915	0·028885
39	0·027874	0·030097	0·030076	0·030062	0·030051	0·030042	0·030007
40	0·028985	0·031292	0·031267	0·031251	0·031238	0·031228	0·031185
41	0·030160	0·032554	0·032526	0·032506	0·032491	0·032479	0·032428
42	0·031408	0·033894	0·033860	0·033837	0·033820	0·033806	0·033744
43	0·032738	0·035320	0·035282	0·035254	0·035234	0·035217	0·035143
44	0·034161	0·036846	0·036801	0·036769	0·036744	0·036724	0·036636
45	0·035690	0·038484	0·038431	0·038393	0·038364	0·038340	0·038234
46	0·037339	0·040248	0·040186	0·040141	0·040107	0·040079	0·039951
47	0·039123	0·042157	0·042083	0·042030	0·041989	0·041955	0·041802
48	0·041062	0·044227	0·044140	0·044077	0·044028	0·043988	0·043804
49	0·043174	0·046481	0·046378	0·046303	0·046244	0·046196	0·045974
50	0·045485	0·048943	0·048820	0·048731	0·048660	0·048602	0·048334
51	0·048019	0·051641	0·051494	0·051386	0·051301	0·051231	0·050907
52	0·050809	0·054605	0·054429	0·054299	0·054196	0·054112	0·053717
53	0·053889	0·057873	0·057660	0·057503	0·057378	0·057276	0·056795
54	0·057299	0·061484	0·061227	0·061036	0·060883	0·060759	0·060171

Table E22c. *(Continued)*

Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *4 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·061088	0·065487	0·065175	0·064941	0·064754	0·064602	0·063881
56	0·065309	0·069937	0·069555	0·069267	0·069038	0·068850	0·067962
57	0·070026	0·074895	0·074426	0·074070	0·073786	0·073554	0·072457
58	0·075312	0·080434	0·079855	0·079413	0·079060	0·078772	0·077410
59	0·081254	0·086639	0·085918	0·085366	0·084925	0·084565	0·082868
60	0·087954	0·093604	0·092702	0·092009	0·091455	0·091003	0·088878
61	0·095530	0·101441	0·100306	0·099430	0·098729	0·098159	0·095485
62	0·104121	0·110279	0·108839	0·107726	0·106835	0·106109	0·102730
63	0·113893	0·120261	0·118425	0·117000	0·115860	0·114933	0·110642
64	0·125040	0·131557	0·129197	0·127363	0·125894	0·124704	0·119230

Table E22d. *Select table of* $ia^d_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *13 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·000863						
17	0·000946	0·001247					
18	0·001034	0·001364	0·001364				
19	0·001128	0·001487	0·001487	0·001487			
20	0·001226	0·001618	0·001618	0·001618	0·001618		
21	0·001331	0·001756	0·001756	0·001756	0·001755	0·001755	0·001755
22	0·001440	0·001901	0·001901	0·001901	0·001901	0·001901	0·001901
23	0·001556	0·002054	0·002054	0·002054	0·002054	0·002054	0·002053
24	0·001678	0·002215	0·002215	0·002215	0·002215	0·002214	0·002214
25	0·001806	0·002384	0·002384	0·002384	0·002384	0·002384	0·002383
26	0·001940	0·002563	0·002562	0·002562	0·002562	0·002562	0·002561
27	0·002082	0·002750	0·002749	0·002749	0·002749	0·002749	0·002748
28	0·002231	0·002947	0·002946	0·002946	0·002945	0·002945	0·002945
29	0·002387	0·003154	0·003153	0·003153	0·003153	0·003152	0·003152
30	0·002553	0·003372	0·003372	0·003371	0·003371	0·003370	0·003369
31	0·002727	0·003603	0·003602	0·003601	0·003600	0·003600	0·003599
32	0·002911	0·003846	0·003844	0·003844	0·003843	0·003843	0·003841
33	0·003105	0·004103	0·004101	0·004100	0·004100	0·004099	0·004097
34	0·003312	0·004375	0·004373	0·004372	0·004371	0·004371	0·004368
35	0·003531	0·004664	0·004662	0·004660	0·004659	0·004659	0·004655
36	0·003763	0·004971	0·004969	0·004967	0·004966	0·004965	0·004961
37	0·004012	0·005299	0·005295	0·005293	0·005292	0·005291	0·005286
38	0·004277	0·005648	0·005645	0·005642	0·005640	0·005639	0·005633
39	0·004562	0·006023	0·006019	0·006016	0·006013	0·006012	0·006004
40	0·004867	0·006425	0·006420	0·006416	0·006414	0·006412	0·006403
41	0·005197	0·006858	0·006852	0·006848	0·006845	0·006842	0·006831
42	0·005552	0·007326	0·007318	0·007313	0·007310	0·007306	0·007293
43	0·005938	0·007832	0·007823	0·007817	0·007813	0·007809	0·007792
44	0·006357	0·008382	0·008372	0·008364	0·008358	0·008354	0·008334
45	0·006814	0·008982	0·008968	0·008959	0·008953	0·008947	0·008922
46	0·007314	0·009636	0·009620	0·009609	0·009601	0·009594	0·009564
47	0·007862	0·010353	0·010334	0·010321	0·010311	0·010303	0·010265
48	0·008465	0·011142	0·011119	0·011103	0·011090	0·011080	0·011033
49	0·009130	0·012011	0·011983	0·011963	0·011948	0·011935	0·011878
50	0·009867	0·012972	0·012938	0·012914	0·012895	0·012880	0·012808
51	0·010686	0·014039	0·013997	0·013967	0·013944	0·013925	0·013836
52	0·011598	0·015226	0·015175	0·015138	0·015109	0·015085	0·014975
53	0·012618	0·016550	0·016488	0·016442	0·016406	0·016377	0·016238
54	0·013763	0·018034	0·017956	0·017899	0·017854	0·017817	0·017644

Table E22d. *(Continued)*
Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *13 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·015050	0·019700	0·019603	0·019532	0·019475	0·019429	0·019212
56	0·016505	0·021577	0·021456	0·021366	0·021295	0·021237	0·020962
57	0·018152	0·023698	0·023546	0·023432	0·023342	0·023269	0·022920
58	0·020024	0·026103	0·025911	0·025766	0·025651	0·025557	0·025114
59	0·022160	0·028837	0·028592	0·028407	0·028260	0·028139	0·027573
60	0·024605	0·031955	0·031641	0·031403	0·031213	0·031058	0·030331
61	0·027412	0·035521	0·035117	0·034808	0·034562	0·034361	0·033423
62	0·030648	0·039610	0·039086	0·038683	0·038362	0·038101	0·036886
63	0·034390	0·044312	0·043627	0·043100	0·042679	0·042337	0·040754
64	0·038732	0·049732	0·048831	0·048135	0·047579	0·047129	0·045058

Table E22e. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *26 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·000141						
17	0·000158	0·000307					
18	0·000176	0·000343	0·000343				
19	0·000196	0·000383	0·000383	0·000382			
20	0·000218	0·000425	0·000425	0·000425	0·000425		
21	0·000242	0·000471	0·000471	0·000471	0·000471	0·000471	0·000471
22	0·000267	0·000521	0·000521	0·000521	0·000521	0·000521	0·000521
23	0·000295	0·000575	0·000575	0·000575	0·000575	0·000575	0·000575
24	0·000325	0·000634	0·000634	0·000634	0·000634	0·000634	0·000634
25	0·000357	0·000697	0·000697	0·000697	0·000697	0·000697	0·000697
26	0·000392	0·000765	0·000765	0·000765	0·000765	0·000765	0·000764
27	0·000429	0·000838	0·000838	0·000838	0·000838	0·000838	0·000838
28	0·000469	0·000917	0·000917	0·000917	0·000917	0·000917	0·000917
29	0·000513	0·001003	0·001002	0·001002	0·001002	0·001002	0·001002
30	0·000560	0·001094	0·001094	0·001094	0·001094	0·001094	0·001093
31	0·000610	0·001194	0·001193	0·001193	0·001193	0·001193	0·001192
32	0·000665	0·001301	0·001300	0·001300	0·001300	0·001300	0·001299
33	0·000724	0·001417	0·001416	0·001416	0·001416	0·001415	0·001415
34	0·000789	0·001542	0·001542	0·001541	0·001541	0·001541	0·001540
35	0·000858	0·001678	0·001677	0·001677	0·001676	0·001676	0·001675
36	0·000934	0·001826	0·001825	0·001824	0·001824	0·001823	0·001822
37	0·001016	0·001986	0·001985	0·001984	0·001984	0·001983	0·001981
38	0·001105	0·002161	0·002159	0·002158	0·002158	0·002157	0·002155
39	0·001203	0·002351	0·002349	0·002348	0·002347	0·002347	0·002344
40	0·001310	0·002560	0·002557	0·002556	0·002555	0·002554	0·002550
41	0·001427	0·002788	0·002785	0·002783	0·002782	0·002781	0·002776
42	0·001556	0·003039	0·003035	0·003033	0·003031	0·003030	0·003024
43	0·001698	0·003315	0·003310	0·003308	0·003306	0·003304	0·003297
44	0·001854	0·003619	0·003614	0·003610	0·003608	0·003606	0·003597
45	0·002028	0·003956	0·003949	0·003945	0·003942	0·003940	0·003929
46	0·002221	0·004329	0·004322	0·004317	0·004313	0·004310	0·004296
47	0·002435	0·004745	0·004735	0·004729	0·004724	0·004720	0·004703
48	0·002675	0·005208	0·005197	0·005189	0·005183	0·005178	0·005156
49	0·002943	0·005727	0·005712	0·005703	0·005695	0·005689	0·005661
50	0·003245	0·006308	0·006290	0·006278	0·006269	0·006261	0·006226
51	0·003585	0·006962	0·006940	0·006925	0·006913	0·006904	0·006860
52	0·003969	0·007701	0·007673	0·007654	0·007639	0·007627	0·007571
53	0·004404	0·008537	0·008502	0·008478	0·008459	0·008444	0·008372
54	0·004899	0·009486	0·009442	0·009412	0·009388	0·009368	0·009277

Table E22e. *(Continued)*

Select table of $\mathrm{ia}^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *26 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·005465	0·010567	0·010512	0·010473	0·010442	0·010417	0·010300
56	0·006112	0·011802	0·011732	0·011682	0·011643	0·011611	0·011460
57	0·006856	0·013218	0·013129	0·013064	0·013014	0·012972	0·012777
58	0·007714	0·014846	0·014732	0·014649	0·014583	0·014529	0·014276
59	0·008707	0·016725	0·016578	0·016469	0·016383	0·016313	0·015983
60	0·009860	0·018901	0·018709	0·018566	0·018453	0·018361	0·017929
61	0·011205	0·021426	0·021175	0·020987	0·020837	0·020716	0·020149
62	0·012778	0·024369	0·024038	0·023788	0·023589	0·023428	0·022679
63	0·014625	0·027808	0·027368	0·027034	0·026769	0·026554	0·025559
64	0·016803	0·031839	0·031251	0·030802	0·030445	0·030157	0·028829

Table E22f. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *52 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·000000						
17	0·000000	0·000104					
18	0·000000	0·000118	0·000118				
19	0·000000	0·000134	0·000134	0·000134			
20	0·000000	0·000152	0·000152	0·000152	0·000152		
21	0·000000	0·000172	0·000172	0·000172	0·000172	0·000172	0·000172
22	0·000000	0·000194	0·000194	0·000194	0·000194	0·000194	0·000194
23	0·000000	0·000218	0·000218	0·000218	0·000218	0·000218	0·000218
24	0·000000	0·000244	0·000244	0·000244	0·000244	0·000244	0·000244
25	0·000000	0·000274	0·000274	0·000274	0·000274	0·000274	0·000273
26	0·000000	0·000306	0·000306	0·000306	0·000306	0·000306	0·000306
27	0·000000	0·000341	0·000341	0·000341	0·000341	0·000341	0·000341
28	0·000000	0·000380	0·000380	0·000380	0·000380	0·000380	0·000380
29	0·000000	0·000423	0·000423	0·000423	0·000423	0·000423	0·000423
30	0·000000	0·000470	0·000470	0·000470	0·000470	0·000470	0·000470
31	0·000000	0·000522	0·000522	0·000522	0·000522	0·000522	0·000521
32	0·000000	0·000579	0·000579	0·000579	0·000578	0·000578	0·000578
33	0·000000	0·000642	0·000641	0·000641	0·000641	0·000641	0·000641
34	0·000000	0·000711	0·000710	0·000710	0·000710	0·000710	0·000709
35	0·000000	0·000787	0·000786	0·000786	0·000786	0·000786	0·000785
36	0·000000	0·000871	0·000870	0·000870	0·000869	0·000869	0·000869
37	0·000000	0·000964	0·000963	0·000962	0·000962	0·000962	0·000961
38	0·000000	0·001067	0·001065	0·001065	0·001064	0·001064	0·001063
39	0·000000	0·001180	0·001179	0·001178	0·001177	0·001177	0·001176
40	0·000000	0·001307	0·001304	0·001304	0·001303	0·001303	0·001301
41	0·000000	0·001447	0·001444	0·001443	0·001443	0·001442	0·001440
42	0·000000	0·001604	0·001600	0·001599	0·001598	0·001597	0·001594
43	0·000000	0·001778	0·001774	0·001772	0·001771	0·001770	0·001767
44	0·000000	0·001973	0·001969	0·001967	0·001965	0·001964	0·001959
45	0·000000	0·002192	0·002187	0·002184	0·002182	0·002181	0·002175
46	0·000000	0·002439	0·002432	0·002428	0·002426	0·002424	0·002416
47	0·000000	0·002716	0·002707	0·002703	0·002701	0·002698	0·002688
48	0·000000	0·003029	0·003019	0·003014	0·003010	0·003007	0·002994
49	0·000000	0·003384	0·003372	0·003365	0·003361	0·003357	0·003340
50	0·000000	0·003788	0·003772	0·003764	0·003758	0·003753	0·003732
51	0·000000	0·004247	0·004228	0·004217	0·004210	0·004204	0·004177
52	0·000000	0·004772	0·004748	0·004735	0·004725	0·004718	0·004682
53	0·000000	0·005374	0·005343	0·005327	0·005315	0·005305	0·005259
54	0·000000	0·006065	0·006027	0·006006	0·005990	0·005978	0·005918

Table E22f. *(Continued)*

Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *52 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·000000	0·006863	0·006815	0·006788	0·006767	0·006751	0·006674
56	0·000000	0·007786	0·007725	0·007690	0·007663	0·007642	0·007541
57	0·000000	0·008857	0·008780	0·008734	0·008699	0·008671	0·008539
58	0·000000	0·010106	0·010006	0·009946	0·009900	0·009863	0·009690
59	0·000000	0·011565	0·011436	0·011357	0·011296	0·011247	0·011018
60	0·000000	0·013276	0·013109	0·013005	0·012924	0·012859	0·012554
61	0·000000	0·015292	0·015073	0·014934	0·014825	0·014738	0·014332
62	0·000000	0·017673	0·017384	0·017198	0·017052	0·016935	0·016390
63	0·000000	0·020497	0·020114	0·019862	0·019665	0·019506	0·018772
64	0·000000	0·023860	0·023348	0·023005	0·022736	0·022519	0·021524

Table E22g. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *104 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·000000						
17	0·000000	0·000000					
18	0·000000	0·000000	0·000048				
19	0·000000	0·000000	0·000056	0·000056			
20	0·000000	0·000000	0·000064	0·000064	0·000064		
21	0·000000	0·000000	0·000074	0·000074	0·000074	0·000074	0·000074
22	0·000000	0·000000	0·000084	0·000084	0·000084	0·000084	0·000084
23	0·000000	0·000000	0·000096	0·000096	0·000096	0·000096	0·000096
24	0·000000	0·000000	0·000110	0·000110	0·000110	0·000110	0·000110
25	0·000000	0·000000	0·000125	0·000125	0·000125	0·000125	0·000125
26	0·000000	0·000000	0·000142	0·000142	0·000142	0·000142	0·000142
27	0·000000	0·000000	0·000161	0·000161	0·000161	0·000161	0·000161
28	0·000000	0·000000	0·000182	0·000182	0·000182	0·000182	0·000182
29	0·000000	0·000000	0·000206	0·000205	0·000205	0·000205	0·000205
30	0·000000	0·000000	0·000232	0·000232	0·000232	0·000232	0·000232
31	0·000000	0·000000	0·000261	0·000261	0·000261	0·000261	0·000261
32	0·000000	0·000000	0·000294	0·000294	0·000294	0·000294	0·000294
33	0·000000	0·000000	0·000331	0·000331	0·000330	0·000330	0·000330
34	0·000000	0·000000	0·000372	0·000371	0·000371	0·000371	0·000371
35	0·000000	0·000000	0·000417	0·000417	0·000417	0·000417	0·000416
36	0·000000	0·000000	0·000468	0·000468	0·000468	0·000468	0·000467
37	0·000000	0·000000	0·000526	0·000525	0·000525	0·000525	0·000524
38	0·000000	0·000000	0·000590	0·000589	0·000588	0·000588	0·000588
39	0·000000	0·000000	0·000661	0·000660	0·000660	0·000660	0·000659
40	0·000000	0·000000	0·000742	0·000741	0·000740	0·000740	0·000739
41	0·000000	0·000000	0·000832	0·000831	0·000830	0·000830	0·000829
42	0·000000	0·000000	0·000934	0·000933	0·000932	0·000932	0·000930
43	0·000000	0·000000	0·001050	0·001047	0·001047	0·001046	0·001044
44	0·000000	0·000000	0·001180	0·001177	0·001176	0·001175	0·001172
45	0·000000	0·000000	0·001327	0·001324	0·001322	0·001321	0·001317
46	0·000000	0·000000	0·001494	0·001490	0·001489	0·001487	0·001482
47	0·000000	0·000000	0·001684	0·001680	0·001677	0·001676	0·001669
48	0·000000	0·000000	0·001901	0·001895	0·001893	0·001891	0·001882
49	0·000000	0·000000	0·002149	0·002142	0·002139	0·002136	0·002125
50	0·000000	0·000000	0·002434	0·002425	0·002420	0·002417	0·002403
51	0·000000	0·000000	0·002761	0·002749	0·002743	0·002739	0·002721
52	0·000000	0·000000	0·003137	0·003123	0·003116	0·003110	0·003086
53	0·000000	0·000000	0·003572	0·003555	0·003545	0·003538	0·003507
54	0·000000	0·000000	0·004077	0·004055	0·004042	0·004033	0·003992

Table E22g. *(Continued)*

Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *104 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·000000	0·000000	0·004664	0·004635	0·004620	0·004608	0·004554
56	0·000000	0·000000	0·005349	0·005312	0·005292	0·005276	0·005205
57	0·000000	0·000000	0·006151	0·006104	0·006077	0·006056	0·005962
58	0·000000	0·000000	0·007092	0·007032	0·006996	0·006969	0·006843
59	0·000000	0·000000	0·008203	0·008124	0·008076	0·008040	0·007873
60	0·000000	0·000000	0·009516	0·009413	0·009350	0·009301	0·009076
61	0·000000	0·000000	0·011075	0·010940	0·010855	0·010789	0·010487
62	0·000000	0·000000	0·012934	0·012754	0·012640	0·012550	0·012141
63	0·000000	0·000000	0·015159	0·014919	0·014762	0·014640	0·014083
64	0·000000	0·000000	0·017832	0·017508	0·017294	0·017126	0·016362

Table E23a. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *0/1 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·1944						
17	0·1978	0·1993					
18	0·2006	0·2022	0·2022				
19	0·2031	0·2046	0·2046	0·2046			
20	0·2050	0·2066	0·2065	0·2065	0·2065		
21	0·2065	0·2081	0·2080	0·2080	0·2080	0·2080	0·2080
22	0·2076	0·2091	0·2091	0·2091	0·2091	0·2091	0·2091
23	0·2082	0·2098	0·2098	0·2098	0·2098	0·2098	0·2097
24	0·2085	0·2101	0·2101	0·2100	0·2100	0·2100	0·2100
25	0·2084	0·2100	0·2100	0·2100	0·2100	0·2100	0·2099
26	0·2081	0·2096	0·2096	0·2096	0·2096	0·2096	0·2095
27	0·2074	0·2090	0·2089	0·2089	0·2089	0·2089	0·2088
28	0·2065	0·2080	0·2080	0·2080	0·2079	0·2079	0·2079
29	0·2054	0·2069	0·2068	0·2068	0·2068	0·2068	0·2067
30	0·2040	0·2055	0·2055	0·2055	0·2054	0·2054	0·2054
31	0·2026	0·2041	0·2040	0·2040	0·2039	0·2039	0·2038
32	0·2010	0·2025	0·2024	0·2024	0·2023	0·2023	0·2022
33	0·1993	0·2008	0·2007	0·2007	0·2006	0·2006	0·2005
34	0·1976	0·1990	0·1989	0·1989	0·1989	0·1988	0·1987
35	0·1959	0·1973	0·1972	0·1971	0·1971	0·1970	0·1969
36	0·1941	0·1955	0·1954	0·1953	0·1953	0·1953	0·1951
37	0·1925	0·1938	0·1937	0·1936	0·1935	0·1935	0·1933
38	0·1908	0·1921	0·1920	0·1919	0·1919	0·1918	0·1916
39	0·1893	0·1906	0·1904	0·1903	0·1903	0·1902	0·1900
40	0·1879	0·1891	0·1890	0·1889	0·1888	0·1887	0·1885
41	0·1866	0·1878	0·1876	0·1875	0·1874	0·1874	0·1871
42	0·1855	0·1866	0·1865	0·1863	0·1862	0·1862	0·1858
43	0·1846	0·1857	0·1855	0·1853	0·1852	0·1851	0·1847
44	0·1838	0·1849	0·1847	0·1845	0·1844	0·1843	0·1839
45	0·1834	0·1844	0·1842	0·1840	0·1838	0·1837	0·1832
46	0·1832	0·1842	0·1839	0·1837	0·1835	0·1834	0·1828
47	0·1833	0·1842	0·1839	0·1836	0·1835	0·1833	0·1826
48	0·1837	0·1845	0·1842	0·1839	0·1837	0·1835	0·1828
49	0·1844	0·1852	0·1848	0·1845	0·1843	0·1841	0·1832
50	0·1855	0·1862	0·1858	0·1854	0·1852	0·1850	0·1839
51	0·1871	0·1877	0·1872	0·1868	0·1865	0·1862	0·1850
52	0·1891	0·1896	0·1890	0·1885	0·1882	0·1879	0·1865
53	0·1915	0·1919	0·1912	0·1907	0·1903	0·1899	0·1884
54	0·1945	0·1948	0·1940	0·1934	0·1929	0·1925	0·1906

Table E23a. *(Continued)*

Select table of $z_{[x]+t}^{a/b}$ *with 5 years selection: methods A and B: sickness period* (a/b) *0/1 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·1981	0·1982	0·1972	0·1965	0·1960	0·1955	0·1933
56	0·2023	0·2022	0·2011	0·2003	0·1996	0·1991	0·1965
57	0·2072	0·2069	0·2056	0·2046	0·2038	0·2032	0·2002
58	0·2129	0·2123	0·2107	0·2096	0·2087	0·2079	0·2043
59	0·2195	0·2185	0·2166	0·2153	0·2142	0·2132	0·2090
60	0·2270	0·2255	0·2233	0·2217	0·2204	0·2193	0·2141
61	0·2356	0·2335	0·2309	0·2289	0·2273	0·2260	0·2198
62	0·2453	0·2426	0·2394	0·2370	0·2350	0·2334	0·2260
63	0·2564	0·2528	0·2489	0·2459	0·2435	0·2416	0·2326
64	0·2690	0·2642	0·2595	0·2558	0·2529	0·2505	0·2395

Table E23b. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *1/3 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0958						
17	0·1001	0·1039					
18	0·1043	0·1083	0·1083				
19	0·1085	0·1126	0·1126	0·1126			
20	0·1125	0·1168	0·1168	0·1168	0·1168		
21	0·1164	0·1209	0·1208	0·1208	0·1208	0·1208	0·1208
22	0·1201	0·1248	0·1248	0·1248	0·1248	0·1248	0·1248
23	0·1238	0·1286	0·1286	0·1286	0·1286	0·1286	0·1286
24	0·1274	0·1324	0·1323	0·1323	0·1323	0·1323	0·1323
25	0·1308	0·1360	0·1360	0·1359	0·1359	0·1359	0·1359
26	0·1342	0·1395	0·1395	0·1394	0·1394	0·1394	0·1394
27	0·1374	0·1429	0·1429	0·1428	0·1428	0·1428	0·1428
28	0·1406	0·1462	0·1462	0·1462	0·1461	0·1461	0·1461
29	0·1437	0·1494	0·1494	0·1494	0·1494	0·1494	0·1493
30	0·1468	0·1526	0·1526	0·1526	0·1526	0·1525	0·1525
31	0·1498	0·1558	0·1557	0·1557	0·1557	0·1557	0·1556
32	0·1527	0·1589	0·1588	0·1588	0·1588	0·1588	0·1587
33	0·1557	0·1620	0·1619	0·1619	0·1619	0·1619	0·1618
34	0·1587	0·1651	0·1650	0·1650	0·1650	0·1650	0·1649
35	0·1617	0·1683	0·1682	0·1682	0·1681	0·1681	0·1680
36	0·1648	0·1715	0·1714	0·1714	0·1713	0·1713	0·1712
37	0·1680	0·1748	0·1747	0·1747	0·1746	0·1746	0·1744
38	0·1713	0·1783	0·1782	0·1781	0·1780	0·1780	0·1778
39	0·1747	0·1819	0·1817	0·1816	0·1816	0·1815	0·1813
40	0·1784	0·1856	0·1855	0·1854	0·1853	0·1853	0·1850
41	0·1822	0·1896	0·1895	0·1894	0·1893	0·1892	0·1889
42	0·1863	0·1939	0·1937	0·1936	0·1935	0·1934	0·1930
43	0·1907	0·1984	0·1982	0·1981	0·1980	0·1979	0·1975
44	0·1954	0·2034	0·2031	0·2029	0·2028	0·2027	0·2022
45	0·2006	0·2087	0·2084	0·2082	0·2080	0·2079	0·2073
46	0·2061	0·2144	0·2141	0·2139	0·2137	0·2135	0·2129
47	0·2122	0·2207	0·2203	0·2201	0·2198	0·2197	0·2189
48	0·2189	0·2276	0·2271	0·2268	0·2266	0·2264	0·2254
49	0·2262	0·2351	0·2346	0·2342	0·2339	0·2337	0·2326
50	0·2342	0·2434	0·2428	0·2423	0·2420	0·2417	0·2404
51	0·2430	0·2525	0·2518	0·2512	0·2508	0·2505	0·2489
52	0·2528	0·2625	0·2617	0·2611	0·2606	0·2602	0·2583
53	0·2637	0·2736	0·2726	0·2719	0·2713	0·2708	0·2686
54	0·2757	0·2859	0·2848	0·2839	0·2832	0·2826	0·2799

Table E23b. *(Continued)*
Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *1/3 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·2891	0·2996	0·2982	0·2971	0·2963	0·2956	0·2923
56	0·3040	0·3148	0·3131	0·3118	0·3108	0·3099	0·3059
57	0·3207	0·3317	0·3296	0·3281	0·3268	0·3258	0·3209
58	0·3393	0·3506	0·3480	0·3461	0·3446	0·3433	0·3374
59	0·3602	0·3716	0·3685	0·3662	0·3643	0·3627	0·3555
60	0·3837	0·3952	0·3914	0·3885	0·3861	0·3842	0·3753
61	0·4102	0·4216	0·4169	0·4133	0·4104	0·4080	0·3969
62	0·4401	0·4512	0·4454	0·4408	0·4372	0·4342	0·4204
63	0·4739	0·4845	0·4771	0·4714	0·4668	0·4631	0·4458
64	0·5123	0·5219	0·5126	0·5053	0·4995	0·4948	0·4730

Table E23c. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *4/9 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·0284						
17	0·0306	0·0351					
18	0·0328	0·0377	0·0377				
19	0·0352	0·0404	0·0404	0·0404			
20	0·0376	0·0432	0·0432	0·0432	0·0432		
21	0·0401	0·0461	0·0461	0·0461	0·0461	0·0461	0·0461
22	0·0427	0·0491	0·0491	0·0491	0·0491	0·0491	0·0491
23	0·0454	0·0522	0·0522	0·0522	0·0522	0·0522	0·0522
24	0·0481	0·0554	0·0554	0·0554	0·0554	0·0554	0·0554
25	0·0509	0·0587	0·0587	0·0587	0·0586	0·0586	0·0586
26	0·0538	0·0620	0·0620	0·0620	0·0620	0·0620	0·0620
27	0·0568	0·0655	0·0655	0·0655	0·0655	0·0655	0·0655
28	0·0599	0·0691	0·0690	0·0690	0·0690	0·0690	0·0690
29	0·0630	0·0727	0·0727	0·0727	0·0727	0·0727	0·0727
30	0·0663	0·0765	0·0765	0·0765	0·0765	0·0765	0·0765
31	0·0697	0·0805	0·0805	0·0804	0·0804	0·0804	0·0804
32	0·0732	0·0846	0·0845	0·0845	0·0845	0·0845	0·0845
33	0·0768	0·0888	0·0888	0·0887	0·0887	0·0887	0·0887
34	0·0806	0·0932	0·0932	0·0932	0·0931	0·0931	0·0931
35	0·0846	0·0978	0·0978	0·0978	0·0977	0·0977	0·0977
36	0·0888	0·1027	0·1026	0·1026	0·1026	0·1026	0·1025
37	0·0932	0·1078	0·1077	0·1077	0·1076	0·1076	0·1075
38	0·0978	0·1131	0·1131	0·1130	0·1130	0·1130	0·1128
39	0·1026	0·1188	0·1187	0·1187	0·1186	0·1186	0·1185
40	0·1078	0·1248	0·1247	0·1247	0·1246	0·1246	0·1244
41	0·1133	0·1313	0·1311	0·1311	0·1310	0·1310	0·1307
42	0·1193	0·1381	0·1380	0·1379	0·1378	0·1378	0·1375
43	0·1256	0·1455	0·1453	0·1452	0·1451	0·1450	0·1447
44	0·1324	0·1534	0·1532	0·1531	0·1530	0·1529	0·1525
45	0·1398	0·1619	0·1617	0·1615	0·1614	0·1613	0·1609
46	0·1478	0·1712	0·1709	0·1707	0·1706	0·1705	0·1699
47	0·1565	0·1813	0·1809	0·1807	0·1805	0·1804	0·1797
48	0·1659	0·1922	0·1919	0·1916	0·1914	0·1912	0·1904
49	0·1763	0·2043	0·2038	0·2035	0·2032	0·2030	0·2020
50	0·1877	0·2175	0·2169	0·2165	0·2162	0·2159	0·2147
51	0·2003	0·2320	0·2313	0·2308	0·2304	0·2301	0·2287
52	0·2142	0·2480	0·2472	0·2466	0·2461	0·2458	0·2440
53	0·2297	0·2658	0·2648	0·2641	0·2635	0·2630	0·2608
54	0·2469	0·2855	0·2843	0·2834	0·2827	0·2821	0·2794

Table E23c. *(Continued)*
Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *4/9 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·2661	0·3076	0·3061	0·3050	0·3041	0·3034	0·3000
56	0·2876	0·3322	0·3303	0·3290	0·3279	0·3270	0·3227
57	0·3117	0·3597	0·3575	0·3558	0·3544	0·3533	0·3480
58	0·3389	0·3908	0·3879	0·3858	0·3841	0·3827	0·3760
59	0·3697	0·4257	0·4222	0·4195	0·4173	0·4155	0·4072
60	0·4047	0·4653	0·4608	0·4573	0·4546	0·4523	0·4417
61	0·4445	0·5101	0·5044	0·4999	0·4964	0·4935	0·4801
62	0·4899	0·5610	0·5537	0·5480	0·5435	0·5398	0·5226
63	0·5420	0·6190	0·6095	0·6022	0·5963	0·5915	0·5694
64	0·6019	0·6852	0·6729	0·6633	0·6557	0·6495	0·6210

Table E23d. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *13/13 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0048						
17	0·0053	0·0080					
18	0·0058	0·0088	0·0088				
19	0·0064	0·0097	0·0097	0·0097			
20	0·0070	0·0106	0·0106	0·0106	0·0106		
21	0·0077	0·0116	0·0116	0·0116	0·0116	0·0116	0·0116
22	0·0084	0·0127	0·0127	0·0127	0·0127	0·0127	0·0127
23	0·0092	0·0139	0·0139	0·0139	0·0139	0·0139	0·0139
24	0·0100	0·0151	0·0151	0·0151	0·0151	0·0151	0·0151
25	0·0108	0·0164	0·0164	0·0164	0·0164	0·0164	0·0164
26	0·0117	0·0178	0·0178	0·0178	0·0178	0·0178	0·0178
27	0·0127	0·0193	0·0193	0·0193	0·0193	0·0193	0·0193
28	0·0137	0·0209	0·0209	0·0209	0·0209	0·0209	0·0209
29	0·0148	0·0226	0·0226	0·0225	0·0225	0·0225	0·0225
30	0·0160	0·0244	0·0243	0·0243	0·0243	0·0243	0·0243
31	0·0173	0·0263	0·0263	0·0263	0·0262	0·0262	0·0262
32	0·0186	0·0283	0·0283	0·0283	0·0283	0·0283	0·0283
33	0·0200	0·0305	0·0305	0·0305	0·0305	0·0305	0·0304
34	0·0215	0·0328	0·0328	0·0328	0·0328	0·0328	0·0328
35	0·0232	0·0353	0·0353	0·0353	0·0353	0·0353	0·0353
36	0·0249	0·0380	0·0380	0·0380	0·0380	0·0380	0·0379
37	0·0268	0·0409	0·0409	0·0409	0·0409	0·0409	0·0408
38	0·0288	0·0441	0·0440	0·0440	0·0440	0·0440	0·0439
39	0·0310	0·0474	0·0474	0·0474	0·0474	0·0473	0·0473
40	0·0334	0·0511	0·0510	0·0510	0·0510	0·0510	0·0509
41	0·0360	0·0551	0·0550	0·0550	0·0550	0·0549	0·0548
42	0·0388	0·0594	0·0593	0·0593	0·0593	0·0592	0·0591
43	0·0419	0·0641	0·0640	0·0640	0·0640	0·0639	0·0638
44	0·0452	0·0693	0·0692	0·0691	0·0691	0·0691	0·0689
45	0·0489	0·0750	0·0749	0·0748	0·0747	0·0747	0·0745
46	0·0530	0·0812	0·0811	0·0810	0·0809	0·0809	0·0806
47	0·0575	0·0881	0·0880	0·0878	0·0878	0·0877	0·0874
48	0·0625	0·0958	0·0956	0·0954	0·0953	0·0952	0·0948
49	0·0680	0·1043	0·1040	0·1038	0·1037	0·1036	0·1031
50	0·0742	0·1137	0·1134	0·1132	0·1130	0·1129	0·1123
51	0·0811	0·1243	0·1239	0·1236	0·1234	0·1233	0·1225
52	0·0889	0·1361	0·1356	0·1353	0·1351	0·1348	0·1339
53	0·0976	0·1494	0·1488	0·1484	0·1481	0·1478	0·1466
54	0·1074	0·1644	0·1637	0·1632	0·1628	0·1624	0·1609

