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THE EXECUTIVE COMMITTEE OF THE CONTINUOUS MORTALITY INVESTIGATION BUREAU

as on March 16th 1976

INTRODUCTION

In 1973 the Councils of the Institute and Faculty presented the first Number of *Continuous Mortality Investigation Reports*, which was prepared under the direction of the Joint Continuous Mortality Investigation Committee and published as a separate issue in place of the previous arrangement under which any such reports were published in both the *Journal* and the *Transactions*. The Councils now have pleasure in presenting this second number which includes the first reports prepared by the Permanent Health Insurance Sub-Committee of the Joint Committee, together with further reports on the experiences of immediate annuitants and of pensioners in the years 1967–70. Our special thanks are due to Mr J. Hamilton-Jones and his Sub-Committee for the preparation of the former, to Mr A. D. Wilkie for his extensive work in the graduations of the latter, and to the contributing offices for their continued support.

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INVESTIGATION OF SICKNESS STATISTICS

INDIVIDUAL POLICIES 1972 AND 1973

1. INTRODUCTION AND GENERAL SURVEY OF THE COMMITTEE'S WORK TO DATE

1.1. In 1970 the Life Offices' Association and the Associated Scottish Life Offices invited the Continuous Mortality Investigation Bureau to undertake an investigation into the sickness rates experienced under Permanent Health Insurance Policies. The Councils of the Institute and Faculty considered the question and on 12 June 1970 they appointed a Sub-Committee to examine the proposal. The constitution of the Bureau was amended so as to extend its activities to the collection of morbidity statistics and to the investigation of permanent health insurance.

1.2. The first United Kingdom insurance company to provide sickness and accident cover commenced to issue policies in 1885 and two life offices have been transacting this type of business for over 85 years. But it was possible to obtain this kind of cover well over a hundred years before the life offices first started to offer it. It was the principal type of cover offered by the Friendly Societies which were part of a growing movement in those days. The State entered the business in 1911 in partnership with the Friendly Society movement but the partnership ended in 1946 with the passing of the National Insurance Act of that year. Since then several life offices have entered the market in competition with the two old established ones.

Throughout the nineteenth and twentieth centuries there has been a need to collect morbidity statistics to enable the various institutions to calculate rates of premium or contribution but there has not been a great deal of pooling of data. The classic data pools in the United Kingdom were assembled by the Independent Order of Oddfellows Manchester Unity Friendly Society for various investigations in the nineteenth century, reaching a climax in the experience of 1893–97 from which standard tables were constructed. These standard tables are still in use, amended if necessary to reflect the experience of the institution using them. During the twentieth century data pools have been assembled under the United Kingdom National Insurance Scheme and there is a large quantity of data from the United States and Canada. It is also believed that some of the larger institutions have made their own investigations but the consensus of opinion is that none of the material available accurately represents the current morbidity experience of Permanent Health Insurance policyholders in the United Kingdom.

1.3. Permanent Health Insurance contracts are issued as ordinary policies on individual lives, or under group policies arranged with the employer of the lives concerned. The underwriting arrangements are different for group business. Benefits may be in the form of income during qualifying disability, lump sums on permanent disablement and waiver of premiums under life policies during disablement. Premiums may be level or varying annual premiums. In the case of group business the recurring single premium system ('current cost') method might be used.

1.4. The Sub-Committee has analysed the experience of individual policies in 1972 and 1973 and communicated the results to the contributing offices. Data for group business are being collected but results are not yet available.

1.5. When it first met, in the presence of Mr J. M. Denholm, the Chairman of the main C.M.I. Committee, the Sub-Committee comprised Messrs J. Hamilton-Jones (Chairman), P. H. Bayliss, D. B. Biggs, D. J. Bond, R. D. Clarke (Secretary), F. W. Eschrich and R. E. White. Unfortunately Mr R. E. White died on 11 February 1972; Mr D. B. Biggs retired from the Sub-Committee on 24 February 1972 and Mr R. D. Clarke retired from his position as Secretary later in 1972. Mr J. A. Cairns succeeded Mr Biggs and Mr R. E. Hayward, Assistant Secretary of the C.M.I., took over the position of Secretary to the Sub-Committee.

Five meetings were held between July 1970 and March 1971 to discuss the founding of the investigation. At the last of these meetings a draft Report was agreed and on 18 June it was submitted to the main C.M.I. Committee who discussed it in detail. After some amendments had been made the Report was circulated to offices in September 1971 with a covering letter inviting offices to comment and to state their estimate of the volume of new business they would be able to submit in 1972. They were also asked to say whether or not they would be able to contribute data for entrants prior to 1972 and whether any data they were able to submit would be on cards or magnetic tape.

The full text of this Report, as amended by subsequent instructions, is given in § 5 of this note.

Draft cards for recording the data were enclosed with the Report and offices were asked to comment or to send a preliminary reply by 15 October 1971. The versions finally adopted are given in § 2 of this note.

The response was both prompt and encouraging. Seventeen offices offered to contribute data and a rough estimate was made that the total 'in-force' business at 1 January 1972 would be of the order of 120,000 individual policies and subsequent new business of about 42,000 policies per year. So the Sub-Committee met on 15 November 1971 to prepare and issue a circular dated December 1971 which contained a revised version of the 'in-force' card.

Further meetings were held on 24 February 1972 and 31 May 1972. Mr H. A. R. Barnett, a senior official of the C.M.I. Bureau, was present at the first of these meetings to present a memorandum on the coding of causes of sickness and this document was accepted as the basis for the instructions to offices which are also given in § 2. Mr G. T. Humphrey, F.I.A., a specialist independent data processing practitioner, was present at the second of these meetings and he presented a preliminary report on the computer system needed for processing the data relating to individual policies. At this meeting a circular to be dated June 1972 was prepared to inform offices of the final version of the 'claims' card.

A meeting took place on 30 November 1972 and there were four meetings in 1973.

During the year 1973 the Sub-Committee was considering the computer systems which Mr Humphrey had submitted and a report outlining the problems of collecting data for group sickness schemes. Circulars were issued in March, July and October to amplify the instructions to offices and to give the instructions for submitting group business data. The climax of the year was the meeting of 12 December 1973 at which Mr Humphrey reported that he had received data for individual policies from nine offices for in-force at 31 December 1971 and claims in 1972 and from ten offices for in-force at 31 December 1972, and it was decided to run the programs and calculate the experience of these policies in 1972.

Before this could be done it was necessary to settle the precise nature of the exposed to risk formula and a meeting was held on 15 February 1974 for this purpose.

1.6. And so, on 3 July 1974, a draft Report on the experience of 1972 was presented. In its approved form it was circulated to all contributing offices in October 1974 and copies of their own results were sent to the offices which had contributed the data.

During 1974 and 1975 the experience of individual policies in 1973 was calculated, summarized and distributed. Meanwhile work is being continued on the collection of group data and it is hoped to issue a report in due course. A full description of the group investigation will also be published and it will include an explanation of the additional problems relating to that type of business.

2. METHOD OF COLLECTING THE DATA

2.1. It was decided to collect the data on 80 column punched cards or, by arrangement with the C.M.I. Bureau, on magnetic tapes with fields to match. A card was to be set up for each policy in force at the beginning of a record year and a card for each claim during the record year.

2.2. The following are the layouts of the in-force and claims cards finally adopted for individual business, incorporating all amendments to date and containing such instructions as are deemed to be necessary. These are followed by the instructions for coding the causes of sickness and, in § 3, by the report on the 1972 experience, in § 4 by the 1973 experience, in § 5 by an up-dated version of the original Report to Offices in 1971 giving the Plan for the Investigation and in § 6 by a description of the computer system as it was when the processing of the data for record year 1973 was completed. It is appropriate to mention here that the matters under discussion in this report are developing as the investigation goes on and as a result the Sub-Committee might change its views of the relative importance of those matters in later investigations.

2.3 Card layouts for individual business:

(1) In-force Card

Field	Columns	Description
Block A		
1	1	Type of Record 1 = individual record (2 = group record)
2	2–4	Contributor's 'office number'
3	5–6	Record Year The last two digits of the calendar year to the end of which the record refers.
4	7	Geographical Location 1 = United Kingdom 2 = Republic of Ireland 3 = Isle of Man 4 = Channel Islands (No other countries outside the British Isles have yet been specified by offices. The Committee will supply further codes on request.)
5	8	Please leave blank or code '0'
6	9	Age Definition Blank or zero if month and year of birth are given in field 11, otherwise 1 = nearest birthday, 2 = next birthday at the date referred to in field 3.
Block B		
7	16	Sex 1 = Male 2 = Female
8	17	Occupational Rating 0 = no rating 1 = rated
9	18–20	Period of Deferment. Code in weeks thus: 001 = 1 week, 052 = 52 weeks, etc., to nearest week, but use code 999 if the period of deferment is one *Calendar month*.
10	21–22	Year of Entry The last two digits of the calendar year in which the policy first went on the books. Code 00 if not known. Note: 'Continuation' policies—that is policies passing from group to individual under a continuation option—should not be included with the individual returns in

Field	Columns	Description
		cases where the disability started before the continuation policy was issued. In other cases the year of entry to be recorded is the year in which the continuation option was exercised. These policies should be coded '1' in column 1 and '3' in column 36
11	23–26	Month and Year of Birth or Office Year of Birth Contributors will have the option of showing the month of birth in columns 23–24 and the last two digits of the year of birth in columns 25–26, or of showing the office year of birth, which allows the calculation of the age next birthday or the nearest age at the date referred to in field 3, in columns 25–26 and zeros in columns 23–24. If possible, offices are requested to adopt the former method, since it is more accurate
12	27–28	Ceasing Year Last two digits of calendar year in which cover will cease.
13	29	Period of Benefit Payment Specify payment period to which rate shown in columns 30–34 relates: 1 = weekly 2 = monthly 3 = yearly 4 = special If the amount of business to which code 4 applies is a large proportion of the whole, the office is requested to approach the Sub-Committee for a separate code to be allocated.
14	30–34	Rate of Benefit Rate of benefit to the nearer £, gross of reinsurance. (*Excluding* waiver amount in every case if possible. Report 00 if the only benefit is waiver of premium, *e.g.* attached to life policy.) Note 1: Where code 2 or 3 applies in field 15, the initial rate of benefit should be shown. Note 2: If it is unnecessarily cumbersome to eliminate amounts of waiver of premium from office records, this need not be done. Please inform the Committee, however. Note 3: Reinsurances ceded to other offices are included in the ceding office's figures. Reinsurances accepted from other offices are not to be included in the investigation.

Field	Columns	Description
15	35	Type of Benefit 1 = level sickness benefit 2 = increasing sickness benefit 3 = decreasing sickness benefit 5 = lump sum benefit 9 = other type of benefit
16	36	Medical Evidence 1 = medical 2 = non-medical (with or without P.M.A. report) 3 = non-selection limit applies part or whole of benefit 4 = unknown (for existing business at 1 January 1972 only) Note: Medically substandard lives (other than those subject only to a special exclusion clause) are not to be included in the investigation.
17	37	Type of Premium 1 = level annual premium 2 = recurrent single premium 3 = increasing annual premium 4 = any other type, but see note for code 4 in field 13
18	38	Underwriting Impairment. (For cases dealt with by exclusions only. For occupational ratings see field 8. Other cases rated for health or dangerous pursuits, etc., should not be included in the investigation at all.) 0 = no extra risk 1 = exclusion relating to hypertension and disease of cardiovascular system 2 = exclusion relating to neurosis, psychoneuroses and psychosis (including anxiety state) 7 = exclusion may or may not be present (for business existing at 1 January 1972 only) 8 = exclusion present but related impairment not known (for business existing at 1 January 1972 only) 9 = all other exclusions Note: Codes 3–6 are being reserved for possible future use.

Block C

Field	Columns	Description
19	71–80	Policy Number Note: This field is reserved for the policy number or any other means by which the particular record can be referred to in any communications between the C.M.I. Bureau and the contributing office for error indications, etc.

Further notes:

1. Block A contains fields which can probably be gang-punched by the contributing offices.

   Block B contains information relating to the particular record, which will have to be individually punched.

   Block C contains only an item of identification, requiring individual punching.
2. Where data are submitted in the form of punched cards, these will be returned by the Bureau after the data have been transferred to tape. It would therefore be possible for the contributing office to use some of the space on the card for its own purposes. Initially offices would be asked not to use columns other than 43–70 in this way and it would not be possible to transfer such data to the claims card because those columns are used for the details of the claim.

(2) Claims Card

Field	Columns	Description
Block A		
1	1	Type of Record 3 = claim under individual policy (4 = claim under group policy)
2–6	2–9	As for In-force Card
Block B		
7–18	16–38	As for In-force Card Offices are asked to ensure that the information shown in Blocks A and B is consistent with that recorded in the corresponding 'in-force' card. If fresh information should come to light when a claim arises, it should be ignored for the purpose of compiling the claims card. For example if code 4 is used in column 36 of the in-force card it should be repeated on the claims card and not amended in accordance with information discovered later.
Block C		
19	44–49	Date of falling sick (*i.e.* beginning of deferred period). If present card relates to an interrupted claim (including a change from total to partial disability) record date of first falling sick. Date to be coded in three groups of two digits, day–month–year.
20	50–53	Date payments commenced (in present record year) in benefit period to which present card relates (day and month only: 0000 if continuation from previous year). A new card should be prepared each time a claim is resumed after an interruption or a change in degree of disability.

Field	Columns	Description
21	54	Mode of commencement of present Benefit 0 = continuation from previous record year 1 = new claim 2 = new claim following interruption of sickness in the deferred period 3 = revival of claim following interruption (whether the benefit rate is the same as before the interruption or different) 4 = continuation of an existing claim but benefit rate changed from date recorded in field 20
22	55–56	Percentage the benefit under the current claim bears to the full rate of benefit (for partial disability claim). Punch zeros if full rate is being paid.
23	57–60	Date payments ceased in benefit period to which present card relates (day and month only: 9999 if claim in force at end of year).
24	61	Mode of cessation 1 = policy expired or void for reason other than death or lump sum payment 2 = death 3 = recovery 4 = lump sum payment terminating contract (add explanatory note) 5 = *ex gratia* commutation (add explanatory note) 6 = benefit rate altered but claim continues (continuation reported on further card) Note: In the case of code 4 or 5 please give amount of payment as well as circumstances, e.g. whether contract was withdrawn. If the *ex gratia* commutation is one month's payment or less punch an adjusted expiry date in field 23 which would give correct total claim. This will not be practicable if the adjusted expiry date is after the current year of claim and in such a case explain in relation to field 24 what has been done.
25	62–65	Cause of disability for current claim. (Abbreviated 'List C' in the eighth revision of the *Manual of the International Statistical Classification of Diseases*. See separate instructions.)
26	71–80	Policy number or other identification. (See note to corresponding field 19 of in-force card.)

2.4. Instructions for coding cause of sickness:

(i) Always follow the latest available diagnosis.

(ii) If only one cause of sickness is shown, refer to the index in volume 2 of the *Manual of the International Statistical Classification of Diseases* (eighth revision). The coding selected from the index should then be confirmed in volume 1, in case there should be any notes modifying the coding. The coding selected should finally be converted to the appropriate number in the abbreviated 'List C' shown in pages 447–9 of volume 1, and where the sickness is due to injury the 'CE' code should be chosen rather than the 'CN' code. In time an experienced coder may in some cases come to know the codes and be able to go direct to 'List C', but this would be a dangerous practice until he is well experienced and, in any cases of doubt, reference should first be made to the main index and full tabular list.

(iii) If more than one cause is shown, and certain of the causes would be likely to cause short-term sickness only, whereas others would be likely to cause long-term sickness, discard the short-term causes.

(iv) If more than one cause is shown it is necessary to determine whether they are connected or unconnected; sometimes a connexion may be presumed from elementary medical knowledge (*e.g.* if both nephritis and uraemia are shown); sometimes a connexion can be detected from the linkage instructions on pages 427–32 of volume 1. In any case of connected causes the coding should be to the underlying cause or to the cause to which the linking instructions direct.

(v) If more than one unconnected cause is shown, discard any which are ill-defined (*e.g.* pyrexia) and any which are trivial (*e.g.* coryza).

(vi) If two diseases of the same site or of related sites are shown, one of which is a general term, the other a more specific one, prefer the specific one.

(vii) If two causes are shown, either of which could have been due to the other, and one is known to have been of longer duration, then assume that cause to be the underlying one.

(viii) If two or more unrelated causes are shown, or two or more ill-defined causes without any better-defined cause, or two or more trivial causes without any more serious cause, then in any of these cases prefer the first named.

(ix) Prefer a trivial but well-defined cause to an ill-defined cause.

(x) If there is any conflict in the above rules, the earlier rule takes precedence; thus, Rule (iii) takes precedence over Rules (iv)–(ix).

Notes

(a) The instructions given on pages 417–36 of volume 1 should be studied, but always remembering that they are based on the form of death certificate which frequently states explicitly cause (a) due to cause (b) due to cause (c), whereas the sickness coding will not usually be based on records given in this form, and therefore the general rule for cause of death coding cannot apply to sickness coding.

(b) The application of Rule (iii) or Rule (vii) will not necessarily give the same coding as in the cause of death classification.

(c) To ensure uniformity of practice, would offices treat the list which runs from C1 to C65 and from CE66 to CE70 as a series running from 0001 to 0070 and record the appropriate code in columns 62–65 of the claims card, omitting all reference to the prefixes 'C' or 'CE'.

(d) 'Depression' is not to be coded as 0065 (other specified ill-health causes) but as 0027 (psychoses and non-psychotic mental disorders) unless there is evidence to show that it falls within 0031 (other diseases of the nervous system and sense organs).

(e) The notes on pages 433–4 on interpretation of highly improbable relationships are very useful, but it must be realized that there are many other combinations of diseases which are unlikely to be related. If in doubt on this or on any other aspect, the C.M.I. Bureau will be glad to give assistance.

3. REPORT ON 1972 EXPERIENCE: INDIVIDUAL POLICIES

3.0. *INTRODUCTION*

3.0.1. The Sub-Committee has now evaluated the data needed for measuring the experience of the calendar year 1972 and results, summarized in quinquennial age groups, are given in Tables 1–10 of this report. Each table shows the exposed to risk, actual number of weeks' claims paid, the actual claim rates and percentages of actual to expected sickness subdivided into the following sickness periods: the notation in brackets indicates how the period is subsequently described in this report.

Over 1 week but not over 4 weeks (1/3)
Over 4 weeks but not over 13 weeks (4/9)
Over 13 weeks but not over 26 weeks (13/13)
Over 26 weeks but not over 52 weeks (26/26)
Over 52 weeks but not over 104 weeks (52/52)
Over 104 weeks (104/all).

3.0.2. The tables are numbered according to the following scheme:

	Table number	
Deferred period (weeks)	Males	Females
1	1	6
4	2	7
13	3	8
26	4	9
52	5	10

Some contributing offices make claim payments relating to the first week of sickness. Minor differences of interpretation of office practice in such cases would have led to difficulties in processing the data. To avoid this, claim payments made within 1 week of falling sick were ignored, the policies concerned being effectively treated as giving 1 week deferred benefits.

3.0.3. Before any attempt can be made to interpret these tables the Sub-Committee must report how, and from what precise data, they were constructed; this is done in § 3.1 of the report. Then, in § 3.2 a comparison is made between the present experience and the Manchester Unity experience of 1893–97, in § 3.3 a first step is taken towards the determining of claim inception rates to which will eventually be applied disability annuity factors when the data for calculating them have been assembled. In § 3.4 the numbers in force and the numbers of claims in the various subdivisions of the data are tabulated.

3.1. NATURE AND SCOPE OF THE DATA

3.1.1. The exposed to risk was calculated from tabulations by age nearest birthday of the business in force on 1 January and 31 December 1972 derived from the returns of ten offices. The following points were taken into consideration for the exposed to risk calculation.

3.1.2. With an investigation into mortality rates we are concerned with a happening which occurs only once for each insured life and where there is no element of duration of death. Death is instantaneous.

Under this investigation the statistic being measured is the extent of disability claim. Disability is a continuing feature—by this we mean that one spell of disability can continue over many years. This fundamental difference between an investigation into morbidity rates and into mortality rates implies that the normal census method used to calculate the exposed to risk in the Continuous Mortality Investigation might not be appropriate for this morbidity investigation.

The Sub-Committee used a method by which the claims for each experience group would be set against the total exposed to risk for that group as accurately as possible. Accordingly the exposed to risk for each experience group was calculated using the calendar year method under which the experience of a group is followed from age x at the beginning of the calendar year under investigation to age $(x+1)$ at the end of that calendar year. By this method, an insured continuously disabled throughout the experience year did not have his claim apportioned to two experience groups as would have been the case if the census method had been employed. The total exposed to risk in each experience group is made up of a full year for those included in the beginning and the end of year data and half a year for those included only at the beginning of the year, but special exposures apply to new business in the current and the two previous years as described in 3.1.3 below. The individual records of claimants were compared with the individual exposed to risk records and any claims which were not matched by exposed to risk were investigated.

3.1.3. Special considerations apply to new business and, depending upon the period of sickness, to business added in the previous year or the year before that. Obviously a new entrant in year y cannot claim in the 104/all period until year $y+2$. The Sub-Committee considered that offices would normally wish to take this into account in calculating their premium rates by adopting a formula for the net cost on the following lines:

Net cost of unit of benefit per week commencing on incapacity, the insurance terminating at age M, entry age x,

$$= \pi_{x:\overline{M-x}|}$$

$$= \left\{ {}^{M}K_{x}^{1/3} + {}^{M}K_{x+1/12}^{4/9} + {}^{M}K_{x+\frac{1}{4}}^{13/13} + {}^{M}K_{x+\frac{1}{2}}^{26/26} + {}^{M}K_{x+1}^{52/52} + {}^{M}K_{x+2}^{104/\text{all}} \right\} \times \frac{1}{N_x - N_M}$$

the notation being derived from that described in volume 1, chapter 17, of *Life and Other Contingencies* by Hooker and Longley-Cook. (Note: This formula is based on the principle that the insurance is included in the appropriate period of sickness to the extent that it is possible for a claim to become payable in that period assuming incapacity immediately following entry.)

The Sub-Committee have therefore adjusted the exposed to risk so that it is consistent with this formula and the average period of exposure at each claim period is shown in Table 11. As with the net premium formula the insurance is included in the exposed to risk in the appropriate period of sickness to the extent that it is possible for a claim to become payable in that period.

As an example of a calculation for Table 11, take the factor to apply to 1971 entrants to derive the exposed to risk in the sickness period 13/13 in 1972. Assume an even flow of new business. Clearly, only policies with deferred periods of 13 weeks or less are considered. So any policy which has been in force for more than 13 weeks at 1 January 1972 can enter the 13/13 claim period at any moment in 1972, *i.e.* three-quarters of the entrants are exposed for the full year. The remainder, who entered during the last quarter of 1971, can enter the 13/13 claim period in 1972 for at least 39 weeks, and at most (entrants at 1 October) 52 weeks—on average, say 7/8 of a year. The overall factor is thus

$$\frac{3}{4} \times 1 + \frac{1}{4} \times \frac{7}{8} = \frac{31}{32}$$

Having calculated the mean number of policies in force in 1972 for the years of entry shown in Table 11 for a group we are processing (*e.g.* males, deferred period 4 weeks, age x) the exposed to risk for the 13/13 period is:

$1 \times$ Mean in force, entry before 1970; $+1 \times$ Mean in force, entry in 1970; $+31/32 \times$ Mean in force, entry in 1971; $+9/16 \times$ Mean in force, entry in 1972 (taken as half the new entrants in force at the end of 1972).

No adjustment has been made for policies with a period of deferment of less than 4 weeks.

3.1.4. Those who are familiar with the details of the Manchester Unity investigation of 1893–97 will recollect that no adjustments of the kind just described were made at that time. The impact of recent entrants was very much less then. For example, the first Manchester Unity investigation covered the years 1846–48 when the exposed to risk was 621,561 compared with 2,995,724 in 1893–97, an average annual increase of about $3\frac{1}{4}\%$, whereas in only one year, 1972, the data for the present investigation has risen by just under 14% from 137,831 at 1 January 1972 to 157,098 at 31 December 1972. The result is that the business in the present investigation is rather heavily weighted with recently written business, as can be seen from any of the Tables 1–10 where exposed to risk at the older ages, where there is less new business, falls less rapidly with regard to claim period than it does at the younger ages where new business is more prominent.

Another indication of the rapid growth in this class of business is the proportion of the business in force which is of less than 3 years' duration. This varies according to the deferred period as follows:

Deferred period (weeks)	Percentage of 'in-force' which is of less than 3 years' duration
1	22
4	44
13	47
26	55
52	52

The method of deriving the exposed to risk leads to the central rates of claim at each age. The Sub-Committee can form no view at this stage of the mortality rates appropriate to lives insured under sickness policies included in the investigation. The central rate of claim, whose value is virtually unaffected by mortality, is therefore considered to be the most appropriate for tabulation of the results. This follows the precedent of the Manchester Unity 1893–97 experience; the tabulated rates in that experience were central rates.

3.1.5. The claims were classified according to age nearest birthday at the beginning of the calendar year in which they occurred and to periods of sickness passed through; sickness which terminated before the end of the deferred period was not recorded in the claims data. This means that the claims under policies with a 52-week deferred period are for relatively serious illnesses which have lasted for a year before they are recorded whilst claims under policies with a 1-week deferred period may well contain a fair number of relatively trivial ailments which would never have been reported had the deferred period been longer.

3.1.6. The following table shows the numbers of policies in force and of claims during the year.

	Males	Females	Total
Number of policies in force on 1 January 1972	132,660	5,171	137,831
Number of policies in force on 31 December 1972	150,681	6,417	157,098
Number of claims during 1972	4,967	249	5,216

Most of the business was for level sickness benefits at level annual premiums, written in the United Kingdom or the Republic of Ireland.

3.2. COMPARISON WITH MANCHESTER UNITY SICKNESS RATES

3.2.1. The expected number of weeks of claim were calculated on the basis of the Manchester Unity (1893–97) sickness rates for males in occupation group AHJ (the least hazardous occupations). The percentages of actual to expected claims shown in Tables 1–10 should be studied in the light of the following remarks.

The percentages shown in the tables were calculated from figures which had been calculated by the computer to two places of decimals and so it may not always be possible to reproduce them exactly from those in the tables; this is also a feature of the actual claim rates in Tables 1–10.

In order to deal with the comparison for the '4 weeks deferred' policies it was thought advisable to break down the shortest tabulated period of sickness (*i.e.* the first 3 months) of the M.U. table into 'first 4 weeks' and 'next 9 weeks' periods. There are so many reasons for hedging the expected rates with reservations that refined methods were avoided. Moreover, no attempt was made to deal separately with the first week of sickness and the next 3 weeks.

As a basis for subdividing the M.U. first 3 months' sickness, it was considered reasonable to derive ratios from current National Insurance statistics. The Government Actuary's Department kindly supplied figures relating to employed and self-employed persons combined for the years (June–May) 1969–70 and 1970–71. The Sub-Committee applied rates for individual ages based on the National Insurance figures given in Table 12. As there are no females in the M.U. experience, only male lives were considered.

3.2.2. The Sub-Committee believes that it is helpful to present, in tabular and graphical form, aggregate rates which summarize the most important results so far. These are the claim rates for males (i) for all periods combined (first 4 weeks, next 9 weeks, etc.) for policies subject to 1 week's deferred period; and (ii) for all periods after 26 weeks combined, relating to all policies contributing to the males experience with a deferred period of 26 weeks or less.

The graphs (Fig. 1) show the central claim rates for age groups 20–24, 25–29, etc., plotted for convenience as points at the mid-ages of the groups. For comparison, they show the Manchester Unity rates, AHJ, for 'all sickness' and 'all sickness after 6 months' respectively, at the mid-ages.

In a theoretical population where l_x is the number living according to a mortality table, the central rate of sickness at age x is the number of weeks of sickness recorded between exact ages x and $x+1$ divided by the central population

$$\int_0^1 l_{x+t}\,dt$$

The standard deviation of a rate of sickness may take the form

$$\sqrt{(\text{expected weeks of sickness})} \times (\text{a factor}).$$

A reference to the problem will be found in *J.I.A.* (1972), **98**, 35 and *J.I.A.* (1949) **75**, 12. The 1949 reference is to a paper by L. E. Coward, who tabulated theoretical values of the ratio of the standard deviation of sickness (in weeks) to the square root of expected sickness (in weeks) using the Manchester Unity sickness experience 1893–97 (Whole Society). These ratios ranged from 2·30 at age 20 to 2·69 at age 70 for the first 3 months' sickness and from 2·84 at age 20 to 5·44 at age 70 for all periods of sickness. He demonstrated that the maximum value this ratio can possibly take is just over 7.

The Sub-Committee believes that the Manchester Unity experience may not be suitable as a standard table for such calculations in the present investigation, but hopes the above remarks are helpful.

3.2.3. The graphs have been plotted on a 3-cycle logarithmic scale for the rates and a unitary scale for the ages. The vertical gap between the Manchester Unity graph and the corresponding 'results' graph is thus log (M.U. rate)/(actual rate).

It is hoped that the visual impression of the graphs adequately reflects the variation of the rates, and of the ratio of actual to expected rate, with age.

3.2.4. In studying the rates for the different deferred periods it should be borne in mind that one cannot prejudge the effect of the deferred period on the experienced sickness. Few offices at present issue immediate benefit policies on a significant scale. If one compares the 'after 26 weeks' rates of sickness for such policies with the 'after 26 weeks' rates for 6-months deferred policies, it is not possible to attribute the differences to the effect of the deferred period. Subject to further evidence in the future, it seems more likely that the differences are largely due to other characteristics of the portfolios of contributing offices. For example, an office may contribute a high proportion of the data for policies with a short deferred period but a lower proportion of the data for longer deferred periods, because the data for longer deferred periods is submitted by a larger number of offices. Thus its claims experience would affect the overall rate of claim for short deferred periods more than the rate for long deferred periods.

3.2.5. Due to the irregularity in the progression of rates, in age groups 40–44 and 50–54 particularly, a special investigation of ages 40–44 was carried out.

The suspicious feature is illustrated in Table 1, where the 52/52 rate is much higher than would be reasonable in relation to the 26/26 or 104/all figure. There are other examples of what would in a mature experience be impossible or, at best, freak results.

