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THE EXECUTIVE COMMITTEE OF THE CONTINUOUS MORTALITY INVESTIGATION BUREAU

as on 17th March 1983

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INTRODUCTION

THE Joint Continuous Mortality Investigation Committee of the Institute and Faculty has pleasure in presenting the sixth number of its *Reports*. The Committee has graduated the mortality of female lives experienced under whole-life and endowment assurances in 1975–78, and this forms the subject matter of the main paper in this number. As the new table is intended to be a standard table it is being published in booklet form separately from this issue; the booklet will contain mortality functions but will not include monetary values.

The history of the Bureau, as described in the first number of these *Reports* (now out of print), is brought up to date in the second paper.

The next item is a reprint of the abridged list of diseases from the Eighth Revision of the *International Classification of Diseases* published by the World Health Organization, whose permission to reprint this list is gratefully acknowledged. The list will be of interest in connexion with a study being undertaken by the P.H.I. Sub-Committee into cause of sickness.

This number also includes a paper on the mortality experienced in 1975–78 by male lives assured under policies issued in the Republic of Ireland; and finally a note on the data collected in respect of duplicate policies amongst the deaths in the linked-life assurance investigation.

In thanking all those who have served on the Committee since I became Chairman, as well as those who have worked behind the scenes, special appreciation should be recorded of two former members of the Committee whose deaths were recently announced. Jo Hamilton-Jones served on the Committee from 1970 to 1979 and became the first Chairman of the P.H.I. Sub-Committee when it was formed in 1970, a position which he held until he retired from the Committee; he died on 17 July 1982. Jim Cairns joined the Committee in 1972 as well as the P.H.I. Sub-Committee; he succeeded Jo Hamilton-Jones as Chairman of the Sub-Committee, a position which he still held when he died suddenly on 13 January 1982; his contribution to the proceedings of the Committee and Sub-Committee are greatly missed.

The new Chairman of the P.H.I. Sub-Committee is R. H. Plumb.

E. B. O. Sherlock
Chairman of the Committee

April, 1983.

GRADUATION OF THE MORTALITY EXPERIENCE OF FEMALE ASSURED LIVES 1975–78

1. INTRODUCTION

RECENTLY the C.M.I. Committee has given serious consideration to the construction of a table reflecting the mortality of female assured lives. The crude data underlying the proposed table relate to the quadrennium 1975–78 and are in respect of whole life and endowment assurances effected at ordinary rates.

The medical and non-medical data have been amalgamated and throughout this note all references are to the combined data. It is difficult to be too precise about the duration of the effect of selection, but preliminary investigations suggest that for practical purposes a 2 year select period will be acceptable. Accordingly, the data have been analysed at durations 0, 1 and 2 and over.

For female lives many offices simply rate down the age (e.g. by 3 or 5 years). However, as will be seen below, the shape of the q_x curve for females is somewhat different from that of the corresponding curve for males. This means that a constant age deduction is not really appropriate. For this reason it may be desirable to have a separate table for female assured lives.

Graduations have been carried out at adult ages by an appropriate mathematical formula. It has been necessary to use *ad hoc* methods to derive rates at young ages. (This was also the case with the A1967–70 data for males.)

2. THE FORMULA TO BE USED

For the A1967–70 table the formula used to produce the graduated rates was

$$q_x = \frac{A + Bc^x + Hx}{1 + A + Bc^x + Hx}.$$

This 4-parameter formula, originally suggested by Mr H.A.R. Barnett, was considered again in relation to the data for females. Unfortunately, it proved impossible to obtain a satisfactory graduation of the ultimate (i.e. durations 2 and over) data by this formula. All the graduations obtained gave too few 'runs'. The graduations produced values of q_x which were consistently too high over a long age interval and then were consistently too low over another long age interval.

Accordingly, attempts were made to use the family of curves adopted to produce the *a* (90) and PA (90) tables for annuitants. The formulae are all of the type

$$q_x = \frac{e^{pol(x)}}{1 + e^{pol\,(x)}},$$

where *pol* (x) is a polynomial in x of low degree. Following the earlier experience, we express this polynomial in the form

$$pol\,(x) = \sum_{r=1}^{n} a_r c_{r-1}\left(\frac{x-70}{50}\right),$$

where $c_i\,(t)$ is the i^{th} Chebycheff polynomial in t. [This celebrated sequence of polynomials may be defined by the initial values $c_0\,(t)=1$, $c_1\,(t)=t$ and by the recurrence relationship $c_k\,(t)=2t\,c_{k-1}\,(t)-c_{k-2}\,(t)$, valid for $k \geqslant 2$.]

The choice of n, the number of parameters to be used, is a matter of judgment, as is the subsequent choice of the values of the parameters themselves. For a given value of n several methods exist to determine the 'best' set of coefficients $a_1, \ldots, a_n$. Two of the most useful methods in practice relate to the values of

$$L^*\,(\boldsymbol{a}) = \sum_{x=x_1}^{x_2} [A_x \log q_x + (ER_x - A_x)\log(1-q_x)]$$

and

$$\chi^2(\boldsymbol{a}) = \sum_{x=x_1}^{x_2} \frac{[A_x - ER_x q_x]^2}{ER_x q_x(1-q_x)}.$$

$L^*\,(\boldsymbol{a})$ is simply the 'log likelihood function' (cf. reference 1) and $\chi^2\,(\boldsymbol{a})$ is the sum of the squares of the standardized deviations at each age (cf. reference 2).

One may choose the coefficient vector $\boldsymbol{a}=(a_1, \ldots, a_n)$ to *maximize* the value of $L^*\,(\boldsymbol{a})$ or to *minimize* the value of $\chi^2\,(\boldsymbol{a})$. Theoretically, these are distinct procedures, but in practice they generally produce similar results. Graduations were in fact produced using both methods (and over several different ranges of ages—determined by x_1 and x_2). To be consistent with the method used in recent tables, we restrict our discussion to graduations based on maximizing $L^*\,(\boldsymbol{a})$.

The test statistics used to summarize the graduations are as described in reference 1. For the ultimate rates the graduation was based on the available data from ages 15 to 95 (nearest birthday) inclusive. For the select rates the graduations were based on the data from ages 15 to 75 inclusive. The relative amounts of data are summarized in Table 1, which gives the total exposure and the total number of deaths for each category.

Table 1. *Exposures and number of deaths*

	Duration 0	Duration 1	Ultimate
Age range (inclusive)	15–75	15–75	15–95
ΣER_x	581,267	504,634	2,080,378
ΣA_x	326	348	4,634

3. GRADUATION OF THE ULTIMATE DATA

Graduations were carried out using formulae with 2, 3, 4, 5 and 6 parameters. The results of these graduations are summarized in Table 2 which gives the principal test statistics for each.

Analysis of these figures indicates that the 2-parameter formula does not produce a totally acceptable graduation. For this formula the number of runs is rather low and the χ^2 value is high. The 4-parameter formula has a negative value for a_4 and the 6-parameter formula has a negative value for a_6. These negative 'leading' coefficients imply that for large x the values of *pol* (x) *decrease* with increasing x and that this is also true for q_x. (For example, with the 6-parameter formula q_x has a maximum value near $x=94$ and thereafter decreases, which is clearly unacceptable!) The remaining formulae both produce reasonable graduations. However, the 3-parameter formula has rather more large 'relative deviations' (i.e. large values of z_x) and correspondingly greater values of χ^2 and t (χ^2). In addition at younger ages (i.e. between ages 15 and 20) the rates of the

Table 2. *Principal statistics for several graduations of the ultimate data*

	Number of parameters in formula				
	2	3	4	5	6
Coefficients					
a_1	−3·920	−3·519	−3·532	−2·965	−7·133
a_2	4·875	5·283	5·255	6·235	−1·504
a_3		·424	·409	1·189	−4·771
a_4			− ·010	·391	−3·507
a_5				·195	−1·663
a_6					− ·700
L^* (a)	−28,970·2	−28,946·3	−28,946·3	−28,944·5	−28,928·9
Graduated rates					
$10^5q_{20\cdot5}$	16	24	24	28	37
$10^5q_{40\cdot5}$	112	115	115	112	114
$10^5q_{60\cdot5}$	780	728	728	739	702
$10^5q_{80\cdot5}$	5,234	5,755	5,762	5,509	6,120
No. of positive (A–E)	49	43	43	42	40
No. of negative (A–E)	32	38	38	39	41
No. of $z_x>2$	15	11	10	9	1
No. of $z_x>3$	4	1	1	0	0
No. of runs r	32	46	46	42	46
$t(r)$	1·81	−1·05	−1·05	− ·12	−1·01
χ^2 (a)	192·4	115·1	114·6	105·7	69·3
$t(\chi^2)$	7·09	2·72	2·77	2·25	− ·43
ρ_{zz}	·37	·14	·14	·09	− ·28
$t(\rho)$	3·34	1·23	1·22	·76	−2·55

(Note: see reference 1 for definitions of above statistics.)

3-parameter formula are unrealistically low. On balance the 5-parameter formula is preferred and henceforth reference to the ultimate rates relates to this formula.

The precise values of the coefficients for the ultimate graduation are as follows:

$$a_1 = -2{\cdot}96537254$$
$$a_2 = 6{\cdot}23522259$$
$$a_3 = 1{\cdot}18884477$$
$$a_4 = 0{\cdot}39070030$$
$$a_5 = 0{\cdot}19540908$$

We refer to this graduation as FA1975–78 ultimate.

It is interesting to compare the values of q_x from this graduation with those of the A1967–70 table. One way of doing this is to determine the necessary age deductions to obtain the corresponding mortality rates on the A1967–70 table. Thus, for a given value of x, we define the age deduction d by the equation

$$q_x \text{ (FA1975–78 ultimate)} = q_{x-d} \text{ (A1967–70 ultimate)}.$$

Values of d as a function of x at quinquennial ages are contained in Table 3.

Below age 30 it is not possible to find a meaningful age deduction to the male age to produce the female rates.

Table 3. *Age deduction necessary to obtain FA1975–78 rates from A1967–70 rates*

Age x	35	40	45	50	55	60	65	70	75	80	85	90	95
Deduction d	2·98	2·67	3·69	4·82	5·83	6·63	7·19	7·47	7·42	6·92	5·85	4·01	1·20

The range of the values of d should be noted. Although at younger ages a deduction of about 3 years is appropriate, between ages 60 and 75 the necessary deduction is of the order of 7 years. Above age 75 the necessary deduction decreases with advancing age. The crude rates and the graduated rates are shown in Figure 1 which, for purposes of comparison, shows also the A1967–70 ultimate rates. The changing value of the age deduction d corresponds to the manner in which the two curves in Figure 1 diverge and then come together again.

One unusual feature should be noted from Figure 1. The two curves intersect between ages 96 and 97. This means that at age 97 and above the FA1975–78 rates are higher than the corresponding A1967–70 rates! In one sense this is undesirable. On the other hand the available data at the very high ages are so small as to make the graduated rates at these ages somewhat artificial. This is particularly so, since—for both curves—the procedure to derive the graduated rates gives much greater weight to the data away from these high ages. The financial consequences of mortality rates around age 100 are insignificant. Moreover, adjustment of the rates for the last few years of life could give rise to considerable practical inconvenience. (If no adjustment is made, the graduated rates—at least above age 20 say—are determined simply by the five coefficients of

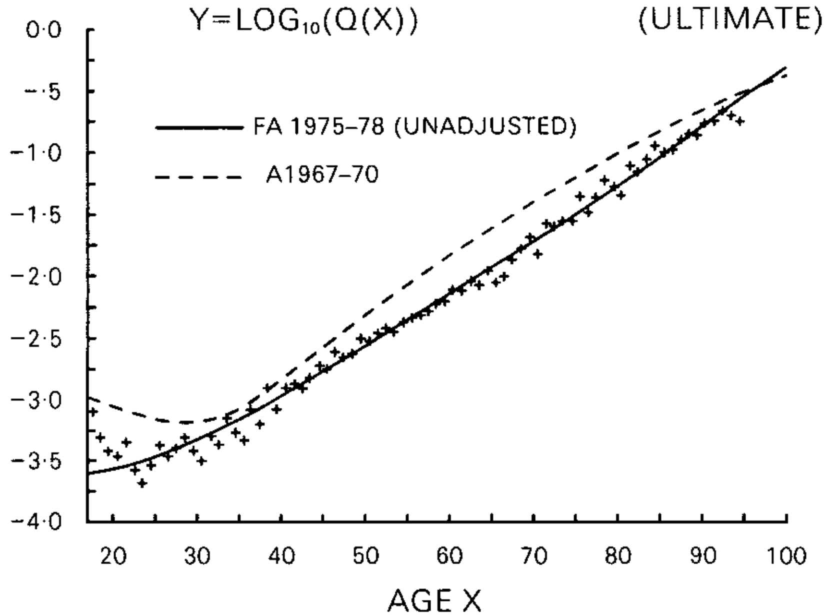

Figure 1

the formula.) Accordingly, it is recommended that the rates defined by the formula be adopted at all adult ages.

4. GRADUATIONS OF THE SELECT DATA

As a first step the same graduation formula (i.e. based on Chebycheff polynomials) was used to graduate the data at durations 0 and 1. Several graduations were obtained, based on formulae with 2, 3 and 4 parameters. The principal features of these graduations are summarized in Tables 5 and 6 below. From these Tables it can be seen that satisfactory graduations can be obtained with 3 and possibly even 2 parameters. For both durations above age 70 the rates of the 2-parameter formula are somewhat low and in addition at young ages the formula produces unrealistic rates at duration 1. Accordingly, as a first step, the 3-parameter rates were considered in greater detail. The rates for this formula, together with the crude rates and the corresponding A1967–70 rates are shown in Figures 2 and 3.

It will be noted that at both durations 0 and 1 the FA1975–78 curve crosses the corresponding A1967–70 curve. At duration 0 the curves intersect between ages 78 and 79. At duration 1 the intersection is between ages 79 and 80.