Table E23d. *(Continued)*

Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *13/13 weeks. One-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·1186	0·1814	0·1805	0·1798	0·1793	0·1789	0·1769
56	0·1313	0·2007	0·1995	0·1987	0·1980	0·1975	0·1949
57	0·1458	0·2226	0·2212	0·2201	0·2192	0·2185	0·2153
58	0·1623	0·2477	0·2458	0·2444	0·2433	0·2424	0·2382
59	0·1813	0·2764	0·2740	0·2722	0·2708	0·2696	0·2642
60	0·2033	0·3093	0·3063	0·3039	0·3021	0·3006	0·2935
61	0·2286	0·3474	0·3434	0·3403	0·3379	0·3359	0·3268
62	0·2581	0·3913	0·3861	0·3821	0·3789	0·3763	0·3643
63	0·2924	0·4423	0·4354	0·4301	0·4259	0·4225	0·4067
64	0·3325	0·5016	0·4924	0·4854	0·4798	0·4752	0·4543

Table E23e. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·0012						
17	0·0014	0·0044					
18	0·0015	0·0050	0·0050				
19	0·0017	0·0056	0·0056	0·0056			
20	0·0019	0·0063	0·0063	0·0063	0·0063		
21	0·0021	0·0070	0·0070	0·0070	0·0070	0·0070	0·0070
22	0·0024	0·0079	0·0078	0·0078	0·0078	0·0078	0·0078
23	0·0026	0·0087	0·0087	0·0087	0·0087	0·0087	0·0087
24	0·0029	0·0097	0·0097	0·0097	0·0097	0·0097	0·0097
25	0·0032	0·0108	0·0108	0·0108	0·0108	0·0108	0·0108
26	0·0036	0·0120	0·0120	0·0119	0·0119	0·0119	0·0119
27	0·0039	0·0132	0·0132	0·0132	0·0132	0·0132	0·0132
28	0·0043	0·0146	0·0146	0·0146	0·0146	0·0146	0·0146
29	0·0047	0·0161	0·0161	0·0161	0·0161	0·0161	0·0161
30	0·0052	0·0177	0·0177	0·0177	0·0177	0·0177	0·0177
31	0·0057	0·0195	0·0195	0·0195	0·0195	0·0195	0·0195
32	0·0063	0·0215	0·0215	0·0215	0·0215	0·0215	0·0215
33	0·0069	0·0236	0·0236	0·0236	0·0236	0·0236	0·0236
34	0·0075	0·0259	0·0259	0·0259	0·0259	0·0259	0·0259
35	0·0083	0·0285	0·0285	0·0285	0·0284	0·0284	0·0284
36	0·0090	0·0313	0·0312	0·0312	0·0312	0·0312	0·0312
37	0·0099	0·0343	0·0343	0·0343	0·0343	0·0343	0·0342
38	0·0109	0·0377	0·0376	0·0376	0·0376	0·0376	0·0376
39	0·0119	0·0414	0·0413	0·0413	0·0413	0·0413	0·0412
40	0·0130	0·0454	0·0454	0·0454	0·0453	0·0453	0·0453
41	0·0143	0·0499	0·0499	0·0498	0·0498	0·0498	0·0497
42	0·0157	0·0549	0·0548	0·0548	0·0548	0·0547	0·0546
43	0·0172	0·0604	0·0603	0·0603	0·0603	0·0602	0·0601
44	0·0189	0·0666	0·0665	0·0664	0·0664	0·0663	0·0661
45	0·0208	0·0734	0·0733	0·0732	0·0731	0·0731	0·0729
46	0·0230	0·0811	0·0809	0·0808	0·0807	0·0807	0·0804
47	0·0254	0·0896	0·0894	0·0893	0·0892	0·0891	0·0888
48	0·0280	0·0993	0·0990	0·0988	0·0987	0·0986	0·0982
49	0·0310	0·1101	0·1098	0·1096	0·1094	0·1093	0·1088
50	0·0344	0·1223	0·1220	0·1217	0·1215	0·1214	0·1207
51	0·0383	0·1362	0·1357	0·1354	0·1352	0·1350	0·1341
52	0·0427	0·1520	0·1514	0·1510	0·1507	0·1505	0·1493
53	0·0477	0·1700	0·1692	0·1687	0·1683	0·1680	0·1666
54	0·0534	0·1905	0·1896	0·1889	0·1884	0·1880	0·1862

Table E23e. *(Continued)*

Select table of $z_{[x]+t}^{a/b}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0599	0·2141	0·2129	0·2121	0·2114	0·2109	0·2085
56	0·0675	0·2412	0·2397	0·2386	0·2378	0·2371	0·2340
57	0·0762	0·2726	0·2706	0·2692	0·2681	0·2673	0·2632
58	0·0863	0·3088	0·3063	0·3045	0·3031	0·3020	0·2967
59	0·0980	0·3510	0·3477	0·3454	0·3435	0·3421	0·3351
60	0·1117	0·4002	0·3959	0·3928	0·3904	0·3884	0·3793
61	0·1278	0·4578	0·4521	0·4480	0·4448	0·4422	0·4300
62	0·1467	0·5254	0·5179	0·5124	0·5081	0·5046	0·4884
63	0·1691	0·6051	0·5950	0·5877	0·5819	0·5772	0·5555
64	0·1956	0·6993	0·6858	0·6759	0·6680	0·6616	0·6325

Table E23f. *Select table of* $z^{a/b}_{[x]+1}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·0000						
17	0·0000	0·0020					
18	0·0000	0·0023	0·0039				
19	0·0000	0·0026	0·0044	0·0044			
20	0·0000	0·0030	0·0051	0·0051	0·0051		
21	0·0000	0·0034	0·0058	0·0058	0·0058	0·0058	0·0058
22	0·0000	0·0038	0·0066	0·0066	0·0066	0·0066	0·0066
23	0·0000	0·0043	0·0074	0·0074	0·0074	0·0074	0·0074
24	0·0000	0·0049	0·0084	0·0084	0·0084	0·0084	0·0084
25	0·0000	0·0055	0·0095	0·0095	0·0095	0·0095	0·0095
26	0·0000	0·0061	0·0107	0·0107	0·0107	0·0107	0·0107
27	0·0000	0·0069	0·0120	0·0120	0·0120	0·0120	0·0120
28	0·0000	0·0077	0·0135	0·0135	0·0135	0·0135	0·0135
29	0·0000	0·0086	0·0151	0·0151	0·0151	0·0151	0·0151
30	0·0000	0·0096	0·0169	0·0169	0·0169	0·0169	0·0169
31	0·0000	0·0107	0·0189	0·0189	0·0189	0·0189	0·0189
32	0·0000	0·0120	0·0212	0·0212	0·0212	0·0212	0·0211
33	0·0000	0·0133	0·0236	0·0236	0·0236	0·0236	0·0236
34	0·0000	0·0149	0·0264	0·0263	0·0263	0·0263	0·0263
35	0·0000	0·0165	0·0294	0·0294	0·0294	0·0294	0·0293
36	0·0000	0·0184	0·0327	0·0327	0·0327	0·0327	0·0327
37	0·0000	0·0204	0·0365	0·0365	0·0365	0·0364	0·0364
38	0·0000	0·0227	0·0407	0·0406	0·0406	0·0406	0·0405
39	0·0000	0·0253	0·0453	0·0453	0·0452	0·0452	0·0452
40	0·0000	0·0281	0·0505	0·0504	0·0504	0·0504	0·0503
41	0·0000	0·0313	0·0563	0·0562	0·0562	0·0562	0·0561
42	0·0000	0·0348	0·0628	0·0627	0·0626	0·0626	0·0625
43	0·0000	0·0388	0·0700	0·0700	0·0699	0·0699	0·0697
44	0·0000	0·0433	0·0782	0·0781	0·0781	0·0780	0·0778
45	0·0000	0·0483	0·0875	0·0873	0·0872	0·0872	0·0869
46	0·0000	0·0540	0·0979	0·0977	0·0976	0·0975	0·0972
47	0·0000	0·0604	0·1097	0·1095	0·1094	0·1093	0·1088
48	0·0000	0·0677	0·1231	0·1228	0·1227	0·1226	0·1220
49	0·0000	0·0760	0·1383	0·1380	0·1378	0·1377	0·1370
50	0·0000	0·0854	0·1557	0·1553	0·1551	0·1549	0·1540
51	0·0000	0·0962	0·1756	0·1751	0·1748	0·1746	0·1734
52	0·0000	0·1086	0·1985	0·1978	0·1974	0·1971	0·1956
53	0·0000	0·1229	0·2247	0·2240	0·2234	0·2230	0·2210
54	0·0000	0·1393	0·2551	0·2541	0·2534	0·2528	0·2503

Table E23f. *(Continued)*

Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. One-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0000	0·1584	0·2902	0·2889	0·2880	0·2872	0·2839
56	0·0000	0·1805	0·3310	0·3293	0·3281	0·3271	0·3228
57	0·0000	0·2063	0·3785	0·3763	0·3747	0·3735	0·3677
58	0·0000	0·2364	0·4340	0·4312	0·4291	0·4274	0·4199
59	0·0000	0·2718	0·4992	0·4954	0·4926	0·4905	0·4804
60	0·0000	0·3135	0·5758	0·5708	0·5672	0·5643	0·5508
61	0·0000	0·3628	0·6664	0·6597	0·6548	0·6509	0·6328
62	0·0000	0·4213	0·7737	0·7648	0·7581	0·7528	0·7285
63	0·0000	0·4911	0·9012	0·8892	0·8802	0·8730	0·8400
64	0·0000	0·5744	1·0535	1·0372	1·0248	1·0150	0·9700

Table E24a. *Sickness rates* $z(x,104/all,x-x_0)$. *One-week deferred period basis*

Attained age x	Entry age x_0 16	17	18	19	20	21	22	23	24	25
16	0·0000									
17	0·0000	0·0000								
18	0·0011	0·0000	0·0000							
19	0·0030	0·0012	0·0000	0·0000						
20	0·0048	0·0034	0·0014	0·0000	0·0000					
21	0·0068	0·0056	0·0040	0·0016	0·0000	0·0000				
22	0·0089	0·0079	0·0064	0·0046	0·0019	0·0000	0·0000			
23	0·0113	0·0103	0·0091	0·0074	0·0053	0·0021	0·0000	0·0000		
24	0·0140	0·0131	0·0120	0·0105	0·0086	0·0060	0·0025	0·0000	0·0000	
25	0·0170	0·0162	0·0151	0·0138	0·0120	0·0098	0·0069	0·0028	0·0000	0·0000
26	0·0203	0·0196	0·0187	0·0174	0·0158	0·0138	0·0113	0·0079	0·0032	0·0000
27	0·0241	0·0234	0·0226	0·0215	0·0200	0·0182	0·0158	0·0129	0·0090	0·0036
28	0·0284	0·0278	0·0270	0·0259	0·0246	0·0230	0·0208	0·0181	0·0147	0·0103
29	0·0332	0·0326	0·0319	0·0310	0·0298	0·0282	0·0263	0·0238	0·0207	0·0167
30	0·0386	0·0381	0·0375	0·0366	0·0355	0·0341	0·0323	0·0300	0·0272	0·0236
31	0·0448	0·0443	0·0437	0·0429	0·0419	0·0406	0·0390	0·0369	0·0343	0·0310
32	0·0518	0·0513	0·0508	0·0500	0·0491	0·0479	0·0464	0·0445	0·0421	0·0390
33	0·0596	0·0592	0·0587	0·0581	0·0572	0·0561	0·0547	0·0530	0·0507	0·0479
34	0·0686	0·0682	0·0677	0·0671	0·0663	0·0653	0·0640	0·0624	0·0604	0·0578
35	0·0787	0·0783	0·0779	0·0773	0·0766	0·0757	0·0745	0·0730	0·0711	0·0687
36	0·0901	0·0898	0·0894	0·0889	0·0882	0·0874	0·0863	0·0849	0·0831	0·0809
37	0·1031	0·1028	0·1025	0·1020	0·1014	0·1006	0·0996	0·0983	0·0966	0·0946
38	0·1179	0·1176	0·1173	0·1168	0·1162	0·1155	0·1145	0·1134	0·1119	0·1100
39	0·1346	0·1343	0·1340	0·1336	0·1331	0·1324	0·1315	0·1304	0·1290	0·1273
40	0·1536	0·1534	0·1531	0·1527	0·1522	0·1515	0·1507	0·1497	0·1484	0·1468
41	0·1752	0·1750	0·1747	0·1743	0·1738	0·1732	0·1725	0·1715	0·1703	0·1688
42	0·1997	0·1995	0·1993	0·1989	0·1985	0·1979	0·1972	0·1963	0·1952	0·1938
43	0·2277	0·2275	0·2272	0·2269	0·2265	0·2260	0·2253	0·2245	0·2234	0·2221
44	0·2595	0·2593	0·2591	0·2588	0·2584	0·2580	0·2573	0·2566	0·2556	0·2543
45	0·2959	0·2957	0·2955	0·2952	0·2949	0·2944	0·2938	0·2931	0·2922	0·2910
46	0·3374	0·3373	0·3371	0·3368	0·3365	0·3360	0·3355	0·3348	0·3339	0·3329
47	0·3850	0·3848	0·3846	0·3844	0·3841	0·3837	0·3832	0·3825	0·3817	0·3807
48	0·4395	0·4393	0·4391	0·4389	0·4386	0·4382	0·4378	0·4372	0·4364	0·4354
49	0·5020	0·5019	0·5017	0·5015	0·5012	0·5009	0·5004	0·4999	0·4991	0·4982
50	0·5740	0·5739	0·5737	0·5735	0·5733	0·5729	0·5725	0·5719	0·5713	0·5704
51	0·6569	0·6568	0·6567	0·6565	0·6562	0·6559	0·6555	0·6550	0·6543	0·6535
52	0·7527	0·7526	0·7525	0·7523	0·7520	0·7517	0·7514	0·7509	0·7503	0·7495
53	0·8636	0·8635	0·8633	0·8631	0·8629	0·8626	0·8623	0·8618	0·8612	0·8605
54	0·9921	0·9920	0·9919	0·9917	0·9915	0·9912	0·9909	0·9905	0·9899	0·9892

Table E24a. *(Continued)*

Sickness rates z(x,*104/all*,x − x_0). *One-week deferred period basis*

Attained age x	Entry age x_0 16	17	18	19	20	21	22	23	24	25
55	1·1416	1·1415	1·1414	1·1412	1·1410	1·1408	1·1404	1·1400	1·1395	1·1388
56	1·3158	1·3157	1·3156	1·3154	1·3152	1·3150	1·3147	1·3143	1·3138	1·3132
57	1·5193	1·5192	1·5191	1·5190	1·5188	1·5186	1·5183	1·5179	1·5174	1·5168
58	1·7578	1·7577	1·7576	1·7574	1·7573	1·7570	1·7568	1·7564	1·7559	1·7554
59	2·0378	2·0378	2·0377	2·0375	2·0374	2·0371	2·0369	2·0365	2·0361	2·0355
60	2·3677	2·3677	2·3676	2·3674	2·3673	2·3671	2·3668	2·3665	2·3661	2·3655
61	2·7574	2·7573	2·7572	2·7571	2·7569	2·7567	2·7565	2·7562	2·7558	2·7553
62	3·2189	3·2188	3·2187	3·2186	3·2185	3·2183	3·2180	3·2177	3·2173	3·2169
63	3·7670	3·7669	3·7669	3·7667	3·7666	3·7664	3·7662	3·7659	3·7655	3·7651
64	4·4198	4·4197	4·4196	4·4195	4·4194	4·4192	4·4190	4·4187	4·4183	4·4179

Table E24b. *Sickness rates* z(x,*104/all*,x − x$_0$). *One-week deferred period basis*

Attained age x	Entry age x_0 26	27	28	29	30	31	32	33	34	35
26	0·0000									
27	0·0000	0·0000								
28	0·0041	0·0000	0·0000							
29	0·0117	0·0047	0·0000	0·0000						
30	0·0190	0·0132	0·0053	0·0000	0·0000					
31	0·0268	0·0216	0·0150	0·0060	0·0000	0·0000				
32	0·0352	0·0305	0·0245	0·0170	0·0067	0·0000	0·0000			
33	0·0444	0·0401	0·0346	0·0278	0·0192	0·0076	0·0000	0·0000		
34	0·0546	0·0505	0·0455	0·0393	0·0315	0·0217	0·0085	0·0000	0·0000	
35	0·0657	0·0620	0·0574	0·0516	0·0445	0·0356	0·0245	0·0096	0·0000	0·0000
36	0·0782	0·0747	0·0704	0·0651	0·0585	0·0504	0·0403	0·0277	0·0108	0·0000
37	0·0921	0·0889	0·0849	0·0800	0·0738	0·0663	0·0570	0·0455	0·0312	0·0122
38	0·1076	0·1046	0·1010	0·0964	0·0907	0·0837	0·0751	0·0645	0·0514	0·0352
39	0·1251	0·1223	0·1189	0·1146	0·1094	0·1029	0·0948	0·0850	0·0729	0·0581
40	0·1447	0·1422	0·1390	0·1350	0·1301	0·1241	0·1166	0·1074	0·0962	0·0825
41	0·1669	0·1645	0·1615	0·1579	0·1533	0·1477	0·1407	0·1322	0·1217	0·1089
42	0·1920	0·1898	0·1870	0·1836	0·1793	0·1740	0·1676	0·1596	0·1498	0·1379
43	0·2205	0·2184	0·2158	0·2126	0·2086	0·2037	0·1976	0·1902	0·1811	0·1699
44	0·2528	0·2508	0·2484	0·2454	0·2417	0·2371	0·2314	0·2244	0·2159	0·2054
45	0·2896	0·2877	0·2855	0·2826	0·2791	0·2748	0·2695	0·2630	0·2550	0·2452
46	0·3315	0·3298	0·3276	0·3250	0·3217	0·3177	0·3127	0·3066	0·2990	0·2898
47	0·3794	0·3778	0·3758	0·3733	0·3702	0·3664	0·3617	0·3559	0·3489	0·3402
48	0·4342	0·4327	0·4308	0·4285	0·4256	0·4220	0·4176	0·4121	0·4055	0·3973
49	0·4971	0·4956	0·4939	0·4917	0·4889	0·4855	0·4814	0·4763	0·4700	0·4623
50	0·5693	0·5680	0·5663	0·5642	0·5616	0·5584	0·5545	0·5497	0·5438	0·5365
51	0·6525	0·6512	0·6497	0·6477	0·6452	0·6422	0·6385	0·6339	0·6283	0·6215
52	0·7485	0·7473	0·7458	0·7439	0·7416	0·7388	0·7353	0·7309	0·7256	0·7191
53	0·8596	0·8584	0·8570	0·8552	0·8530	0·8503	0·8470	0·8429	0·8379	0·8317
54	0·9883	0·9873	0·9859	0·9842	0·9821	0·9796	0·9764	0·9725	0·9677	0·9619
55	1·1380	1·1370	1·1357	1·1341	1·1321	1·1296	1·1266	1·1229	1·1184	1·1128
56	1·3124	1·3114	1·3102	1·3086	1·3067	1·3044	1·3015	1·2980	1·2937	1·2884
57	1·5161	1·5151	1·5139	1·5125	1·5107	1·5085	1·5057	1·5024	1·4982	1·4932
58	1·7546	1·7537	1·7526	1·7512	1·7495	1·7474	1·7448	1·7416	1·7376	1·7328
59	2·0349	2·0340	2·0329	2·0316	2·0300	2·0279	2·0254	2·0223	2·0186	2·0140
60	2·3649	2·3641	2·3630	2·3618	2·3602	2·3582	2·3558	2·3529	2·3493	2·3449
61	2·7546	2·7539	2·7529	2·7516	2·7501	2·7483	2·7460	2·7431	2·7397	2·7354
62	3·2163	3·2155	3·2146	3·2134	3·2119	3·2101	3·2079	3·2052	3·2019	3·1978
63	3·7645	3·7638	3·7628	3·7617	3·7603	3·7586	3·7565	3·7539	3·7507	3·7468
64	4·4173	4·4166	4·4158	4·4147	4·4133	4·4117	4·4097	4·4072	4·4041	4·4003

Table E24c. *Sickness rates* $z(x, 104/all, x-x_0)$. *One-week deferred period basis*

Attained age x	Entry age x_0 36	37	38	39	40	41	42	43	44	45
36	0·0000									
37	0·0000	0·0000								
38	0·0137	0·0000	0·0000							
39	0·0396	0·0154	0·0000	0·0000						
40	0·0656	0·0447	0·0173	0·0000	0·0000					
41	0·0932	0·0741	0·0504	0·0195	0·0000	0·0000				
42	0·1233	0·1055	0·0837	0·0568	0·0219	0·0000	0·0000			
43	0·1562	0·1396	0·1193	0·0946	0·0641	0·0247	0·0000	0·0000		
44	0·1927	0·1771	0·1581	0·1351	0·1070	0·0724	0·0278	0·0000	0·0000	
45	0·2332	0·2186	0·2008	0·1792	0·1530	0·1211	0·0818	0·0314	0·0000	0·0000
46	0·2786	0·2649	0·2482	0·2279	0·2033	0·1734	0·1371	0·0925	0·0354	0·0000
47	0·3297	0·3168	0·3011	0·2820	0·2589	0·2308	0·1967	0·1555	0·1048	0·0401
48	0·3874	0·3753	0·3605	0·3425	0·3207	0·2943	0·2622	0·2234	0·1764	0·1188
49	0·4529	0·4415	0·4276	0·4107	0·3901	0·3652	0·3349	0·2984	0·2541	0·2005
50	0·5276	0·5168	0·5037	0·4877	0·4683	0·4448	0·4162	0·3817	0·3399	0·2893
51	0·6131	0·6029	0·5905	0·5753	0·5570	0·5347	0·5078	0·4751	0·4356	0·3878
52	0·7112	0·7015	0·6898	0·6754	0·6581	0·6370	0·6114	0·5805	0·5431	0·4978
53	0·8242	0·8150	0·8038	0·7903	0·7738	0·7538	0·7296	0·7002	0·6648	0·6218
54	0·9547	0·9460	0·9354	0·9225	0·9069	0·8879	0·8649	0·8370	0·8033	0·7626
55	1·1060	1·0977	1·0876	1·0754	1·0605	1·0424	1·0206	0·9941	0·9620	0·9233
56	1·2819	1·2740	1·2644	1·2527	1·2385	1·2213	1·2005	1·1753	1·1447	1·1078
57	1·4870	1·4795	1·4703	1·4592	1·4456	1·4292	1·4094	1·3853	1·3562	1·3210
58	1·7269	1·7197	1·7110	1·7003	1·6874	1·6717	1·6527	1·6298	1·6020	1·5683
59	2·0083	2·0014	1·9931	1·9829	1·9705	1·9556	1·9374	1·9154	1·8888	1·8567
60	2·3395	2·3329	2·3249	2·3151	2·3033	2·2889	2·2715	2·2505	2·2250	2·1943
61	2·7303	2·7240	2·7163	2·7069	2·6956	2·6818	2·6652	2·6450	2·6206	2·5911
62	3·1929	3·1868	3·1794	3·1705	3·1596	3·1464	3·1304	3·1110	3·0876	3·0593
63	3·7420	3·7362	3·7291	3·7205	3·7100	3·6973	3·6820	3·6634	3·6409	3·6137
64	4·3957	4·3901	4·3833	4·3750	4·3650	4·3528	4·3380	4·3201	4·2985	4·2724

Table E24d. *Sickness rates* z(x,*104/all*,x − x_0). *One-week deferred period basis*

Attained age x	Entry age x_0 46	47	48	49	50	51	52	53	54	55
46	0·0000									
47	0·0000	0·0000								
48	0·0453	0·0000	0·0000							
49	0·1348	0·0514	0·0000	0·0000						
50	0·2282	0·1532	0·0583	0·0000	0·0000					
51	0·3300	0·2600	0·1745	0·0663	0·0000	0·0000				
52	0·4431	0·3770	0·2969	0·1990	0·0755	0·0000	0·0000			
53	0·5700	0·5072	0·4314	0·3396	0·2274	0·0862	0·0000	0·0000		
54	0·7133	0·6537	0·5817	0·4947	0·3893	0·2605	0·0986	0·0000	0·0000	
55	0·8764	0·8198	0·7513	0·6685	0·5684	0·4472	0·2990	0·1130	0·0000	0·0000
56	1·0632	1·0093	0·9441	0·8653	0·7700	0·6547	0·5149	0·3441	0·1299	0·0000
57	1·2784	1·2270	1·1648	1·0897	0·9988	0·8889	0·7558	0·5944	0·3970	0·1498
58	1·5277	1·4786	1·4192	1·3475	1·2607	1·1558	1·0287	0·8748	0·6879	0·4592
59	1·8178	1·7708	1·7141	1·6455	1·5626	1·4622	1·3408	1·1936	1·0152	0·7983
60	2·1570	2·1121	2·0577	1·9921	1·9127	1·8166	1·7004	1·5596	1·3888	1·1815
61	2·5554	2·5123	2·4602	2·3973	2·3212	2·2291	2·1177	1·9828	1·8192	1·6206
62	3·0250	2·9837	2·9337	2·8733	2·8003	2·7120	2·6051	2·4757	2·3188	2·1283
63	3·5808	3·5411	3·4931	3·4350	3·3649	3·2801	3·1775	3·0532	2·9026	2·7197
64	4·2407	4·2026	4·1564	4·1007	4·0333	3·9518	3·8531	3·7337	3·5890	3·4133

Table E24e. *Sickness rates* $z(x, 104/all, x-x_0)$. *One-week deferred period basis*

Attained age x	Entry age x_0: 56	57	58	59	60	61	62	63	64
56	0·0000								
57	0·0000	0·0000							
58	0·1731	0·0000	0·0000						
59	0·5327	0·2007	0·0000	0·0000					
60	0·9292	0·6198	0·2334	0·0000	0·0000				
61	1·3791	1·0848	0·7235	0·2723	0·0000	0·0000			
62	1·8967	1·6148	1·2705	0·8473	0·3187	0·0000	0·0000		
63	2·4975	2·2269	1·8968	1·4930	0·9958	0·3745	0·0000	0·0000	
64	3·1999	2·9400	2·6230	2·2356	1·7606	1·1745	0·4417	0·0000	0·0000

Table E25. *Average probabilities,* pi(x,d), *of survival while sick from duration* 0 *to duration* d, *sickness commencing between ages* x *and* x + *1, using values conditional on entry at age 16. One-week deferred period basis*

Age	Duration *d* (weeks)					
x	1	4	13	26	52	104
16	0·293380	0·030977	0·003455	0·000861	0·000308	0·000142
17	0·298746	0·032709	0·003745	0·000953	0·000347	0·000163
18	0·304204	0·034537	0·004059	0·001055	0·000391	0·000186
19	0·309754	0·036464	0·004399	0·001168	0·000440	0·000212
20	0·315399	0·038497	0·004767	0·001292	0·000496	0·000243
21	0·321139	0·040642	0·005165	0·001429	0·000558	0·000277
22	0·326977	0·042904	0·005597	0·001581	0·000628	0·000316
23	0·332915	0·045290	0·006064	0·001749	0·000707	0·000361
24	0·338953	0·047807	0·006570	0·001935	0·000796	0·000412
25	0·345094	0·050461	0·007117	0·002140	0·000896	0·000470
26	0·351340	0·053260	0·007710	0·002366	0·001008	0·000537
27	0·357691	0·056212	0·008352	0·002617	0·001134	0·000612
28	0·364151	0·059325	0·009046	0·002893	0·001276	0·000698
29	0·370720	0·062608	0·009798	0·003199	0·001435	0·000796
30	0·377401	0·066070	0·010611	0·003537	0·001614	0·000908
31	0·384195	0·069721	0·011492	0·003909	0·001815	0·001035
32	0·391105	0·073570	0·012444	0·004321	0·002040	0·001180
33	0·398133	0·077629	0·013475	0·004776	0·002294	0·001345
34	0·405280	0·081909	0·014591	0·005278	0·002578	0·001533
35	0·412548	0·086422	0·015798	0·005833	0·002898	0·001746
36	0·419940	0·091180	0·017104	0·006445	0·003257	0·001989
37	0·427458	0·096197	0·018517	0·007121	0·003660	0·002265
38	0·435104	0·101486	0·020045	0·007867	0·004112	0·002580
39	0·442880	0·107062	0·021699	0·008691	0·004619	0·002937
40	0·450788	0·112942	0·023488	0·009600	0·005189	0·003344
41	0·458831	0·119140	0·025424	0·010604	0·005828	0·003806
42	0·467010	0·125674	0·027517	0·011712	0·006545	0·004332
43	0·475329	0·132563	0·029781	0·012934	0·007349	0·004929
44	0·483790	0·139825	0·032231	0·014283	0·008252	0·005608
45	0·492395	0·147481	0·034880	0·015771	0·009264	0·006379
46	0·501146	0·155551	0·037745	0·017413	0·010398	0·007255
47	0·510047	0·164058	0·040843	0·019225	0·011671	0·008250
48	0·519099	0·173026	0·044194	0·021224	0·013097	0·009380
49	0·528306	0·182480	0·047818	0·023429	0·014697	0·010663
50	0·537670	0·192444	0·051737	0·025861	0·016490	0·012119
51	0·547194	0·202948	0·055974	0·028543	0·018499	0·013773
52	0·556880	0·214020	0·060556	0·031502	0·020751	0·015649
53	0·566732	0·225691	0·065510	0·034764	0·023274	0·017778
54	0·576752	0·237993	0·070867	0·038362	0·026102	0·020194

Table E25. *(Continued)*

Average probabilities, pi(x,d), *of survival while sick from duration* 0 *to duration* d, *sickness commencing between ages* x *and* x + *1, using values conditional on entry at age 16. One-week deferred period basis*

Age	Duration d (weeks)					
x	1	4	13	26	52	104
55	0·586943	0·250959	0·076658	0·042330	0·029269	0·022935
56	0·597309	0·264626	0·082919	0·046704	0·032817	0·026043
57	0·607852	0·279031	0·089689	0·051526	0·036791	0·029568
58	0·618575	0·294214	0·097007	0·056843	0·041241	0·033564
59	0·629481	0·310217	0·104918	0·062704	0·046225	0·038094
60	0·640574	0·327083	0·113469	0·069165	0·051805	0·043228
61	0·651857	0·344860	0·122713	0·076285	0·058051	0·049047
62	0·663333	0·363595	0·132705	0·084134	0·065043	0·055639
63	0·675005	0·383341	0·143505	0·092782	0·072869	0·063107
64	0·686877	0·404151	0·155177	0·102313	0·081627	0·071566

Table E26. *Select table of* $q_{[x]+t}$ *with 5 years selection. Four-week deferred period basis*

Attained Age x	Select durations (t) 0	1	2	3	4	5 and over
16	0·000948					
17	0·000912	0·000924				
18	0·000879	0·000892	0·000893			
19	0·000849	0·000864	0·000865	0·000865		
20	0·000822	0·000839	0·000841	0·000841	0·000841	
21	0·000798	0·000818	0·000820	0·000820	0·000820	0·000821
22	0·000779	0·000801	0·000803	0·000803	0·000803	0·000804
23	0·000762	0·000788	0·000789	0·000790	0·000790	0·000791
24	0·000750	0·000778	0·000780	0·000781	0·000782	0·000782
25	0·000742	0·000773	0·000776	0·000777	0·000777	0·000778
26	0·000738	0·000773	0·000776	0·000777	0·000778	0·000779
27	0·000739	0·000777	0·000781	0·000782	0·000783	0·000785
28	0·000745	0·000787	0·000791	0·000793	0·000794	0·000797
29	0·000756	0·000803	0·000808	0·000810	0·000811	0·000814
30	0·000773	0·000824	0·000830	0·000832	0·000834	0·000838
31	0·000796	0·000853	0·000859	0·000862	0·000863	0·000868
32	0·000826	0·000888	0·000895	0·000899	0·000900	0·000906
33	0·000863	0·000931	0·000939	0·000943	0·000945	0·000952
34	0·000908	0·000982	0·000992	0·000996	0·000999	0·001007
35	0·000962	0·001042	0·001053	0·001058	0·001061	0·001072
36	0·001024	0·001112	0·001125	0·001130	0·001134	0·001146
37	0·001096	0·001192	0·001207	0·001213	0·001217	0·001232
38	0·001179	0·001284	0·001300	0·001308	0·001313	0·001330
39	0·001273	0·001388	0·001406	0·001415	0·001421	0·001441
40	0·001379	0·001505	0·001526	0·001536	0·001542	0·001567
41	0·001499	0·001637	0·001661	0·001672	0·001679	0·001708
42	0·001633	0·001784	0·001811	0·001824	0·001833	0·001866
43	0·001782	0·001949	0·001980	0·001994	0·002004	0·002043
44	0·001949	0·002132	0·002167	0·002184	0·002195	0·002241
45	0·002133	0·002336	0·002375	0·002395	0·002408	0·002461
46	0·002338	0·002562	0·002607	0·002629	0·002644	0·002706
47	0·002564	0·002812	0·002864	0·002889	0·002906	0·002978
48	0·002814	0·003090	0·003148	0·003178	0·003197	0·003281
49	0·003089	0·003397	0·003464	0·003498	0·003520	0·003617
50	0·003393	0·003738	0·003814	0·003853	0·003879	0·003991
51	0·003727	0·004115	0·004203	0·004247	0·004277	0·004407
52	0·004095	0·004533	0·004634	0·004685	0·004720	0·004870
53	0·004501	0·004997	0·005114	0·005173	0·005213	0·005387
54	0·004948	0·005513	0·005648	0·005717	0·005764	0·005964

Table E26. *(Continued)*

Select table of $q_{[x]+t}$ *with 5 years selection. Four-week deferred period basis*

Attained	Select durations (t)					
Age x	0	1	2	3	4	5 and over
55	0·005443	0·006088	0·006246	0·006326	0·006379	0·006611
56	0·005989	0·006730	0·006915	0·007008	0·007070	0·007338
57	0·006595	0·007451	0·007667	0·007776	0·007848	0·008157
58	0·007267	0·008262	0·008515	0·008643	0·008727	0·009084
59	0·008017	0·009179	0·009478	0·009627	0·009726	0·010137
60	0·008856	0·010220	0·010574	0·010750	0·010865	0·011338
61	0·009798	0·011410	0·011831	0·012038	0·012172	0·012715
62	0·010863	0·012779	0·013280	0·013525	0·013681	0·014300
63	0·012072	0·014364	0·014963	0·015250	0·015432	0·016134
64	0·013456	0·016212	0·016929	0·017267	0·017477	0·018266

