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Income Protection Committee

WORKING PAPER 47

The Graduation of Sickness Rates for the CMI Individual Income Protection Experience for 1991-98 of Males, Occupation Class 1

This paper forms part of a series of papers which are summarised in CMI Working Paper 48: "*An overview of the Graduation of Sickness Inception and Termination Rates for the CMI Individual Income Protection Experience for 1991-98 of Males, Occupation Class 1*".

It is recommended that Working Paper 48 is read before this paper.

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for the CMI Individual Income Protection Experience for 1991-98
of Males, Occupation Class 1

1. INTRODUCTION

1.1. This paper forms part of a series of papers covering the graduation of the CMI Individual Income Protection (IP) experience, of Males in Occupation Class 1, for the period 1991-98. A separate (and much shorter) paper [CMI Working Paper 48: "*An overview of the Graduation of Sickness Inception and Termination Rates for the CMI Individual Income Protection Experience for 1991-98 of Males, Occupation Class 1*"] has been issued alongside this paper to provide an overview of the whole series of papers. It is recommended that Working Paper 48 is read before this paper; in particular this paper does not contain an Executive Summary – this is effectively provided by Working Paper 48.

1.2. In CMI Working Paper 5, "*The Graduation of Claim Recovery and Mortality Intensities for the Individual Income Protection Experience for 1991-98 of Males, Occupation Class 1*", the graduation of rates for Terminations (recoveries and deaths) based on the experience for 1991 to 1998 for Males, Occupation Class 1 (OC1), was described. We now turn to the graduation of Sickness rates (from which we can derive Claim Inception rates) for the corresponding experience.

1.3. This investigation has taken very much longer than we would have liked, in part because the method used is complex and the calculations needed careful checking at each stage, but mainly because of discrepancies that appeared in the first results, both internally and in comparison with previous investigations of this data. We therefore had to reconsider a number of aspects, which are described in full in two separate papers:

- "*Note on Exclusions and some other features of the Claims data*"
  (Part A of CMI Working Paper 46, which we will refer to as the "Exclusions Note"); and
- "*The Identification of Duplicates*"
  (Part B of CMI Working Paper 46, which we will refer to as the "Duplicates Note").

In particular we have redefined the way in which "duplicate claims" are identified, introduced an analysis of cases with Deferred Periods 0, 2 and 8 weeks, and reviewed carefully the way in which Claim records that appear to be not wholly consistent are dealt with.

1.4. This paper is lengthy because we explain fully exactly how the calculations have been done. The structure of the paper is described in the next five paragraphs. Readers who are interested only in the results can skip directly to Section 7.8.

1.5. The framework for the investigation is covered in Sections 2 and 3:

- In Section 2 we set out an overview of the data used for the investigation.
- In Section 3 we summarise the general principles we have adopted.

1.6. The subsequent three Sections present the detail of the calculations to determine the number of Claim Inceptions, and the corresponding exposed to risk measure, for each cell in the data:

- In Section 4 we set out the calculation of the number of Claim Inceptions, both with and without Duplicates.

- In Section 5 we explain a number of auxiliary functions, which are used in the calculation of the exposed to risk, and calculate the required values. These functions all relate to the early durations of Sickness, during the Deferred Period and any run-in period.
- In Section 6 we set out the calculation of the exposed to risk. This calculation is more complex than for a mortality investigation as we need to restrict the exposure to 'healthy' lives by removing any time spent during the Investigation period as Sick, whether claiming IP benefits or not (for example, during the Deferred Period).

1.7. The graduation methodology and results, for Males in Occupation Class 1, are then set out in Section 7. We refer to the new graduations, of both Inceptions and Terminations, as IPM 1991-98.

1.8. We present some comparisons in Sections 8 and 9.
- In Section 8 we compare the IPM 1991-98 graduated rates of Sickness and of Claim Inceptions against the corresponding rates in SM1975-78 (the previous set of graduated CMI Individual IP experience, based on data for 1975-78 and presented in CMIR 12).
- In Section 9 we illustrate the effect of the changes in methodology introduced for this Investigation, by comparing, for each of the quadrennia 1991-94, 1995-98 and 1999-2002, high-level results (counts of Inceptions and 100 × *A*/*E* using SM1975-78) based on the revised methodology against those previously published (in CMIRs 18, 20 and 22 respectively).

1.9. Finally, in Section 10, we examine the Claim Inceptions experience of other Occupation Classes, and of Females, for 1991-98, by comparing actual Inceptions against those expected on the basis of IPM 1991-98.

1.10. The CMI IP Committee intends next to produce an analysis of the experience for 1999-2006, calculating expected Claim Inceptions and Terminations on the basis of the IPM 1991-98 graduations, for publication as a CMI Working Paper in the Autumn of 2010.

1.11. No formal consultation process is planned on these graduations, and no specific questions are posed within this paper, but the Committee would be pleased to receive any comments on these graduations. The Committee intends to propose the IPM 1991-98 graduations for adoption by the UK Actuarial Profession in due course, but will not do so until after the publication of the next Working Paper, comparing the 1999-2006 experience with the new graduations. The Committee would be pleased to receive any comments by 30 November 2010.

2. THE DATA

2.1. As for the graduated Claim Termination rates, we use the Individual IP business experience for the years 1991 to 1998. However, as further data is now available we also include some statistics for the years 1999-2002 in this Section, and some high-level results for the same additional period in Section 9. Further, we plan to investigate the experience of the years 1999-2006 in comparison with our new Claim Inception and Termination rates in a separate paper to follow this one.

2.2. We have records for the "In force" at the beginning and end of each of the twelve years 1991 to 2002. Sometimes the ending In force for one year is identical to the beginning In force for the following year, but in some years it is not. This is because offices may enter or leave the investigation. Even if the overall number of cases in force is the same, the details may have changed, for example because an office may have started coding occupation more fully, so that the details for the start of the year differ from those at the end of the previous year.

2.3. We also have records of Claims for each of the years 1991 to 2002. We have these in two forms: a complete set, which includes many Claims identified as "duplicate Claims", and a subset which keeps only one of each set of apparent Duplicates. We explain in the Duplicates Note exactly how Duplicates cases have been defined. We need to use both of these sets of Claim data, because the In force includes Duplicates (we have no way of identifying them and excluding them) and yet we wish to analyse Claim Inceptions excluding Duplicates. We refer to the two sets as the "cumD" and "exD" sets. We also exclude certain other Claim records, including False One-day Claims, and do this in a somewhat different way than previously, as explained in the Exclusions Note. There are relatively small numbers of these.

2.4. We note in the Exclusions Note that we have identified new Deferred Periods DP0, DP2 and DP8 separately. We give some statistics for these, but have deferred any further analysis of their experience. We also now omit cases with Deferred Periods which are not given as 0, 1, 2, 4, 8, 13, 26, or 52 weeks or "one month" (code 999); these are denoted as cases with "Odd Deferred Periods".

2.5. We wish to graduate Sickness rates, yet we have no record of all Sicknesses. We know only about those Sicknesses that have given rise to a Claim. We call the commencement of a Claim an "Inception"; note however, that it is always a Claim Inception, not the beginning of a period of Sickness. Our definition of an Inception is discussed further in the Exclusions Note.

2.6. Also as for the graduation of Claim Termination rates, we restrict ourselves to the Standard* (or Standard Star) data. The Standard* data is smaller than the total (Aggregate) data because of the exclusion of non-UK cases, special benefit types and policies with identifiable underwriting exclusions. These datasets are defined in the Exclusions note,

2.7. In due course we use the data for Males, Occupation Class 1 (OC1), for the graduation of Sickness rates, but in the meantime we analyse the data for both Males and Females and for all Occupation Classes (1, 2, 3 and 4 and also "5" or "unclassified"). The data in Occupation Classes 1 to 4 that we can use is less than that used for the analysis of Terminations, because some offices are able to classify Claims by Occupation Class, but are not able to classify the In force similarly. The Claims data for such offices is identified, so that it can be included as Class 1 or 2 or 3 or 4 in the Terminations analysis, but excluded from that Class and included in Class 5 (unclassified) for the Inceptions analysis (to match the classification of the corresponding In force). A few offices (small ones) are unable to supply In force data at all, although they do supply data for Claims, so their Claims are omitted entirely from this Claim Inceptions investigation.

2.8. We base the graduated Sickness rates on this data for Males, Occupation Class 1. However, we compare the experience for other Occupation Classes for Males and for all Occupation Classes for Females, in Section 10.

2.9. In Table 2.1 we show the overall number of eligible cases (Standard*, both Sexes, Occupation Classes 1 to 5) in the In force files and the numbers of Claims, and of Claim Inceptions, in both files of Claims for each Year. In Table 2.2 we show the corresponding numbers for Males OC1. As noted above, besides restricting the Claims to Standard* cases, we exclude those for which no In force records are available, those with Odd Deferred Periods, and False One-day Claims.

2.10. We can see directly from Tables 2.1 and 2.2 that:

(a) the number in force at the start of one year is the same as the number at the end of the previous year for 1993/94, 1995/96, 2000/01 and 2001/02; this is true in both tables;
(b) the total number in force fell for the first few years, and has since risen, especially in the most recent years shown; by contrast the number of Males OC1 has risen fairly steadily; this is because the proportion of cases with the Occupation Class coded has risen substantially;
(c) the number of Claims, both cumD and exD, has behaved similarly to the In force;
(d) the number of Inceptions has generally fallen, in both tables;
(e) the number in force at the end of each year is quite close to the number at the start of each year; the decline and later growth noted in (b) has mostly been over year-ends, indicating that it is mainly because of changes in the offices participating.