As before, it may seem that the intersection of the curves is an undesirable

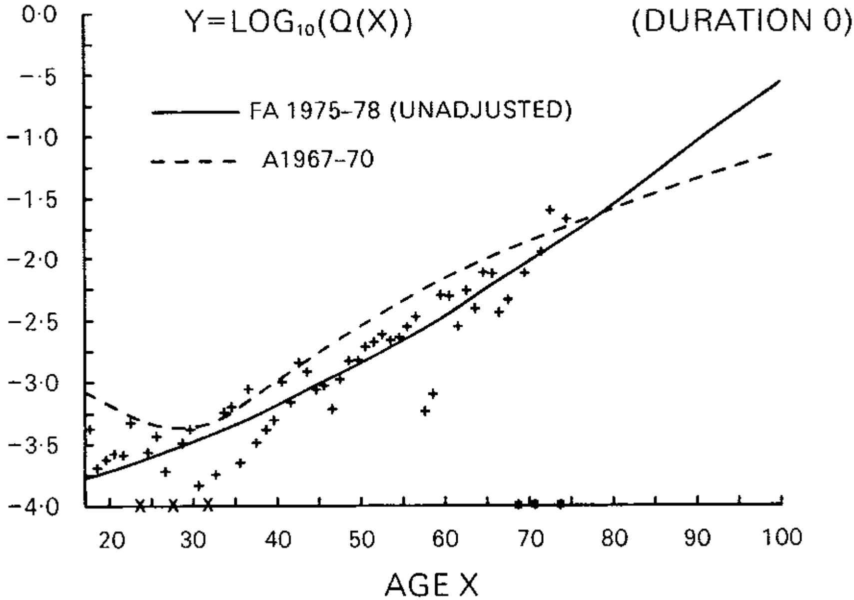

Figure 2

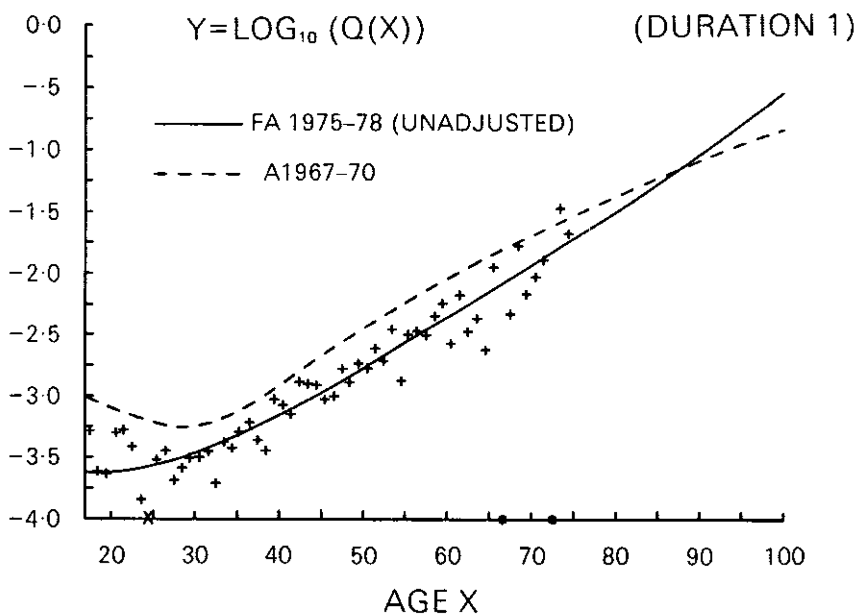

Figure 3

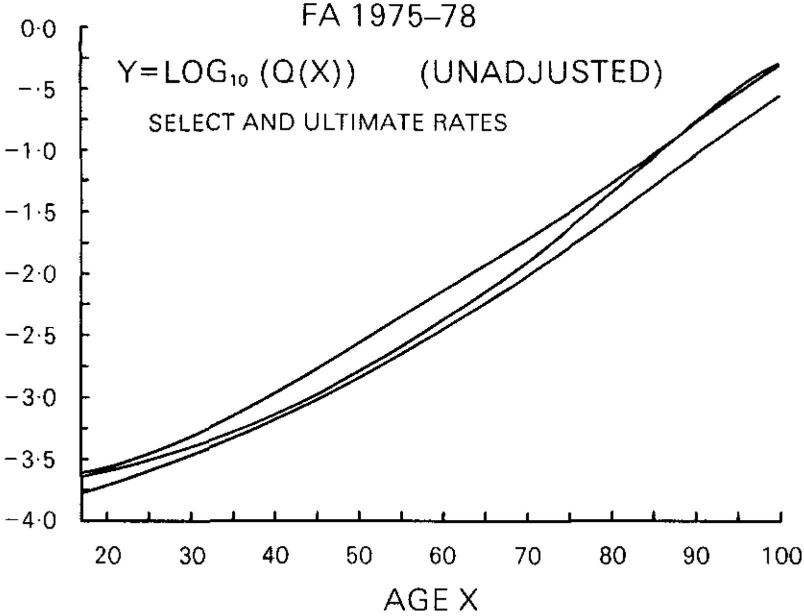

Figure 4

feature of the graduation. However, it must be borne in mind that the A1967–70 select rates at advanced ages were criticized as being too low, which may be a significant point in this context.

The select rates (by the 3-parameter formula) and the ultimate rates (by the 5-parameter formula) are shown in Figure 4. One unsatisfactory feature of these rates is that the duration 1 curve crosses the ultimate curve between ages 96 and 97. This is clearly unrealistic.

An alternative approach to graduating the select data was to consider a simple age adjustment to the previously obtained ultimate rates. At each duration, therefore, a graduation was attempted using the formula

$$q_x \text{ (FA1975–78 select)} = q_{x-a_1} \text{ (FA1975–78 ultimate)}.$$

The 'best' deduction a_1 was determined by maximizing the likelihood function described in § 2 above. The age deductions which gave the best fits are shown in Table 4.

Table 4. *Age deductions to obtain select rates*

Duration	Age deduction from ultimate
0	6·51725591 years
1	5·07221888 years

Table 5. *Principal statistics for several graduations of data at duration* 0

	Formula 2 Parameter	3 Parameter	4 Parameter	Ultimate $-a_1$ years
Coefficients				
a_1	−4·993	−3·971	−8·147	6·517
a_2	3·729	5·257	−1·689	
a_3		·688	−3·154	
a_4			−1·203	
$L^*(\boldsymbol{a})$	−2,633·9	−2,632·2	−2,630·1	−2,631·1
Graduated rates				
$10^5\, q_{20\cdot5}$	17	20	22	24
$10^5\, q_{40\cdot5}$	75	69	65	64
$10^5\, q_{60\cdot5}$	333	366	386	394
$10^5\, q_{70\cdot5}$	699	990	687	1,037
No. of positive (A−E)	34	33	31	30
No. of negative (A−E)	27	28	30	31
No. of $z_x > 2$	4	5	3	4
No. of $z_x > 3$	0	0	0	0
No. of runs r	28	26	34	28
$t(r)$	·81	1·38	− ·65	·90
$\chi^2(\boldsymbol{a})$	73·3	65·4	64·1	62·0
$t(\chi^2)$	1·29	·71	·69	·23
ρ_{zz}	·08	·06	− ·03	·02
$t(\rho)$	·69	·43	− ·21	·15

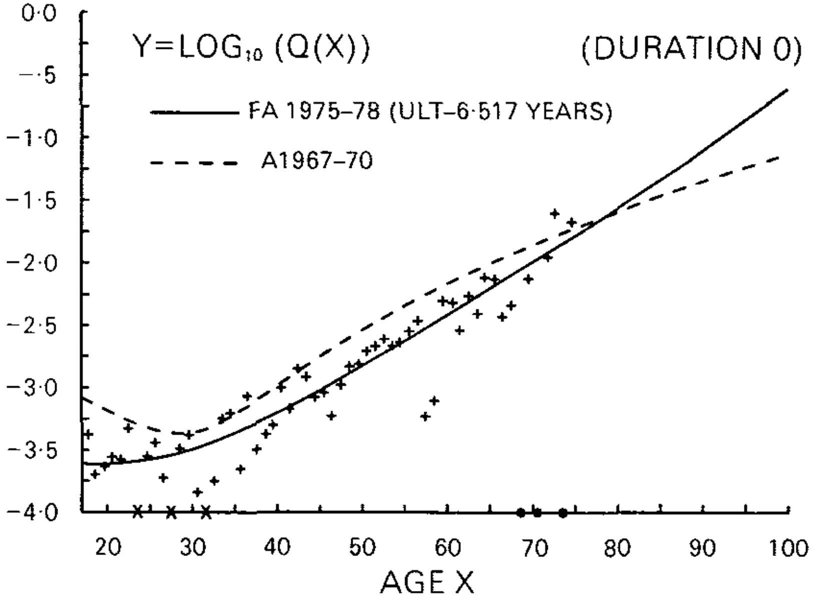

Figure 5

Table 6. *Principal statistics for several graduations of data at duration* 1

	Formula			
	2 Parameter	3 Parameter	4 Parameter	Ultimate $-a_1$ years
Coefficients				
a_1	−4·863	−3·325	−8·698	5·072
a_2	3·719	6·009	−2·957	
a_3		1·056	−3·921	
a_4			−1·593	
$L^*(\boldsymbol{a})$	−2,742·5	−2,738·1	−2,734·4	−2,736·1
Graduated rates				
$10^5 q_{20\cdot5}$	19	26	31	24
$10^5 q_{40\cdot5}$	86	75	69	72
$10^5 q_{60\cdot5}$	380	429	467	453
$10^5 q_{70\cdot5}$	795	1,312	850	1,194
No. of positive (A–E)	29	25	29	28
No. of negative (A–E)	32	36	32	33
No. of $z_x > 2$	3	3	0	3
No. of $z_x > 3$	1	0	0	0
No. of runs r	28	31	36	32
$t(r)$	·89	− ·13	−1·18	− ·18
$\chi^2(\boldsymbol{a})$	68·7	49·4	42·9	47·4
$t(\chi^2)$	·90	− ·78	−1·37	−1·18
ρ_{zz}	·22	·11	− ·05	·06
$t(\rho)$	1·68	·86	− ·39	·43

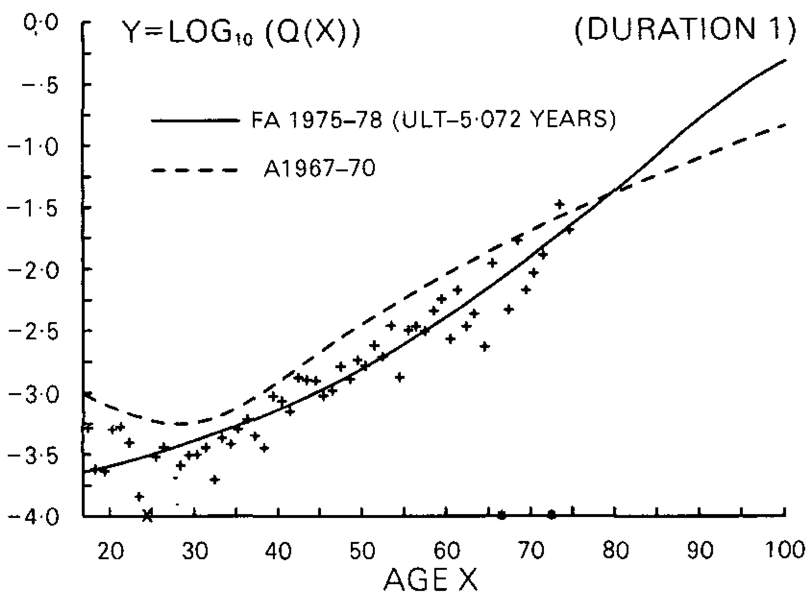

Figure 6

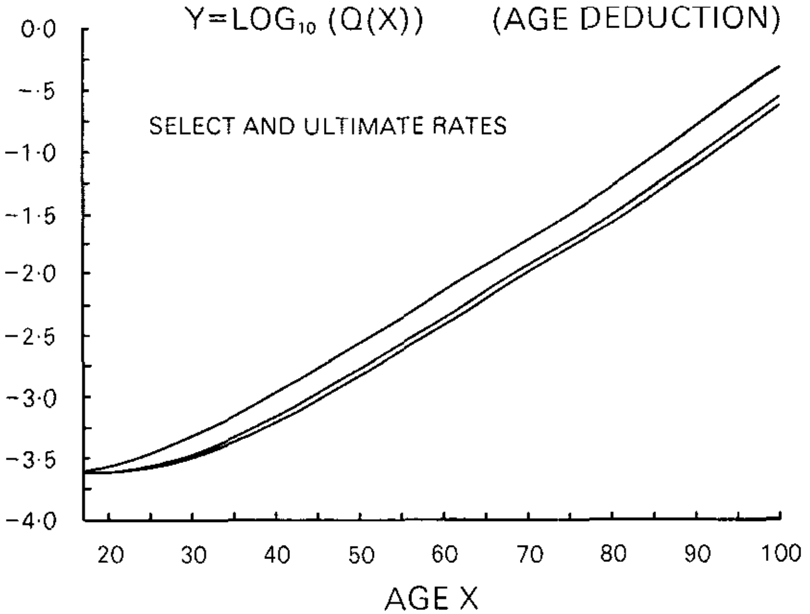

Figure 7

The principal statistics for these graduations are given in Tables 5 and 6.

It will be seen that both graduations based on the age deductions are acceptable. Graphs of the select rates by these formulae are given in Figures 5 and 6 and a comparison of the select and ultimate rates is shown in Figure 7.

Again it will be noted that the select curves obtained by the age deduction graduation cross the corresponding A1967–70 select curves. At duration 0 the intersection is still between ages 78 and 79, but at duration 1 the intersection is now between ages 87 and 88.

In view of the uncertainty attaching to the A1967–70 select rates at advanced ages, the relative financial insignificance of the select rates at these ages, and the considerable practical convenience of having a single formula (at least at adult ages), it is recommended that the age adjustment method be adopted to define the select rates. On this basis the FA1975–78 Table would be defined completely (at adult ages) by seven parameters, the five coefficients of the ultimate formula together with the two age deductions necessary for the select rates.

Table 7 shows the select rates (based on the age deductions found above) at each duration as a percentage of the corresponding ultimate rates. For comparison purposes figures are given for the A1967–70 Table.

Specimen values of the graduated rates are contained in Table 8.

Details of the graduations (at adult ages) are given in Appendices 1–3.

Table 7. *Select rates as a percentage of ultimate rates*

	Duration 0		Duration 1	
Age	FA1975–78	A1967–70	FA1975–78	A1967–70
25	75	68	79	85
35	61	70	68	86
45	56	66	63	79
55	54	53	62	68
65	53	40	61	56
75	52	30	60	46
85	49	23	57	39
95	47	19	56	35

5. EXTRAPOLATION TO VERY YOUNG AGES

Although there are virtually no data for female assured lives at very young ages, for practical purposes it is convenient that the published table be extended to age 0—even if the resulting rates of mortality are somewhat artificial at the youngest ages.