Every claim for the 1-week deferred class was, therefore, inspected for the test ages 40–44; the effect described was due to some multiple policies on individual lives. At age 42, three policies on one life contributed 284 days to the 52/52 period and 81 days to the 104/all period during 1972.

This special investigation underlines the provisional nature of the results of a single year's experience. The Sub-Committee is convinced of its duty to present these results with strong warnings that they lead to no firm conclusions.

3.2.6. It appears worthwhile to comment on one likely trend. Comparison of the results in the 52/52 and 104/all periods demonstrates that for the relationship between these two results to remain at the present level, approximately 50% of the claims in the 52/52 period and 25% of the claims in the 104/all period must terminate within the following year. For the longer periods of claim one of the main causes of termination of claim is mortality, the rate of recovery varying inversely with the length of the period of incapacity. Except at the older ages where the claim terminates with the expiry of the insurance, it is not likely that these rates of recovery will be attained so that we may expect to see an overall increase in the 104/all claims rate in future years.

3.2.7. The data for females are relatively scanty but Tables 1–10 give the general impression that the females' experience is heavier than the males' experience. A rough measure was obtained by calculating for both sexes and all deferment types separately the overall claim rates for all ages and all durations combined, *i.e.* the sum of the rates for all sickness periods following the deferred period stated. The results are shown below:

Deferred period (weeks)	Overall claim rate Males (1)	Females (2)	(2)/(1)%
1	1·03	1·88	183
4	0·39	0·60	154
13	0·22	0·48	218
26	0·23	0·14	61
52	0·07	0·37	529

These overall claim rates include the data for lives over age 60, but in the case of females there were comparatively few such lives.

3.3. CLAIM INCEPTION RATES

3.3.1. Claim inception rates for males were calculated by dividing the numbers of new claims in 1972 by the appropriate exposed to risk, namely for 1 week deferred benefits, the exposed to risk for period 1/3, for 4 weeks deferred benefits.

the exposed to risk for period 4/9, for the 13 weeks deferred benefits, the exposed to risk for period 13/13, for the 26 weeks deferred benefits the exposed to risk for period 26/26, and for the 52 weeks deferred benefits, the exposed to risk for period 52/52. The male results are shown in Table 13 in quinquennial age groups and the female results are shown for all ages combined.

3.3.2. There are three features which seem worthy of comment. First for 1 week deferred benefits up to age group 55–59 the male claim inception rate is surprisingly constant. Secondly, there is a high inception rate for the age groups 20–24 for almost all deferred periods. It is probably premature to investigate why this should be, but recent mortality investigations have shown a similar feature. Thirdly, the overall inception rate for females is higher than for males. In view of the small numbers of females involved any breakdown of this comparison into age groups is probably superfluous this year.

3.3.3. The Sub-Committee plans to calculate disability annuities later on, but the data of several more years will be needed before a start can be made on this.

3.4. OTHER TYPES OF SUBDIVISION AVAILABLE

3.4.1. The three previous sections of this report have dealt with the primary subdivision of the data into male and female. The Sub-Committee has collected details to enable other subdivisions to be made and the following table gives details of what is available according to the various attributes:

	Number of Policies at		
Attribute	1 January 1972	31 December 1972	Number of Claims in 1972
Sex			
Male	132,660	150,681	4,967
Female	5,171	6,417	249
Country			
United Kingdom	134,276	152,835	5,102
Republic of Ireland	3,464	4,140	114
Isle of Man	1	4	0
Channel Islands	90	119	0
Occupation Rating			
No rating	117,000	140,163	4,558
Rated	20,831	16,935	658
Type of Benefit			
Level	126,415	142,945	3,806
Increasing	3,694	6,782	45
Decreasing	7,678	7,312	1,365
Waiver only	0	0	0
Lump sum	44	59	0
Medical Evidence			
Medical	9,828	18,271	154

(*cont.*)

Attribute	Number of Polices at 1 January 1972	Number of Polices at 31 December 1972	Number of Claims in 1972
Non-medical	5,217	15,270	154
Non-selection limit applies	5	19	0
Unknown	122,781	123,538	4,908
Type of Premium			
Level Annual	137,799	157,064	5,216
Recurring single	3	3	0
Increasing annual	0	0	0
Any other type	29	31	0
Underwriting Impairment			
No extra risk	84,099	99,313	4,162
Exclusion exists for hypertension or for cardiovascular reasons	28	45	7
for neurosis	438	634	67
Unknown whether exclusion exists	48,745	51,959	538
Exclusion known to exist but condition unknown	1,956	1,793	130
All other exclusions	2,565	3,354	312

3.4.2. These figures show that there is a large proportion of policies for which important attributes at the outset were unknown. Until the requirements of the present investigation were made known, the contributing offices were not recording data in the appropriate form. But from then onwards the offices did start to record the required data and it would have been unreasonable to ask offices to reopen their old files, the more so as the business is growing rapidly and the old files will assume a lesser significance as time goes on. The above table shows how the 'unknown' class is tending to diminish in importance and it may well be of only minor significance when the investigation is fully under way. The Sub-Committee considers that the crude claim rates, obtained by dividing the number of claims for each attribute by the mean in-force figures, are of little value because of the changing nature of the age and duration structure of the business and so no figures have been shown.

3.4.3. The following figures describing the claims are interesting because they indicate areas of the data which might be explored from the point of view of the setting up or adjusting of office administration routines.

3.4.4. It is clear that the claims fall into two main entry groups and that in the main they either recover or continue as claims. But there are small pockets of complicated cases which must produce more than a proportionate share of administrative problems. From a purely statistical point of view it is tempting

Attribute	Number of claims in 1972
Mode of commencement	
Continuation from previous record year	992
New claim	4,093
New claim following interruption of sickness in deferred period	2
Revival of claim following interruption	86
Continuation of claim but benefit rate changed	43
Rate of benefit	
Full rate being paid	5,130
Reduced rate being paid	86
Mode of cessation	
No cessation by 31 December 1972	1,003
Policy expired or void	45
Death	68
Recovery	4,055
Lump sum paid	4
Ex gratia commutation	0
Benefit rate altered but claim continues	41

This analysis does not include a small number of late claims submitted with the 1973 data whereas Tables 1–10 and 13 do include these late claims.

to concentrate upon the main causes of entry and exit, but if the Sub-Committee is to give a worthwhile service to the offices as opposed to providing a summary of the statistically significant data only, then it has a duty to pursue matters which, whilst relating to small numbers of policies, are important both to the offices and their claimants. It is hoped that offices will feel able to continue to supply statistics for the minority classes even though, as yet, the Sub-Committee is not able to interpret them fully.

3.5. ACKNOWLEDGMENTS

3.5.1. Having reached this stage the Sub-Committee must place on record its indebtedness to offices who have provided data for investigation and to the many individuals who have worked to establish it. A list of the offices covered by the 1972 investigation appears below. Thanks are also due to the Government Actuary's Department for supplying the information which enabled us to calculate expected claims in a suitable form.

J. Hamilton-Jones
Chairman

Offices which supplied data for 1972:

Clerical, Medical & General
Commercial Union
Eagle Star
Friends' Provident
Guardian
Legal & General
Medical Sickness Group
Norwich Union
Yorkshire General

INDIVIDUAL PHI POLICIES 1972 EXPERIENCE

Users of these tables should note that the experience is heavily weighted with recently written business. See paragraph 3.1.4 and §3.2.

In Tables 1 and 6, expected weeks of sickness for period 1/3 were based on estimated M.U. sickness in the first 4 weeks. No attempt was made to treat the first week separately. See paragraph 3.2.1.

Table 1. *Males—Deferred period* 1 *week* (1972 *experience*)

	Exposed to risk						Actual weeks of sickness						Actual claim rate						Actual weeks of sickness/expected (%)					
Age	1/3	4/9	13/13	26/26	52/52	104/all	1/3	4/9	13/13	26/26	52/52	104/all	1/3	4/9	13/13	26/26	52/52	104/all	1/3	4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20–24	404	381	337	278	180	65	80	19	—	—	—	—	·198	·050	—	—	—	—	40	34	—	—	—	—
25–29	2,310	2,263	2,168	2,025	1,736	1,179	366	181	59	30	—	—	·158	·080	·027	·015	—	—	34	47	32	25	—	—
30–34	2,527	2,504	2,459	2,391	2,257	1,971	542	312	122	50	3	—	·214	·124	·049	·021	·001	—	46	64	51	29	2	—
35–39	2,686	2,671	2,641	2,597	2,507	2,308	468	329	187	180	88	197	·174	·123	·071	·069	·035	·085	36	55	58	76	51	57
40–44	3,190	3,176	3,148	3,107	3,026	2,855	688	497	179	162	323	255	·216	·156	·057	·052	·107	·089	41	56	36	42	108	54
45–49	3,528	3,520	3,504	3,480	3,430	3,329	749	700	284	153	241	620	·212	·199	·081	·044	·070	·186	38	58	40	26	51	46
50–54	2,660	2,656	2,648	2,636	2,612	2,563	617	426	226	177	711	725	·232	·160	·085	·067	·272	·283	38	36	30	27	125	40
55–59	2,651	2,649	2,645	2,640	2,632	2,612	817	975	394	375	615	1,305	·308	·368	·149	·142	·234	·500	44	65	37	35	58	59
60–64	1,637	1,636	1,636	1,635	1,632	1,627	590	758	412	559	596	2,327	·360	·463	·252	·342	·365	1·430	46	64	42	51	48	61
Total	21,593	21,456	21,186	20,789	20,012	18,509	4,917	4,197	1,863	1,686	2,577	5,429							40	56	39	39	64	47

Table 2. *Males—Deferred period 4 weeks (1972 experience)*

Age	Exposed to risk 4/9	13/13	26/26	52/52	104/all	Actual weeks of sickness 4/9	13/13	26/26	52/52	104/all	Actual claim rate 4/9	13/13	26/26	52/52	104/all	Actual weeks sickness/expected (%) 4/9	13/13	26/26	52/52	104/all
18–19	51	44	35	20	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20–24	1,570	1,421	1,208	823	325	131	41	35	—	—	·083	·029	·029	—	—	56	37	59	—	—
25–29	4,843	4,591	4,222	3,518	2,339	305	134	28	—	—	·063	·029	·007	—	—	37	34	11	—	—
30–34	4,934	4,739	4,453	3,903	2,971	418	152	78	64	16	·085	·032	·018	·016	·005	44	33	25	31	6
35–39	5,164	5,017	4,799	4,376	3,612	533	186	42	85	15	·103	·037	·009	·019	·004	46	30	10	28	3
40–44	4,628	4,535	4,394	4,113	3,603	648	346	304	130	304	·140	·076	·069	·032	·084	50	48	56	32	33
45–49	3,399	3,552	3,280	3,129	2,837	731	385	214	255	377	·215	·115	·065	·081	·133	63	57	40	59	31
50–54	1,925	1,906	1,876	1,815	1,699	415	245	232	253	434	·216	·129	·124	·139	·256	49	46	51	65	36
55–59	936	931	924	909	876	241	153	172	233	503	·257	·164	·186	·256	·574	46	41	46	65	45
60–64	276	276	275	273	266	129	86	74	79	285	·469	·313	·271	·289	1·070	66	54	42	39	47
Total	27,726	27,012	25,466	22,879	18,531	3,551	1,728	1,179	1,099	1,934						49	43	37	11	32

Table 3. *Males—Deferred period* 13 *weeks* (1972 *experience*)

Age	Exposed to risk				Actual weeks of sickness				Actual claim rate				Actual weeks sickness/expected (%)			
	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all
18–19	25	21	12	2	—	—	—	—	—	—	—	—	—	—	—	—
20–24	900	769	529	201	19	26	7	—	·021	·034	·013	—	27	69	45	—
25–29	3,915	3,553	2,845	1,652	55	20	83	19	·014	·006	·029	·011	16	9	73	22
30–34	4,760	4,477	3,914	2,881	42	42	44	—	·009	·009	·011	—	9	13	21	—
35–39	5,560	5,312	4,801	3,845	132	57	—	52	·024	·011	—	·013	20	12	—	9
40–44	5,278	5,096	4,724	4,000	130	123	121	401	·025	·024	·025	·100	15	20	26	39
45–49	4,036	3,935	3,733	3,313	91	71	74	349	·023	·018	·019	·105	11	11	14	26
50–54	2,296	2,260	2,181	2,014	221	200	124	376	·096	·089	·056	·186	34	37	27	26
55–59	1,361	1,350	1,325	1,277	249	356	152	601	·183	·264	·114	·470	46	66	29	37
60–64	502	501	500	497	80	95	283	454	·160	·189	·565	·913	28	29	78	41
Total	28,633	27,274	24,564	19,682	1,019	990	888	2,252					22	26	30	30

Table 4. *Males—Deferred period* 26 *weeks* (1972 *experience*)

Age	Exposed to risk			Actual weeks of sickness			Actual claim rate			Actual weeks sickness/expected (%)		
	26/26	52/52	104/all	26/26	52/52	104/all	26/26	52/52	104/all	26/26	52/52	104/all
18–19	43	25	5	—	—	—	—	—	—	—	—	—
20–24	2,516	1,558	374	51	2	—	·020	·001	—	41	4	—
25–29	6,788	5,055	2,368	44	28	28	·006	·005	·011	11	14	24
30–34	6,590	5,481	3,545	70	106	2	·010	·019	—	15	36	1
35–39	6,810	5,985	4,489	111	92	—	·016	·015	—	18	22	—
40–44	6,847	6,293	5,205	119	117	153	·017	·018	·029	14	19	11
45–49	5,469	5,157	4,542	219	226	245	·040	·043	·053	24	32	14
50–54	3,946	3,814	3,542	244	270	1,299	·061	·070	·366	25	33	52
55–59	2,562	2,535	2,471	345	451	1,287	·134	·177	·520	34	45	41
60–64	1,010	1,008	996	193	314	1,072	·191	·310	1·075	29	43	48
Total	42,581	36,911	27,537	1,396	1,606	4,086				23	33	33

Table 5. *Males—Deferred period 52 weeks* (1972 *experience*)

Age	Exposed to risk 52/52	Exposed to risk 104/all	Actual weeks of Sickness 52/52	Actual weeks of Sickness 104/all	Actual claim rate 52/52	Actual claim rate 104/all	Actual weeks Sickness expected (%) 52/52	Actual weeks Sickness expected (%) 104/all
18–19	8	2	—	—	—	—	—	—
20–24	152	48	—	—	—	—	—	—
25–29	890	470	—	—	—	—	—	—
30–34	1,511	958	32	—	·021	—	39	—
35–39	2,141	1,581	—	—	—	—	—	—
40–44	2,475	1,974	—	—	—	—	—	—
45–49	1,949	1,662	126	246	·064	·147	47	37
50–54	1,117	979	29	36	·026	·036	12	5
55–59	551	521	58	59	·106	·112	27	9
60–64	126	121	12	40	·092	·333	13	15
Total	10,920	8,316	257	381	—	—	19	12

Table 6. *Females—Deferred period* 1 *week* (1972 *experience*)

Age	Exposed to risk 1/3	4/9	13/13	26/26	52/52	104/all	Actual weeks of sickness 1/3	4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—	—	—
20–24	37	35	30	24	14	3	8	19	—	—	—	—
25–29	88	86	82	75	61	33	12	12	2	—	—	—
30–34	70	69	67	64	58	47	25	16	2	—	—	—
35–39	87	86	84	82	78	70	16	18	—	—	—	—
40–44	96	95	94	91	88	83	25	41	26	32	32	52
45–49	175	174	174	173	170	164	74	118	43	1	19	85
50–54	89	89	89	89	89	87	34	40	6	26	19	—
55–59	69	69	69	69	69	68	55	65	51	52	—	104
60–64	15	15	15	15	15	15	6	3	—	—	—	104
Total	726	718	704	682	642	570	255	332	130	111	70	345

Age	Actual claim rate 1/3	4/9	13/13	26/26	52/52	104/all	Actual weeks sickness/expected (%) 1/3	4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—	—	—
20–24	·223	·530	·004	—	—	—	46	357	—	—	—	—
25–29	·133	·134	·020	—	—	—	28	80	24	—	—	—
30–34	·353	·231	·025	—	—	—	75	120	26	—	—	—
35–39	·181	·213	—	—	—	—	37	97	—	—	—	—
40–44	·261	·424	·272	·350	·367	·622	50	151	170	279	368	236
45–49	·424	·672	·245	·003	·112	·515	76	196	122	2	82	130
50–54	·376	·446	·068	·291	·211	—	62	103	25	123	101	—
55–59	·796	·931	·731	·754	—	1·527	114	166	184	190	—	121
60–64	·371	·180	—	—	—	6·622	47	25	—	—	—	293
Total							64	145	93	95	70	121

Table 7. *Females—Deferred period* 4 *weeks* (1972 *experience*)

	Exposed to risk					Actual weeks of sickness					Actual claim rate					Actual weeks sickness/expected (%)				
Age	4/9	13/13	26/26	52/52	104/all	4/9	13/13	26/26	52/52	104/all	4/9	13/13	26/26	52/52	104/all	4/9	13/13	26/26	52/52	104/all
18–19	2	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20–24	103	91	75	48	19	20	4	—	—	—	·191	·048	—	—	—	130	63	—	—	—
25–29	244	233	218	188	137	33	23	19	—	—	·134	·097	·086	—	—	79	115	144	—	—
30–34	196	188	177	158	129	49	3	—	—	—	·249	·018	—	—	—	129	19	—	—	—
35–39	198	192	184	167	131	30	9	—	4	—	·152	·044	—	·025	—	69	36	—	37	—
40–44	194	190	184	171	146	28	13	13	37	35	·145	·068	·072	·216	·239	52	43	59	219	92
45–49	146	143	139	131	117	45	15	18	8	96	·306	·102	·125	·061	·818	89	50	76	45	204
50–54	78	76	75	72	67	26	9	26	22	—	·328	·116	·345	·296	—	75	42	144	139	—
55–59	32	32	31	31	29	23	12	—	—	—	·696	·376	—	—	—	124	95	—	—	—
60–64	9	9	9	9	9	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
Total	1,202	1,156	1,093	975	784	254	88	76	71	131						84	53	59	73	58

Table 8. *Females—Deferred period* 13 *weeks* (1972 *experience*)

Age	Exposed to risk 13/13	26/26	52/52	104/all	Actual weeks of sickness 13/13	26/26	52/52	104/all	Actual claim rate 13/13	26/26	52/52	104/all	Actual weeks sickness/expected (%) 13/13	26/26	52/52	104/all
18–19	3	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—
20–24	56	46	30	13	—	—	—	—	—	—	—	—	—	—	—	—
25–29	166	150	124	80	8	26	16	—	·045	·173	·126	—	53	288	—	—
30–34	161	164	137	94	10	26	16	—	·056	·158	·114	—	59	224	216	—
35–39	228	218	197	154	18	12	—	—	·077	·056	—	—	64	62	—	—
40–44	260	248	224	182	33	4	—	52	·128	·014	—	·284	81	11	—	103
45–49	240	231	216	187	25	—	—	139	·106	—	—	·742	53	—	—	187
50–54	130	127	122	110	17	26	2	—	·129	·203	·018	—	46	83	9	—
55–59	52	51	50	48	4	—	—	52	·070	—	—	1·069	18	—	—	84
60–64	18	18	18	17	—	—	—	14	—	—	—	·772	—	—	—	31
Total	1,314	1,255	1,119	885	115	94	34	257					52	53	24	75

Table 9. *Females—Deferred period* 26 *weeks* (1972 *experience*)

Age	Exposed to risk			Actual weeks of sickness			Actual claim rate			Actual weeks sickness/expected (%)		
	26/26	52/52	104/all	26/26	52/52	104/all	26/26	52/52	104/all	26/26	52/52	104/all
18–19	2	1	—	—	—	—	—	—	—	—	—	—
20–24	43	27	11	—	—	—	—	—	—	—	—	—
25–29	103	71	34	—	—	—	—	—	—	—	—	—
30–34	193	156	104	—	—	—	—	—	—	—	—	—
35–39	283	235	168	36	8	—	·125	·035	—	137	51	—
40–44	378	339	265	25	—	—	·064	—	—	52	1	—
45–49	291	267	225	8	—	—	·026	—	—	16	—	—
50–54	162	152	134	—	—	—	—	—	—	—	—	—
55–59	93	91	87	—	48	57	—	·522	·649	—	140	53
60–64	14	14	13	—	—	—	—	—	—	—	—	—
Total	1,562	1,353	1,041	69	56	57				30	32	13

Table 10. *Females—Deferred period* 52 *weeks* (1972 *experience*)

Age	Exposed to risk		Actual weeks of sickness		Actual claim rate		Actual weeks sickness/expected (%)	
	52/52	104/all	52/52	104/all	52/52	104/all	52/52	104/all
18–19	—	—	—	—	—	—	—	—
20–24	6	1	—	—	—	—	—	—
25–29	26	14	—	—	—	—	—	—
30–34	40	24	—	—	—	—	—	—
35–39	76	56	—	—	—	—	—	—
40–44	103	77	39	65	·377	·837	378	314
45–49	76	57	—	—	—	—	—	—
50–54	54	46	—	9	—	·185	—	27
55–59	26	25	—	9	—	·338	—	28
60–64	2	2	—	—	—	—	—	—
Total	409	302	39	83			75	67

Table 11. *Factors to apply to the mean in-force,* 1972, *to correct for recent entry* (1972 *experience*)

Year of entry	Factor (fraction of a year) for sickness period 1/3	4/9	13/13	26/26	52/52	104/all
Before 1970	1	1	1	1	1	1
1970	1	1	1	1	1	$\frac{1}{2}$
1971	1	$\frac{287}{288}$	$\frac{31}{32}$	$\frac{7}{8}$	$\frac{1}{2}$	—
1972	1	$\frac{121}{144}$	$\frac{9}{16}$	$\frac{1}{4}$	—	—

Table 12. *Proportion of first* 3 *months' sickness* (*Manchester Unity*) *falling in first* 4 *weeks* (*see assumptions in* 3.2.1) (1972 *experience*)

Age	Proportion	Age	Proportion
18	·82	52	·58
22	·78	57	·55
27	·73	62	·52
32	·71	67	·50
37	·69		
42	·65		
47	·62		

Values for other ages were interpolated.

Table 13. *Claim inception rates per thousand exposed to risk* (1972 *experience*)

Age	*Deferred Period* 1 week M	1 week F	4 weeks M	4 weeks F	13 weeks M	13 weeks F	26 weeks M	26 weeks F	52 weeks M	52 weeks F
20–24	153	106	18	29	2	—	2	—	—	—
25–29	122	73	11	20	2	—	1	—	—	—
30–34	136	263	15	46	1	—	—	—	1	—
35–39	125	126	19	30	2	9	—	7	—	—
40–44	133	155	24	46	4	12	1	3	—	—
45–49	117	194	35	55	3	17	2	—	1	—
50–54	128	162	30	51	8	15	3	—	—	—
55–59	161	380	43	123	13	19	5	—	1	—
60–64	178	190	56	—	20	—	8	—	—	—
Total	135	182	22	40	4	9	2	2	—	—

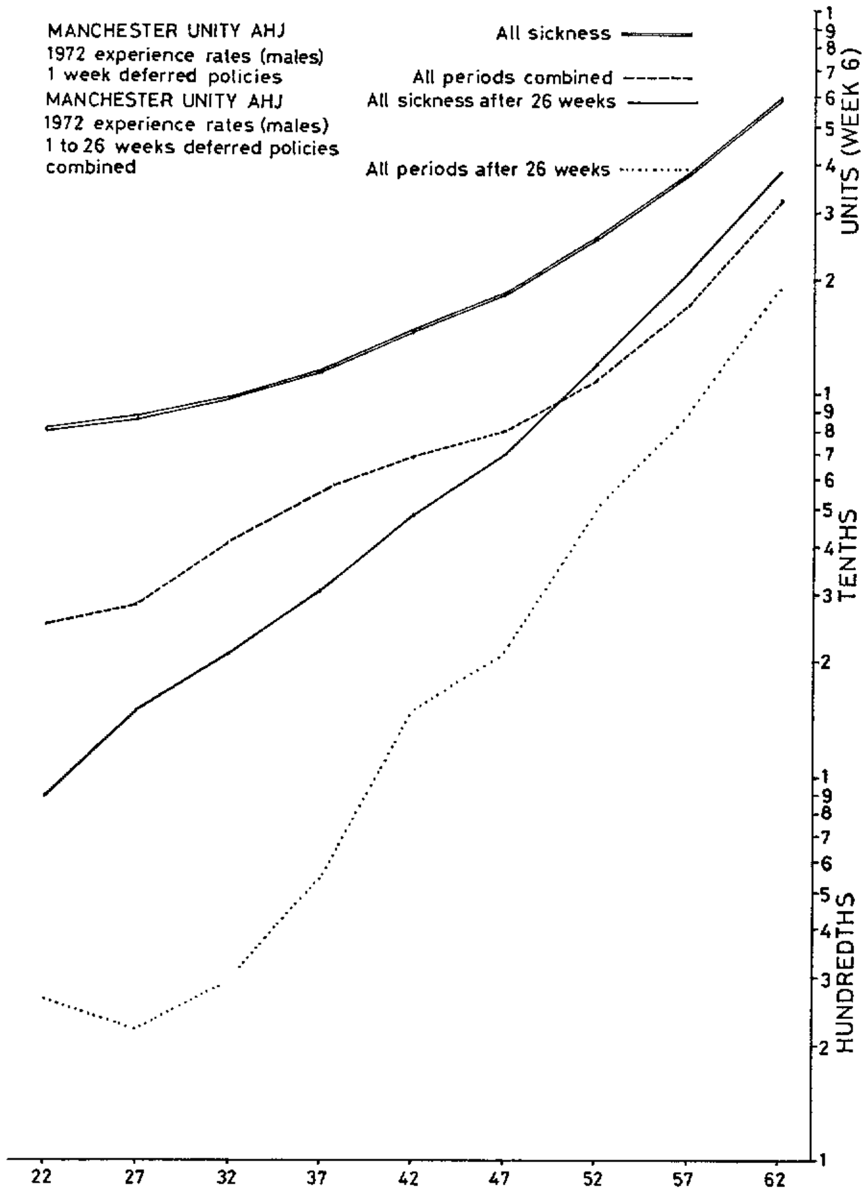

Figure 1. M.U. Sickness rates and C.M.I. 1972 claim rates

4. EXPERIENCE OF INDIVIDUAL POLICIES 1973

4.1. The data for 1973 comprised the following numbers of policies and claims:

	Males	Females	Total
Number of policies in force on 1 January 1973	150,681	6,417	157,098
Number of policies in force on 31 December 1973	156,754	6,445	163,199
Number of claims during 1973	4,992	242	5,234

As in the previous year, the majority of the policies were on male lives issued in the United Kingdom for level benefits at level annual premiums. An analysis of the experience is given in the following tables.

INDIVIDUAL PHI POLICIES 1973 EXPERIENCE

Users of these tables should note that the experience is heavily weighted with recently written business. See paragraph 3.1.4 and § 3.2 of the 1972 Report.

In Tables 1 and 6, expected weeks of sickness for period 1/3 were based on estimated M.U. sickness in the first 4 weeks. No attempt was made to treat the first week separately. See paragraph 3.2.1.

The following tables are not strictly comparable with those for 1972. Late notified claims are not included in 1973 whereas they were included in 1972.