2.11. Calculation of various ratios of the data in Tables 2.1 and 2.2 and for “the Rest”, i.e. Table 2.1 minus Table 2.2, shows further:

(a) the ratio of numbers cumD to numbers exD, both for Claims (which include continuation Claims) and for Inceptions, has remained fairly similar over the years, but it is much higher for Males OC1 than for the Rest and is rather higher for Inceptions than for Claims;
(b) the number of Inceptions as a percentage of the total number of Claims has fallen, both for the total, for Males OC1 and for the Rest; further, that percentage is higher for Males OC1 than for the Rest;
(c) the crude Inception rate based on cumD Inceptions only, divided by the mean In force, has fallen substantially over the years;
(d) the crude Inception rate is very much higher for Males OC1 than for the Rest.

2.12. These last three observations may be misleading as they stand. The crude Inception rate is very much higher for policies with Deferred Period 1 week (DP1) than for policies with longer Deferred Periods (DP4, DP13, DP26 or DP52). DP1 consists almost entirely of Occupation Class 1 cases and has quite few of the other Occupation Classes. And

DP1 has been falling as a proportion of the total over the years. In combination these could explain observations (b) to (d) in paragraph 2.11.

Table 2.1: Numbers of eligible cases, in the In force and Claims files, for each year 1991-2002 for Standard*, both Sexes, Occupation Classes 1 to 5, excluding Odd Deferred Periods.

Year	In force at start	In force at end	Claims cumD	Inceptions cumD	Claims exD	Inceptions exD
1991	414,081	419,356	14,810	7,564	10,332	4,820
1992	416,735	408,918	14,982	6,866	10,626	4,438
1993	374,926	367,964	14,082	6,553	9,423	3,929
1994	367,964	366,914	13,622	5,645	9,364	3,471
1995	387,041	388,731	14,058	5,985	9,595	3,594
1996	388,731	391,919	14,079	5,302	9,552	3,157
1997	443,625	453,200	15,388	5,278	10,951	3,318
1998	538,149	552,876	16,284	5,530	11,882	3,586
1999	635,063	654,202	17,712	5,404	13,544	3,591
2000	609,015	630,376	17,597	5,260	13,410	3,482
2001	630,376	649,637	17,829	4,827	13,707	3,225
2002	649,637	661,102	18,085	4,612	14,083	3,254
Total	5,855,343	5,945,195	188,528	68,826	136,469	43,865

Table 2.2: Numbers of eligible cases, in the In force and Claims files, for each year 1991-2002 for Standard*, Males, Occupation Class 1, excluding Odd Deferred Periods.

Year	In force at start	In force at end	Claims cumD	Inceptions cumD	Claims exD	Inceptions exD
1991	129,693	128,075	7,584	4,435	3,693	1,979
1992	147,485	143,997	7,716	3,981	3,965	1,825
1993	160,135	156,215	8,279	4,392	4,217	2,043
1994	156,215	154,656	7,689	3,703	4,021	1,750
1995	182,633	181,833	8,507	4,182	4,639	2,047
1996	181,833	182,103	8,545	3,702	4,612	1,756
1997	195,723	196,351	8,621	3,444	4,827	1,721
1998	230,828	235,810	8,712	3,366	4,958	1,656
1999	282,700	283,265	8,896	3,288	5,424	1,708
2000	260,666	264,075	8,704	3,129	5,252	1,561
2001	264,075	262,718	8,517	2,822	5,179	1,433
2002	262,718	262,593	8,394	2,514	5,239	1,347
Total	2,454,704	2,451,691	100,164	42,958	56,026	20,826

3. PRINCIPLES

3.1. We start by describing the general principles we have adopted. Imagine a group of individuals who are exposed to the risk of becoming Sick for R years (or other units of time) in total; they are subject to a continuous transition intensity (force) of becoming Sick of σ per year (regardless of age); under suitable assumptions [see reference below *] the number of Sicknesses is Poisson distributed with parameter $R.\sigma$, so the expected number of Sicknesses is also $R.\sigma$. If Sicknesses are not observed until a Claim starts at the end of the Deferred Period (if the Sick person remains Sick for that long), the probability of a Sickness continuing to the end of the Deferred Period is π, and the probability that a Claim is then made is η, then the number of Claim Inceptions is Poisson distributed with parameter $R.\sigma.\pi.\eta$, and the expected number of Inceptions is also $R.\sigma.\pi.\eta$. If R, π and η are known and the observed number of Inceptions is I, then the maximum likelihood estimator of σ is $I/(R.\pi.\eta)$. This is just like the way we analyse a mortality investigation, except that the exposed to risk, R is reduced by the factor $\pi.\eta$.

[* See CMIR 12, Part A, paragraph 4.4; or refer to Macdonald, A S (1996) "*An Actuarial Survey of Statistical Models for Decrement and Transition Data I: Multiple State, Poisson and Binomial Models.*" (BAJ, Vol. 2, Part 1, No. 6, pages 129-155, particularly sections 3.5, 3.6 and 4.3).]

3.2. In practice we assume that σ is a function of age, x, i.e. $\sigma(x)$. We assume that π and η are also functions of age, x, and also of the Deferred Period, d, i.e. $\pi(x,d)$ and $\eta(x,d)$. We then subdivide the data into separate Deferred Periods (since we expect that σ will turn out to be a function of Deferred Period as well as of age) and then into small age intervals (in practice years of age), so that we can assume that σ, π and η are nearly constant over that age interval. In practice we subdivide data into years of age, defined as age last birthday, and thus from exact age x to exact age x+1. We then use $\sigma(x+½)$, $\pi(x+½,d)$ and $\eta(x+½,d)$ as approximations to the constant values over the year of age.

3.3. We have chosen to define all the data by age last birthday (or our best estimate of that) at the date the Sickness commences. Thus we classify Inceptions by age last birthday at Commencement of Sickness, and we start our calculation of the exposed to risk by taking the In force policies aged x last birthday at the beginning and end of each year. This is different from the previous method used by the IP Committee, as we discuss in paragraph 6.1.4.

3.4. For the insured to count as being exposed to the risk of falling Sick in a particular period, he or she must be healthy at the outset of that period. However, the data we have available for the In force includes all policies, and therefore includes those who were claiming and also those who may have been Sick but were not (or not yet) claiming (for example, those in a Deferred Period). We can count the days spent claiming (described in Section 6.2) and we need to estimate the period Sick but not yet claiming (described in Section 6.3). For certain Deferred Periods (DP4 and DP13 on this occasion, but including DP26 previously) there is evidence that the recovery rates in the first four (or for DP4 perhaps twelve) weeks from the end of the Deferred Period are unreasonably low, and we follow the assumption made previously by the IP Committee (see CMIR 12, Part B, Section 3.3) that many insured whose Sickness extends only a short way beyond the Deferred Period do not bother to make a Claim. We need to include two adjustments for this, one affecting the duration of exposures (because we assume that some insured are Sick during the four-week "run-in" period as well as during the Deferred Period) and one allowing for the proportion of unrecorded Claims (the complement of the η factors mentioned above). We describe all these calculations in Section 6.3.

3.5. The Poisson model we described in paragraph 3.1 assumes that each individual at risk is independent of the others. This would not be true if we included duplicate Claims. We therefore base the counts of Inceptions on the exD Claims files, that is with Duplicates removed. However, we have the In force recorded only with Duplicates included, so further adjustments are necessary, as described in Section 6.4.

3.6. We subdivide all the data by Sex (Males and Females), by Occupation Class (OCs 1 to 5) and by Deferred Period (DP0, DP1, DP2, DP4, DP8, DP13, DP26 and DP52). The Deferred Periods need different treatment in some respects. The subdivisions by Sex and Occupation Class can be treated identically. We also keep the data separate for each Year initially, though we aggregate Years towards the end of the process.

4. INCEPTIONS

4.1. As noted in Section 3 we wish to count the Inceptions that take place in each calendar Year, for each subdivision (by Sex, Occupation Class and Deferred Period) and by each Age. What is counted as an Inception is defined in the Exclusions Note. We count the number of Inceptions both cumD and exD.

4.2. We define Age as age last birthday at Commencement of Sickness. We do not have exact day, month and year of birth for any policies. The records show an Age Definition code, which may be either 0 or 1 in this data (there used to be an Age Definition code 2, but no such cases appear in the data for 1991 and later, and in fact Age Definition code 1 ceased to appear from 1999 onwards. The two codes have the following meanings:

0 an “exact” date is given; this is in the form of month and year of birth (but no day);

1 an “office year of birth” is given so that “year of investigation” minus “year of birth” equals the age nearest birthday at the end of the investigation year.