Table E27. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *4 weeks. Four-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·004542						
17	0·005122	0·005523					
18	0·005734	0·006184	0·006184				
19	0·006374	0·006877	0·006876	0·006876			
20	0·007039	0·007596	0·007595	0·007595	0·007595		
21	0·007724	0·008337	0·008337	0·008336	0·008336	0·008336	0·008336
22	0·008426	0·009096	0·009096	0·009095	0·009095	0·009095	0·009095
23	0·009139	0·009868	0·009867	0·009867	0·009867	0·009867	0·009866
24	0·009861	0·010649	0·010648	0·010647	0·010647	0·010647	0·010647
25	0·010586	0·011434	0·011433	0·011432	0·011432	0·011431	0·011431
26	0·011312	0·012219	0·012218	0·012217	0·012217	0·012216	0·012216
27	0·012036	0·013002	0·013001	0·013000	0·012999	0·012999	0·012998
28	0·012755	0·013780	0·013778	0·013777	0·013776	0·013776	0·013774
29	0·013467	0·014551	0·014548	0·014547	0·014546	0·014545	0·014543
30	0·014171	0·015313	0·015310	0·015308	0·015307	0·015306	0·015304
31	0·014867	0·016066	0·016063	0·016061	0·016059	0·016058	0·016055
32	0·015556	0·016811	0·016807	0·016805	0·016803	0·016802	0·016797
33	0·016238	0·017549	0·017544	0·017541	0·017539	0·017537	0·017532
34	0·016916	0·018281	0·018276	0·018272	0·018270	0·018268	0·018261
35	0·017592	0·019011	0·019005	0·019001	0·018998	0·018996	0·018987
36	0·018270	0·019743	0·019736	0·019731	0·019728	0·019725	0·019715
37	0·018954	0·020482	0·020473	0·020468	0·020463	0·020460	0·020448
38	0·019649	0·021232	0·021222	0·021215	0·021210	0·021207	0·021191
39	0·020362	0·022000	0·021989	0·021981	0·021975	0·021971	0·021952
40	0·021100	0·022794	0·022781	0·022772	0·022765	0·022760	0·022738
41	0·021869	0·023623	0·023608	0·023597	0·023589	0·023583	0·023556
42	0·022680	0·024495	0·024477	0·024465	0·024456	0·024448	0·024416
43	0·023542	0·025422	0·025401	0·025387	0·025376	0·025367	0·025329
44	0·024468	0·026416	0·026392	0·026375	0·026363	0·026352	0·026306
45	0·025469	0·027490	0·027463	0·027444	0·027428	0·027416	0·027361
46	0·026562	0·028662	0·028630	0·028607	0·028589	0·028575	0·028509
47	0·027762	0·029948	0·029911	0·029884	0·029863	0·029846	0·029767
48	0·029090	0·031371	0·031327	0·031295	0·031270	0·031250	0·031155
49	0·030570	0·032954	0·032902	0·032864	0·032834	0·032810	0·032696
50	0·032226	0·034724	0·034663	0·034617	0·034582	0·034553	0·034415
51	0·034092	0·036717	0·036643	0·036588	0·036545	0·036510	0·036344
52	0·036203	0·038969	0·038880	0·038814	0·038762	0·038720	0·038517
53	0·038605	0·041528	0·041420	0·041340	0·041276	0·041224	0·040977
54	0·041349	0·044448	0·044316	0·044217	0·044139	0·044076	0·043771

Table E27. *(Continued)*

Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *4 weeks. Four-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·044500	0·047795	0·047632	0·047510	0·047413	0·047334	0·046958
56	0·048134	0·051649	0·051446	0·051293	0·051172	0·051074	0·050606
57	0·052345	0·056106	0·055850	0·055657	0·055505	0·055381	0·054794
58	0·057247	0·061281	0·060955	0·060710	0·060516	0·060358	0·059617
59	0·062983	0·067318	0·066899	0·066583	0·066332	0·066130	0·065186
60	0·069725	0·074390	0·073845	0·073432	0·073107	0·072844	0·071632
61	0·077691	0·082710	0·081992	0·081449	0·081020	0·080676	0·079105
62	0·087151	0·092541	0·091583	0·090857	0·090287	0·089830	0·087776
63	0·098444	0·104202	0·102908	0·101927	0·101159	0·100546	0·097832
64	0·111996	0·118089	0·116315	0·114973	0·113924	0·113091	0·109471

Table E28a. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *4/9 weeks. Four-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0134						
17	0·0152	0·0174					
18	0·0172	0·0197	0·0197				
19	0·0193	0·0221	0·0221	0·0221			
20	0·0215	0·0247	0·0247	0·0247	0·0247		
21	0·0238	0·0274	0·0274	0·0274	0·0274	0·0274	0·0274
22	0·0263	0·0301	0·0301	0·0301	0·0301	0·0301	0·0301
23	0·0288	0·0330	0·0330	0·0330	0·0330	0·0330	0·0330
24	0·0313	0·0360	0·0360	0·0360	0·0360	0·0360	0·0360
25	0·0340	0·0391	0·0391	0·0391	0·0391	0·0391	0·0391
26	0·0366	0·0422	0·0422	0·0422	0·0422	0·0422	0·0422
27	0·0394	0·0453	0·0453	0·0453	0·0453	0·0453	0·0453
28	0·0421	0·0485	0·0485	0·0485	0·0485	0·0485	0·0485
29	0·0449	0·0518	0·0518	0·0518	0·0518	0·0518	0·0517
30	0·0477	0·0550	0·0550	0·0550	0·0550	0·0550	0·0550
31	0·0505	0·0584	0·0583	0·0583	0·0583	0·0583	0·0583
32	0·0534	0·0617	0·0617	0·0617	0·0617	0·0617	0·0616
33	0·0563	0·0651	0·0651	0·0650	0·0650	0·0650	0·0650
34	0·0592	0·0685	0·0685	0·0685	0·0685	0·0684	0·0684
35	0·0622	0·0720	0·0720	0·0719	0·0719	0·0719	0·0719
36	0·0653	0·0755	0·0755	0·0755	0·0755	0·0755	0·0754
37	0·0684	0·0792	0·0792	0·0791	0·0791	0·0791	0·0791
38	0·0716	0·0830	0·0829	0·0829	0·0829	0·0829	0·0828
39	0·0750	0·0869	0·0868	0·0868	0·0868	0·0867	0·0867
40	0·0785	0·0910	0·0909	0·0909	0·0908	0·0908	0·0907
41	0·0822	0·0953	0·0952	0·0952	0·0951	0·0951	0·0950
42	0·0861	0·0998	0·0998	0·0997	0·0997	0·0996	0·0995
43	0·0903	0·1047	0·1046	0·1046	0·1045	0·1045	0·1043
44	0·0948	0·1100	0·1099	0·1098	0·1098	0·1097	0·1095
45	0·0998	0·1157	0·1156	0·1155	0·1154	0·1154	0·1151
46	0·1051	0·1219	0·1218	0·1217	0·1216	0·1216	0·1213
47	0·1110	0·1288	0·1286	0·1285	0·1284	0·1283	0·1280
48	0·1176	0·1364	0·1362	0·1360	0·1359	0·1358	0·1354
49	0·1248	0·1448	0·1446	0·1444	0·1443	0·1442	0·1437
50	0·1330	0·1543	0·1540	0·1538	0·1536	0·1535	0·1529
51	0·1422	0·1649	0·1645	0·1643	0·1641	0·1640	0·1632
52	0·1526	0·1769	0·1765	0·1762	0·1760	0·1758	0·1749
53	0·1645	0·1906	0·1901	0·1897	0·1895	0·1892	0·1881
54	0·1781	0·2063	0·2057	0·2052	0·2048	0·2045	0·2031

Table E28a. *(Continued)*

Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *4/9 weeks. Four-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·1937	0·2243	0·2235	0·2229	0·2225	0·2221	0·2203
56	0·2118	0·2450	0·2441	0·2433	0·2428	0·2423	0·2401
57	0·2328	0·2692	0·2679	0·2670	0·2663	0·2657	0·2628
58	0·2574	0·2973	0·2957	0·2945	0·2935	0·2928	0·2892
59	0·2863	0·3302	0·3282	0·3266	0·3254	0·3244	0·3198
60	0·3205	0·3691	0·3663	0·3643	0·3627	0·3614	0·3553
61	0·3611	0·4150	0·4114	0·4086	0·4065	0·4048	0·3969
62	0·4096	0·4696	0·4648	0·4611	0·4582	0·4559	0·4454
63	0·4678	0·5349	0·5283	0·5232	0·5193	0·5161	0·5022
64	0·5383	0·6133	0·6040	0·5971	0·5916	0·5873	0·5685

Table E28b. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *13/13 weeks. Four-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·0022						
17	0·0026	0·0039					
18	0·0030	0·0045	0·0045				
19	0·0035	0·0052	0·0052	0·0052			
20	0·0040	0·0060	0·0060	0·0060	0·0060		
21	0·0046	0·0068	0·0068	0·0068	0·0068	0·0068	0·0068
22	0·0052	0·0077	0·0077	0·0077	0·0077	0·0077	0·0077
23	0·0058	0·0087	0·0087	0·0087	0·0087	0·0087	0·0087
24	0·0065	0·0098	0·0098	0·0098	0·0098	0·0098	0·0098
25	0·0072	0·0109	0·0109	0·0109	0·0109	0·0109	0·0109
26	0·0080	0·0121	0·0121	0·0121	0·0121	0·0121	0·0121
27	0·0088	0·0133	0·0133	0·0133	0·0133	0·0133	0·0133
28	0·0097	0·0146	0·0146	0·0146	0·0146	0·0146	0·0146
29	0·0106	0·0160	0·0160	0·0160	0·0160	0·0160	0·0160
30	0·0115	0·0175	0·0175	0·0175	0·0175	0·0175	0·0175
31	0·0125	0·0190	0·0190	0·0190	0·0190	0·0190	0·0190
32	0·0136	0·0206	0·0206	0·0206	0·0206	0·0206	0·0206
33	0·0147	0·0223	0·0223	0·0223	0·0223	0·0223	0·0223
34	0·0158	0·0241	0·0241	0·0241	0·0241	0·0241	0·0241
35	0·0170	0·0260	0·0260	0·0260	0·0260	0·0260	0·0260
36	0·0183	0·0280	0·0280	0·0280	0·0280	0·0279	0·0279
37	0·0197	0·0301	0·0301	0·0301	0·0301	0·0300	0·0300
38	0·0211	0·0323	0·0323	0·0323	0·0323	0·0323	0·0322
39	0·0227	0·0347	0·0347	0·0347	0·0347	0·0346	0·0346
40	0·0243	0·0373	0·0372	0·0372	0·0372	0·0372	0·0372
41	0·0261	0·0400	0·0400	0·0400	0·0399	0·0399	0·0399
42	0·0280	0·0430	0·0429	0·0429	0·0429	0·0429	0·0428
43	0·0301	0·0462	0·0462	0·0461	0·0461	0·0461	0·0460
44	0·0324	0·0497	0·0497	0·0496	0·0496	0·0496	0·0495
45	0·0349	0·0536	0·0535	0·0535	0·0535	0·0534	0·0533
46	0·0377	0·0579	0·0578	0·0578	0·0577	0·0577	0·0576
47	0·0408	0·0626	0·0625	0·0625	0·0624	0·0624	0·0622
48	0·0443	0·0679	0·0678	0·0678	0·0677	0·0677	0·0675
49	0·0482	0·0739	0·0738	0·0737	0·0736	0·0736	0·0733
50	0·0526	0·0806	0·0805	0·0804	0·0803	0·0802	0·0799
51	0·0575	0·0883	0·0881	0·0879	0·0878	0·0877	0·0873
52	0·0633	0·0970	0·0967	0·0966	0·0964	0·0963	0·0958
53	0·0698	0·1070	0·1067	0·1065	0·1063	0·1062	0·1055
54	0·0774	0·1185	0·1182	0·1179	0·1177	0·1175	0·1167

Table E28b *Continued)*

Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *13/13 weeks. Four-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0862	0·1319	0·1315	0·1311	0·1308	0·1306	0·1296
56	0·0965	0·1476	0·1470	0·1465	0·1462	0·1459	0·1445
57	0·1086	0·1659	0·1651	0·1645	0·1641	0·1637	0·1620
58	0·1230	0·1876	0·1865	0·1858	0·1852	0·1847	0·1824
59	0·1400	0·2133	0·2119	0·2109	0·2101	0·2094	0·2064
60	0·1605	0·2439	0·2421	0·2407	0·2396	0·2388	0·2348
61	0·1851	0·2808	0·2783	0·2764	0·2749	0·2737	0·2684
62	0·2149	0·3252	0·3218	0·3192	0·3172	0·3156	0·3083
63	0·2513	0·3792	0·3744	0·3708	0·3679	0·3657	0·3558
64	0·2961	0·4450	0·4382	0·4331	0·4292	0·4260	0·4124

Table E28c. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. Four-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0006						
17	0·0007	0·0021					
18	0·0008	0·0025	0·0025				
19	0·0009	0·0030	0·0030	0·0030			
20	0·0011	0·0035	0·0035	0·0035	0·0035		
21	0·0013	0·0041	0·0041	0·0041	0·0041	0·0041	0·0041
22	0·0014	0·0047	0·0047	0·0047	0·0047	0·0047	0·0047
23	0·0017	0·0054	0·0054	0·0054	0·0054	0·0054	0·0054
24	0·0019	0·0062	0·0062	0·0062	0·0062	0·0062	0·0062
25	0·0021	0·0071	0·0071	0·0071	0·0071	0·0071	0·0071
26	0·0024	0·0080	0·0080	0·0080	0·0080	0·0080	0·0080
27	0·0027	0·0091	0·0091	0·0091	0·0091	0·0091	0·0091
28	0·0030	0·0102	0·0102	0·0102	0·0102	0·0102	0·0102
29	0·0034	0·0114	0·0114	0·0114	0·0114	0·0114	0·0114
30	0·0037	0·0127	0·0127	0·0127	0·0127	0·0127	0·0127
31	0·0041	0·0141	0·0141	0·0141	0·0141	0·0141	0·0141
32	0·0046	0·0156	0·0156	0·0156	0·0156	0·0156	0·0156
33	0·0050	0·0173	0·0173	0·0173	0·0172	0·0172	0·0172
34	0·0055	0·0190	0·0190	0·0190	0·0190	0·0190	0·0190
35	0·0061	0·0209	0·0209	0·0209	0·0209	0·0209	0·0209
36	0·0067	0·0230	0·0230	0·0230	0·0230	0·0230	0·0230
37	0·0073	0·0252	0·0252	0·0252	0·0252	0·0252	0·0252
38	0·0080	0·0277	0·0276	0·0276	0·0276	0·0276	0·0276
39	0·0087	0·0303	0·0303	0·0303	0·0302	0·0302	0·0302
40	0·0095	0·0332	0·0331	0·0331	0·0331	0·0331	0·0331
41	0·0104	0·0363	0·0363	0·0363	0·0362	0·0362	0·0362
42	0·0113	0·0398	0·0397	0·0397	0·0397	0·0397	0·0396
43	0·0124	0·0436	0·0435	0·0435	0·0435	0·0435	0·0434
44	0·0136	0·0478	0·0478	0·0477	0·0477	0·0477	0·0476
45	0·0149	0·0525	0·0525	0·0524	0·0524	0·0524	0·0523
46	0·0163	0·0578	0·0577	0·0577	0·0576	0·0576	0·0575
47	0·0180	0·0637	0·0636	0·0636	0·0635	0·0635	0·0633
48	0·0198	0·0704	0·0703	0·0702	0·0701	0·0701	0·0699
49	0·0220	0·0780	0·0778	0·0777	0·0777	0·0776	0·0773
50	0·0244	0·0867	0·0865	0·0863	0·0862	0·0862	0·0858
51	0·0271	0·0966	0·0964	0·0962	0·0961	0·0960	0·0955
52	0·0303	0·1081	0·1077	0·1076	0·1074	0·1073	0·1067
53	0·0340	0·1213	0·1210	0·1207	0·1205	0·1204	0·1196
54	0·0384	0·1368	0·1363	0·1360	0·1358	0·1356	0·1346

Table E28c. *(Continued)*
Select table of $z_{[x]+t}^{a/b}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. Four-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0434	0·1550	0·1544	0·1539	0·1536	0·1534	0·1521
56	0·0494	0·1764	0·1756	0·1750	0·1746	0·1743	0·1727
57	0·0566	0·2018	0·2008	0·2000	0·1995	0·1990	0·1969
58	0·0651	0·2322	0·2307	0·2298	0·2290	0·2284	0·2256
59	0·0754	0·2686	0·2666	0·2653	0·2643	0·2635	0·2597
60	0·0878	0·3125	0·3099	0·3081	0·3067	0·3056	0·3005
61	0·1029	0·3660	0·3624	0·3599	0·3580	0·3565	0·3495
62	0·1215	0·4313	0·4264	0·4229	0·4202	0·4181	0·4084
63	0·1444	0·5117	0·5048	0·4999	0·4961	0·4930	0·4797
64	0·1730	0·6114	0·6015	0·5944	0·5889	0·5846	0·5658

Table E28d. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. Four-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0000						
17	0·0000	0·0009					
18	0·0000	0·0011	0·0019				
19	0·0000	0·0014	0·0023	0·0023			
20	0·0000	0·0016	0·0027	0·0027	0·0027		
21	0·0000	0·0019	0·0033	0·0033	0·0033	0·0033	0·0033
22	0·0000	0·0023	0·0039	0·0039	0·0039	0·0039	0·0039
23	0·0000	0·0026	0·0045	0·0045	0·0045	0·0045	0·0045
24	0·0000	0·0031	0·0053	0·0053	0·0053	0·0053	0·0053
25	0·0000	0·0035	0·0061	0·0061	0·0061	0·0061	0·0061
26	0·0000	0·0041	0·0071	0·0071	0·0071	0·0071	0·0071
27	0·0000	0·0047	0·0081	0·0081	0·0081	0·0081	0·0081
28	0·0000	0·0053	0·0093	0·0093	0·0093	0·0093	0·0093
29	0·0000	0·0061	0·0106	0·0106	0·0106	0·0106	0·0106
30	0·0000	0·0069	0·0120	0·0120	0·0120	0·0120	0·0120
31	0·0000	0·0077	0·0136	0·0136	0·0136	0·0136	0·0136
32	0·0000	0·0087	0·0153	0·0153	0·0153	0·0153	0·0153
33	0·0000	0·0097	0·0172	0·0172	0·0172	0·0172	0·0172
34	0·0000	0·0109	0·0193	0·0193	0·0193	0·0193	0·0193
35	0·0000	0·0121	0·0216	0·0216	0·0216	0·0216	0·0215
36	0·0000	0·0135	0·0241	0·0241	0·0241	0·0241	0·0240
37	0·0000	0·0150	0·0268	0·0268	0·0268	0·0268	0·0268
38	0·0000	0·0167	0·0299	0·0299	0·0298	0·0298	0·0298
39	0·0000	0·0185	0·0332	0·0332	0·0332	0·0332	0·0331
40	0·0000	0·0205	0·0369	0·0369	0·0369	0·0369	0·0368
41	0·0000	0·0228	0·0410	0·0410	0·0410	0·0410	0·0409
42	0·0000	0·0252	0·0456	0·0455	0·0455	0·0455	0·0454
43	0·0000	0·0280	0·0507	0·0506	0·0506	0·0506	0·0505
44	0·0000	0·0311	0·0564	0·0563	0·0563	0·0562	0·0561
45	0·0000	0·0346	0·0627	0·0627	0·0626	0·0626	0·0625
46	0·0000	0·0385	0·0700	0·0699	0·0698	0·0698	0·0696
47	0·0000	0·0429	0·0781	0·0780	0·0780	0·0779	0·0777
48	0·0000	0·0480	0·0874	0·0873	0·0872	0·0872	0·0869
49	0·0000	0·0538	0·0981	0·0979	0·0978	0·0977	0·0974
50	0·0000	0·0604	0·1103	0·1101	0·1100	0·1099	0·1094
51	0·0000	0·0681	0·1244	0·1242	0·1240	0·1239	0·1233
52	0·0000	0·0770	0·1408	0·1405	0·1403	0·1401	0·1393
53	0·0000	0·0874	0·1599	0·1595	0·1592	0·1590	0·1580
54	0·0000	0·0996	0·1824	0·1818	0·1815	0·1812	0·1799

Table E28d. *(Continued)*

Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. Four-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·0000	0·1140	0·2088	0·2081	0·2077	0·2073	0·2056
56	0·0000	0·1311	0·2402	0·2393	0·2387	0·2382	0·2360
57	0·0000	0·1516	0·2776	0·2765	0·2757	0·2750	0·2720
58	0·0000	0·1761	0·3226	0·3210	0·3199	0·3191	0·3150
59	0·0000	0·2059	0·3769	0·3748	0·3733	0·3721	0·3666
60	0·0000	0·2420	0·4428	0·4399	0·4379	0·4362	0·4288
61	0·0000	0·2863	0·5234	0·5195	0·5166	0·5143	0·5041
62	0·0000	0·3409	0·6226	0·6171	0·6130	0·6098	0·5957
63	0·0000	0·4088	0·7455	0·7376	0·7318	0·7273	0·7074
64	0·0000	0·4937	0·8986	0·8873	0·8790	0·8724	0·8442

Table E29a. *Sickness rates* $z(x, 104/all, x-x_0)$. *Four-week deferred period basis*

Attained age x	Entry age x_0 16	17	18	19	20	21	22	23	24	25
16	0·0000									
17	0·0000	0·0000								
18	0·0005	0·0000	0·0000							
19	0·0014	0·0006	0·0000	0·0000						
20	0·0024	0·0017	0·0007	0·0000	0·0000					
21	0·0035	0·0029	0·0021	0·0009	0·0000	0·0000				
22	0·0048	0·0042	0·0035	0·0026	0·0011	0·0000	0·0000			
23	0·0062	0·0058	0·0051	0·0043	0·0031	0·0013	0·0000	0·0000		
24	0·0079	0·0075	0·0069	0·0061	0·0051	0·0036	0·0015	0·0000	0·0000	
25	0·0099	0·0095	0·0090	0·0083	0·0073	0·0060	0·0043	0·0018	0·0000	0·0000
26	0·0122	0·0118	0·0113	0·0107	0·0098	0·0087	0·0071	0·0051	0·0021	0·0000
27	0·0148	0·0145	0·0140	0·0134	0·0127	0·0116	0·0102	0·0084	0·0059	0·0024
28	0·0178	0·0175	0·0171	0·0166	0·0159	0·0149	0·0136	0·0120	0·0098	0·0069
29	0·0213	0·0210	0·0207	0·0202	0·0195	0·0186	0·0175	0·0159	0·0140	0·0114
30	0·0253	0·0250	0·0247	0·0242	0·0236	0·0228	0·0218	0·0204	0·0186	0·0162
31	0·0298	0·0296	0·0293	0·0289	0·0283	0·0276	0·0266	0·0253	0·0237	0·0215
32	0·0350	0·0348	0·0345	0·0341	0·0336	0·0330	0·0321	0·0309	0·0293	0·0274
33	0·0409	0·0407	0·0405	0·0401	0·0397	0·0390	0·0382	0·0371	0·0357	0·0339
34	0·0476	0·0474	0·0472	0·0469	0·0465	0·0459	0·0451	0·0441	0·0428	0·0411
35	0·0552	0·0551	0·0548	0·0545	0·0541	0·0536	0·0529	0·0520	0·0508	0·0492
36	0·0638	0·0637	0·0635	0·0632	0·0628	0·0623	0·0617	0·0608	0·0597	0·0583
37	0·0736	0·0734	0·0732	0·0730	0·0726	0·0722	0·0716	0·0708	0·0698	0·0684
38	0·0846	0·0844	0·0843	0·0840	0·0837	0·0833	0·0827	0·0820	0·0810	0·0798
39	0·0970	0·0969	0·0967	0·0965	0·0962	0·0958	0·0953	0·0946	0·0937	0·0926
40	0·1110	0·1109	0·1107	0·1105	0·1103	0·1099	0·1094	0·1088	0·1080	0·1069
41	0·1269	0·1268	0·1266	0·1264	0·1262	0·1258	0·1254	0·1248	0·1240	0·1230
42	0·1448	0·1447	0·1446	0·1444	0·1442	0·1438	0·1434	0·1429	0·1422	0·1412
43	0·1651	0·1650	0·1649	0·1647	0·1645	0·1642	0·1638	0·1633	0·1626	0·1617
44	0·1881	0·1880	0·1879	0·1877	0·1875	0·1872	0·1869	0·1864	0·1858	0·1849
45	0·2141	0·2141	0·2140	0·2138	0·2136	0·2134	0·2130	0·2126	0·2120	0·2112
46	0·2438	0·2437	0·2436	0·2435	0·2433	0·2431	0·2427	0·2423	0·2418	0·2411
47	0·2776	0·2775	0·2774	0·2773	0·2771	0·2769	0·2766	0·2762	0·2757	0·2750
48	0·3162	0·3161	0·3160	0·3159	0·3157	0·3155	0·3152	0·3148	0·3144	0·3137
49	0·3603	0·3603	0·3602	0·3601	0·3599	0·3597	0·3594	0·3591	0·3586	0·3580
50	0·4111	0·4110	0·4109	0·4108	0·4107	0·4105	0·4102	0·4099	0·4095	0·4089
51	0·4695	0·4695	0·4694	0·4693	0·4692	0·4690	0·4687	0·4684	0·4680	0·4675
52	0·5371	0·5371	0·5370	0·5369	0·5368	0·5366	0·5364	0·5361	0·5357	0·5352
53	0·6156	0·6155	0·6155	0·6154	0·6153	0·6151	0·6149	0·6146	0·6142	0·6138
54	0·7070	0·7070	0·7069	0·7068	0·7067	0·7066	0·7064	0·7061	0·7057	0·7053

Table E29a. *(Continued)*
Sickness rates z(x,*104/all*,x − x$_0$). *Four-week deferred period basis*

Attained age x	Entry age x_0 16	17	18	19	20	21	22	23	24	25
55	0·8141	0·8140	0·8140	0·8139	0·8138	0·8136	0·8134	0·8132	0·8128	0·8124
56	0·9399	0·9398	0·9398	0·9397	0·9396	0·9395	0·9393	0·9390	0·9387	0·9383
57	1·0885	1·0884	1·0884	1·0883	1·0882	1·0881	1·0879	1·0877	1·0874	1·0870
58	1·2649	1·2648	1·2648	1·2647	1·2646	1·2645	1·2643	1·2641	1·2638	1·2634
59	1·4753	1·4753	1·4753	1·4752	1·4751	1·4750	1·4748	1·4746	1·4743	1·4740
60	1·7278	1·7278	1·7277	1·7277	1·7276	1·7275	1·7273	1·7271	1·7268	1·7265
61	2·0323	2·0323	2·0322	2·0322	2·0321	2·0320	2·0318	2·0316	2·0314	2·0311
62	2·4017	2·4017	2·4016	2·4016	2·4015	2·4014	2·4012	2·4010	2·4008	2·4005
63	2·8524	2·8524	2·8523	2·8523	2·8522	2·8521	2·8519	2·8518	2·8515	2·8512
64	3·4055	3·4055	3·4054	3·4054	3·4053	3·4052	3·4051	3·4049	3·4047	3·4044

Table E29b. *Sickness rates* $z(x, 104/all, x-x_0)$. *Four-week deferred period basis*

Attained age x	Entry age x_0: 26	27	28	29	30	31	32	33	34	35
26	0·0000									
27	0·0000	0·0000								
28	0·0028	0·0000	0·0000							
29	0·0080	0·0032	0·0000	0·0000						
30	0·0132	0·0092	0·0037	0·0000	0·0000					
31	0·0187	0·0152	0·0106	0·0042	0·0000	0·0000				
32	0·0248	0·0216	0·0175	0·0121	0·0048	0·0000	0·0000			
33	0·0315	0·0286	0·0248	0·0200	0·0139	0·0055	0·0000	0·0000		
34	0·0390	0·0362	0·0327	0·0284	0·0228	0·0158	0·0062	0·0000	0·0000	
35	0·0472	0·0447	0·0415	0·0374	0·0323	0·0260	0·0179	0·0070	0·0000	0·0000
36	0·0565	0·0541	0·0511	0·0474	0·0427	0·0368	0·0295	0·0203	0·0080	0·0000
37	0·0667	0·0646	0·0618	0·0583	0·0540	0·0485	0·0418	0·0334	0·0229	0·0090
38	0·0782	0·0762	0·0737	0·0704	0·0664	0·0614	0·0551	0·0474	0·0378	0·0259
39	0·0911	0·0892	0·0869	0·0839	0·0801	0·0754	0·0696	0·0624	0·0536	0·0427
40	0·1055	0·1038	0·1016	0·0988	0·0953	0·0910	0·0856	0·0789	0·0706	0·0605
41	0·1218	0·1201	0·1181	0·1155	0·1122	0·1082	0·1032	0·0969	0·0893	0·0798
42	0·1400	0·1385	0·1366	0·1342	0·1312	0·1274	0·1227	0·1169	0·1097	0·1009
43	0·1606	0·1592	0·1574	0·1552	0·1523	0·1488	0·1444	0·1390	0·1323	0·1241
44	0·1839	0·1826	0·1809	0·1788	0·1761	0·1728	0·1687	0·1636	0·1574	0·1497
45	0·2103	0·2090	0·2074	0·2055	0·2030	0·1999	0·1960	0·1913	0·1854	0·1782
46	0·2401	0·2390	0·2375	0·2356	0·2333	0·2304	0·2268	0·2223	0·2168	0·2101
47	0·2741	0·2730	0·2717	0·2699	0·2677	0·2650	0·2616	0·2574	0·2522	0·2459
48	0·3129	0·3119	0·3106	0·3089	0·3069	0·3043	0·3011	0·2971	0·2923	0·2863
49	0·3573	0·3563	0·3551	0·3535	0·3516	0·3491	0·3461	0·3424	0·3378	0·3322
50	0·4082	0·4073	0·4061	0·4046	0·4028	0·4005	0·3977	0·3941	0·3898	0·3845
51	0·4668	0·4659	0·4648	0·4634	0·4617	0·4595	0·4568	0·4535	0·4494	0·4444
52	0·5345	0·5337	0·5327	0·5314	0·5297	0·5277	0·5251	0·5220	0·5181	0·5133
53	0·6131	0·6124	0·6114	0·6101	0·6086	0·6066	0·6042	0·6012	0·5975	0·5930
54	0·7047	0·7040	0·7030	0·7019	0·7004	0·6985	0·6962	0·6934	0·6899	0·6856
55	0·8118	0·8111	0·8103	0·8091	0·8077	0·8060	0·8038	0·8011	0·7977	0·7937
56	0·9378	0·9371	0·9362	0·9352	0·9338	0·9322	0·9301	0·9275	0·9243	0·9204
57	1·0865	1·0858	1·0850	1·0840	1·0827	1·0811	1·0791	1·0767	1·0736	1·0699
58	1·2630	1·2623	1·2616	1·2606	1·2594	1·2578	1·2559	1·2536	1·2507	1·2472
59	1·4735	1·4729	1·4722	1·4712	1·4701	1·4686	1·4668	1·4646	1·4618	1·4584
60	1·7260	1·7255	1·7248	1·7239	1·7228	1·7214	1·7196	1·7175	1·7148	1·7116
61	2·0306	2·0301	2·0294	2·0285	2·0275	2·0261	2·0245	2·0224	2·0199	2·0167
62	2·4001	2·3996	2·3989	2·3981	2·3970	2·3958	2·3942	2·3922	2·3897	2·3867
63	2·8508	2·8503	2·8497	2·8489	2·8479	2·8467	2·8451	2·8432	2·8409	2·8380
64	3·4040	3·4035	3·4029	3·4022	3·4012	3·4000	3·3985	3·3967	3·3944	3·3917

Table E29c. *Sickness rates* z(x,*104/all*,x − x_0). *Four-week deferred period basis*

Attained age *x*	Entry age x_0 36	37	38	39	40	41	42	43	44	45
36	0·0000									
37	0·0000	0·0000								
38	0·0101	0·0000	0·0000							
39	0·0291	0·0113	0·0000	0·0000						
40	0·0482	0·0328	0·0127	0·0000	0·0000					
41	0·0683	0·0543	0·0369	0·0142	0·0000	0·0000				
42	0·0902	0·0771	0·0612	0·0415	0·0160	0·0000	0·0000			
43	0·1141	0·1018	0·0869	0·0689	0·0466	0·0179	0·0000	0·0000		
44	0·1403	0·1289	0·1149	0·0981	0·0776	0·0524	0·0201	0·0000	0·0000	
45	0·1694	0·1587	0·1456	0·1298	0·1106	0·0874	0·0590	0·0226	0·0000	0·0000
46	0·2018	0·1917	0·1795	0·1646	0·1466	0·1249	0·0986	0·0664	0·0254	0·0000
47	0·2381	0·2286	0·2171	0·2031	0·1862	0·1658	0·1411	0·1113	0·0749	0·0286
48	0·2790	0·2701	0·2592	0·2461	0·2302	0·2109	0·1877	0·1597	0·1259	0·0846
49	0·3253	0·3169	0·3067	0·2943	0·2793	0·2611	0·2392	0·2128	0·1810	0·1426
50	0·3780	0·3700	0·3604	0·3487	0·3345	0·3174	0·2967	0·2718	0·2417	0·2055
51	0·4382	0·4307	0·4216	0·4105	0·3971	0·3809	0·3614	0·3378	0·3094	0·2752
52	0·5075	0·5004	0·4917	0·4813	0·4686	0·4532	0·4347	0·4124	0·3855	0·3531
53	0·5875	0·5807	0·5726	0·5626	0·5506	0·5360	0·5185	0·4973	0·4718	0·4410
54	0·6803	0·6739	0·6662	0·6567	0·6453	0·6315	0·6148	0·5947	0·5704	0·5412
55	0·7887	0·7826	0·7752	0·7662	0·7553	0·7422	0·7263	0·7072	0·6841	0·6563
56	0·9157	0·9099	0·9028	0·8943	0·8839	0·8714	0·8563	0·8380	0·8161	0·7896
57	1·0654	1·0599	1·0531	1·0450	1·0351	1·0231	1·0087	0·9913	0·9704	0·9451
58	1·2428	1·2375	1·2311	1·2233	1·2139	1·2024	1·1887	1·1721	1·1520	1·1279
59	1·4542	1·4492	1·4430	1·4356	1·4265	1·4156	1·4024	1·3865	1·3674	1·3443
60	1·7076	1·7027	1·6968	1·6897	1·6810	1·6706	1·6579	1·6427	1·6244	1·6023
61	2·0129	2·0083	2·0026	1·9958	1·9874	1·9774	1·9653	1·9507	1·9331	1·9119
62	2·3831	2·3786	2·3732	2·3666	2·3586	2·3490	2·3373	2·3233	2·3064	2·2860
63	2·8345	2·8302	2·8250	2·8186	2·8109	2·8017	2·7905	2·7770	2·7608	2·7412
64	3·3883	3·3841	3·3791	3·3730	3·3656	3·3567	3·3459	3·3330	3·3173	3·2985

Table E29d. *Sickness rates* z(x,*104/all*,x − x_0). *Four-week deferred period basis*