As a first step, the 5-parameter formula for the ultimate rates (cf. § 3 above) was considered in greater detail. Although the formula was derived from an analysis of the data between ages 15 and 95, it produces rates of mortality at the youngest ages which may reasonably be used for the published table. Initially, the

Table 8. $10^6 q_x$

Age x	Duration 0	Duration 1	Ultimate
20	237	240	273
25	259	272	346
30	318	344	477
35	429	474	699
40	619	695	1,071
45	938	1,063	1,685
50	1,465	1,674	2,695
55	2,334	2,676	4,347
60	3,758	4,317	7,042
65	6,081	6,993	11,448
70	9,874	11,368	18,728
75	16,115	18,595	30,989
80	26,555	30,761	52,223
85	44,470	51,822	90,157
90	76,182	89,433	159,331
95	133,798	158,009	282,805
100	238,070	280,519	478,119

trend of the 'formula' rates is similar to that of the mortality of the general female population, in that the rates of mortality *decrease* with age until age 13. Above age 13 the formula rates increase with age. (For the general female population mortality rates between ages 20 and 25 are virtually constant. At these ages, however, the rates are so small that the fact that the formula does not mirror this feature is of no great importance.)

At adult ages (i.e. at age 20 and above) the select rates at durations 0 and 1 are obtained from the ultimate rates by age deductions of approximately $6\frac{1}{2}$ and 5 years respectively (see § 4 above). Since over the first 13 years of life the ultimate rates decrease with age, these age deductions cannot be used to produce select rates at the youngest ages. (The duration 0 rates would exceed the duration 1 rates up to age 18.) Accordingly, below age 20 the select rates at each age were calculated as a constant proportion of the corresponding ultimate rates, the constants of proportionality for each duration being taken as the values which result from the select and ultimate rates at age 20.

Thus, for $x < 20$, the select rates are given by the equations

$$q_{[x]} = (q_{[20]}/q_{20})q_x$$

and

$$q_{[x-1]+1} = (q_{[19]+1}/q_{20})q_x.$$

The resulting select mortality rates, although derived on an *ad hoc* basis, appear satisfactory for practical purposes.

6. SUMMARY

It is perhaps convenient to summarize the above results, which completely specify the FA 1975–78 mortality table.

(i) The table has a select period of two years.

(ii) *Ultimate rates of mortality.* For all $x \geqslant 0$ the ultimate mortality rate q_x is given by the formula

$$q_x = \frac{e^{pol(x)}}{1+e^{pol(x)}},$$

where

$$pol(x) = a_1 + a_2 t + a_3(2t^2-1) + a_4(4t^3-3t) + a_5(8t^4-8t^2+1),$$

$$t = \frac{x-70}{50},$$

$$a_1 = -2{\cdot}96537254,$$
$$a_2 = 6{\cdot}23522259,$$
$$a_3 = 1{\cdot}18884477,$$
$$a_4 = {\cdot}39070030,$$
$$\text{and } a_5 = {\cdot}19540908.$$

(iii) *Duration 0 select mortality rates.* These are defined as

$$q_{[x]} = \begin{cases} q_{x-\alpha} & \text{if } x \geqslant 20 \\ (q_{[20]}/q_{20})q_x \cdot & \text{if } x < 20, \end{cases}$$

where $\alpha = 6{\cdot}51725591$

(iv) *Duration 1 select mortality rates.* These are defined as

$$q_{[x-1]+1} = \begin{cases} q_{x-\beta} & \text{if } x \geqslant 20 \\ (q_{[19]+1}/q_{20})q_x & \text{if } x < 20, \end{cases}$$

where $\beta = 5{\cdot}07221888$.

The complete set of mortality rates is given in Appendix 4.

7. COMPARISONS OF CERTAIN FINANCIAL FUNCTIONS (FA 1975–78/A 1967–70)

In § 3 above it was noted that the ultimate mortality rates of the FA 1975–78 table may be obtained by 'rating down' the rates of the A1967–70 table (which relates to male assured lives), but that the necessary deduction from the attained male age itself depends on the attained age and varies from about 3 to $7\frac{1}{2}$ years (cf. Table 3). It should be noted that the mortality rates of the FA 1975–78 table are derived from a quadrennium 8 years later than that used for the (male) A 1967–70 table.

In Table 9 below, which relates to whole life assurances at 4% per annum interest, we give the values of $1{,}000P_{[x]}$ on the FA 1975–78 table at quinquennial ages from 15 to 70. We give also the appropriate age deduction d (to the nearest integer) such that $P_{[x]}$ (FA 1975–78) $= P_{[x-d]}$ (A1967–70). It is to be noted that at each age the appropriate deduction is either 6 or 7 years. Accordingly, we give the value of $1{,}000P_{[x-6]}$ (A1967–70). Finally, at each age $P_{[x]}$ (FA 1975–78) is expressed as a percentage of $P_{[x-6]}$ (A1967–70). Since the percentage varies from 96 to 99, for female whole life assurances the use of the A1967–70 table rated down by 6 years incorporates a small safety margin in the calculation of premiums. (This feature is to be found also at other rates of interest.)

Although for female whole life assurances the use of the A1967–70 rated down by a constant age deduction (e.g. 6 years) produces satisfactory premiums, no one age deduction is really appropriate for the calculation of term assurance premiums.

Tables 10, 11 and 12 below relate to term assurances for 5, 10 and 15 years respectively on the basis of an interest rate of 6% per annum. For each term n we give the value of $10{,}000P^{1}_{[x]:\overline{n}|}$ at quinquennial ages from 15 to 50. At each age x we give also the appropriate age deduction d (again to the nearest integer) such that $P^{1}_{[x]:\overline{n}|}$ (FA 1975–78) $= P^{1}_{[x-d]:\overline{n}|}$ (A1967–70). It should be noted, however, that it is

Table 9. *Whole life assurances: interest 4%*

(1) Age	(2) $1000P_{[x]}$ (FA 1975–78)	(3) Age deductions (years) to get A 1967–70	(4) $1000P_{[x-6]}$ (A 1967–70)	(5) 100 (2)/(4)
x				
15	3·68	7	3·85	96
20	4·51	7	4·70	96
25	5·56	7	5·69	98
30	6·88	6	6·94	99
35	8·56	6	8·61	99
40	10·70	6	10·83	99
45	13·47	6	13·75	98
50	17·08	7	17·59	97
55	21·83	7	22·64	96
60	28·18	7	29·31	96
65	36·82	7	38·13	97
70	48·82	6	49·79	98

Table 10. *5-year term assurance: interest 6%*

(1) Age	(2) $10{,}000P^{\,1}_{[x]:\overline{5}|}$ (FA 1975–78)	(3) Age deduction to get A 1967–70	(4) $10{,}000P^{\,1}_{[x-4]:\overline{5}|}$ (A1967–70)	(5) 100 (2)/(4)
x				
15	2·23	—	3·90	57
20	2·64	—	8·32	32
25	3·32	—	6·55	51
30	4·58	—	5·52	83
35	6·71	3	6·37	105
40	10·27	4	10·02	102
45	16·16	5	17·83	91
50	25·83	6	31·88	81

Table 11. *10-year term assurance: interest 6%*

(1) Age	(2) $10{,}000P^{\,1}_{[x]:\overline{10}|}$ (FA 1975–78)	(3) Age deduction to get A1967–70	(4) $10{,}000P^{\,1}_{[x-4]:\overline{10}|}$ (A1967-70)	(5) 100 (2)/(4)
x				
15	2·47	—	6·01	41
20	3·08	—	7·86	39
25	4·13	—	6·39	65
30	5·94	6	6·16	96
35	8·99	3	8·30	108
40	14·04	4	14·06	100
45	22·33	5	25·39	88
50	35·87	6	45·23	79

Table 12. *15-year term assurance: interest 6%*

(1) Age x	(2) $10{,}000P^{\;1}_{[x]:\overline{15}|}$ (FA 1975–78)	(3) Age deduction to get A1967–70	(4) $10{,}000P^{\;1}_{[x-4]:\overline{15}|}$ (A1967–70)	(5) 100 (2)/(4)
15	2·76	—	6·31	44
20	3·60	—	7·45	48
25	5·01	—	6·55	76
30	7·42	4	7·30	102
35	11·42	4	10·99	104
40	18·02	5	19·22	94
45	28·80	6	34·40	84
50	46·30	6	60·20	77

not always possible to find an appropriate age deduction at the youngest ages. (For example, in relation to 5-year term assurances, no meaningful age deduction can be found when the age at entry is less than 33.) Moreover, the range of appropriate deductions varies from 3 to 6 years, with an average value of between 4 and 5 years.

Rating the A1967–70 down by 5 years produces premiums which are generally too low for female entrants between the ages of 30 and 40, but one might consider the use of an age deduction of 4 years. Accordingly, in Tables 10, 11 and 12 we give the appropriate values of $10{,}000P^{\;1}_{[x-4]:\overline{n}|}$ (A1967–70) and also express the value of $P^{\;1}_{[x]:\overline{n}|}$ (FA 1975–78) as a percentage of $P^{\;1}_{[x-4]:\overline{n}|}$ (A1967–70).

The unsatisfactory nature of such an age adjustment for term assurances is immediately apparent. At the youngest entry ages the net premium on the basis of FA 1975–78 mortality is less than half the corresponding net premium on the basis of A1967–70 rated down by 4 years. However, for entry ages between 30 and 40, the FA 1975–78 net premium exceeds the corresponding rated down A1967–70 value for certain terms of assurance.

In relation to net annual premiums for endowment assurances it is easily verified that the use of the A1967–70 table with the age rated down by 5 or 6 years (independent of the term of the policy) produces satisfactory results in comparison with the actual net premiums of the FA 1975–78 table.

It must, however, be borne in mind that when allowance is made for expenses, commission, and other loadings the age deductions and ratios described above may change considerably. It is the actuary's responsibility to allow appropriately for such factors when using the table.

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APPENDIX 1

Duration 0. *Exposed to risk and comparison of actual deaths with those expected according to the graduated table*

Age x	Exposed to risk ER_x	Graduated rate q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$
20·5	22,854·75	·00023781	6	5·44	·56	·2423
21·5	23,975·00	·00024008	6	5·76	·24	·1018
22·5	23,902·75	·00024373	11	5·83	5·17	2·1440
23·5	23,556·00	·00024876	2	5·86	−3·86	−1·5947
24·5	22,780·25	·00025520	6	5·81	·19	·0773
25·5	22,402·00	·00026309	8	5·89	2·11	·8677
26·5	21,591·25	·00027248	4	5·88	−1·88	− ·7765
27·5	21,565·25	·00028345	1	6·11	−5·11	−2·0682
28·5	21,984·75	·00029610	7	6·51	·49	·1922
29·5	22,271·75	·00031054	9	6·92	2·08	·7925
30·5	21,360·50	·00032691	3	6·98	−3·98	−1·5075
31·5	18,634·75	·00034537	1	6·44	−5·44	−2·1431
32·5	17,208·25	·00036611	3	6·30	−3·30	−1·3150
33·5	16,033·00	·00038931	9	6·24	2·76	1·1042
34·5	14,752·25	·00041522	9	6·13	2·87	1·1617
35·5	13,822·25	·00044410	3	6·14	−3·14	−1·2670
36·5	13,003·25	·00047622	11	6·19	4·81	1·9324
37·5	12,618·00	·00051193	4	6·46	−2·46	− ·9680
38·5	12,173·75	·00055158	5	6·71	−1·71	− ·6619
39·5	12,171·00	·00059557	6	7·25	−1·25	− ·4639
40·5	11,363·50	·00064434	11	7·32	3·68	1·3597
41·5	10,393·50	·00069839	7	7·26	− ·26	− ·0961
42·5	10,016·75	·00075826	14	7·60	6·40	2·3248
43·5	9,373·00	·00082456	11	7·73	3·27	1·773
44·5	9,358·00	·00089794	8	8·40	− ·40	− ·1391
45·5	8,962·00	·00097914	8	8·78	− ·78	− ·2618
46·5	8,415·00	·00106898	5	9·00	−4·00	−1·3329

APPENDIX 1 (*continued*)

Age x	Exposed to risk ER_x	Graduated rate q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$
47·5	7,853·50	·00116833	8	9·18	−1·18	− ·3883
48·5	7,503·50	·00127819	11	9·59	1·41	·4553
49·5	7,483·75	·00139963	11	10·47	·53	·1625
50·5	6,319·00	·00153384	12	9·69	2·31	·7418
51·5	5,273·00	·00168213	11	8·87	2·13	·7158
52·5	4,593·00	·00184593	11	8·48	2·52	·8668
53·5	4,174·50	·00202682	9	8·46	·54	·1855
54·5	3,970·75	·00222655	9	8·84	·16	·0535
55·5	3,239·25	·00244702	9	7·93	1·07	·3818
56·5	2,426·50	·00269034	8	6·53	1·47	0·5769
57·5	1,739·00	·00295883	1	5·15	−4·15	−1·8302
58·5	1,296·50	·00325503	1	4·22	−3·22	−1·5701
59·5	1,255·25	·00358175	6	4·50	1·50	·7106
60·5	848·00	·00394210	4	3·34	·66	·3601
61·5	720·00	·00433950	2	3·12	−1·12	− ·6375
62·5	579·25	·00477774	3	2·77	·23	·1401
63·5	522·75	·00526099	2	2·75	− ·75	− ·4536
64·5	550·50	·00579389	4	3·19	·81	·4551
65·5	421·25	·00638160	3	2·69	·31	·1908
66·5	279·25	·00702982	1	1·96	− ·96	− ·6898
67·5	223·75	·00774493	1	1·73	− ·73	− ·5589
68·5	196·00	·00853401	0	1·67	−1·67	−1·2989
69·5	137·50	·00940499	1	1·29	− ·29	− ·2590
70·5	105·00	·01036675	0	1·09	−1·09	−1·0488
71·5	90·00	·01142921	1	1·03	− ·03	− ·0284
72·5	81·25	·01260355	2	1·02	·98	·9706
73·5	64·75	·01390232	0	·90	− ·90	− ·9554
74·5	47·50	·01533969	1	·73	·27	·3204
Total	508,536·75		310	312·2		

$$\chi^2 = \sum z_x^2 = 57{\cdot}76$$

APPENDIX 2

Duration 1. Exposed to risk and comparison of actual deaths with those expected according to the graduated table.