4.3. We therefore define an age for Sicknesses, Age1, which is the age last birthday at the Commencement of Sickness, as best as can be estimated, thus:

if Age Definition = 0 [exact month and year of birth are known]

then *if* Month of Sickness > Month of Birth
then Age1 = Year of Sickness – Year of Birth

if Month of Sickness < Month of Birth
then Age1 = Year of Sickness – Year of Birth – 1

if Month of Sickness = Month of Birth
then *if* Day of Sickness ≤ 15
then Age1 = Year of Sickness – Year of Birth – 1
if Day of Sickness > 15
then Age1 = Year of Sickness – Year of Birth;

if Age Definition = 1 [only age nearest birthday at end year is known]

then Age1 = Year of Sickness – Year of Birth – 1

4.4. For those Claim records with Age Definition 0 (which form about 90% of all cases in the 1991-98 period), where the months given are not equal, this algorithm gives the correct age last birthday at the Commencement of Sickness. Where the months are equal a little calculation, assuming an even spread of birthdays over a 30-day month, shows that “on average” the correct Age is given for about three quarters of cases, is one too big for one eighth of cases and one too small for one eighth of cases, thus being “right” overall. Similarly, for those with Age Definition 1 (which form the remaining 10% of cases) this algorithm gives the correct Age last birthday for about three quarters of cases, is one too big for one eighth of cases and one too small for one eighth of cases, thus being a reasonable approximation overall and the best we can do with the available data.

4.5. In Tables 4.1a, 4.1b and 4.1c we show the total numbers of Inceptions, exD, for all Ages combined and for the Years 1991 to 1998 combined, for Males, Females and both Sexes respectively, for each subdivision (OC and DP).

Table 4.1a. Numbers of Inceptions, ex Duplicates, Males, 1991-98, all Ages combined, for each combination of Occupation Class and Deferred Period.

	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	Total
OC1	0	9,697	37	2,304	5	1,125	1,107	502	14,777
OC2	0	7	0	510	8	329	174	62	1,090
OC3	0	3	0	873	19	319	149	55	1,418
OC4	0	2	0	757	31	286	108	34	1,218
OC5	168	68	0	3,372	71	2,106	889	379	7,053
All OCs	168	9,777	37	7,816	134	4,165	2,427	1,032	25,556

Table 4.1b. Numbers of Inceptions, ex Duplicates, Females, 1991-98, all Ages combined, for each combination of Occupation Class and Deferred Period.

	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	Total
OC1	0	1,639	2	777	4	353	371	198	3,344
OC2	0	2	0	147	4	98	78	38	367
OC3	0	0	0	38	0	23	23	17	101
OC4	0	0	0	1	0	3	6	1	11
OC5	0	8	0	308	6	273	235	104	934
All OCs	0	1,649	2	1,271	14	750	713	358	4,757

Table 4.1c. Numbers of Inceptions, ex Duplicates, both Sexes, 1991-98, all Ages combined, for each combination of Occupation Class and Deferred Period.

	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	Total
OC1	0	11,336	39	3,081	9	1,478	1,478	700	18,121
OC2	0	9	0	657	12	427	252	100	1,457
OC3	0	3	0	911	19	342	172	72	1,519
OC4	0	2	0	758	31	289	114	35	1,229
OC5	168	76	0	3,680	77	2,379	1,124	483	7,987
All OCs	168	11,426	39	9,087	148	4,915	3,140	1,390	30,313

4.6. From these tables we see that:

(a) the contributions of DP0, DP2 and DP8 are small, though not wholly negligible; DP0 is entirely, and DP8 is mostly, in OC5, and DP2 is entirely in OC1; our other comments ignore these Deferred Periods;

(b) DP1 is overwhelmingly OC1 and also contributes the largest total number of Inceptions;

(c) the number of Inceptions reduces (not surprisingly) as the Deferred Period increases, almost without exception;

(d) number of Inceptions for Males is much larger than for Females, especially in DP1, and in OC3 and OC4;

(e) the number of Inceptions is small (no more than 100) for both Sexes in OC2, OC3 and OC4 in both DP1 and DP52;

(f) the number of Inceptions is also small in these OCs for Females for DP4, DP13 and DP26 except the combination DP4 with OC2.

(g) much the largest numbers of Inceptions are found for Males, OC1, for DP1, DP26 and DP52. For DP4 and DP13, OC5 is the largest.

4.7. The last point justifies the use of the Males OC1 data as the basis for the graduations of Sickness rates, since OC5 contains a mixture of different Occupation Classes.

4.8. We require the numbers of cumD Inceptions only to calculate ratios of exD to cumD numbers for each "cell" in order to ratio down the exposures, as described in Section 6.4. The cumD numbers do not enter the "numerators" of the estimates of Sickness rates at all.

4.9. In Tables 4.2a, 4.2b and 4.2c we show the number of cumD Inceptions, exD Inceptions and the ratio of the cumD to exD numbers for Standard*, Males, Occupation Class 1, in each Year for each Deferred Period. The totals for each Year are the same as already shown in Table 2.2. From here on we generally omit reference to DP0, DP2 and DP8, though where totals for all DPs are shown we also show a combined total for these less common DPs, so that the results balance.

4.10. We can observe from Tables 4.2a and 4.2b that the number of Inceptions in DP1 is much greater than in all other Deferred Periods together. Also that the numbers in DP1 have been falling, the numbers in DP4 have been falling, but by less, and the numbers in the other DPs have been rising. The ratios of cumD to exD numbers have been very stable, except that the ratios for DP52 have been falling.

Table 4.2a: Inceptions, cum Duplicates, for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	3,664	455	131	129	50	6	4,435
1992	3,126	445	184	130	92	4	3,981
1993	3,595	411	176	161	41	8	4,392
1994	2,870	365	184	203	73	8	3,703
1995	3,070	461	255	261	129	6	4,182
1996	2,771	370	219	235	101	6	3,702
1997	2,464	403	208	240	127	2	3,444
1998	2,405	424	216	217	98	6	3,366
Total	23,965	3,334	1,573	1,576	711	46	31,205

Table 4.2b: Inceptions, ex Duplicates, for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	1,470	308	91	76	29	5	1,979
1992	1,254	304	124	94	46	3	1,825
1993	1,456	300	130	114	35	8	2,043
1994	1,172	254	122	144	50	8	1,750
1995	1,251	328	182	185	96	5	2,047
1996	1,111	259	154	156	70	6	1,756
1997	1,016	284	150	175	94	2	1,721
1998	967	267	172	163	82	5	1,656
Total	9,697	2,304	1,125	1,107	502	42	14,777

Table 4.2c: Ratios of cumD to exD Inceptions, for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	2.49	1.48	1.44	1.70	1.72	1.20	2.24
1992	2.49	1.46	1.48	1.38	2.00	1.33	2.18
1993	2.47	1.37	1.35	1.41	1.17	1.00	2.15
1994	2.45	1.44	1.51	1.41	1.46	1.00	2.12
1995	2.45	1.41	1.40	1.41	1.34	1.20	2.04
1996	2.49	1.43	1.42	1.51	1.44	1.00	2.11
1997	2.43	1.42	1.39	1.37	1.35	1.00	2.00
1998	2.49	1.59	1.26	1.33	1.20	1.20	2.03
Total	2.47	1.45	1.40	1.42	1.42	1.10	2.11

5. CALCULATION OF AUXILIARY FUNCTIONS

5.1. *What is required*

5.1.1. In order to carry out the calculations described in Section 3 we need values of the probability of a Sickness that commences at age $x+½$ "surviving" as Sick to duration d, $\pi(x+½,d)$, for each value of the Deferred Period, d. In earlier work, the values of d often have been taken as simple fractions of a year e.g. $^1/_{52}$, $^1/_{12}$, $^1/_4$, $^1/_2$, etc, but on this occasion we count throughout in weeks of 7 days, with the exception of DP1, where we allow for a Deferred Period of 6 days (see Section 4 of the Exclusions Note), so we have:

DP1	d	= 6/365	= 0.016438
DP4	d	= 28/365	= 0.076712
DP13	d	= 91/365	= 0.249315
DP26	d	= 182/365	= 0.498630
DP52	d	= 364/365	= 0.997260

Inspection showed that most DP52 Claims start on the 365th day, counting the day of Sickness as the first. This implies that the Deferred Period really is 52 weeks of 7 days, and is not a calendar year of 365 days, as had previously been assumed.

5.1.2. We also require the values of the probability that a Sickness that commences at age $x+½$ and continues to the end of the Deferred Period is then claimed for, which we denote $\eta(x+½,d)$. For many Deferred Periods this value is unity. But for those Deferred Periods where there is a run-in period it is less than unity, and needs to be calculated.