Table 1. *Males—Deferred period* 1 *week* (1973 *experience*)

Age	Exposed to risk 1/3	4/9	13/13	26/26	52/52	104/all	Actual weeks of sickness 1/3	4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—	—	—
20–24	375	352	309	253	164	66	69	25	—	—	—	—
25–29	2,517	2,470	2,376	2,233	1,937	1,340	309	120	25	—	52	—
30–34	2,581	2,561	2,519	2,453	2,313	2,023	445	193	36	25	—	—
35–39	2,792	2,778	2,749	2,706	2,616	2,437	495	270	88	39	51	131
40–44	3,099	3,089	3,067	3,031	2,947	2,775	679	422	147	247	278	488
45–49	3,520	3,514	3,500	3,479	3,432	3,322	773	673	379	265	220	551
50–54	2,835	2,832	2,825	2,814	2,789	2,738	724	556	151	199	239	988
55–59	2,519	2,517	2,514	2,509	2,499	2,480	731	858	508	548	659	1,882
60–64	1,624	1,624	1,624	1,623	1,621	1,615	691	911	584	616	764	2,297
Total	21,862	21,737	21,483	21,101	20,318	18,796	4,916	4,028	1,918	1,939	2,263	6,337

Age	Actual claim rate 1/3	4/9	13/13	26/26	52/52	104/all	Actual weeks sickness/expected (%) 1/3	4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—	—	—
20–24	·185	·072	—	—	—	—	38	48	—	—	—	—
25–29	·123	·049	·010	—	·027	—	26	28	12	—	68	—
30–34	·172	·076	·014	·010	—	—	37	39	15	15	—	—
35–39	·177	·097	·032	·015	·019	·054	36	44	26	16	28	36
40–44	·219	·137	·048	·082	·094	·176	42	49	40	65	95	67
45–49	·220	·191	·108	·076	·064	·166	39	55	53	46	46	41
50–54	·256	·197	·054	·071	·086	·361	42	45	19	29	39	50
55–59	·290	·341	·202	·218	·264	·759	42	60	49	53	65	58
60–64	·425	·561	·360	·379	·471	1·422	54	78	61	57	63	61
Total							40	55	41	47	60	58

Table 2. *Males—Deferred period* 4 *weeks* (1973 *experience*)

Age	Exposed to risk 4/9	13/13	26/26	52/52	104/all	Actual weeks of sickness 4/9	13/13	26/26	52/52	104/all
18–19	42	37	29	18	5	—	—	—	—	—
20–24	1,356	1,224	1,045	753	348	54	2	—	—	—
25–29	5,119	4,882	4,530	3,838	2,580	388	150	53	50	—
30–34	5,175	5,005	4,749	4,230	3,263	491	146	79	53	33
35–39	5,411	5,293	5,111	4,724	3,960	597	286	206	17	30
40–44	4,963	4,885	4,764	4,510	3,984	685	328	304	364	364
45–49	3,663	3,626	3,567	3,438	3,157	649	317	215	209	395
50–54	2,218	2,201	2,173	2,113	1,976	436	267	192	203	558
55–59	1,058	1,055	1,050	1,038	1,004	363	274	211	187	607
60–64	334	334	333	331	325	167	133	142	110	246
Total	29,339	28,542	27,351	24,993	20,602	3,830	1,903	1,402	1,193	2,233

Age	Actual claim rate 4/9	13/13	26/26	52/52	104/all	Actual weeks sickness/expected (%) 4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—
20–24	·010	·002	—	—	—	27	2	—	—	—
25–29	·076	·031	·012	·013	—	44	36	19	33	—
30–34	·095	·029	·017	·012	·010	49	30	23	23	12
35–39	·175	·054	·040	·004	·008	50	44	44	5	5
40–44	·138	·067	·064	·081	·091	49	42	51	82	35
45–49	·177	·087	·060	·061	·125	51	43	36	44	31
50–54	·197	·121	·089	·096	·282	45	43	36	45	40
55–59	·343	·260	·201	·180	·604	61	65	50	45	47
60–64	·499	·399	·427	·331	·757	70	68	65	45	33
Total						48	41	37	39	29

Table 3. *Males—Deferred period* 13 *weeks* (1973 *experience*)

Age	Exposed to risk				Actual weeks of sickness				Actual claim rate				Actual weeks sickness/expected (%)			
	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all
18–19	20	16	10	4	—	—	—	—	—	—	—	—	—	—	—	—
20–24	767	666	491	230	23	—	—	—	·029	—	—	—	38	—	—	—
25–29	4,107	3,770	3,110	1,922	49	49	15	67	·012	·013	·005	·035	14	21	12	67
30–34	5,292	5,006	4,443	3,345	96	34	80	27	·018	·007	·018	·008	19	10	34	9
35–39	5,811	5,595	5,150	4,223	138	155	101	52	·024	·028	·020	·012	19	30	29	8
40–44	5,574	5,419	5,084	4,384	135	118	129	158	·024	·022	·025	·036	15	17	26	14
45–49	4,423	4,340	4,149	3,735	323	198	21	635	·073	·046	·005	·170	36	28	4	43
50–54	2,582	2,551	2,479	2,311	192	213	256	436	·074	·084	·103	·189	26	34	48	26
55–59	1,453	1,445	1,426	1,374	130	190	385	599	·090	·131	·270	·436	22	32	68	34
60–64	565	565	563	559	116	152	333	910	·205	·268	·591	1·627	35	41	82	73
Total	30,594	29,373	26,905	22,087	1,202	1,109	1,320	2,884					24	27	40	35

Table 4. *Males—Deferred period* 26 *weeks* (1973 *experience*)

Age	Exposed to risk			Actual weeks of sickness			Actual claim rate			Actual weeks sickness/expected (%)		
	26/26	52/52	104/all	26/26	52/52	104/all	26/26	52/52	104/all	26/26	52/52	104/all
18–19	25	16	6	—	—	—	—	—	—	—	—	—
20–24	2,562	1.956	777	26	50	2	·010	·026	·002	20	84	8
25–29	8,626	7,266	4,133	26	—	52	·003	—	·013	5	—	25
30–34	8,074	7,145	4,963	90	48	109	·011	·007	·022	16	13	26
35–39	7,908	7,234	5,658	139	182	99	·018	·025	·018	19	37	12
40–44	7,483	7,012	5,905	176	132	213	·024	·019	·036	19	19	14
45–49	6,138	5,894	5,269	229	382	409	·037	·065	·078	23	47	19
50–54	4,394	4,279	3,984	356	452	1,334	·081	·106	·335	33	48	46
55–59	2,660	2,633	2,567	247	349	1,581	·093	·133	·616	23	33	48
60–64	1,138	1,136	1,131	192	336	1,368	·168	·296	1·209	26	41	54
Total	49,008	44,571	34,393	1,481	1,931	5,167				22	35	37

Table 5. *Males—Deferred period* 52 *weeks* (1973 *experience*)

Age	Exposed to risk		Actual weeks of sickness		Actual claim rate		Actual weeks sickness expected (%)	
	52/52	104/all	52/52	104/all	52/52	104/all	52/52	104/all
18–19	9	3	—	—	—	—	—	—
20–24	169	76	—	—	—	—	—	—
25–29	1,066	629	—	—	—	—	—	—
30–34	1,813	1,240	—	—	—	—	—	—
35–39	2,498	1,920	—	—	—	—	—	—
40–44	2,842	2,341	—	—	—	—	—	—
45–49	2,242	1,928	49	313	·022	·162	16	41
50–54	1,395	1,257	43	104	·031	·083	14	12
55–59	612	582	57	130	·093	·223	24	18
60–64	171	167	2	52	·013	·311	2	14
Total	12,817	10,143	151	599			10	16

Table 6. *Females—Deferred period* 1 *week* (1973 *experience*)

Age	Exposed to risk						Actual weeks of sickness					
	1/3	4/9	13/13	26/26	52/52	104/all	1/3	4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—	—	—
20–24	33	30	26	20	12	4	7	1	—	—	—	—
25–29	115	113	108	101	86	52	43	7	—	—	—	—
30–34	82	81	78	74	66	52	17	13	1	—	—	—
35–39	92	91	90	89	84	76	23	11	—	—	—	—
40–44	90	89	88	87	82	73	16	12	—	—	20	32
45–49	170	169	169	168	166	162	71	100	25	2	—	156
50–54	105	105	104	104	104	103	34	32	2	—	—	—
55–59	66	65	65	65	65	64	29	40	—	—	104	104
60–64	13	13	13	13	13	13	16	6	—	—	—	72
Total	766	756	741	721	678	599	256	222	28	2	124	364

Age	Actual claim rate						Actual weeks sickness/expected (%)					
	1/3	4/9	13/13	26/26	52/52	104/all	1/3	4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—	—	—
20–24	·219	·042	—	—	—	—	45	—	—	—	—	—
25–29	·369	·063	·004	—	—	—	79	37	5	—	—	—
30–34	·204	·160	·011	—	—	—	43	83	11	—	—	—
35–39	·248	·121	—	—	—	—	51	55	—	—	—	—
40–44	·173	·130	—	—	·240	·443	33	46	—	—	241	168
45–49	·419	·587	·149	·014	—	·963	75	168	73	8	—	235
50–54	·321	·307	·016	—	—	—	52	70	6	—	—	—
55–59	·442	·609	—	—	1·604	1·630	63	107	—	—	392	125
60–64	1·264	·439	—	—	—	5·561	160	61	—	—	—	239
Total							61	93	20	2	120	125

Table 7. *Females—Deferred period* 4 *weeks* (1973 *experience*)

Age	Exposed to risk 4/9	*13/13*	*26/26*	*52/52*	*104/all*	Actual weeks of sickness *4/9*	*13/13*	*26/26*	*52/52*	*104/all*
18–19	7	5	3	2	—	—	—	—	—	—
20–24	111	98	81	54	20	—	—	—	—	—
25–29	259	250	235	203	140	3	—	4	—	—
30–34	197	192	183	164	130	36	54	89	25	—
35–39	206	203	197	183	153	38	6	—	—	—
40–44	202	200	195	185	159	37	26	4	—	52
45–49	158	156	153	146	132	29	2	9	44	52
50–54	99	98	96	92	84	13	2	—	34	70
55–59	34	34	34	34	32	3	9	7	—	52
60–64	8	8	8	8	8	—	—	—	—	—
Total	1,281	1,244	1,185	1,071	858	159	99	113	103	226

Age	Actual claim rate *4/9*	*13/13*	*26/26*	*52/52*	104/all	Actual weeks sickness/expected (%) 4/9	13/13	26/26	52/52	104/all
18–19	—	—	—	—	—	—	—	—	—	—
20–24	—	—	—	—	—	—	—	—	—	—
25–29	·012	·002	·017	—	—	7	2	28	—	—
30–34	·182	·280	·482	·149	—	94	288	680	280	—
35–39	·183	·030	—	—	—	82	25	—	—	—
40–44	·181	·131	·019	—	·326	65	82	15	—	125
45–49	*·186*	*·015*	*·056*	·297	·393	54	7	34	214	*98*
50–54	·133	·021	—	·373	·827	30	7	—	175	117
55–59	·087	·261	·204	—	1·592	15	64	50	—	124
60–64	—	—	—	—	—	—	—	—	—	—
Total						49	55	79	94	87

Table 8. *Females—Deferred period* 13 *weeks* (1973 *experience*)

Age	Exposed to risk				Actual weeks of sickness				Actual claim rate				Actual weeks sickness/expected (%)			
	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all	13/13	26/26	52/52	104/all
18–19	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—
20–24	62	53	35	12	13	15	—	—	·208	·273	—	—	—	—	—	—
25–29	173	158	131	83	6	4	—	—	·037	·026	—	—	43	44	—	—
30–34	234	219	183	123	6	4	18	8	·027	·019	·099	·064	28	27	188	78
35–39	230	219	201	165	13	17	18	8	·056	·077	·090	·048	46	83	131	31
40–44	252	243	224	187	36	48	5	—	·141	·198	·022	—	89	160	23	—
45–49	264	258	244	216	—	—	—	156	—	—	—	·722	—	—	—	183
50–54	149	146	141	131	31	61	64	—	·210	·413	·454	—	75	171	212	—
55–59	65	64	62	57	17	7	—	52	·268	·115	—	·911	68	29	—	72
60–64	20	20	19	19	9	—	—	—	·419	—	—	—	71	—	—	—
Total	1,450	1,381	1,240	993	131	156	105	224					54	80	69	58

Table 9. *Females—Deferred period* 26 *weeks* (1973 *experience*)

Age	Exposed to risk			Actual weeks of sickness			Actual claim rate			Actual weeks sickness/expected (%)		
	26/26	52/52	104/all	26/26	52/52	104/all	25/26	52/52	104/all	26/26	52/52	104/all
18–19	1	1	—	—	—	—	—	—	—	—	—	—
20–24	57	43	20	—	—	—	—	—	—	—	—	—
25–29	146	109	48	—	—	—	—	—	—	—	—	—
30–34	244	201	129	—	—	—	—	—	—	—	—	—
35–39	355	302	203	28	36	—	·079	·118	—	86	170	—
40–44	415	377	300	—	—	—	—	—	—	—	—	—
45–49	337	317	271	7	—	—	·021	—	—	13	—	—
50–54	213	201	177	33	—	—	·157	·002	—	65	1	—
55–59	95	94	91	6	—	76	·063	—	·838	16	—	66
60–64	16	16	16	4	—	24	·264	—	1·454	41	—	66
Total	1,879	1,661	1,255	78	36	100				29	17	20

Table 10. *Females—Deferred period 52 weeks* (1973 *experience*)

Age	Exposed to risk		Actual weeks of sickness		Actual claim rate		Actual weeks sickness/expected (%)	
	52/52	104/all	52/52	104/all	52/52	104/all	52/52	104/all
18–19	—	—	—	—	—	—	—	—
20–24	9	2	—	—	—	—	—	—
25–29	33	18	—	—	—	—	—	—
30–34	56	33	—	—	—	—	—	—
35–39	89	62	—	—	—	—	—	—
40–44	123	92	—	—	—	—	—	—
45–49	104	84	35	30	·335	·360	245	91
50–54	70	56	—	—	—	—	—	—
55–59	31	30	29	—	·919	—	230	—
60–64	1	1	—	—	—	—	—	—
Total	516	378	64	30	—	—	96	20

5. REPORT DATED SEPTEMBER 1971

5.1. The following is the text of the Circular and Report prepared in September 1971 amended where necessary to conform with instructions contained in later circulars.

5.2. Circular dated September 1971.

The Life Offices' Association and the Associated Scottish Life Offices have invited the Continuous Mortality Investigation Bureau to undertake an investigation into sickness rates experienced under Permanent Health Insurance policies.

The Councils of the Institute and Faculty have considered the question, and have amended the Bureau's constitution so as to extend its activities to the collection of morbidity statistics, and to the investigation of health insurance. An Advisory Committee has been set up and has now prepared its first report.

The report is enclosed. Most of the contents particularly concern the offices now transacting health insurance. It will however, be seen that all offices contributing to the finances of the Bureau are concerned because it is recommended that the cost of the investigation should be apportioned in the same manner as the cost of other investigations.

Offices are invited to comment on the report, but those intending or hoping to contribute data are requested to write to the Secretary, Institute of Actuaries, indicating:

(i) The estimated number of cases they would submit for new business arising in 1972.
(ii) Whether it seems possible to contribute data relating to 1971 new business or earlier years, and if so approximately what volume (data deficient in some minor respect(s) may well be acceptable; offices should indicate in suitable cases what the deficiencies would be).
(iii) Whether the information will be submitted on punched cards or magnetic tape.
(iv) Any comments they wish to make on the report or the draft cards.

It is hoped that offices will be able to send at least a preliminary reply by 15 October 1971 so that the work of the Sub-Committee can proceed.

5.3. Report of the Advisory Sub-Committee for the Investigation of Sickness Statistics. (Condensed version.)

Scope of the Investigation

If a sufficient number of offices is able to contribute data in the form required, the investigation will cover the following classes of business:

(i) Individual policies (including disability income and lump sum benefits attached to Life policies).
(ii) Group policies.

(iii) Waiver of premium benefits attached to life policies.

It is intended to process the data by computer; the computer would not, of course, be under the control of any individual life insurance office.

Basically the data will be required, for each class, on 80 column punched cards or on tape with the corresponding number of fields covered. For each individual policy or, in group schemes, each life, an 'in-force' card or tape record is needed. In addition, for each life concerned in a claim, a separate claim card or tape record will be set up. In view of the comparatively large number of subdivisions into which the data will be classified for initial study, the committee proposes to set up a program for summarizing the data centrally. The offices will, it is proposed, submit data for individual cases. An additional advantage of this procedure is the control which can be applied to ensure that valid data are submitted. Although it is intended that each year's experience should be compiled and analysed by itself, both for offices separately and in combination, it is realized that, to provide a sufficient volume of data to enable clear results to emerge, it will be necessary to build up a cumulative experience. It follows that a 'data bank' will have to be set up as a store of accumulated information. This will be used in the early years to provide reports on the cumulative experience by Manchester Unity type methods and, at a later stage, for an investigation on a 'Disability Annuity' basis. It should be noted that disability annuities have to be derived from 'select' data with a very long period of selection (15 years was used in the U.S.A.) and a number of years' experience must be amalgamated to produce results which are statistically reliable. Offices will appreciate that if they contribute data for existing business as well as new business, results will be achieved earlier. It is, of course, recognized that some offices might be able to contribute data in respect of new business only.

The data submitted by each office will be treated as confidential, and each office will receive summary reports on its own business which are confidential and not revealed to any other office. The reports on the combined investigation, possibly more detailed than the individual offices' reports, will be made available to all contributors.

Method

The census method will be used and initially there is not expected to be enough information to produce a graduated table of rates. The first results to be published will, therefore, be a comparison of actual and expected weeks of sickness using Manchester Unity AHJ rates for the expected. It has been noted that the investigation of sickness on the basis of amounts of benefit is liable to distort the experience. If the data provide adequate evidence, an early opportunity will be taken to investigate the effect; nevertheless it is not expected that results on an 'amounts' basis will be published for some time.

The census date will be 1 January in each year for individual business but the scheme revision date may be used for group business by offices which find it more convenient to do so.

Results on a 'Disability Annuity' basis

It has already been noted that a long period of investigation is required before reliable values of disability annuities can be established. The basis of calculation of these annuities is a combined termination rate for sickness terminating by recovery or death at each age. Reverse selection applies for some years after the commencement of the attack.

However, the investigation of the exposed lives (including those claiming benefit) will produce observed 'rates of starting a claim at age x' separately for each waiting period and this information is useful for international comparison as well as to enable trends to be followed from an early stage. It is therefore proposed to publish crude rates in this form at the same time as the 'Manchester Unity' type results.

Sub-divisions of data

Besides age and sex, rates of sickness depend on the waiting period (deferment period) to an important extent. For each class of business investigated (individual, group and possibly waiver of premium) it will, therefore, be necessary to publish a separate experience for each of the most common waiting periods, probably in quinquennial age groups. If there is enough information, some indication of the experience for females may also be published at an early stage. Group schemes involving a non-selection limit may exhibit special features and it is hoped to investigate the experience separately if certain technical difficulties can be overcome.

A complete analysis of occupations is not practicable and simple methods of classification would involve *a priori* decisions on the degree of extra risk involved, if any. The Sub-Committee could not put forward an authoritative opinion on the way underwriting of the occupational risk should be done. It is, therefore, recommended that offices submit cases, whether rated for occupation or not. They would be classified in two groups only, namely, (1) accepted at ordinary rates and (2) accepted with an extra premium for occupational risk. Medically substandard risks will, of course, be excluded, except under group business or where an individual life is rated solely by means of a special exclusion rather than an extra premium.

The underlying cause of sickness for claims should also be reported. The Sub-Committee decided to adopt abbreviated 'List C' of the eighth revision of the *Manual of the International Statistical Classification of Diseases* for this purpose. The rules for applying these codes are clearly set out in two volumes published by H.M.S.O. and detailed instructions would be supplied to offices. For classification of sickness due to injury, where the list offers two alternative sets of codes, the 'CE' set, based on the cause of injury rather than its clinical symptoms, would be adopted.

The cost of collecting this additional information about occupation and claims is not identifiable but it is obviously small and will eventually, it is hoped, yield helpful results.

The Sub-Committee considers that some offices which issue policies outside the U.K. (*e.g.* in the Republic of Ireland) may consider that the data would be appropriate for comparison (and possibly amalgamation) with U.K. data and would be worth recording. It would not be suitable to include business written against a completely different market background. Individual offices are asked to comment on their own situation in this respect, and to estimate how much data they would submit relating to business outside the U.K.

Apportionment of cost

The Sub-Committee was asked for its recommendations on apportionment of cost. First the question of initial costs of setting up the investigation was considered. It was pointed out that the offices with the largest amounts of business had least to gain from the information to be derived from the investigation, while small offices, or those newly venturing into the field of permanent health insurance, had most to gain. The possibility of a flat charge per contributing office was suggested but, in view of the potential usefulness of the investigation to other offices, this solution is not free from difficulty. As the initial impetus for the investigation came from the Life Offices' Association and the Associated Scottish Life Offices, it seemed more appropriate that these bodies should contribute to the cost of setting up the investigation.

The offices contributing to the main finances of the C.M.I. Bureau are largely the same as the members of the L.O.A. and A.S.L.O. The Sub-Committee therefore recommends that C.M.I. funds be employed for the initial expenses of the investigation.

It is also recommended that the subsequent running costs should be met by an appropriate addition to the normal contributions of all offices contributing to the C.M.I. Bureau.

6. DESCRIPTION OF COMPUTER SYSTEM

The investigation falls into two distinct parts having different levels of priority. The first is an analysis along Manchester Unity lines and the second is an investigation into claim inception and termination rates leading to the calculation of disability annuities. It will not be possible to obtain useful results from the second part of the investigation for a number of years and this means that it has a lower level of priority than the first.

The computer system used for the experience under individual policies in 1972 and 1973 was concerned mainly with the Manchester Unity type investigation but it was designed so as not to prejudice the second part of the investigation.

Data is accepted on punched cards or on IBM nine-track magnetic tape or in other forms with the agreement of the Sub-Committee.

If an office were supplying data as at 31 October instead of 31 December then for this office an event which occurs on 1 December 1972, for example, would be coded and treated as a 1973 event, the only inaccuracy being a slight discrepancy in the year of experience as compared with other offices.

Detailed descriptions of card layouts have been given in paragraph 2.3.

The variables by which the data can be analysed separately are:

Contributing office
Age
Sex
Year of experience
Country
Whether or not rated for occupation
Deferred period
Benefit rate
Benefit type
Medical evidence type
Premium type
Medical exclusion type

and additionally for claims:

Whether full rate or reduced for partial disability
Commencement mode
Cessation mode
Cause of disability
Number of days of sickness in any period

The main reports required by the system are: (1) analysis of the data for all offices combined subdivided by age, sex and other variables, singly or in combination, which may affect the level of sickness claims; and (2) analysis for individual offices to compare with the results for all offices combined. The individual office figures will be analysed in less detail than the all office data.

The number of subdivisions is far too large for an analysis by all variables simultaneously so the computer system is designed to allow the Sub-Committee to change the combinations of variables as necessary when results are obtained. The exceptions to this general rule are concerned with age, where a fixed range will be used for all analyses; deferment period, where standard periods of 1, 4, 13, 26 and 52 weeks will be analysed separately and any other deferment period will be taken as the next higher period and any claims up to the deemed deferment period will be ignored; periods of sickness, where standard periods of sickness 1/3, 4/9, 13/13, 26/26, 52/52, 104/all (measured in weeks) will be used; and benefit rate, where it has been agreed that for lump sum benefits and special periods of benefit payments the rate of benefit will not be used but will be held in the system as zero. In other cases the amount to the next higher multiple of £100 per annum will be held in the system.

Since the requirements of a disability annuity investigation have not yet been specified, it is essential that none of the information supplied to the C.M.I. is lost. It is impracticable to keep copies of all input tapes submitted, so consolidated tapes for each end year's in force for all offices combined and for each year's claims will be held indefinitely.

Apart from the analyses to be performed there are other areas where generality is desirable. They are:

(i) Years to be analysed. The system is written so that the data for any number of years, which need not be consecutive, may be combined.
(ii) Comparison to be made. Initially all sickness rates will be compared with Manchester Unity AHJ. Later the basis of comparison may well be with a standard table derived from C.M.I. data, so the standard rates are read in as data.
(iii) Printing. Results may be required by individual ages, by individual ages with quinquennial sub-totals, or by quinquennial sub-totals only. Also the quinquennial groupings must be capable of variation. It must be possible to suppress the printing of A/E if E is less than a certain amount and this amount should be variable. So all these facilities are controlled by parameters read by the print program.

The following is a list of programs included in the system:

A	Input Edit
B	In-Force Extract Create
C	Claims Extract Create
D	In-Force Extract Sort
E	Claims Extract Sort
F	Aggregation
G	Summaries Create
H	Summaries Extraction
J	Results Print
K	Statistical Summary
L	Extract File Sort
M	Results Extraction
N	Claims Extract Print
P	Claims Records Sort
Q	Claims Records Analysis
R	Delete Office

Data enter the system as card or tape files containing in-force or claim records for one or more offices. The files are read by the Input Edit program which checks each record for obvious errors and makes a list of any it finds. Records not in error are copied on to a tape called the 'brought forward' tape containing the records which have passed through previous input edit runs. Error records are corrected and submitted in card form to a later edit run. If an input file contains a large number of errors, the output file is scrapped and the data are returned to the office concerned for correction.

When all input edit runs for 1 year have been completed there will be two tapes holding in-force and claims for all offices combined. These tapes are the basic data of the investigation and will be held indefinitely.

These individual records are converted into a form suitable for later processing by the In-Force Extract Create and the Claims Extract Create programs. For example, dates of birth are converted to age nearest birthday, benefit rates are converted to annual amounts, non-standard deferred periods are converted to standard ones and so on. This puts all records into a standard form regardless of what the office year is or how ages are defined. Any records over age 65 or under age 15 are listed and are not passed on to the next stage of the work. Records for ages 15–17 are replaced by similar records at deemed age 18. The Claims Extract Create program also notes whether any claim ranks as a new claim for the calculation of the Claim Inception Rate.

The output tapes from this process are sorted by the In-Force Extract Sort and the Claims Extract Sort programs by the values of the several variables and these extracts will be retained indefinitely.

The extracts for which all variables have the same value are combined by the Aggregation program which takes 1 year's claims extracts and the in-force extracts for the beginning and end of the year and calculates the exposed to risk on the assumption that new policies were issued in the middle of the year of entry and allowing for the fact that no claims could arise before the end of the deferment period. It produces an Aggregates Tape consisting of the exposed to risk and claims subdivided into sickness periods 1/3, 4/9, 13/13, 26/26, 52/52 and 104/all.

Up to this stage no material information has been discarded and programs (except for input data) will normally be run once per year as an automatic process. From this stage onwards programs will be run at the request of the Sub-Committee to carry out any analyses that seem to be necessary and parameters must be specified for each run.

The next stage is to compress the aggregates into summaries by combining all records with variables other than age that are not required in a particular analysis. This is done by the Summaries Create program and the process is governed by Data Combination Cards which are coded as follows to add a record from the Aggregates Tape into a summary according to whether a particular variable (1) has a given value, (2) does not have a given value, (3) is not less than a given value, (4) is not greater than a given value.

The variables which can be referred to in this way by Data Combination Cards are as follows:

Reference No.		Values
01	Office	000 to 999
02	Sex	1 = male, 2 = female
03	Country	1 = U.K., 2 = Republic of Ireland, 3 = Isle of Man, 4 = Channel Islands

Reference No.		Values
04	Occupation	1 = Occupation rated, else 0
05	Deferment Period	1 = 1 week, 2 = 4 weeks, 3 = 13 weeks, 4 = 26 weeks, 5 = 52 weeks
06	Curtate period	Curtate duration at end of year of experience (maximum = 8, 99 = unknown)
07	Payment type of benefit	1 = level, 2 = increasing, 3 = decreasing, 5 = lump sum, 9 = other types
08	Medical evidence	1 = medical, 2 = non-medical, 3 = non-selection limit applies, 4 = unknown
09	Premium type	1 = level annual, 2 = recurrent single, 3 = increasing annual, 4 = other
10	Medical exclusions	0 = no exclusions, 1, 2, 9 = specific types; 7 = exclusion may or may not be present; 8 = exclusion present, but reason unknown
11	Level of benefit	00 to 99 (in £100s taken to the nearest higher multiple of £100 per annum)
12	Full rate code (claims only)	0 = benefit reduced; 1 = benefit not reduced
13	Disability code (claims only)	01–70 (classification code omitting C or CE)

No Data Combination Card is required if all values of a given variable are to be included. Any number of sets of Data Combination Cards can be handled in one run leading to separate summaries. A maximum of fifteen Data Combination Cards may be used in any one set.

The first card in each set is a header card, giving the set number and the number of Data Combination Cards that will follow it. Each card in the set gives the set number, the reference number of the variable to be tested, the code for the test required and up to seven values for which the test is to be applied. If the test is (3) or (4), only one value can be given.

As an example of how the Summaries Create program works, suppose that the Sub-Committee wish to analyse the experience of all offices combined, males only, deferred 1 week, durations 5 and over (but excluding 'unknown'), excluding recurrent single premium cases and including only those that are known to have medical exclusions and disability codes 20–27. This operation would need one header card and eight Data Combination Cards. The header card would be punched as follows:

Column	Data punched	Description
1–4	0123, say	Set number
5–6	000	Type of card
7–8	08	Number of Data Combination Cards in set

The next card in the set would be as follows:

Column	Data punched	Description
1–4	0123	Set number
5–6	02	Number of variable to be tested
7	1	Selection code
8–19	001	Value of field to be tested

This card selects aggregates which are coded 001 in field 2, *i.e.* males.

The second card tests variable 5 for value equal to 001 and so selects deferred period of 1 week from the aggregates selected by the previous card. The third card tests variable 6 for value not less than 005 and so selects duration 5 and over from the aggregates selected by the previous cards. The fourth card also operates on variable 6 to select, by code 2, those aggregates whose field value is not 099 and so selects only known durations. The fifth card selects aggregates which are not coded 002 in vaiiable 9, *i.e* recurrent single premium. The sixth card operates on variable 10 to select aggregates with values 001, 002, 008 or 009, *i.e.* those with specific types of exclusion. The seventh card selects aggregates with not less than code 20 in variable 13 and the final card selects those not greater than 27 in variable 13, thereby separating the sickness causes required.

Thus the data to be analysed are separated from the whole body of data and the summary derived from the set of Data Combination Cards is referred to by its year/set number—thus set 0123 of 1972 data is designated 720123. The output from the Summaries Create Program is a Summaries Tape. Each summary on the tape contains a copy of the Data Combination Cards that were used to create it.

When results are required from the system, any required summaries are extracted by the Summaries Extraction program; similar summaries for different years being added together if required and written into a Summaries Extracted File. This is achieved by means of results parameter cards which are laid out in the following form:

Column	Description
1–4	Results set number
5–6	Card number in set
7–10	Data combination set number
11–30	Marker for years 1972–1991: for each year 1 = required 0 = not required

The program also checks that the details of the Data Combination Cards are the same on each summary to be combined. A warning is printed if they are not.

The following Results Print program calculates expected claims on a basis to be defined, ratios of actual to expected claims and, optionally, actual rates of sickness for the six sickness periods referred to earlier and prints the results. The sickness rates for the calculation of the expected claims are read in from a set of Basis Cards and the rates actually used so far are Manchester Unity AHJ Rates 1893–97. The details of the printing of the results are defined by a Print Parameter Card which allows the following choices to be made:

Printing of individual ages and/or quinquennnial sub-totals
Choice of quinquennial groupings
Minimum value of expected sickness below which A/E is suppressed
Whether actual sickness rates are to be calculated and printed.

All tapes used by the system are retained until the end of the year's running, except results tapes which are discarded immediately. When each year's running is complete all tapes are discarded except for:

In-force records (all offices combined)
Claims records (all offices combined)
Aggregates
Summaries.

The in-force and claims tapes contain all data submitted, so that no basic information is ever lost. The aggregate tape contains all the information that the present system is designed to use and it will only be necessary to go back to the in-force and claims records in the event of a major modification of the system. The claims records will eventually be needed for the disability annuity investigation.

The summaries tape contains information relating to all years and is updated annually, so that the current tape includes all past information.

There is always the possibility that a magnetic tape will become unreadable if it is stored for any long period of time, and so the procedure adopted (except for the summaries tape, which is never more than a year old) is to copy each tape at the end of a year's running and to keep both copies. Once a year thereafter each copy is again copied and the copy made 2 years previously is discarded. Thus two copies of each tape are always held, one created within the last year and one a year earlier.

The last three generations of the summaries tape are held and a copy of this tape is taken at the end of each year.

This description of the computer system is a condensed version of a full report prepared by Mr G. T. Humphrey. It omits the description of some facilities built into the system for such special purposes as correction of data errors, control prints, etc. The Sub-Committee wishes to acknowledge the valuable work done by Mr Humphrey and Pensions and Insurance Computer Services.