Age x	Exposed to risk ER_x	Graduated rate q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$
20·5	18,018·50	·00024153	9	4·35	4·65	2·2283
21·5	19,408·00	·00024580	10	4·77	5·23	2·3946
22·5	20,636·00	·00025145	8	5·19	2·81	1·2342
23·5	20,942·50	·00025853	3	5·41	−2·41	−1·0377
24·5	20,594·50	·00026708	2	5·50	−3·50	−1·4927
25·5	20,032·75	·00027716	6	5·55	·45	·1901
26·5	19,916·25	·00028886	7	5·75	1·25	·5200
27·5	19,546·75	·00030229	4	5·91	−1·91	− ·7854
28·5	19,460·25	·00031758	5	6·18	−1·18	− ·4748
29·5	19,184·25	·00033486	6	6·42	− ·42	− ·1673
30·5	19,149·50	·00035431	6	6·78	− ·78	− ·3014
31·5	17,239·75	·00037611	6	6·48	− ·48	− ·1902
32·5	15,377·75	·00040049	3	6·16	−3·16	−1·2731
33·5	14,450·00	·00042769	6	6·18	− ·18	− ·0725
34·5	13,323·50	·00045797	5	6·10	−1·10	− ·4461
35·5	12,135·00	·00049165	6	5·97	·03	·0138
36·5	11,759·25	·00052907	7	6·22	·78	·3122
37·5	11,451·00	·00057060	5	6·53	−1·53	− ·6002
38·5	11,143·25	·00061665	4	6·87	−2·87	−1·0958
39·5	10,679·50	·00066771	10	7·13	2·87	1·0748
40·5	10,758·25	·00072428	9	7·79	1·21	·4329
41·5	9,902·50	·00078693	7	7·79	− ·79	− ·2840
42·5	9,146·75	·00085630	12	7·83	4·17	1·4898
43·5	8,770·75	·00093307	11	8·18	2·82	·9849
44·5	8,228·25	·00101801	10	8·38	1·62	·5613
45·5	8,509·75	·00111196	8	9·46	−1·46	− ·4757
46·5	7,908·00	·00121587	8	9·62	−1·62	− ·5212
47·5	7,368·50	·00133074	12	9·81	2·19	·7013
48·5	6,893·75	·00145771	9	10·05	−1·05	− ·3312

APPENDIX 2 (*continued*)

Age x	Exposed to risk ER_x	Graduated rate q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$
49·5	6,640·25	·00159802	12	10·61	1·39	·4267
50·5	6,606·00	·00175302	11	11·58	− ·58	− ·1707
51·5	5,529·75	·00192422	13	10·64	2·36	·7240
52·5	4,723·50	·00211327	9	9·98	− ·98	− ·3112
53·5	4,148·50	·00232199	14	9·63	4·37	1·4088
54·5	3,757·50	·00255235	5	9·59	−4·59	−1·4842
55·5	3,490·50	·00280657	11	9·80	1·20	·3851
56·5	2,734·25	·00308706	9	8·44	·56	·1928
57·5	1,947·00	·00339648	6	6·61	− ·61	− ·2388
58·5	1,345·50	·00373777	6	5·03	·97	·4337
59·5	1,062·25	·00411417	6	4·37	1·63	·7812
60·5	1,150·50	·00452925	3	5·21	−2·21	− ·9707
61·5	767·75	·00498698	5	3·83	1·17	·6001
62·5	601·00	·00549172	2	3·30	−1·30	− ·7178
63·5	469·25	·00604835	2	2·84	− ·84	− ·4990
64·5	431·00	·00666224	1	2·87	−1·87	−1·1081
65·5	463·25	·00733941	5	3·40	1·60	·8709
66·5	343·25	·00808651	0	2·78	−2·78	−1·6728
67·5	216·25	·00891101	1	1·93	− ·93	− ·6708
68·5	184·75	·00982124	3	1·81	1·19	·8845
69·5	150·00	·01082651	1	1·62	− ·62	− ·4923
70·5	110·00	·01193731	1	1·31	− ·31	− ·2749
71·5	79·25	·01316538	1	1·04	− ·04	− ·0427
72·5	71·75	·01452398	0	1·04	−1·04	−1·0283
73·5	61·50	·01602806	2	·99	1·01	1·0299
74·5	48·75	·01769451	1	·86	·14	·1493
Total	459,068		334	329·5		

$$\chi^2 = \sum z_x^2 = 42{\cdot}96$$

APPENDIX 3

Durations 2 and over. Exposed to risk and comparison of actual deaths with those expected according to the graduated table

Age x	Exposed to risk ER_x	Graduated rate q_x	Actual deaths A_x	Expected deaths E_x	Deviation $A_x - E_x$	$z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$
20·5	26330·25	·00027795	9	7·32	1·68	·6217
21·5	31956·00	·00028977	14	9·26	4·74	1·5579
22·5	37506·75	·00030333	10	11·38	−1·38	− ·4083
23·5	43621·00	·00031876	9	13·90	−4·90	−1·3155
24·5	49018·75	·00033619	14	16·48	−2·48	−·6109
25·5	53938·00	·00035580	23	19·19	3·81	·8696
26·5	58133·50	·00037779	20	21·96	−1·96	− ·4187
27·5	62582·25	·00040236	25	25·18	− ·18	− ·0360
28·5	67630·00	·00042977	33	29·07	3·93	·7300
29·5	70568·75	·00046029	27	32·48	−5·48	− ·9621
30·5	71431·75	·00049423	22	35·30	−13·30	−2·2396
31·5	66124·75	·00053192	33	35·17	−2·17	− ·3666
32·5	63718·25	·00057376	27	36·56	−9·56	−1·5814
33·5	62111·50	·00062017	44	38·52	5·48	·8833
34·5	58353·75	·00067161	31	39·19	−8·19	−1·3088
35·5	53381·75	·00072860	25	38·89	−13·89	−2·2286
36·5	51667·25	·00079171	42	40·91	1·09	·1712
37·5	52869·25	·00086158	33	45·55	−12·55	−1·8605
38·5	53326·50	·00093892	66	50·07	15·93	2·2525
39·5	52501·50	·00102443	44	53·79	−9·79	−1·3351
40·5	51581·75	·00111912	65	57·73	7·27	·9579
41·5	50698·75	·00122378	67	62·04	4·96	·6295
42·5	49841·75	·00133949	62	66·76	−4·76	− ·5833
43·5	49343·25	·00146738	74	72·41	1·59	·1875
44·5	49382·75	·00160870	92	79·44	12·56	1·4101
45·5	49262·25	·00176482	86	86·94	− ·94	− ·1008
46·5	49336·75	·00193726	119	95·58	23·42	2·3981
47·5	48457·00	·00212767	107	103·10	3·90	·3845
48·5	47580·50	·00233787	110	111·24	−1·24	− ·1174
49·5	46099·50	·00256989	144	118·47	25·53	2·3485
50·5	44964·25	·00282592	135	127·07	7·93	·7049
51·5	44422·50	·00310841	154	138·08	15·92	1·3566
52·5	43563·75	·00342003	161	148·99	12·01	·9857
53·5	42193·50	·00376374	150	158·81	−8·81	− ·7001
54·5	40209·50	·00414281	167	166·58	·42	·0326
55·5	37865·25	·00456083	172	172·70	− ·70	− ·0532
56·5	35048·25	·00502180	169	176·01	−7·01	− ·5294

APPENDIX 3 (*continued*)

Age x	Exposed to risk ER_x	Graduated rate q_x	Actual deaths A_x	Expected deaths E_x	Deviation A_x-E_x	$z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$
57·5	30841·25	·00553012	158	170·56	−12·56	− ·9641
58·5	25695·00	·00609070	154	156·50	−2·50	− ·2005
59·5	19395·25	·00670895	119	130·12	−11·12	− ·9783
60·5	16235·25	·00739093	124	119·99	4·01	·3671
61·5	15177·75	·00814337	115	123·60	−8·60	− ·7765
62·5	13917·25	·00897377	125	124·89	·11	·0099
63·5	12106·50	·00989053	101	119·74	−18·74	−1·7211
64·5	7691·50	·01090307	82	83·86	−1·86	− ·2043
65·5	4924·50	·01202192	43	59·20	−16·20	−2·1185
66·5	4213·25	·01325895	41	55·86	−14·86	−2·0019
67·5	3689·75	·01462754	50	53·97	−3·97	− ·5447
68·5	3104·50	·01614275	51	50·12	·88	·1260
69·5	2510·75	·01782164	51	44·75	6·25	·9434
70·5	2096·25	·01968351	31	41·26	−10·26	−1·6135
71·5	1850·25	·02175028	48	40·24	7·76	1·2362
72·5	1629·75	·02404684	41	39·19	1·81	·2926
73·5	1441·75	·02660151	40	38·35	1·65	·2696
74·5	1228·00	·02944658	34	36·16	−2·16	− ·3647
75·5	1071·00	·03261888	46	34·93	11·07	1·9034
76·5	917·00	·03616045	30	33·16	−3·16	− ·5588
77·5	802·25	·04011937	35	32·19	2·81	·5063
78·5	728·75	·04455056	43	32·47	10·53	1·8913
79·5	654·75	·04951682	34	32·42	1·58	·2844
80·5	604·00	·05508989	27	33·27	−6·27	−1·1190
81·5	551·75	·06135168	42	33·85	8·15	1·4457
82·5	513·50	·06839549	36	35·12	·88	·1537
83·5	439·25	·07632739	38	33·53	4·47	·8038
84·5	370·75	·08526748	41	31·61	9·39	1·7456
85·5	326·00	·09535115	32	31·08	·92	·1726
86·5	278·75	·10673001	29	29·75	− ·75	− ·1457
87·5	255·50	·11957241	32	30·55	1·45	·2794
88·5	228·75	·13406319	32	30·67	1·33	·2587
89·5	203·25	·15040236	28	30·57	−2·57	− ·5042
90·5	170·25	·16880213	29	28·74	·26	·0535
91·5	132·75	·18948171	23	25·15	−2·15	− ·4770
92·5	104·00	·21265928	22	22·12	− ·12	− ·0279
93·5	72·50	·23854031	14	17·29	−3·29	− ·9078
94·5	57·00	·26730180	10	15·24	−5·24	−1·5672
Total	2,042,853		4616	4624·2		

$$\chi^2 = \sum z_x^2 = 90{\cdot}78$$

APPENDIX 4

Age	Graduated rates			Age	Graduated rates		
x	$q_{[x]}$	$q_{[x-1]+1}$	q_x	x	$q_{[x]}$	$q_{[x-1]+1}$	q_x
0	·00038070	·00038505	·00043759	30	·00031847	·00034430	·00047681
1	·00034492	·00034887	·00039647	31	·00033587	·00036490	·00051258
2	·00031557	·00031918	·00036273	32	·00035544	·00038797	·00055230
3	·00029145	·00029479	·00033501	33	·00037738	·00041372	·00059637
4	·00027164	·00027474	·00031223	34	·00040191	·00044243	·00064523
5	·00025540	·00025832	·00029357	35	·00042927	·00047437	·00069937
6	·00024217	·00024494	·00027836	36	·00045973	·00050987	·00075935
7	·00023150	·00023415	·00026610	37	·00049361	·00054929	·00082576
8	·00022305	·00022560	·00025638	38	·00053124	·00059303	·00089927
9	·00021652	·00021900	·00024888	39	·00057301	·00064153	·00098062
10	·00021172	·00021414	·00024336	40	·00061933	·00069527	·00107061
11	·00020847	·00021086	·00023963	41	·00067067	·00075481	·00117014
12	·00020664	·00020901	·00023753	42	·00072756	·00082073	·00128019
13	·00020615	·00020850	·00023695	43	·00079057	·00089371	·00140184
14	·00020692	·00020928	·00023784	44	·00086032	·00097446	·00153628
15	·00020891	·00021130	·00024013	45	·00093751	·00106380	·00168482
16	·00021210	·00021453	·00024380	46	·00102293	·00116261	·00184890
17	·00021651	·00021898	·00024886	47	·00111741	·00127186	·00203011
18	·00022213	·00022467	·00025533	48	·00122189	·00139264	·00223017
19	·00022901	·00023163	·00026324	49	·00133739	·00152611	·00245102
20	·00023720	·00023992	·00027265	50	·00146506	·00167359	·00269475
21	·00023877	·00024349	·00028365	51	·00160614	·00183650	·00296370
22	·00024173	·00024845	·00029633	52	·00176199	·00201641	·00326040
23	·00024607	·00025481	·00031080	53	·00193413	·00221505	·00358768
24	·00025181	·00026262	·00032721	54	·00212422	·00243433	·00394864
25	·00025896	·00027192	·00034571	55	·00233407	·00267634	·00434671
26	·00026759	·00028280	·00036648	56	·00256569	·00294337	·00478568
27	·00027776	·00029535	·00038973	57	·00282129	·00323798	·00526975
28	·00028955	·00030970	·00041569	58	·00310329	·00356295	·00580355
29	·00030308	·00032596	·00044462	59	·00341439	·00392137	·00639225

APPENDIX 4 (*continued*)

Age	Graduated rates			Age	Graduated rates		
x	$q_{[x]}$	$q_{[x-1]+1}$	q_x	x	$q_{[x]}$	$q_{[x-1]+1}$	q_x
60	·00375752	·00431664	·00704157	90	·07618233	·08943267	·15933106
61	·00413594	·00475252	·00775789	91	·08510392	·10005088	·17884317
62	·00455326	·00523318	·00854832	92	·09516662	·11203423	·20074494
63	·00501346	·00576323	·00942079	93	·10652176	·12555822	·22524992
64	·00552092	·00634778	·01038420	94	·11933738	·14081444	·25255157
65	·00608055	·00699252	·01144849	95	·13379806	·15800857	·28280534
66	·00669776	·00770377	·01262486	96	·15010357	·17735661	·31610634
67	·00737859	·00848859	·01392590	97	·16846593	·19907851	·35246360
68	·00812975	·00935485	·01536580	98	·18910430	·22338867	·39177289
69	·00895873	·01031137	·01696058	99	·21223697	·25048258	·43379139
70	·00987393	·01136802	·01872840	100	·23806974	·28051910	·47811900
71	·01088472	·01253589	·02068981	101	·26678026	·31359854	·52419179
72	·01200164	·01382747	·02286815	102	·29849786	·34973706	·57129273
73	·01323653	·01525683	·02528999	103	·33327948	·38883951	·61858298
74	·01460272	·01683985	·02798556	104	·37108284	·43067380	·66515332
75	·01611527	·01859451	·03098933	105	·41173968	·47485160	·71009054
76	·01779118	·02054119	·03434065	106	·45493298	·52082068	·75254968
77	·01964972	·02270302	·03808448	107	·50018340	·56787426	·79182033
78	·02171275	·02510631	·04227220	108	·54685054	·61518082	·82737629
79	·02400511	·02778100	·04696254	109	·59415336	·66183410	·85890133
80	·02655507	·03076126	·05222264	110	·64121152	·70691880	·88628937
81	·02939482	·03408605	·05812915	111	·68710496	·74958265	·90962244
82	·03256113	·03779989	·06476952	112	·73094450	·78910361	·92913344
83	·03609595	·04195368	·07224327	113	·77194265	·82494101	·94516191
84	·04004722	·04660558	·08066332	114	·80947310	·85676314	·95811002
85	·04446975	·05182208	·09015728	115	·84310938	·88444926	·96840417
86	·04942619	·05767912	·10086855	116	·87263822	·90806893	·97646472
87	·05498814	·06426331	·11295709	117	·89804820	·92784560	·98268484
88	·06123728	·07167326	·12659958	118	·91949947	·94411260	·98741745
89	·06826674	·08002088	·14198870	119	·93728205	·95726898	·99096905

HISTORY AND DEVELOPMENT

A PAPER with the above title appeared in *C.M.I.R.* **1**, as an opening to the C.M.I.R. publications which have now reached their sixth number. The purpose of the present paper is to bring the history up to date, without repeating any items which have little bearing on current or on possible future practice. Relevant items are repeated as *C.M.I.R.* **1** is now out of print, although reference to it can of course be made in the Institute and Faculty Libraries.