5.1.3. Other auxiliary functions are required, and these are described in the Sections which follow.

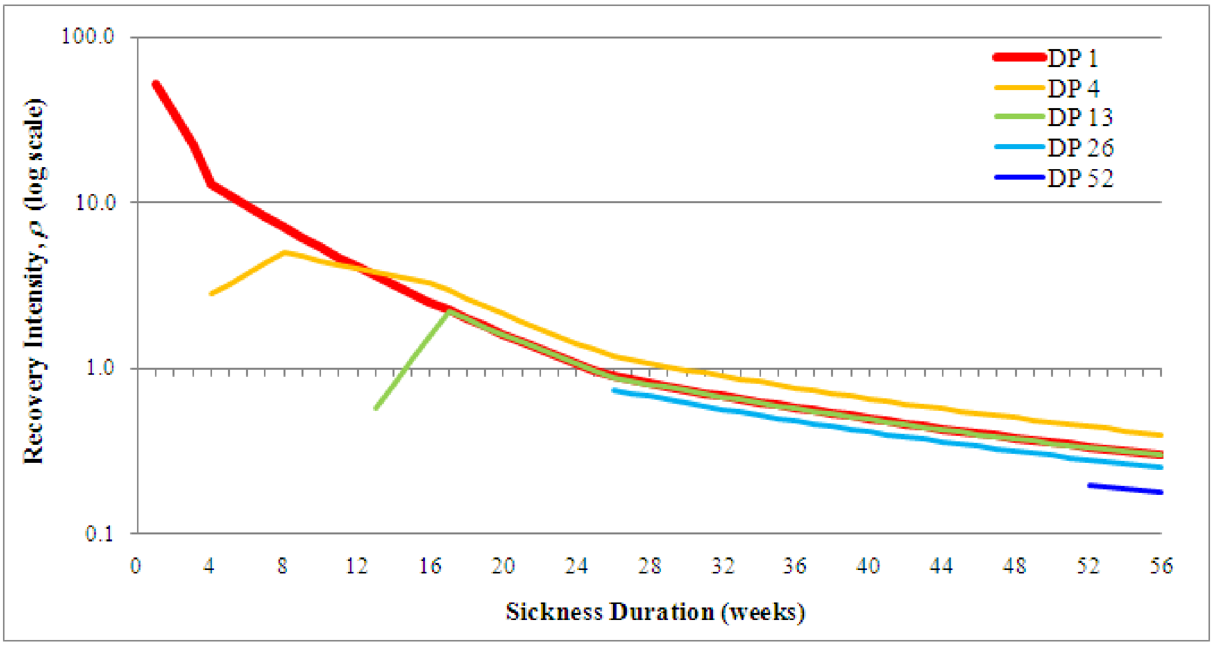
5.2. *Recovery and mortality rates*

5.2.1. In order to calculate $\pi(x+½, d)$ we require values of the intensities of recovery and death for all ages between $x+½$ and $x+½+d$. These have been graduated for the Males, OC1, 1991-98 data as described in "*The Graduation of Claim Recovery and Mortality Intensities for the Individual Income Protection Experience for 1991-98 of Males, Occupation Class 1*" (CMI Working Paper 5), for each Deferred Period separately, and we base our calculations on these graduated intensities. We denote recovery intensities as $\rho(x, z)$, where x is the policyholder's attained age and z is the duration of his current Sickness, and mortality intensities as $v(x, z)$. Although strictly speaking these are intensities, akin to a force of mortality, we use the shorter term "rates" in the remainder of this paper.

5.2.2. However, we cannot use these graduated rates unthinkingly. First, they are not based on any data before the end of the relevant Deferred Period; but this is precisely the period for which we need them. Secondly, for DP4 and DP13 (on this occasion) the graduated rates allow for either a four-week or (for DP4) possibly a twelve-week run-in period. But for later calculations we require rates both allowing for the run-in period, which we may describe as "rates of Claim Termination", and rates not allowing for the run-in period, which we may describe as "rates of Sickness Termination". The distinction applies only to recovery rates, and not to mortality rates for which there is no evidence of a run-in period, although it is quite likely that one exists. If an insured were to die shortly after the end of the Deferred Period, it might then fall to his or her executors to make a Claim under an IP policy, and this might well be overlooked, or not be thought worth the trouble for a small amount, or the policy might not be known about. However, the graduated mortality rates pay no attention to such considerations.

5.2.3. In Figure 5.1 we show the graduated recovery rates, $\rho(x+z, z)$, for 1991-98, for the five main Deferred Periods, for Age at Falling Sick x = 40 and for durations of Sickness from 0 to 56 weeks (just over one year). The rates for other Ages are at different levels, but the overall pattern is very similar. This graph shows only the rates of recovery from Claim, starting at the end of the Deferred Period. The run-in adjustments, which exist only for DP4 and DP13, are clearly visible.

Figure 5.1. Graduated recovery rates (rho or $\rho(x+z, z)$) for Age at Falling Sick 40

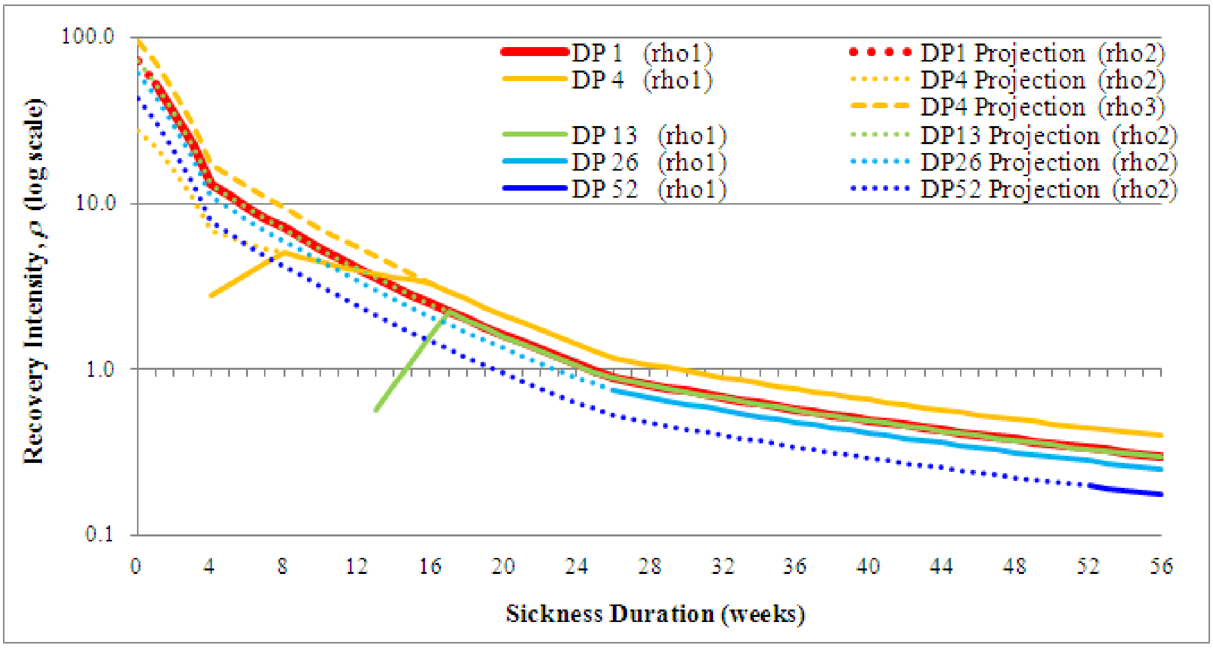


5.2.4. Next we need estimates of the early duration Sickness (not Claim) recovery rates, that is: recovery rates during the Deferred Period and any run-in period, with the allowance for run-in effects removed. We take the fitted graduation formula, subject to the modifications described below, and assume (in the absence of other observations) the values it gives for recovery rates at early Sickness durations are reasonable. Thus, in essence, we apply the graduation formula to ‘extrapolate’ the rates back to duration zero.

5.2.5. For DP13 the run-in adjustment lasts only for four weeks from durations 13 weeks to 17 weeks. Omitting the adjustment would allow the rates to run back to duration 0 very close to the rates of DP1. For DP4 it is more complicated. There is a clear run-in period of four weeks from durations 4 weeks to 8 weeks, but then another, lesser, adjustment that runs from durations 8 weeks to 16 weeks. We could remove either both of these or only the first in order to give assumed Sickness recovery rates.

5.2.6. In Figure 5.2 we show the same recovery rates as in Figure 5.1, but also now projected back to duration zero, using the formula for the graduated rates and removing the allowance for run-in effects. We call the original, graduated rates ρ_1 (rho1), the graduated rates adjusted to remove the assumed four-week run-in period ρ_2 (rho2), and the rates adjusted to remove the assumed twelve-week run-in period for DP4 ρ_3 (rho3). One can see that the rates for early durations of Sickness for DP1, which are projected back only one week, look reasonable, and that the rates for DP26 and DP52, for which no run-in period was assumed, are quite similar. The rates for DP13 ρ_2 are very close to those for DP1. There are two versions, ρ_2 and ρ_3, for DP4 giving significantly different possible assumed Sickness recovery rates during the first 16 weeks of Sickness.

Figure 5.2: Graduated and adjusted recovery rates (rho or $\rho(x+z, z)$) for Age at Falling Sick 40
Solid lines show Claim recovery rates, including the run-in periods;
Dotted lines show the projection back to duration 0, ignoring run-in effects.



5.2.7. We note the changes to the formula to produce these altered rates. The run-in is controlled by two parameters for DP4, denoted r_1 and r_2, and by one parameter for DP13, denoted r_3. The values of these used for ρ_1, as shown in CMI Working Paper 5, and also the values used for ρ_2 and ρ_3, are all shown in Table 5.1. For ρ_2, the value of r_2 (applying between durations 4 and 8) is set such that the adjustment at these durations is simply the extrapolation back of the adjustment made by r_1 between durations 8 and 16. The value of r_2 is half that for r_1 because it applies for half the period, four weeks instead of eight.

Table 5.1. Parameters to control run-in periods.

	r_1 (DP4)	r_2 (DP4)	r_3 (DP13)
ρ_1 (rho1 or Original)	0.622543	1.197880	1.830356
ρ_2 (rho2)	0.622543	0.3112715	0.0
ρ_3 (rho3)	0.0	0.0	0.0

5.2.8. It is a matter of judgement as to which of the two possible adjustments for DP4 should be used. For DP4 we have done the calculations on both adjustments, distinguishing them as DP4(2) (using the ρ_2 rates) and DP4(3) (using the ρ_3 rates).