THE GRADUATION OF PENSIONERS' AND OF ANNUITANTS' MORTALITY EXPERIENCE, 1967–70

1. INTRODUCTION

1.1. In the paper 'Considerations Affecting the Preparation of Standard Tables of Mortality' (*J.I.A.* **101,** 133 and *T.F.A.* **34,** 135), the Committee presented a new graduated table for assured lives, 'A1967–70'. In the same paper, and in a report 'Mortality of Immediate Annuitants' (*C.M.I.R.* **1**, 29), the Committee explained why they did not at that time think that it was appropriate to prepare a new standard table for the annuitants. Also in the paper the Committee examined the pensioners' experience and recommended 'that two graduated tables of mortality be constructed, one for males and the other for females to exhibit faithfully the experience in 1967–70 of "lives" who retired at or after the normal age'. Nothing was said in the discussions at the Institute and the Faculty to encourage the Committee to change its mind about the pensioners' experience but the Committee nevertheless feels that it should explore the possibility of giving further guidance to the profession and this matter is referred to again in paragraph 3.16. On the other hand the Committee gained the impression that new graduated tables of the experience of male and female annuitants who have purchased annuities since 1956 might be welcomed even though the experience to date for this apparently new class of lives is heavily weighted with the shorter durations. This paper presents new graduated tables of mortality rates for male and female pensioners and for male and female annuitants derived directly from the experience of the years 1967–70. In none of the cases does the graduated table make any explicit allowance for future improvements in mortality.

2. THE CHOICE OF A FORMULA

2.1. In the Institute discussion on the A1967–70 graduation, Professor R. E. Beard had criticized the formula used on the grounds that it did not readily allow comparison of its parameters with those of other tables at different times (such as A1949–52) or in other countries. For the same reasons the Committee felt too that the use of an *ad hoc* formula for each graduation, however well it fitted the experience, was not wholly satisfactory. The formulae that had been used for various recent graduations included Beard's variation of Perks' formula (for A1949–52):

$$q_x = A + . \frac{B \,.\, c^x}{E \,.\, c^{-2x} + 1 + D \,.\, c^x}$$

Barnett's variation of Makeham's formula (for A1967–70):

$$q_x/p_x = A+B\,.\,c^x-H\,.\,x$$

Logistic plus Normal (for ELT11 and 12):

$$m_x = a+\frac{b}{1+e^{-\alpha\,.\,(x-x_1)}}+c\,.\,e^{-\beta\,.\,(x-x_2)^2}$$

It can be shown that each of these is a variation of a straightforward logistic curve:

$$(m_x \text{ or})q_x = A+B\,.\,\frac{1}{1+\exp(-C-Dx)}$$

and it was thought that a more powerful variation would be to allow the exponential term to contain, not just a linear function of x, but a polynomial of whatever degree was required. First experiments with the pensioners' experience showed that in fact satisfactory results could be obtained with $A = 0{\cdot}0$ and $B = 1{\cdot}0$, so the formula could be rearranged yet again to give:

$$q_x = \frac{\exp(F(x))}{1+\exp(F(x))}$$

$$p_x = \frac{1}{1+\exp(F(x))}$$

or $\log(q_x/p_x) = F(x)$ where $F(x)$ is a polynomial in x of whatever degree is appropriate for the data. The item $\log(q_x/p_x)$ can also be termed the log of the odds of dying.

The above form, namely:

$$\log(q_x/p_x) = F(x) = \sum_{r=0}^{n} a_r x^r$$

has therefore been used for all the graduations, with the degree of the polynomial, n, being adjusted appropriately. It can be seen that, since the range of $\log(q_x/p_x)$ is potentially from $-\infty$ to $+\infty$, the function can be appropriately represented by a polynomial function of x, which is less appropriate for the representation of q_x, whose maximum range is from 0 to 1.

2.2. The method used to determine the parameters, $\{a_r\}$, for any value of n for each set of data was the same as for the A1967–70 data, namely the method of maximum likelihood. The parameters are chosen so as to maximize the log likelihood function:

$$\mathrm{L}' = \sum_x \{\mathrm{A}_x\,.\,\log q_x+(\mathrm{ER}_x-\mathrm{A}_x)\,.\,\log(p_x)\}$$

where A_x is the number of actual deaths at age x and ER_x is the exposed to risk at that age. The function L′ is described by Edwards (in *Likelihood*, Cambridge, 1972) as the 'support function' and he shows that a difference of 2·0 in this function is equivalent to a deviation of two standard deviations about the mean, roughly implying significance at the 5% level. This observation can be used in determining the order of the polynomial to be used. If an additional term improves the level of support by 2·0 or more then it shows a significant improvement in the fit of the curve, and the extra term should be included. However, if the improvement in the level of support is less than 2·0 then the extra term does not add significantly to the fit of the curve, and it should be rejected.

2.3. An incidental convenience of the form of curve and the method used is that the maximum value of the support function is found when

$$\sum_x x^r \{A_x - ER_x\, q_x\} = 0 \text{ for } r = 0 \text{ to } n*$$

so that at the maximum value the totals of actual and expected deaths are equal, and so are the moments of actual and expected deaths up to as many moments as there are terms in the polynomial. The method, therefore, produces the same results as the method of moments for the logistic form of curve though not necessarily for other formulae for q_x. However, this fact cannot be used to facilitate the calculation of the values.

2.4. In addition to the level of support test mentioned above the same tests of a satisfactory graduation as were described in the previous paper have been used again for these graduations, namely: a runs test, $t(r)$, and a serial correlation coefficient test, $t(\rho)$, to test whether the graduation goes satisfactorily down the middle of the data; and a χ^2 test, $t(\chi^2)$, to test whether the fit of the data to the binomial model is satisfactory. For 'amounts' data in the pensioners' experience an assumption of independence for each pound of annuity is of course quite inappropriate, so that values of χ^2 and of the level of support are unsuitable; how this was dealt with is explained in paragraph 3.6.

3. THE PENSIONERS' EXPERIENCE

3.1. The basic statistics for the pensioners' experience were published in the paper 'Considerations . . .'. Tables in that paper showed exposed to risk and actual deaths for individual ages for eight groups of data, classified in pairs:

$$\left.\begin{matrix}\text{Males}\\ \text{Females}\end{matrix}\right\}\text{by}\left\{\begin{matrix}\textit{Retirements at or after normal retiring age}\\ \text{Retirements before the normal age}\end{matrix}\right\}\text{by}\left\{\begin{matrix}\text{Lives}\\ \text{Amounts}\end{matrix}\right.$$

* This can be shown as follows. The log likelihood function L′ is at a maximum when $\partial L'/\partial a_r = 0$ for all a_r. But L′

$$= \sum_x \{A_x \,.\, \log q_x + (ER_x - A_x) \,.\, \log p_x\}$$

$$= \sum_x \{A_x \,.\, \log (q_x/p_x) - ER_x \,.\, \log (1/p_x)\}$$

$$= \sum_x \{A_x \,.\, F(x) - ER_x \,.\, \log (1+\exp(F(x)))\}$$

since $\log (q_x/p_x) = F(x)$ and $1/p_x = 1+\exp(F(x))$. But

$$\frac{\partial F(x)}{\partial a_r} = x^r \text{ and } \frac{\partial}{\partial a_r} \,.\, \log (1+\exp(F(x))) = \frac{\exp(F(x))}{1+\exp(F(x))} \,.\, x^r = q_x \,.\, x^r.$$

So

$$\frac{\partial L'}{\partial a_r} = \sum_x \{x^r (A_x - ER_x \,.\, q_x)\} \quad r = 0 \text{ to } n.$$

The statistics for ages up to nearest age 50 and for ages from 101 onwards are not available for single years of age, and they have therefore been ignored. This involves the loss of 40 actual deaths and 936 exposed to risk in the Males, Early, Lives data and 9 actual deaths and 272·5 exposed to risk in the Females, Early, Lives data. The statistics for each single year up to age 107 will be collected from offices from 1975 onwards. The statistics for single years of age from 51 to 60 were available to the Committee and have been used for the graduations, although they were shown grouped in the earlier paper.

3.2. In the eight sets of data, by far the largest exposures are those for Males, Normal, as can be seen from the following table:

	Lives		Amounts (£)	
	Exposed to risk	Actual deaths	Exposed to risk	Actual deaths
Males, Normal	833,442	45,860	125,639,566	5,754,895
Males, Early	162,856·5	8,784	28,640,701	1,165,059
Females, Normal	170,200·5	3,829	13,794,287	255,435
Females, Early	30,368·5	638	2,286,171	37,945

The bulk of the exposure for normal retirements is in the years after 65 for males and 60 for females; for early retirements there is a reasonable volume for both sexes down to age 55. However, the exposure is very scanty for both Males and Females, Normal, below age 59 and above about age 94 and for Males and Females, Early, above about age 88. This restricts the range of validity of the tables, especially in the lower direction.

3.3. The restricted lower age range is no disadvantage if the tables are used only for the mortality of pensioners retiring at or after a normal retirement age of 60 or more. Of greater importance for early retirements is that the statistics show that almost certainly mortality in the years immediately following early retirement is high, and that, in aggregate, it declines with advancing age. This feature could be caused either by the mortality of those retiring at a certain age actually falling in the years subsequent to retirement, or by the high mortality of those who retire early at an early age (and therefore presumably because of severe ill-health) being diluted by the addition to the experience of others retiring only shortly before their normal age (and not being, on average, in such poor health). Which is the case cannot be determined without having the data subdivided by duration since retirement, and offices are contributing data in this form from 1976 onwards. But whichever is the case, a table based on such aggregate data is quite inappropriate for calculating annuity values for a person retiring at any specific age.

3.4. The discrepancy between the experience by lives and the experience by amounts was fully discussed in the earlier paper, and the conclusion there was that there was probably a genuine difference between the mortality by lives and by amounts, but that the statistics available possibly exaggerated the difference because of various 'time-trends' in the size of annuities being paid.

3.5. In spite of the arguments against using the amounts data and the early retirements data for standard tables, the Committee felt that it was useful to prepare graduations of each of the eight sets of data, purely for the purpose of comparing the levels of mortality one with another. The formula discussed above was used, namely:

$$\log (q_x/p_x) = F(x)$$

using a quadratic function for $F(x)$. In fact, by a transformation of scale of x and the use of Chebyshev polynomials the formula for $F(x)$ used was:

$$A+B\,.\,t+C\,.\,(2t^2-1),$$

where $t = (x-70)/50$, so that $t = -1, 0, +1$ at $x = 20, 70, 120$ respectively. The advantages of using a formula expressed in Chebyshev polynomials in this way are that the values of A, B and C are a convenient order of size, are easily comparable from one graduation to another, change only to a moderate extent when the second-degree term is omitted or when a third-degree term, $D\,.\,(4t^3-3t)$, is added, and can be thought of as representing a level, a slope, and a second-degree adjustment respectively, all centred on age 70.

3.6. In order that the amounts data might be of the same scale as the lives data, the numbers of exposed to risk and of actual deaths were divided by a factor, the same for all ages, but different for each set of data, so that the total number of exposed to risk was the same for amounts as for lives. The factors and the adjusted total data are shown in the table below:

	Factor (average pension per life exposed to risk) (£)	Exposed to risk (£)	Exposed to risk (Units)	Actual deaths (£)	Actual deaths (Units)
Males, Normal	150·74782	125,639,566	833,442	5,754,895	38,175·6
Males, Early	175·86465	28,640,701	162,856·5	1,165,059	6,624·7
Females, Normal	81·04728	13,794,287	170,200·5	255,435	3,151·7
Females, Early	75·28100	2,286,171	30,368·5	37,945	504·0

This allowed values of χ^2 and of the support function for amounts data to be of the right order of size.

3.7. The results of the graduations using a quadratic function (three parameters) for $F(x)$ are summarized in Table 1. The values of Σ (A–E) and $\Sigma\Sigma$ (A–E) are near zero in each case, confirming that the repeated approximation method has produced a result very close to the exact maximum. The results of the tests can be summarized:

(a) Runs test: the number of runs is always just below the number expected, i.e. $t(r)$ is positive in every case, but not substantially so except in one case, that of Females, Normal, Amounts, where $t(r)$ at 1·77 exceeds the desired value of 1·65 (but not by much).

(b) Serial correlation test: $t(\rho)$ is no greater than 0·88 at the maximum, which is quite satisfactory.

(c) χ^2-test: for lives, the values of χ^2, grouped to eliminate cases where the expected deaths are less than 5, are quite satisfactory in three cases out of four. In the fourth case, Males, Normal, Lives, the values are too big. A study of the individual ages shows that this high value derives substantially from the ages from 64 nearest to 66 nearest, which contribute 20·74 to a grouped χ^2 of 70·25. The crude values of $q_{x-\frac{1}{2}}$ in this area are:

x	$q_{x-\frac{1}{2}}$
63	0·02792
64	0·03366
65	0·03322
66	0·03177
67	0·03595

It is clear that no satisfactorily smooth curve for q_x will be anything else than low for age 64 and high for age 66 as compared with these crude values. It is not clear what the reasons might be for this irregularity in the experience. It is possible that some 'normal' retirements at age 63 or 64 in fact take place because of some degree of ill-health, though perhaps after a full 40 years service. While, if actual retirements are not evenly spread across the calendar year, the census method of calculating the exposed to risk, which assumes an average exposure of half a year for new entrants, will not satisfactorily represent the true exposure; however, there is no reason to suppose that such an irregular distribution of retirements in fact occurs, except to the extent that retirements may occur on the first of a month, so that the average exposure in the first year is 6½ months rather than exactly 6.

3.8. Each of the normal retirement graduated curves slopes upwards with age, and the parameters are comparable in size: A is between $-3{\cdot}1$ and $-3{\cdot}4$; B is between $+4{\cdot}3$ and $+5{\cdot}4$; and C is fairly small, being less than 0·53 in absolute value. But for the males, both lives and amounts, C is small and negative, whereas for the females C is positive. A negative C implies that $F(x)$, and hence q_x, has a maximum value at some high age, and a positive C implies correspondingly a minimum value at some low age; but in these four cases the turning points are well outside the age range in question. However, the female rates do exceed the male rates at high ages, overlapping at age 98 for normal and age 88 for early retirements, whereas otherwise the female rates are lower than the males. In both cases these crossing points are outside the range where there is any substantial amount of data.

The early retirement curves show a quite different set of shapes. In each case C is positive and quite large, ranging from $+1{\cdot}6$ to $+4{\cdot}9$, indicating a strong second-degree effect. In fact all four curves have a minimum within the useful age-range—at 56 for Males, Early, Lives; 46 for Males, Early, Amounts; 58 for Females, Early, Lives; and 60 for Females, Early, Amounts. For Males, Early, Lives this minimum is amply justified by the data as the following table of Actual/ Exposed to risk shows:

Nearest ages	Actual deaths	Actual/Exposed to risk
51–55	108	·05512
56–60	819	·04438
61–65	3,077	·04741

For females the quantity of exposure is less, but the same feature clearly exists to some extent:

Nearest ages	Actual deaths	Actual/Exposed to risk
51–55	25	·01575
56–60	118	·01642
61–65	164	·01613
66–70	127	·01886

3.9. The value of χ^2 for amounts data do not have the same use as for lives data, but the calculated values are of some interest. If all lives had the same amount of pension, then the amounts experience would be identical with the lives experience. If all lives had the same probability of death, but differing amounts, then the average mortality rates experienced in the amounts investigation would be the same as for lives, but the divergences from that average at individual ages would be expected to be greater, and hence the values of ${z_x}^2$ at individual ages, and also χ^2 would also be greater, the amount of the increase depending on the distribution of amounts. If the lives at any one age do not have the same probabilities of death, then the variances (the ${z_x}^2$) of the data for individual ages will also be increased—both for lives data and amounts data. The increase in χ^2 for the amounts data over the χ^2 for lives data, therefore, gives some indication of the distribution of amounts per life and of the lack of homogeneity in the probabilities of death. The table below shows the values of χ^2 for the different graduations (using ungrouped data):

	Lives χ^2	Amounts χ^2	Ratio A/L
Males, Normal	77·51	223·14	2·88
Males, Early	55·09	86·28	1·57
Females, Normal	47·94	83·75	1·75
Females, Early	35·43	39·89	1·13

The implication of the figures in the final column is that the variation in size of amount is greater for males than for females, and greater for normal retirements than for early retirements. One has to take into account also the 'time-trend' of average amounts, but these implications are not unreasonable, although they would require solider evidence from more complete data before being accepted as fact.

3.10. Although the quadratic function for $F(x)$ seemed to provide satisfactory results, it was appropriate to investigate whether a polynomial of different degree would be more satisfactory. Accordingly, a cubic function for $F(x)$ was tried:

$$A+B.t+C.(2t^2-1)+D.(4t^3-3t)$$

and the results of this were compared with the three-parameter graduations. In five cases out of the eight there was no substantial increase in the value of the support function, and no substantial reduction in the value of χ^2. The values of D were generally small (less than 0·5 in absolute magnitude). However, in three cases, the improvement was significant. These were: Males, Early, Lives; Males, Early, Amounts; Females, Normal, Amounts. But in every case the improvement was gained by having a substantial negative D, which resulted in the curve of q_x rising to a rather low maximum between 95 and 100 and then falling away sharply.

3.11. In the other direction, trials were then made with a simpler, linear, function for $F(x)$:

$$A+B.t,$$

and the results again compared with the three-parameter graduations. As might be expected, the graduations of the Early retirement data were all very much poorer with only two parameters. The second-degree term is essential to represent the real decline in the curve in the 50s. But for each of the sets of Normal data the two-parameter graduations produced results not significantly worse than the three-parameter ones. Indeed, in the case of Males, Normal, Lives, the value of $t(\chi^2)$, being based on more degrees of freedom, was slightly reduced. The values of A and B were not substantially changed from the three-parameter graduations, and the shapes of the curves of q_x were quite satisfactory. The results of the graduations for all sets of data are summarized in Table 2, and shown in detail for Males, Normal, Lives; Males, Normal, Amounts; Females, Normal, Lives; and Females, Normal, Amounts in Tables 3–6 and in Figures 1–4.

3.12. Table 7 shows the ratios (as percentages) of the graduated qs for each quinquennial age for each of the possible paired comparisons of corresponding data, using the two-parameter graduations for Normal and the three-parameter graduations for Early. First are shown qs for Amounts as a percentage of Lives for four combinations; then Early as a percentage of Normal; then Females as a percentage of Males. The results of course confirm the general tendencies evident in the crude data. Amounts rates are lower than Lives rates, but the rates converge at higher ages, and for Females the Amounts rates from age 95 (Normal) or 85 (Early) are higher. Early rates at the young ages are much higher than Normal, but the differences reduce up to 75–80, and then widen again;

indeed for Females, Early, rates are below Normal at ages 70–75 (Lives) or 80 (Amounts). The sharp rise in the ratio at the highest ages results from the quadratic term in the formula that is needed to fit the lower ages and is not justified by the data. Female rates are substantially lower than Males except at the highest ages (Early), where again the exposure is scanty.

3.13. The Committee felt that there was no justification in using a more complicated formula than was necessary to fit the data for the published tables and therefore suggest that the two-parameter formulae should form the basis of graduation of the data for Males and Females, Normal, using both lives and amounts. The tables can be referred to for convenience as Peg 1967–70 (mL), (mA), (fL) and (fA) respectively, *i.e.* 'Pensioners' experience graduated . . . males Lives, etc.' Values of q_x for these tables for ages from 50 to 114 inclusive are shown in Table 8, and depicted in Figure 5.

3.14. The values of q_x for the graduated experience tables for pensioners are compared with corresponding values for some other tables (including the new annuitants experience graduated—*aeg* 1967–70—see § 4) in Tables 9 and 10. For males, Peg 1967–70 (mL) is seen to be substantially higher than *a*(55) ultimate at ages up to 85, and higher than A1967–70—the 1967–70 assured lives table—at ages up to 90. It is higher even than ELT 12 (Males) below age 60, but rather lower thereafter except at the highest ages. The result is to show pensioners' mortality beyond the common retirement ages as being intermediate between assured lives' and population mortality, and below the common retirement ages as being noticeably high. The Amounts rates, Peg 1967–70 (mA), are lower than the Lives, but show the same sort of features. For females, pensioners' mortality is relatively better. Peg 1967–70 (fL) is below ELT 12 (Females) at all ages, but it is above *a*(55) ultimate at ages above 55. No female assured lives table is available for comparison.

Female Amounts rates, Peg 1967–70 (fA), are better, being lower than *a*(55) up to age 75, and substantially lower than ELT 12 (Females) except at age 100. The results are consistent with female pensioners being a rather more select group than male pensioners, possibly including proportionately more women from professional and clerical occupations.

3.15. Tables 11 and 12 show selected annuity values using Peg 1967–70 mortality and also *a*(55) ultimate (and *aeg* 1967–70) for males and females, at rates of interest of 0%, 5%, 10% and 15%. From this it can be seen that the Peg 1967–70 (mL) and (fL) Lives annuity values are generally *lower* than those for *a*(55) ultimate, except for males above about age 85. For males at age 65 the Peg 1967–70 (mL) values are 90–95% of the *a*(55) values, depending on the rate of interest used, while for females at the same age the values are closer, Peg 1967–70 values being some 97–99% of the *a*(55) values. The Amounts annuity values are closer to *a*(55) for both sexes. For males, the (mA) values exceed *a*(55) above about age 75, and at age 65 the values are 97–98% of *a*(55). For females, the (fA) values are very close to *a*(55), the values in the table being between 97·6% and 100·6% of *a*(55) throughout. The use of the new Lives tables by life offices for calculation of premiums and reserves would result in a reduction of premium rates and a weakening of reserves as compared with $a(55)$; the use of the Amounts tables would have less effect, but on balance the change would also be towards weakening the basis.

3.16. In paragraph 5.15 of the paper 'Considerations . . .', the Committee suggested that offices might be willing to use monetary functions derived from the experience tables and to adjust them in the light of the comparison of their own experience with that of the experience of all offices in 1967–70. The Committee now feels, however, that for a number of reasons the production of monetary functions based on the graduated tables derived from that experience might itself lead offices to believe that they had an authority which is not justified. The Committee has decided, therefore, to publish the rates set out in this report but to undertake further investigations to see whether it is possible in the reasonably near future to offer for consideration by the profession some better set of tables for use by offices than is available at the present time. The difficulties which will need to be solved include questions relating to allowance for future improvements in mortality, doubts about the appropriateness of using either the lives table, which is relatively heavily weighted with the lower amounts with which are presumably associated heavier mortality, or the amounts table, which contains a number of confusing features to which reference was made in the earlier paper. The Committee cannot stress too strongly that the present tables of graduated rates have been produced solely as a standard of comparison for offices. They do not purport to provide a suitable tool for the calculation of premiums or reserves without adjustment being made for one or more of a number of possible reasons. It would, in the opinion of the Committee, be quite inappropriate for an office which is currently using lighter mortality than appears in the present tables to change to a heavier basis for calculation of premiums or reserves.

4. THE ANNUITANTS EXPERIENCE

4.1. The note 'Mortality of Immediate Annuitants' (*C.M.I.R.* **1**, 29) describes the mortality experience of immediate annuitants in the period 1967–70, and compares it with that of the earlier periods 1959–62 and 1963–66. Comparisons of annuitants' mortality in recent years have been confused by the break in the class of person effecting annuities after the Finance Act, 1956. All the pre-1957 contracts were at least 10 years old by the beginning of 1967; the post-1956 contracts are no more than 14 years old by the end of 1970. Statistics were collected by individual years of complete duration from 0 to 4 and for '5 and over' split into pre-1957 and post-1956 data, for males and females separately. We therefore have 14 sets of data:

$$\left.\begin{array}{l}\text{Males}\\ \text{Females}\end{array}\right\}\text{ by }\left\{\begin{array}{l}\text{Durations 0, 1, 2, 3, 4, 5 and over (post-1956),}\\ \text{5 and over (pre-1957)}\end{array}\right.$$

It was decided to use only the post-1956 data for the construction of standard tables. The pre-1957 5 and over experience was also graduated but the results are used only for comparison with the post-1956 data.

4.2. Statistics are available by single ages for nearest ages 51–100 inclusive. For ages below 51 or above 100 the returns from offices were grouped; these groups contain very little exposure, except for the pre-1957 females where 81 deaths occur in the over 100 group, and they have not been used in the graduations.

4.3. The same general methods were used for annuitants as for pensioners. The problems that required solving were:

(a) What select period to use? A select period of only 1 year had been found convenient for the $a(55)$ tables.
(b) What order of polynomial to use in the logistic formula? Would only two parameters be sufficient, as for the pensioners?
(c) What to do about ages below 50, where there is no experience, or even between 50 and 60 where there is very little exposure? The $a(55)$ tables were extended down to age 20 on the basis of current assured lives mortality, and rates for ages as low as this are in fact used by life offices for the calculation of, for example, reversionary annuities.

4.4. The female exposure is considerably larger than the male. The exposed to risk and actual deaths for each experience are shown in the following table.

	Male		Female	
Duration	Exposed to risk	Actual deaths	Exposed to risk	Actual deaths
0	8,231	256	16,273	348
1	8,036	339	16,518	483
2	7,338·5	368	16,001	516
3	6,586·0	315	15,496·5	583
4	6,065·5	331	15,145·5	618
5 and over (post-1956)	26,472	1,985	71,704·5	3,473
5 and over (pre-1957)	11,568	1,464	75,969·5	7,133

With such numbers we find that the 'gates' (to use the terminology of the paper 'Considerations . . .') are rather wide, so it is not difficult to find a curve which fits the data; but the confidence limits one can place on the curve are also rather wide, so that any one of quite a number of curves would fit.

4.5. *Females.* The females were considered first. A preliminary graduation with a quadratic function for $F(x)$, *i.e.* three parameters, gave satisfactory graduations at all durations. The graduated rates for 5 and over (post-1956) were then used as a basis for comparing actual and expected deaths for the other durations.

The following table shows the percentage ratios of actual to expected on this basis:

Duration	0	1	2	3	4	5 and over (post-1956)	5 and over (pre-1957)
100 A/E	64·3	83·2	86·3	96·3	99·5	100·0	119·5

It was clear from this that durations 3 and 4 should certainly be amalgamated with the 5 and over (post-1956) data. The graduated rates for durations 1 and 2 overlapped, and there was therefore no justification for separating them. The value of χ^2 for the data for durations 1 and 2 combined, as compared with the 5 and over (post-1956) data, showed that mortality at these durations was significantly lower, but not exceptionally so. It therefore seemed reasonable to amalgamate all durations from 1 upwards (excluding the pre-1957 data) and to prepare tables on the basis of 1 year's selection.

4.6. Alternative graduations were then carried out using only two parameters, *i.e.* a linear form for $F(x)$. For duration 0 the value of the support function was only slightly smaller than with three parameters, and the other tests were satisfactory. For durations 1 and over the support function increased by 2·3, which is more than the acceptable 2·0.

4.7. Then, in the other direction, the number of parameters was increased to four, giving a cubic form for $F(x)$. For duration 0 there was an improvement of 1·3 in the support function, at the expense of a curiously shaped curve immediately outside the main age-range. There was no advantage here. However, for durations 1 and over the value of the support function improved by 2·8, $t(\chi^2)$ reduced from 2·45 to 1·93 for the grouped data, and the shape of the curve was not unreasonable within the age-range 55–115. But q_x reached a minimum at age 49 (rising below this age) and a maximum at age 115 (falling above this age). The drop at high ages is of no significance and can be eliminated by taking q_x as constant above age 115. At the lower end it is clear that the graduated values for ages 50–54 or so are unreasonably high in comparison with other tables, and need adjustment. But overall the fit is sufficiently improved to prefer this four-parameter graduation to the three-parameter one.

4.8. Increasing the number of parameters yet further for the data for durations 1 and over, *i.e.* to five parameters, giving a quartic form for $F(x)$ produced no significant improvement in the support function. It therefore seemed best to use the two-parameter graduation for duration 0 and the four-parameter one for durations 1 and over; the ratios of select $q_{[x]}$ to ultimate q_x on these bases were for the most part reasonable, except at the extreme ages.

4.9. The data and the graduation results are shown in Tables 13–17. Table 13 shows the graduation parameters and test results for the graduations using two, three and four parameters for duration 0 and for durations 1 and over. Tables 14 and 15 show the details of exposed to risk and actual and expected deaths for the duration 0 two-parameter graduation and the duration 1 and over four-parameter graduation respectively which are shown graphically in Figures 6 and 7. Tables 16 and 17 give the basic data of exposed to risk and actual deaths for all the other separate durations, namely: 1, 2, 3, 4, 5 and over (post-1956) and 5 and over (pre-1957).

4.10. *Males*. Before describing how the female tables were completed it is convenient to discuss the graduations of the male data. The first graduations showed exceptionally high values of χ^2 for both sets of 5 and over data. This was accounted for by values of z_x $(= (A_x - E_x)/\sqrt{E_x p_x})$ at age 94 nearest of 5·9 and 6·0 in the two sets of data. A closer look at the original data showed the following numbers of deaths in each calendar year:

Male annuitants: durations 5 and over (post-1956)

Age (nearest)	(1967)	(1968)	(1969)	(1970)
92	6	10	12	7
93	5	11	13	14
94	2	4	6	47
95	4	6	9	2
96	0	2	0	1

Durations 5 and over (pre-1957)

	(1967)	(1968)	(1969)	(1970)
92	7	16	8	3
93	17	14	21	9
94	9	11	5	49
95	9	8	9	7
96	4	2	7	3

It was clear that some unusual feature was present at age 94 in the year 1970, and it was not accounted for by errors of transcription of the data. Offices who had contributed to these deaths were asked if they could identify the annuitants by name. Many offices could do so, and it transpired that of 54 identified deaths in the combined pre-1957 and post-1956 data some 41 were of a Mr A and 3 of a Mr B, so there were only 12 separate lives for the 54 policies. The Committee previously imagined that duplicate lives in the annuitants investigation were insignificantly few in numbers. This investigation showed that on certain occasions they were not. It was decided to reduce the number of deaths recorded at age 94 by 30 for each of the sets of 5 and over data, and to reduce the exposed to risk by 15 at age 90, and by 30 at each age from 91 to 94 inclusive, on the assumption that all these policies must have been in force for the whole period of the investigation. The adjusted statistics were used thereafter, and are quoted in all the tables that follow.