CHAIRMEN

The earlier paper outlined the system whereby prior to 1923 various investigations were undertaken by *ad hoc* Committees. Since the formation of the Joint Mortality Investigation Committee the following have served as Chairmen:

Sir W. P. Elderton	1923–48
R.Ll. Gwilt	1948–61
F. M. Redington	1961–68
J. M. Denholm	1968–74
E. B. O. Sherlock	1974–83

It will be seen that the alternation of Institute and Faculty representatives as Chairmen of the Committee has been maintained throughout.

ADMINISTRATION

Formal minutes were not kept before the end of World War II, when F. L. Bradshaw was appointed the first Superintending Actuary. There was no Secretary to the Committee until July 1950 when R. D. Clarke was appointed. The clerical work of the Bureau was carried out by a small staff, paid by the hour. At the end of 1955 F. L. Bradshaw was succeeded by H. A. R. Barnett.

When R. D. Clarke retired as Secretary in September 1972 the position of Superintending Actuary disappeared; H. A. R. Barnett became Secretary with R. E. Hayward as Assistant Secretary. At the same time R. E. Hayward became Secretary to the (then new) P.H.I. Sub-Committee to whom R. D. Clarke had acted as Secretary during its opening stages.

Since the work became computerized, *c.* 1975, Mrs J. V. Evans has been responsible for routine liaison with the computer company and may perhaps be regarded as having resuscitated the position of Superintending Actuary. There is still a small clerical staff responsible for dealing with queries from contributing offices, circulating letters, output, questionnaires, etc., to the offices and the Committee, coding cause-of-death sheets and, occasionally, making calculations if the Committee request additional information for which it would be uneconomic to order a computerized re-run of data.

STATISTICS

Statistics are submitted in a form suitable for the use of the 'Census Method', that is to say the offices submit particulars of policies in force on 1 January each year (a different date can be substituted if more convenient to the office) and of policies becoming claims by death notified during the year. Statistics are usually submitted according to age nearest birthday, but for some offices a different age classification is more convenient, and in these cases the Bureau makes appropriate adjustments to the figures to approximate to an age nearest birthday classification. For some investigations there are also subdivisions according to curtate duration of policy, all durations beyond the select period being combined. Data are received from about sixty offices, some who have joined recently cancelling out numerically some who have lost their separate identities through mergers.

Originally, female lives were excluded from the assured lives investigation but offices were permitted to include them if exclusion was difficult and if they were only a small proportion of the total. Since a separate investigation was started for female assured lives it is believed there are now very few, if any, female lives still included in the male investigation.

Mention has been made of 'policies' and this word was used deliberately. The investigations have generally been on the basis of policies rather than of lives, although in the assured lives' experience 'concurrent duplicates' have been excluded, so that a batch of policies effected at the same time on the same life would only be counted as one. Where possible all duplicates in the assured lives over age 80 have also been excluded, and the Bureau has recently asked those offices who can do so to make similar exclusions from annuity data.

The effect of duplicate policies was investigated by the Committee who invited the contributing offices to analyse the death claims in 1954 showing the number of lives at each age having 1, 2, 3 . . . policies. The returns were of duplicates within offices, no attempt being made to trace duplicates on the same life in several offices. The purpose of this investigation was to ascertain whether it would be possible to improve the estimates of the standard deviations needed to test the differences between actual and expected deaths; it was based on statistics for durations 3 and over.

INVESTIGATIONS

Whole Life and Endowment Assurances—Males

The assured lives' experience was originally subdivided into eight sections according to whether the policies were whole life or endowment assurance, with or without profits, and effected with or without medical examination. Policies effected at higher than normal rates were excluded, and similar exclusions have subsequently been made in all assured lives' investigations except that into impaired assured lives. However, when standard tables were prepared based on the experiences of 1924–29 and 1949–52, it was found that the Whole Life section was insufficient to give reliable results at young ages and the Endowment Assurance section was insufficient at the older ages. Furthermore, the Non-Profit Whole Life section was insufficient at the older ages. Variations between individual offices were found to be more significant than variations between the different types of policy and consequently the A 1924–29 and A 1949–52 tables were constructed from the whole of the data. After the publication of the A 1924–29 table, the experience was subdivided into offices exhibiting the lightest and heaviest mortality and 'Light' and 'Heavy' tables were constructed; similar tables were not prepared from the 1949–52 experience.

Despite the knowledge that variations between types of policy were of little significance, the subdivisions of the data in this way continued up to and including 1958, after which the only division (apart from age and duration and, later, sex) has been between medical business and business accepted without medical examination.

For convenience, the A 1924–29 tables were based on a three-year select period even though data had been collected on a five-year select basis. The A 1949–52 and A 1967–70 tables were based on a two-year select period, but when the latter were compiled a further table, A 1967–70(5) was compiled with a five-year select period. Details of the graduation methods employed are described in the official publications.

Periodical reports, at first published in the *Journal of the Institute* and in the *Transactions of the Faculty* but since 1972 in *Continuous Mortality Investigation Reports*, have included comparisons of trends with those of the national mortality; these were originally published as separate reports but are now incorporated with the main reports on whole life and endowment assurances.

Although comparisons with national mortality can show overall differences, they cannot analyse these differences in detail, and accordingly from 1964 a subsidiary investigation has been undertaken according to cause of death, which is described in a later section.

Children's Deferred Assurances

From 1924 to 1960 there was a separate investigation into mortality under Children's Deferred Assurances. Interest in the experience under this class of policy declined and the investigation was closed.

Immediate Annuitants

The other investigation which has been continuing ever since the Bureau was set up is that on immediate annuitants. Data for males and females are kept separate and there has generally been a five-year select period; however, periodical scrutiny of the results in the preparation of reports indicated that from

1957 onwards there appeared to be some change in the class of lives effecting immediate annuities, possibly arising from the effects of the 1956 Finance Act, and accordingly from 1963 onwards the select period was extended by one year every year up to and including 1968. The select period has now reverted to five years, but pre-1957 business has been kept separate from post-1956. The pre-1957 business remaining on offices' books now relates only to lives at very advanced ages, and the collection of data in respect of these annuities has been discontinued from 1982.

From 1960 the offices were asked to submit, in the case of immediate annuitants, lists of deaths notified in the first half of a year which had taken place in the previous year. This enabled the 'in force' figures to be adjusted, in order to cut down systematic distortion which can otherwise be quite serious at the advanced ages which figure prominently in this class of business. Since the returns have been computerized by the Bureau, those offices who can do so have been waiting until the middle of the following year to make up a year's immediate annuitant statistics, so that in effect most of the offices now make the adjustments themselves.

From 1975 statistics have been collected by 'amounts' as well as by 'lives'.

A standard table was prepared on the basis of the 1947–48 experience, projected to give rates which might be expected to apply to lives purchasing annuities in 1955. A 'forecast' generation table was also prepared on a projection based on expected improvements in mortality which, in the event, did not materialize.

A further standard table was prepared on the basis of the 1967–70 'lives' experience and, by taking note of earlier experiences, projected to give rates applicable to the year of experience 1990 and therefore suitable for use in the 1980s. The projection was devised in such a way that a fixed year of progression in time was equated to a fixed deduction from age, and the publication included a double entry table of mortality rates by age and calendar year of birth, as well as a table of mortality functions applicable to lives born in the calendar year 1925.

Annuities payable for a term certain and life thereafter

In 1948 a separate investigation was started into the experience of this type of annuity. This was concluded in 1957.

Pensioners under Life Office Pension Schemes

Also in 1948, collection of data was started for pensioners under life office pension schemes. Originally this investigation was based on lives, but since 1958 particulars based on amounts have been submitted, and the two sets of data are still being collected.

A standard table was prepared similar to the latest table for immediate annuitants, but based on 'amounts' experiences. As in the case of the annuitants' table, experiences up to 1967–70 were projected to 1990 so that the table would be suitable for use in the 1980s; a similar projection was made, and the table also included mortality rates according to age and calendar year of birth, and mortality functions applicable to lives born in 1925.

Between 1965 and 1974 separate data were submitted by five offices based on 'Works' pension schemes, these also being included in the main pensioners' data; this separate investigation is now discontinued, having been virtually superseded by the 'amounts' investigation.

Since 1976 data have been collected subdivided by duration, based on a ten-year select period since retirement.

Retirement Annuities

Retirement annuities effected under the 1956 Finance Act have been the subject of an *ab initio* investigation, both during the period of deferment and after retirement, and will therefore form the basis of the only investigation into which all possible data will have been collected, apart from the fact that there are some offices who write this business but do not submit data.

Group Life Schemes

A special investigation based on seven offices and three years only (1958–60) was undertaken into mortality under group life assurance schemes. This was carried out on a 'policy year' basis, and the results were presented according to both lives and amounts. The returns of policies in force were by age nearest birthday on the scheme anniversary which occurred in the calendar year for which the return was being made. Deaths and withdrawals were tabulated by age nearest birthday on the scheme anniversary prior to death or withdrawal no matter in what calendar year the exit took place. New entrants and increments, where they took place at dates other than the scheme anniversary, were tabulated by age nearest birthday on the scheme anniversary prior to entry.

Whole Life and Endowment Assurances—Republic of Ireland

From 1970 a separate investigation into the mortality of male lives covered by whole life or endowment assurances issued in the Republic of Ireland was started. From 1982 a similar investigation into the mortality of female lives is being made. At present thirteen Irish offices, or offices with Irish branches, are contributing data. The select period is five years.

Temporary Assurances

Also from 1970, a new investigation was started into the mortality of male lives under certain temporary assurances, with a similar investigation into female lives starting from 1982. Level and decreasing policies are kept separate, and again the select period is five years.

Female Lives (Whole Life and Endowment Assurances)

The first investigation into the mortality of female assured lives was started in 1973, and related to lives assured under whole life and endowment assurances issued in the United Kingdom. The select period is five years, and the Committee have just completed a graduation of the latest experience, relating to the years 1975–78.

Linked Life Assurances

From 1976 an investigation was started into the mortality experience under linked life assurances. Data for both male and female lives are submitted, and there is a select period of five years. Offices were asked to submit data for the medical and non-medical sections separately, with the option of submitting all their data as 'combined' if they could not separate the two sections; in the event most offices have taken up this option, so that there are very few policies recorded in the separate medical and non-medical sections. From 1982 the Irish offices are also contributing data under this class of assurance issued in the Republic of Ireland.

Joint-Life-First-Death

From 1983 a new investigation has been started into the mortality of lives assured under joint-life-first-death policies. Similar policies where one or both lives are impaired are included in the Impaired Assured Lives Investigation (see later), the data for which should thus be substantially increased. In order to keep an independent check that, when a claim occurs on a first death, the second life is removed from the in-force data, this new class has been limited to policies on one male and one female life, so that the in-force for any office at any time should consist of equal total numbers from each sex.

Whole Life and Endowment Assurances without selection

Also from 1983 a new investigation has been inaugurated into the mortality of lives accepted without any medical evidence whatsoever, under whole life and endowment assurances completed in connexion with mortgages. Such non-selection cases should not be confused with the non-medical data in the main investigation, where the proposal forms include certain questions of a medical nature.

Cause of Death

From 1964 the mortality experience of male lives under whole life and endowment assurances issued in the U.K. have been analysed according to cause of death. Following a discussion on an Institute paper in 1976, the causes of death among immediate annuitants and pensioners are also being investigated from 1979 to enable comparisons to be made between these classes and the lives covered by whole life and endowment assurances. To make these comparisons complete, causes of death are also being analysed in the experience of whole life and endowment assurances on female lives, also from 1979.

From 1982 the cause of death investigations are being extended to include temporary assurances, linked life assurances and assurances issued in the

Republic of Ireland, in order to have a complete 'control' for the Impaired Assured Lives Investigation described in the next section.

Impaired Assured Lives

From 1982 an investigation has been started into the mortality of assured lives known, at the time of effecting a policy, to be suffering from certain impairments. There is an extensive impairment coding list, each impairment being recorded at the outset as a separate investigation. It is expected that the volume of data relating to some of the impairments will be small, but until a number of years' statistics have been collected the Committee are unable to determine which impairments will need to be grouped to avoid fragmentation of the data. The classes of assurance included are the same as those in the main assured lives investigations.

Permanent Health Insurance

In 1970 a sub-committee was set up to plan and conduct an investigation into sickness rates under permanent health insurance policies. Data were collected from 1972, and separate investigations have been made into experiences under individual policies and group policies. Development is described in the relevant papers listed in the bibliography which follows.

CONFIDENTIALITY

It should be mentioned that strict anonymity of offices is preserved in all data and statistics. Each contributing office is allocated a number, and the name of the office does not appear on any data sheets.

GENERAL

The historical note in *C.M.I.R.* 1 included three Appendices. The first showed the Constitution and Rules of the C.M.I. Bureau; these are not repeated in the present paper as the only change has been the alteration of the financial year, which now runs from 1 March to 28 or 29 February. The second gave the Consolidated Rules of the investigations, as issued to offices; these are now in loose-leaf form which is not suitable for reprinting as an appendix, but anyone wishing for a copy may obtain one on request to the Bureau subject to payment; many items from the Rules have been incorporated in the above historical outline. The third appendix was a bibliography, to which an addendum is given below.

ADDENDUM TO BIBLIOGRAPHY

Considerations affecting the preparation of standard tables of mortality (*Joint Mortality Investigation Committee*, 1974) *J.I.A.* **101,** 133 & *T.F.A.* **34,** 135.