Attained age x	Entry age x_0 46	47	48	49	50	51	52	53	54	55
46	0·0000									
47	0·0000	0·0000								
48	0·0322	0·0000	0·0000							
49	0·0957	0·0364	0·0000	0·0000						
50	0·1619	0·1086	0·0412	0·0000	0·0000					
51	0·2339	0·1842	0·1235	0·0469	0·0000	0·0000				
52	0·3140	0·2670	0·2102	0·1408	0·0534	0·0000	0·0000			
53	0·4040	0·3594	0·3056	0·2405	0·1611	0·0611	0·0000	0·0000		
54	0·5061	0·4637	0·4126	0·3509	0·2763	0·1850	0·0701	0·0000	0·0000	
55	0·6229	0·5826	0·5340	0·4753	0·4044	0·3185	0·2132	0·0808	0·0000	0·0000
56	0·7578	0·7194	0·6731	0·6172	0·5497	0·4679	0·3686	0·2468	0·0935	0·0000
57	0·9148	0·8781	0·8340	0·7807	0·7162	0·6383	0·5437	0·4286	0·2870	0·1087
58	1·0989	1·0639	1·0217	0·9708	0·9093	0·8348	0·7445	0·6346	0·5006	0·3354
59	1·3165	1·2831	1·2427	1·1940	1·1352	1·0639	0·9775	0·8725	0·7444	0·5877
60	1·5757	1·5436	1·5049	1·4583	1·4019	1·3337	1·2509	1·1503	1·0277	0·8777
61	1·8864	1·8556	1·8185	1·7738	1·7197	1·6543	1·5749	1·4785	1·3609	1·2171
62	2·2615	2·2320	2·1964	2·1534	2·1015	2·0387	1·9625	1·8698	1·7569	1·6189
63	2·7176	2·6892	2·6550	2·6137	2·5638	2·5033	2·4301	2·3411	2·2326	2·1000
64	3·2758	3·2485	3·2155	3·1758	3·1278	3·0696	2·9992	2·9135	2·8092	2·6816

Table E29e. *Sickness rates* $z(x, 104/all, x-x_0)$. *Four-week deferred period basis*

Attained age x	Entry age x_0: 56	57	58	59	60	61	62	63	64
56	0·0000								
57	0·0000	0·0000							
58	0·1271	0·0000	0·0000						
59	0·3940	0·1493	0·0000	0·0000					
60	0·6936	0·4654	0·1765	0·0000	0·0000				
61	1·0407	0·8233	0·5529	0·2099	0·0000	0·0000			
62	1·4496	1·2411	0·9831	0·6610	0·2512	0·0000	0·0000		
63	1·9373	1·7370	1·4893	1·1814	0·7953	0·3027	0·0000	0·0000	
64	2·5251	2·3325	2·0944	1·7986	1·4291	0·9635	0·3673	0·0000	0·0000

Table E30. *Select table of* $q_{[x]+t}$ *with 5 years selection. Thirteen-week deferred period basis*

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
16	0·000990					
17	0·000949	0·000964				
18	0·000910	0·000926	0·000927			
19	0·000874	0·000892	0·000893	0·000893		
20	0·000840	0·000860	0·000861	0·000862	0·000862	
21	0·000810	0·000831	0·000833	0·000833	0·000834	0·000834
22	0·000783	0·000806	0·000808	0·000808	0·000809	0·000809
23	0·000759	0·000784	0·000786	0·000787	0·000787	0·000788
24	0·000739	0·000767	0·000769	0·000770	0·000770	0·000771
25	0·000723	0·000753	0·000756	0·000757	0·000757	0·000758
26	0·000712	0·000744	0·000747	0·000748	0·000749	0·000750
27	0·000705	0·000740	0·000743	0·000745	0·000745	0·000747
28	0·000703	0·000741	0·000745	0·000746	0·000747	0·000750
29	0·000707	0·000748	0·000752	0·000754	0·000755	0·000758
30	0·000717	0·000761	0·000765	0·000768	0·000769	0·000773
31	0·000733	0·000780	0·000786	0·000788	0·000789	0·000794
32	0·000756	0·000807	0·000813	0·000815	0·000817	0·000823
33	0·000786	0·000841	0·000848	0·000851	0·000853	0·000859
34	0·000824	0·000883	0·000891	0·000894	0·000897	0·000904
35	0·000870	0·000934	0·000943	0·000947	0·000950	0·000959
36	0·000925	0·000995	0·001005	0·001009	0·001012	0·001023
37	0·000991	0·001066	0·001077	0·001082	0·001085	0·001098
38	0·001066	0·001148	0·001160	0·001166	0·001170	0·001185
39	0·001153	0·001241	0·001256	0·001262	0·001267	0·001284
40	0·001252	0·001348	0·001364	0·001372	0·001377	0·001397
41	0·001364	0·001468	0·001487	0·001495	0·001501	0·001525
42	0·001489	0·001604	0·001624	0·001634	0·001641	0·001668
43	0·001630	0·001755	0·001778	0·001790	0·001797	0·001829
44	0·001787	0·001924	0·001950	0·001963	0·001972	0·002008
45	0·001961	0·002112	0·002142	0·002156	0·002166	0·002209
46	0·002154	0·002320	0·002354	0·002371	0·002382	0·002431
47	0·002367	0·002551	0·002589	0·002608	0·002621	0·002678
48	0·002602	0·002806	0·002849	0·002871	0·002886	0·002951
49	0·002861	0·003088	0·003137	0·003162	0·003179	0·003254
50	0·003145	0·003398	0·003455	0·003484	0·003503	0·003590
51	0·003457	0·003741	0·003806	0·003839	0·003861	0·003961
52	0·003799	0·004118	0·004193	0·004231	0·004257	0·004372
53	0·004175	0·004535	0·004621	0·004665	0·004695	0·004828
54	0·004587	0·004995	0·005094	0·005145	0·005180	0·005333

Table E30. *(Continued)*

Select table of $q_{[x]+t}$ *with 5 years selection. Thirteen-week deferred period basis*

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
55	0·005038	0·005503	0·005618	0·005677	0·005717	0·005894
56	0·005534	0·006065	0·006199	0·006268	0·006314	0·006518
57	0·006078	0·006688	0·006844	0·006924	0·006978	0·007215
58	0·006676	0·007380	0·007563	0·007657	0·007719	0·007993
59	0·007335	0·008151	0·008366	0·008476	0·008549	0·008865
60	0·008062	0·009013	0·009266	0·009395	0·009481	0·009846
61	0·008867	0·009980	0·010280	0·010432	0·010532	0·010953
62	0·009760	0·011070	0·011426	0·011605	0·011722	0·012208
63	0·010754	0·012305	0·012728	0·012940	0·013077	0·013636
64	0·011866	0·013711	0·014217	0·014467	0·014627	0·015267

Table E31. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *13 weeks. Thirteen-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·000529						
17	0·000584	0·000769					
18	0·000642	0·000846	0·000846				
19	0·000703	0·000927	0·000927	0·000927			
20	0·000767	0·001012	0·001012	0·001011	0·001011		
21	0·000833	0·001100	0·001100	0·001099	0·001099	0·001099	0·001099
22	0·000902	0·001191	0·001191	0·001191	0·001191	0·001191	0·001191
23	0·000974	0·001286	0·001286	0·001286	0·001286	0·001286	0·001286
24	0·001047	0·001384	0·001384	0·001384	0·001383	0·001383	0·001383
25	0·001124	0·001485	0·001485	0·001485	0·001485	0·001485	0·001484
26	0·001202	0·001589	0·001589	0·001589	0·001589	0·001589	0·001589
27	0·001283	0·001697	0·001697	0·001696	0·001696	0·001696	0·001696
28	0·001367	0·001808	0·001807	0·001807	0·001807	0·001807	0·001807
29	0·001453	0·001922	0·001922	0·001922	0·001922	0·001921	0·001921
30	0·001542	0·002040	0·002040	0·002040	0·002040	0·002039	0·002039
31	0·001634	0·002163	0·002162	0·002162	0·002162	0·002162	0·002161
32	0·001730	0·002289	0·002289	0·002289	0·002288	0·002288	0·002288
33	0·001830	0·002421	0·002420	0·002420	0·002420	0·002420	0·002419
34	0·001933	0·002558	0·002558	0·002557	0·002557	0·002557	0·002556
35	0·002042	0·002702	0·002701	0·002701	0·002700	0·002700	0·002699
36	0·002156	0·002853	0·002852	0·002851	0·002851	0·002851	0·002849
37	0·002276	0·003012	0·003011	0·003010	0·003009	0·003009	0·003007
38	0·002403	0·003180	0·003179	0·003178	0·003177	0·003177	0·003175
39	0·002539	0·003359	0·003358	0·003357	0·003356	0·003355	0·003353
40	0·002684	0·003550	0·003549	0·003547	0·003547	0·003546	0·003543
41	0·002840	0·003756	0·003754	0·003752	0·003751	0·003751	0·003747
42	0·003008	0·003977	0·003975	0·003973	0·003972	0·003971	0·003967
43	0·003190	0·004217	0·004215	0·004213	0·004211	0·004210	0·004205
44	0·003389	0·004479	0·004476	0·004473	0·004472	0·004470	0·004464
45	0·003607	0·004765	0·004761	0·004758	0·004756	0·004755	0·004747
46	0·003846	0·005079	0·005075	0·005072	0·005069	0·005068	0·005058
47	0·004111	0·005427	0·005422	0·005418	0·005415	0·005413	0·005401
48	0·004405	0·005812	0·005806	0·005802	0·005798	0·005795	0·005782
49	0·004733	0·006242	0·006234	0·006229	0·006225	0·006221	0·006205
50	0·005101	0·006723	0·006714	0·006707	0·006702	0·006698	0·006677
51	0·005514	0·007264	0·007253	0·007245	0·007238	0·007233	0·007208
52	0·005982	0·007875	0·007861	0·007851	0·007843	0·007837	0·007806
53	0·006513	0·008568	0·008551	0·008539	0·008529	0·008521	0·008482
54	0·007119	0·009357	0·009336	0·009321	0·009309	0·009299	0·009251

Table E31. *(Continued)*
Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *13 weeks. Thirteen-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·007813	0·010260	0·010234	0·010215	0·010200	0·010187	0·010127
56	0·008612	0·011299	0·011266	0·011241	0·011222	0·011206	0·011130
57	0·009536	0·012497	0·012456	0·012425	0·012400	0·012380	0·012282
58	0·010610	0·013888	0·013834	0·013794	0·013762	0·013736	0·013612
59	0·011863	0·015507	0·015438	0·015386	0·015345	0·015311	0·015151
60	0·013334	0·017403	0·017313	0·017244	0·017190	0·017146	0·016938
61	0·015068	0·019633	0·019513	0·019423	0·019351	0·019292	0·019019
62	0·017123	0·022267	0·022107	0·021985	0·021889	0·021811	0·021451
63	0·019571	0·025393	0·025177	0·025012	0·024882	0·024778	0·024297
64	0·022503	0·029120	0·028825	0·028600	0·028422	0·028279	0·027634

Table E32a. *Select table of* $z_{[x]+t}^{a/b}$ *with 5 years selection: methods A and B: sickness period* (a/b) *13/13 weeks. Thirteen-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0029						
17	0·0033	0·0049					
18	0·0036	0·0054	0·0054				
19	0·0040	0·0060	0·0060	0·0060			
20	0·0044	0·0066	0·0066	0·0066	0·0066		
21	0·0048	0·0073	0·0073	0·0073	0·0073	0·0073	0·0073
22	0·0053	0·0080	0·0080	0·0080	0·0080	0·0080	0·0080
23	0·0057	0·0087	0·0087	0·0087	0·0087	0·0087	0·0087
24	0·0062	0·0094	0·0094	0·0094	0·0094	0·0094	0·0094
25	0·0067	0·0102	0·0102	0·0102	0·0102	0·0102	0·0102
26	0·0073	0·0110	0·0110	0·0110	0·0110	0·0110	0·0110
27	0·0078	0·0119	0·0119	0·0119	0·0119	0·0119	0·0119
28	0·0084	0·0128	0·0128	0·0128	0·0128	0·0128	0·0128
29	0·0090	0·0138	0·0138	0·0138	0·0138	0·0138	0·0137
30	0·0097	0·0147	0·0147	0·0147	0·0147	0·0147	0·0147
31	0·0103	0·0158	0·0158	0·0158	0·0158	0·0158	0·0158
32	0·0111	0·0169	0·0169	0·0169	0·0169	0·0169	0·0169
33	0·0118	0·0180	0·0180	0·0180	0·0180	0·0180	0·0180
34	0·0126	0·0192	0·0192	0·0192	0·0192	0·0192	0·0192
35	0·0134	0·0205	0·0205	0·0205	0·0205	0·0205	0·0205
36	0·0143	0·0218	0·0218	0·0218	0·0218	0·0218	0·0218
37	0·0152	0·0233	0·0233	0·0233	0·0233	0·0233	0·0233
38	0·0162	0·0248	0·0248	0·0248	0·0248	0·0248	0·0248
39	0·0173	0·0265	0·0265	0·0265	0·0265	0·0264	0·0264
40	0·0184	0·0283	0·0282	0·0282	0·0282	0·0282	0·0282
41	0·0197	0·0302	0·0302	0·0302	0·0301	0·0301	0·0301
42	0·0210	0·0323	0·0323	0·0322	0·0322	0·0322	0·0322
43	0·0225	0·0346	0·0345	0·0345	0·0345	0·0345	0·0344
44	0·0241	0·0370	0·0370	0·0370	0·0370	0·0370	0·0369
45	0·0259	0·0398	0·0398	0·0397	0·0397	0·0397	0·0396
46	0·0279	0·0428	0·0428	0·0428	0·0428	0·0427	0·0427
47	0·0301	0·0462	0·0462	0·0461	0·0461	0·0461	0·0460
48	0·0325	0·0500	0·0499	0·0499	0·0499	0·0498	0·0497
49	0·0353	0·0542	0·0541	0·0541	0·0540	0·0540	0·0539
50	0·0384	0·0589	0·0589	0·0588	0·0587	0·0587	0·0585
51	0·0419	0·0643	0·0642	0·0641	0·0641	0·0640	0·0638
52	0·0458	0·0704	0·0703	0·0702	0·0701	0·0700	0·0698
53	0·0503	0·0773	0·0772	0·0770	0·0770	0·0769	0·0765
54	0·0555	0·0853	0·0851	0·0849	0·0848	0·0847	0·0843

Table E32a. *(Continued)*
Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *13/13 weeks. Thirteen-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0615	0·0944	0·0941	0·0940	0·0938	0·0937	0·0932
56	0·0684	0·1050	0·1046	0·1044	0·1042	0·1041	0·1034
57	0·0765	0·1172	0·1168	0·1165	0·1163	0·1161	0·1152
58	0·0859	0·1315	0·1310	0·1306	0·1303	0·1301	0·1289
59	0·0970	0·1483	0·1476	0·1471	0·1467	0·1464	0·1449
60	0·1100	0·1681	0·1672	0·1665	0·1660	0·1656	0·1635
61	0·1255	0·1915	0·1903	0·1894	0·1887	0·1881	0·1855
62	0·1439	0·2193	0·2177	0·2165	0·2155	0·2148	0·2112
63	0·1661	0·2526	0·2504	0·2487	0·2474	0·2464	0·2416
64	0·1928	0·2925	0·2895	0·2873	0·2855	0·2840	0·2775

Table E32b. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. Thirteen-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0007						
17	0·0008	0·0027					
18	0·0009	0·0031	0·0031				
19	0·0011	0·0035	0·0035	0·0035			
20	0·0012	0·0039	0·0039	0·0039	0·0039		
21	0·0013	0·0044	0·0044	0·0044	0·0044	0·0044	0·0044
22	0·0015	0·0049	0·0049	0·0049	0·0049	0·0049	0·0049
23	0·0016	0·0055	0·0055	0·0055	0·0055	0·0055	0·0055
24	0·0018	0·0061	0·0061	0·0061	0·0061	0·0061	0·0061
25	0·0020	0·0067	0·0067	0·0067	0·0067	0·0067	0·0067
26	0·0022	0·0074	0·0074	0·0074	0·0074	0·0074	0·0074
27	0·0024	0·0082	0·0082	0·0082	0·0082	0·0082	0·0082
28	0·0026	0·0090	0·0090	0·0090	0·0090	0·0090	0·0090
29	0·0029	0·0098	0·0098	0·0098	0·0098	0·0098	0·0098
30	0·0032	0·0108	0·0108	0·0108	0·0108	0·0108	0·0108
31	0·0034	0·0118	0·0118	0·0118	0·0118	0·0118	0·0118
32	0·0037	0·0128	0·0128	0·0128	0·0128	0·0128	0·0128
33	0·0041	0·0140	0·0140	0·0140	0·0140	0·0140	0·0140
34	0·0044	0·0152	0·0152	0·0152	0·0152	0·0152	0·0152
35	0·0048	0·0166	0·0166	0·0166	0·0166	0·0166	0·0165
36	0·0052	0·0180	0·0180	0·0180	0·0180	0·0180	0·0180
37	0·0056	0·0196	0·0196	0·0196	0·0196	0·0196	0·0196
38	0·0061	0·0213	0·0213	0·0213	0·0213	0·0213	0·0213
39	0·0066	0·0232	0·0232	0·0231	0·0231	0·0231	0·0231
40	0·0072	0·0252	0·0252	0·0252	0·0252	0·0252	0·0251
41	0·0078	0·0274	0·0274	0·0274	0·0274	0·0274	0·0274
42	0·0085	0·0299	0·0299	0·0299	0·0299	0·0299	0·0298
43	0·0093	0·0326	0·0326	0·0326	0·0326	0·0326	0·0325
44	0·0101	0·0357	0·0356	0·0356	0·0356	0·0356	0·0355
45	0·0110	0·0390	0·0390	0·0390	0·0390	0·0390	0·0389
46	0·0121	0·0428	0·0428	0·0427	0·0427	0·0427	0·0426
47	0·0133	0·0471	0·0470	0·0470	0·0469	0·0469	0·0468
48	0·0146	0·0518	0·0518	0·0517	0·0517	0·0517	0·0515
49	0·0161	0·0572	0·0572	0·0571	0·0571	0·0570	0·0569
50	0·0178	0·0634	0·0633	0·0632	0·0632	0·0631	0·0629
51	0·0197	0·0704	0·0703	0·0702	0·0702	0·0701	0·0699
52	0·0220	0·0785	0·0783	0·0782	0·0781	0·0781	0·0778
53	0·0245	0·0878	0·0876	0·0874	0·0873	0·0873	0·0869
54	0·0275	0·0985	0·0983	0·0981	0·0980	0·0979	0·0974

Table E32b. *(Continued)*
Select table of $z_{[x]+t}^{a/b}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. Thirteen-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0310	0·1110	0·1107	0·1105	0·1103	0·1102	0·1095
56	0·0351	0·1257	0·1252	0·1250	0·1247	0·1246	0·1237
57	0·0398	0·1428	0·1423	0·1419	0·1416	0·1414	0·1403
58	0·0455	0·1631	0·1624	0·1619	0·1615	0·1612	0·1597
59	0·0522	0·1871	0·1862	0·1855	0·1850	0·1846	0·1827
60	0·0602	0·2158	0·2145	0·2136	0·2130	0·2124	0·2098
61	0·0698	0·2501	0·2484	0·2472	0·2463	0·2455	0·2420
62	0·0814	0·2914	0·2892	0·2875	0·2863	0·2852	0·2805
63	0·0954	0·3415	0·3384	0·3362	0·3344	0·3330	0·3265
64	0·1126	0·4026	0·3982	0·3950	0·3926	0·3906	0·3817

Table E32c. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. Thirteen-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·0000						
17	0·0000	0·0012					
18	0·0000	0·0014	0·0024				
19	0·0000	0·0016	0·0028	0·0028			
20	0·0000	0·0018	0·0032	0·0032	0·0032		
21	0·0000	0·0021	0·0036	0·0036	0·0036	0·0036	0·0036
22	0·0000	0·0024	0·0041	0·0041	0·0041	0·0041	0·0041
23	0·0000	0·0027	0·0047	0·0047	0·0047	0·0047	0·0047
24	0·0000	0·0030	0·0053	0·0053	0·0053	0·0053	0·0053
25	0·0000	0·0034	0·0059	0·0059	0·0059	0·0059	0·0059
26	0·0000	0·0038	0·0067	0·0067	0·0067	0·0067	0·0067
27	0·0000	0·0043	0·0075	0·0075	0·0075	0·0075	0·0075
28	0·0000	0·0048	0·0083	0·0083	0·0083	0·0083	0·0083
29	0·0000	0·0053	0·0093	0·0093	0·0093	0·0093	0·0093
30	0·0000	0·0059	0·0103	0·0103	0·0103	0·0103	0·0103
31	0·0000	0·0065	0·0115	0·0115	0·0115	0·0115	0·0115
32	0·0000	0·0072	0·0127	0·0127	0·0127	0·0127	0·0127
33	0·0000	0·0079	0·0141	0·0141	0·0141	0·0141	0·0141
34	0·0000	0·0087	0·0156	0·0156	0·0156	0·0156	0·0156
35	0·0000	0·0096	0·0172	0·0172	0·0172	0·0172	0·0172
36	0·0000	0·0106	0·0190	0·0190	0·0190	0·0190	0·0190
37	0·0000	0·0117	0·0210	0·0210	0·0210	0·0210	0·0209
38	0·0000	0·0129	0·0231	0·0231	0·0231	0·0231	0·0231
39	0·0000	0·0142	0·0255	0·0255	0·0255	0·0255	0·0255
40	0·0000	0·0156	0·0282	0·0282	0·0282	0·0282	0·0281
41	0·0000	0·0172	0·0311	0·0311	0·0311	0·0311	0·0311
42	0·0000	0·0190	0·0344	0·0344	0·0344	0·0344	0·0343
43	0·0000	0·0210	0·0381	0·0380	0·0380	0·0380	0·0380
44	0·0000	0·0232	0·0422	0·0421	0·0421	0·0421	0·0420
45	0·0000	0·0257	0·0468	0·0467	0·0467	0·0467	0·0466
46	0·0000	0·0286	0·0520	0·0519	0·0519	0·0519	0·0518
47	0·0000	0·0317	0·0578	0·0578	0·0577	0·0577	0·0576
48	0·0000	0·0354	0·0645	0·0644	0·0644	0·0644	0·0642
49	0·0000	0·0395	0·0721	0·0721	0·0720	0·0720	0·0718
50	0·0000	0·0442	0·0809	0·0808	0·0807	0·0807	0·0804
51	0·0000	0·0497	0·0909	0·0908	0·0907	0·0906	0·0903
52	0·0000	0·0560	0·1026	0·1024	0·1023	0·1022	0·1018
53	0·0000	0·0633	0·1160	0·1158	0·1157	0·1156	0·1150
54	0·0000	0·0718	0·1317	0·1315	0·1313	0·1311	0·1304

Table E32c. *(Continued)*

Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. Thirteen-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·0000	0·0818	0·1501	0·1498	0·1496	0·1494	0·1484
56	0·0000	0·0935	0·1718	0·1713	0·1710	0·1708	0·1696
57	0·0000	0·1074	0·1974	0·1968	0·1964	0·1961	0·1945
58	0·0000	0·1239	0·2278	0·2270	0·2265	0·2260	0·2239
59	0·0000	0·1437	0·2641	0·2631	0·2623	0·2617	0·2589
60	0·0000	0·1674	0·3077	0·3063	0·3053	0·3045	0·3007
61	0·0000	0·1960	0·3603	0·3584	0·3570	0·3559	0·3508
62	0·0000	0·2308	0·4241	0·4215	0·4196	0·4181	0·4111
63	0·0000	0·2733	0·5020	0·4984	0·4957	0·4935	0·4839
64	0·0000	0·3255	0·5974	0·5924	0·5886	0·5856	0·5721

Table E33a. *Sickness rates* $z(x, 104/all, x-x_0)$. *Thirteen-week deferred period basis*

Attained age x	Entry age x_0: 16	17	18	19	20	21	22	23	24	25
16	0·0000									
17	0·0000	0·0000								
18	0·0006	0·0000	0·0000							
19	0·0018	0·0008	0·0000	0·0000						
20	0·0030	0·0021	0·0009	0·0000	0·0000					
21	0·0042	0·0035	0·0025	0·0010	0·0000	0·0000				
22	0·0055	0·0049	0·0040	0·0029	0·0012	0·0000	0·0000			
23	0·0071	0·0065	0·0057	0·0046	0·0033	0·0013	0·0000	0·0000		
24	0·0087	0·0082	0·0075	0·0065	0·0054	0·0038	0·0015	0·0000	0·0000	
25	0·0106	0·0101	0·0095	0·0086	0·0075	0·0062	0·0043	0·0018	0·0000	0·0000
26	0·0127	0·0122	0·0117	0·0109	0·0099	0·0086	0·0070	0·0049	0·0020	0·0000
27	0·0150	0·0146	0·0141	0·0134	0·0125	0·0114	0·0099	0·0080	0·0056	0·0023
28	0·0177	0·0173	0·0168	0·0162	0·0154	0·0143	0·0130	0·0113	0·0091	0·0064
29	0·0206	0·0203	0·0198	0·0193	0·0185	0·0175	0·0163	0·0148	0·0128	0·0104
30	0·0239	0·0236	0·0232	0·0227	0·0220	0·0211	0·0200	0·0186	0·0168	0·0145
31	0·0277	0·0274	0·0270	0·0265	0·0259	0·0251	0·0240	0·0227	0·0211	0·0190
32	0·0318	0·0315	0·0312	0·0308	0·0302	0·0294	0·0285	0·0273	0·0258	0·0239
33	0·0365	0·0362	0·0359	0·0355	0·0350	0·0343	0·0334	0·0323	0·0309	0·0292
34	0·0417	0·0415	0·0412	0·0408	0·0403	0·0397	0·0389	0·0379	0·0366	0·0350
35	0·0476	0·0474	0·0471	0·0467	0·0463	0·0457	0·0450	0·0440	0·0429	0·0414
36	0·0541	0·0540	0·0537	0·0534	0·0530	0·0524	0·0517	0·0509	0·0498	0·0484
37	0·0615	0·0613	0·0611	0·0608	0·0604	0·0599	0·0593	0·0585	0·0575	0·0562
38	0·0698	0·0696	0·0694	0·0691	0·0688	0·0683	0·0677	0·0670	0·0661	0·0649
39	0·0791	0·0789	0·0787	0·0785	0·0781	0·0777	0·0772	0·0765	0·0756	0·0745
40	0·0895	0·0894	0·0892	0·0890	0·0887	0·0883	0·0878	0·0871	0·0863	0·0853
41	0·1013	0·1012	0·1010	0·1008	0·1005	0·1001	0·0996	0·0990	0·0983	0·0973
42	0·1145	0·1144	0·1142	0·1140	0·1138	0·1134	0·1130	0·1124	0·1117	0·1108
43	0·1295	0·1294	0·1292	0·1290	0·1288	0·1284	0·1280	0·1275	0·1268	0·1260
44	0·1464	0·1463	0·1461	0·1459	0·1457	0·1454	0·1450	0·1445	0·1439	0·1431
45	0·1655	0·1654	0·1653	0·1651	0·1649	0·1646	0·1642	0·1638	0·1632	0·1625
46	0·1872	0·1871	0·1870	0·1868	0·1866	0·1863	0·1860	0·1856	0·1850	0·1844
47	0·2119	0·2118	0·2117	0·2115	0·2113	0·2111	0·2108	0·2104	0·2099	0·2092
48	0·2400	0·2399	0·2398	0·2397	0·2395	0·2393	0·2390	0·2386	0·2381	0·2375
49	0·2722	0·2721	0·2720	0·2719	0·2717	0·2715	0·2712	0·2709	0·2704	0·2698
50	0·3091	0·3090	0·3089	0·3088	0·3086	0·3084	0·3082	0·3078	0·3074	0·3069
51	0·3515	0·3514	0·3514	0·3512	0·3511	0·3509	0·3506	0·3503	0·3499	0·3494
52	0·4005	0·4004	0·4003	0·4002	0·4000	0·3999	0·3996	0·3993	0·3989	0·3984
53	0·4571	0·4570	0·4569	0·4568	0·4567	0·4565	0·4563	0·4560	0·4556	0·4552
54	0·5229	0·5228	0·5227	0·5226	0·5225	0·5223	0·5221	0·5218	0·5215	0·5211

Table E33a. *(Continued)*

Sickness rates $z(x,104/all,x-x_0)$. *Thirteen-week deferred period basis*

Attained age x	Entry age x_0: 16	17	18	19	20	21	22	23	24	25
55	0·5996	0·5995	0·5994	0·5993	0·5992	0·5991	0·5988	0·5986	0·5983	0·5978
56	0·6893	0·6893	0·6892	0·6891	0·6890	0·6888	0·6886	0·6884	0·6881	0·6877
57	0·7948	0·7948	0·7947	0·7946	0·7945	0·7943	0·7942	0·7939	0·7936	0·7932
58	0·9193	0·9192	0·9192	0·9191	0·9190	0·9188	0·9186	0·9184	0·9181	0·9178
59	1·0668	1·0667	1·0667	1·0666	1·0665	1·0664	1·0662	1·0660	1·0657	1·0654
60	1·2424	1·2424	1·2423	1·2422	1·2421	1·2420	1·2418	1·2416	1·2414	1·2410
61	1·4525	1·4524	1·4524	1·4523	1·4522	1·4521	1·4519	1·4517	1·4515	1·4511
62	1·7049	1·7049	1·7048	1·7048	1·7047	1·7045	1·7044	1·7042	1·7040	1·7037
63	2·0098	2·0098	2·0097	2·0097	2·0096	2·0095	2·0093	2·0091	2·0089	2·0086
64	2·3799	2·3799	2·3798	2·3798	2·3797	2·3796	2·3794	2·3792	2·3790	2·3787

Table E33b. *Sickness rates* z(x,*104/all*,x − x_0). *Thirteen-week deferred period basis*

Attained age x	Entry age x_0 26	27	28	29	30	31	32	33	34	35
26	0·0000									
27	0·0000	0·0000								
28	0·0025	0·0000	0·0000							
29	0·0072	0·0029	0·0000	0·0000						
30	0·0117	0·0081	0·0032	0·0000	0·0000					
31	0·0165	0·0132	0·0092	0·0036	0·0000	0·0000				
32	0·0215	0·0186	0·0149	0·0103	0·0041	0·0000	0·0000			
33	0·0270	0·0243	0·0209	0·0168	0·0116	0·0045	0·0000	0·0000		
34	0·0330	0·0305	0·0274	0·0236	0·0189	0·0130	0·0051	0·0000	0·0000	
35	0·0395	0·0372	0·0344	0·0308	0·0265	0·0212	0·0145	0·0057	0·0000	0·0000
36	0·0467	0·0446	0·0419	0·0387	0·0347	0·0297	0·0237	0·0162	0·0063	0·0000
37	0·0546	0·0527	0·0502	0·0472	0·0435	0·0389	0·0334	0·0266	0·0181	0·0070
38	0·0634	0·0616	0·0593	0·0565	0·0531	0·0488	0·0437	0·0374	0·0297	0·0202
39	0·0732	0·0715	0·0693	0·0667	0·0635	0·0596	0·0548	0·0490	0·0419	0·0333
40	0·0840	0·0824	0·0805	0·0781	0·0751	0·0714	0·0670	0·0615	0·0549	0·0469
41	0·0961	0·0947	0·0928	0·0906	0·0878	0·0844	0·0803	0·0752	0·0691	0·0616
42	0·1097	0·1084	0·1067	0·1046	0·1020	0·0988	0·0949	0·0902	0·0845	0·0775
43	0·1250	0·1237	0·1221	0·1202	0·1177	0·1148	0·1112	0·1067	0·1014	0·0949
44	0·1422	0·1410	0·1395	0·1376	0·1354	0·1326	0·1292	0·1251	0·1201	0·1140
45	0·1616	0·1604	0·1590	0·1573	0·1552	0·1526	0·1494	0·1456	0·1409	0·1351
46	0·1835	0·1825	0·1811	0·1795	0·1775	0·1751	0·1721	0·1685	0·1641	0·1587
47	0·2084	0·2074	0·2062	0·2047	0·2028	0·2005	0·1977	0·1943	0·1901	0·1851
48	0·2368	0·2358	0·2347	0·2332	0·2315	0·2293	0·2267	0·2235	0·2196	0·2148
49	0·2691	0·2682	0·2672	0·2658	0·2641	0·2621	0·2596	0·2566	0·2529	0·2484
50	0·3062	0·3054	0·3043	0·3031	0·3015	0·2996	0·2972	0·2944	0·2909	0·2867
51	0·3488	0·3480	0·3470	0·3458	0·3443	0·3425	0·3403	0·3376	0·3343	0·3303
52	0·3978	0·3971	0·3962	0·3950	0·3936	0·3919	0·3898	0·3873	0·3841	0·3804
53	0·4546	0·4539	0·4530	0·4520	0·4506	0·4490	0·4470	0·4446	0·4416	0·4380
54	0·5205	0·5199	0·5190	0·5180	0·5167	0·5152	0·5133	0·5110	0·5082	0·5048
55	0·5973	0·5967	0·5959	0·5949	0·5937	0·5923	0·5904	0·5883	0·5856	0·5823
56	0·6872	0·6866	0·6858	0·6849	0·6838	0·6824	0·6806	0·6785	0·6760	0·6729
57	0·7928	0·7922	0·7915	0·7906	0·7895	0·7882	0·7865	0·7845	0·7821	0·7791
58	0·9173	0·9168	0·9161	0·9152	0·9142	0·9129	0·9113	0·9094	0·9071	0·9043
59	1·0649	1·0644	1·0637	1·0629	1·0619	1·0607	1·0592	1·0574	1·0552	1·0525
60	1·2406	1·2401	1·2395	1·2387	1·2377	1·2366	1·2351	1·2334	1·2313	1·2287
61	1·4508	1·4503	1·4497	1·4489	1·4480	1·4469	1·4455	1·4438	1·4418	1·4393
62	1·7033	1·7028	1·7022	1·7015	1·7006	1·6995	1·6982	1·6966	1·6946	1·6923
63	2·0082	2·0078	2·0072	2·0065	2·0057	2·0046	2·0034	2·0018	1·9999	1·9976
64	2·3784	2·3780	2·3774	2·3767	2·3759	2·3749	2·3737	2·3722	2·3704	2·3682

Table E33c. *Sickness rates* z(x,*104/all*,x − x₀). *Thirteen-week deferred period basis*