4.11. As for the females, the first graduations used a quadratic function for $F(x)$, *i.e.* three parameters. The graduated rates for 5 and over (post-1956) were again used as a basis for comparing actual and expected deaths for the other durations. The following table shows the percentage ratios of actual to expected on this basis:

Duration	0	1	2	3	4	5 and over (post-1956)	5 and over (pre-1957)
100 A/E	62·8	80·5	89·7	80·1	85·4	100·0	115·6

Although there was less sign of the select rates running quickly into the 5 and over rates than was the case with the females, the number of deaths in each case was sufficiently small and the χ^2 values were also sufficiently low for it to be difficult to justify using a different select period for males than for females. The statistics for durations 1 and over (post-1956) were therefore amalgamated.

4.12. The initial three-parameter graduation showed satisfactory results with only one exception: the number of runs for the duration 0 data was uncomfortably low, giving $t(r) = 3{\cdot}25$ (only 13 runs over 50 ages). However, inspection of the data (shown graphically in Figure 8) indicated that a much higher order of polynomial would be needed to provide a satisfactory fit, and the gates were so wide that a very simple curve would readily pass between them. Alternative graduations with only two parameters were therefore tried, and these gave results very similar to the three-parameter ones, with only a small increase in the value of the support function in each case. The two-parameter graduations seemed, therefore, to be satisfactory and no further trials were made.

4.13. The data and the graduation results for males are shown in Tables 18–22, corresponding to Tables 13–17 for the females. The graduation parameters and test results are shown in Table 18 only for the two-parameter and three-parameter graduations, and Tables 19 and 20 show details of exposed to risk and actual and expected deaths for durations 0 and 1 and over for the two-parameter graduations, also shown graphically in Figures 8 and 9.

4.14. *Extensions to young and old ages.* There is insufficient exposure for either sex below about age 60 to construct any satisfactory mortality rates based on the actual experience. Nevertheless it is convenient, if only for a comparison of future experience (*e.g.* of pensioners' widows) with some standard, for the published mortality rates to extend down to younger ages. Experience with the existing tables has also shown that it is inconsistent to have annuitant mortality rates at the younger ages at a higher level than the rates shown in the assured lives table relating to similar years of investigation, as is the case with $a(55)$ and A1949–52. For this reason it is felt that the A1967–70 tables provide the most suitable basis for extension to the younger ages of annuitants. They do, of course, relate only to male lives and can therefore only be used as they are to extend the male annuitants table. A comparison of the graduated rates for durations 2 and over from A1967–70 with those for durations 1 and over for male annuitants showed that the curves crossed between ages 65 and 66, the annuitants rates being higher below this age and the assured lives' higher above it.

A smooth blend between the two curves was obtained by using a quintic blending function over the ages 50–65, thus taking gradually increasing account of the annuitant data as the exposure increases. The function used was:

$$k_x\,(\text{annuitants } q_x) + (1 - k_x)\,(\text{A1967–70 } q_x)$$

where

$$k_x = (8 + 15t - 10t^3 + 3t^5)/16 \text{ and } t = (x - 57{\cdot}5)/7{\cdot}5$$

The graduated rates for male annuitants for duration 0 are so far above the A1967–70 rates for either duration 0 or duration 1 that the assured lives rates gave no useful guidance. The graduated male annuitants duration 0 rate at age 65 was 70·7% of the duration 1 and over rate, and the same ratio was used for all lower ages to obtain artificial duration 0 rates for the male annuitants.

4.15. For the females there are as yet no assured lives rates, so an artificial table was constructed by using population rates from 'English Life Table No. 12—Males and Females', and by reducing the A1967–70 proportionately, *i.e.*

$$q_x \text{ A1967–70} \times \frac{q_x \text{ ELT12F}}{q_x \text{ ELT12M}}$$

These artificial rates crossed the female annuitants rates between ages 66 and 67 in a similar manner as for the males, so the same system of blending was used, again between ages 50 and 65. The female duration 0 rates at age 65 were 49·7% of the duration 1 and over rates, and the same ratio was used for all lower ages to obtain artificial duration 0 rates for the female annuitants.

4.16. At the highest ages the graduated male rates were quite well behaved and no adjustments were necessary. But for the females, first, the graduated duration 1 and over rates reached a maximum around age 115 and declined thereafter, and, secondly, the duration 0 rates rose above the male duration 0 rates at age 91 and even above the female duration 1 and over rates at age 99. The necessary adjustments were, in the first place, to make $q_x = q_{115}$ for all $x \geq 115$, and in the second, to make $q_{[x]} = 0{\cdot}75\, q_x$ for all $x \geq 86$. The consequences of these adjustments are not material.

4.17. The final tables, so graduated, extended and adjusted are shown in Table 23 and in Figure 10, for both sexes, both select and ultimate. They can be denoted the *aeg* 1967–70 males and *aeg* 1967–70 females tables—*i.e.* annuitants' experience graduated 1967–70. It should be noted that the graduated rates are quoted to eight decimal places, whereas any rates based on arbitrary adjustments are quoted to only six decimal places. In neither case are more than about the first three figures of any significance. Select rates, $q_{[x]}$ are not quoted above age 100. The extended and adjusted rates are also shown on each of Figures 6–9, from which it can be seen that in no case do the adjustments seriously disagree with the actual experience.

4.18. *Comparisons.* Table 24 shows various comparisons between the mortality rates of the *aeg* 1967–70 tables: select rates as a percentage of the ultimate for each sex, and the female rates as a percentage of the male rates for each duration. Tables 9 and 10 show comparisons between the *aeg* 1967–70 tables and various other tables, in particular the *a*(55) tables. For males the *aeg* 1967–70 ultimate rates are *higher* than those of *a*(55) from ages 55 to 65, and the select rates are higher from ages 50 to 75. This is not inconsistent with changes in population mortality, where the improvement has been least at these middling male ages. For females the *aeg* 1967–70 tables are lower than *a*(55) throughout.

4.19. Tables 11 and 12 show selected annuity values for *aeg* 1967–70 select and ultimate in comparison with *a*(55) and Peg 1967–70 values. The new tables show rather higher annuity values than *a*(55) at most ages, the increase being greatest for males at high ages (11–15% up at age 85, 4–9% up at age 75) and then for females at high ages (7–11% up at age 85, 4–9% up at age 75). For males at age 55 there is a small decline in the annuity value at higher rates of interest. It is clearly not possible to approximate to *aeg* 1967–70 mortality by a constant age adjustment to the *a*(55) tables, because the relative changes at different ages have been so different.

4.20. The Committee considers that it should pursue with urgency its search for a suitable way to prepare standard tables with full monetary functions broadly corresponding to the present *a*(55) tables, particularly since there is no simple way of making use of the existing tables while allowing for changes in mortality since they were produced. As soon as the Committee has some proposals to put to the profession they will be exposed to scrutiny. The Committee is also aware of the particular responsibility for producing justifiable mortality rates for annuitants since they are required for a variety of purposes including the statutory calculation of capital content factors for immediate annuities. Meanwhile, the Committee feels that it is right for the graduated rates to be published so that they can be used as a standard of comparison by life offices and they will be so used by the Committee in reporting on annuitants' experience starting with the combined 4-year experience of 1971–74. Nevertheless, the Committee must repeat its warning that the tables are not necessarily appropriate for the calculation of premiums or reserves since they make no explicit allowance for possible future improvements in mortality.

INDEX TO TABLES

Table 1. *Pensioners* 1967–70: *graduation parameters, test statistics and values of* q_x *for preliminary three-parameter graduations*

Formula: $\log (q_x/p_x) = A+Bt+C(2t^2-1)$ where $t = (x-70)/50$

Experience	Males, Normal, Lives	Males, Normal, Amounts	Males, Early, Lives	Males, Early, Amounts	Females, Normal, Lives	Females, Normal, Amounts	Females, Early, Lives	Females, Early, Amounts
Ages used (nearest)	51–100	51–100	51–97	51–97	51–99	51–99	51–93	51–93
Parameters								
A	−3·1569	−3·3655	−0·5171	−1·2381	−3·3911	−3·2483	+0·4177	+1·0220
B	+4·2865	+4·5647	+2·4797	+3·2021	+5·1817	+5·3487	+3·8937	+4·0570
C	−0·1875	−0·2533	+2·2173	+1·6726	+0·2849	+0·5229	+4·1242	+4·8896
Actual deaths	45,860	—	8,784	—	3,829	—	638	—
Σ(A–E)	0·03	0·05	−0·02	−0·08	0·00	−0·02	0·00	0·00
ΣΣ(A–E)	1·04	0·57	−0·59	−2·76	−0·28	−0·65	−0·08	−0·12
−L′	172,675·9	150,547·6	33,920·4	27,431·2	17,477·6	14,980·5	3,003·3	2,517·4
Using single ages:								
Runs, r	25	25	20	20	23	19	21	17
$t(r)$	0·27	0·11	1·26	1·20	0·59	1·77	0·46	1·56
$t(\rho)$	0·88	−1·48	−0·08	0·29	0·02	0·22	0·02	0·84
Using grouped ages:								
χ^2	70·2	*214·2*	50·5	*82·9*	40·9	*69·5*	31·5	*35·2*
Degrees of freedom	39	40	39	37	33	33	34	28
$t(\chi^2)$	3·08	*11·81*	1·27	*4·33*	0·98	*3·73*	−0·25	*0·98*

Values of χ^2 and $t(\chi^2)$ based on (scaled) amounts data are shown in italics.

$10^5 \times q_x$: Ages	Males, Normal, Lives	Males, Normal, Amounts	Males, Early, Lives	Males, Early, Amounts	Females, Normal, Lives	Females, Normal, Amounts	Females, Early, Lives	Females, Early, Amounts
50	*863*	*657*	*4,668*	*2,518*	*348*	*319*	*1,900*	*1,935*
55	*1,353*	*1,070*	4,398	2,738	*560*	*506*	1,579	1,471
60	2,101	1,720	4,509	3,176	911	817	1,544	1,355
65	3,224	2,729	5,031	3,926	1,494	1,345	1,775	1,514
70	4,883	4,261	6,097	5,163	2,470	2,251	2,397	2,048
75	7,280	6,533	8,002	7,195	4,101	3,820	3,788	3,344
80	10,649	9,801	11,294	10,560	6,806	6,540	6,928	6,507
85	15,223	14,327	16,919	16,124	11,202	11,180	14,233	14,550
90	21,168	20,304	26,251	25,075	18,062	18,780	*30,379*	*33,624*
95	28,497	27,757	*40,471*	*38,382*	*28,035*	*30,251*	*57,504*	*64,696*
100	*36,993*	*36,452*	*58,659*	*55,350*	*41,051*	*45,376*	*83,190*	*88,965*

Values of q_x which are in a region where actual deaths at each age are generally less than 5 are shown in italics.

Table 2. *Pensioners* 1967–70: *graduation parameters, test statistics and values of q_x for two-parameter graduations*

Formula: $\log(q_x/p_x) = A + Bt$ where $t = (x-70)/50$

Experience	Males, Normal, Lives	Males, Normal, Amounts	Males, Early, Lives	Males, Early, Amounts	Females, Normal, Lives	Females, Normal, Amounts	Females, Early, Lives	Females, Early, Amounts
Ages used (nearest)	51–100	51–100	51–97	51–97	51–99	51–99	51–93	51–93
Parameters								
A	−2·9719	−3·1164	−2·6713	−2·8707	−3·6666	−3·7520	−3·5034	−3·6870
B	+4·2143	+4·4803	+2·2464	+2·9869	+5·2448	+5·4434	+3·5383	+2·9278
Actual deaths	45,860	—	8,784	—	3,829	—	638	—
Σ(A–E)	0·00	0·02	−0·11	−0·02	0·00	0·02	0·00	−0·01
ΣΣ(A–E)	0·14	0·49	−3·23	−0·18	−0·03	0·40	−0·24	−0·10
−L′	172,676·8	150,549·0	33,957·4	27,444·5	17,478·0	14,981·5	3,023·0	2,534·8
Using single ages:								
Runs, r	25	21	20	22	23	19	16	8
$t(r)$	0·27	1·08	0·86	0·22	0·66	1·82	1·98	4·27
$t(\rho)$	0·96	−1·43	2·84	1·09	0·12	0·32	3·06	2·70
Using grouped ages:								
χ^2	72·7	*213·6*	129·8	*107·5*	42·4	*72·9*	75·3	*69·2*
Degrees of freedom	41	41	37	35	34	34	30	24
$t(\chi^2)$	3·06	*11·67*	7·57	*6·36*	1·02	*3·89*	4·59	*4·91*

Values of χ^2 and $t(\chi^2)$ based on (scaled) amounts data are shown in italics.

$10^5 \times q_x$:

Ages	Males, Normal, Lives	Males, Normal, Amounts	Males, Early, Lives	Males, Early, Amounts	Females, Normal, Lives	Females, Normal, Amounts	Females, Early, Lives	Females, Early, Amounts
50	*940*	*733*	*2,739*	*1,687*	*313*	*265*	*726*	*771*
55	*1,426*	*1,143*	3,405	2,260	*527*	*456*	1,030	1,030
60	2,157	1,777	4,227	3,023	888	784	1,461	1,375
65	3,251	2,753	5,236	4,033	1,490	1,343	2,069	1,835
70	4,871	4,244	6,469	5,362	2,493	2,293	2,922	2,443
75	7,240	6,487	7,969	7,096	4,140	3,888	4,111	3,248
80	10,631	9,794	9,779	9,336	6,801	6,517	5,756	4,305
85	15,348	14,526	11,948	12,189	10,977	10,726	8,003	5,686
90	21,650	21,011	14,521	15,764	17,241	17,155	*11,026*	*7,475*
95	29,635	29,396	*17,537*	*20,145*	*26,035*	*26,301*	*15,004*	*9,769*
100	*39,095*	*39,456*	*21,025*	*25,378*	*37,293*	*38,083*	*20,094*	*12,671*

Values of q_x which are in a region where actual deaths at each age are generally less than 5 are shown in italics.

Table 3. *Pensioners* 1967–70: *Males, Normal, Lives*: *exposed to risk and comparison of actual deaths with those expected according to the graduated table, namely:*

$$\log (q_x/p_x) = -2{\cdot}9718602 + 4{\cdot}2142613\ (x-70)/50$$

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \dfrac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	35	·009801	0	0·34	−0·34		50½
51½	40	·010654	1	0·43	+0·57		51½
52½	48	·011580	0	0·56	−0·56	−0·84	52½
53½	72	·012585	1	0·91	+0·09		53½
54½	100·5	·013677	2	1·37	+0·63		54½
55½	165	·014862	0	2·45	−2·45		55½
56½	235·5	·016148	3	3·80	−0·80	−1·22	56½
57½	270·5	·017543	2	4·75	−2·75		57½
58½	316	·019056	9	6·02	+2·98	+1·23	58½
59½	939·5	·020697	21	19·44	+1·56	+0·36	59½
60½	2,070	·022476	48	46·53	+1·47	+0·22	60½
61½	2,570·5	·024404	55	62·73	−7·73	−0·99	61½
62½	3,080	·026494	86	81·60	+4·40	+0·49	62½
63½	3,683·5	·028756	124	105·92	+18·08	+1·78	63½
64½	41,232·5	·031206	1,370	1,286·72	+83·28	+2·36	64½
65½	90,159	·033858	2,864	3,052·58	−188·58	−3·47	65½
66½	91,679	·036726	3,296	3,366·98	−70·98	−1·25	66½
67½	85,674·5	·039827	3,355	3,412·14	−57·14	−1·00	67½
68½	76,582·5	·043178	3,378	3,306·67	+71·33	+1·27	68½
69½	67,050·5	·046797	3,206	3,137·78	+68·22	+1·25	69½
70½	57,747	·050704	2,971	2,928·00	+43·00	+0·82	70½
71½	49,322	·054918	2,701	2,708·66	−7·66	−0·15	71½
72½	42,466	·059460	2,575	2,525·04	+49·96	+1·03	72½
73½	36,716	·064353	2,414	2,362·77	+51·23	+1·09	73½
74½	31,940	·069618	2,330	2,223·59	+106·41	+2·34	74½
75½	27,836	·075279	2,044	2,095·46	−51·46	−1.17	75½
76½	23,907	·081360	1,917	1,945·08	−28·08	−0·66	76½
77½	20,304·5	·087886	1,762	1,784·49	−22·49	−0·56	77½
78½	16,926	·094882	1,607	1,605·97	+1·03	+0·03	78½
79½	13,790	·102371	1,422	1,411·70	+10·30	+0·29	79½
80½	11,263	·110380	1,191	1,243·21	−52·21	−1·57	80½
81½	8,995	·118932	1,092	1,069·80	+22·20	+0·72	81½
82½	6,966·5	·128052	870	892·07	−22·07	−0·79	82½
83½	5,376	·137761	742	740·60	+1·40	+0·06	83½
84½	4,007·5	·148082	583	593·44	−10·44	−0·46	84½

Table 3 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
85½	3,004	·159033	433	477·74	−44·74	−2·23	85½
86½	2,165	·170632	362	369·42	−7·42	−0·42	86½
87½	1,528	·182893	286	279·46	+6·54	+0·43	87½
88½	1,067·5	·195827	231	209·05	+21·95	+1·69	88½
89½	731	·209441	149	153·10	−4·10	−0·37	89½
90½	523	·223738	136	117·02	+18·98	+1·99	90½
91½	327·5	·238717	81	78·18	+2·82	+0·37	91½
92½	219·5	·254369	61	55·83	+5·17	+0·80	92½
93½	130	·270684	40	35·19	+4·81	+0·95	93½
94½	79	·287640	22	22·72	−0·72	−0·18	94½
95½	47·5	·305215	8	14·50	−6·50	−2·05	95½
96½	23·5	·323376	4	7·60	−3·60	−1·59	96½
97½	16	·342085	1	5·47	−4·47	−2·36	97½
98½	11	·361299	4	3·97	+0·03	−0·62	98½
99½	3	·380967	0	1·14	−1·14		99½
Total	833,442	—	45,860	45,860·00	0·00	—	

χ^2 = Total z_x^2 = 72·74

Table 4. *Pensioners* 1967–70: *Males, Normal, Amounts: exposed to risk and comparison of actual deaths with those expected according to the graduated table, namely:*

$$\log(q_x/p_x) = -3{\cdot}1163671 + 4{\cdot}4802968\,(x-70)/50$$

(Exposed to risk and deaths are measured in units of £150·75)

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	78·79	·007663	0·00	0·60	−0·60		50½
51½	93·59	·008375	0·75	0·78	−0·03		51½
52½	136·48	·009153	0·00	1·25	−1·25	+0·44	52½
53½	166·66	·010003	1·79	1·67	+0·12		53½
54½	299·96	·010930	6·25	3·28	+2·97		54½
55½	414·92	·011942	0·00	4·96	−4·96	−2·04	55½
56½	587·58	·013047	5·41	7·67	−2·26		56½
57½	678·40	·014253	7·59	9·67	−2·08	−0·67	57½
58½	838·88	·015568	8·14	13·06	−4·92	−1·37	58½
59½	2,597·24	·017003	41·30	44·16	−2·86	−0·43	59½

Table 4 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
60½	5,854·56	·018567	91·38	108·70	−17·32	−1·68	60½
61½	6,897·41	·020272	114·23	139·83	−25·60	−2·19	61½
62½	7,601·30	·022131	165·48	168·22	−2·75	−0·21	62½
63½	8,118·53	·024155	117·92	196·10	−78·19	−5·65	63½
64½	46,887·30	·026360	1,469·48	1,235·94	+233·54	+6·73	64½
65½	97,785·49	·028760	2,736·83	2,812·30	−75·47	−1·44	65½
66½	96,404·17	·031371	3,081·20	3,024·32	+56·87	+1·05	66½
67½	88,006·90	·034212	2,918·74	3,010·85	−92·12	−1·71	67½
68½	76,470·45	·037299	2,851·06	2,852·27	−1·21	−0·02	68½
69½	65,078·59	·040653	2,523·50	2,645·67	−122·17	−2·42	69½
70½	55,547·07	·044296	2,487·42	2,460·49	+26·93	+0·56	70½
71½	47,126·35	·048248	2,221·63	2,273·73	−52·10	−1·12	71½
72½	39,688·31	·052533	2,218·08	2,084·94	+133·14	+3·00	72½
73½	33,699·28	·057176	1,973·65	1,926·79	+46·86	+1·10	73½
74½	28,679·25	·062202	1,903·03	1,783·92	+119·11	+2·91	74½
75½	24,467·26	·067639	1,674·39	1,654·94	+19·45	+0·50	75½
76½	20,711·69	·073514	1,486·44	1,522·59	−36·16	−0·96	76½
77½	17,275·25	·079855	1,420·10	1,379·51	+40·59	+1·14	77½
78½	14,111·09	·086692	1,115·90	1,223·31	−107·41	−3·21	78½
79½	11,547·58	·094054	1,083·16	1,086·10	−2·94	−0·09	79½
80½	9,229·11	·101972	898·48	941·11	−42·63	−1·47	80½
81½	7,166·11	·110475	723·13	791·68	−68·54	−2·58	81½
82½	5,538·74	·119593	689·20	662·39	+26·81	+1·11	82½
83½	4,122·54	·129354	538·63	533·27	+5·37	+0·25	83½
84½	3,049·72	·139784	469·59	426·30	+43·29	+2·26	84½
85½	2,199·10	·150911	297·00	331·87	−34·87	−2·08	85½
86½	1,513·87	·162755	261·94	246·39	+15·55	+1·08	86½
87½	916·33	·175337	161·04	160·67	+0·38	+0·03	87½
88½	621·79	·188673	132·54	117·32	+15·22	+1·56	88½
89½	402·63	·202773	70·22	81·64	−11·42	−1·42	89½
90½	298·68	·217645	78·20	65·01	+13·20	+1·85	90½
91½	186·95	·233288	52·39	43·61	+8·78	+1·52	91½
92½	125·43	·249696	29·36	31·32	−1·96	−0·40	92½
93½	72·66	·266857	33·82	19·39	+14·44	+3·83	93½
94½	44·33	·284750	6·24	12·62	−6·38	−2·12	94½
95½	45·75	·303346	2·57	13·88	−11·31	−3·64	95½
96½	28·18	·322609	2·88	9·09	−6·21	−2·50	96½
97½	18·55	·342493	0·31	6·35	−6·04		97½
98½	9·88	·362947	3·24	3·58	−0·35	−2.65	98½
99½	1·35	·383908	0·00	0·52	−0·52		99½
Total	833,442	—	38,175·64	38,175·62	+0·02	—	

χ^2 = Total z_x^2 = 213·57

Table 5. *Pensioners* 1967–70: *Females, Normal, Lives: exposed to risk and comparison of actual deaths with those expected according to the graduated table, namely:*

$$\log (q_x/p_x) = -3{\cdot}6665812 + 5{\cdot}2448241\,(x-70)/50$$

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \dfrac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	14·5	·003295	0	0·05	−0·05		50½
51½	10	·003658	0	0·04	−0·04		51½
52½	21	·004061	0	0·09	−0·09		52½
53½	23	·004508	0	0·10	−0·10		53½
54½	89·5	·005004	0	0·45	−0·45	+0·08	54½
55½	165	·005554	0	0·92	−0·92		55½
56½	211·5	·006165	3	1·30	+1·70		56½
57½	277·5	·006842	2	1·90	+0·10		57½
58½	385·5	·007593	3	2·93	+0·07		58½
59½	4,926·5	·008426	54	41·51	+12·49	+1·95	59½
60½	11,535	·009349	115	107·84	+7·16	+0·69	60½
61½	12,416·5	·010372	131	128·79	+2·21	+0·20	61½
62½	12,772·5	·011506	129	146·96	−17·96	−1·49	62½
63½	12,835	·012762	145	163·81	−18·81	−1·48	63½
64½	13,102·5	·014154	200	185·45	+14·55	+1·08	64½
65½	13,110	·015695	195	205·76	−10·76	−0·76	65½
66½	12,206	·017400	234	212·39	+21·61	+1·50	66½
67½	11,072	·019288	197	213·55	−16·55	−1·14	67½
68½	9,784·5	·021375	209	209·14	−0·14	−0·01	68½
69½	8,658	·023683	229	205·05	+23·95	+1·69	69½
70½	7,607·5	·026234	200	199·57	+0·43	+0·03	70½
71½	6,718	·029051	195	195·16	−0·16	−0·01	71½
72½	5,887	·032160	194	189·33	+4·67	+0·35	72½
73½	5,130·5	·035590	186	182·60	+3·40	+0·26	73½
74½	4,403	·039371	163	173·35	−10·35	−0·80	74½
75½	3,687·5	·043536	131	160·54	−29·54	−2·38	75½
76½	3,022	·048119	147	145·42	+1·58	+0·13	76½
77½	2,403	·053158	115	127·74	−12·74	−1·16	77½
78½	1,879·5	·058692	117	110·31	+6·69	+0·66	78½
79½	1,410·5	·064763	99	91·35	+7·65	+0·83	79½
80½	1,090	·071414	76	77·84	−1·84	−0·22	80½
81½	825	·078691	58	64·92	−6·92	−0·89	81½
82½	649·5	·086639	53	56·27	−3·27	−0·46	82½
83½	513·5	·095308	54	48·94	+5·06	+0·76	83½
84½	390	·104745	46	40·85	+5·15	+0·85	84½

Table 5 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
85½	297·5	·114997	47	34·21	+12·79	+2·32	85½
86½	203	·126111	21	25·60	−4·60	−0·97	86½
87½	151	·138132	21	20·86	+0·14	+0·03	87½
88½	114	·151100	22	17·23	+4·77	+1·25	88½
89½	76	·165053	12	12·54	−0·54	−0·17	89½
90½	50·5	·180021	11	9·09	+1·91	+0·70	90½
91½	28·5	·196028	5	5·59	−0·59	−0·28	91½
92½	15	·213088	6	3·20	+2·80	+1·27	92½
93½	9·5	·231206	2	2·20	−0·20		93½
94½	8·5	·250374	0	2·13	−2·13		94½
95½	8	·270572	2	2·16	−0·16		95½
96½	3	·291766	0	0·88	−0·88	−2·01	96½
97½	2·5	·313905	0	0·78	−0·78		97½
98½	1	·336924	0	0·34	−0·34		98½
Total	170,200·5	—	3,829	3,829·00	0·00	—	

χ^2 = Total z_x^2 = 42·36

Table 6. *Pensioners* 1967–70: *Females, Normal, Amounts: exposed to risk and comparison of actual deaths with these expected according to the graduated table, namely:*

$$\log (q_x/p_x) = -3{\cdot}7520391 + 5{\cdot}4433637\ (x-70)/50$$

(Exposed to risk and deaths are measured in units of £81.05)

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	38·67	·002801	0·00	0·11	−0·11		50½
51½	16·58	·003122	0·00	0·05	−0·05		51½
52½	28·24	·003480	0·00	0·10	−0·10		52½
53½	48·27	·003879	0·00	0·19	−0·19	+0·82	53½
54½	163·93	·004323	0·00	0·71	−0·71		54½
55½	336·96	·004818	0·00	1·62	−1·62		55½
56½	504·79	·005369	7·40	2·71	+4·69		56½
57½	586·20	·005983	3·01	3·51	−0·50	−0·67	57½
58½	773·50	·006666	3·70	5·16	−1·45		58½
59½	6,501·51	·007427	77·39	48·29	+29·10	+4·20	59½

Table 6 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
60½	14,289·36	·008274	125·40	118·24	+7·16	+0·66	60½
61½	14,618·49	·009217	131·26	134·74	−3·49	−0·30	61½
62½	14,333·02	·010267	140·39	147·15	−6·76	−0·56	62½
63½	14,095·97	·011434	140·40	161·17	−20·77	−1·65	63½
64½	14,136·59	·012732	174·80	179·99	−5·19	−0·39	64½
65½	13,879·74	·014176	203·47	196·76	+6·72	+0·48	65½
66½	12,409·19	·015780	216·36	195·82	+20·53	+1·48	66½
67½	10,806·83	·017564	171·23	189·81	−18·57	−1·36	67½
68½	9,066·77	·019544	190·47	177·20	+13·27	+1·01	68½
69½	7,620·12	·021743	169·35	165·69	+3·66	+0·29	69½
70½	6,329·57	·024183	145·85	153·07	−7·22	−0·59	70½
71½	5,436·85	·026890	142·78	146·20	−3·42	−0·29	71½
72½	4,706·98	·029890	109·89	140·69	−30·81	−2·64	72½
73½	4,043·57	·033214	124·68	134·30	−9·62	−0·84	73½
74½	3,361·52	·036893	114·75	124·02	−9·27	−0·85	74½
75½	2,696·72	·040963	88·13	110·46	−22·33	−2·17	75½
76½	2,206·32	·045460	103·15	100·30	+2·85	+0·29	76½
77½	1,658·66	·050425	86·02	83·64	+2·39	+0·27	77½
78½	1,367·57	·055900	115·09	76·45	+38·65	+4·55	78½
79½	997·00	·061931	63·74	61·75	+2·00	+0·26	79½
80½	799·20	·068566	54·55	54·80	−0·25	−0·03	80½
81½	605·87	·075854	46·85	45·96	+0·89	+0·14	81½
82½	470·49	·083846	43·76	39·45	+4·32	+0·72	82½
83½	352·16	·092597	35·51	32·61	+2·90	+0·53	83½
84½	261·30	·102159	28·84	26·69	+2·14	+0·44	84½
85½	197·74	·112586	26·89	22·26	+4·62	+1·04	85½
86½	149·99	·123930	17·16	18·59	−1·43	−0·35	86½
87½	94·23	·136242	12·52	12·84	−0·31	−0·09	87½
88½	76·15	·149568	16·55	11·39	+5·16	+1·66	88½
89½	46·66	·163950	4·39	7·65	−3·26	−1·29	89½
90½	30·35	·179423	5·59	5·45	+0·14	+0·07	90½
91½	15·20	·196014	3·55	2·98	+0·57		91½
92½	7·01	·213740	1·72	1·50	+0·22	+0·49	92½
93½	5·02	·232605	1·41	1·17	+0·24		93½
94½	8·50	·232600	0·00	2·15	−2·15		94½
95½	9·97	·273701	3·69	2·73	+0·96		95½
96½	2·21	·295867	0·00	0·65	−0·65	−1·94	96½
97½	6·01	·319040	0·00	1·92	−1·92		97½
98½	2·94	·343142	0·00	1·01	−1·01		98½
Total	170,200·51	—	3,151·68	3,151·66	+0·02	—	

$\chi^2 = \text{Total } z_x^2 = 72{\cdot}92$

Table 7. *Pensioners* 1967–70. *Ratios (as a percentage) of graduated rates, using two-parameter graduations for Normal, three-parameter graduations for Early, for each comparison of corresponding pairs of data*

	Amounts as per cent of Lives				Early as per cent of Normal				Females as per cent of Males				
Age	Males, Normal	Males, Early	Females, Normal	Females, Early	Males, Lives	Males, Amounts	Females, Lives	Females, Amounts	Normal, Lives	Normal, Amounts	Early, Lives	Early, Amounts	Age
50	*78·0*	*53·9*	*84·7*	*101·8*	*496·6*	*343·5*	*607·0*	*730·2*	*33·3*	*36·2*	*40·7*	*76·8*	50
55	*80·2*	62·3	*86·5*	93·2	*308·4*	*239·5*	*299·6*	*322·6*	*37·0*	*39·9*	35·9	53·7	55
60	82·4	70·4	88·3	87·8	209·0	178·7	173·9	172·8	41·2	44·1	34·2	42·7	60
65	84·7	78·0	90·1	85·3	154·8	142·6	119·1	112·7	45·8	48·8	35·3	38·6	65
70	87·1	84·7	92·0	85·4	125·2	135·8	96·1	89·3	51·2	54·0	39·3	39·7	70
75	89·6	89·9	93·9	88·3	110·5	110·9	91·5	86·0	57·2	59·9	47·3	46·5	75
80	92·1	93·5	95·8	93·9	106·2	107·8	101·9	99·8	64·0	66·5	61·3	61·6	80
85	94·6	95·3	97·7	102·2	110·2	111·0	129·7	135·7	71·5	73·8	84·1	90·2	85
90	97·0	95·5	99·5	*110·7*	121·3	119·3	*176·2*	*196·0*	79·6	81·6	*115·7*	*134·1*	90
95	99·2	*94·8*	*101·0*	*112·5*	*136·6*	*130·6*	*220·9*	*246·0*	*87·9*	*89·5*	*142·1*	*168·6*	95
100	*100·9*	*94·4*	*102·1*	*106·9*	*150·0*	*140·3*	*223·1*	*233·6*	*95·4*	*96·5*	*141·8*	*160·7*	100

Ratios based on rates in the regions where actual deaths at each age are generally less than 5 are shown in italics.