Correction to parameters for A 1967–70(5) (*A. D. Wilkie*) *J.I.A.* **102,** 215

Continuous Mortality Investigation Bureau Reports

	Period covered by report	*C.M.I.R.* Vol.	Page
Annuitants (Immediate)	1967–70	1	29
	" (graduation)	2	57
	Standard tables	3	1
	1971–74	3	53
	1975–78	5	19
	a(55) at high ages	5	47
Assured Lives			
Whole life and endowment assurances, males	1967–70	1	19
	1971–74	3	31
	1975–78	5	1
" " " " " females	1973–74	3	42
	1975–78	5	9
" " " " " males	1971–74	3	40
(Republic of Ireland)	1975–78	5	7
" " " " " males	1967–70	1	49
(Cause of death)	1971–74	3	77
	1975–78	5	37
Temporary assurances, males	1971–74	3	47
	1975–78	5	12
Linked contracts, males and females	1976–78	5	15
Pensioners	1967–70	1	35
	" (graduation)	2	57
	Standard tables	3	1
	1971–74	3	59
	1975–78	5	27
Retirement Annuities	1967–70	1	45
	1971–74	3	69
	1975–78	5	35
Sickness			
Individual policies	1971 and 1973	2	1
	1974 and 1975	3	91
	1972–75	4	1
Group policies	1973–76	5	51

ABRIDGED LIST OF DISEASES

(Reprinted with kind permission of the World Health Organization)

List of 70 Causes for Tabulation of Morbidity

	Cause Groups	*Detailed List Numbers*
C 1	Typhoid, paratyphoid fever, other salmonella infections	001-003
C 2	Bacillary dysentery and amoebiasis	004, 006
C 3	Enteritis and other diarrhoeal diseases	008,009
C 4	Tuberculosis of respiratory system	010-012
C 5	Other tuberculosis, including late effects	013-019
C 6	Brucellosis	023
C 7	Diphtheria	032
C 8	Whooping cough	033
C 9	Streptococcal sore throat and scarlet fever	034
C10	Smallpox	050
C11	Measles	055
C12	Viral encephalitis	062-065
C13	Infectious hepatitis	070
C14	Typhus and other rickettsioses	080-083
C15	Malaria	084
C16	Syphilis and its sequelae	090-097
C17	Gonococcal infections	098
C18	Helminthiases	120-129
C19	All other infective and parasitic diseases	Remainder of 000-136
C20	Malignant neoplasms, including neoplasms of lymphatic and haematopoietic tissue	140-209
C21	Benign neoplasms and neoplasms of unspecified nature	210-239
C22	Thyrotoxicosis with or without goitre	242
C23	Diabetes mellitus	250
C24	Avitaminoses and-other nutritional deficiency	260-269
C25	Other endocrine and metabolic diseases	240, 241 243-246 251-258 270-279
C26	Anaemias	280-285
C27	Psychoses and non psychotic mental disorders	290-309

	Cause Groups	*Detailed List Numbers*
C28	Inflammatory diseases of eye	360-369
C29	Cataract	374
C30	Otitis media and mastoiditis	381-383
C31	Other diseases of nervous system and sense organs	320-358 370-373 375-380 384-389
C32	Active rheumatic fever	390-392
C33	Chronic rheumatic heart disease	393-398
C34	Hypertensive disease	400-404
C35	Ischaemic heart disease	410-414
C36	Cerebrovascular disease	430-438
C37	Venous thrombosis and embolism	450-453
C38	Other diseases of circulatory system	420-429 440-448 454-458
C39	Acute respiratory infections	460-466
C40	Influenza	470-474
C41	Pneumonia	480-486
C42	Bronchitis, emphysema and asthma	490-493
C43	Hypertrophy of tonsils and adenoids	500
C44	Pneumoconioses and related diseases	515, 516
C45	Other diseases of respiratory system	501-514 517-519
C46	Diseases of teeth and supporting structures	520-525
C47	Peptic ulcer	531-533
C48	Appendicitis	540-543
C49	Intestinal obstruction and hernia	550-553 560
C50	Cholelithiasis and cholecystitis	574, 575
C51	Other diseases of digestive system	526-530 534-537 561-573 576, 577
C52	Nephritis and nephrosis	580-584
C53	Calculus of urinary system	592, 594
C54	Hyperplasia of prostate	600

	Cause Groups	Detailed List Numbers
C55	Other diseases of genito-urinary system	590, 591 593 595-599 601-629
C56	Abortion	640-645
C57	Other complications of pregnancy, childbirth and the puerperium	630-639 651-678
C58	Delivery without mention of complication	650
C59	Infections of skin and subcutaneous tissue	680-686
C60	Other diseases of skin and subcutaneous tissue	690-709
C61	Arthritis and spondylitis	710-715
C62	Other diseases of musculoskeletal system and connective tissue	716-738
C63	Congenital anomalies	740-759
C64	Certain causes of perinatal morbidity	760-779
C65	Other specified and ill-defined diseases	286-289 310-315 780-796
	External Cause of Injury	
CE66	Road transport accidents	E810-E819 E825-E827
CE67	All other accidents	E800-E807 E820-E823 E830-E949
CE68	Attempted suicide and self-inflicted injuries	E950-E959
CE69	Attempted homicide and injury purposely inflicted by other persons; legal intervention	E960-E978
CE70	All other external causes	E980-E999
	Nature of Injury	
CN66	Fractures	N800-N829
CN67	Intracranial and internal injuries	N850-N869
CN68	Burn	N940-N949
CN69	Adverse effects of chemical substances	N960-N989
CN70	All other injuries	N830-N848 N870-N939 N950-N959 N990-N999

ON A POSSIBLE GRADUATION OF THE IRISH ASSURED LIVES MORTALITY EXPERIENCE

1. The results of the investigations into the 1971–74 and 1975–78 experiences under policies issued in the Republic of Ireland, relating to whole-life and endowment assurances on male lives, were published in *C.M.I.R.*, **3**, 40 and *C.M.I.R.*, **5**, 7 respectively. This investigation has a much smaller exposed-to-risk than the corresponding investigation in the United Kingdom, and is rather smaller in size than the experience of female lives effecting whole-life and endowment assurances in the U.K., as the following table for combined medical and non-medical data shows:

Assured lives, 1975–78

	Irish lives (exposed-to-risk)	Irish lives (deaths)	Female lives (exposed-to-risk)	Female lives (deaths)
Duration 0	73,330	64	582,676	329
Duration 1	65,331	85	505,575	352
Durations 2 and over	879,382	4,467	2,080,947	4,666

2. The number of deaths at durations 0 and 1 is rather small to attempt an independent graduation, but the numbers in the ultimate section, at durations 2 and over, are quite sufficient for a formula graduation to be attempted. We therefore consider first the ultimate data.

3. Inspection of the crude rates of mortality at individual ages (see Table 1) shows rates of mortality that decline between age 17 and the mid-20s, rising in the usual exponential way thereafter, though levelling off in the manner of a logistic curve as the highest ages are reached. This general pattern is very similar to that of the A 1967–70 table. A comparison of the actual deaths and those expected according to this standard table, first shown in *C.M.I.R.*, **5**, 9 is repeated below:

Age group (nearest ages)	Actual deaths	100 A/E
Up to 25	43	154
26–30	56	100
31–35	88	97
36–40	128	88
41–45	260	99
46–50	398	92
51–55	781	120
56–60	884	101
61–65	857	97
66–70	336	96
71–75	221	98
76–80	216	118
81–85	138	113
86–90	48	101
91 and over	13	73
All ages	4,467	102

4. The level of mortality is thus very close to that of A 1967–70 overall, and not so very different at individual age groups. The first two formulae to try, therefore, were:

(i) $q_x = A + Bq_x^{\text{A 1967–70}}$ and
(ii) $q_x/p_x = Bc^{(x-Y)} - H(x-Y)$, the formula used for A 1967–70.

5. The graduated rates for A 1967–70 only apply to age 17 upwards. The trivial amount of data below age 18 nearest was therefore omitted from this graduation. The data that are gathered go up to age 100 nearest. When the A 1967–70 tables were constructed, the data in the 90s of age were so irregular that the graduated rates were based only on the data up to age 90 nearest. In this case, however, the data in the 90s are sparse (only 13 actual deaths) and show no obvious significant irregularities; in the circumstances it was worth including them in the graduation.

6. The results of applying formula (i) above, *i.e.* a linear function of A 1967–70, show no significant improvement over the use of A 1967–70 itself. The maximum likelihood estimates of A and B are $-\cdot00002490$ and $1\cdot0270901$, respectively, which are hardly different from 0 and 1. The log likelihood function is reduced by 1·22, at the expense of fitting two parameters, which is not a worth-while trade-off (one needs a reduction of 2 in the log likelihood function to justify an extra parameter). The value of χ^2 for data, grouped to ensure that the expected in each group is at least 5, is reduced from 174·7, with 71 degrees of freedom to 169·7 with 69 degrees of freedom, not a very worth-while change.

7. The value of χ^2 is far too high for one to accept the hypothesis that the

deaths are independently binomially distributed; this will be discussed further below. However, the test of runs of deviations of the same sign, and of the serial correlation coefficients of the standardized deviations show clearly that either the original A 1967–70 table, or the rates derived by formula (i) give a satisfactory fit to the data.

8. The maximum likelihood estimates of the parameters for formula (ii), compared with those for the A 1967–70 table, are shown below:

	Formula (ii)	A 1967–70
Y	24·113643	24·794921
$B \times 10^3$	·85416220	·70074652
c^5	1·5504987	1·5840485
$H \times 10^4$	1·2999562	·91650991

9. The reduction in the log likelihood function is 3·09, insufficient to justify the fitting of four parameters. The tests of runs and of the serial correlations are again quite satisfactory. The value of χ^2, with suitable groupings, is reduced from 174·7 to 161·9, with 67 degrees of freedom. This is an improvement, but the value of χ^2 is still far too high.

10. The values of $10^5 q_x$ are given in the table below, from which it can be seen that, over the main range of the table, there is not a great deal of difference between the mortality rates according to the different formulae. Details of the individual graduations are shown in Table 1.

Values of $10^5 q_x$

Age	A 1967–70	Formula (i)	Formula (ii)
20	89	89	113
25	69	69	81
30	65	65	67
35	86	85	80
40	144	146	137
45	264	268	261
50	479	489	488
55	844	865	874
60	1,443	1,480	1,500
65	2,403	2,466	2,489
70	3,911	4,014	4,017
75	6,229	6,395	6,325
80	9,703	9,963	9,721
85	14,727	15,124	14,552
90	21,651	22,235	21,121
95	30,593	31,419	29,540
100	41,229	42,343	39,557

Table 1. *Irish assured lives, 1975–78, durations 2 and over*

Age x	Exposed to risk ER_x	Actual deaths A_x	Crude $q_x = A_x/ER_x$	A 1967–70: Expected deaths E_x	A 1967–70: Deviations A_x-E_x	A 1967–70: $z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Formula (i): Expected deaths E_x	Formula (i): Deviations A_x-E_x	Formula (i): $z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Formula (ii): Expected deaths E_x	Formula (ii): Deviations A_x-E_x	Formula (ii): $z_x = \frac{A_x-E_x}{\sqrt{E_x(1-q_x)}}$	Age x
9·5	1·5	0	·0										
10·5	23·0	0	·0										
11·5	10·5	0	·0										
12·5	19·0	0	·0										
13·5	26·7	0	·0										
14·5	41·4	0	·0										
15·5	61·1	0	·0										
16·5	90·1	1	·01110										
17·5	270·2	0	·0	0·28	−0·28		0·28	−0·28		0·36	−0·36		17·5
18·5	668·8	2	·00299	0·65	1·35		0·65	1·35		0·84	1·16		18·5
19·5	1,523·3	0	·0	1·39	−1·39	2·09	1·39	−1·39	2·10	1·78	−1·78	1·09	19·5
20·5	3,024·8	7	·00231	2·62	4·38		2·61	4·39		3·30	3·70		20·5
21·5	4,714·8	6	·00127	3·86	2·14		3·85	2·15		4·80	1·20	1·50	21·5
22·5	6,543·1	10	·00153	5·08	4·92	2·18	5·06	4·94	2·20	6·22	3·78		22·5
23·5	8,481·9	2	·00024	6·28	−4·28	−1·71	6·24	−4·24	−1·70	7·54	−5·54	−2·02	23·5
24·5	10,537·2	15	·00142	7·47	7·53	2·76	7·41	7·59	2·79	8·77	6·23	2·10	24·5
25·5	12,579·3	4	·00032	8·59	−4·59	−1·57	8·51	−4·51	−1·55	9·86	−5·86	−1·87	25·5
26·5	14,810·7	4	·00027	9·82	−5·82	−1·86	9·72	−5·72	−1·83	10·99	−6·99	−2·11	26·5
27·5	17,177·0	9	·00052	11·17	−2·17	−0·65	11·05	−2·05	−0·62	12·18	−3·18	−0·91	27·5
28·5	19,539·6	11	·00056	12·61	−1·61	−0·45	12·47	−1·47	−0·42	13·37	−2·37	−0·65	28·5
29·5	21,672·6	28	·00129	14·06	13·94	3·72	13·90	14·10	3·78	14·51	13·49	3·54	29·5
30·5	23,378·8	18	·00077	15·46	2·54	0·65	15·29	2·71	0·69	15·55	2·45	0·62	30·5
31·5	24,523·4	14	·00057	16·76	−2·76	−0·68	16·61	−2·61	−0·64	16·48	−2·48	−0·61	31·5
32·5	25,326·8	17	·00067	18·16	−1·16	−0·27	18·02	−1·02	−0·24	17·52	−0·52	−0·12	32·5
33·5	25,528·2	26	·00102	19·46	6·54	1·48	19·36	6·64	1·51	18·51	7·49	1·74	33·5
34·5	25,560·4	13	·00051	20·99	−7·99	−1·74	20·92	−7·92	−1·73	19·77	−6·77	−1·52	34·5
35·5	25,681·5	20	·00078	22·96	−2·96	−0·62	22·95	−2·95	−0·62	21·52	−1·52	−0·33	35·5
36·5	25,871·0	22	·00085	25·44	−3·44	−0·68	25·48	−3·48	−0·69	23·82	−1·82	−0·37	36·5
37·5	26,145·1	27	·00103	28·49	−1·49	−0·28	28·61	−1·61	−0·30	26·73	0·27	0·05	37·5
38·5	26,292·7	28	·00106	31·94	−3·94	−0·70	32·15	−4·15	−0·73	30·12	−2·12	−0·39	38·5
39·5	26,417·7	31	·00117	35·96	−4·96	−0·83	36·27	−5·27	−0·88	34·13	−3·13	−0·54	39·5

Table 1 (*continued*)