Attained age x	Entry age x_0: 36	37	38	39	40	41	42	43	44	45
36	0·0000									
37	0·0000	0·0000								
38	0·0078	0·0000	0·0000							
39	0·0226	0·0087	0·0000	0·0000						
40	0·0372	0·0252	0·0097	0·0000	0·0000					
41	0·0526	0·0416	0·0282	0·0108	0·0000	0·0000				
42	0·0691	0·0589	0·0466	0·0315	0·0121	0·0000	0·0000			
43	0·0870	0·0775	0·0660	0·0522	0·0352	0·0135	0·0000	0·0000		
44	0·1066	0·0977	0·0870	0·0740	0·0584	0·0394	0·0151	0·0000	0·0000	
45	0·1282	0·1199	0·1098	0·0977	0·0831	0·0656	0·0441	0·0169	0·0000	0·0000
46	0·1522	0·1444	0·1349	0·1236	0·1099	0·0934	0·0736	0·0495	0·0189	0·0000
47	0·1790	0·1717	0·1628	0·1521	0·1392	0·1237	0·1052	0·0828	0·0556	0·0212
48	0·2091	0·2022	0·1938	0·1837	0·1716	0·1571	0·1396	0·1186	0·0933	0·0626
49	0·2430	0·2365	0·2286	0·2191	0·2077	0·1940	0·1775	0·1577	0·1339	0·1054
50	0·2816	0·2754	0·2679	0·2590	0·2482	0·2352	0·2197	0·2010	0·1785	0·1516
51	0·3255	0·3196	0·3126	0·3041	0·2939	0·2817	0·2670	0·2493	0·2281	0·2026
52	0·3758	0·3703	0·3636	0·3556	0·3459	0·3343	0·3204	0·3036	0·2835	0·2594
53	0·4337	0·4284	0·4221	0·4145	0·4054	0·3943	0·3811	0·3653	0·3462	0·3233
54	0·5006	0·4957	0·4897	0·4824	0·4737	0·4633	0·4507	0·4356	0·4175	0·3958
55	0·5784	0·5737	0·5680	0·5611	0·5528	0·5429	0·5309	0·5166	0·4994	0·4787
56	0·6692	0·6647	0·6592	0·6527	0·6448	0·6353	0·6239	0·6103	0·5939	0·5742
57	0·7756	0·7713	0·7661	0·7598	0·7523	0·7433	0·7324	0·7194	0·7037	0·6849
58	0·9009	0·8968	0·8918	0·8858	0·8787	0·8700	0·8596	0·8472	0·8322	0·8143
59	1·0492	1·0453	1·0405	1·0348	1·0279	1·0196	1·0097	0·9978	0·9835	0·9663
60	1·2256	1·2218	1·2172	1·2117	1·2052	1·1972	1·1877	1·1763	1·1626	1·1461
61	1·4363	1·4327	1·4283	1·4230	1·4167	1·4091	1·4000	1·3890	1·3758	1·3600
62	1·6894	1·6859	1·6817	1·6766	1·6706	1·6632	1·6545	1·6439	1·6313	1·6161
63	1·9949	1·9915	1·9875	1·9826	1·9767	1·9697	1·9613	1·9511	1·9389	1·9243
64	2·3655	2·3623	2·3584	2·3537	2·3480	2·3413	2·3331	2·3233	2·3116	2·2975

Table E33d. *Sickness rates* z(x,*104/all*,x − x_0). *Thirteen-week deferred period basis*

Attained age x	Entry age x_0 46	47	48	49	50	51	52	53	54	55
46	0·0000									
47	0·0000	0·0000								
48	0·0238	0·0000	0·0000							
49	0·0706	0·0268	0·0000	0·0000						
50	0·1192	0·0799	0·0303	0·0000	0·0000					
51	0·1720	0·1353	0·0905	0·0343	0·0000	0·0000				
52	0·2305	0·1957	0·1538	0·1029	0·0389	0·0000	0·0000			
53	0·2959	0·2629	0·2232	0·1755	0·1173	0·0444	0·0000	0·0000		
54	0·3697	0·3384	0·3008	0·2554	0·2008	0·1342	0·0507	0·0000	0·0000	
55	0·4539	0·4241	0·3883	0·3452	0·2932	0·2305	0·1540	0·0582	0·0000	0·0000
56	0·5505	0·5222	0·4880	0·4470	0·3975	0·3378	0·2656	0·1775	0·0670	0·0000
57	0·6624	0·6353	0·6028	0·5636	0·5164	0·4595	0·3907	0·3073	0·2053	0·0775
58	0·7927	0·7669	0·7358	0·6983	0·6532	0·5989	0·5331	0·4535	0·3569	0·2384
59	0·9457	0·9209	0·8912	0·8554	0·8122	0·7602	0·6973	0·6211	0·5287	0·4163
60	1·1264	1·1026	1·0741	1·0398	0·9984	0·9486	0·8883	0·8154	0·7269	0·6192
61	1·3411	1·3183	1·2909	1·2580	1·2183	1·1705	1·1127	1·0427	0·9577	0·8545
62	1·5979	1·5760	1·5497	1·5180	1·4799	1·4339	1·3784	1·3111	1·2295	1·1303
63	1·9068	1·8857	1·8604	1·8300	1·7933	1·7490	1·6956	1·6309	1·5524	1·4569
64	2·2806	2·2603	2·2359	2·2066	2·1713	2·1286	2·0771	2·0148	1·9392	1·8472

Table E33e. *Sickness rates* z(x,*104/all*,x − x_0). *Thirteen-week deferred period basis*

Attained age x	Entry age x_0: 56	57	58	59	60	61	62	63	64
56	0·0000								
57	0·0000	0·0000							
58	0·0900	0·0000	0·0000						
59	0·2782	0·1050	0·0000	0·0000					
60	0·4878	0·3260	0·1230	0·0000	0·0000				
61	0·7285	0·5743	0·3840	0·1450	0·0000	0·0000			
62	1·0093	0·8613	0·6795	0·4547	0·1717	0·0000	0·0000		
63	1·3405	1·1982	1·0235	0·8083	0·5412	0·2045	0·0000	0·0000	
64	1·7351	1·5980	1·4298	1·2228	0·9667	0·6478	0·2449	0·0000	0·0000

Table E34. *Select table of* $q_{[x]+t}$ *with 5 years selection. Twenty-six-week deferred period basis*

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
16	0·000984					
17	0·000930	0·000944				
18	0·000879	0·000894	0·000895			
19	0·000832	0·000847	0·000848	0·000848		
20	0·000788	0·000803	0·000804	0·000805	0·000805	
21	0·000748	0·000764	0·000765	0·000766	0·000766	0·000766
22	0·000712	0·000729	0·000730	0·000730	0·000731	0·000731
23	0·000680	0·000698	0·000699	0·000700	0·000700	0·000701
24	0·000653	0·000671	0·000673	0·000673	0·000674	0·000675
25	0·000630	0·000650	0·000651	0·000652	0·000652	0·000653
26	0·000613	0·000633	0·000635	0·000636	0·000636	0·000638
27	0·000600	0·000622	0·000624	0·000625	0·000626	0·000627
28	0·000594	0·000617	0·000619	0·000620	0·000621	0·000623
29	0·000593	0·000618	0·000620	0·000621	0·000622	0·000624
30	0·000599	0·000625	0·000628	0·000629	0·000630	0·000633
31	0·000611	0·000639	0·000643	0·000644	0·000645	0·000648
32	0·000631	0·000661	0·000665	0·000667	0·000668	0·000671
33	0·000658	0·000691	0·000695	0·000697	0·000698	0·000703
34	0·000694	0·000729	0·000734	0·000736	0·000738	0·000743
35	0·000738	0·000777	0·000782	0·000785	0·000786	0·000792
36	0·000792	0·000834	0·000840	0·000843	0·000845	0·000852
37	0·000855	0·000902	0·000909	0·000912	0·000914	0·000922
38	0·000930	0·000981	0·000989	0·000992	0·000995	0·001004
39	0·001016	0·001072	0·001081	0·001085	0·001088	0·001098
40	0·001114	0·001176	0·001186	0·001191	0·001194	0·001206
41	0·001225	0·001294	0·001305	0·001311	0·001315	0·001329
42	0·001350	0·001427	0·001440	0·001447	0·001451	0·001468
43	0·001491	0·001576	0·001591	0·001599	0·001604	0·001623
44	0·001647	0·001742	0·001760	0·001769	0·001775	0·001798
45	0·001821	0·001928	0·001949	0·001959	0·001965	0·001992
46	0·002014	0·002134	0·002158	0·002169	0·002177	0·002208
47	0·002226	0·002362	0·002390	0·002403	0·002412	0·002449
48	0·002461	0·002614	0·002646	0·002662	0·002672	0·002715
49	0·002719	0·002892	0·002929	0·002948	0·002960	0·003010
50	0·003001	0·003198	0·003242	0·003263	0·003278	0·003336
51	0·003311	0·003536	0·003586	0·003612	0·003628	0·003697
52	0·003651	0·003906	0·003966	0·003995	0·004015	0·004096
53	0·004022	0·004314	0·004383	0·004418	0·004442	0·004536
54	0·004427	0·004763	0·004843	0·004885	0·004912	0·005023

Table E34. *(Continued)*

Select table of $q_{[x]+t}$ *with 5 years selection. Twenty-six-week deferred period basis*

Attained age x	Select durations (t) 0	1	2	3	4	5 and over
55	0·004870	0·005255	0·005350	0·005399	0·005431	0·005561
56	0·005353	0·005797	0·005908	0·005965	0·006003	0·006156
57	0·005880	0·006392	0·006524	0·006591	0·006635	0·006815
58	0·006456	0·007048	0·007202	0·007281	0·007334	0·007546
59	0·007084	0·007770	0·007951	0·008045	0·008107	0·008355
60	0·007769	0·008565	0·008780	0·008890	0·008963	0·009255
61	0·008517	0·009443	0·009696	0·009826	0·009912	0·010255
62	0·009335	0·010413	0·010712	0·010865	0·010966	0·011368
63	0·010229	0·011487	0·011840	0·012021	0·012139	0·012608
64	0·011207	0·012677	0·013095	0·013307	0·013446	0·013992

Table E35a. *Select table of* $ia^{d}_{[x]+1}$ *with 5 years selection: methods A and B: deferred period* (d) *26 weeks. Twenty-six-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
16	0·000085						
17	0·000089	0·000176					
18	0·000094	0·000185	0·000185				
19	0·000099	0·000196	0·000196	0·000196			
20	0·000105	0·000207	0·000207	0·000207	0·000207		
21	0·000111	0·000219	0·000219	0·000219	0·000219	0·000219	0·000219
22	0·000118	0·000232	0·000232	0·000232	0·000232	0·000232	0·000232
23	0·000125	0·000246	0·000246	0·000246	0·000246	0·000246	0·000246
24	0·000133	0·000262	0·000262	0·000262	0·000262	0·000262	0·000262
25	0·000142	0·000280	0·000280	0·000280	0·000280	0·000280	0·000280
26	0·000152	0·000299	0·000299	0·000299	0·000299	0·000299	0·000299
27	0·000163	0·000320	0·000320	0·000320	0·000320	0·000320	0·000320
28	0·000175	0·000343	0·000343	0·000343	0·000343	0·000343	0·000343
29	0·000188	0·000369	0·000369	0·000368	0·000368	0·000368	0·000368
30	0·000202	0·000397	0·000397	0·000397	0·000397	0·000397	0·000396
31	0·000218	0·000428	0·000428	0·000428	0·000428	0·000428	0·000428
32	0·000236	0·000462	0·000462	0·000462	0·000462	0·000462	0·000462
33	0·000255	0·000500	0·000500	0·000500	0·000500	0·000500	0·000500
34	0·000277	0·000542	0·000542	0·000542	0·000542	0·000542	0·000542
35	0·000301	0·000589	0·000589	0·000589	0·000589	0·000589	0·000589
36	0·000328	0·000641	0·000641	0·000641	0·000641	0·000641	0·000641
37	0·000357	0·000699	0·000699	0·000699	0·000699	0·000699	0·000699
38	0·000391	0·000764	0·000764	0·000764	0·000763	0·000763	0·000763
39	0·000428	0·000836	0·000836	0·000836	0·000836	0·000835	0·000835
40	0·000469	0·000917	0·000916	0·000916	0·000916	0·000916	0·000916
41	0·000516	0·001007	0·001007	0·001007	0·001006	0·001006	0·001006
42	0·000568	0·001108	0·001108	0·001108	0·001107	0·001107	0·001107
43	0·000627	0·001222	0·001222	0·001221	0·001221	0·001221	0·001220
44	0·000693	0·001350	0·001350	0·001349	0·001349	0·001348	0·001347
45	0·000767	0·001494	0·001494	0·001493	0·001493	0·001492	0·001491
46	0·000851	0·001657	0·001656	0·001655	0·001655	0·001654	0·001652
47	0·000946	0·001841	0·001839	0·001839	0·001838	0·001837	0·001835
48	0·001053	0·002049	0·002047	0·002046	0·002045	0·002044	0·002041
49	0·001175	0·002284	0·002282	0·002281	0·002279	0·002279	0·002275
50	0·001313	0·002552	0·002549	0·002547	0·002546	0·002544	0·002539
51	0·001471	0·002856	0·002852	0·002850	0·002848	0·002846	0·002840
52	0·001650	0·003202	0·003197	0·003194	0·003192	0·003190	0·003181
53	0·001854	0·003596	0·003591	0·003586	0·003583	0·003581	0·003569
54	0·002088	0·004047	0·004039	0·004034	0·004030	0·004027	0·004012

Table E35a. *(Continued)*
Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *26 weeks. Twenty-six-week deferred period basis*

	Duration t						
Age x	*0*	*1*	*2*	*3*	*4*	*5 and over* B	*5 and over* A
55	0·002356	0·004563	0·004553	0·004545	0·004540	0·004536	0·004516
56	0·002662	0·005153	0·005140	0·005131	0·005123	0·005118	0·005092
57	0·003015	0·005830	0·005813	0·005801	0·005791	0·005784	0·005750
58	0·003420	0·006608	0·006586	0·006570	0·006557	0·006547	0·006502
59	0·003887	0·007503	0·007474	0·007453	0·007436	0·007422	0·007363
60	0·004426	0·008535	0·008496	0·008468	0·008445	0·008427	0·008347
61	0·005048	0·009725	0·009674	0·009635	0·009605	0·009581	0·009474
62	0·005769	0·011099	0·011032	0·010980	0·010940	0·010907	0·010764
63	0·006604	0·012690	0·012599	0·012531	0·012476	0·012432	0·012239
64	0·007574	0·014532	0·014411	0·014318	0·014244	0·014185	0·013923

Table E35b. *Select table of* $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *52 weeks. Twenty-six-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·000000						
17	0·000000	0·000061					
18	0·000000	0·000066	0·000066				
19	0·000000	0·000071	0·000071	0·000071			
20	0·000000	0·000076	0·000076	0·000076	0·000076		
21	0·000000	0·000082	0·000082	0·000082	0·000082	0·000082	0·000082
22	0·000000	0·000088	0·000088	0·000088	0·000088	0·000088	0·000088
23	0·000000	0·000095	0·000095	0·000095	0·000095	0·000095	0·000095
24	0·000000	0·000103	0·000103	0·000103	0·000103	0·000103	0·000103
25	0·000000	0·000111	0·000111	0·000111	0·000111	0·000111	0·000111
26	0·000000	0·000121	0·000121	0·000121	0·000121	0·000121	0·000121
27	0·000000	0·000132	0·000132	0·000132	0·000132	0·000132	0·000132
28	0·000000	0·000144	0·000144	0·000144	0·000144	0·000144	0·000144
29	0·000000	0·000157	0·000157	0·000157	0·000157	0·000157	0·000157
30	0·000000	0·000172	0·000171	0·000171	0·000171	0·000171	0·000171
31	0·000000	0·000188	0·000188	0·000188	0·000188	0·000188	0·000188
32	0·000000	0·000206	0·000206	0·000206	0·000206	0·000206	0·000206
33	0·000000	0·000227	0·000227	0·000227	0·000227	0·000227	0·000227
34	0·000000	0·000250	0·000250	0·000250	0·000250	0·000250	0·000250
35	0·000000	0·000276	0·000276	0·000276	0·000276	0·000276	0·000276
36	0·000000	0·000306	0·000305	0·000305	0·000305	0·000305	0·000305
37	0·000000	0·000339	0·000338	0·000338	0·000338	0·000338	0·000338
38	0·000000	0·000376	0·000376	0·000376	0·000376	0·000375	0·000375
39	0·000000	0·000418	0·000418	0·000418	0·000418	0·000418	0·000417
40	0·000000	0·000466	0·000465	0·000465	0·000465	0·000465	0·000465
41	0·000000	0·000520	0·000519	0·000519	0·000519	0·000519	0·000519
42	0·000000	0·000581	0·000581	0·000581	0·000581	0·000580	0·000580
43	0·000000	0·000651	0·000651	0·000650	0·000650	0·000650	0·000650
44	0·000000	0·000731	0·000730	0·000730	0·000730	0·000730	0·000729
45	0·000000	0·000822	0·000821	0·000821	0·000820	0·000820	0·000819
46	0·000000	0·000926	0·000925	0·000924	0·000924	0·000924	0·000923
47	0·000000	0·001045	0·001043	0·001043	0·001043	0·001042	0·001041
48	0·000000	0·001181	0·001180	0·001179	0·001178	0·001178	0·001176
49	0·000000	0·001338	0·001336	0·001335	0·001334	0·001334	0·001331
50	0·000000	0·001518	0·001516	0·001514	0·001514	0·001513	0·001510
51	0·000000	0·001726	0·001723	0·001721	0·001720	0·001719	0·001715
52	0·000000	0·001966	0·001962	0·001960	0·001958	0·001957	0·001951
53	0·000000	0·002243	0·002238	0·002235	0·002233	0·002231	0·002224
54	0·000000	0·002564	0·002557	0·002554	0·002551	0·002549	0·002539

Table E35b. *(Continued)*

Select table of $ia^{d}_{[x]+t}$ *with 5 years selection: methods A and B: deferred period* (d) *52 weeks. Twenty-six-week deferred period basis*

	Duration t						
Age x	0	1	2	3	4	5 and over B	5 and over A
55	0·000000	0·002936	0·002927	0·002923	0·002919	0·002916	0·002903
56	0·000000	0·003369	0·003357	0·003351	0·003346	0·003342	0·003325
57	0·000000	0·003872	0·003857	0·003848	0·003842	0·003837	0·003814
58	0·000000	0·004458	0·004439	0·004427	0·004419	0·004412	0·004381
59	0·000000	0·005143	0·005117	0·005102	0·005090	0·005081	0·005040
60	0·000000	0·005943	0·005910	0·005889	0·005873	0·005860	0·005805
61	0·000000	0·006880	0·006836	0·006808	0·006787	0·006770	0·006694
62	0·000000	0·007980	0·007921	0·007884	0·007854	0·007830	0·007727
63	0·000000	0·009272	0·009194	0·009142	0·009102	0·009069	0·008928
64	0·000000	0·010792	0·010687	0·010617	0·010561	0·010517	0·010322

Table E36a. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. Twenty-six-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·0007						
17	0·0008	0·0026					
18	0·0008	0·0027	0·0027				
19	0·0009	0·0029	0·0029	0·0029			
20	0·0009	0·0031	0·0031	0·0031	0·0031		
21	0·0010	0·0033	0·0033	0·0033	0·0033	0·0033	0·0033
22	0·0010	0·0035	0·0035	0·0035	0·0035	0·0035	0·0035
23	0·0011	0·0038	0·0038	0·0038	0·0038	0·0038	0·0038
24	0·0012	0·0041	0·0041	0·0041	0·0041	0·0041	0·0041
25	0·0013	0·0044	0·0044	0·0044	0·0044	0·0044	0·0044
26	0·0014	0·0047	0·0047	0·0047	0·0047	0·0047	0·0047
27	0·0015	0·0051	0·0051	0·0051	0·0051	0·0051	0·0051
28	0·0016	0·0055	0·0055	0·0055	0·0055	0·0055	0·0055
29	0·0017	0·0059	0·0059	0·0059	0·0059	0·0059	0·0059
30	0·0019	0·0064	0·0064	0·0064	0·0064	0·0064	0·0064
31	0·0020	0·0070	0·0070	0·0070	0·0070	0·0070	0·0070
32	0·0022	0·0076	0·0076	0·0076	0·0076	0·0076	0·0076
33	0·0024	0·0083	0·0083	0·0083	0·0083	0·0083	0·0083
34	0·0026	0·0091	0·0091	0·0091	0·0091	0·0091	0·0091
35	0·0029	0·0100	0·0100	0·0100	0·0100	0·0100	0·0100
36	0·0032	0·0110	0·0110	0·0110	0·0110	0·0110	0·0110
37	0·0035	0·0121	0·0121	0·0121	0·0121	0·0121	0·0121
38	0·0038	0·0133	0·0133	0·0133	0·0133	0·0133	0·0133
39	0·0042	0·0147	0·0147	0·0147	0·0147	0·0147	0·0147
40	0·0047	0·0162	0·0162	0·0162	0·0162	0·0162	0·0162
41	0·0052	0·0180	0·0180	0·0180	0·0180	0·0180	0·0180
42	0·0057	0·0200	0·0200	0·0200	0·0200	0·0200	0·0199
43	0·0063	0·0222	0·0222	0·0222	0·0222	0·0222	0·0222
44	0·0071	0·0248	0·0247	0·0247	0·0247	0·0247	0·0247
45	0·0079	0·0276	0·0276	0·0276	0·0276	0·0276	0·0276
46	0·0088	0·0309	0·0309	0·0309	0·0309	0·0309	0·0308
47	0·0098	0·0346	0·0346	0·0346	0·0346	0·0346	0·0345
48	0·0110	0·0389	0·0389	0·0388	0·0388	0·0388	0·0387
49	0·0124	0·0437	0·0437	0·0437	0·0436	0·0436	0·0435
50	0·0139	0·0493	0·0492	0·0492	0·0492	0·0491	0·0490
51	0·0157	0·0556	0·0556	0·0555	0·0555	0·0554	0·0553
52	0·0177	0·0629	0·0628	0·0628	0·0627	0·0627	0·0625
53	0·0200	0·0713	0·0712	0·0711	0·0710	0·0710	0·0708
54	0·0227	0·0810	0·0808	0·0807	0·0806	0·0805	0·0802

Table E36a. *(Continued)*
Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *26/26 weeks. Twenty-six-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0258	0·0921	0·0918	0·0917	0·0916	0·0915	0·0911
56	0·0293	0·1049	0·1046	0·1044	0·1043	0·1042	0·1036
57	0·0334	0·1197	0·1194	0·1191	0·1189	0·1187	0·1181
58	0·0382	0·1369	0·1364	0·1361	0·1358	0·1356	0·1347
59	0·0437	0·1569	0·1562	0·1558	0·1554	0·1551	0·1539
60	0·0500	0·1800	0·1792	0·1786	0·1781	0·1777	0·1760
61	0·0574	0·2070	0·2058	0·2050	0·2044	0·2038	0·2016
62	0·0661	0·2384	0·2369	0·2357	0·2349	0·2342	0·2311
63	0·0762	0·2751	0·2730	0·2715	0·2703	0·2693	0·2652
64	0·0879	0·3179	0·3151	0·3131	0·3115	0·3102	0·3044

Table E36b. *Select table of* $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. Twenty-six-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
16	0·0000						
17	0·0000	0·0012					
18	0·0000	0·0013	0·0022				
19	0·0000	0·0014	0·0024	0·0024			
20	0·0000	0·0015	0·0026	0·0026	0·0026		
21	0·0000	0·0016	0·0028	0·0028	0·0028	0·0028	0·0028
22	0·0000	0·0017	0·0030	0·0030	0·0030	0·0030	0·0030
23	0·0000	0·0019	0·0033	0·0033	0·0033	0·0033	0·0033
24	0·0000	0·0021	0·0036	0·0036	0·0036	0·0036	0·0036
25	0·0000	0·0022	0·0039	0·0039	0·0039	0·0039	0·0039
26	0·0000	0·0024	0·0043	0·0043	0·0043	0·0043	0·0043
27	0·0000	0·0027	0·0047	0·0047	0·0047	0·0047	0·0047
28	0·0000	0·0029	0·0051	0·0051	0·0051	0·0051	0·0051
29	0·0000	0·0032	0·0057	0·0057	0·0057	0·0057	0·0057
30	0·0000	0·0035	0·0062	0·0062	0·0062	0·0062	0·0062
31	0·0000	0·0039	0·0069	0·0069	0·0069	0·0069	0·0069
32	0·0000	0·0043	0·0076	0·0076	0·0076	0·0076	0·0076
33	0·0000	0·0047	0·0084	0·0084	0·0084	0·0084	0·0084
34	0·0000	0·0052	0·0093	0·0093	0·0093	0·0093	0·0093
35	0·0000	0·0058	0·0103	0·0103	0·0103	0·0103	0·0103
36	0·0000	0·0064	0·0115	0·0115	0·0115	0·0115	0·0115
37	0·0000	0·0072	0·0128	0·0128	0·0128	0·0128	0·0128
38	0·0000	0·0080	0·0143	0·0143	0·0143	0·0143	0·0143
39	0·0000	0·0089	0·0160	0·0160	0·0160	0·0160	0·0160
40	0·0000	0·0100	0·0179	0·0179	0·0179	0·0179	0·0179
41	0·0000	0·0112	0·0201	0·0201	0·0201	0·0201	0·0201
42	0·0000	0·0126	0·0227	0·0227	0·0227	0·0226	0·0226
43	0·0000	0·0142	0·0255	0·0255	0·0255	0·0255	0·0255
44	0·0000	0·0160	0·0288	0·0288	0·0288	0·0288	0·0288
45	0·0000	0·0181	0·0326	0·0326	0·0326	0·0326	0·0326
46	0·0000	0·0204	0·0370	0·0369	0·0369	0·0369	0·0369
47	0·0000	0·0232	0·0420	0·0419	0·0419	0·0419	0·0419
48	0·0000	0·0263	0·0477	0·0477	0·0477	0·0477	0·0476
49	0·0000	0·0299	0·0544	0·0543	0·0543	0·0543	0·0542
50	0·0000	0·0341	0·0621	0·0620	0·0620	0·0620	0·0618
51	0·0000	0·0390	0·0710	0·0709	0·0709	0·0708	0·0707
52	0·0000	0·0446	0·0814	0·0812	0·0812	0·0811	0·0809
53	0·0000	0·0511	0·0934	0·0932	0·0931	0·0931	0·0928
54	0·0000	0·0587	0·1074	0·1072	0·1071	0·1070	0·1066

Table E36b. *(Continued)*
Select table of $z^{a/b}_{[x]+t}$ *with 5 years selection: methods A and B: sickness period* (a/b) *52/52 weeks. Twenty-six-week deferred period basis*

Age x	Duration t 0	1	2	3	4	5 and over B	5 and over A
55	0·0000	0·0675	0·1237	0·1234	0·1233	0·1232	0·1226
56	0·0000	0·0778	0·1427	0·1424	0·1422	0·1420	0·1413
57	0·0000	0·0899	0·1650	0·1646	0·1643	0·1641	0·1631
58	0·0000	0·1040	0·1911	0·1906	0·1902	0·1899	0·1885
59	0·0000	0·1205	0·2217	0·2210	0·2205	0·2200	0·2183
60	0·0000	0·1399	0·2577	0·2567	0·2560	0·2554	0·2530
61	0·0000	0·1627	0·3001	0·2987	0·2978	0·2970	0·2937
62	0·0000	0·1896	0·3500	0·3482	0·3469	0·3458	0·3412
63	0·0000	0·2214	0·4090	0·4065	0·4047	0·4033	0·3969
64	0·0000	0·2589	0·4787	0·4754	0·4729	0·4709	0·4621

Table E37a. *Sickness rates* $z(x,104/all,x-x_0)$. *Twenty-six-week deferred period basis*

Attained age x	Entry age x_0: 16	17	18	19	20	21	22	23	24	25
16	0·0000									
17	0·0000	0·0000								
18	0·0006	0·0000	0·0000							
19	0·0017	0·0007	0·0000	0·0000						
20	0·0027	0·0019	0·0007	0·0000	0·0000					
21	0·0036	0·0029	0·0020	0·0008	0·0000	0·0000				
22	0·0046	0·0040	0·0032	0·0022	0·0009	0·0000	0·0000			
23	0·0057	0·0051	0·0044	0·0035	0·0025	0·0010	0·0000	0·0000		
24	0·0068	0·0063	0·0056	0·0048	0·0039	0·0027	0·0011	0·0000	0·0000	
25	0·0080	0·0075	0·0069	0·0062	0·0053	0·0043	0·0030	0·0012	0·0000	0·0000
26	0·0093	0·0088	0·0083	0·0077	0·0069	0·0059	0·0047	0·0033	0·0013	0·0000
27	0·0107	0·0103	0·0098	0·0092	0·0085	0·0076	0·0065	0·0053	0·0036	0·0014
28	0·0122	0·0118	0·0114	0·0109	0·0102	0·0094	0·0084	0·0073	0·0058	0·0040
29	0·0139	0·0136	0·0132	0·0127	0·0121	0·0114	0·0105	0·0094	0·0081	0·0065
30	0·0158	0·0155	0·0151	0·0147	0·0141	0·0135	0·0126	0·0117	0·0105	0·0090
31	0·0179	0·0176	0·0173	0·0169	0·0164	0·0158	0·0150	0·0141	0·0130	0·0117
32	0·0203	0·0200	0·0197	0·0193	0·0189	0·0183	0·0176	0·0168	0·0158	0·0145
33	0·0229	0·0227	0·0224	0·0221	0·0216	0·0211	0·0205	0·0197	0·0188	0·0177
34	0·0260	0·0257	0·0255	0·0252	0·0248	0·0243	0·0237	0·0230	0·0221	0·0211
35	0·0294	0·0292	0·0289	0·0286	0·0283	0·0278	0·0273	0·0266	0·0259	0·0249
36	0·0333	0·0331	0·0329	0·0326	0·0322	0·0318	0·0313	0·0307	0·0300	0·0291
37	0·0377	0·0375	0·0373	0·0371	0·0367	0·0364	0·0359	0·0354	0·0347	0·0338
38	0·0427	0·0426	0·0424	0·0422	0·0419	0·0415	0·0411	0·0406	0·0400	0·0392
39	0·0485	0·0484	0·0482	0·0480	0·0477	0·0474	0·0470	0·0465	0·0459	0·0452
40	0·0552	0·0550	0·0549	0·0547	0·0544	0·0541	0·0537	0·0533	0·0528	0·0521
41	0·0628	0·0627	0·0625	0·0623	0·0621	0·0618	0·0615	0·0611	0·0605	0·0599
42	0·0716	0·0715	0·0713	0·0711	0·0709	0·0707	0·0703	0·0699	0·0695	0·0689
43	0·0817	0·0816	0·0815	0·0813	0·0811	0·0808	0·0805	0·0802	0·0797	0·0792
44	0·0934	0·0933	0·0932	0·0930	0·0928	0·0926	0·0923	0·0920	0·0916	0·0911
45	0·1070	0·1069	0·1067	0·1066	0·1064	0·1062	0·1059	0·1056	0·1052	0·1048
46	0·1227	0·1226	0·1225	0·1223	0·1222	0·1220	0·1217	0·1214	0·1210	0·1206
47	0·1409	0·1408	0·1407	0·1406	0·1404	0·1402	0·1400	0·1397	0·1394	0·1390
48	0·1621	0·1621	0·1620	0·1618	0·1617	0·1615	0·1613	0·1610	0·1607	0·1603
49	0·1869	0·1868	0·1867	0·1866	0·1865	0·1863	0·1861	0·1859	0·1855	0·1852
50	0·2158	0·2157	0·2156	0·2155	0·2154	0·2152	0·2150	0·2148	0·2145	0·2142
51	0·2496	0·2495	0·2494	0·2493	0·2492	0·2491	0·2489	0·2487	0·2484	0·2481
52	0·2892	0·2891	0·2890	0·2890	0·2888	0·2887	0·2885	0·2883	0·2881	0·2877
53	0·3357	0·3356	0·3355	0·3354	0·3353	0·3352	0·3350	0·3348	0·3346	0·3343
54	0·3903	0·3902	0·3902	0·3901	0·3900	0·3898	0·3897	0·3895	0·3893	0·3890

Table 37a. *(Continued)*

Sickness rates $z(x, 104/all, x-x_0)$. *Twenty-six-week deferred period basis*

Attained age x	Entry age x_0 16	17	18	19	20	21	22	23	24	25
55	0·4546	0·4546	0·4545	0·4544	0·4543	0·4542	0·4540	0·4539	0·4536	0·4534
56	0·5305	0·5304	0·5304	0·5303	0·5302	0·5301	0·5299	0·5297	0·5295	0·5293
57	0·6201	0·6200	0·6200	0·6199	0·6198	0·6197	0·6196	0·6194	0·6192	0·6190
58	0·7262	0·7261	0·7261	0·7260	0·7259	0·7258	0·7257	0·7255	0·7253	0·7251
59	0·8519	0·8518	0·8518	0·8517	0·8516	0·8515	0·8514	0·8513	0·8511	0·8509
60	1·0012	1·0011	1·0011	1·0010	1·0009	1·0008	1·0007	1·0006	1·0004	1·0002
61	1·1787	1·1786	1·1786	1·1785	1·1785	1·1784	1·1783	1·1781	1·1779	1·1777
62	1·3902	1·3901	1·3901	1·3900	1·3899	1·3898	1·3897	1·3896	1·3894	1·3892
63	1·6424	1·6424	1·6424	1·6423	1·6422	1·6421	1·6420	1·6419	1·6418	1·6416
64	1·9439	1·9439	1·9438	1·9438	1·9437	1·9436	1·9435	1·9434	1·9432	1·9430

Table E37b. *Sickness rates* z(x,*104*/*all*,x − x_0). *Twenty-six-week deferred period basis*