Table 8. *Peg* 1967–70: *values of* q_x

Formula: $\log (q_x/p_x) = A + B\,(x-70)/50$

Males, Lives (mL)	$A = -2{\cdot}9718602$	$B = +4{\cdot}2142613$
Males, Amounts (mA)	$A = -3{\cdot}1163671$	$B = +4{\cdot}4802968$
Females, Lives (fL)	$A = -3{\cdot}6665812$	$B = +5{\cdot}2448241$
Females, Amounts (fA)	$A = -3{\cdot}7520391$	$B = +5{\cdot}4433637$

Age x	Males, Lives (mL)	Males, Amounts (mA)	Females, Lives (fL)	Females, Amounts (fA)	Age x
50	·00940034	·00732954	·00312712	·00265307	50
51	·01021855	·00801113	·00347177	·00295730	51
52	·01110718	·00875554	·00385425	·00329631	52
53	·01207214	·00956846	·00427869	·00367404	53
54	·01311983	·01045606	·00474964	·00409488	54
55	·01425713	·01142505	·00527216	·00456370	55
56	·01549146	·01248270	·00585183	·00508592	56
57	·01683084	·01363691	·00649481	·00566756	57
58	·01828387	·01489624	·00720793	·00631529	58
59	·01985981	·01626994	·00799872	·00703653	59
60	·02156859	·01776803	·00887550	·00783948	60
61	·02342089	·01940135	·00984742	·00873326	61
62	·02542813	·02118157	·01092460	·00972794	62
63	·02760254	·02312129	·01211818	·01083467	63
64	·02995717	·02523406	·01344038	·01206578	64
65	·03250594	·02753445	·01490467	·01343487	65
66	·03526368	·03003808	·01652582	·01495697	66
67	·03824614	·03276169	·01832003	·01664859	67
68	·04147000	·03572317	·02030500	·01852794	68
69	·04495290	·03894156	·02250012	·02061499	69
70	·04871345	·04243715	·02492650	·02293164	70
71	·05277120	·04623143	·02760716	·02550184	71
72	·05714666	·05034711	·03056706	·02835176	72
73	·06186121	·05480813	·03383327	·03150987	73
74	·06693709	·05963960	·03743501	·03500709	74
75	·07239733	·06486775	·04140374	·03887688	75
76	·07826560	·07051983	·04577321	·04315532	76
77	·08456618	·07662404	·05057948	·04788114	77
78	·09132371	·08320933	·05586087	·05309576	78
79	·09856309	·09030523	·06165792	·05884319	79
80	·10630920	·09794161	·06801323	·06516994	80
81	·11458670	·10614838	·07497127	·07212481	81
82	·12341970	·11495519	·08257806	·07975857	82
83	·13283145	·12439098	·09088084	·08812356	83
84	·14284397	·13448360	·09992750	·09727312	84

Table 8 (*continued*).

Age x	Males, Lives (mL)	Males, Amounts (mA)	Females, Lives (fL)	Females, Amounts (fA)	Age x
85	·15347762	·14525925	·10976596	·10726092	85
86	·16475073	·15674195	·12044345	·11814002	86
87	·17667903	·16895294	·13200559	·12996192	87
88	·18927527	·18191000	·14449531	·14277528	88
89	·20254861	·19562683	·15795171	·15662452	89
90	·21650421	·21011234	·17240871	·17154830	90
91	·23114263	·22536992	·18789367	·18757781	91
92	·24645947	·24139684	·20442588	·20473497	92
93	·26244482	·25818364	·22201504	·22303062	93
94	·27908299	·27571351	·24065979	·24246276	94
95	·29635215	·29396196	·26034636	·26301484	95
96	·31422412	·31289643	·28104729	·28465439	96
97	·33266435	·33247618	·30272055	·30733188	97
98	·35163185	·35265231	·32530892	·33098012	98
99	·37107947	·37336798	·34873979	·35551406	99
100	·39095410	·39455885	·37292549	·38083138	100
101	·41119725	·41615369	·39776407	·40681358	101
102	·43174552	·43807526	·42314067	·43332783	102
103	·45253144	·46024130	·44892941	·46022942	103
104	·47348426	·48256574	·47499568	·48736473	104
105	·49453089	·50495999	·50119892	·51457473	105
106	·51559691	·52733437	·52739558	·54169860	106
107	·53660766	·54959946	·55344225	·56857756	107
108	·55748922	·57166766	·57919877	·59505854	108
109	·57816952	·59345442	·60453126	·62099756	109
110	·59857925	·61487956	·62931473	·64626276	110
111	·61865282	·63586839	·65343543	·67073673	111
112	·63832909	·65635263	·67679269	·69431835	112
113	·65755210	·67627121	·69930017	·71692384	113
114	·67627156	·69557075	·72088667	·73848727	114

Table 9. *Values of* 10^5q_x and $10^5q_{[x]}$ *by various tables: Males*

Age	Peg 1967–70			10^5q_x (ultimate)			$10^5q_{[x]}$ (select)			Age
				A		*aeg*		A	*aeg*	
x	(mL)	(mA)	*a*(55)	1967–70	ELT 12	1967–70	*a*(55)	1967–70	1967–70	x
20			118	89	119	89	71	66	63	20
25			128	69	99	69	77	47	49	25
30			139	65	115	65	83	44	46	30
35			168	86	155	86	101	60	60	35
40			230	144	235	144	138	102	102	40
45			349	264	399	264	209	174	186	45
50	940	733	547	479	728	479	328	286	338	50
55	1,426	1,143	870	844	1,331	875	522	447	619	55
60	2,157	1,777	1,402	1,443	2,287	1,527	841	670	1,079	60
65	3,251	2,753	2,297	2,403	3,648	2,407	1,378	969	1,701	65
70	4,871	4,244	3,776	3,911	5,566	3,724	2,266	1,363	2,561	70
75	7,240	6,487	6,164	6,229	8,434	5,717	3,698	1,875	3,838	75
80	10,631	9,794	9,861	9,703	12,747	8,683	6,212	2,531	5,715	80
85	15,348	14,526	15,246	14,727	18,659	12,974	10,291		8,429	85
90	21,650	21,011	22,413	21,651	25,593	18,947	16,137		12,265	90
95	29,635	29,396	30,903	30,593	32,385	26,821	22,250		17,512	95
100	39,095	39,456	39,668	41,229	37,983	36,495			24,381	100

Table 10. *Values of* 10^5q_x *and* $10^5q_{[x]}$ *by various tables: Females*

Age	Peg 1967–70		10^5q_x (ultimate)			$10^5q_{[x]}$(select)		Age
					aeg		*aeg*	
x	(fL)	(fA)	*a*(55)	ELT 12	1967–70	*a*(55)	1967–70	x
20			117	44	33	70	16	20
25			123	54	38	74	19	25
30			132	75	43	79	21	30
35			153	114	63	92	31	35
40			191	180	111	115	55	40
45			261	284	188	157	93	45
50	313	265	376	439	289	226	144	50
55	527	456	553	682	484	332	241	55
60	888	784	855	1,088	836	513	416	60
65	1,490	1,343	1,385	1,808	1,262	831	627	65
70	2,493	2,293	2,307	3,104	1,971	1,384	1,152	70
75	4,140	3,888	3,881	5,370	3,250	2,329	2,107	75
80	6,801	6,517	6,495	9,108	5,490	4,092	3,825	80
85	10,977	10,726	10,628	14,729	9,207	7,174	6,844	85
90	17,241	17,155	16,694	22,128	14,839	12,020	11,129	90
95	26,035	26,301	24,688	30,323	22,323	17,775	16,742	95
100	37,293	38,083	33,846	37,788	30,729		23,047	100

Table 11. *Comparison of annuity values (a_x) by various tables: Males*

Rate of interest	Age x	Peg 1967–70 (mL)	Peg 1967–70 (mA)	a(55) (ult.)	*aeg* 1967–70 (ult.)	a(55) (select)	*aeg* 1967–70 (select)
0%	55	19·158	20·420	21·369	21·686	21·445	21·742
	65	12·555	13·391	13·807	14·351	13·936	14·455
	75	7·507	7·944	7·891	8·639	8·098	8·811
	85	4·078	4·228	4·007	4·663	4·241	4·907
5%	55	11·192	11·694	12·122	12·149	12·164	12·180
	65	8·393	8·823	9·081	9·269	9·166	9·336
	75	5·646	5·925	5·915	6·336	6·071	6·462
	85	3·367	3·481	3·321	3·792	3·515	3·990
10%	55	7·552	7·795	8·017	8·001	8·045	8·022
	65	6·136	6·386	6·555	6·625	6·617	6·673
	75	4·468	4·658	4·665	4·923	4·787	5·021
	85	2·854	2·943	2·823	3·176	2·988	3·342
15%	55	5·595	5·732	5·862	5·844	5·882	5·859
	65	4·774	4·934	5·051	5·078	5·098	5·114
	75	3·670	3·807	3·820	3·990	3·921	4·070
	85	2·470	2·541	2·448	2·722	2·591	2·864

Table 12. *Comparison of annuity values (a_x) by various tables: Females*

Rate of interest	Age x	Peg 1967–70 (fL)	Peg 1967–70 (fA)	a(55) (ult.)	*aeg* 1967–70 (ult.)	a(55) (select)	*aeg* 1967–70 (select)
0%	55	24·843	25·417	25·293	26·473	25·349	26·538
	65	16·653	17·053	17·089	18·269	17·185	18·386
	75	9·927	10·126	10·205	11·163	10·370	11·294
	85	5·133	5·169	5·295	5·859	5·500	6·011
5%	55	13·263	13·464	13·390	13·698	13·420	13·731
	65	10·362	10·552	10·553	11·021	10·612	11·092
	75	7·116	7·239	7·272	7·796	7·389	7·886
	85	4·137	4·167	4·244	4·627	4·408	4·748
10%	55	8·489	8·576	8·532	8·634	8·551	8·655
	65	7·222	7·325	7·320	7·539	7·361	7·587
	75	5·434	5·516	5·531	5·848	5·620	5·917
	85	3·440	3·465	3·514	3·791	3·650	3·889
15%	55	6·097	6·143	6·115	6·156	6·128	6·171
	65	5·441	5·504	5·498	5·615	5·529	5·651
	75	4·347	4·405	4·412	4·620	4·484	4·675
	85	2·931	2·951	2·985	3·193	3·101	3·276

Table 13. *Female annuitants* 1967–70: *graduation parameters, test statistics, and values of q_x for graduations of data for durations* 0 *and* 1 *and over* (*post*-1956) *using using two, three and four parameters*

Formula: $\log(q_x/p_x) = A+Bt+C\,(2t^2-1)+D(4t^3-3t)$ where $t = (x-70)/50$

Experience	Duration 0			Durations 1 and over		
Graduation	2-pars	3-pars	4-pars	2-pars	3-pars	4-pars
Ages used (nearest)	51–100	51–100	51–100	51–100	51–100	51–100
Parameters						
A	−4·4521	−3·8484	−6·4936	−3·8814	−3·4054	−2·4116
B	+6·1370	+5·7613	+14·8741	+5·2600	+4·9226	+1·7242
C	—	+0·6071	−2·1301	—	+0·4688	+1·4950
D	—	—	+3·0390	—	—	−1·0501
Actual deaths	348	348	348	5673	5673	5673
Σ(A–E)	0·00	0·00	−0·01	−0·02	0·01	0·04
ΣΣ(A–E)	0·01	0·00	−0·12	−0·92	0·27	0·25
−L′	1506·2	1506·0	1504·7	21625·6	21623·3	21620·6
Using single ages:						
Runs, r	22	22	22	28	24	24
$t(r)$	1·10	1·10	1·10	−0·58	0·57	0·57
$t(\rho)$	−0·31	−0·33	−0·75	0·68	0·34	−0·06
Using grouped ages:						
χ^2	28·9	28·2	26·1	70·5	66·8	59·7
Degrees of freedom	29	28	28	42	42	41
$t(\chi^2)$	0·05	0·09	−0·20	2·77	2·45	1·93
$10^5 \times q_x$: Ages						
50	*100*	*141*	*30*	*251*	*336*	*600*
55	*185*	*230*	*111*	*424*	*514*	*678*
60	*340*	*384*	*307*	715	799	876
65	*627*	*656*	*673*	1,204	1,265	1,262
70	1,152	1,148	1,257	2,020	2,035	1,971
75	2,107	2,049	2,149	3,372	3,317	3,250
80	3,825	3,716	3,608	5,576	5,457	5,490
85	6,844	6,800	6,347	*9,084*	*9,005*	*9,207*
90	11,949	12,382	12,300	14,462	14,741	14,839
95	*20,044*	*21,903*	*26,299*	22,246	23,533	22,323
100	*31,651*	*36,317*	*54,557*	*32,621*	*35,823*	*30,729*

Values of q_x which are in a region where actual deaths at each age are generally less than 5 are shown in italics.

Table 14. *Female annuitants* 1967–70: *duration* 0: *exposed to risk and comparison of actual deaths with those expected according to the graduated table, namely:*

$$\log(q_x/p_x) = -4{\cdot}4520890 + 6{\cdot}1370401\,(x-70)/50$$

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	34·5	·001063	0	0·04	−0·04		50½
51½	39	·001202	0	0·05	−0·05		51½
52½	48·5	·001358	0	0·07	−0·07		52½
53½	65	·001536	0	0·10	−0·10		53½
54½	101	·001736	0	0·18	−0·18		54½
55½	142·5	·001962	1	0·28	+0·72	+0·05	55½
56½	118·5	·002218	0	0·26	−0·26		56½
57½	175	·002507	0	0·44	−0·44		57½
58½	197	·002833	3	0·56	+2·44		58½
59½	491	·003202	2	1·57	+0·43		59½
60½	646·5	·003618	0	2·34	−2·34		60½
61½	452·5	·004089	3	1·85	+1·15		61½
62½	504·5	·004620	1	2·33	−1·33	−0·36	62½
63½	527	·005221	2	2·75	−0·75		63½
64½	717	·005898	2	4·23	−2·23	−1·18	64½
65½	817	·006664	4	5·44	−1·44		65½
66½	681	·007527	3	5·13	−2·13	−0·94	66½
67½	640·5	·008502	8	5·45	+2·55	+1·10	67½
68½	663·5	·009601	11	6·37	+4·63	+1·84	68½
69½	710·5	·010842	7	7·70	−0·70	−0·25	69½
70½	741·5	·012240	8	9·08	−1·08	−0·36	70½
71½	668	·013817	16	9·23	+6·77	+2·24	71½
72½	651	·015593	9	10·15	−1·15	−0·36	72½
73½	672·5	·017593	13	11·83	+1·17	+0·34	73½
74½	601	·019845	17	11·93	+5·07	+1·48	74½
75½	624·5	·022379	14	13·98	+0·02	+0·01	75½
76½	557·5	·025227	11	14·06	−3·06	−0·83	76½
77½	519	·028428	14	14·75	−0·75	−0·20	77½
78½	448	·032022	13	14·35	−1·35	−0·36	78½
79½	454·5	·036053	13	16·39	−3·39	−0·85	79½
80½	415	·040570	15	16·84	−1·84	−0·46	80½
81½	375·5	·045626	14	17·13	−3·13	−0·77	81½
82½	301	·051279	16	15·44	+0·56	+0·15	82½
83½	294·5	·057590	19	16·96	+2·04	+0·51	83½
84½	272	·064625	18	17·58	+0·42	+0·10	84½

Table 14 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \dfrac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
85½	228	·072453	17	16·52	+0·48	+0·12	85½
86½	174	·081147	9	14·12	−5·12	−1·42	86½
87½	125·5	·090782	11	11·39	−0·39	−0·12	87½
88½	125	·101435	14	12·68	+1·32	+0·39	88½
89½	68·5	·113182	8	7·75	+0·25	+0·09	89½
90½	48·5	·126099	5	6·12	−1·12	−0·48	90½
91½	44·5	·140256	9	6·24	+2·76	+1·19	91½
92½	40	·155720	2	6·23	−4·23	−1·84	92½
93½	23	·172547	5	3·97	+1·03		93½
94½	12	·190782	3	2·29	+0·71		94½
95½	7·5	·210453	5	1·58	+3·42		95½
96½	3	·231572	1	0·69	+0·31	+2·05	96½
97½	3·5	·254128	0	0·89	−0·89		97½
98½	1·5	·278086	1	0·42	+0·58		98½
99½	1	·303384	1	0·30	+0·70		99½
Total	16,273	—	348	348·00	0·00	—	

$\chi^2 = \text{Total } z_x^2 = 28{\cdot}87$

Table 15. *Female annuitants* 1967–70: *durations* 1 *and over* (*post*-1956): *exposed to risk and comparison of actual deaths with those expected according to the graduated table, namely:*

$$\log(q_x/p_x) = -2{\cdot}4115886 + 1{\cdot}7242068t + 1{\cdot}4950480(2t^2-1) - 1{\cdot}0500534(4t^3-3t)$$

where $t = (x-70)/50$

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \dfrac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	111	·006038	1	0·67	+0·33		50½
51½	166·5	·006132	0	1·02	−1·02		51½
52½	224	·006266	3	1·40	+1·60	+1·27	52½
53½	290·5	·006441	3	1·87	+1·13		53½
54½	386	·006658	4	2·57	+1·43		54½
55½	513·5	·006921	2	3·55	−1·55	−0·84	55½
56½	676·5	·007231	4	4·89	−0·89		56½
57½	820·5	·007594	6	6·23	−0·23	−0·09	57½
58½	984	·008014	8	7·89	+0·11	+0·04	58½
59½	1,246	·008497	15	10·59	+4·41	+1·36	59½
60½	1,813	·009049	8	16·41	−8·41	−2·08	60½
61½	2,497·5	·009677	16	24·17	−8·17	−1·67	61½
62½	2,948·5	·010391	28	30·64	−2·64	−0·48	62½
63½	3,361·5	·011200	40	37·65	+2·35	+0·39	63½
64½	3,758	·012116	42	45·53	−3·53	−0·53	64½

Table 15 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Age x
65½	4,232·5	·013151	71	55·66	+15·34	+2·07	65½
66½	4,770	·014319	79	68·30	+10·70	+1·30	66½
67½	5,082	·015637	78	79·47	−1·47	−0·17	67½
68½	5,314	·017123	89	90·99	−1·99	−0·21	68½
69½	5,611	·018797	124	105·47	+18·53	+1·82	69½
70½	5,821	·020682	112	120·39	−8·39	−0·77	70½
71½	5,931	·022802	98	135·24	−37·24	−3·24	71½
72½	5,950	·025186	147	149·86	−2·86	−0·24	72½
73½	5,884·5	·027862	159	163·96	−4·96	−0·39	73½
74½	5,916·5	·030865	204	182·61	+21·39	+1·61	74½
75½	5,750·5	·034229	208	196·83	+11·17	+0·81	75½
76½	5,569·5	·037991	217	211·59	+5·41	+0·38	76½
77½	5,361·5	·042193	234	226·22	+7·78	+0·53	77½
78½	5,173	·046877	241	242·49	−1·49	−0·10	78½
79½	4,997·5	·052086	269	260·30	+8·70	+0·55	79½
80½	4,816·5	·057864	280	278·70	+1·30	+0·08	80½
81½	4,490·5	·064256	284	288·54	−4·54	−0·28	81½
82½	4,137	·071306	268	294·99	−26·99	−1·63	82½
83½	3,692·5	·079055	278	291·91	−13·91	−0·85	83½
84½	3,287	·087540	269	287·75	−18·75	−1·16	84½
85½	2·872	·096794	297	277·99	+19·01	+1·20	85½
86½	2,414·5	·106842	258	257·97	+0·03	0·00	86½
87½	1,980	·117699	255	233·04	+21·96	+1·53	87½
88½	1,534·5	·129371	186	198·52	−12·52	−0·95	88½
89½	1,202·5	·141852	168	170·58	−2·58	−0·21	89½
90½	938	·155120	167	145·50	+21·50	+1·94	90½
91½	703	·169140	111	118·91	−7·91	−0·80	91½
92½	541·5	·183858	110	99·56	+10·44	+1·16	92½
93½	398	·199208	67	79·28	−12·28	−1·54	93½
94½	270	·215105	56	58·08	−2·08	−0·31	94½
95½	173	·231449	42	40·04	+1·96	+0·35	95½
96½	108	·248128	27	26·80	+0·20	+0·04	96½
97½	71·5	·265019	21	18·95	+2·05	+0·55	97½
98½	43·5	·281989	15	12·27	+2·73	+0·92	98½
99½	30·5	·298900	4	9·12	−5·12	−2·02	99½
Total	134,865·5	—	5,673	5,672·96	+0·04	—	

$\chi^2 = \text{Total } z_x^2 = 59{\cdot}74$

Table 16. *Female annuitants* 1967–70: *exposed to risk and actual deaths*

Age	Duration 1		Duration 2		Duration 3		Age
x	ER_x	A_x	ER_x	A_x	ER_x	A_x	x
50½	24	0	13·5	0	17·5	0	50½
51½	40·5	0	30·5	0	20	0	51½
52½	47	0	45	0	42	3	52½
53½	49	1	48	1	55	0	53½
54½	75	0	51	0	51	1	54½
55½	113	0	80·5	0	50	0	55½
56½	153·5	1	123·5	0	85	0	56½
57½	126	1	162	1	140·5	1	57½
58½	180	1	127·5	0	169·5	1	58½
59½	214	1	181	2	157	2	59½
60½	523·5	2	223	1	188·5	2	60½
61½	675·5	3	525	4	244·5	2	61½
62½	482	2	684	3	525	11	62½
63½	521	5	488	6	685·5	7	63½
64½	557·5	4	518·5	6	490·5	6	64½
65½	727·5	8	541·5	10	518·5	9	65½
66½	828	9	703	13	525	9	66½
67½	710·5	8	777	15	697·5	15	67½
68½	659·5	12	697	7	728·5	14	68½
69½	660·5	11	650·5	10	695·5	14	69½
70½	730·5	17	644	17	657	12	70½
71½	731·5	14	685	8	619·5	6	71½
72½	698	17	708·5	21	646·5	10	72½
73½	663	13	692·5	18	678·5	22	73½
74½	682	17	647	18	683·5	18	74½
75½	586·5	14	650·5	14	623	19	75½
76½	620·5	24	570·5	25	628·5	26	76½
77½	539·5	19	590·5	23	545	26	77½
78½	532	15	524·5	18	563·5	17	78½
79½	473·5	25	518	19	510	32	79½
80½	473·5	30	471	21	485	31	80½
81½	410	24	440·5	22	448·5	29	81½
82½	382·5	19	381·5	14	425	30	82½
83½	303	25	375	25	373·5	32	83½
84½	293	24	284·5	17	327·5	31	84½
85½	241	22	273·5	25	260·5	22	85½
86½	210·5	22	211·5	24	246	29	86½
87½	166·5	16	174	23	175	16	87½
88½	105	10	147·5	19	141	20	88½
89½	105·5	12	90·5	14	115	12	89½

Table 16 (*continued*).

Age x	Duration 1 ER_x	Duration 1 A_x	Duration 2 ER_x	Duration 2 A_x	Duration 3 ER_x	Duration 3 A_x	Age x
90½	61	13	90·5	14	79·5	16	90½
91½	50	4	54·5	11	67·5	5	91½
92½	31·5	9	48	12	43	8	92½
93½	31·5	2	19	5	28	7	93½
94½	12·5	4	17·5	4	17·5	5	94½
95½	7	2	10·5	3	8	3	95½
96½	4·5	1	5·5	1	4	0	96½
97½	0·5	0	2·5	1	5	1	97½
98½	4·5	0	1	1	3	1	98½
99½			1·5	0	2·5	0	99½
Total	16,518	483	16,001	516	15,496·5	583	

Table 17. *Female annuitants* 1967–70: *exposed to risk and actual deaths*

Age x	Duration 4 ER_x	Duration 4 A_x	Durations 5 and over (post-1956) ER_x	Durations 5 and over (post-1956) A_x	Durations 5 and over (pre-1957) ER_x	Durations 5 and over (pre-1957) A_x	Age x
50½	13·5	0	42·5	1	27·5	1	50½
51½	18·5	0	57	0	38·5	0	51½
52½	23·5	0	66·5	0	48·5	0	52½
53½	48·5	1	90	0	67	0	53½
54½	64	1	145	2	84·5	0	54½
55½	54·5	0	215·5	2	96	1	55½
56½	58	1	256·5	2	122·5	0	56½
57½	89	0	303	3	149	1	57½
58½	157	2	350	4	204·5	3	58½
59½	187·5	1	506·5	9	245	3	59½
60½	184·5	0	693·5	3	286	0	60½
61½	205·5	1	847	6	356·5	3	61½
62½	261·5	3	996	9	415	5	62½
63½	531·5	6	1,135·5	16	495	4	63½
64½	696	12	1,495·5	14	580	13	64½
65½	488	6	1,957	38	665·5	19	65½
66½	513·5	12	2,200·5	36	757·5	6	66½
67½	519	10	2,378	30	893·5	17	67½
68½	680	9	2,549	47	1,040·5	25	68½
69½	743	22	2,861·5	67	1,183	29	69½
70½	672	8	3,117·5	58	1,334·5	30	70½
71½	649·5	16	3,245·5	54	1,559·5	47	71½
72½	634	12	3,263	87	1,828·5	78	72½
73½	631·5	20	3,219	86	2,132·5	77	73½
74½	680	20	3,224	131	2,426·5	107	74½

Table 17 (*continued*).

Age	Duration 4		Durations 5 and over (post-1956)		Durations 5 and over (pre-1957)		Age
x	ER_x	A_x	ER_x	A_x	ER_x	A_x	x
75½	660·5	28	3,230	133	2,707	131	75½
76½	607·5	25	3,142·5	117	2,971·5	170	76½
77½	612	27	3,074·5	139	3,194	175	77½
78½	534·5	32	3,018·5	159	3,427·5	196	78½
79½	550	31	2,946	162	3,663	276	79½
80½	496	30	2,891	168	3,747·5	256	80½
81½	440	29	2,751·5	180	3,827·5	297	81½
82½	413·5	27	2,534·5	178	3,885	356	82½
83½	384·5	21	2,256·5	175	3,860·5	354	83½
84½	342	29	2,040	168	3,825	397	84½
85½	276·5	31	1,820·5	197	3,682	481	85½
86½	239	26	1,507·5	157	3,419	393	86½
87½	209·5	29	1,255	171	3,196	486	87½
88½	145·5	16	995·5	121	2,787	426	88½
89½	120	19	771·5	111	2,373	397	89½
90½	93·5	11	613·5	113	2,000	367	90½
91½	63	12	468	79	1,643·5	340	91½
92½	58	12	361	69	1,314·5	287	92½
93½	40·5	5	279	48	1,055·5	262	93½
94½	24·5	5	198	38	769	204	94½
95½	14	5	133·5	29	574·5	141	95½
96½	6	3	88	22	404	92	96½
97½	3	1	60·5	18	284	78	97½
98½	5·5	1	29·5	12	197·5	72	98½
99½	3	0	23·5	4	124·5	30	99½
Total	15,145·5	618	71,704·5	3,473	75,969·5	7,133	

Table 18. *Male annuitants* 1967–70: *graduation parameters, test statistics and values of* q_x *for graduations of data for durations* 0 *and* 1 *and over* (*post*-1956) *using two and three parameters*

Formula: $\log (q_x/p_x) = A+Bt+C(2t^2-1)$ where $t = (x-70)/50$

Experience	Duration 0		Durations 1 and over	
Graduation	2-pars	3-pars	2-pars	3-pars
Ages used (nearest)	51–96	51–96	51–100	51–100
Parameters				
A	−3·6389	−2·5372	−3·2526	−2·8294
B	+4·1783	+3·7167	+4·4977	+4·2391
C	—	+1·1454	—	+0·4250
Actual deaths	256	256	3,338	3,338
Σ(A–E)	0·00	0·00	0·02	−0·01
ΣΣ(A–E)	0·01	−0·03	0·62	−0·12
−L′	1,080·7	1,080·0	11,752·3	11,751·2
Using single ages:				
Runs, r	13	13	23	21
$t(r)$	3·27	3·25	0·85	1·39
$t(\rho)$	1·67	1·42	1·66	1·62
Using grouped ages:				
χ^2	29·5	27·4	64·2	61·5
Degrees of freedom	27	27	39	39
$t(\chi^2)$	0·40	0·12	2·56	2·31
$10^5 \times q_x$: Ages				
50	*492*	*814*	*636*	*805*
55	*745*	*1,004*	*993*	*1,155*
60	*1,127*	*1,294*	1,549	1,682
65	1,701	1,744	2,407	2,485
70	2,561	2,454	3,724	3,717
75	3,838	3,598	5,717	5,615
80	5,715	5,480	8,683	8,529
85	8,429	8,616	12,974	12,942
90	*12,265*	*13,831*	18,947	19,424
95	*17,512*	*22,243*	26,821	28,449
100	*24,381*	*34,798*	*36,495*	*40,014*

Values of q_x which are in a region where actual deaths at each age are generally less than 5 are shown in italics.