5.2.9. We call the rates of Claim Termination "Type 1" rates. These are the original, graduated rates, the ρ_1 rates. The adjusted rates are the rates of Sickness Termination and we generally call them "Type 2" rates. The Type 2 rates are the same as the Type 1 rates for DP1, DP26 and DP52. For DP4(2) and DP13, the Type 2 rates are the ρ_2 rates, all assuming a four-week run-in period. For DP4(3) we also have the ρ_3 rates, assuming a twelve-week run-in period, and we refer to them as "Type 3 rates". Where necessary we use the subscripts 1, 2 and 3 to differentiate the Type of rate calculated.

5.2.10. The graduated mortality rates, $\nu(x, z)$, make no allowance for a run-in period, and in fact are the same for Deferred Periods DP4, DP13, DP26 and DP52. It is convenient to denote the sum of the recovery and mortality rates as the "Termination rates" and to use the symbol τ for them so that:

$$\tau(x, z) \quad = \quad \rho(x, z) \quad + \quad \nu(x, z)$$

The mortality rates are very much smaller than the recovery rates for at least the first year (though they rise well above them at long durations of Sickness) so that graphs of Termination rates for the first year look very similar to the graphs of recovery rates.

5.3. *Calculation of probabilities of "survival"*

5.3.1. We now go on to the calculation of the probabilities of "survival", $\pi(x+½, d)$. By survival we mean the survival of a Sickness. If the Sick person recovers he or she may survive as a person for a long time, but we do not refer to that. We start by generalising a little, and denote the probability of a Sickness that started when the affected life was aged x and has survived (remained Sick) to duration z_1 then surviving (remaining Sick) further to duration z_2 as $\pi(x, z_1, z_2)$. This can be readily expressed in terms of the Termination rates:

$$\pi(x, z_1, z_2) = \exp\left(-\int_{z_1}^{z_2} \tau(x+z, z)\,dz\right)$$

5.3.2. To calculate this we require to integrate $\tau(x, z)$ from z_1 to z_2. In general this has to be done by approximate integration. We use the "superSimpson" formula, of the form:

$$\int_0^1 f(x)dx \approx (7f(0) + 32f(¼) + 12f(½) + 32f(¾) + 7f(1)) \,/\, 90$$

applied to as many sub-divisions of the time interval as we need. We double the number of sub-divisions at each iteration and find the estimates generally converge very rapidly.

5.3.3. We first use this method to calculate $\pi(x+½, 0, d)$, which we can abbreviate to $\pi(x+½, d)$, for x = 15 to 65, and for each Deferred Period, using the values of d given in paragraph 5.1.1. We need this only for rates of Sickness Termination, i.e. with no allowance for the run-in periods. The results, for selected Ages, from 20 to 65, for all Deferred Periods, are shown in Table 5.2.

Table 5.2: Probabilities of a Sickness starting at age x+ ½ surviving to the end of the Deferred Period, $\pi(x+½, d)$, using Type 2 Termination rates; also Type 3 rates for DP4.

Age	DP1	DP4(2)	DP4(3)	DP13(2)	DP26	DP52
20	0.3167	0.2979	0.0230	0.0226	0.0278	0.0631
25	0.2752	0.2464	0.0129	0.0118	0.0151	0.0402
30	0.2719	0.2324	0.0109	0.0093	0.0120	0.0341
35	0.3010	0.2492	0.0136	0.0109	0.0140	0.0387
40	0.3583	0.2928	0.0227	0.0174	0.0218	0.0544
45	0.4379	0.3597	0.0431	0.0327	0.0394	0.0853
50	0.5301	0.4429	0.0823	0.0630	0.0730	0.1358
55	0.6230	0.5326	0.1458	0.1140	0.1273	0.2061
60	0.7064	0.6183	0.2313	0.1857	0.2012	0.2891
65	0.7748	0.6922	0.3282	0.2700	0.2849	0.3714

5.3.4. We can observe that, apart from DP1, and DP4 with Type 2 rates, and at older ages, the probabilities of survival as Sick to the end of the Deferred Period are very low. The probabilities dip across the middle ages, being a little higher at younger ages, and a lot higher at older ages. Generally the probabilities fall with increasing duration of Sickness (length of Deferred Period), but not always. Perhaps counter-intuitively the probabilities rise from DP13 to DP26 and again to DP52. This is because the graduated recovery rates for the different Deferred Periods are not all at the same level, with those for DP52 in particular being noticeably lower as shown in Figure 5.1. We can also see that if we use the Type 3 rates for DP4 we get very different numerical values, very much lower than when we use the Type 2 rates.

5.3.5. For further calculations for DP4 and DP13 we also require the values of the probabilities of surviving from the end of the Deferred Period to the end of the run-in period, $\pi(x+½, d, d_2)$ where d is the end of the Deferred Period and d_2 is the end of the run-in period. As for the Deferred Periods, we measure each run-in period in days and then express it as a proportion of a 365-day year. For DP13 it is therefore 28 days (28/365 = 0.076712) and for DP4 it is either 28 days (for Type 2 rates) or 84 days (84/365 = 0.230137) (for Type 3 rates). We also need these probabilities for the relevant run-in periods using the Type 1 rates for each DP. We can calculate all these in just the same way. Note that the Sickness is still assumed to start at age x+½. The results, for selected ages, are shown in Table 5.3.

5.3.6. We can see the same shape by age as for $\pi(x+½, d)$. We also see that, when the run-in period is allowed for (Type 1), the probability of survival to the end of the run-in period is higher than when the run-in period is not allowed for (Types 2 and 3). As expected, Claims are more likely than Sicknesses to survive to the end of the run-in period.

Tables 5.3: Probabilities of a Sickness starting at age x+½, which has already survived to the end of the Deferred Period, surviving to the end of the run-in period, $\pi(x+½, d, d_2)$, using Type 1, 2 and 3 Termination rates for DP4, with run-in periods of 28 days (1 and 2) and 84 days (1 and 3); Type 1 and Type 2 rates for DP13 with a run-in period of 28 days.

	28 days run-in		84 days run-in		28 days run-in	
Age	DP4(1)	DP4(2)	DP4(1)	DP4(3)	DP13(1)	DP13(2)
20	0.7981	0.7088	0.4687	0.2235	0.9152	0.8209
25	0.7582	0.6548	0.4010	0.1617	0.9014	0.7922
30	0.7364	0.6257	0.3703	0.1359	0.8957	0.7801
35	0.7340	0.6221	0.3723	0.1357	0.8982	0.7848
40	0.7482	0.6404	0.4017	0.1564	0.9071	0.8028
45	0.7740	0.6744	0.4522	0.1973	0.9197	0.8287
50	0.8053	0.7167	0.5164	0.2572	0.9331	0.8573
55	0.8371	0.7608	0.5855	0.3317	0.9455	0.8843
60	0.8657	0.8013	0.6513	0.4128	0.9555	0.9071
65	0.8888	0.8351	0.7073	0.4906	0.9628	0.9246

5.4. *Calculation of proportions not claiming for DP4 and DP13*

5.4.1. Our next calculation is of the estimated proportion of Sicknesses that reach the end of the Deferred Period, but for which no Claim is made. We define three groups among those who reach the end of the Deferred Period:

- A those whose Sickness lasts for the run-in period (who all Claim);
- B those whose Sickness terminates within the run-in period and Claim;
- C those whose Sickness terminates within the run-in period and do not Claim.

We have no actual records for those in group C. We are inferring their existence by the observation that the recovery rates for those who do Claim appear to be unduly low for the first few weeks of Sickness.

5.4.2. For a specified age at Commencement of Sickness, we denote the probabilities of a Sickness that reaches the end of the Deferred Period and is in group A, B or C by pa, pb and pc (omitting age subscripts). Clearly $pa + pb + pc = 1$. We can see also that:

$\pi_2(x+½, d, d_2)$ using Type 2 (or 3), Sickness recovery, rates = $pa / (pa + pb + pc) = pa$

$\pi_1(x+½, d, d_2)$ using Type 1, Claim recovery, rates = $pa / (pa + pb)$

Hence (omitting age and duration arguments on π_1 and π_2):

$$pa = \pi_2$$

$$pb = pa.(1 - \pi_1) / \pi_1 = \pi_2.(1 - \pi_1) / \pi_1 = \pi_2 / \pi_1 - \pi_2$$

$$pc = 1 - (pa + pb) = 1 - pa / [\, pa/(pa + pb)\,] = 1 - \pi_2 / \pi_1$$

and, as a check:

$$pb + pc = (\pi_2 / \pi_1 - \pi_2) + (1 - \pi_2 / \pi_1) = 1 - \pi_2 = 1 - pa$$

5.4.3. In Table 5.4 we show the probabilities of surviving to the end of the Deferred Period and not claiming, *pc*, using both the Type 1 combined with the Type 2 rates and the Type 1 combined with the Type 3 rates for DP4, and the Type 1 combined with the Type 2 rates for DP13. The probability is not large for DP13 and for DP4 on the basis of Type 2 rates, but it is large for DP4 on the basis of Type 3 rates. This reflects the large gap between the rates marked rho2/DP4 and rho3/DP4 in Figure 5.2 (noting the log scale).