Attained	Entry age x_0									
age x	26	27	28	29	30	31	32	33	34	35
26	0·0000									
27	0·0000	0·0000								
28	0·0016	0·0000	0·0000							
29	0·0045	0·0018	0·0000	0·0000						
30	0·0072	0·0050	0·0020	0·0000	0·0000					
31	0·0100	0·0080	0·0055	0·0022	0·0000	0·0000				
32	0·0130	0·0112	0·0090	0·0062	0·0024	0·0000	0·0000			
33	0·0163	0·0146	0·0125	0·0100	0·0069	0·0027	0·0000	0·0000		
34	0·0198	0·0183	0·0164	0·0141	0·0112	0·0077	0·0030	0·0000	0·0000	
35	0·0237	0·0223	0·0205	0·0184	0·0158	0·0126	0·0086	0·0034	0·0000	0·0000
36	0·0280	0·0267	0·0251	0·0231	0·0207	0·0178	0·0142	0·0097	0·0038	0·0000
37	0·0328	0·0316	0·0301	0·0283	0·0261	0·0233	0·0200	0·0160	0·0109	0·0043
38	0·0382	0·0371	0·0357	0·0340	0·0320	0·0294	0·0264	0·0226	0·0180	0·0123
39	0·0444	0·0433	0·0420	0·0404	0·0385	0·0362	0·0333	0·0298	0·0256	0·0204
40	0·0513	0·0503	0·0491	0·0476	0·0459	0·0437	0·0410	0·0378	0·0338	0·0290
41	0·0592	0·0583	0·0571	0·0558	0·0541	0·0521	0·0496	0·0466	0·0429	0·0384
42	0·0682	0·0673	0·0663	0·0650	0·0635	0·0616	0·0593	0·0565	0·0530	0·0488
43	0·0785	0·0777	0·0768	0·0756	0·0741	0·0724	0·0702	0·0676	0·0644	0·0605
44	0·0905	0·0897	0·0888	0·0877	0·0863	0·0847	0·0827	0·0802	0·0772	0·0735
45	0·1042	0·1035	0·1026	0·1016	0·1003	0·0988	0·0969	0·0946	0·0918	0·0883
46	0·1201	0·1194	0·1186	0·1176	0·1164	0·1150	0·1132	0·1110	0·1084	0·1052
47	0·1385	0·1379	0·1371	0·1362	0·1351	0·1337	0·1320	0·1300	0·1275	0·1245
48	0·1598	0·1593	0·1586	0·1577	0·1566	0·1554	0·1538	0·1519	0·1495	0·1467
49	0·1847	0·1842	0·1835	0·1827	0·1817	0·1805	0·1790	0·1772	0·1750	0·1723
50	0·2138	0·2132	0·2126	0·2118	0·2109	0·2097	0·2084	0·2066	0·2046	0·2020
51	0·2477	0·2472	0·2466	0·2458	0·2450	0·2439	0·2426	0·2409	0·2390	0·2366
52	0·2874	0·2869	0·2863	0·2856	0·2848	0·2838	0·2825	0·2810	0·2791	0·2769
53	0·3339	0·3335	0·3329	0·3323	0·3315	0·3305	0·3293	0·3279	0·3261	0·3240
54	0·3886	0·3882	0·3877	0·3871	0·3863	0·3854	0·3843	0·3829	0·3812	0·3792
55	0·4530	0·4526	0·4522	0·4516	0·4508	0·4500	0·4489	0·4476	0·4460	0·4440
56	0·5290	0·5286	0·5281	0·5276	0·5269	0·5260	0·5250	0·5238	0·5223	0·5204
57	0·6187	0·6183	0·6179	0·6173	0·6167	0·6159	0·6149	0·6137	0·6123	0·6105
58	0·7248	0·7244	0·7240	0·7235	0·7229	0·7221	0·7212	0·7201	0·7187	0·7170
59	0·8506	0·8502	0·8498	0·8494	0·8488	0·8480	0·8471	0·8460	0·8447	0·8431
60	0·9999	0·9996	0·9992	0·9987	0·9982	0·9975	0·9966	0·9956	0·9943	0·9927
61	1·1775	1·1772	1·1768	1·1764	1·1758	1·1751	1·1743	1·1733	1·1721	1·1706
62	1·3890	1·3887	1·3884	1·3879	1·3874	1·3867	1·3860	1·3850	1·3838	1·3824
63	1·6413	1·6411	1·6407	1·6403	1·6398	1·6392	1·6384	1·6375	1·6363	1·6350
64	1·9428	1·9426	1·9422	1·9418	1·9413	1·9407	1·9400	1·9391	1·9380	1·9367

Table E37c. *Sickness rates* z(x,*104/all*,x − x_0). *Twenty-six-week deferred period basis*

Attained age x	Entry age x_0 36	37	38	39	40	41	42	43	44	45
36	0·0000									
37	0·0000	0·0000								
38	0·0048	0·0000	0·0000							
39	0·0139	0·0054	0·0000	0·0000						
40	0·0231	0·0158	0·0061	0·0000	0·0000					
41	0·0329	0·0262	0·0179	0·0069	0·0000	0·0000				
42	0·0437	0·0375	0·0298	0·0203	0·0079	0·0000	0·0000			
43	0·0557	0·0498	0·0427	0·0340	0·0231	0·0089	0·0000	0·0000		
44	0·0691	0·0636	0·0569	0·0487	0·0388	0·0263	0·0102	0·0000	0·0000	
45	0·0841	0·0790	0·0728	0·0651	0·0557	0·0443	0·0301	0·0116	0·0000	0·0000
46	0·1012	0·0964	0·0906	0·0834	0·0746	0·0639	0·0507	0·0344	0·0132	0·0000
47	0·1208	0·1163	0·1107	0·1040	0·0957	0·0856	0·0733	0·0582	0·0394	0·0151
48	0·1432	0·1389	0·1337	0·1274	0·1196	0·1101	0·0985	0·0843	0·0669	0·0452
49	0·1690	0·1650	0·1601	0·1541	0·1468	0·1379	0·1269	0·1135	0·0971	0·0770
50	0·1989	0·1951	0·1905	0·1849	0·1779	0·1695	0·1591	0·1465	0·1310	0·1120
51	0·2336	0·2301	0·2257	0·2203	0·2138	0·2058	0·1960	0·1841	0·1694	0·1515
52	0·2741	0·2707	0·2666	0·2615	0·2553	0·2477	0·2385	0·2271	0·2133	0·1963
53	0·3213	0·3181	0·3142	0·3094	0·3035	0·2963	0·2875	0·2768	0·2636	0·2475
54	0·3767	0·3736	0·3699	0·3653	0·3597	0·3529	0·3446	0·3344	0·3219	0·3066
55	0·4417	0·4388	0·4352	0·4309	0·4256	0·4191	0·4111	0·4014	0·3895	0·3750
56	0·5181	0·5154	0·5120	0·5078	0·5028	0·4966	0·4890	0·4798	0·4685	0·4546
57	0·6083	0·6057	0·6025	0·5985	0·5937	0·5878	0·5806	0·5717	0·5609	0·5477
58	0·7149	0·7124	0·7093	0·7055	0·7009	0·6953	0·6884	0·6799	0·6696	0·6570
59	0·8411	0·8387	0·8357	0·8321	0·8277	0·8223	0·8157	0·8077	0·7978	0·7857
60	0·9908	0·9885	0·9857	0·9822	0·9780	0·9728	0·9665	0·9588	0·9493	0·9377
61	1·1688	1·1666	1·1638	1·1605	1·1565	1·1515	1·1454	1·1380	1·1289	1·1178
62	1·3806	1·3785	1·3759	1·3727	1·3688	1·3640	1·3582	1·3510	1·3423	1·3316
63	1·6333	1·6312	1·6287	1·6256	1·6219	1·6173	1·6117	1·6048	1·5964	1·5861
64	1·9351	1·9331	1·9307	1·9277	1·9241	1·9196	1·9142	1·9076	1·8995	1·8896

Table E37d. *Sickness rates* $z(x,104/all,x-x_0)$. *Twenty-six-week deferred period basis*

Attained age x	Entry age x_0 46	47	48	49	50	51	52	53	54	55
46	0·0000									
47	0·0000	0·0000								
48	0·0174	0·0000	0·0000							
49	0·0520	0·0199	0·0000	0·0000						
50	0·0888	0·0600	0·0229	0·0000	0·0000					
51	0·1295	0·1026	0·0692	0·0264	0·0000	0·0000				
52	0·1755	0·1500	0·1188	0·0800	0·0305	0·0000	0·0000			
53	0·2278	0·2037	0·1741	0·1377	0·0927	0·0353	0·0000	0·0000		
54	0·2878	0·2649	0·2368	0·2023	0·1600	0·1076	0·0409	0·0000	0·0000	
55	0·3572	0·3353	0·3086	0·2758	0·2356	0·1863	0·1251	0·0475	0·0000	0·0000
56	0·4376	0·4168	0·3913	0·3601	0·3218	0·2749	0·2172	0·1458	0·0553	0·0000
57	0·5315	0·5117	0·4873	0·4576	0·4210	0·3762	0·3212	0·2537	0·1701	0·0644
58	0·6415	0·6225	0·5993	0·5708	0·5359	0·4931	0·4406	0·3761	0·2969	0·1989
59	0·7709	0·7527	0·7305	0·7032	0·6698	0·6289	0·5786	0·5169	0·4412	0·3481
60	0·9235	0·9061	0·8848	0·8587	0·8267	0·7874	0·7393	0·6802	0·6076	0·5185
61	1·1041	1·0875	1·0670	1·0419	1·0112	0·9735	0·9273	0·8706	0·8010	0·7155
62	1·3185	1·3025	1·2828	1·2587	1·2292	1·1930	1·1485	1·0940	1·0271	0·9450
63	1·5735	1·5580	1·5391	1·5159	1·4875	1·4526	1·4099	1·3574	1·2930	1·2140
64	1·8774	1·8625	1·8443	1·8219	1·7945	1·7609	1·7197	1·6692	1·6071	1·5309

Table E37e. *Sickness rates* $z(x,104/all,x-x_0)$. *Twenty-six-week deferred period basis*

Attained age x	Entry age x_0 56	57	58	59	60	61	62	63	64
56	0·0000								
57	0·0000	0·0000							
58	0·0752	0·0000	0·0000						
59	0·2330	0·0880	0·0000	0·0000					
60	0·4089	0·2735	0·1031	0·0000	0·0000				
61	0·6104	0·4812	0·3215	0·1211	0·0000	0·0000			
62	0·8440	0·7200	0·5674	0·3787	0·1424	0·0000	0·0000		
63	1·1168	0·9975	0·8508	0·6702	0·4470	0·1679	0·0000	0·0000	
64	1·4373	1·3224	1·1810	1·0071	0·7930	0·5285	0·1982	0·0000	0·0000

CMIR **12** (1991) 227–252

PART F: CALCULATION OF MONETARY FUNCTIONS

SUMMARY

In this Part the calculation of monetary functions, mainly annuity values of various types, is discussed. First, in Section 1, an example of the calculation of 'exact' annuity values using the detailed methods of Part E, is shown. In Sections 2 and 3 the calculation of corresponding annuity values using ζ and z functions is considered, and in Section 4 the use of select tables of z functions and the size of error introduced by their use are discussed. Then in Section 5 the values of '*current claim annuities*' are described, preparatory in Section 6 to describing two ways of calculating the corresponding annuity values using inception rates, of types (*a*) and (*b*), and appropriate average values of current claim annuities. In Section 7 the use of select tables of inception rates is considered. In Section 8 specimen premium rates are shown, and Section 9 contains some concluding observations.

1. ANNUITIES PAYABLE IN A GIVEN STATUS—TOTAL SICKNESS ANNUITIES

1.1 In Section 7 of Part D it was described how the expected present values of annuities payable while in a given status could be calculated to a high degree of accuracy (within the approximations inherent in the whole calculation method); we describe these as the 'exact' values. The expected present values of annuities payable monthly in advance provided that the life is in a particular status at the due date can be calculated exactly. The expected present values of annuities payable continuously can be calculated approximately, to a high degree of accuracy, by the trapezium rule applied over steps of $\frac{1}{156}$ of a year. Continuous annuities approximate closely to annuities payable in strict proportion to the duration of sickness, or to the duration of sickness beyond any deferred period.

It is convenient to calculate the expected present values of annuities for each term of years commencing at a given starting age. We revert, for the purposes of example, to the life who is in the healthy state at age 30, and we calculate annuities for terms of 1 to 35 years, on the 1 week deferred period basis. Table F1 shows the expected present values of annuities payable continuously in each status and Table F2 shows the expected present values of annuities payable monthly in advance provided that the life is in the given status on the due date. Results are shown for the following statuses: healthy; sick (all sickness periods); living (healthy plus sick); and sick during specified sickness periods. If we were to include the status dead and sum the annuities payable in various statuses to give

Table F1. *Present values of annuities payable continuously in each status for each term from starting age: conditional on starting at age 30 with initial status healthy: rate of interest 6%. One-week deferred period basis*

					Sick for given sickness periods								
Term	End age	Healthy	Sick	Living	0/1 wks	1/3 wks	4/9 wks	13/13 wk	26/26 wk	52/52 wk	104/all	0/all	1/all
1	31	0·9628	0·0082	0·9710	0·0038	0·0027	0·0012	0·0003	0·0001	0·0000	0·0000	0·0082	0·0044
2	32	1·8692	0·0169	1·8862	0·0074	0·0055	0·0026	0·0008	0·0004	0·0002	0·0000	0·0169	0·0095
3	33	2·7230	0·0256	2·7486	0·0108	0·0081	0·0041	0·0012	0·0008	0·0005	0·0001	0·0256	0·0148
4	34	3·5272	0·0342	3·5614	0·0139	0·0106	0·0054	0·0017	0·0012	0·0009	0·0004	0·0342	0·0203
5	35	4·2846	0·0426	4·3272	0·0168	0·0131	0·0068	0·0022	0·0015	0·0013	0·0009	0·0426	0·0258
6	36	4·9979	0·0509	5·0489	0·0196	0·0154	0·0082	0·0027	0·0019	0·0017	0·0015	0·0509	0·0314
7	37	5·6696	0·0592	5·7288	0·0221	0·0176	0·0095	0·0032	0·0023	0·0021	0·0023	0·0592	0·0371
8	38	6·3020	0·0673	6·3693	0·0245	0·0198	0·0108	0·0037	0·0028	0·0026	0·0032	0·0673	0·0428
9	39	6·8973	0·0754	6·9727	0·0267	0·0219	0·0121	0·0042	0·0032	0·0030	0·0042	0·0754	0·0486
10	40	7·4576	0·0834	7·5410	0·0288	0·0238	0·0134	0·0047	0·0037	0·0035	0·0054	0·0834	0·0546
11	41	7·9848	0·0913	8·0762	0·0307	0·0257	0·0147	0·0052	0·0041	0·0041	0·0067	0·0913	0·0606
12	42	8·4808	0·0993	8·5801	0·0325	0·0276	0·0160	0·0058	0·0046	0·0046	0·0082	0·0993	0·0668
13	43	8·9473	0·1073	9·0546	0·0342	0·0293	0·0172	0·0063	0·0051	0·0052	0·0099	0·1073	0·0730
14	44	9·3859	0·1152	9·5011	0·0358	0·0310	0·0185	0·0068	0·0056	0·0058	0·0117	0·1152	0·0794
15	45	9·7981	0·1233	9·9214	0·0373	0·0327	0·0197	0·0074	0·0062	0·0064	0·0136	0·1233	0·0859
16	46	10·1854	0·1314	10·3167	0·0387	0·0342	0·0209	0·0080	0·0067	0·0071	0·0157	0·1314	0·0926
17	47	10·5491	0·1395	10·6886	0·0400	0·0358	0·0222	0·0085	0·0073	0·0078	0·0180	0·1395	0·0995
18	48	10·8905	0·1479	11·0383	0·0412	0·0372	0·0234	0·0091	0·0079	0·0085	0·0205	0·1479	0·1066
19	49	11·2107	0·1563	11·3670	0·0424	0·0387	0·0246	0·0097	0·0085	0·0093	0·0232	0·1563	0·1139
20	50	11·5109	0·1650	11·6759	0·0435	0·0400	0·0258	0·0103	0·0091	0·0101	0·0261	0·1650	0·1215

Table F1 *(Continued)*

Term	End age	Healthy	Sick	Living	Sick for given sickness periods 0/1 wks	1/3 wks	4/9 wks	13/13 wk	26/26 wk	52/52 wk	104/all	0/all	1/all
21	51	11·7922	0·1738	11·9660	0·0445	0·0414	0·0270	0·0110	0·0098	0·0109	0·0292	0·1738	0·1293
22	52	12·0554	0·1829	12·2384	0·0455	0·0427	0·0282	0·0116	0·0105	0·0118	0·0326	0·1829	0·1374
23	53	12·3016	0·1923	12·4939	0·0464	0·0440	0·0294	0·0123	0·0113	0·0128	0·0363	0·1923	0·1459
24	54	12·5316	0·2020	12·7336	0·0473	0·0452	0·0306	0·0129	0·0120	0·0138	0·0402	0·2020	0·1547
25	55	12·7462	0·2121	12·9583	0·0481	0·0464	0·0318	0·0136	0·0128	0·0149	0·0444	0·2121	0·1640
26	56	12·9462	0·2225	13·1687	0·0489	0·0476	0·0330	0·0144	0·0137	0·0160	0·0490	0·2225	0·1736
27	57	13·1322	0·2334	13·3656	0·0496	0·0487	0·0342	0·0151	0·0146	0·0173	0·0539	0·2334	0·1838
28	58	13·3049	0·2449	13·5498	0·0503	0·0499	0·0354	0·0159	0·0155	0·0186	0·0593	0·2449	0·1945
29	59	13·4649	0·2568	13·7218	0·0510	0·0510	0·0367	0·0166	0·0165	0·0200	0·0651	0·2568	0·2058
30	60	13·6129	0·2694	13·8823	0·0517	0·0521	0·0379	0·0175	0·0175	0·0214	0·0713	0·2694	0·2178
31	61	13·7492	0·2827	14·0319	0·0523	0·0532	0·0392	0·0183	0·0186	0·0230	0·0781	0·2827	0·2304
32	62	13·8745	0·2967	14·1712	0·0529	0·0542	0·0405	0·0192	0·0197	0·0247	0·0855	0·2967	0·2439
33	63	13·9890	0·3115	14·3006	0·0534	0·0553	0·0418	0·0201	0·0210	0·0265	0·0935	0·3115	0·2581
34	64	14·0934	0·3273	14·4206	0·0540	0·0563	0·0431	0·0210	0·0222	0·0285	0·1021	0·3273	0·2733
35	65	14·1878	0·3439	14·5318	0·0545	0·0573	0·0444	0·0220	0·0236	0·0305	0·1116	0·3439	0·2895

Table F2. *Present values of annuities payable monthly in advance in each status for each term from starting age: conditional on starting at age 30 with initial status healthy: rate of interest 6%. One-week deferred period basis*

	End				Sick for given sickness periods								
Term	age	Healthy	Sick	Living	0/1 wks	1/3 wks	4/9 wks	13/13 wk	26/26 wk	52/52 wk	104/all	0/all	1/all
1	31	0·9657	0·0077	0·9734	0·0035	0·0026	0·0012	0·0003	0·0001	0·0000	0·0000	0·0077	0·0041
2	32	1·8744	0·0164	1·8908	0·0071	0·0054	0·0026	0·0007	0·0004	0·0002	0·0000	0·0164	0·0093
3	33	2·7303	0·0251	2·7554	0·0105	0·0080	0·0040	0·0012	0·0008	0·0005	0·0001	0·0251	0·0146
4	34	3·5365	0·0337	3·5702	0·0136	0·0105	0·0054	0·0017	0·0011	0·0009	0·0004	0·0337	0·0200
5	35	4·2958	0·0421	4·3379	0·0166	0·0130	0·0067	0·0022	0·0015	0·0013	0·0009	0·0421	0·0255
6	36	5·0109	0·0504	5·0614	0·0193	0·0153	0·0081	0·0027	0·0019	0·0017	0·0015	0·0504	0·0311
7	37	5·6843	0·0587	5·7430	0·0219	0·0175	0·0094	0·0032	0·0023	0·0021	0·0022	0·0587	0·0368
8	38	6·3183	0·0668	6·3851	0·0243	0·0197	0·0108	0·0037	0·0028	0·0026	0·0031	0·0668	0·0426
9	39	6·9151	0·0749	6·9900	0·0265	0·0218	0·0121	0·0042	0·0032	0·0030	0·0042	0·0749	0·0484
10	40	7·4768	0·0829	7·5597	0·0286	0·0238	0·0134	0·0047	0·0036	0·0035	0·0054	0·0829	0·0543
11	41	8·0053	0·0909	8·0962	0·0305	0·0257	0·0147	0·0052	0·0041	0·0040	0·0067	0·0909	0·0603
12	42	8·5026	0·0988	8·6014	0·0323	0·0275	0·0159	0·0057	0·0046	0·0046	0·0082	0·0988	0·0665
13	43	8·9703	0·1068	9·0771	0·0340	0·0293	0·0172	0·0063	0·0051	0·0051	0·0098	0·1068	0·0727
14	44	9·4100	0·1148	9·5248	0·0356	0·0310	0·0184	0·0068	0·0056	0·0057	0·0116	0·1148	0·0791
15	45	9·8233	0·1228	9·9461	0·0371	0·0326	0·0197	0·0074	0·0061	0·0064	0·0135	0·1228	0·0857
16	46	10·2116	0·1309	10·3425	0·0385	0·0342	0·0209	0·0079	0·0067	0·0070	0·0156	0·1309	0·0924
17	47	10·5762	0·1391	10·7153	0·0398	0·0357	0·0221	0·0085	0·0073	0·0077	0·0179	0·1391	0·0992
18	48	10·9185	0·1474	11·0659	0·0411	0·0372	0·0233	0·0091	0·0079	0·0085	0·0204	0·1474	0·1063
19	49	11·2396	0·1558	11·3954	0·0422	0·0386	0·0245	0·0097	0·0085	0·0092	0·0231	0·1558	0·1136
20	50	11·5406	0·1645	11·7051	0·0433	0·0400	0·0257	0·0103	0·0091	0·0100	0·0260	0·1645	0·1211

Table F2 *(Continued)*

	End				Sick for given sickness periods								
Term	age	Healthy	Sick	Living	0/1 wks	1/3 wks	4/9 wks	13/13 wk	26/26 wk	52/52 wk	104/all	0/all	1/all
21	51	11·8227	0·1733	11·9960	0·0443	0·0413	0·0269	0·0109	0·0098	0·0109	0·0291	0·1733	0·1290
22	52	12·0867	0·1824	12·2691	0·0453	0·0426	0·0281	0·0116	0·0105	0·0118	0·0325	0·1824	0·1371
23	53	12·3336	0·1918	12·5253	0·0462	0·0439	0·0293	0·0122	0·0112	0·0128	0·0361	0·1918	0·1455
24	54	12·5642	0·2014	12·7657	0·0471	0·0451	0·0305	0·0129	0·0120	0·0138	0·0400	0·2014	0·1543
25	55	12·7794	0·2115	12·9909	0·0479	0·0464	0·0317	0·0136	0·0128	0·0149	0·0442	0·2115	0·1636
26	56	12·9800	0·2219	13·2019	0·0487	0·0475	0·0329	0·0143	0·0136	0·0160	0·0488	0·2219	0·1732
27	57	13·1666	0·2328	13·3994	0·0495	0·0487	0·0341	0·0151	0·0145	0·0172	0·0537	0·2328	0·1834
28	58	13·3398	0·2442	13·5840	0·0502	0·0498	0·0354	0·0158	0·0154	0·0185	0·0591	0·2442	0·1941
29	59	13·5004	0·2562	13·7566	0·0508	0·0509	0·0366	0·0166	0·0164	0·0199	0·0648	0·2562	0·2053
30	60	13·6488	0·2687	13·9175	0·0515	0·0520	0·0379	0·0174	0·0175	0·0214	0·0711	0·2687	0·2173
31	61	13·7856	0·2820	14·0676	0·0521	0·0531	0·0391	0·0183	0·0185	0·0230	0·0778	0·2820	0·2299
32	62	13·9113	0·2960	14·2073	0·0527	0·0542	0·0404	0·0191	0·0197	0·0246	0·0852	0·2960	0·2433
33	63	14·0263	0·3108	14·3371	0·0533	0·0552	0·0417	0·0200	0·0209	0·0265	0·0931	0·3108	0·2575
34	64	14·1311	0·3264	14·4575	0·0538	0·0563	0·0430	0·0210	0·0222	0·0284	0·1018	0·3264	0·2726
35	65	14·2259	0·3431	14·5690	0·0543	0·0573	0·0444	0·0219	0·0235	0·0305	0·1112	0·3431	0·2888

the value of an annuity payable in any status, this would equal the present value of an annuity certain; such a calculation has been carried out and it provides a useful check on the calculation method.

In practice continuous annuities are applicable to benefits payable during periods of sickness, and annuities payable monthly in advance are applicable to premiums, which are normally paid while not claiming, i.e. either healthy or sick but within the deferred period.

2. APPROXIMATE CALCULATION OF ANNUITY VALUES USING ζ FUNCTIONS

2.1 The annuity values discussed in Section 1 have been described as calculated exactly, but using calculation steps of $\frac{1}{156}$ of a year. Calculation with smaller steps, which would provide any desired greater accuracy, would be possible, but not computationally worth while. We now turn to a series of methods for obtaining present values of annuities by simpler approximate calculations, and we discuss the degree of approximation involved.

2.2 We first consider formulae using the ζ functions defined in Section 6.1 of Part D, which are discussed further in Section 3 of Part E, and for which specimen values are shown in Table E16. We also need the life table discussed in Section 4 of Part E; the values of l_x for this are shown in Table E17.

We start with the calculation of the present value of an annuity payable continuously while alive to a life who was healthy at age x_0, for a maximum term of n years, i.e. to a maximum age of x_0+n years. In accordance with the notation introduced in Section 7.2 of Part D we give this the symbol

$$\bar{a}^{HL}_{x_0:\overline{n}|}$$

and we calculate it using the approximate formula

$$\bar{a}^{HL}_{x_0:\overline{n}|} \doteqdot \tfrac{1}{2} + \sum_{t=1}^{n-1} v^t \frac{l_{x_0+t}}{l_{x_0}} + \tfrac{1}{2} v^n \frac{l_{x_0+n}}{l_{x_0}} \tag{1}$$

For the exact calculations above we have used the same trapezium rule but with steps of $\frac{1}{156}$ of a year.

The expected present value of an annuity payable continuously while sick, during sickness period a/b, conditional on being healthy at age x_0, can be denoted by the function on the left of the approximation below, and calculated by the expression on the right.

$$\bar{a}^{HS(a/b)}_{x_0:\overline{n}|} \doteqdot \tfrac{1}{2}\zeta\,(x_0,a/b)/52 + \sum_{t=1}^{n-1} v^t \frac{l_{x_0+t}}{l_{x_0}}\zeta\,(x_0+t,a/b)/52$$

$$+ \tfrac{1}{2} v^n \frac{l_{x_0+n}}{l_{x_0}}\zeta\,(x_0+n,a/b)/52 \tag{2}$$

Note that, for symmetry, we show the first term in the calculation as $\frac{1}{2}\zeta\,(x_0,a/b)/52$, but in practice the value of this is zero, since it is assumed that all those at x_0 are healthy.

The expected present value of an annuity payable continuously during all sickness periods can be calculated as the summation of the appropriate annuities payable during each sickness period:

$$\bar{a}^{HS}_{x_0:\overline{n}|} = \Sigma\,\bar{a}^{HS\,(\text{period})}_{x_0:\overline{n}|} \tag{3}$$

where the summation is over all relevant periods.

The expected present value of an annuity payable whilst healthy can be calculated by subtraction of the annuity payable while sick from the annuity payable while alive as:

$$\bar{a}^{HH}_{x_0:\overline{n}|} = \bar{a}^{HL}_{x_0:\overline{n}|} - \bar{a}^{HS}_{x_0:\overline{n}|} \tag{4}$$

2.3 Extensive calculations using these approximations were made. The method of calculating the annuities while living, $\bar{a}^{HL}_{x_0:\overline{n}|}$, is satisfactory, producing errors, as compared with the more exact method, of no more than 3 parts in 10,000. The formula for sickness annuities, $\bar{a}^{HS(a/b)}_{x_0:\overline{n}|}$, was found to be unsatisfactory. The problem arises in the first year. For example, for sickness period 0/1 weeks, the value of ζ at the start of the first year is zero, but the average value of ζ is nearly the same as the value at the end of the first year. For a term of one year, therefore, the calculated sickness annuities fall substantially short of the exact values, by as much as 50%; the same absolute error persists for longer durations.

Rather than make special adjustments for the first year it seems better to use the sickness rates, the z functions as described in Section 3.

2.4 Annuities payable monthly in advance can be calculated using approximations such as

$$\ddot{a}^{HL(12)}_{x_0:\overline{n}|} \doteqdot \frac{13}{24} + \sum_{t=1}^{n-1} v^t\,\frac{l_{x_0+t}}{l_{x_0}} + \frac{11}{24}\,v^n\,\frac{l_{x_0+n}}{l_{x_0}} \tag{5}$$

$$\doteqdot \bar{a}^{HL}_{x_0:\overline{n}|} + \frac{1}{24}\left(1 - v^n\,\frac{l_{x_0+n}}{l_{x_0}}\right) \tag{6}$$

and

$$\ddot{a}^{HS(a/b)(12)}_{x_0:\overline{n}|} \doteqdot \bar{a}^{HS(a/b)}_{x_0:\overline{n}|} - \frac{1}{24}\,v^n\,\frac{l_{x_0+n}}{l_{x_0}}\,\zeta\,(x_0+n,a/b)/52 \tag{7}$$

Calculations using these formula show that the approximation for $\ddot{a}^{HL(12)}_{x_0:\overline{n}|}$ is just as good as for the continuous annuity, and that the approximation for $\ddot{a}_{x_0:\overline{n}|}^{HS(a/b)(12)}$ is just as bad as that for $\bar{a}_{x_0:\overline{n}|}^{HS(a/b)}$.

This shows, however, that the adjustment to convert from continuous to monthly payments is satisfactory.

3. APPROXIMATE CALCULATION OF ANNUITY VALUES USING MANCHESTER-UNITY-STYLE SICKNESS RATES

3.1 Rather than use the ζ functions used in Section 2 we can carry out similar calculations using the sickness rates z. The ζ functions are available for integral ages, whereas the z functions apply over a year of age: $(x, x+1)$, $(x+1, x+2)$, etc. The formulae required are therefore slightly different.

3.2 Again we start with the calculation of the expected present value of an annuity payable while alive to a life who is healthy at age x_0, for a maximum term of n years, i.e. to a maximum age of x_0+n years. We can calculate this using the approximate formula

$$\bar{a}_{x_0:\overline{n}|}^{HL} \doteqdot \sum_{t=0}^{n-1} v^{t+\frac{1}{2}} \frac{L_{x_0+t}}{l_{x_0}} \tag{8}$$

where

$$L_{x+t} \doteqdot \tfrac{1}{2}(l_{x+t} + l_{x+t+1}) \tag{9}$$

The expected present value of an annuity payable continuously while sick, during sickness period a/b, conditional on being healthy at age x_0, can be calculated approximately by the familiar formula

$$\bar{a}_{x_0:\overline{n}|}^{HS(a/b)} \doteqdot \sum_{t=0}^{n-1} v^{t+\frac{1}{2}} \frac{L_{x_0+t}}{l_{x_0}} z(x_0+t, a/b)/52 \tag{10}$$

It will be seen that this is equivalent to the traditional 'Manchester-Unity' type of formula.

As before, the expected present value of an annuity payable continuously in all sickness periods can be calculated as the sum of the appropriate annuities payable during each sickness period, using formula (3).

The expected present value of an annuity payable whilst healthy can be calculated as before by subtraction of the annuity payable while sick from the annuity payable while living, using formula (4).

3.3 Annuities payable monthly in advance can be calculated using formula (6) for the annuity payable whilst living, and by

$$\ddot{a}_{x_0:\overline{n}|}^{HS(a/b)(12)} \doteqdot \bar{a}_{x_0:\overline{n}|}^{HS(a/b)} - \frac{1}{24} v^n \frac{l_{x_0+n}}{l_{x_0}} \left(z(x_0+n, a/b) + z(x_0+n+1, a/b)\right)/104 \tag{11}$$

for the annuity payable during sickness period a/b. The term involving the average of two z's is a substitute for the value of ζ at the terminal age of annuity; i.e. the adjustment is the same as in formula (7). At the end of a table the value of $z(x_0+n+1,a/b)$ may not be available. In that case the approximation given by extrapolation: $2z(x_0+n,a/b)-z(x_0+n-1,a/b)$ could be used instead.

Note that all the above are conditional on being healthy at age x_0, and are calculated using the full table of sickness rates conditional on that entry age.

3.4 Extensive calculations show that the approximations for $\bar{a}^{HL}_{x_0:\overline{n}|}$ and $\ddot{a}^{HL(12)}_{x_0:\overline{n}|}$ are only a little less accurate than those derived from the similar, but slightly different, formula (1). The errors are generally very small, but at high ages of entry rise to nearly 1 part per 1,000.

The values of $\bar{a}^{HS(a/b)}_{x_0:\overline{n}|}$ are also very close to the exact values, being generally less than 1 part in 1,000 different from the exact value, and often very much closer. The exceptions occur for short duration and higher sickness periods, where the percentage errors approach 2% at the maximum; the absolute values, however, are very small, and the errors are always less than 0·001 in the annuity value per unit annual benefit.

The values of $\ddot{a}^{HS(a/b)(12)}_{x_0:\overline{n}|}$ calculated using formula (11) are a little less accurate than those of the corresponding continuous annuities, with errors generally less than 1 part in 1,000, except for the cases where the continuous annuity is less accurate, and for sickness period 0/1 week, where the error may be as large as 1% (the approximation being too high) at short durations.

4. APPROXIMATE CALCULATION OF ANNUITY VALUES USING SELECT TABLES OF SICKNESS RATES

4.1 Instead of using the full set of tables of sickness rates as discussed in Section 3, we can alternatively make use of the select tables of sickness rates constructed in Section 9 of Part E. In the specimen calculations below we assume a 5-year select period, both for the life table and for the sickness rates; we have used the select table calculated using Methods *A* and *B* of Section 9 of Part E. One must not expect this method to be satisfactory for calculating the values of annuities payable during sickness period 104/all, for the reason already explained in Section 9 of Part E.

The formulae exactly parallel those of Section 3, except that the functions are picked from the appropriate select table.

Thus we have approximately

$$\bar{a}^{HL}_{[x_0]:\overline{n}|} \doteqdot \sum_{t=0}^{n-1} v^{t+\frac{1}{2}} \frac{L_{[x_0]+t}}{l_{[x_0]}} \tag{12}$$

where
$$L_{[x_0]+t} \doteqdot \tfrac{1}{2}(l_{[x_0]+t} + l_{[x_0]+t+1}) \tag{13}$$

$$\bar{a}^{HS(a/b)}_{[x_0]:\overline{n}|} \doteqdot \sum_{t=0}^{n-1} v^{t+\frac{1}{2}} \frac{L_{[x_0]+t}}{l_{[x_0]}} \cdot z^{a/b}_{[x_0]+t}/52 \tag{14}$$

and

$$\ddot{a}^{HS(a/b)(12)}_{[x_0]:\overline{n}|} \doteqdot \bar{a}^{HS(a/b)}_{[x_0]:\overline{n}|} - \frac{1}{24} v^n \frac{l_{[x_0]+n}}{l_{[x_0]}} (z^{a/b}_{[x_0]+n} + z^{a/b}_{[x_0]+n+1})/104 \tag{15}$$

4.2 Calculations using this method show very satisfactory results. Values of $\bar{a}^{HL}_{x_0:\overline{n}|}$ have about the same proportionate error as those calculated using formula (8), but in the other direction, being slightly lower than the exact values rather than slightly higher.

Values of $\bar{a}^{HS(a/b)}_{x_0:\overline{n}|}$ using Method *A* are slightly lower than those using formula (10), i.e. z's with an unlimited select period, and those using Method *B* are higher. Since formula (10) gives results a little higher than the exact values, Method *A* gives results that are in some cases closer to the exact values than those from formula (10), whereas the results on Method *B* are always further away from the exact values, the error sometimes being up to 2% too high.

As already noted, a short select period table is unsatisfactory for calculating values for 104/all (or for any unlimited period contained within 104/all, e.g. 156/all), and is also unsatisfactory for any longer sickness period that includes this sickness period. For example, for sickness period 1/all the results using Method A with a 5-year select period for 104/all would be up to 5% too high, and using Method B would be up to 20% too low.