Table 19. *Male annuitants* 1967–70: *duration* 0: *exposed to risk and comparison of actual deaths with those expected according to the graduated table, namely:*

$$\log(q_x/p_x) = -3{\cdot}6388848 + 4{\cdot}1782890\,(x-70)/50$$

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	7·5	·005125	0	0·04	−0·04		50½
51½	13·5	·005570	0	0·08	−0·08		51½
52½	22·5	·006052	0	0·14	−0·14		52½
53½	24·5	·006576	0	0·16	−0·16		53½
54½	45·5	·007145	1	0·33	+0·67		54½
55½	50	·007763	1	0·39	+0·61	+2·97	55½
56½	59	·008434	1	0·50	+0·50		56½
57½	81	·009162	1	0·74	+0·26		57½
58½	114·5	·009953	4	1·14	+2·86		58½
59½	217·5	·010811	5	2·35	+2·65		59½
60½	365	·011742	5	4·29	+0·71	−0·57	60½
61½	255·5	·012752	1	3·26	−2·26		61½
62½	239	·013849	4	3·31	+0·69	−0·45	62½
63½	330·5	·015037	3	4·97	−1·97		63½
64½	642	·016327	8	10·48	−2·48	−0·77	64½
65½	776	·017724	10	13·75	−3·75	−1·02	65½
66½	399·5	·019239	10	7·69	+2·31	+0·84	66½
67½	362·5	·020881	9	7·57	+1·43	+0·53	67½
68½	309·5	·022660	7	7·01	−0·01	−0·01	68½
69½	328	·024586	4	8·06	−4·06	−1·45	69½
70½	339·5	·026672	8	9·06	−1·06	−0·36	70½
71½	292·5	·028930	10	8·46	+1·54	+0·54	71½
72½	280·5	·031372	9	8·80	+0·20	+0·07	72½
73½	253	·034013	10	8·61	+1·39	+0·48	73½
74½	227	·036868	10	8·37	+1·63	+0·57	74½
75½	275	·039953	12	10·99	+1·01	+0·31	75½
76½	242	·043285	8	10·47	−2·47	−0·78	76½
77½	259	·046880	11	12·14	−1·14	−0·34	77½
78½	230	·050759	11	11·67	−0·67	−0·20	78½
79½	207	·054940	8	11·37	−3·37	−1·03	79½
80½	163	·059444	9	9·69	−0·69	−0·23	80½
81½	151·5	·064291	4	9·74	−5·74	−1·90	81½
82½	142	·069505	13	9·87	+3·13	+1·03	82½
83½	117	·075108	9	8·79	+0·21	+0·07	83½
84½	95	·081123	14	7·71	+6·29	+2·36	84½

Table 19 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
85½	73	·087575	8	6·39	+1·61	+0·67	85½
86½	59	·094486	6	5·57	+0·43	+0·19	86½
87½	53·5	·101882	7	5·45	+1·55	+0·70	87½
88½	31·5	·109787	4	3·46	+0·54	+0·50	88½
89½	28	·118225	4	3·31	+0·69		89½
90½	24	·127218	1	3·05	−2·05	−0·97	90½
91½	14·5	·136789	1	1·98	−0·98		91½
92½	12·5	·146959	1	1·84	−0·84		92½
93½	10	·157747	3	1·58	+1·42		93½
94½	6	·169170	1	1·02	−0·02		94½
95½	2	·181242	0	0·36	−0·36		95½
Total	8,231	—	256	256·00	0·00	—	

$\chi^2 = \text{Total } z_x^2 = 29{\cdot}46$

Table 20. *Male annuitants* 1967–70: *durations* 1 *and over* (*post*-1956) *exposed to risk and comparison of actual deaths with those expected according to the graduated table, namely:*

$$\log (q_x/p_x) = -3{\cdot}2525555 + 4{\cdot}4976687\,(x-70)/50$$

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
50½	50	·006649	1	0·33	+0·67	+0·64	50½
51½	72	·007270	3	0·52	+2·48		51½
52½	82	·007949	1	0·65	+0·35		52½
53½	112·5	·008691	0	0·98	−0·98		53½
54½	133	·009501	1	1·26	−0·26		54½
55½	169·5	·010386	1	1·76	−0·76		55½
56½	220	·011352	7	2·50	+4·50	+1·26	56½
57½	278·5	·012407	2	3·46	−1·46		57½
58½	368·5	·013559	5	5·00	0·00	+0·42	58½
59½	509	·014817	9	7·54	+1·46		59½
60½	743·5	·016189	7	12·04	−5·04	−1·46	60½
61½	1,071·5	·017686	16	18·95	−2·95	−0·68	61½
62½	1,225	·019318	26	23·66	+2·34	+0·48	62½
63½	1,339	·021098	24	28·25	−4·25	−0·81	63½
64½	1,563·5	·023038	22	36·02	−14·02	−2·36	64½

Table 20 (*continued*).

Age x	Exposed to risk ER_x	Graduated q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Age x
65½	2,081·5	·025152	58	52·35	+5·65	+0·79	65½
66½	2,574	·027454	81	70·67	+10·33	+1·25	66½
67½	2,576	·029961	75	77·18	−2·18	−0·25	67½
68½	2,573·5	·032689	95	84·12	+10·88	+1·21	68½
69½	2,566	·035656	101	91·49	+9·51	+1·01	69½
70½	2,509	·038882	108	97·55	+10·45	+1·08	70½
71½	2,507	·042386	95	106·26	−11·26	−1·12	71½
72½	2,487·5	·046191	121	114·90	+6·10	+0·58	72½
73½	2,384	·050320	122	119·96	+2·04	+0·19	73½
74½	2,351·5	·054797	128	128·86	−0·86	−0·08	74½
75½	2,277·5	·059647	138	135·85	+2·15	+0·19	75½
76½	2,161	·064897	144	140·24	+3·76	+0·33	76½
77½	2,019·5	·070574	140	142·52	−2·52	−0·22	77½
78½	1,938·5	·076707	138	148·70	−10·70	−0·91	78½
79½	1,823	·083325	147	151·90	−4·90	−0·42	79½
80½	1,724·5	·090459	141	156·00	−15·00	−1·26	80½
81½	1,601·5	·098138	123	157·17	−34·17	−2·87	81½
82½	1,435	·106392	155	152·67	+2·33	+0·20	82½
83½	1,271	·115252	159	146·48	+12·52	+1·10	83½
84½	1,091	·124746	132	136·10	−4·10	−0·38	84½
85½	967·5	·134904	136	130·52	+5·48	+0·52	85½
86½	839·5	·145751	143	122·36	+20·64	+2·02	86½
87½	682	·157311	120	107·29	+12·71	+1·34	87½
88½	553·5	·169607	80	93·88	−13·88	−1·57	88½
89½	457·5	·182655	77	83·56	−6·56	−0·79	89½
90½	342·5	·196469	51	67·29	−16·29	−2·22	90½
91½	275·5	·211058	53	58·15	−5·15	−0·76	91½
92½	211·5	·226426	65	47·89	+17·11	+2·81	92½
93½	118	·242568	40	28·62	+11·38	+2·44	93½
94½	75	·259475	25	19·46	+5·54	+1·46	94½
95½	42·5	·277130	9	11·78	−2·78	−0·95	95½
96½	23·5	·295506	5	6·94	−1·94	−0·88	96½
97½	11·5	·314570	3	3·62	−0·62		97½
98½	6	·334281	3	2·01	+0·99	+0·81	98½
99½	2	·354588	2	0·71	+1·29		99½
Total	54,498	—	3,338	3,337·98	+0·02	—	

$\chi^2 = \text{Total } z_x^2 = 64{\cdot}21$

Table 21. *Male annuitants* 1967–70: *exposed to risk and actual deaths*

Age	Duration 1		Duration 2		Duration 3		Age
x	ER_x	A_x	ER_x	A_x	ER_x	A_x	x
50½	8	0	5·5	0	8	0	50½
51½	9	0	11	0	5	2	51½
52½	15·5	0	11·5	0	11·5	1	52½
53½	24·5	0	19·5	0	16	0	53½
54½	24·5	0	27·5	0	21·5	0	54½
55½	45·5	0	24	0	27	1	55½
56½	53·5	3	42·5	2	24·5	0	56½
57½	59·5	1	58	0	39·5	1	57½
58½	81	1	60·5	1	65	1	58½
59½	125·5	1	83·5	2	63	1	59½
60½	236	3	123·5	0	92	0	60½
61½	362	5	226	2	113·5	0	61½
62½	252	3	326	13	212·5	3	62½
63½	232	2	243	8	287	4	63½
64½	334·5	1	229	5	222·5	5	64½
65½	623·5	22	313	3	223·5	15	65½
66½	735	22	538·5	23	275	5	66½
67½	396·5	10	618·5	13	464	24	67½
68½	365	11	376	11	537	24	68½
69½	313	17	328·5	12	344·5	12	69½
70½	319	14	273	6	275	7	70½
71½	332·5	11	285	11	255·5	8	71½
72½	274	8	322	19	253·5	11	72½
73½	275·5	9	242	13	283	10	73½
74½	236·5	8	266	10	221	9	74½
75½	222·5	14	213	14	248·5	14	75½
76½	252·5	13	205	11	190	8	76½
77½	249·5	19	230·5	12	190·5	12	77½
78½	259·5	20	238·5	25	209·5	17	78½
79½	214	9	231	15	207	11	79½
80½	211	10	194	17	192	11	80½
81½	159·5	12	197·5	13	162·5	11	81½
82½	156·5	14	150·5	14	175·5	16	82½
83½	120	12	143	17	131	11	83½
84½	104	12	96·5	13	117	3	84½
85½	84	9	92	17	86	12	85½
86½	70	6	70·5	12	84·5	9	86½
87½	52	7	68·5	10	58·5	10	87½
88½	44·5	6	43	8	51	3	88½
89½	28	5	37·5	3	39·5	5	89½

Table 21 (*continued*).

Age x	Duration 1 ER_x	Duration 1 A_x	Duration 2 ER_x	Duration 2 A_x	Duration 3 ER_x	Duration 3 A_x	Age x
90½	19	2	21·5	1	42·5	2	90½
91½	22·5	4	19	2	23	6	91½
92½	15	8	14	3	17	5	92½
93½	10	2	10	3	10·5	4	93½
94½	4·5	1	3·5	1	6	0	94½
95½	2·5	1	4·5	3	2	1	95½
96½	1·5	1	1·5	0	1	0	96½
97½					0·5	0	97½
Total	8,036	339	7,338·5	368	6,586	315	

Table 22. *Male annuitants* 1967–70: *exposed to risk and actual deaths*

Age x	Duration 4 ER_x	Duration 4 A_x	Durations 5 and over (post-1956) ER_x	Durations 5 and over (post-1956) A_x	Durations 5 and over (pre-1957) ER_x	Durations 5 and over (pre-1957) A_x	Age x
50½	6	0	22·5	1	13	1	50½
51½	10·5	0	36·5	1	14·5	0	51½
52½	2·5	0	41	0	14·5	0	52½
53½	11·5	0	41	0	24·5	0	53½
54½	17	0	42·5	1	33	0	54½
55½	22	0	51	0	35·5	0	55½
56½	24·5	1	75	1	51	2	56½
57½	27·5	0	94	0	52	0	57½
58½	44	1	118	1	53	0	58½
59½	73·5	0	163·5	5	55·5	2	59½
60½	66	1	226	3	53	2	60½
61½	95·5	0	274·5	9	64	8	61½
62½	103·5	1	331	6	76·5	2	62½
63½	209·5	3	367·5	7	100	2	63½
64½	267·5	4	510	7	119·5	3	64½
65½	210·5	4	711	14	137	13	65½
66½	208	8	817·5	23	146·5	6	66½
67½	234	9	863	19	156	3	67½
68½	388	11	907·5	38	182	20	68½
69½	489·5	14	1,090·5	46	181·5	15	69½
70½	315	13	1,327	68	187	10	70½
71½	256·5	12	1,377·5	53	205·5	10	71½
72½	241·5	10	1,396·5	73	251·5	15	72½
73½	240	15	1,343·5	75	297	15	73½
74½	258·5	9	1,369·5	92	354·5	29	74½

Table 22 (*continued*).

Age x	Duration 4 ER_x	Duration 4 A_x	Durations 5 and over (post-1956) ER_x	Durations 5 and over (post-1956) A_x	Durations 5 and over (pre-1957) ER_x	Durations 5 and over (pre-1957) A_x	Age x
75½	211	11	1,382·5	85	400·5	30	75½
76½	228·5	18	1,285	94	461	45	76½
77½	186	13	1,163	84	518	42	77½
78½	193	13	1,038	63	564·5	49	78½
79½	185·5	15	985·5	97	601·5	60	79½
80½	180·5	13	947	90	615·5	75	80½
81½	174	13	908	74	619	68	81½
82½	130	11	822·5	100	653·5	92	82½
83½	151	19	726	100	627	95	83½
84½	121·5	21	652	83	600·5	79	84½
85½	106·5	11	599	87	570·5	96	85½
86½	77	17	537·5	99	482	104	86½
87½	75·5	10	427·5	83	426	69	87½
88½	44·5	6	370·5	57	355·5	78	88½
89½	52·5	4	300	60	278	57	89½
90½	39·5	3	220	43	234	55	90½
91½	43	6	168	35	186	34	91½
92½	22	6	143·5	43	168	61	92½
93½	6·5	2	81	29	111·5	44	93½
94½	8	2	53	21	94·5	33	94½
95½	4·5	1	29	3	56	16	95½
96½	0·5	0	19	4	35·5	10	96½
97½	1·5	0	9·5	3	24	4	97½
98½	0·5	0	5·5	3	18	6	98½
99½			2	2	9·5	4	99½
Totals	6,065·5	331	26,472	1,985	11,568	1,464	

Table 23. *aeg* 1967–70: *values of* $q_{[x]}$ *and* q_x

	Males		Females		
Age	Duration 0	Durations 1 and over	Duration 0	Durations 1 and over	Age
x	$q_{[x]}$	q_x	$q_{[x]}$	q_x	x
20	·000629	·000889	·000163	·000328	20
21	·000595	·000841	·000160	·000323	21
22	·000564	·000797	·000163	·000328	22
23	·000536	·000758	·000171	·000343	23
24	·000511	·000724	·000180	·000363	24
25	·000491	·000695	·000188	·000378	25
26	·000475	·000672	·000194	·000390	26
27	·000463	·000656	·000198	·000398	27
28	·000457	·000647	·000199	·000401	28
29	·000456	·000646	·000204	·000410	29
30	·000462	·000654	·000212	·000426	30
31	·000474	·000671	·000223	·000448	31
32	·000494	·000699	·000238	·000478	32
33	·000522	·000738	·000257	·000517	33
34	·000558	·000790	·000281	·000566	34
35	·000605	·000856	·000311	·000626	35
36	·000662	·000937	·000346	·000697	36
37	·000731	·001034	·000388	·000780	37
38	·000812	·001150	·000435	·000875	38
39	·000908	·001285	·000489	·000984	39
40	·001020	·001443	·000549	·001105	40
41	·001148	·001624	·000615	·001238	41
42	·001294	·001831	·000687	·001382	42
43	·001461	·002068	·000764	·001537	43
44	·001650	·002335	·000846	·001703	44
45	·001864	·002637	·000933	·001877	45
46	·002104	·002977	·001024	·002060	46
47	·002373	·003358	·001119	·002253	47
48	·002674	·003784	·001219	·002454	48
49	·003010	·004259	·001325	·002666	49
50	·003384	·004789	·001436	·002891	50
51	·003803	·005382	·001560	·003139	51
52	·004283	·006061	·001711	·003443	52
53	·004837	·006845	·001901	·003826	53
54	·005471	·007742	·002133	·004294	54
55	·006187	·008754	·002406	·004842	55
56	·006979	·009876	·002713	·005461	56
57	·007842	·011097	·003049	·006137	57
58	·008769	·012408	·003406	·006855	58
59	·009752	·013800	·003777	·007602	59

Table 23 (*continued*).

	Males		Females		
Age	Duration 0	Durations 1 and over	Duration 0	Durations 1 and over	Age
x	$q_{[x]}$	q_x	$q_{[x]}$	q_x	x
60	·010788	·015266	·004156	·008365	60
61	·011880	·016810	·004542	·009141	61
62	·013032	·018442	·004936	·009933	62
63	·014260	·020179	·005345	·010758	63
64	·015580	·022046	·005784	·011641	64
65	·01701140	·02407240	·00626935	·01261751	65
66	·01846668	·02627863	·00708228	·01371712	66
67	·02004392	·02868112	·00799976	·01495816	67
68	·02175288	·03129620	·00903502	·01635778	68
69	·02360403	·03414133	·01020288	·01793516	69
70	·02560860	·03723517	·01151994	·01971166	70
71	·02777856	·04059758	·01300478	·02171098	71
72	·03012671	·04424967	·01467816	·02395933	72
73	·03266668	·04821379	·01656326	·02648544	73
74	·03541298	·05251352	·01868586	·02932074	74
75	·03838100	·05717367	·02107464	·03249925	75
76	·04158704	·06222021	·02376141	·03605763	76
77	·04504835	·06768022	·02678134	·04003492	77
78	·04878308	·07358177	·03017322	·04447237	78
79	·05281032	·07995378	·03397969	·04941296	79
80	·05715004	·08682589	·03824742	·05490085	80
81	·06182311	·09422816	·04302729	·06098067	81
82	·06685120	·10219088	·04837447	·06769657	82
83	·07225673	·11074423	·05434842	·07509107	83
84	·07806277	·11991786	·06101282	·08320371	84
85	·08429296	·12974053	·06843529	·09206954	85
86	·09097132	·14023958	·076288	·10171744	86
87	·09812210	·15144041	·084126	·11216829	87
88	·10576956	·16336584	·092575	·12343316	88
89	·11393775	·17603556	·101634	·13551151	89
90	·12265022	·18946534	·111292	·14838957	90
91	·13192971	·20366643	·121529	·16203894	91
92	·14179780	·21864482	·132312	·17641559	92
93	·15227453	·23440052	·143595	·19145935	93
94	·16337797	·25092698	·155320	·20709392	94
95	·17512378	·26821043	·167421	·22322755	95
96	·18752477	·28622946	·179816	·23975422	96
97	·20059035	·30495457	·192417	·25655555	97
98	·21432613	·32434803	·205127	·27350301	98
99	·22873336	·34436375	·217846	·29046072	99

Table 23 (*continued*).

	Males		Females		
Age	Duration 0	Durations 1 and over	Duration 0	Durations 1 and over	Age
x	$q_{[x]}$	q_x	$q_{[x]}$	q_x	x
100	·24380853	·36494748	·230466	·30728820	100
101		·38603713		·32384339	101
102		·40756336		·33998542	102
103		·42945033		·35557708	103
104		·45161668		·37048698	104
105		·47397669		·38459117	105
106		·49644156		·39777420	106
107		·51892080		·40992974	107
108		·54132371		·42096062	108
109		·56356079		·43077847	109
110		·58554521		·43930308	110
111		·60719409		·44646141	111
112		·62842974		·45218655	112
113		·64918065		·45641662	113
114		·66938238		·45909381	114

Table 24. *aeg 1967–70 tables for annuitants: comparison of rates*

	Select as percentage of Ultimate		Females as percentage of Males		
Age	Males	Females	Select	Ultimate	Age
20	70·7	49·7	25·9	36·8	20
25	70·7	49·7	38·2	54·4	25
30	70·7	49·7	45·8	65·1	30
35	70·7	49·7	51·4	73·1	35
40	70·7	49·7	53·8	76·6	40
45	70·7	49·7	50·0	71·2	45
50	70·7	49·7	42·4	60·4	50
55	70·7	49·7	38·9	55·3	55
60	70·7	49·7	38·5	54·8	60
65	70·7	49·7	36·9	52·4	65
70	68·8	58·4	45·0	52·9	70
75	67·1	64·8	54·9	56·8	75
80	65·8	69·7	66·9	63·2	80
85	65·0	74·3	81·2	71·0	85
90	64·7	75·0	90·7	78·3	90
95	65·3	75·0	95·6	83·2	95
100	66·8	75·0	94·5	84·2	100

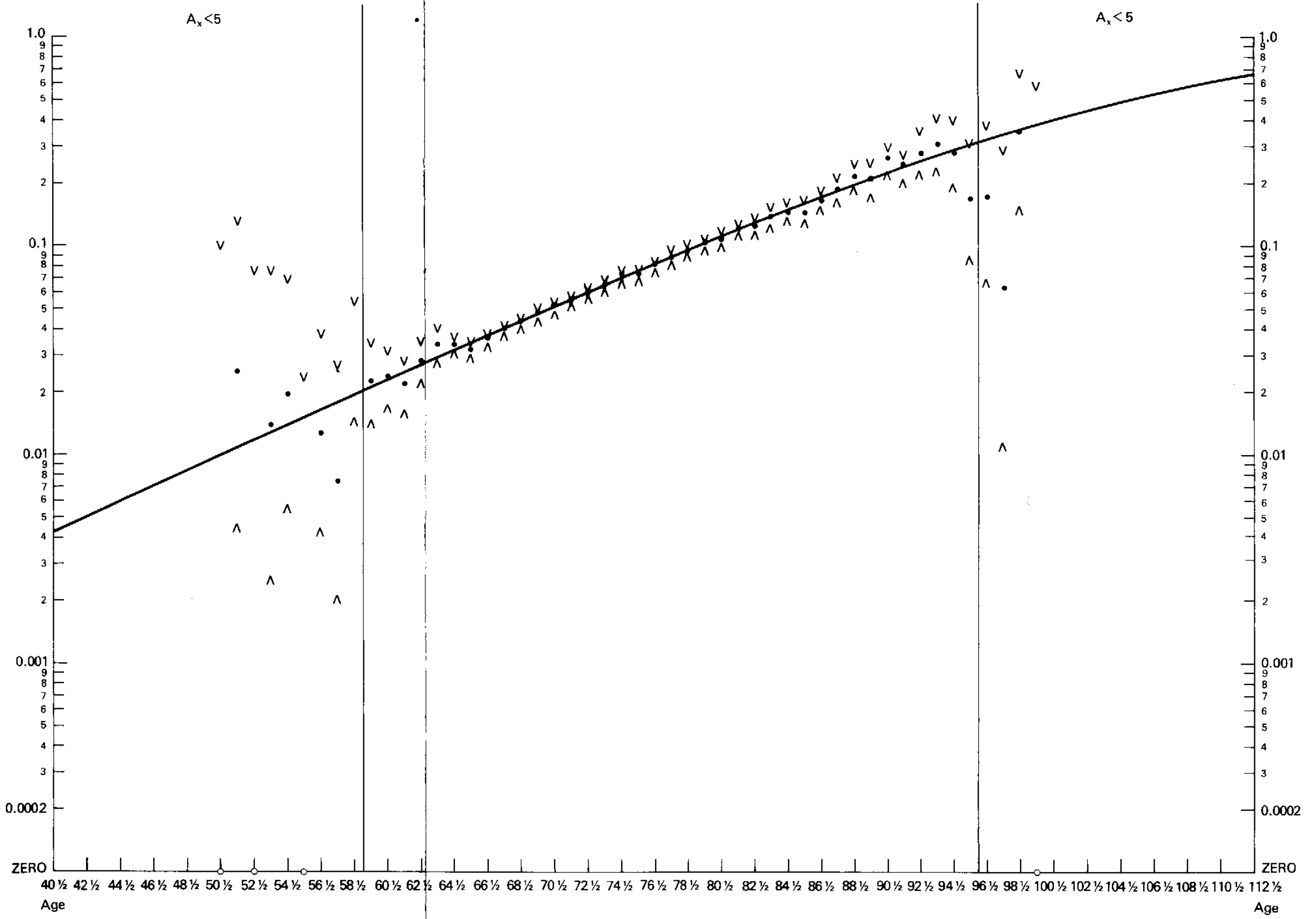

Figure 1. Pensioners 1967–70: Males, Normal, Lives: ql_x = ∧ qa_x = · qh_x = ∨ Graduated q_x = — $ql_x = qa_x$ = zero = ○

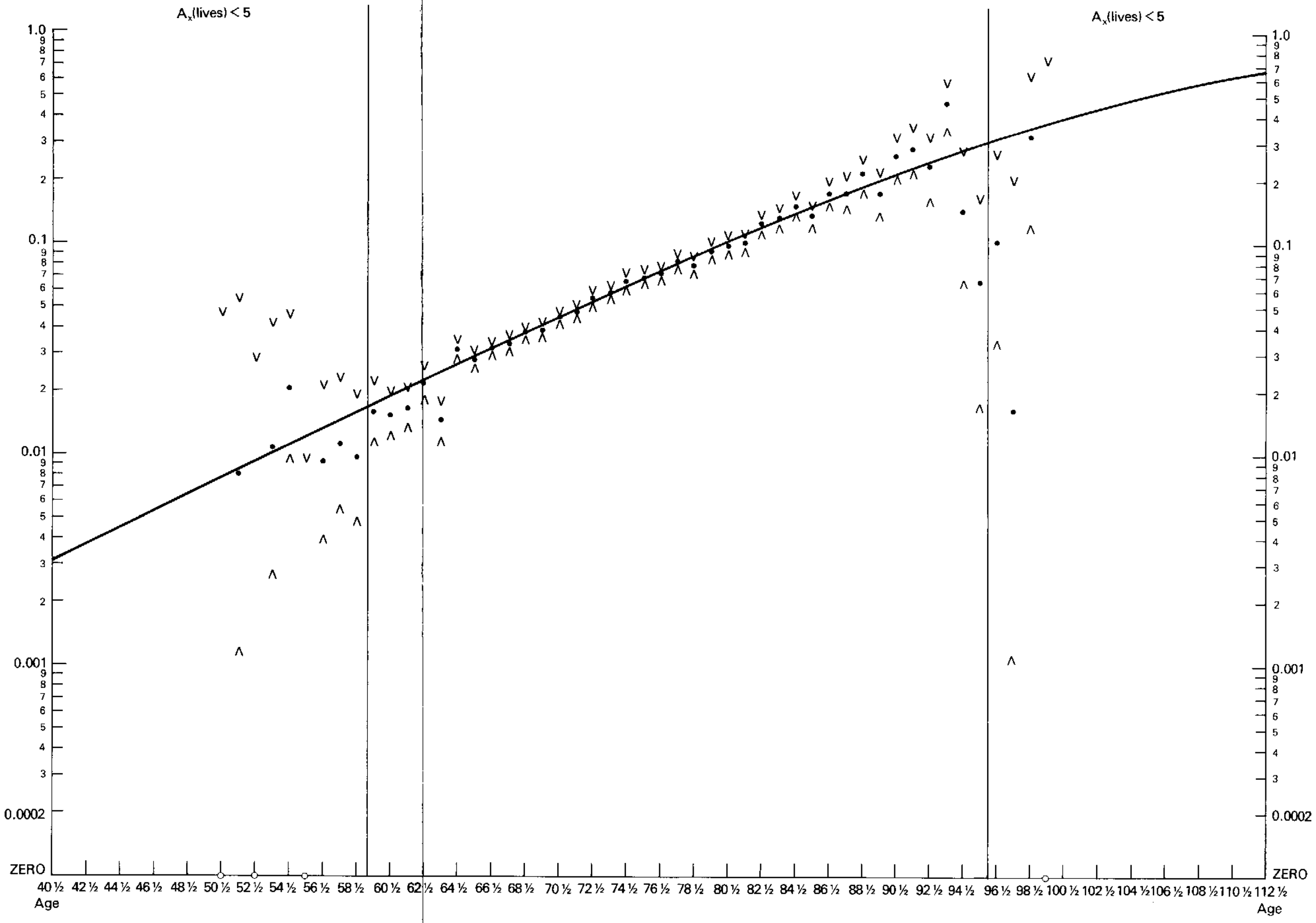

Figure 2. Pensioners 1967–70: Males, Normal, Amounts: ql_x = ∧ qa_x = · qh_x = ∨ Graduated q_x = —— $ql_x = qa_x$ = zero = ○