5.4.4. We now denote the probability that a Sickness, starting at age x+½, that reaches the end of the Deferred Period, d, does claim by $\eta(x+½,d)$; this is equal to $pa + pb$ (or equivalently $1 – pc$). The value of $\eta(x+½,d)$ is unity for Deferred Periods where there is no run-in, DP1, DP26 and DP52, and is the complement of the value given in Table 5.4 for DP4 and DP13. This factor is used later on.

Table 5.4: Probabilities of a Sickness starting at age x+ ½, which has already survived to the end of the Deferred Period, not claiming, *pc*, using Type 1 and Type 2 Termination rates for DP4 and DP13; also Type 1 and Type 3 rates for DP4.

Age	DP4(2)	DP4(3)	DP13(2)
20	0.1119	0.5233	0.1030
25	0.1364	0.5968	0.1212
30	0.1503	0.6329	0.1290
35	0.1524	0.6357	0.1262
40	0.1440	0.6107	0.1150
45	0.1287	0.5638	0.0988
50	0.1100	0.5020	0.0812
55	0.0912	0.4334	0.0647
60	0.0743	0.3662	0.0507
65	0.0605	0.3063	0.0396

5.5. *Calculation of expected period spent Sick during Deferred Period*

5.5.1. We next wish to calculate the time spent Sick during the Deferred Period by those who fall Sick and either terminate their Sickness or reach the end of the Deferred Period. We note that, among those whose Sickness starts at age x+½, the probability that the Sickness lasts till the end of the Deferred Period, d, is $\pi(x+½ ,0, d)$, or say π. These Sicknesses necessarily last for period d within the Deferred Period. The probability that the Sickness terminates before the end of the Deferred Period is $1 – \pi$. We define the average period spent Sick among these as $e(x+½, 0, d)$ or say e.

5.5.2. We can calculate $e(x+½, 0, d)$ in either of two different ways. The first is to calculate the total time spent by all those who fall Sick, which can be calculated by integrating over the survivor probabilities as:

$$T = \int_0^d \pi(x + ½, 0, z)dz$$

which can also be expressed as:

$$T = \pi.d + (1 – \pi).e$$

Hence:

$$e(x+½,0,d) = \frac{\left(\int_0^d \pi(x+½,0,z)dz - \pi(x+½,0,d).d\right)}{\left(1-\pi(x+½,0,d)\right)}$$

5.5.3. The second method is to calculate the time spent by each Termination, integrating over the Terminations:

$$e(x+½,0,d) = \frac{\left(\int_0^d z.[-d\pi(x+½,0,z)/dz]\,dz\right)}{\left(\int_0^d [-d\pi(x+½,0,z)/dz]\,dz\right)}$$

The integral in the denominator equals (1 – π(x+½ ,0, d)) and the two calculations give the same result.

5.5.4. Again, the integrals can be calculated approximately using the superSimpson (or any other suitable) formula. In Table 5.5 we show these average times (in days) for all Deferred Periods (for DP4 both on the Type 2 and Type 3 bases).

Table 5.5: Average time (in days) spent Sick for a Sickness starting at age x+ ½, for those whose Sickness terminates before the end of the Deferred Period, e(x+½, 0, d), using Type 2 Termination rates; also Type 3 rates for DP4.

Age	DP1	DP4(2)	DP4(3)	DP13(2)	DP26	DP52
20	2.26	7.66	3.81	6.47	9.30	17.57
25	2.21	7.52	3.45	5.62	7.84	14.67
30	2.22	7.60	3.43	5.51	7.57	13.92
35	2.27	7.93	3.72	6.01	8.24	14.84
40	2.37	8.44	4.30	7.13	9.81	17.20
45	2.48	9.05	5.14	8.88	12.28	20.84
50	2.58	9.70	6.16	11.16	15.53	25.44
55	2.67	10.32	7.24	13.76	19.23	30.67
60	2.75	10.89	8.27	16.37	22.99	36.36
65	2.81	11.39	9.16	18.77	26.60	42.90

5.5.5. We note that the average time spent Sick before Termination within the Deferred Period is in general much less than the Deferred Period itself. The assumed recovery rates, as can be seen from Figure 5.2, are very high initially and drop steadily to much lower levels. Thus on these assumptions many Sicknesses last for a very short time before recovering. However, for DP4 the Type 2 recovery rates are a lot lower than the Type 3 rates, so Sickness can last a bit longer on that basis, and for some ages last longer than for much longer Deferred Periods.

5.6. *Calculation of time spent Sick among those who do not claim*

5.6.1. We next need the average time spent after the end of the Deferred Period by those who reach the end of the Deferred Period but are assumed not to claim. We can calculate this indirectly. Reverting to the notation of Section 5.4 we need the average time spent Sick (after the end of the Deferred Period) by those in group C. Using Type 2 (or 3) rates, we can calculate the average time spent Sick for those in groups B and C combined. Using Type 1 rates we can calculate the average time spent Sick for those in group B. By subtraction we can derive the average time for those in group C.

5.6.2. We define $e(x+½, d_1, d_2)$ in general as the average time spent Sick between durations d_1 and d_2 among those who become Sick at age $x+½$ and who remain Sick at duration d_1 and whose Sickness terminates before duration d_2. In particular we take d_1 as the end of the Deferred Period, d, and d_2 as the end of the run-in period. We differentiate between $eb(x+½, d, d_2)$, using Type 1 rates, which gives the average time for the fraction pb in group B, and $ebc(x+½, d, d_2)$, using Type 2 (or Type 3) rates, which gives the average time for the combined fractions pb and pc in groups B and C. We abbreviate these to eb and ebc.

5.6.3. We can then obtain ec, the average time spent by those in group C by:

$$pc.ec + pb.eb = (pb + pc).ebc$$

or

$$ec = \{(pb + pc).ebc – pb.eb\} / pc$$

Note that those in group A remain Sick for the whole period from d to d_2, so are necessarily Sick for $d_2 – d$, but they do not enter the calculations.

5.6.4. We calculate the average times eb and ebc in the same way as is described in Section 5.5. The results are shown in Tables 5.6(a), (b) and (c) for (a) DP4 using Type 1 and Type 2 rates, (b) DP4 using Type 1 and Type 3 rates and (c) DP13 using Type 1 and Type 2 rates. For completeness, we show pb, $(pb + pc)$ and pc, and eb, ebc and ec in each case. The values of pc have already been shown in Table 5.4.

5.6.5. The implication of these tables is that the expected days of Sickness for those who do not claim (ec) is quite a bit lower than the expected days of Sickness for those who do claim, but still recover within the run-in period (eb). This is an accidental, but not an unreasonable result. Note that the average time spent for DP4 using Type 3 rates shown in Table 5.6(b) should be compared with a period of 12 weeks (84 days) from d to d_2, whereas in the other two cases the period is 4 weeks (28 days).

5.7. *Estimation of the number of Sicknesses that commence*

5.7.1. Our final piece of theoretical foundation is that we need to be able to derive the expected number of Sicknesses that have commenced, given the observed number of Inceptions. This is a matter of manipulating probabilities, and we show the process in Appendix A. The result is that, given I, the number of Claim Inceptions, and p, the probability of a Sickness becoming an Inception, the number of Sicknesses that do not become Inceptions, J, is distributed according to the negative binomial distribution, with

$$P[J = j \mid I] = (I + j)!/\{I!\, j!\}\, p^{I+1} q^j = ({}_{I+j}C_j).p^{I+1}.q^j$$

where $q = 1 – p$.

5.7.2. We then get the mean of J, $E[J|I] = (I+1)\, q / p$. So the mean of $S = I + J$, or

$$E[S|I] = I + (I + 1)\, q / p = (I + 1) / p – 1.$$

5.7.3. The naïve, deterministic, assumption would be that, given I and p, $S = I / p$. However, this is equivalent to asserting that $E[X|Y] = E[X|Y = E[Y|X]]$, which is in general not true. Thus, given S, the expected value of I is $p.S$. But it is not correct to say that, given I, the expected value of S is I / p. If I is large, and p is not very small, then I / p and $(I + 1) / p – 1$ are numerically not very different, so there is sometimes quite a small adjustment by using the more correct method. However, the difference is noticeable when I is small. Thus when there are no observed Inceptions, so $I = 0$, we get $E[S|I{=}0] = 1 / p – 1$. Since p is often quite small, this may make a big difference. It is more realistic, when there are no Inceptions, to say that the expected number of Sicknesses is non-zero, but that it so happens that all of them terminated before they could become Inceptions, rather than assuming that no Inceptions implies no earlier Sicknesses.

Table 5.6(a): Proportions *pb*, (*pb* + *pc*) and *pc*, also expected durations of Sickness *eb*, *ebc* and *ec* (in days), using Type 1 and Type 2 Termination rates for DP4.

Age	*pb*	*pb+pc*	*pc*	*eb*	*ebc*	*ec*
20	0.1793	0.2912	0.1119	15.01	12.69	8.96
25	0.2088	0.3452	0.1364	14.85	12.46	8.79
30	0.2240	0.3743	0.1503	14.74	12.31	8.68
35	0.2255	0.3779	0.1524	14.69	12.25	8.64
40	0.2155	0.3596	0.1440	14.69	12.28	8.66
45	0.1970	0.3256	0.1287	14.72	12.35	8.72
50	0.1733	0.2833	0.1100	14.77	12.45	8.80
55	0.1480	0.2392	0.0912	14.82	12.55	8.88
60	0.1243	0.1987	0.0743	14.85	12.64	8.94
65	0.1045	0.1649	0.0605	14.86	12.71	8.99

Table 5.6(b): Proportions *pb*, (*pb* + *pc*) and *pc*, also expected durations of Sickness *eb*, *ebc* and *ec* (in days), using Type 1 and Type 3 Termination rates for DP4.