5. VALUES OF CURRENT CLAIM ANNUITIES

5.1 We now consider "current claim annuities", i.e. annuities payable only during the current period of sickness, commencing when someone is already sick. We distinguish these from "future claim annuities", i.e. annuities payable during future periods of sickness, either to someone who is at present healthy, for whom the current claim annuity value is zero, or to someone who is at present sick, but in respect of future sickness periods, not the current one.

Current claim annuities have already been introduced in Section 7 of Part D. They can be readily calculated. We first introduce the notation $\pi(x,z_1,z_2)$ to represent the probability that an individual currently aged x and sick with exact duration z_1 continues while remaining sick to exact duration z_2 ($z_2 \geqslant z_1$) (the same as ${}_{z_2-z_1}p^{\overline{SS}}_{x,z}$). We have already introduced the notation $\pi_{x,z}$ in Section 10 of Part E to represent the probability that a life who has just become sick at age x continues while sick to duration z, so that we can treat $\pi_{x,z}$ as an abbreviation for $\pi(x,0,z)$.

5.2 We can now calculate the expected present value of an annuity payable continuously to someone who is currently aged x and sick with exact duration of sickness z, payable so long as he remains sick, for a maximum duration of n, which has been defined in Section 7 of Part D as

$$\bar{a}^{\overline{SS}}_{x,z:\overline{n}|}$$

An exact expression for this is given by

$$\bar{a}^{\overline{SS}}_{x,z:\overline{n}|} = \int_0^n v^t\,\pi(x,z,z+t)dt \tag{16}$$

where v is the usual discounting factor.

It is possible to calculate this function as accurately as one likes by a number of methods, usually requiring successive approximation. For example, one can calculate values of $\pi(x,z,z+t)$ from the formula

$$\pi(x,z,z+t) = \exp\left(-\int_0^t \rho_{x+u,z+u} + \nu_{x+u,z+u}du\right) \tag{17}$$

and calculate the integral in the formula by an approximate integration formula such as the trapezium rule or Simpson's rule, each value being calculated to any desired accuracy by reducing the step size until successive approximations are close enough together. This method can be used for any required value of t. One can then calculate the integral

$$\int_0^n v^t\,\pi(x,z,z+t)dt$$

also by approximate integration, using the same, or a different approximation method, and the same or a different step size. It may be convenient to use the same step size in the two approximations and to combine the process to calculate values of $\bar{a}^{\overline{SS}}_{x,z:\overline{n}|}$ for a given combination of x and z and for various values of n, e.g. $n = 1, 2, 3 \ldots$ up to $x + n = 65$ say, all in the same sequence of approximations.

5.3 An alternative method, which is consistent with the calculations in Part E and in the earlier section of this Part is to calculate values of $\pi(x,z,z+t)$ by the recursive approximate formula:

$$\pi(x,z,z+u+h) \doteqdot \pi(x,z,z+u)\frac{(1-\frac{1}{2}h(\rho_{x+u,z+u}+\nu_{x+u,z+u}))}{(1+\frac{1}{2}h(\rho_{x+u+h,z+u+h}+\nu_{x+u+h,z+u+h}))} \tag{18}$$

with $\pi_{(x,z,z)} = 1$, and steps of $h = \frac{1}{156}$ of a year, and then to calculate $\bar{a}^{\overline{SS}}_{x,z:\overline{n}|}$ using the trapezium rule, also using steps of $h = \frac{1}{156}$ of a year:

$$\bar{a}^{\overline{SS}}_{x,z:\overline{n}|} \doteqdot \frac{h}{2} + \sum_{j=1}^{n/h-1} hv^t\,\pi(x,z,z+jh) + \frac{h}{2}v^n\,\pi(x,z,z+n) \tag{19}$$

Table F3. *Present values of continuous current claim annuities of 1 per year ceasing at age 65: rate of interest 6%. One-week deferred period basis*

		Duration in weeks									
		0	1	4	13	26	52	104	156	208	260
Age	Term										
16	49	0·0206	0·0296	0·1036	0·3990	1·1376	2·4930	4·2595	5·4875	6·1210	6·2988
17	48	0·0211	0·0307	0·1072	0·4161	1·1782	2·5574	4·3348	5·5604	6·1898	6·3662
18	47	0·0216	0·0318	0·1111	0·4342	1·2205	2·6237	4·4116	5·6342	6·2594	6·4342
19	46	0·0222	0·0331	0·1153	0·4533	1·2646	2·6919	4·4897	5·7088	6·3296	6·5029
20	45	0·0229	0·0344	0·1198	0·4736	1·3106	2·7621	4·5692	5·7842	6·4004	6·5720
21	44	0·0236	0·0358	0·1246	0·4951	1·3585	2·8343	4·6499	5·8604	6·4716	6·6415
22	43	0·0243	0·0374	0·1298	0·5179	1·4085	2·9085	4·7318	5·9372	6·5432	6·7113
23	42	0·0251	0·0390	0·1353	0·5420	1·4606	2·9846	4·8149	6·0145	6·6150	6·7813
24	41	0·0260	0·0409	0·1413	0·5675	1·5148	3·0627	4·8990	6·0922	6·6869	6·8512
25	40	0·0270	0·0428	0·1477	0·5944	1·5712	3·1427	4·9841	6·1701	6·7587	6·9210
26	39	0·0280	0·0450	0·1546	0·6230	1·6298	3·2247	5·0700	6·2481	6·8302	6·9904
27	38	0·0292	0·0473	0·1619	0·6532	1·6908	3·3085	5·1565	6·3259	6·9012	7·0593
28	37	0·0304	0·0498	0·1699	0·6851	1·7540	3·3941	5·2435	6·4034	6·9714	7·1272
29	36	0·0318	0·0526	0·1785	0·7188	1·8197	3·4814	5·3308	6·4802	7·0407	7·1941
30	35	0·0333	0·0557	0·1877	0·7544	1·8877	3·5702	5·4180	6·5561	7·1085	7·2594
31	34	0·0350	0·0590	0·1976	0·7919	1·9580	3·6604	5·5051	6·6307	7·1746	7·3230
32	33	0·0368	0·0626	0·2082	0·8314	2·0307	3·7519	5·5915	6·7036	7·2386	7·3843
33	32	0·0388	0·0666	0·2197	0·8730	2·1056	3·8443	5·6769	6·7745	7·3001	7·4429
34	31	0·0410	0·0710	0·2319	0·9168	2·1827	3·9375	5·7610	6·8427	7·3584	7·4983
35	30	0·0435	0·0758	0·2451	0·9627	2·2619	4·0311	5·8432	6·9079	7·4131	7·5499
36	29	0·0462	0·0811	0·2593	1·0108	2·3430	4·1246	5·9230	6·9693	7·4635	7·5970
37	28	0·0492	0·0868	0·2744	1·0610	2·4259	4·2178	5·9997	7·0263	7·5089	7·6391
38	27	0·0525	0·0932	0·2906	1·1135	2·5102	4·3099	6·0728	7·0781	7·5486	7·6752
39	26	0·0562	0·1002	0·3078	1·1680	2·5956	4·4006	6·1413	7·1239	7·5816	7·7046

Table F3 *(Continued)*

		Duration in weeks									
		0	1	4	13	26	52	104	156	208	260
Age	Term										
40	25	0·0603	0·1078	0·3262	1·2246	2·6818	4·4889	6·2044	7·1628	7·6070	7·7262
41	24	0·0649	0·1162	0·3458	1·2830	2·7682	4·5743	6·2612	7·1937	7·6239	7·7391
42	23	0·0699	0·1253	0·3665	1·3432	2·8544	4·6557	6·3106	7·2155	7·6309	7·7420
43	22	0·0754	0·1353	0·3884	1·4047	2·9396	4·7321	6·3512	7·2270	7·6270	7·7338
44	21	0·0815	0·1462	0·4114	1·4673	3·0230	4·8023	6·3819	7·2267	7·6106	7·7129
45	20	0·0883	0·1581	0·4355	1·5305	3·1036	4·8651	6·4011	7·2131	7·5803	7·6780
46	19	0·0957	0·1709	0·4606	1·5939	3·1804	4·9189	6·4071	7·1846	7·5343	7·6273
47	18	0·1038	0·1847	0·4865	1·6566	3·2521	4·9619	6·3981	7·1393	7·4710	7·5590
48	17	0·1126	0·1995	0·5131	1·7179	3·3170	4·9923	6·3721	7·0753	7·3883	7·4712
49	16	0·1221	0·2153	0·5400	1·7768	3·3736	5·0080	6·3269	6·9903	7·2840	7·3617
50	15	0·1324	0·2319	0·5670	1·8320	3·4198	5·0064	6·2599	6·8820	7·1558	7·2282
51	14	0·1433	0·2494	0·5936	1·8820	3·4532	4·9849	6·1686	6·7477	7·0011	7·0681
52	13	0·1549	0·2674	0·6193	1·9251	3·4711	4·9405	6·0498	6·5846	6·8173	6·8787
53	12	0·1670	0·2858	0·6432	1·9593	3·4706	4·8697	5·9004	6·3897	6·6012	6·6570
54	11	0·1793	0·3041	0·6645	1·9819	3·4482	4·7688	5·7169	6·1595	6·3497	6·3998
55	10	0·1917	0·3217	0·6820	1·9902	3·3998	4·6337	5·4952	5·8904	6·0591	6·1036
56	9	0·2035	0·3379	0·6943	1·9807	3·3210	4·4596	5·2312	5·5785	5·7257	5·7646
57	8	0·2144	0·3518	0·6998	1·9493	3·2066	4·2414	4·9202	5·2195	5·3455	5·3787
58	7	0·2234	0·3620	0·6963	1·8914	3·0508	3·9733	4·5571	4·8087	4·9138	4·9417
59	6	0·2295	0·3669	0·6813	1·8015	2·8470	3·6490	4·1363	4·3411	4·4261	4·4486
60	5	0·2312	0·3643	0·6517	1·6733	2·5878	3·2614	3·6518	3·8113	3·8770	3·8945
61	4	0·2267	0·3515	0·6037	1·4992	2·2647	2·8029	3·0970	3·2134	3·2609	3·2738
62	3	0·2134	0·3246	0·5325	1·2699	1·8673	2·2643	2·4648	2·5410	2·5720	2·5807
63	2	0·1875	0·2786	0·4314	0·9725	1·3815	1·6336	1·7469	1·7872	1·8037	1·8086
64	1	0·1429	0·2044	0·2889	0·5853	0·7839	0·8925	0·9315	0·9441	0·9491	0·9509

5.4 Table F3 shows approximate values of current claim annuities, payable continuously, at 6% interest, ceasing at age 65, for integral values of x from 16 to 64, and for values of z of 0, 1, 4, 13, 26, 52 and 104 weeks calculated using the method just described. The values of these annuities have also been calculated using the more exact method described above. In general the approximate method produces values a little lower than the exact method. For $z=0$, i.e. annuities starting immediately a period of sickness starts, the error is as much as 4%, but for higher values of z the error is much smaller, being no more than about 0·5% for $z=1$ week, and much less than this for high values of z. The error is greater at those durations of sickness when the recovery rates change rapidly with duration.

6. APPROXIMATE CALCULATION OF ANNUITY VALUES USING INCEPTION RATES AND VALUES OF CURRENT CLAIM ANNUITIES

6.1 An alternative approach to the calculation of the expected present values of payments during sickness is by the use of inception rates and current claim annuity values. The difference between this method and the sickness rate method can be demonstrated in Figures F1, F2 and F3. All of these use the same format as Figure E1. They relate to a life who is healthy at age x_0 at point A, and to a benefit that commences after a 13-week deferred period. The duration of the policy is restricted in the figures to four years.

In all three figures the whole shaded area represents the region within which sickness benefit may be payable. In Figure F1 this is subdivided into mainly rectangular regions by horizontal and vertical lines. The horizontal lines represent years of age passed through, and the vertical lines represent different sickness durations passed through. Sickness periods 13/13, 26/26, 52/52 and 104/all are represented. To each rectangle there is a corresponding $z(x,a/b)$, and the present value of sickness benefit is calculated first by multiplying these $z(x,a/b)$ functions by an appropriate ratio $L(x)/l(x_0)$ and a discount factor, then by summing in columns to give the values of sickness benefit payable during a particular sickness period, and then by summing across sickness periods.

A second approach is shown in Figure F2. The shaded area is subdivided into diagonal stripes. Once a claim commences by the period of sickness passing across the 13-week claim inception boundary, the line representing this period of sickness remains within a single diagonal stripe until the life either recovers, dies or reaches the end of the period of cover, which in this case is age x_0+4. The inception rates of type (a), $ia(x,d)$ discussed in Section 5 of Part E, represent the proportion of lives among the L_x survivors who cross the line for duration d between ages x and $x+1$ at each age, and the current claim annuities, commencing at duration d, for an appropriate average age, represent the values of benefit within each diagonal stripe.

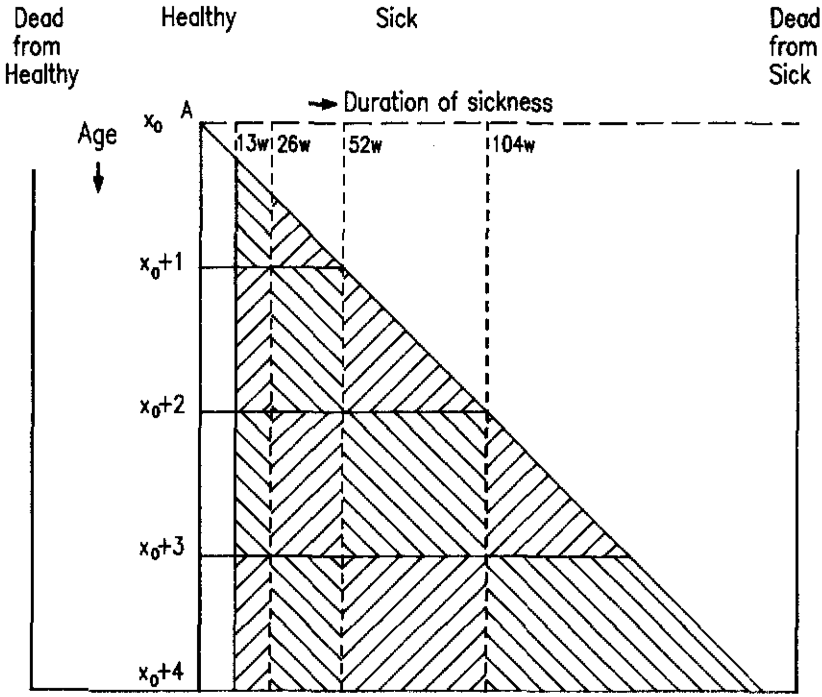

Figure F1. Calculating sickness benefit using sickness rates.

It can be noted that the stripes are not of equal width. The top stripe is only 39/52 or 3/4 of a year wide. This occurs because a life who is healthy at age x_0 cannot reach 13 weeks of sickness until at least age $x_0 + 13/52$. With a 26-week deferred period the top stripe would only be half the width of the lower ones; for a 52-week deferred period the top stripe would disappear to nothing, and for longer deferred periods the second or lower stripes would also be affected.

A third way of subdividing the shaded region is shown in Figure F3, in which the diagonal lines start at the integral ages on the vertical line representing healthy. In this case the higher stripes are all the same width, representing one year, and the lowest one is narrower, in this case only 3/4 of a year wide. For this partitioning one uses the claim inception rates of type (*b*), $ib(x,d)$ discussed in Section 5 of Part E.

This third approach may be useful when calculating 1-year cover sickness benefit for group PHI policies, where the cover granted is for sickness that commences in the year of age $(x_0, x_0 + 1)$ and continues for a duration of at least d, the claim annuity then being payable up to some maximum age.

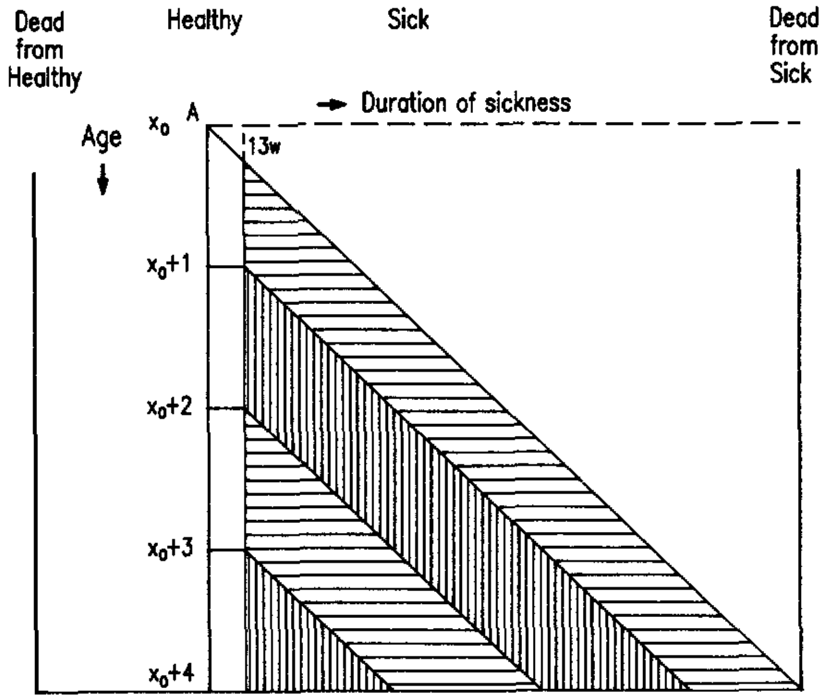

Figure F2. Calculating sickness benefit using inception rates of type (*a*).

6.2 In Section 5 of Part D it was shown how the inception rates $ia(x,d)$ were derived from the numbers of claim inceptions $ca(x,d)$. The latter can be reconstructed from the former by the formula

$$ca(x,d) = ia(x,d) \cdot L_x \tag{20}$$

and the alternative claim inceptions $cb(x,d)$ can be approximated by

$$cb(x,d) \doteqdot (1-d)ia(x,d) \cdot L_x + d \cdot ia(x+1,d) \cdot L_{x+1} \tag{21}$$

It will be noted that for the partitioning in Figure F2, appropriate to the inception rates of type (*a*) the line representing deferred period d (13 weeks in the figure) is subdivided into intervals (x_0+d,x_0+1), (x_0+1,x_0+2), (x_0+2,x_0+3), (x_0+3,x_0+4), whereas in Figure F3, appropriate to inception rate of type (*b*), the corresponding line is subdivided into intervals (x_0+d,x_0+1+d), (x_0+1+d,x_0+2+d), (x_0+2+d,x_0+3+d),(x_0+3+d,x_0+4). These can be

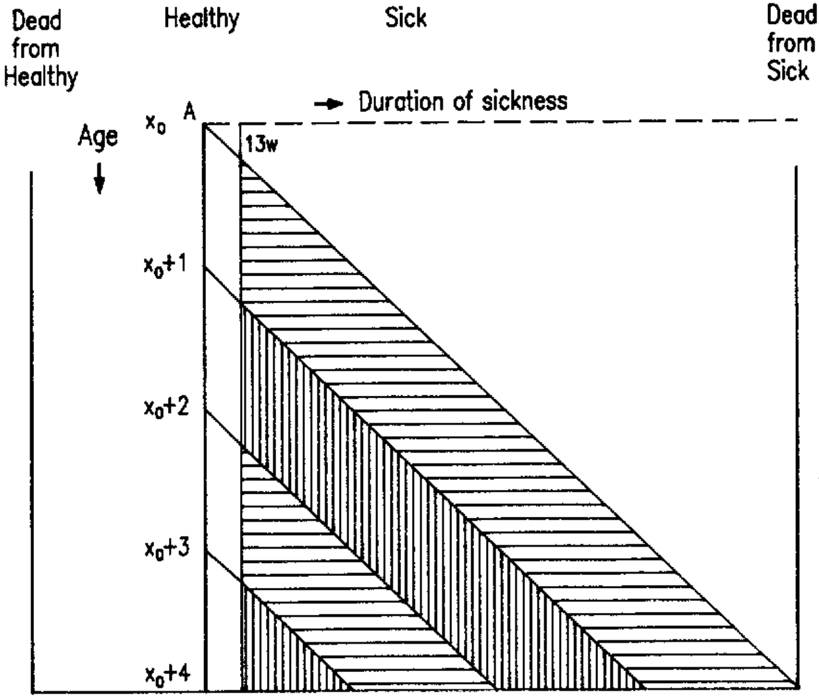

Figure F3. Calculating sickness benefit using inception rates of type (*b*).

combined, as shown in Figure F4 where the subdivision is (x_0+d, x_0+1), (x_0+1, x_0+1+d), (x_0+1+d, x_0+2), etc. with sections alternatively d and $(1-d)$ wide, and labelled alternately '2' and '1'.

6.3 We define two types of average annuity value, each being the average of current claim annuities, commencing at duration d, but the first averaging annuities commencing between ages x and $x+d$, and the second averaging annuities commencing between ages $x+d$ and $x+1$, in both cases for integral values of x, and ceasing at age $x+n$. We define these for $0<d<52$ weeks as

$$\bar{a}^1_{x,d:\overline{n}|} \doteqdot \int_0^d \bar{a}^{\overline{SS}}_{x+t,d:\overline{n-t}|}\, dt/d \tag{22}$$

and

$$\bar{a}^2_{x,d:\overline{n}|} \doteqdot \int_d^1 \bar{a}^{\overline{SS}}_{x+t,d:\overline{n-t}|}\, dt/(1-d) \tag{23}$$

Table F4. *Average values of continuous current claim annuities of 1 per year, payable while sick commencing at given age and given duration of sickness and ceasing at age 65: types (1) and (2): rate of interest 6%. One-week deferred period basis*

Duration		0 weeks		1 week		4 weeks		13 weeks		26 weeks		52 weeks		104 weeks	
Age	Term	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
16	49	0·0208	0·0	0·0296	0·0302	0·1037	0·1055	0·4010	0·4096	1·1476	1·1679	2·5250	0·0	4·2970	0·0
17	48	0·0214	0·0	0·0307	0·0313	0·1074	0·1093	0·4183	0·4273	1·1886	1·2097	2·5904	0·0	4·3731	0·0
18	47	0·0219	0·0	0·0319	0·0325	0·1113	0·1134	0·4365	0·4460	1·2313	1·2534	2·6577	0·0	4·4506	0·0
19	46	0·0226	0·0	0·0331	0·0337	0·1155	0·1177	0·4558	0·4659	1·2759	1·2989	2·7269	0·0	4·5294	0·0
20	45	0·0232	0·0	0·0344	0·0351	0·1200	0·1224	0·4763	0·4869	1·3224	1·3464	2·7981	0·0	4·6094	0·0
21	44	0·0239	0·0	0·0358	0·0366	0·1248	0·1274	0·4979	0·5092	1·3709	1·3958	2·8712	0·0	4·6908	0·0
22	43	0·0247	0·0	0·0374	0·0382	0·1300	0·1327	0·5208	0·5328	1·4213	1·4474	2·9464	0·0	4·7733	0·0
23	42	0·0256	0·0	0·0391	0·0400	0·1355	0·1385	0·5451	0·5578	1·4739	1·5010	3·0235	0·0	4·8569	0·0
24	41	0·0265	0·0	0·0409	0·0419	0·1415	0·1447	0·5708	0·5842	1·5287	1·5569	3·1026	0·0	4·9415	0·0
25	40	0·0275	0·0	0·0429	0·0439	0·1479	0·1513	0·5979	0·6121	1·5856	1·6150	3·1836	0·0	5·0270	0·0
26	39	0·0286	0·0	0·0450	0·0461	0·1548	0·1585	0·6267	0·6417	1·6449	1·6753	3·2665	0·0	5·1132	0·0
27	38	0·0298	0·0	0·0473	0·0486	0·1622	0·1662	0·6571	0·6730	1·7064	1·7380	3·3512	0·0	5·2000	0·0
28	37	0·0311	0·0	0·0499	0·0512	0·1702	0·1745	0·6892	0·7060	1·7703	1·8031	3·4376	0·0	5·2871	0·0
29	36	0·0325	0·0	0·0526	0·0541	0·1788	0·1834	0·7231	0·7408	1·8365	1·8705	3·5257	0·0	5·3744	0·0
30	35	0·0341	0·0	0·0557	0·0573	0·1880	0·1929	0·7589	0·7776	1·9051	1·9402	3·6152	0·0	5·4616	0·0
31	34	0·0359	0·0	0·0590	0·0608	0·1980	0·2032	0·7967	0·8164	1·9760	2·0123	3·7061	0·0	5·5483	0·0
32	33	0·0378	0·0	0·0627	0·0646	0·2086	0·2143	0·8365	0·8572	2·0492	2·0867	3·7981	0·0	5·6343	0·0
33	32	0·0399	0·0	0·0666	0·0688	0·2201	0·2262	0·8784	0·9002	2·1247	2·1633	3·8909	0·0	5·7191	0·0
34	31	0·0422	0·0	0·0710	0·0734	0·2324	0·2390	0·9224	0·9453	2·2024	2·2420	3·9843	0·0	5·8023	0·0
35	30	0·0448	0·0	0·0758	0·0784	0·2457	0·2527	0·9686	0·9925	2·2821	2·3226	4·0779	0·0	5·8833	0·0
36	29	0·0477	0·0	0·0811	0·0840	0·2598	0·2673	1·0169	1·0420	2·3636	2·4050	4·1712	0·0	5·9616	0·0
37	28	0·0509	0·0	0·0869	0·0900	0·2750	0·2830	1·0675	1·0936	2·4468	2·4890	4·2639	0·0	6·0366	0·0
38	27	0·0544	0·0	0·0933	0·0967	0·2912	0·2998	1·1202	1·1474	2·5315	2·5742	4·3554	0·0	6·1074	0·0
39	26	0·0583	0·0	0·1002	0·1040	0·3085	0·3177	1·1750	1·2032	2·6171	2·6602	4·4450	0·0	6·1733	0·0

Table F4 *(Continued)*

Duration		0 weeks		1 week		4 weeks		13 weeks		26 weeks		52 weeks		104 weeks	
Age	Term	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
40	25	0·0626	0·0	0·1079	0·1120	0·3270	0·3367	1·2318	1·2610	2·7034	2·7466	4·5319	0·0	6·2334	0·0
41	24	0·0673	0·0	0·1163	0·1208	0·3466	0·3568	1·2905	1·3205	2·7898	2·8329	4·6153	0·0	6·2866	0·0
42	23	0·0726	0·0	0·1254	0·1304	0·3673	0·3782	1·3508	1·3815	2·8758	2·9184	4·6943	0·0	6·3317	0·0
43	22	0·0784	0·0	0·1354	0·1408	0·3893	0·4007	1·4125	1·4437	2·9606	3·0023	4·7678	0·0	6·3675	0·0
44	21	0·0849	0·0	0·1463	0·1522	0·4123	0·4243	1·4752	1·5068	3·0434	3·0837	4·8344	0·0	6·3925	0·0
45	20	0·0919	0·0	0·1582	0·1645	0·4364	0·4489	1·5385	1·5701	3·1232	3·1616	4·8928	0·0	6·4053	0·0
46	19	0·0997	0·0	0·1710	0·1778	0·4615	0·4744	1·6018	1·6332	3·1988	3·2346	4·9414	0·0	6·4040	0·0
47	18	0·1081	0·0	0·1848	0·1921	0·4875	0·5007	1·6644	1·6951	3·2689	3·3014	4·9783	0·0	6·3866	0·0
48	17	0·1173	0·0	0·1996	0·2074	0·5141	0·5276	1·7254	1·7550	3·3320	3·3603	5·0015	0·0	6·3512	0·0
49	16	0·1272	0·0	0·2154	0·2237	0·5411	0·5546	1·7839	1·8117	3·3861	3·4092	5·0087	0·0	6·2953	0·0
50	15	0·1378	0·0	0·2321	0·2408	0·5681	0·5814	1·8385	1·8638	3·4293	3·4460	4·9974	0·0	6·2164	0·0
51	14	0·1491	0·0	0·2496	0·2585	0·5947	0·6076	1·8878	1·9097	3·4590	3·4681	4·9647	0·0	6·1116	0·0
52	13	0·1609	0·0	0·2676	0·2768	0·6202	0·6323	1·9299	1·9474	3·4727	3·4725	4·9074	0·0	5·9778	0·0
53	12	0·1732	0·0	0·2860	0·2951	0·6441	0·6549	1·9627	1·9746	3·4670	3·4558	4·8219	0·0	5·8117	0·0
54	11	0·1855	0·0	0·3042	0·3131	0·6652	0·6743	1·9838	1·9886	3·4384	3·4143	4·7043	0·0	5·6094	0·0
55	10	0·1977	0·0	0·3219	0·3301	0·6826	0·6892	1·9900	1·9861	3·3828	3·3435	4·5501	0·0	5·3669	0·0
56	9	0·2091	0·0	0·3381	0·3453	0·6947	0·6980	1·9780	1·9633	3·2956	3·2384	4·3544	0·0	5·0798	0·0
57	8	0·2191	0·0	0·3519	0·3574	0·6999	0·6988	1·9435	1·9159	3·1713	3·0935	4·1117	0·0	4·7432	0·0
58	7	0·2267	0·0	0·3621	0·3651	0·6960	0·6894	1·8820	1·8385	3·0041	2·9023	3·8161	0·0	4·3517	0·0
59	6	0·2308	0·0	0·3669	0·3663	0·6804	0·6668	1·7876	1·7253	2·7871	2·6576	3·4608	0·0	3·8996	0·0
60	5	0·2296	0·0	0·3643	0·3588	0·6502	0·6277	1·6541	1·5692	2·5127	2·3512	3·0384	0·0	3·3806	0·0
61	4	0·2209	0·0	0·3513	0·3392	0·6014	0·5677	1·4736	1·3616	2·1720	1·9734	2·5407	0·0	2·7877	0·0
62	3	0·2017	0·0	0·3243	0·3031	0·5292	0·4811	1·2366	1·0912	1·7538	1·5111	1·9572	0·0	2·1134	0·0
63	2	0·1672	0·0	0·2780	0·2437	0·4268	0·3592	0·9295	0·7407	1·2426	0·9444	1·2733	0·0	1·3480	0·0
64	1	0·1006	0·0	0·2035	0·1377	0·2823	0·1722	0·5280	0·2606	0·6097	0·2230	0·4621	0·0	0·4766	0·0

We then put

$$\bar{a}^{1,2}_{x,d:\overline{n}|} = da^{1}_{x,d:\overline{n}|} + (1-d)\bar{a}^{2}_{x,d:\overline{n}|}$$
$$= \int_0^1 \bar{a}^{\overline{SS}}_{x+t,d:\overline{n-t}|}\,dt \tag{24}$$

and

$$\bar{a}^{2,1}_{x,d:\overline{n}|} = (1-d)a^{2}_{x,d:\overline{n}|} + da^{1}_{x+1,d:\overline{n-1}|}$$
$$= \int_d^{1+d} \bar{a}^{\overline{SS}}_{x+t,d:\overline{n-t}|}\,dt \tag{25}$$

Where $d=0$ weeks, 52 weeks or 104 weeks annuities only of one type are defined, say a^1, with the other, a^2, being zero.

Table F4 shows approximate values of average current claim annuities of types 1 and 2, payable continuously, at 6% interest, ceasing at age 65, for values of x from 16 to 64 and for values of $d=0$, 1, 4, 13, 26, 52 and 104 weeks. These have

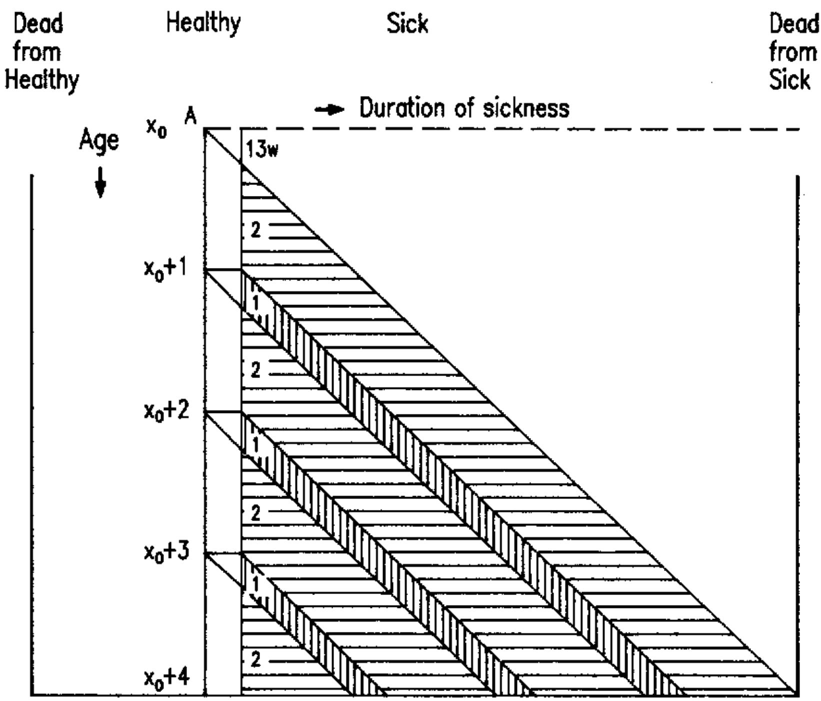

Figure F4. Calculating sickness benefit using inception rates of either type.

been calculated using the method described in Section 5.3 for each value of $x+t$ used, and with the integrals in formula (22) and (23) approximated by using Simpson's rule or the three-eighths rule repeated, with the number of repeats shown in the table below, for terms greater than one year, and using the trapezium rule with 156 repeats for a term of one year. It was found that these approximations for terms greater than one year were satisfactorily close to approximations with many more repeats. The number of repeats and the type of formula (S = Simpson, T = three-eighths rule) are:

	Type 1	Type 2
d = 0 weeks	2S	—
1 week	1T	1T
4 weeks	2S	2S
13 weeks	1T	1T
26 weeks	1T	1T
52 weeks	2S	—
104 weeks	2S	—

6.4 We are now able to calculate the expected present value of an annuity payable continuously while sick with duration greater than d, to a life who is currently healthy at age x_0, ceasing at age x_0+n, for $0<d<52$ weeks, by the general formulae:

$$\bar{a}_{x:\overline{n}|}^{\overline{HS}(d/\text{all})} \doteq v^{(1+d)/2} \frac{L_{x_0}}{l_{x_0}} \cdot ia(x_0,d)\, \bar{a}^{\,2}_{x_0,d:\overline{n}|} + \sum_{t=1}^{n-1} v^{t+1/2} \frac{L_{x_0+t}}{l_{x_0}} \cdot ia(x_0+t,d)\, \bar{a}^{\,1,2}_{x_0+t,d:\overline{n}|} \qquad (26)$$

or

$$\bar{a}_{x:\overline{n}|}^{\overline{HS}(d/\text{all})} \doteq \sum_{t=0}^{n-2} v^{t+d+1/2} \frac{L_{x_0+t}}{l_{x_0}} \cdot ib(x_0+t,d)\, \bar{a}^{\,2,1}_{x_0+t,d:\overline{n}|} + v^{n-1+(1+d)/2} \frac{L_{x_0+n-1}}{l_{x_0}} (1-d) \cdot ib(x_0+n-1,d)\, \bar{a}^{\,2}_{x_0+n-1,d:\overline{n}|} \qquad (27)$$

with obvious modifications if $n=1$. Note that in the final year the value of $ib(x_0+n-1,d)$ requires reducing by a factor $(1-d)$. For $d=0$, $d=52$ weeks and $d=104$ weeks there is only one formula:

$$\bar{a}_{x:\overline{n}|}^{\overline{HS}(d/\text{all})} \doteq \sum_{t=1}^{n} v^{t+1/2} \frac{L_{x_0+t}}{l_{x_0}} \cdot ia(x_0+t,d)\, \bar{a}^{\,1}_{x_0,d:\overline{n}|} \qquad (28)$$

For $d=52$ weeks the first term of this formula is zero, since $ia(x_0,d)$ is zero, and for $d=104$ weeks the first two terms are zero.