Age x	Exposed to risk ER_x	Actual deaths A_x	Crude $q_x = A_x/ER_x$	A 1967–70 Expected deaths E_x	A 1967–70 Deviations $A_x - E_x$	A 1967–70 $z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Formula (i) Expected deaths E_x	Formula (i) Deviations $A_x - E_x$	Formula (i) $z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Formula (ii) Expected deaths E_x	Formula (ii) Deviations $A_x - E_x$	Formula (ii) $z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Age x
40·5	26,631·6	41	·00154	40·75	0·25	0·04	41·19	−0·19	−0·03	38·97	2·03	0·33	40·5
41·5	26,566·2	50	·00188	45·81	4·19	0·62	46·39	3·61	0·53	44·16	5·84	0·88	41·5
42·5	26,804·8	44	·00164	52·16	−8·16	−1·13	52·90	−8·90	−1·23	50·70	−6·70	−0·94	42·5
43·5	26,591·6	56	·00211	58·43	−2·43	−0·32	59·35	−3·35	−0·44	57·25	−1·25	−0·17	43·5
44·5	26,303·7	69	·00262	65·28	3·72	0·46	66·39	2·61	0·32	64·46	4·54	0·57	44·5
45·5	25,793·3	60	·00233	72·28	−12·28	−1·45	73·59	−13·59	−1·59	71·89	−11·89	−1·40	45·5
46·5	25,019·3	74	·00296	79·11	−5·11	−0·58	80·63	−6·63	−0·74	79·21	−5·21	−0·59	46·5
47·5	24,208·7	85	·00351	86·30	−1·30	−0·14	88·04	−3·04	−0·32	86·93	−1·93	−0·21	47·5
48·5	23,265·1	85	·00365	93·41	−8·41	−0·87	95·36	−10·36	−1·06	94·60	−9·60	−0·99	48·5
49·5	22,306·7	94	·00421	100·76	−6·76	−0·67	102·93	−8·93	−0·88	102·54	−8·54	−0·84	49·5
50·5	21,579·9	108	·00500	109·53	−1·53	−0·15	111·96	−3·96	−0·37	111·93	−3·93	−0·37	50·5
51·5	20,990·2	119	·00567	119·55	−0·55	−0·05	122·27	−3·27	−0·30	122·62	−3·62	−0·33	51·5
52·5	20,324·2	174	·00856	129·74	44·26	3·90	132·75	41·25	3·59	133·49	40·51	3·52	52·5
53·5	19,832·4	202	·01019	141·70	60·30	5·08	145·05	56·95	4·75	146·19	55·81	4·63	53·5
54·5	18,968·8	178	·00938	151·51	26·49	2·16	155·15	22·85	1·84	156·64	21·36	1·71	54·5
55·5	18,065·3	171	·00947	161·11	9·89	0·78	165·02	5·98	0·47	166·85	4·15	0·32	55·5
56·5	17,142·7	179	·01044	170·49	8·51	0·66	174·68	4·32	0·33	176·80	2·20	0·17	56·5
57·5	16,051·2	180	·01121	177·80	2·20	0·17	182·22	−2·22	−0·17	184·57	−4·57	−0·34	57·5
58·5	14,714·1	197	·01339	181·34	15·66	1·17	185·88	11·12	0·82	188·35	8·65	0·63	58·5
59·5	13,277·6	157	·01182	181·85	−24·85	−1·86	186·45	−29·45	−2·17	188·94	−31·94	−2·34	59·5
60·5	11,844·6	192	·01621	180·09	11·91	0·89	184·67	7·33	0·54	187·11	4·89	0·36	60·5
61·5	10,860·3	214	·01970	183·12	30·88	2·30	187·81	26·19	1·93	190·20	23·80	1·74	61·5
62·5	9,972·8	174	·01745	186·29	−12·29	−0·91	191·09	−17·09	−1·25	193·39	−19·39	−1·41	62·5
63·5	9,070·7	178	·01962	187·54	−9·54	−0·70	192·39	−14·39	−1·05	194·53	−16·53	−1·20	63·5
64·5	6,522·3	99	·01518	149·11	−50·11	−4·15	152·98	−53·98	−4·42	154·51	−55·51	−4·52	64·5
65·5	3,463·0	93	·02686	87·46	5·54	0·60	89·74	3·26	0·35	90·52	2·48	0·26	65·5
66·5	2,720·3	74	·02720	75·83	−1·83	−0·21	77·81	−3·81	−0·44	78·37	−4·37	−0·50	66·5
67·5	2,249·8	60	·02667	69·16	−9·16	−1·12	70·97	−10·97	−1·32	71·36	−11·36	−1·37	67·5
68·5	1,828·2	53	·02899	61·92	−8·92	−1·15	63·55	−10·55	−1·35	63·78	−10·78	−1·37	68·5
69·5	1,485·2	56	·03770	55·37	0·63	0·09	56·84	−0·84	−0·11	56·93	−0·93	−0·13	69·5
70·5	1,138·0	38	·03392	46·67	−8·67	−1·30	47·90	−9·90	−1·46	47·89	−9·89	−1·46	70·5
71·5	1,022·9	45	·04399	46·10	−1·10	−0·17	47·32	−2·32	−0·35	47·20	−2·20	−0·33	71·5
72·5	939·2	41	·04365	46·48	−5·48	−0·82	47·72	−6·72	−1·00	47·48	−6·48	−0·97	72·5
73·5	813·3	49	·06025	44·16	4·84	0·75	45·33	3·67	0·56	45·00	4·00	0·61	73·5
74·5	699·6	48	·06861	41·64	6·36	1·02	42·75	5·25	0·83	42·33	5·67	0·90	74·5

Table 1 (*continued*)

Age x	Exposed to risk ER_x	Actual deaths A_x	Crude $q_x = A_x/ER_x$	A 1967–70: Expected deaths E_x	A 1967–70: Deviations $A_x - E_x$	A 1967–70: $z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Formula (i): Expected deaths E_x	Formula (i): Deviations $A_x - E_x$	Formula (i): $z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Formula (ii): Expected deaths E_x	Formula (ii): Deviations $A_x - E_x$	Formula (ii): $z_x = \frac{A_x - E_x}{\sqrt{E_x(1-q_x)}}$	Age x
75·5	599·2	46	·07677	39·06	6·94	1·15	40·10	5·90	0·96	39·61	6·39	1·05	75·5
76·5	522·0	44	·08430	37·23	6·77	1·15	38·22	5·78	0·97	37·66	6·34	1·07	76·5
77·5	495·0	59	·11920	38·59	20·41	3·42	39·63	19·37	3·21	38·93	20·07	3·35	77·5
78·5	418·0	36	·08613	35·60	0·40	0·07	36·55	−0·55	−0·10	35·81	0·19	0·03	78·5
79·5	344·5	31	·08999	32·01	−1·01	−0·19	32·87	−1·87	−0·34	32·12	−1·12	−0·21	79·5
80·5	308·9	25	·08094	31·29	−6·29	−1·19	32·13	−7·13	−1·33	31·30	−6·30	−1·19	80·5
81·5	265·5	40	·15066	29·28	10·72	2·10	30·07	9·93	1·92	29·22	10·78	2·12	81·5
82·5	204·4	38	·18594	24·52	13·48	2·90	25·18	12·82	2·73	24·39	13·61	2·94	82·5
83·5	153·4	18	·11736	19·99	−1·99	−0·48	20·53	−2·53	−0·60	19·83	−1·83	−0·44	83·5
84·5	119·7	17	·14197	16·94	0·06	0·02	17·39	−0·39	−0·10	16·76	0·24	0·06	84·5
85·5	100·0	15	·15000	15·33	−0·33	−0·09	15·74	−0·74	−0·20	15·13	−0·13	−0·04	85·5
86·5	66·0	16	·24242	10·95	5·05	1·67	11·25	4·75	1·56	10·78	5·22	1·74	86·5
87·5	50·0	5	·10000	8·97	−3·97	−1·46	9·21	−4·21	−1·54	8·80	−3·80	−1·41	87·5
88·5	39·0	8	·20513	7·55	0·45	0·18	7·75	0·25	0·10	7·39	0·61	0·25	88·5
89·5	22·5	4	·17778	4·70	−0·70	−0·90	4·82	−0·82	−0·97	4·59	−0·59	−0·84	89·5
90·5	10·9	1	·09200	2·44	−1·44		2·51	−1·51		2·38	−1·38		90·5
91·5	10·9	1	·09200	2·62	−1·62	−1·69	2·69	−1·69	−1·75	2·55	−1·55	−1·63	91·5
92·5	10·8	1	·09302	2·78	−1·78		2·86	−1·86		2·70	−1·70		92·5
93·5	10·5	3	·28571	2·91	0·09	−0·03	2·99	0·01	−0·13	2·82	0·18	0·11	93·5
94·5	6·5	3	·46154	1·92	1·08		1·98	1·02		1·86	1·14		94·5
95·5	4·5	1	·22222	1·42	−0·42		1·46	−0·46		1·37	−0·37		95·5
96·5	3·5	1	·28571	1·18	−0·18		1·21	−0·21		1·13	−0·13		96·5
97·5	2·0	1	·50000	0·71	0·29		0·73	0·27		0·69	0·31		97·5
98·5	3·5	1	·28571	1·33	−0·33		1·36	−0·36		1·27	−0·27		98·5
99·5	1·5	0	·0	0·60	−0·60		0·62	−0·62		0·58	−0·58		99·5
Total 17·5–99·5:	879,109·1	4,466		4,369·23	96·77	$\chi^2 = 174·7$ (71 d.f.)	4,465·71	0·29	$\chi^2 = 169·72$ (71 d.f.)	4,465·99	0·01	$\chi^2 = 161·9$ (71 d.f.)	
Total All ages:	879,382·4	4,467											

11. Careful inspection of the values for individual ages in Table 1 shows that no reasonable formula is likely to produce substantially lower values of χ^2. The actual numbers of deaths at ages 29·5, 52·5, 53·5 and 77·5 are exceptionally high. It is possible that the numbers of duplicate cases (*i.e.* multiple policies on one life) are unusually high. The Committee has no evidence about the distribution of duplicates in the Irish experience, but the variance ratios used in the tests of the A 1967–70 graduation are not sufficient to account for these very large values. By contrast the actual number of deaths at age 64·5 is exceptionally low. At this age the exposed-to-risk is changing rapidly with age, presumably because of endowment assurances maturing at age 65, so the assumption that the exposed-to-risk is evenly spread over the year of age may not be valid.

12. These arguments, and the facts that the other graduation tests are satisfactory, and the two formula graduations give results very little better than those for A 1967–70 itself, indicate that the A 1967–70 table can be taken as a satisfactory representation of Irish assured lives mortality for 1975–78, at least for duration 2 and over.

13. We now turn to the select durations. At duration 0 there were only 64 deaths, all between the ages of 18 and 76 nearest. The actual number of deaths was 106% of those expected according to A 1967–70, duration 0, which were 60·4. The grouped χ^2 was 7·0, with 10 degrees of freedom, well within an acceptable range; the tests of runs and serial correlations were also quite satisfactory. There is therefore no reason to reject the hypothesis that A 1967–70 duration 0 mortality is a satisfactory fit to the Irish 1975–78 duration 0 experience.

14. At duration 1 the picture is similar. There were 85 deaths, between ages 18 and 76. This was 120% of those expected according to A 1967–70 duration 1, which were 70·7. However, the grouped χ^2 was 11·8 with 12 degrees of freedom, well within what might be expected, and the other tests were quite satisfactory. Again, there is no reason to reject the hypothesis that A 1967–70 duration 1 rates are a satisfactory fit to the corresponding Irish 1975–78 experience.

15. We therefore conclude that the A 1967–70 table is, for all durations, a satisfactory representation of the Irish assured lives experience 1975–78. There appears to be no case for preparing and publishing any other standard table. However, it should be remembered that the Irish experience applies to a period eight years later than that on which the U.K. A 1967–70 table was based. The report in *C.M.I.R.*, **5**, 1 shows that the corresponding U.K. experience for 1975–78 is lower than that for 1967–70. The Irish assured lives show higher mortality than those in the U.K. when the same time periods are compared.

16. Finally, it is of interest to observe that, as can be seen from the table in paragraph 1 above, the exposed-to-risk for the Irish experience is comparable with that of U.K. females at durations 2 and over, but is very much less at durations 0 and 1. Indeed, the figures for the early durations seem so low that one must suspect that the experience is declining in volume; alternatively, the U.K. females' experience is rising rapidly in volume. However, it seems more likely that the true position is that very many new policies in Ireland are being written as linked ones, and it will be interesting to see the first reports of the experience into Irish linked policies, which commenced in 1982.

NOTE ON DUPLICATES AMONG LINKED POLICIES

1. A new investigation was started in 1976 into the mortality experienced under linked life assurance contracts in the United Kingdom. The results of the first three years' experience, 1976–78, were published in *C.M.I.R.* **5**, 15 ('the original report'). Offices contributing data were asked also to repeat the data submitted for deaths, but excluding all policies except the first for each life at each separate duration. This note reports on the findings for the same period, 1976–78.

2. The instructions to offices were to exclude all policies except the first at each separate duration. Data are submitted for durations 0, 1, 2, 3, 4 and 5 and over separately. A life who had, for example, effected one policy a year for several years would be counted as 1 at each of these six durations, all policies except one at durations 5 and over being omitted. However, if the durations were aggregated to provide figures for '2 and over', duplicates between durations would not then be eliminated, and our specimen life would then count as 4 in the 2 and over experience. Thus, by adding together the numbers of deaths for durations 2, 3, 4 and 5 and over, not all duplicate policies on the same life would be eliminated. It is therefore necessary to consider each duration separately.

3. Although a few offices submit separate data for the medical and non-medical sections, the vast majority of the experience has been contributed on a combined basis only. The same is true for the analyses of deaths excluding duplicates. We therefore consider only the combined experience.

4. The exposed-to-risk at each duration, for males and females separately, together with the numbers of deaths (policies), are shown in the table below.

	Males		Females	
Duration	Exposed-to-risk	Deaths (Policies)	Exposed-to-risk	Deaths (Policies)
0	120,734·5	215	39,654·5	152
1	100,234	225	29,347	165
2	92,778·5	280	27,544·5	120
3	92,614	424	32,160	228
4	85,502	520	32,539	315
5 and over	132,713	735	38,762	312

5. The exposed-to-risk at durations 5 and over is hardly larger than that at each of the early durations, reflecting the very young duration of this whole class of business. Indeed for females the exposed-to-risk at duration 0 is greater than that for durations 5 and over.

6. Tables 1 and 2 show, for males and females respectively, for durations 0, 1, 2, 3, 4 and 5 and over in each case, the numbers of deaths in the basic investigation (Policies) and the numbers excluding duplicates (Lives), together with the ratio of Policies to Lives, P/L.