Age	*pb*	*pb+pc*	*pc*	*eb*	*ebc*	*ec*
20	0.2533	0.7765	0.5233	38.09	22.98	15.67
25	0.2415	0.8383	0.5968	36.77	20.96	14.57
30	0.2312	0.8641	0.6329	35.95	19.83	13.93
35	0.2287	0.8643	0.6357	35.68	19.56	13.77
40	0.2329	0.8436	0.6107	35.86	20.02	13.98
45	0.2390	0.8027	0.5638	36.32	20.96	14.46
50	0.2409	0.7428	0.5020	36.89	22.14	15.07
55	0.2348	0.6683	0.4334	37.44	23.33	15.69
60	0.2210	0.5872	0.3662	37.89	24.39	16.24
65	0.2030	0.5094	0.3063	38.21	25.26	16.67

Table 5.6(c): Proportions *pb*, (*pb* + *pc*) and *pc*, also expected durations of Sickness *eb*, *ebc* and *ec* (in days), using Type 1 and Type 2 Termination rates for DP13.

Age	*pb*	*pb+pc*	*pc*	*eb*	*ebc*	*ec*
20	0.0761	0.1791	0.1030	16.79	12.60	9.51
25	0.0866	0.2078	0.1212	16.75	12.48	9.42
30	0.0909	0.2199	0.1290	16.72	12.41	9.37
35	0.0890	0.2152	0.1262	16.69	12.39	9.36
40	0.0822	0.1972	0.1150	16.67	12.41	9.37
45	0.0724	0.1713	0.0988	16.63	12.46	9.40
50	0.0615	0.1427	0.0812	16.59	12.52	9.44
55	0.0510	0.1157	0.0647	16.52	12.57	9.47
60	0.0422	0.0929	0.0507	16.40	12.63	9.49
65	0.0357	0.0754	0.0396	16.22	12.69	9.50

5.7.4. We need to use for p the probability that a Sickness lasts the length of the Deferred Period, and that a Claim is then made. This is the product of the probability denoted $\pi(x+½, d)$ shown in Table 5.2 and the probability defined as $\eta(x+½,d)$ which is either unity or the complement of the value shown in Table 5.4. The values are shown in Table 5.7.

Table 5.7: Probabilities that a Sickness starting at age x+ ½ survives to the end of the Deferred Period, $\pi(x+½, d)$, and that a Claim is then made, $\eta(x+½,d)$, using Type 2 Termination rates; also Type 3 rates for DP4.

Age	DP1	DP4(2)	DP4(3)	DP13(2)	DP26	DP52
20	0.3167	0.2646	0.0110	0.0203	0.0278	0.0631
25	0.2752	0.2128	0.0052	0.0104	0.0151	0.0401
30	0.2719	0.1975	0.0040	0.0081	0.0120	0.0341
35	0.3010	0.2112	0.0050	0.0095	0.0140	0.0387
40	0.3583	0.2507	0.0088	0.0154	0.0218	0.0543
45	0.4379	0.3134	0.0188	0.0295	0.0394	0.0852
50	0.5301	0.3942	0.0410	0.0579	0.0730	0.1357
55	0.6230	0.4840	0.0826	0.1066	0.1273	0.2060
60	0.7064	0.5723	0.1466	0.1763	0.2012	0.2890
65	0.7748	0.6503	0.2276	0.2593	0.2849	0.3714

5.7.5. We also need an estimate of the number of Sicknesses that reach the end of the Deferred Period and do not claim, denoted N. We can estimate this in the same way, using as p the probability that a Sickness, having reached the end of the Deferred Period, is then claimed for. Denote the probability that a Sickness survives the Deferred Period by π, and the probability that a Claim is then made by η. We can calculate $E[N|I]$ as $(I + 1) / \eta - (I + 1)$ and $E[S|I]$ either as $(I + 1) / (\pi.\ \eta) - 1$ or as $(E[N|I] + (I + 1)) / \pi - 1$, both giving the same answer.

5.8. *Type 2 or Type 3 rates*

5.8.1. The results shown for Deferred Periods other than DP4 look reasonable. However, for DP4 we have had two choices in the adjustments needed to get the underlying Sickness recovery rates from the Claim recovery rates, adjusting for an assumed short run-in period of 4 weeks, which we have denoted the Type 2 rates and adjusting for an assumed long run-in period of 12 weeks, which we have denoted the Type 3 rates. The Type 3 recovery rates, shown in Figure 5.2, look more compatible with the graduated recovery rates (which we assume apply to both Sickness and Claims), based on the 1991-98 experience, for DP1, DP26 and DP52, and what we have assumed as the Type 2 rates for DP13. However, if these are the true underlying rates of recovery, the proportion of Sicknesses that pass the end of the Deferred Period and yet for which no Claim is made, seem very large, between about 30% and about 64% of all such Claims, as shown in Table 5.4, and as a percentage of those that do recover within the (12-week) run-in period, are even larger as can be seen by comparing *pc* with *pb*+*pc* in Table 5.6(b). Yet the graduated rates do correctly reflect the recovery experience of DP4 policies. We leave unsettled at this point which rates to use, and show subsequent results on both bases.

6. Calculation of Exposed to Risk

6.1. *Stage1: In force*

6.1.1. We are now ready to return to the In force data which we described in Section 2. We keep separate, throughout, the data for each Sex, Occupation Class and Deferred Period. We keep separate, for the time being, the data for each Year, though data for the various Years will later be combined. And we wish to consider each Age separately until we reach the stage of the graduations. By a "cell" we mean some combination of Sex, Occupation Class, Deferred Period, Year and Age.

6.1.2. First, however, we need to classify the In force by Age. The algorithm given in paragraph 4.3 applies for Inceptions and we now need the corresponding algorithm for the In force. We need to deal separately with the different Age definitions, and we know we are working at a year-end. We define a second integral age, Age2, as at the end of year Y (beginning of year Y+1):

if Age Definition = 0 [exact month and year of birth are known]

then Age2 = Y – Year of Birth
add 1 to In force at age Age2

if Age Definition = 1 [only age nearest birthday at end year is known]

then Age2 = Y – Year of Birth
add ½ to In force at age Age2 – 1
add ½ to In force at age Age2

6.1.3. This allows us to accumulate the numbers in force within each cell, for each Age, x, at the beginning and the end of each Year which we can denote as $F_0(x)$ and $F_1(x)$. If we were dealing with a mortality investigation we would naturally calculate the central exposed-to-risk by averaging these two values, giving:

$$R(x) \quad = \quad (F_0(x) + F_1(x)) / 2$$

This assumes that we are classifying events by Age at the date of the event (date of death in a mortality investigation, date of Sickness in this case) and that we would really like to calculate the in force at that Age for each day of the year (a day count) summing (integrating) over the year. However, as we only have beginning and end In force, and no further information on the movement dates for policies entering or exiting the investigation during the year, the best we can do is average the values so using a "trapezium rule" approximation to the integral.

6.1.4. An alternative method, which has been used in previous CMI IP Inceptions analyses, including that in CMIR 12, is to classify events by age last birthday at the beginning of the year, calculating the exposed to risk by averaging what we would denote $F_0(x)$ and $F_1(x+1)$, and to classify Inceptions by the Age at beginning of the year. This however, spreads the Sicknesses within one Age cell over ages x to $x+2$, whereas the method used here spreads over ages x to $x+1$. It is preferable to keep the age span within one cell as small as practicable.

6.1.5. We note that for Deferred Period d an Inception that occurs at some point during one calendar year, say between 01/01/Y and 31/12/Y, inclusive, must relate to a Sickness that commenced period d previously, that is, between (01/01/Y – d) and (31/12/Y – d), inclusive. We therefore need to estimate the exposure over this period. To do this we need to project $F_0(x)$ and $F_1(x)$ backwards to estimate the In force at times $(0 - d)$ and $(1 - d)$, say (omitting age subscripts) $F_{(0-d)}$ and $F_{(1-d)}$ and we could then average these values. Noting again that we have no information on policy movement dates during the year, and no guarantee of continuity between the data for different years, there are three ways in which we could do this.

6.1.6. The first method is by linear extrapolation. We put:

$$F_{(0-d)} = F_0 - d.(F_1 - F_0) = (1 + d).F_0 - d.F_1$$

and

$$F_{(1-d)} = F_1 - d.(\mathrm{F}_1 - F_0) = d.F_0 + (1-d).F_1$$

and we could then average these values giving:

$$R = (F_{0-d} + F_{1-d}) /2 = (F_0 + F_1) /2 - d\,(F_1 - F_0)$$

6.1.7. However, in some cases $\mathrm{F}_{(0-d)}$ may be negative, and it is desirable to avoid negative values so that it is better to put:

$$R = (\mathrm{Max}(F_{0-d},0) + F_{(1-d)})/2$$

However, if d = 52 weeks, and F_0 is zero, we might still get R as zero, or almost zero.