Table F5. *Annuity values and annual premium rates for a benefit of 1,000 per year, with selected deferred periods for each term from starting age, conditional on starting at age 30 with initial status healthy: premiums payable 12 times a year in advance: benefits treated as payable continuously: rate of interest 6%. One-week deferred period basis*

			Deferred period 1 week			Deferred period 4 weeks			Deferred period 13 weeks			Deferred period 26 weeks		
Term	End age	Healthy in adv.	Sick 0/1 in adv.	Sick 1/all cont.	AP 1/all in adv.	Sick 0/4 in adv.	Sick 4/all cont.	AP 4/all in adv.	Sick 0/13 in adv.	Sick 13/all cont.	AP 13/all in adv.	Sick 0/26 in adv.	Sick 26/all cont.	AP 26/all in adv.
1	31	0·9657	0·0035	4·362	4·500	0·0061	1·625	1·672	0·0073	0·392	0·402	0·0076	0·096	0·098
2	32	1·8744	0·0071	9·512	5·055	0·0125	4·034	2·138	0·0151	1·385	0·733	0·0158	0·627	0·332
3	33	2·7303	0·0105	14·834	5·412	0·0185	6·722	2·445	0·0225	2·671	0·970	0·0237	1·444	0·524
4	34	3·5365	0·0136	20·264	5·708	0·0242	9·622	2·702	0·0295	4·184	1·173	0·0312	2·481	0·695
5	35	4·2958	0·0166	25·781	5·978	0·0295	12·708	2·938	0·0363	5·899	1·362	0·0385	3·713	0·857
6	36	5·0109	0·0193	31·377	6·238	0·0346	15·972	3·166	0·0427	7·807	1·545	0·0454	5·131	1·015
7	37	5·6843	0·0219	37·052	6·493	0·0394	19·408	3·391	0·0489	9·903	1·727	0·0520	6·731	1·173
8	38	6·3183	0·0243	42·808	6·749	0·0440	23·015	3·617	0·0547	12·185	1·912	0·0584	8·510	1·335
9	39	6·9151	0·0265	48·650	7·008	0·0483	26·792	3·848	0·0603	14·653	2·101	0·0645	10·468	1·500
10	40	7·4768	0·0286	54·582	7·272	0·0523	30·743	4·083	0·0657	17·308	2·295	0·0704	12·606	1·670
11	41	8·0053	0·0305	60·613	7·543	0·0562	34·869	4·325	0·0708	20·153	2·495	0·0760	14·928	1·847
12	42	8·5026	0·0323	66·753	7·821	0·0598	39·177	4·575	0·0758	23·195	2·704	0·0815	17·437	2·031
13	43	8·9703	0·0340	73·012	8·109	0·0633	43·675	4·835	0·0805	26·437	2·921	0·0868	20·140	2·224
14	44	9·4100	0·0356	79·404	8·406	0·0666	48·370	5·104	0·0850	29·890	3·148	0·0918	23·045	2·425
15	45	9·8233	0·0371	85·943	8·716	0·0697	53·275	5·385	0·0894	33·562	3·386	0·0968	26·160	2·637
16	46	10·2116	0·0385	92·645	9·038	0·0727	58·400	5·679	0·0936	37·464	3·635	0·1015	29·496	2·860
17	47	10·5762	0·0398	99·528	9·375	0·0755	63·761	5·986	0·0976	41·610	3·898	0·1061	33·065	3·095
18	48	10·9185	0·0411	106·613	9·728	0·0782	69·374	6·309	0·1015	46·014	4·175	0·1107	36·882	3·344
19	49	11·2396	0·0422	113·921	10·098	0·0808	75·257	6·648	0·1053	50·693	4·468	0·1150	40·962	3·608
20	50	11·5406	0·0433	121·476	10·487	0·0833	81·431	7·005	0·1090	55·667	4·778	0·1193	45·324	3·887

Table F5 *(Continued)*

			Deferred period 1 week			Deferred period 4 weeks			Deferred period 13 weeks			Deferred period 26 weeks		
Term	End age	Healthy in adv.	Sick 0/1 in adv.	Sick 1/all cont.	AP 1/all in adv.	Sick 0/4 in adv.	Sick 4/all cont.	AP 4/all in adv.	Sick 0/13 in adv.	Sick 13/all cont.	AP 13/all in adv.	Sick 0/26 in adv.	Sick 26/all cont.	AP 26/all in adv.
21	51	11·8227	0·0443	129·305	10·896	0·0857	87·918	7·383	0·1126	60·956	5·107	0·1235	49·987	4·184
22	52	12·0867	0·0453	137·435	11·328	0·0879	94·744	7·782	0·1161	66·585	5·457	0·1276	54·974	4·501
23	53	12·3336	0·0462	145·898	11·785	0·0901	101·939	8·205	0·1194	72·580	5·828	0·1317	60·312	4·838
24	54	12·5642	0·0471	154·729	12·269	0·0922	109·532	8·654	0·1228	78·971	6·225	0·1357	66·027	5·199
25	55	12·7794	0·0479	163·965	12·782	0·0943	117·558	9·132	0·1260	85·791	6·648	0·1396	72·152	5·585
26	56	12·9800	0·0487	173·647	13·328	0·0963	126·057	9·640	0·1292	93·076	7·100	0·1435	78·722	5·999
27	57	13·1666	0·0495	183·818	13·909	0·0982	135·070	10·183	0·1323	100·867	7·585	0·1474	85·775	6·443
28	58	13·3398	0·0502	194·528	14·528	0·1000	144·644	10·762	0·1354	109·209	8·104	0·1512	93·354	6·920
29	59	13·5004	0·0508	205·828	15·189	0·1018	154·828	11·383	0·1384	118·149	8·663	0·1550	101·507	7·433
30	60	13·6488	0·0515	217·775	15·896	0·1035	165·678	12·047	0·1414	127·742	9·263	0·1588	110·285	7·987
31	61	13·7856	0·0521	230·430	16·652	0·1052	177·253	12·760	0·1444	138·047	9·910	0·1627	119·745	8·585
32	62	13·9113	0·0527	243·856	17·463	0·1069	189·617	13·526	0·1473	149·125	10·607	0·1665	129·949	9·231
33	63	14·0263	0·0533	258·123	18·333	0·1085	202·837	14·350	0·1502	161·046	11·360	0·1703	140·963	9·929
34	64	14·1311	0·0538	273·299	19·267	0·1101	216·984	15·236	0·1531	173·879	12·173	0·1741	152·858	10·685
35	65	14·2259	0·0543	289·457	20·270	0·1116	232·131	16·190	0·1560	187·699	13·051	0·1779	165·707	11·504

6.5 It will be noted that this method does not allow calculation directly of the expected present value of a sickness benefit payable only during a specific sickness period, a/b. However, this can be calculated by subtraction:

$$\bar{a}^{HS(a/b)}_{x_0:\overline{n}|} = \bar{a}^{HS(a/\text{all})}_{x_0:\overline{n}|} - \bar{a}^{HS(b/\text{all})}_{x_0:\overline{n}|} \tag{29}$$

6.6 Calculations using these methods have been carried out and the results compared with the exact method described in Section 1. The results for d/all annuities are reasonably close for $d=1$ week or more, being no worse than 1% out over most of the range of possible values, and often much less. The values calculated using this method are typically almost 1% too small for 1/all, a little too small for 4/all and 13/all, a little too large for 26/all, 52/all and 104/all. It is not obvious why the errors should be in opposite directions.

For 0/all annuities, however, the values calculated by this method are significantly too low, by up to 6%.

When differences are taken, to give annuities for 0/1, 1/4 etc. the errors are increased. For 0/1 the calculated values are substantially too low, for 1/4 up to 1·5% too low. Only for the intermediate sickness periods are they tolerably accurate.

6.7 It is not difficult to calculate the values of current claim annuities payable monthly in advance, but these are not what is required for the calculation of sickness annuities payable monthly in advance. The payment dates of the former commence with the date the claim commences, whereas the latter require a sequence of dates commencing when the policy commences. The equivalent of formulae (22) and (23) therefore require the average value of annuities with possible payment dates at ages x, $x+\frac{1}{12}$, $x+\frac{2}{12}$ etc., not at ages $x+t$, $x+t+\frac{1}{12}$ etc. Calculations on these lines have not been carried out. Such calculations would normally only be required for waiver of premium benefit, and the methods using sickness rates is more satisfactory for the period of sickness ($0/d$).

7. APPROXIMATE CALCULATION OF ANNUITY VALUES USING SELECT TABLES OF INCEPTION RATES

7.1 The formula described in Section 6 for calculating annuities using inception rates and average values of current claim annuities can be readily adapted when the life table and inception rate tables are arranged as select tables with a limited select period, rather than being conditional on the life being healthy at age x_0. The values of current claim annuities do not depend on what earlier age a life was healthy, so they remain as already calculated in Section 6. The only alterations required are to interpret the functions L, l and i in formulae (26), (27) and (28) appropriately to give, for $0<d<52$ weeks:

$$\bar{a}_{x:\overline{n}|}^{HS(d/\text{all})} \doteqdot v^{(1+d)/2}\frac{L_{[x_0]}}{l_{[x_0]}}\cdot ia^d_{[x_0]}\bar{a}^{\,2}_{x_0,d:\overline{n}|} + \sum_{t=1}^{n-1} v^{t+1/2}\frac{L_{[x_0]+t}}{l_{[x_0]}}\cdot ia^d_{[x_0]+t}\bar{a}^{\,1,2}_{x_0+t,d:\overline{n}|} \quad (30)$$

or

$$\bar{a}_{x:\overline{n}|}^{HS(d/\text{all})} \doteqdot \sum_{t=0}^{n-2} v^{t+d}\frac{L_{[x_0]+t}}{l_{[x_0]}}\cdot ib^d_{[x_0]+t}\;\bar{a}^{\,2,1}_{x_0+t,d:\overline{n}|} + v^{n-1+(1+d)/2}\frac{L_{[x_0]+n-1}}{l_{[x_0]}}(1-d)\cdot ib^d_{[x_0]+n-1}\;\bar{a}^{\,2}_{x_0+n-1,d:\overline{n}|} \quad (31)$$

with suitable modifications when $n=1$, and

$$\bar{a}_{x:\overline{n}|}^{HS(d/\text{all})} =:= \sum_{t=1}^{n} v^{t+1/2}\frac{L_{[x_0]+t}}{l_{[x_0]}}\cdot ia^d_{[x_0]+t}\;\bar{a}^{\,1}_{x_0,d:\overline{n}|} \quad (32)$$

for $d=0$, $d=52$ weeks or $d=104$ weeks.

Calculations show that the present values of sickness annuities are up to 1% lower using select inception rates of type (a), and up to 1% higher using select inception rates of type (b), as compared with the calculations using a full table of inception rates. These errors are generally larger than the differences between using the exact method and using the full inception rates table.

8. PREMIUMS

8.1 The formula appropriate for the calculation of net premium rates depends on the precise definition of the benefits and on the conditions under which premiums are payable. Table F5 shows specimen rates, calculated using the exact values from Tables F1 and F2, for a life who is healthy at entry at age 30, for a policy for terms from 1 year to 35 years, providing a sickness benefit, payable continuously at the rate of 1,000 per year, for sickness following deferred periods of 1, 4, 13 and 26 weeks, the premiums being payable monthly in advance while not claiming, i.e. while healthy or sick with duration of sickness less than the deferred periods. The relevant premium is thus calculated as:

$$1{,}000\;\bar{a}_{x_0:\overline{n}|}^{HS(d/\text{all})} \Big/ \left(\ddot{a}_{x_0:\overline{n}|}^{HH(12)} + \ddot{a}_{x_0:\overline{n}|}^{HS(0/d)(12)}\right) \quad (33)$$

This formulation allows for a monthly premium being waived if the assured is sick and claiming on the due date. Alternative policy conditions might allow no waiver, or a pro-rata waiver proportionate to the duration of sickness. Alternative formulae would apply in these cases.

9. COMPARISON OF SICKNESS RATE AND INCEPTION RATE METHODS

9.1 It has been shown that the approximations involved in the calculation of sickness annuities by means of inception rates and current claim annuities are substantially greater than in the calculation using sickness rates. In the calculation using z functions the only approximation as compared with the 'exact' method is in applying a discount function as at the middle of the year to the benefits payable during that year. Using inception rates and current claim annuities there is an additional approximation in the calculations of average values of current claim annuities. For the calculation of 0/all and 1/all benefits the errors of approximation are considerable, whereas for larger values of d the errors in the calculation of d/all benefits are tolerable.

For the calculation of d/all benefits the fact that inception rates can be conveniently condensed into an appropriate select table, whereas sickness rates for 104/all cannot be so conveniently condensed, gives a computational advantage to the inception rate method.

However, when it comes to calculating benefits for a limited sickness period, such as $0/d$ benefits, a select sickness rate table can readily be calculated, and the sickness rate method is both computationally convenient and considerably more accurate.

Both approaches therefore have their place. It is a strength of the present approach that it encompasses both the traditional methods, showing that each can be treated as simply a method of calculation within the same model of sickness.

REFERENCES

AALEN, O. O. & HOEM, J. M. (1978). Random time changes for multivariate counting processes. *Scandinavian Actuarial Journal*, 1978, 81–101.

BEARD, R. E. & PERKS, W. (1949). The relation between the distribution of sickness and the effect of duplicates on the distribution of deaths. *J.I.A.*, **75**, 75–86.

BORGAN, Ø. (1984). Maximum likelihood estimation in parametric counting process models, with applications to censored failure time data. *Scandinavian Journal of Statistics*, **11**, 1–16.

BOND, D. J. (1963). Permanent sickness insurance. *J.I.A.S.S.*, **17**, 195–224.

CMI COMMITTEE (1976). Investigation of Sickness Statistics—Individual Policies 1972 and 1973. *C.M.I.R.*, **2**, 1–55.

CMI COMMITTEE (1979). Sickness Experience 1972–75 for Individual Policies. *C.M.I.R.*, **4**, 1–91.

CMI COMMITTEE (1984). Sickness Experience 1975–78 for Individual Policies. *C.M.I.R.*, **7**, 1–161.

CMI COMMITTEE (1988). Graduation of the 1979–82 Mortality Experiences. *C.M.I.R.*, **9**, 1–102.

COX, D. R. & MILLER, H. D. (1965). *The Theory of Stochastic Processes*. Methuen & Co., London.

DAW, R. H. (1951). Duplicate policies in mortality data. *J.I.A.*, **77**, 262–267.

DILLNER, C-G. (1969). New bases for non-cancellable sickness insurance in Sweden. *Skandinavisk Aktuarietidskrift*, **52**, 113–124.

DU PASQUIER, L. G. (1912, 1913). Mathematische Theorie der Invaliditätsversicherung. *Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker*, **7**, 1–7; **8**, 1–153.

FORFAR, D. O., MCCUTCHEON, J. J. & WILKIE, A. D. (1988). On graduation by mathematical formula. *J.I.A.*, **115**, 1–149 and *T.F.A.*, **41**, 97–269.

HOEM, J. M. (1969). Some notes on the qualifying period in disability insurance. II. Problems of maximum likelihood estimation. *Mitteilungen der Vereinigung schweizerischer Versicherungsmathematiker*, **69**, 301–317.

HOEM, J. M. (1972). Inhomogeneous semi-Markov processes, select actuarial tables and duration dependence in demography. In GREVILLE, T. N. E. (ed.) *Population Dynamics*. Academic Press, New York.

HOEM, J. M. (1976). The statistical theory of demographic rates: a review of current developments. *Scandinavian Journal of Statistics*, **3**, 169–185.

HOEM, J. M. (1987). Statistical analysis of a multiplicative model and its application to the standardization of vital rates: a review. *International Statistical Review*, **55**, 119–152.

HOEM, J. M. & FUNCK JENSEN, U. (1982). Multistate life table methodology: a probabilist critique. In LAND, K. C. & ROGERS, A. (eds.) *Multidimensional Mathematical Demography*. Academic Press, New York.

MEDIN, K. (1952). Some theoretical aspects of permanent sickness insurance. *J.I.A.S.S.*, **11**, 76–103.

NEILL, A. (1977). *Life Contingencies*. Heinemann, London.

SCHOU, G. & VAETH, M. (1980). A small sample study of occurrence/exposure rates for rare events. *Scandinavian Actuarial Journal*, 1980, 209–225.

STARKE, L. G. K. (1980). The duration of sickness. *J.I.A.*, **107**, 365–372.

SVERDRUP, E. (1965). Estimates and test procedures in connection with stochastic models for deaths, recoveries and transfers between different states of health. *Scandinavian Actuarial Journal*, 1965, 184–211.

SANDERS, A. J. & SILBY, N. F. (1988). Actuarial aspects of PHI in the UK. *J.I.A.S.S.*, **31**, 1–57.

SANSOM, R. (1978). Practical PHI. *J.I.A.S.S.*, **22**, 64–133.

WATERS, H. R. (1984). An approach to the study of multiple state models. *J.I.A.*, **111**, 363–374.

WATERS, H. R. & WILKIE, A. D. (1987). A short note on the construction of life tables and multiple decrement tables. *J.I.A.*, **114**, 569–580.

GLOSSARY OF NOTATION

This glossary gives a reference to the Part and Section wherein the notation is first defined. Note that the superscript $^{(1)}$ in a comment indicates that the probability or expected value is conditional on the individual or group of R individuals being healthy at age x_0. Some conventional actuarial notation is not included.

Notation	*Reference*	*Comment*
A	Appendix A	The average exposure per individual policyholder.
a/b	D6.1	The sickness period from a weeks to $a+b$ weeks.
$\bar{a}^{HH}_{x_0:\overline{n}\vert}$	D7.2	Present value of an annuity payable continuously while healthy, for a maximum term of n years.$^{(1)}$
$\bar{a}^{HL}_{x_0:\overline{n}\vert}$	D7.2	Present value of an annuity payable continuously while alive, for a maximum term of n years.$^{(1)}$
$\bar{a}^{HS}_{x_0:\overline{n}\vert}$	D7.2	Present value of an annuity payable continuously while sick, for a maximum term of n years.$^{(1)}$
$\bar{a}^{HS(a/b)}_{x_0:\overline{n}\vert}$	D7.2	Present value of an annuity payable continuously while sick, with sickness period a/b, for a maximum term of n years.$^{(1)}$
$\bar{a}^{SS}_{x,z:\overline{n}\vert}$	D7.3	Present value of an annuity payable continuously while still sick to a life sick at age x with duration of sickness z, for a maximum term of n years.
$\ddot{a}^{(12)HH}_{x_0:\overline{n}\vert}$	D7.5	Present value of an annuity payable monthly in advance while healthy, for a maximum term of n years.$^{(1)}$
$\ddot{a}^{(12)HL}_{x_0:\overline{n}\vert}$	D7.5	Present value of an annuity payable monthly in advance while alive, for a maximum term of n years.$^{(1)}$
$\ddot{a}^{(12)HS}_{x_0:\overline{n}\vert}$	D7.5	Present value of an annuity payable monthly in advance while sick, for a maximum term of n years.$^{(1)}$
$\ddot{a}^{(12)HS(a/b)}_{x_0:\overline{n}\vert}$	D7.5	Present value of an annuity payable monthly in advance while sick, with sickness period a/b, for a maximum term of n years.$^{(1)}$

Notation	*Reference*	*Comment*
$\ddot{a}^{(12)SS}_{x,z:\overline{n}\vert}$	D7.4	Present value of an annuity payable monthly in advance while still sick to a life sick at age x with duration of sickness z, for a maximum term of n years.
CL_x	C4.1	Total time (measured in weeks) spent claiming at age x last birthday.
$ca(x,d)$	D5.3	The expected number of claim inceptions between ages x and $x+1$ for deferred period d among the R individuals healthy at age x_0.
$cb(x,d)$	D5.5	The expected number of claim inceptions between ages $x+d$ and $x+d+1$ for deferred period d among the R individuals healthy at age x_0.
D	A2.1	The state 'dead'.
$D1$	B1.4	Deferred period 1 week.
$D4$	B1.4	Deferred period 4 weeks.
$D13$	B1.4	Deferred period 13 weeks.
$D26$	B1.4	Deferred period 26 weeks.
$D52$	B1.4	Deferred period 52 weeks.
d	C1.2	The deferred period for a policy.
$d(y,z)$	B7.1	Number of deaths in the sickness period z to $z+1$ among those who fell sick at age y.
$dHD(x)$	D4.3	The number of transitions from healthy to dead between ages x and $x+1$.(1)
$dHS(x)$	D4.3	The number of transitions from healthy to sick between ages x and $x+1$.(1)
$dSD(x)$	D4.3	The number of transitions from sick to dead between ages x and $x+1$.(1)
$dSH(x)$	D4.3	The number of transitions from sick to healthy between ages x and $x+1$.(1)
E	A3.3	Total time spent sick in the observation period by individual policies, counting only the time when the policyholders were aged between x_1 and x_2 and when the duration of their sickness was between z_1 and z_2.
E'	A4.2	A period of exposure where duplicate policies have not been eliminated.
$\hat{E}$	C9.4	A period of exposure.

Notation	*Reference*	*Comment*
$E(x_1,x_2)$	C1.2	Total time spent in the observation period by individuals who are healthy and aged between x_1 and x_2.
$E(y,z)$	B3.2	The exposed to risk for the data cell for mean age y and mean sickness duration z.
EH_x	C4.1	Time spent in 1975–78 as healthy by policyholders aged x last birthday.
E_i	C3.3	The period of exposure in the ith week following the end of the deferred period.
E_t	Appendix A	A period of exposure for individuals having exactly t policies.
f_1,f_2	C2.1	Factors used to calculate Manchester-Unity-type exposures exactly.
f_t	A4.2	The proportion of policyholders having exactly t policies.
f_y	B3.2	Function of age y at sickness inception, being a factor of recovery intensity $\rho_{y+z,z}$.
g	C4.4	Annual rate of growth of the number of policies in force.
g_z	B3.2	Function of sickness duration z, being a factor of recovery intensity $\rho_{y+z,z}$.
H	A2.1	The state 'healthy'.
h	D2.1	The step size for the numerical algorithms.
I	A4.4	The number of sicknesses among policies with deferred period d which start in the observation period and for which the policyholder is aged between x_1 and x_2 at the start of the sickness and which last beyond the deferred period.
I'	A4.5	A number of claim inceptions, where duplicate policies have not been eliminated.
$I(x_1,x_2)$	C1.2	Number of sicknesses which start in the observation period, with age at start of sickness between x_1 and x_2, and which last beyond the deferred period of the policy.
$I\hat{E}$	C9.4	Expected number of claim inceptions.
IE^*	C7.2	Expected number of sicknesses lasting 4 weeks beyond the end of the deferred period.
IN	C3.2	The number of claim inceptions not reported.
IR	C3.2	The number of claim inceptions reported.
IR^*	C7.2	The number of claims lasting 4 weeks beyond the end of the deferred period.

Notation	*Reference*	*Comment*
i	D7.2	The rate of interest for calculating monetary functions.
$ia(x,d)$	D5.4	The claim inception rate of type (a) at age x for deferred period d.(1)
$ia(x,d,t)$	D5.4	The claim inception rate of type (a) at age x for deferred period d for a life who was healthy at age $x-t$.(1)
ia_x^d	E8.4	The ultimate claim inception rate of type (a) at age x for deferred period d.
$ia^d_{[x-t]+t}$	E8.4	The select claim inception rate of type (a) at attained age x for deferred period d, for a life who was healthy at age $x-t$.
$ib(x,d)$	D5.5	The claim inception rate of type (b) at age x for deferred period d.(1)
$k(y)$	C9.9	A function linking claim termination rates for $D1$ and $D26$.
L_x	D5.2	The expected number of years lived between ages x and $x+1$ by the R individuals who were healthy at age x_0.(1)
$L(x,x_0)$	E8.5	The value of L_x conditional on being healthy at age x_0.
$L_{[x_0]+t}$	E8.5	The select value of L_{x_0+t} for a life who was healthy at age x_0.
$l(y,z)$	B7.1	Number of claims remaining in force at exact sickness duration z for lives aged exactly y at the date of falling sick.
$lD(x)$	D4.2	The number dead at age x.(1)
$lDH(x)$	D4.2	The number dead at age x having died as healthy.(1)
$lDS(x)$	D4.2	The number dead at age x, having died as sick.(1)
$lH(x)$	D4.2	The number healthy at age x.(1)
$lL(x)$	D4.2	The number alive at age x.(1)
$lS(x)$	D4.2	The number sick at age x.(1)
l_x	D4.2	The number alive at age x; the same as $lL(x)$.(1)
$M(t)$	A4.4	The number of policyholders at time t who are healthy, aged between x_1 and x_2 and have policies with deferred period d.
$m(x)$	D2.2	The transition intensity from healthy to dead at age x; the same as μ_x.

Notation	*Reference*	*Comment*
$mL(x)$	D3.3	Average force of mortality of the living; the same as $\mu L_{[x_0]+x-x_0}$.[1]
$mS(x)$	D3.3	Average force of mortality of the sick, weighted by duration.[1]
m_i	A4.2	The i-th moment about zero of the distribution of policies per individual.
N	D2.3	The duration of sickness, in terms of the number of steps of size h, beyong which ρ and v depend only on attained age.
N	Appendix A	The number of individuals contributing to an exposure.
N'	Appendix A	The number of policies contributing to an exposure.
$N(t)$	A4.4	The number of policyholders at time t who are aged between x_1 and x_2 and have policies with deferred period d.
$n(x,m)$	D2.2	The transition intensity from sick to dead at age x, duration of sickness $(m-\frac{1}{2})h$; the same as $v_{x,(m-\frac{1}{2})h}$.
O	A3.3	Observed number of recoveries by policyholders who, at the time of recovery, were aged between x_1 and x_2 and whose duration of sickness was between z_1 and z_2.
$O(y,z)$	B3.2	Number of recoveries for the data cell for mean age y and mean sickness duration z.
O'	A4.2	An observed number of recoveries, where duplicate policies have not been eliminated.
O_t	Appendix A	An observed number of recoveries among policyholders having exactly t policies.
$p(y;x_0)$	C4.1	Probability that a policyholder healthy at age x_0 is alive at age $y(>x_0)$ and sick but not yet claiming.[1]
$pD(x)$	D2.2	The probability of dying before age x.[1]
$pDH(x)$	D2.2	The probability of dying as healthy before age x.[1]
$pDS(x)$	D2.2	The probability of dying as sick before age x.[1]
$pH(x)$	D2.2	The probability of being healthy at age x.[1]
$pL(x)$	D3.4	The probability of being alive at age x.[1]
$pS(x)$	D2.2	The probability of being sick at age x.[1]

Notation	*Reference*	*Comment*
$pS(x,m)$	D2.2	The probability of being sick at age x, with duration of sickness between $(m-1)h$ and mh.[(1)]
$pS(x,N^+)$	D2.4	The probability of being sick at age x, with duration of sickness greater than Nh.[(1)]
$pS(x,a/b)$	D6.1	The probability of being sick at age x, with duration of sickness between a and $a+b$.[(1)]
$pi(x,d)$	E10.1	An average value of $\pi_{y,d}$ over the year of age x to $x+1$.
p_x	C4.1	The proportion of TE_x spent as sick but not yet claiming.
${}_tp^{\overline{HH}}{}_x$	A2.4	The probability of remaining healthy from age x to age $x+t$.
${}_tp_x^{\overline{SS}}$	A2.4	The same as ${}_tp_{x,0}^{\overline{SS}}$
${}_tp_{x,z}^{\overline{SS}}$	A2.4	The probability of remaining sick from age x to age $x+t$ given duration of sickness z at age x.
${}_tp_{x,z}^{j,k}$	A2.4	The probability of being in state k at age $x+t$, conditional on being in state j at age x with duration z.
${}_tp_{[x]}$	E7.3	Probability of surviving for t years of a select life entering at age x.
${}_{w,t}p_x^{HS}$	A2.4	The probability of being sick at age $x+t$ with duration not more than w, conditional on being healthy at age x.
$Q_1(t)$	A4.4	The proportion of policyholders at time t who are sick and claiming.
$Q_2(t)$	A4.4	The proportion of policyholders at time t who are sick but not yet claiming.
$q(x,t)$	E7.2	The value of q_x for attained age x for a life who was healthy at age $x-t$.
q_x	E4.3	The probability of death within 1 year for those living at age x[(1)]; or (E7.3) the ultimate mortality rate at age x.
$q_{[x-t]+t}$	E7.3	The select mortality rate at age x for a life who was healthy at age $x-t$.
${}_hq_{y+z,z}^d$	B7.1	Independent rate of death in the short interval h.
${}_hq_{y+z,z}^r$	B7.1	Independent rate of recovery in the short interval h.
${}_tq_{[x]}$	E7.3	Probability of death within t years of a life who was healthy at age x.
R	D4.2	The radix for the increment–decrement table.

Notation	*Reference*	*Comment*
R_i	C3.3	The number of reported recoveries in the i-th week following the end of the deferred period.
$r(x,m)$	D2.2	The transition intensity from sick to healthy at age x, duration of sickness $(m-\frac{1}{2})h$; the same as $\rho_{x,(m-\frac{1}{2})h}$.
$r(y,z)$	B7.1	Number of recoveries in the sickness period z to $z+1$ among those who fell sick at age y.
S	A2.1	The state 'sick'.
$s(x)$	D2.2	The transition intensity from healthy to sick at age x; the same as σ_x.
T	E7.3	The number of years selection in a select table of mortality rates, inception rates or sickness rates.
T_1	A4.4	The start of an observation period.
T_2	A4.4	The end of an observation period.
TE_x	C4.1	Total time spent in the observation period 1975–78 at age x last birthday.
$Tca(x_0,d,n)$	E8.5	Total number of claim inceptions for deferred period d, for a life healthy at age x_0, within n years.
$Tca^*(x_0,d,n)$	E8.5	Equivalent version of $Tca(x_0,d,n)$ using select tables.
V	C1.2	The factor by which the variance of the estimator for σ is inflated by the presence of duplicate policies in the data.
v	D7.2	The usual discounting factor $(1+i)^{-1}$.
XR_i	C3.3	The number of 'non-reported recoveries' in the i-th week following the end of the deferred period.
x	A2.2	The attained age of a policyholder.
x_0	C4.1	An age at which the policyholder is healthy.
$Y(x)$	A2.2	The state of the individual (healthy, sick or dead) at age x.
y	B3.2	Age at start of sickness.
$Z(x)$	A2.2	Duration of the sojourn so far in the current state, $Y(x)$.
z	A2.2	The duration so far of the policyholder's current sickness.
$z(x,a/b)$	D6.2	The central rate of sickness at age x, with duration of sickness between a and $a+b$.[1]

Notation	*Reference*	*Comment*
$z(x,a/b,t)$	E9.1	The value of $z(x,a/b)$ for a life aged x who was healthy at age $x-t$.
$z_x^{a/b}$	A1.2	A (Manchester-Unity-type) rate of sickness or (E9.2) the ultimate rate of sickness for attained age x, sickness period a/b.
$z_{[x-t]+t}^{a/b}$	E9.2	The select rate of sickness for attained age x, sickness period a/b, for a life who was healthy at age $x-t$.
α_x	C2.1	Sum over each of the 4 years of the number of policies at the start of the year for which the policyholders were either aged x nearest birthday and entered in the previous calendar year or were aged $x+1$ nearest birthday and entered between 1 and 2 years ago.
β_x	C2.1	Sum over each of the 4 years of the number of policies at the start of each year for which the policyholders were aged $x+1$ nearest birthday and who entered in the previous year.
$\zeta(x,a/b)$	D6.1	52 times the probability that an individual is sick at age x with duration of sickness between a and $a+b$.(1)
μ_x	A2.1	The transition intensity from healthy to dead at age x.
$\mu L_{[x_0]+t}$	D3.2	The transition intensity from living to dead at age x_0+t.(1)
$\nu_{x,z}$	A2.1	The transition intensity from sick to dead at current age x and current duration of sickness z.
$\nu_{x,z}^{(1)}$	C9.9	The value of $\nu_{x,z}$ for a $D\,1$ policyholder.
$\nu_{x,z}^{(26)}$	C9.9	The value of $\nu_{x,z}$ for a $D\,26$ policyholder.
π	A4.4	The assumed constant value of ${}_dp_x^{\overline{SS}}$ over the interval $[x_1,x_2]$.
π'	C7.2	The probability that a sickness lasts for at least 4 weeks beyond the deferred period.
$\pi_{x,d}$	C1.2	The probability that an individual who falls sick at age x will remain sick for at least period d; the same as ${}_dp_x^{\overline{SS}}$.
$\pi_x^{(26)}$	C9.9	The value of π_x with $d=26$ weeks for a $D\,26$ policyholder.
ρ	A3.3	The assumed constant value of $\rho_{x,z}$ over the rectangle $[x_1,x_2]\times[z_1,z_2]$.

Notation	*Reference*	*Comment*
ρ'	A4.2	The maximum likelihood estimate of ρ where duplicate policies have not been eliminated.
$\hat{\rho}$	A3.3	The maximum likelihood estimate of ρ where $\hat{\rho}=O/E$.
ρ_i	C3.3	An estimate of ρ in the i-th week following the end of the deferred period.
$\hat{\rho}_i^*$	C3.3	An estimate of ρ in the i-th week following the end of the deferred period allowing for 'non-reported recoveries'.
$\rho_{x,z}$	A2.1	The transition intensity from sick to healthy at current age x and current duration of sickness z.
$\rho_{x,z}^{(1)}$	C9.9	The value of $\rho_{x,z}$ for a $D\,1$ policyholder.
$\rho^{(26)}{}_{x,z}$	C9.9	The value of $\rho_{x,z}$ for a $D\,26$ policyholder.
σ	A4.4	The assumed constant value of σ_x over the interval $[x_1,x_2]$.
σ_x	A2.1	The transition intensity from healthy to sick at age x.
σ'	A4.5	An estimator for σ where duplicates have not been eliminated.
$\hat{\sigma}$	A4.4	The maximum likelihood estimator for σ.
$\sigma_x^{(1)}$	C9.9	The value of σ_x for a $D\,1$ policyholder.
$\sigma_x^{(26)}$	C9.9	The value of σ_x for a $D\,26$ policyholder.

CONTINUOUS MORTALITY INVESTIGATION REPORTS

NUMBER 12

Printed in Great Britain at The Alden Press, Oxford