Table 1. *Linked Contracts,* 1976–78, *males; numbers of deaths in basic investigation (Policies) and excluding duplicates (Lives). Combined data*

Age group (nearest)	Duration 0 Deaths (Policies)	Duration 0 Deaths (Lives)	Duration 0 Ratio P/L	Duration 1 Deaths (Policies)	Duration 1 Deaths (Lives)	Duration 1 Ratio P/L
–25	15	12	1·25	14	9	1·56
26–30	14	7	2·00	11	8	1·38
31–35	11	9	1·22	14	10	1·40
36–40	11	10	1·10	10	9	1·11
41–45	16	9	1·78	16	12	1·33
46–50	7	7	1·00	16	11	1·45
51–55	17	16	1·06	18	15	1·20
56–60	7	4	1·75	19	12	1·58
61–65	49	19	2·58	19	14	1·36
66–70	13	7	1·86	27	15	1·80
71–75	21	10	2·10	8	8	1·00
76–80	15	11	1·36	27	17	1·59
81–	19	9	2·11	26	14	1·86
–30	29	19	1·53	25	17	1·47
31–45	38	28	1·36	40	31	1·29
46–60	31	27	1·15	53	38	1·39
61–75	83	36	2·31	54	37	1·46
76–	34	20	1·70	53	31	1·71
All ages	215	130	1·65	225	154	1·46

Table 1 (*continued*)

Age group (nearest)	Duration 2 Deaths (Policies)	Deaths (Lives)	Ratio P/L	Duration 3 Deaths (Policies)	Deaths (Lives)	Ratio P/L
–25	10	4	2·50	10	7	1·43
26–30	7	7	1·00	9	7	1·29
31–35	7	3	2·33	12	7	1·71
36–40	13	8	1·62	14	10	1·40
41–45	12	11	1·09	21	16	1·31
46–50	18	15	1·20	20	14	1·43
51–55	29	26	1·12	46	35	1·31
56–60	23	22	1·05	28	27	1·04
61–65	34	29	1·17	52	47	1·11
66–70	31	27	1·15	68	54	1·26
71–75	50	28	1·79	50	47	1·06
76–80	18	17	1·06	42	31	1·35
81–85	22	14	1·57	38	33	1·15
86–	6	6	1·00	14	14	1·00
–30	17	11	1·55	19	14	1·36
31–45	32	22	1·45	47	33	1·42
46–60	70	63	1·11	94	76	1·24
61–75	115	84	1·37	170	148	1·15
76–	46	37	1·24	94	78	1·21
All ages	280	217	1·29	424	349	1·21

Table 1 (*continued*)

Age group (nearest)	Duration 4			Durations 5 and over		
	Deaths (Policies)	Deaths (Lives)	Ratio P/L	Deaths (Policies)	Deaths (Lives)	Ratio P/L
–30	15	9	1·67	8	3	2·67
31–35	7	5	1·40	12	5	2·40
36–40	7	4	1·75	24	10	2·40
41–45	23	17	1·35	26	15	1·73
46–50	25	17	1·47	58	25	2·32
51–55	24	20	1·20	69	36	1·92
56–60	47	40	1·18	80	46	1·74
61–65	63	59	1·07	70	56	1·25
66–70	115	111	1·04	118	110	1·07
71–75	89	82	1·09	123	116	1·06
76–80	45	43	1·05	74	69	1·07
81–85	37	34	1·09	43	43	1·00
86–90	17	17	1·00	22	21	1·05
91–	6	6	1·00	8	7	1·14
–30	15	9	1·67	8	3	2·67
31–45	37	26	1·42	62	30	2·07
46–60	96	77	1·25	207	107	1·93
61–75	267	252	1·06	311	282	1·10
76–	105	100	1·05	147	140	1·05
All ages	520	464	1·12	735	562	1·31

Table 2. *Linked Contracts, 1976–78, females; numbers of deaths in basic investigation (Policies) and excluding duplicates (Lives). Combined data*

Age group (nearest)	Duration 0			Duration 1		
	Deaths (Policies)	Deaths (Lives)	Ratio P/L	Deaths (Policies)	Deaths (Lives)	Ratio P/L
–25	3	3	1·00	1	1	1·00
26–30	0	0	—	2	2	1·00
31–35	2	2	1·00	0	0	—
36–40	2	2	1·00	2	2	1·00
41–45	0	0	—	1	1	1·00
46–50	1	1	1·00	3	3	1·00
51–55	1	1	1·00	3	3	1·00
56–60	3	2	1·50	11	3	3·67
61–65	5	5	1·00	9	8	1·12
66–70	9	6	1·50	6	6	1·00
71–75	13	7	1·86	28	11	2·55
76–80	50	18	2·78	30	16	1·88
81–85	39	18	2·17	35	17	2·06
86–	24	15	1·60	34	17	2·00
–30	3	3	1·00	3	3	1·00
31–45	4	4	1·00	3	3	1·00
46–60	5	4	1·25	17	9	1·89
61–75	27	18	1·50	43	25	1·72
76–	113	51	2·22	99	50	1·98
All ages	152	80	1·90	165	90	1·83

Table 2 (*continued*)

Age group (nearest)	Duration 2 Deaths (Policies)	Duration 2 Deaths (Lives)	Duration 2 Ratio P/L	Duration 3 Deaths (Policies)	Duration 3 Deaths (Lives)	Duration 3 Ratio P/L
–30	3	3	1·00	5	4	1·25
31–35	0	0	—	1	1	1·00
36–40	3	3	1·00	0	0	—
41–45	1	1	1·00	3	3	1·00
46–50	3	3	1·00	6	4	1·50
51–55	3	3	1·00	11	8	1·38
56–60	3	3	1·00	14	12	1·17
61–65	6	5	1·20	24	22	1·09
66–70	17	10	1·70	23	21	1·10
71–75	14	13	1·08	38	35	1·09
76–80	23	17	1·35	36	34	1·06
81–85	20	16	1·25	36	34	1·06
86–90	11	9	1·22	26	25	1·04
91–	13	6	2·17	5	4	1·25
–30	3	3	1·00	5	4	1·25
31–45	4	4	1·00	4	4	1·00
46–60	9	9	1·00	31	24	1·29
61–75	37	28	1·32	85	78	1·09
76–	67	48	1·40	103	97	1·06
All ages	120	92	1·30	228	207	1·10

Table 2 (*continued*)

Age group (nearest)	Duration 4			Durations 5 and over		
	Deaths (Policies)	Deaths (Lives)	Ratio P/L	Deaths (Policies)	Deaths (Lives)	Ratio P/L
–30	3	3	1·00	0	0	—
31–35	1	1	1·00	2	1	2·00
36–40	0	0	—	1	1	1·00
41–45	4	4	1·00	4	4	1·00
46–50	3	3	1·00	6	5	1·20
51–55	12	12	1·00	16	14	1·14
56–60	13	13	1·00	12	12	1·00
61–65	30	28	1·07	34	29	1·17
66–70	43	41	1·05	38	37	1·03
71–75	58	57	1·02	60	55	1·09
76–80	57	53	1·08	47	44	1·07
81–85	42	41	1·02	47	46	1·02
86–90	33	31	1·06	30	29	1·03
91–	16	15	1·07	15	15	1·00
–30	3	3	1·00	0	0	—
31–45	5	5	1·00	7	6	1·17
46–60	28	28	1·00	34	31	1·10
61–75	131	126	1·04	132	121	1·09
76–	148	140	1·06	139	134	1·04
All ages	315	302	1·04	312	292	1·07

The table below gives the totals

	Males			Females		
Duration	Deaths (Policies)	Deaths (Lives)	Ratio P/L	Deaths (Policies)	Deaths (Lives)	Ratio P/L)
0	215	130	1·65	152	80	1·90
1	225	154	1·46	165	90	1·83
2	280	217	1·29	120	92	1·30
3	424	349	1·21	228	207	1·10
4	520	464	1·12	315	302	1·04
5 and over	735	562	1·31	312	292	1·07
Total	2,399	1,876	1·28	1,292	1,063	1·22

7. The overall ratio for males, at 1·28 policies per life, is somewhat higher than that for females, at 1·22. But the female ratio is higher than the male at early durations, falling below it to only a little above 1·0 at the later durations. For both sexes the ratios fall with increasing duration, and this effect is stronger for females than for males. These, if not chance results, imply that multiple policies are being increasingly sold, and to a greater extent to females than to males. But it could also indicate an increasing tendency for offices not to eliminate simultaneous duplicates. On the other hand, the figures for individual ages do not suggest that, for example, multiple sets of 20 or so policies are being incorrectly included in the data with a count of 20. The excess policies are typically in quite small numbers.

8. For both sexes at most durations there is some tendency for the ratios to rise with age, though the number of deaths among females at young ages is very small. In fact for females there is an exact match between Policies and Lives at the majority of individual ages up to about age 55. However, for males for the higher durations the ratios tend to decrease with age.

9. In the original report it was noted that: 'The male ratios of actual to expected deaths at duration 0 fluctuated considerably from age to age, with the overall actual deaths slightly in excess of the expected, and with an unfavourable experience at the oldest ages. Similarly at duration 1, where the total actual deaths were slightly below the expected, the most unfavourable part of the experience was above age 75. . . . The female results were similar, with very high ratios at durations 0 and 1 above age 75.'

10. Expected deaths were calculated according to A1967–70 select (−4 years for females), and the results for durations 2 and over, both combined and for individual durations, show the actual deaths some 70% or so of expected, which is considerably lighter than the experience for the main assured lives investigation (whole-life and endowment assurances) for 1975–78. The unusual feature of this experience was the presence of high actual deaths at early durations for both sexes. The ratios are summarized in the Table below.

	Males			Females		
Duration	Actual deaths (Policies)	Expected deaths	100 A/E	Actual deaths (Policies)	Expected deaths	100 A/E
0	215	210·3	102	152	110·0	138
1	225	227·5	99	165	107·9	153
2	280	387·0	72	120	205·6	58
3	424	559·6	76	228	330·0	69
4	520	684·2	76	315	408·9	77
5 and over	735	1,052·3	70	312	505·3	62
2 and over	1,959	2,683·1	73	975	1,449·9	67

11. One of the purposes of obtaining data about duplicate policies was to discover whether the high ratios at early durations, and in particular at certain single ages, were attributable to an unusually large number of duplicate policies among the deaths. Inspection of the results for individual ages shows that this is at least a partial cause, though not necessarily the whole one.

12. We can replace the actual deaths counted by Policies (now denoted A, and the same as P above and in Tables 1 and 2) by the actual deaths counted by Lives (similarly denoted B or L); it is also appropriate to reduce the expected number of deaths by Policies (E) to give an adjusted expected by Lives (F). We do not know whether the distribution of duplicates among the exposed-to-risk is the same as the distribution among the deaths, though there is no reason to suppose a regular bias in either direction. Chance, however, may have resulted in an unduly high proportion of the deaths having duplicate policies, though the fact that somewhat similar ratios are found for both sexes makes this doubtful. The best we can do with the available data is to assume that the average number of Policies per Life among the exposed-to-risk is the same at each age as the overall ratio within one sex and duration category. The adjusted expected deaths (F) have therefore been calculated by dividing the original expected (E) by the overall ratios for each sex and duration.

13. The results for selected single ages and quinary age groups are shown in Table 3, which selects each age or age group where the number of expected deaths (E) was at least 2, and the z score (i.e. $(\text{A}-\text{E})/\sqrt{\text{ER}qp}$) is greater than 3·0. It can be seen that the ratios 100 B/F are in many cases much smaller than the original ratios 100 A/E. The z score is also in fact reduced in many cases. However, there are still quite a number of age groups, particularly among females duration 0, where the ratio 100 B/F remains large, and indeed the z score would be also high. Whenever the ratio of Policies to Lives for a particular age is lower than the average for the particular sex and duration category, the ratio of actual to expected is increased by the adjustments. It will be noted that no single ages or groups with unusually high z scores occurred at durations 3, 4 or 5 and over for either sex.

14. Not noted in Table 3 are ages or age groups where the ratio of Policies to

Table 3. *Linked Contracts,* 1976–78; *adjusted Actual and Expected for selected ages and age groups*

Age group (nearest)	Actual Deaths Policies (A)	Expected Deaths Policies (E)	100 A/E	Actual Deaths Lives (B)	Adjusted Expected Deaths (F)	100 B/F
Males, Duration 0 (F = E/1·65)						
62	19	5·89	323	5	3·57	140
63	17	5·11	332	4	3·10	129
72	10	3·13	319	3	1·90	158
61–65	49	28·05	175	19	17·00	112
81–85	18	6·10	295	8	3·70	216
Males, Duration 1 (F = E/1·46)						
77	11	2·84	387	9	1·95	463
78	13	2·81	462	5	1·92	260
76–80	27	10·63	254	17	7·28	233
86–90	18	4·02	448	7	2·75	254
Males, Duration 2 (F = E/1·29)						
72	25	9·64	259	6	7·47	80
Females, Duration 0 (F = E/1·90)						
77	8	2·69	297	4	1·42	282
79	17	3·16	538	4	1·66	241
80	16	2·37	673	6	1·25	481
83	12	2·41	498	6	1·27	473
76–80	50	13·35	375	18	7·00	257
81–85	39	10·25	380	18	5·39	334
86–	24	7·24	331	15	3·81	394
Females, Duration 1 (F = E/1·83)						
74	10	3·03	330	3	1·66	181
76	10	2·30	435	6	1·26	477
78	8	2·91	275	3	1·59	189
80	8	2·73	292	3	1·49	201
81	9	2·45	366	3	1·34	224
71–75	28	13·43	208	11	7·34	150
76–80	30	13·22	227	16	7·22	221
81–85	35	10·60	330	17	5·79	293
86–	34	9·30	366	17	5·08	334

Lives is high, but the ratio of Actual to Expected deaths is low. For example, among males, duration 0, ages 66–70 there were 13 actual deaths compared with an expected of 25·11; the 13 Policies came from only seven Lives, which should be compared with an adjusted expected of 15·22. This is still a very low rate.

15. The original report, referring to the males, pointed out that '... at durations 2 and over the actual deaths were below the expected throughout, except for ages 21–25': for this particular group there were 25 actual deaths (Policies), compared with 18·60 expected, giving a ratio 100 A/E of 134. These 25 Policy deaths came fom 13 Lives; when the expected deaths are reduced by the ratio 1/1·23, the ratio applicable to all ages for males, duration 2 and over, we get an adjusted expected of 15·12. Thus the numbers of deaths appear not unduly high. It can be seen from Table 1 that these excess policies were spread over all durations from 2 upwards.

16. The Committee's only previous investigation into duplicate policies was published in *J.I.A.* **83,** 34, and *T.F.A.* **24,** 94, 'Memorandum on a Special Inquiry into the Distribution of Duplicate Policies', which related to policies becoming claims by death during 1954, for all policies at duration 3 and over (at that time only males). The average number of policies per life was about 1·2 overall, the ratio showing a peak of about 1·3 around age 50. The results from this investigation suggest that the average number of policies per life among linked contracts is somewhat higher and may be increasing.

17. The Committee wishes to continue the additional investigation into policies excluding duplicates, in particular to see whether the unfavourable experience at high ages at low durations is a continuing feature of the investigation and, if so, whether it can be explained by a continuing high ratio of duplicate policies.

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