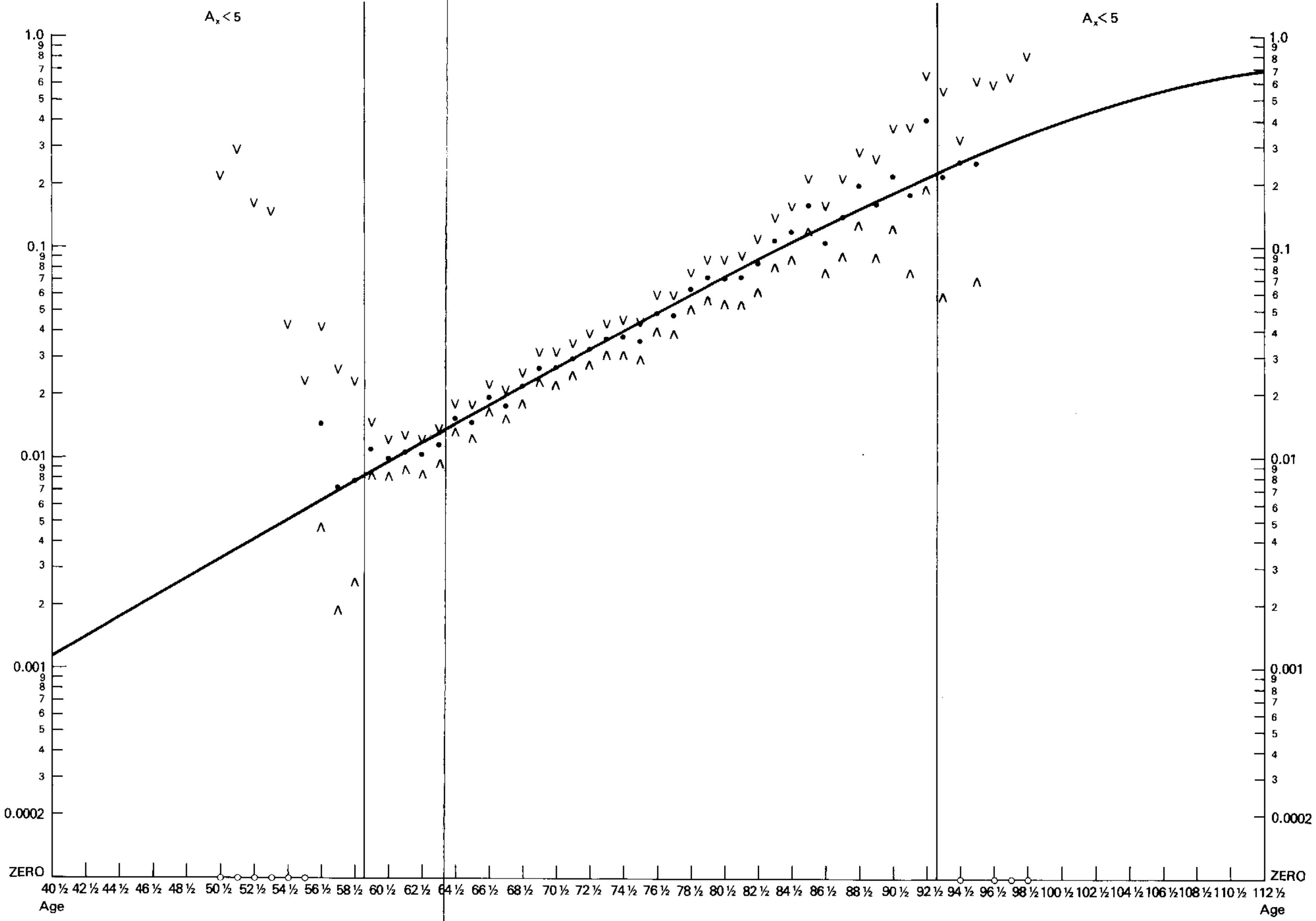

Figure 3. Pensioners 1967–70: Females, Normal, Lives: ql_x = ∧ qa_x = · qh_x = ∨ Graduated q_x = — $ql_x = qa_x$ = zero = ○

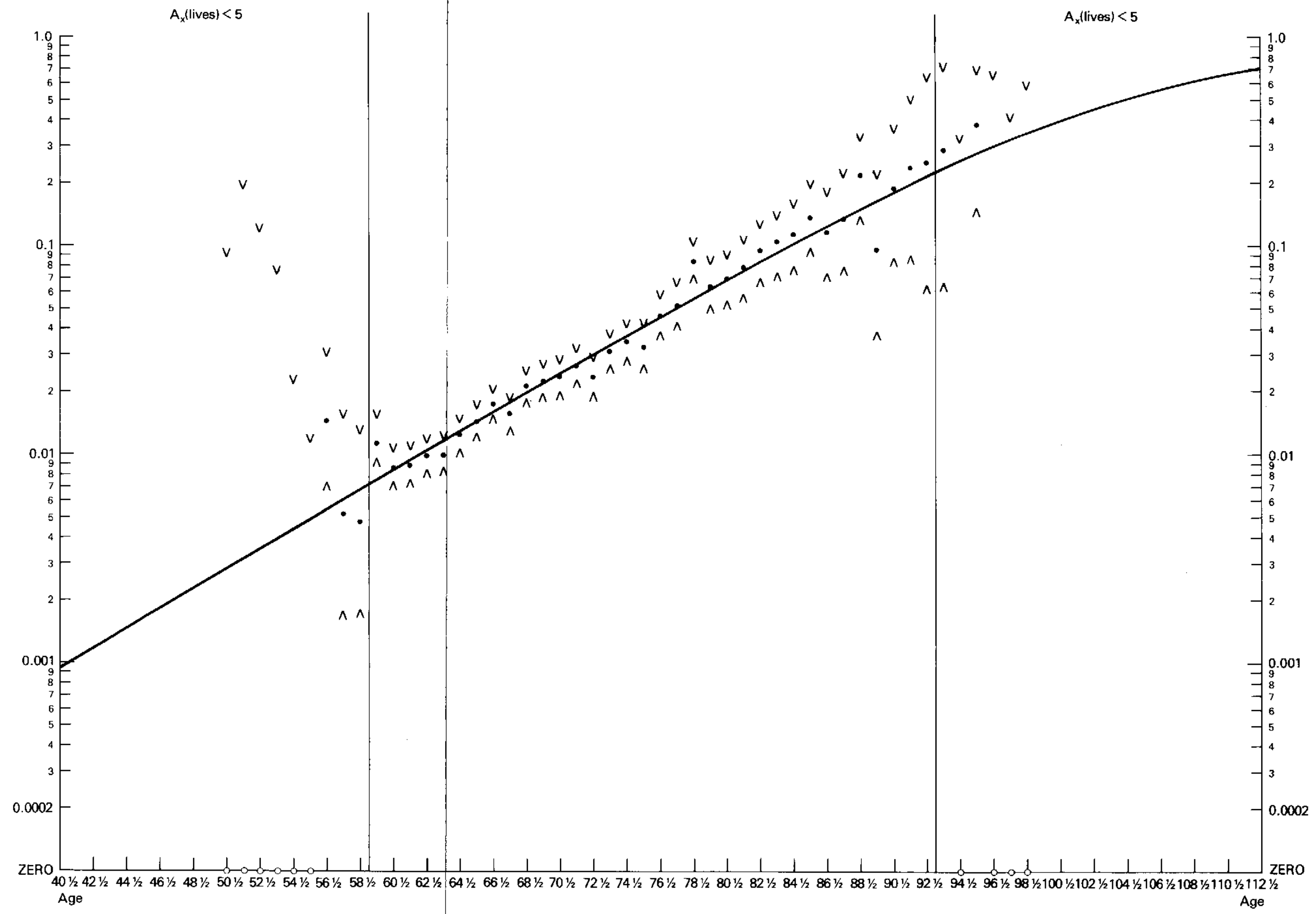

Figure 4. Pensioners 1967–70: Females, Normal, Amounts: ql_x = ∧ qa_x = · qh_x = ∨ Graduated q_x = — $ql_x = qa_x$ = zero = ○

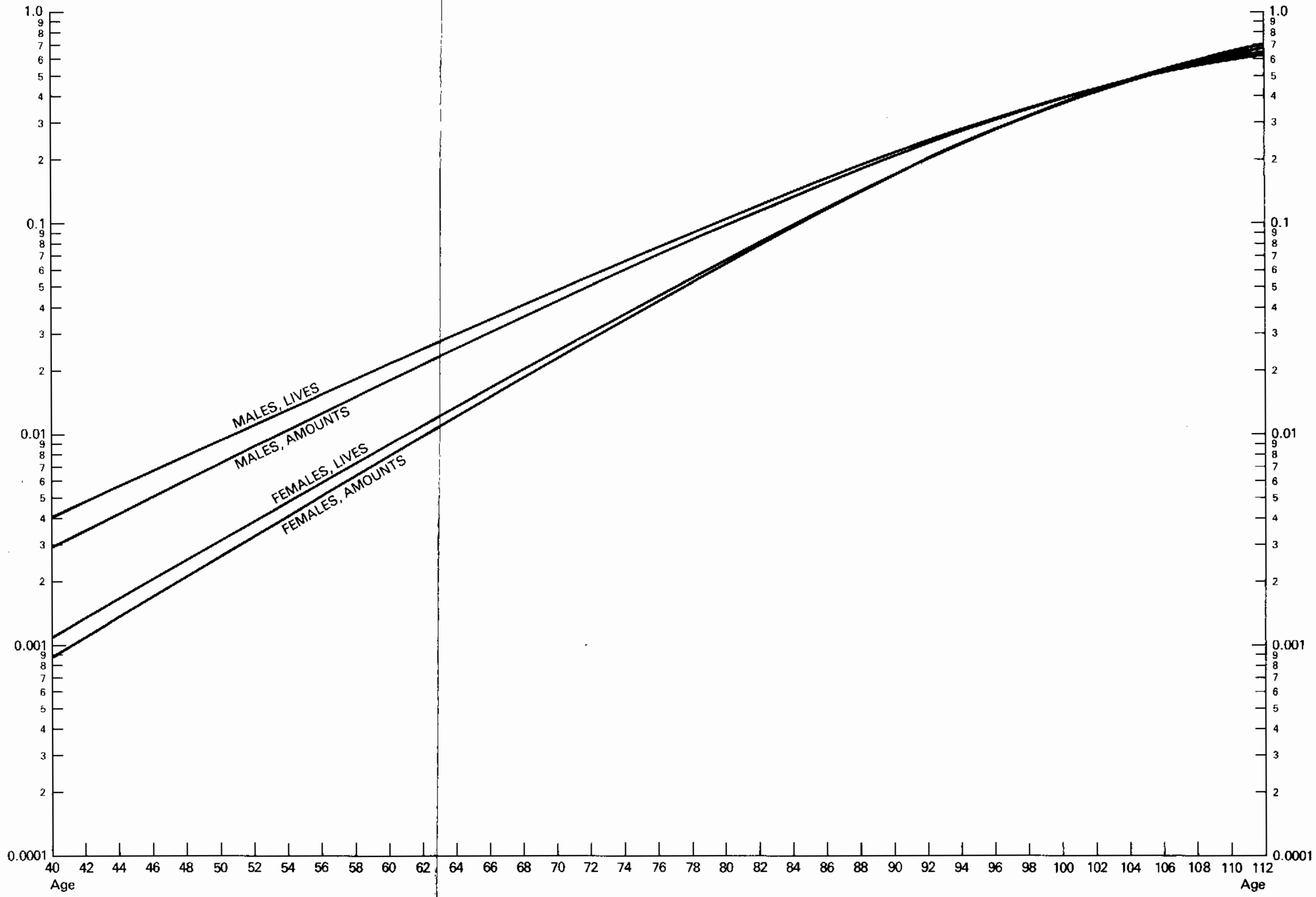

Figure 5. Pensioners Peg 1967–70: Male and Female, Lives and Amounts: q_x on graduated basis

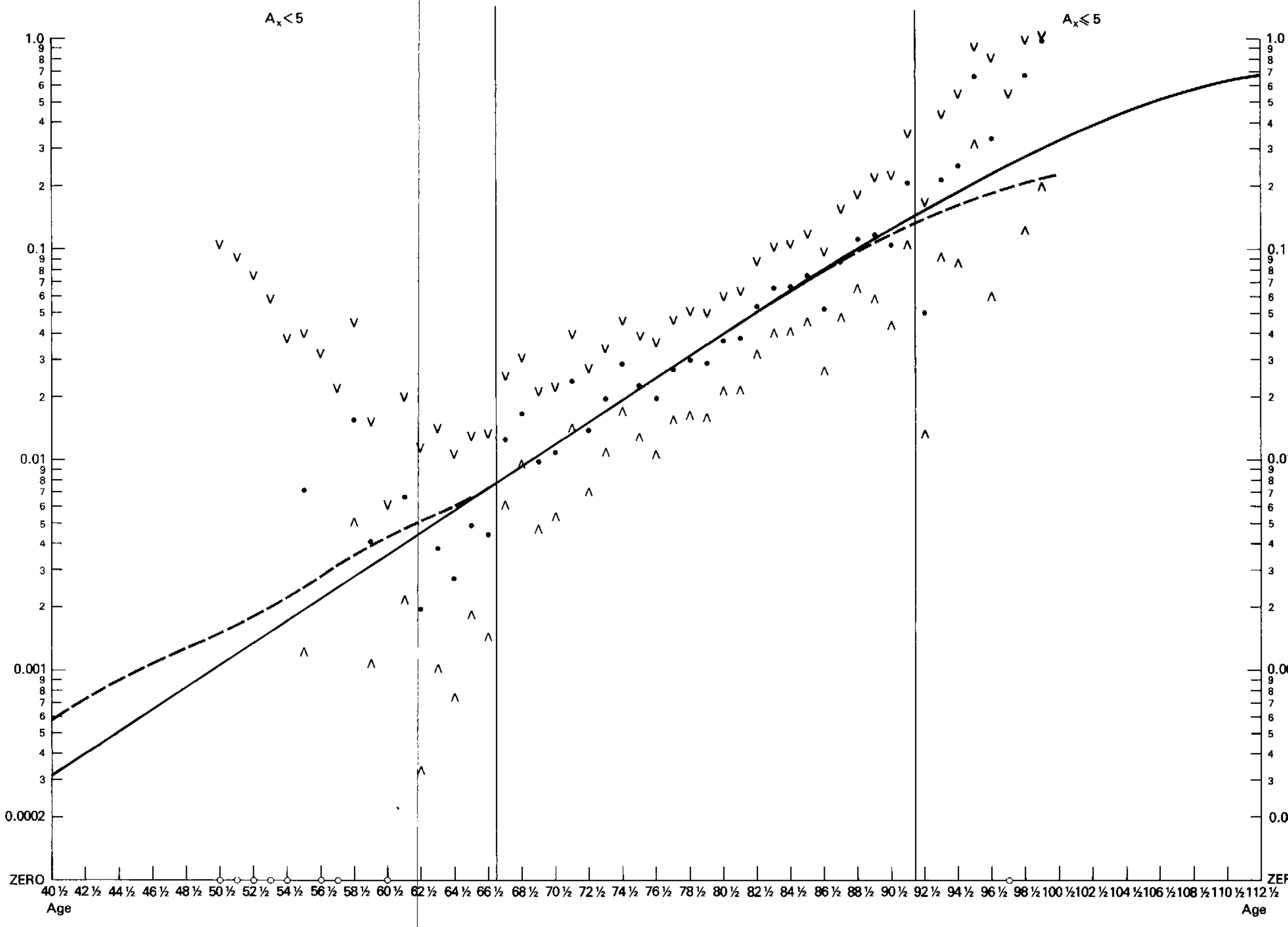

Figure 6. Female annuitants 1967–70: Duration 0: ql_x = ∧ qa_x = · qh_x = ∧ Graduated q_x = — $ql_x = qa_x$ = zero = ○ Adjusted q_x = — — — — —

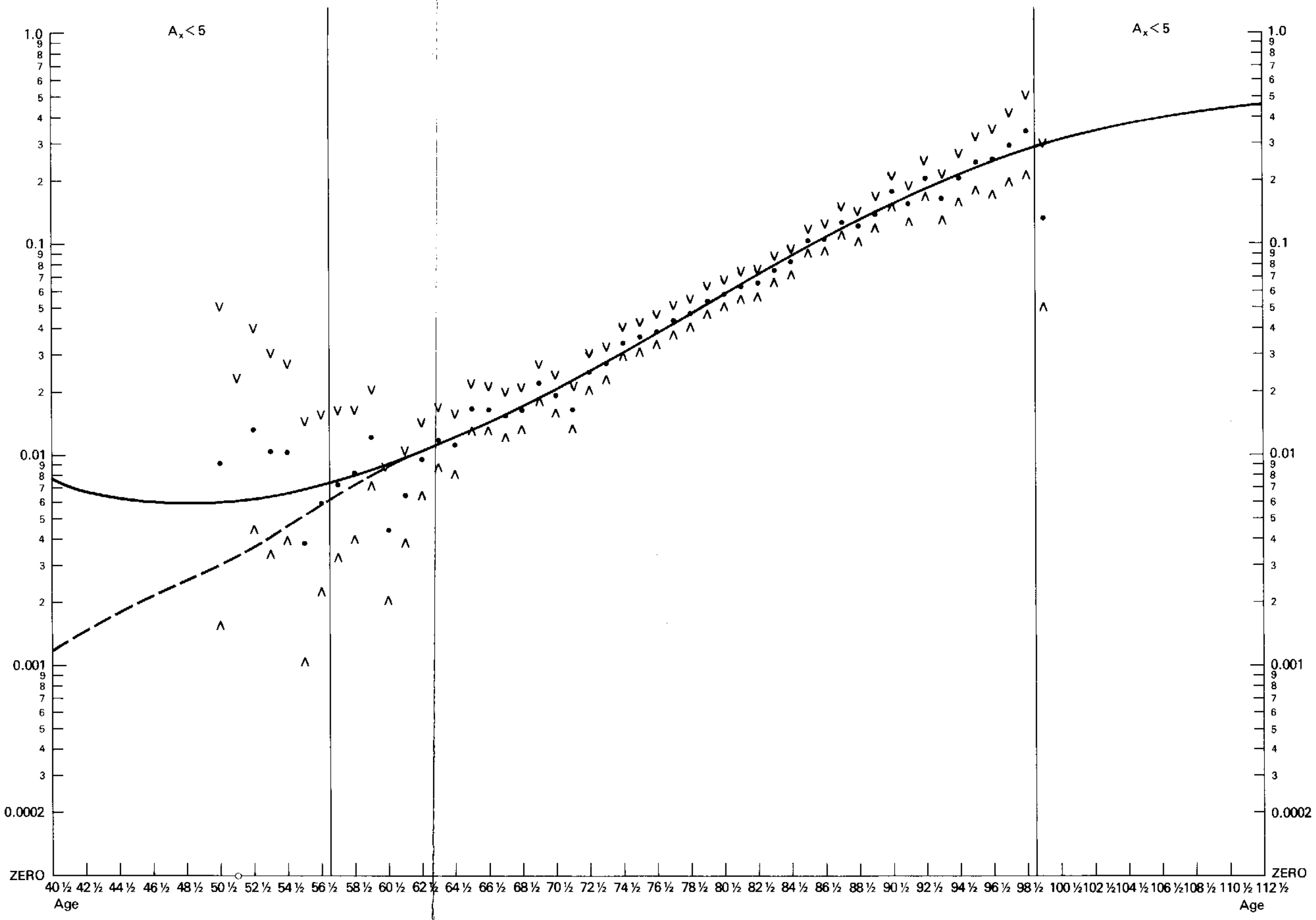

Figure 7. Female annuitants 1967–70: Durations 1 and over: ql_x = ∧ qa_x = · qh_x = ∨ Graduated q_x = — $ql_x = qa_x$ = zero = ○ Adjusted q_x = — — — —

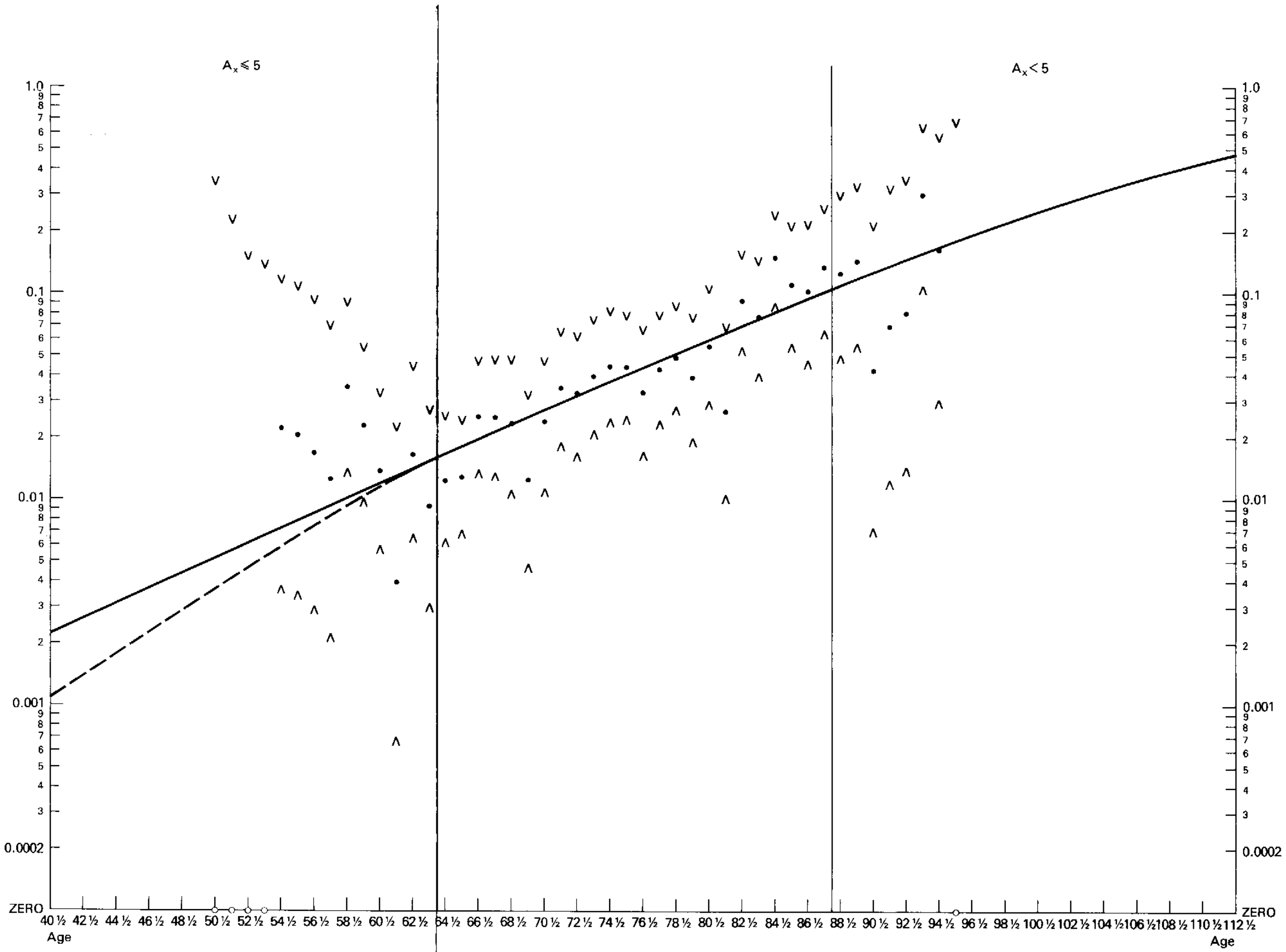

Figure 8. Male annuitants 1967–70: Duration 0: $ql_x = \wedge$ $qa_x = \cdot$ $qh_x = \vee$ Graduated q_x = —— $ql_x = qa_x$ = zero = ○ Adjusted q_x = — — — — —

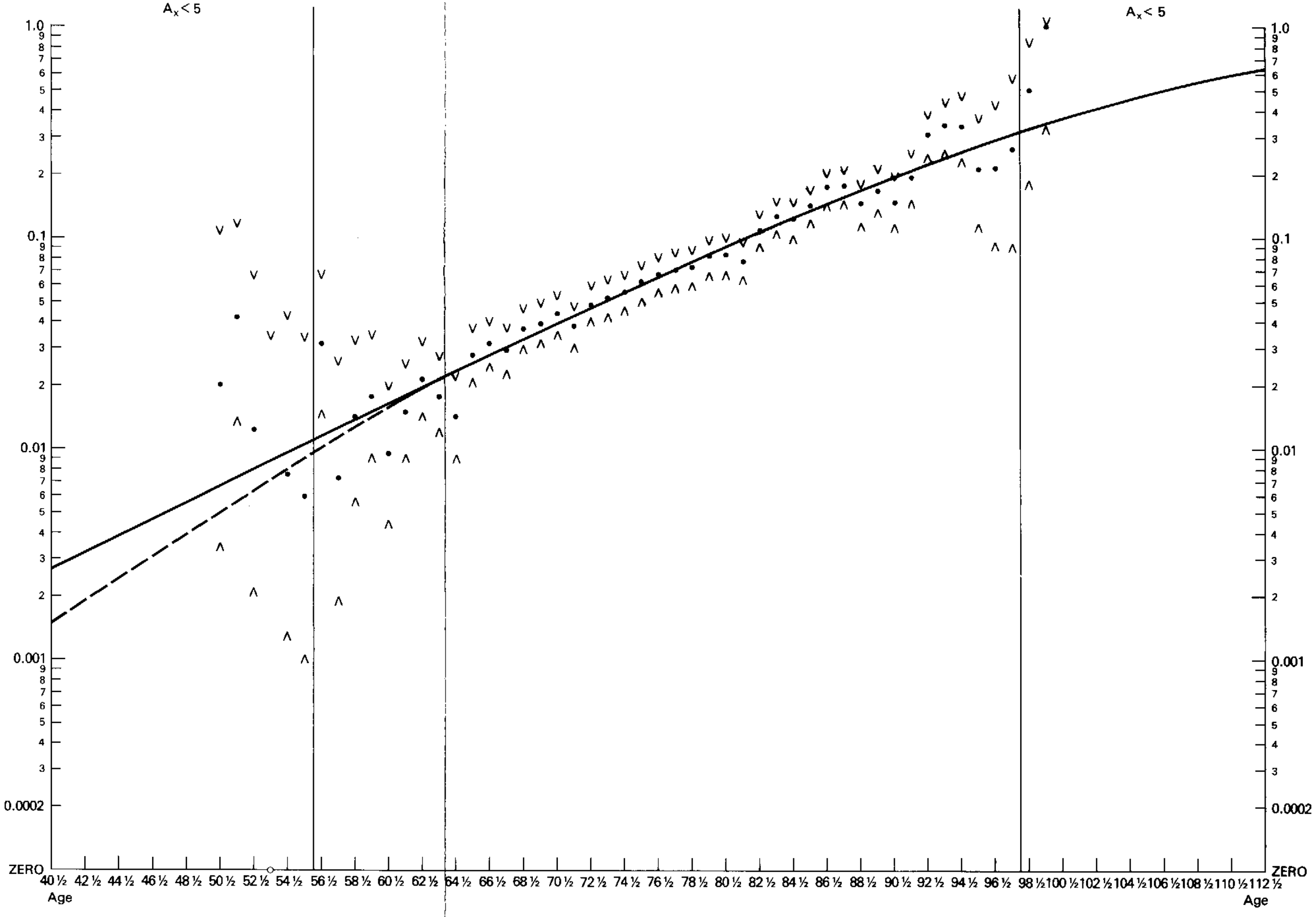

Figure 9. Male annuitants 1967–70: Durations 1 and over: $ql_x = \wedge$ $qa_x = \cdot$ $qh_x = \vee$ Graduated q_x = ― $ql_x = qa_x$ = zero = ○ Adjusted q_x = — — — — —

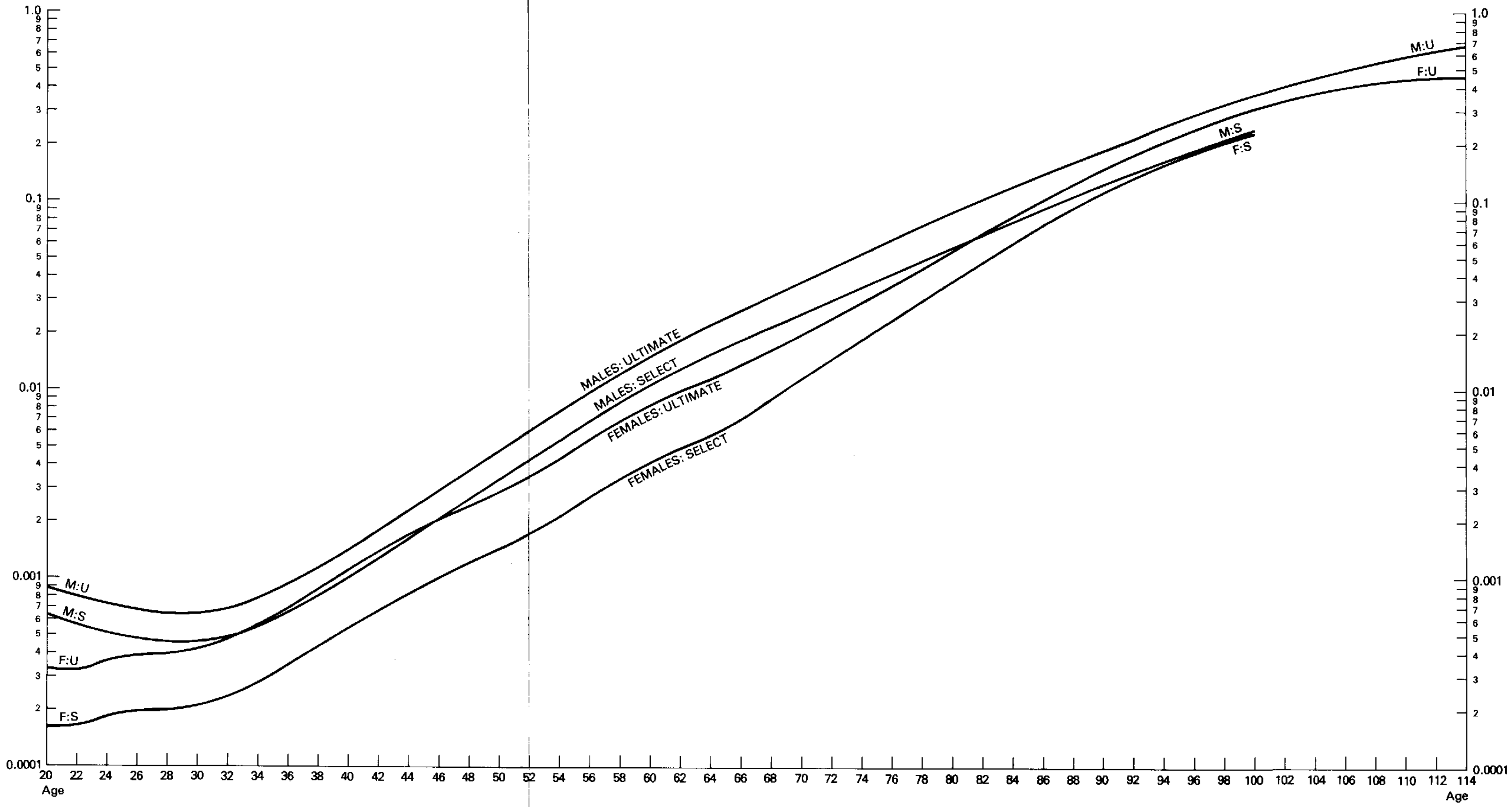

Figure 10. Annuitants *aeg* 1967–70: Male and Female: Select $q_{[x]}$ and ultimate q_x on graduated and adjusted basis

CONTINUOUS MORTALITY INVESTIGATION REPORTS

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