6.1.8. Method two reduces this problem. We assume exponential growth in the In force between the beginning and the end of the year, and we project the same growth rate backwards to time $0-d$. Thus we put:

$$r = \ln F_1 - \ln F_0$$

and estimate

$$F'_{0-d} = F_0 \exp(-r.d)$$

and

$$F'_{1-d} = F_1 \exp(-r.d)$$

6.1.9. We can now estimate R by integrating over the exponential function giving:

$$R' = \int_{0-d}^{1-d} F_0 \exp(r.y)\,dy = F_0 \exp(-r.d)\,(\exp(r)-1) / r$$

6.1.10. Thirdly, and more simply, but less consistently and less satisfactorily, we could interpolate linearly between these estimated values of F'_{0-d} and F'_{1-d} giving:

$$R'' = (F_0 \exp(-r.d) + F_1 \exp(-r.d)) / 2$$
$$= (F_0 + F_1) \exp(-r.d) / 2 = F_0\,(1 + \exp(r)) \exp(-r.d) / 2$$

6.1.11. Both the exponential methods fail if either F_0 or F_1 is zero, when we have to revert to the linear method. This leaves the problem of a possible zero exposure for DP52 if $F_0 = 0$. However, if there is an Inception during the year for DP52, the policy ought to have been In force at the beginning of the year. However, the In force and the Inceptions are taken from different data files so errors are not impossible.

6.1.12. Specific examples of the calculations may be of interest and are shown in Table 6.1, under Method 3. All are taken from Males, Occupation Class 1, DP26, for the Year 1991. The value of d is taken as 182/365 = 0.498630. Methods 1, 2 and 3 are:

Method 1:	R	=	$(\mathrm{Max}(F_{0-d},0) + F_{(1-d)}) / 2$
Method 2	R'	=	$F_0 \exp(-r.d) (\exp(r)-1) / r$
Method 3	R''	=	$F_0 (1 + \exp(r)) \exp(-r.d) / 2$

Table 6.1. Specimen calculations of exposed to risk from In force at year-ends.

Age	F_0	F_1	$F_{(0-d)}$	$F_{(1-d)}$	Method 1	r	$F'_{(0-d)}$	$F'_{(1-d)}$	Method 2	Method 3
20	1	0	1.50	0.50	1.00					
22	2	14	–3.98	8.02	4.01	1.9459	0.76	5.31	2.34	3.03
25	58	197	–11.31	127.69	63.85	1.2228	31.52	107.07	61.78	69.30
39	1,049	1,097	1,025.1	1,073.1	1,049.1	0.0447	1,025.9	1,072.8	1,049.2	1,049.3
64	411	376	428.45	393.45	410.95	–0.0890	429.65	393.06	411.09	411.36

6.1.13. Initially we compare the results of Method 1 with those of Method 2. At age 39 the values of F_0 and F_1 are large, and not very different ($r = 0.0447$). The two methods give quite similar answers, actually (because this is DP26) very similar to F_0. At age 25 the values of F_0 and F_1 are smaller, but increasing considerably ($r = 1.2228$), so that the interpolated value of F_{0-d} is negative and is replaced by zero for Method 1. Noticeably different answers are produced by the two methods at age 25, 63.85 and 61.78. At age 22 the values of F_0 and F_1 are increasing even more ($r = 1.9459$) and the two methods produce, proportionately, even more diverse answers, 4.01 and 2.34. At age 20, the value of F_1 is zero, and the exponential methods cannot be used, so the result for Method 1 is used. At age 64 the values of F_0 and F_1 are decreasing, but not enormously, and the methods give similar answers.

6.1.14. Method 3 gives answers that are always larger than those of Method 2, sometimes, as for age 25, almost unreasonably so.

6.1.15. We adopt Method 2 as our preferred approach. In Table 6.2 we show the exposed to risk (denoted *R1*), based on the In force and calculated using Method 2, for Standard*, Males, Occupation Class 1, for each Year and each Deferred Period. Note that the totals for each Year are quite similar to the In force at the beginning and the end of each Year, as shown in Table 2.2. We can observe that the exposure in DP1 has declined steadily over the period, that that for DP4 has fallen and then risen again, and that the exposures for DP13, DP26 and DP52 have risen considerably. One might imagine that this is because of a rise in the amount of IP business written for these longer Deferred Periods, but it might alternatively be because certain offices that write longer Deferred Period business have entered the investigation, or have begun to code Occupation Class in more detail. All three factors are present within the data but we have not investigated further to determine their relative contributions.

Table 6.2: Exposed to risk (*R1*), at Stage 1, calculated using Method 2,
for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52.

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	24,479.4	26,590.4	27,771.4	33,729.9	15,134.9	94.2	127,800.1
1992	23,429.2	26,716.5	31,145.5	44,038.9	19,440.1	254.5	145,024.6
1993	22,360.4	27,081.2	34,028.3	49,089.3	24,827.1	246.5	157,632.8
1994	21,393.4	25,378.3	32,966.2	48,344.6	26,341.7	241.4	154,665.6
1995	20,618.8	25,026.5	43,282.1	59,424.1	32,599.4	240.4	181,191.2
1996	19,810.9	24,496.0	42,878.5	59,633.8	33,792.6	251.8	180,863.6
1997	19,274.8	27,292.2	44,626.0	63,811.1	38,621.6	1,436.4	195,062.1
1998	18,433.0	30,025.6	56,165.9	76,628.4	48,174.2	1,372.9	230,800.0
Total	169,799.9	212,606.7	312,863.9	434,700.0	238,931.5	4,138.1	1,373,040.0

6.1.16. For many cells, especially at the extreme ages, and in investigations where the total amount of business is small, the exposure at this stage is zero. In a few cases there are Inceptions or Claims in such a cell. This may arise quite correctly. Imagine that there is no In force at the start of a year at age *x*, but there is one policy at age *x*–1. The policyholder passes his x^{th} birthday, falls Sick, claims, and then either he dies or the policy expires before the end of the year, so that the In force at age *x* at the end of the year is zero. This is a defect of any "census method", which a "day count" is able to remedy. In such a case we set to zero the number of Inceptions, the duration of Claims, the expected number of Sicknesses, etc.

6.2. *Stage 2: Current Claims*

6.2.1. Stage 1 of the calculation of exposures gives us the total exposed from the In force. The In force, however, includes policies that are currently Claims and those where the policyholder is Sick but has not yet claimed. Our first adjustment is to calculate the actual days for which claims are being made. We have exact dates for Claim Commencement and Cessation and so we know for each Claim in each calendar year the days for which a Claim is made. This may be as long as the full year of 365 or 366 days, or may be for a shorter period.

6.2.2. We need to attribute each day of Claim to the appropriate age, the age at the time that the Claim is being paid. This is not the same age as either of those we have used already, so we define a third age, Age3. For those cases with Age Definition = 0 we know the month and year of birth, but not the day. We assume for this purpose that all birthdays are on the 16th of the month, and that the insured reaches the next age at "0001 hours" on that day. For Age Definition = 1, we know only the age nearest birthday at the end of the year, so we have at best an estimate of the year of birth, and we assume that all births are on 1 January. We then calculate "Days Claim" for each Age3 from each Claim by the following algorithm. Note that for Claim records which are continuations from the previous year Date of Commencement is taken as 1 January, and for continuations into the subsequent year Date of Cessation is taken as 31 December. Both the Date of Commencement and the Date of Cessation contribute a full day to "Days Claim".

if Age Definition = 0 [exact month and year of birth are known]

then calculate Birthday in Year = {16, Month of Birth, Year of Claim}
Age3 = Year of Claim – Year of Birth

if Date of Commencement ≥ Birthday [Commences after Birthday]
then add Date of Cessation – Date of Commencement + 1 to Days Claim (Age3)

if Date of Commencement < Birthday [Commences before Birthday]
then *if* Date of Cessation < Birthday [Ceases before Birthday]
then add Date of Cessation – Date of Commencement + 1 to Days Claim (Age3–1)

if Date of Cessation ≥ Birthday [Spans Birthday; so split into two]
then add Birthday – Date of Commencement to Days Claim (Age3 – 1)
add Date of Cessation – Birthday + 1 to Days Claim (Age3)

if Age Definition = 1 [only age nearest birthday at end year is known]

then Age3 = Year of Claim – Year of Birth – 1
add Date of Cessation – Date of Commencement + 1 to Days Claim (Age3)

6.2.3. Because the In force includes Duplicates, we use the cum Duplicates file of Claims for the adjustments here and in the following Stages 3 and 4. We also need to use the Occupation Class coding that corresponds to the Class coding in the In force for the corresponding office.

6.2.4. In Table 6.3 we show the number of Claims for Standard*, Males, Occupation Class 1, for each Year and each Deferred Period. The number of Claims in total is the same as already shown in Table 2.2.

Table 6.3: Number of Claims, cum Duplicates, for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52.

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	5,171	1,048	555	596	200	14	7,584
1992	4,709	1,154	726	825	292	10	7,716
1993	5,205	1,142	735	877	306	14	8,279
1994	4,479	1,094	779	972	347	18	7,689
1995	4,627	1,189	1,045	1,157	475	14	8,507
1996	4,503	1,167	1,069	1,245	548	13	8,545
1997	4,189	1,262	1,094	1,396	659	21	8,621
1998	4,091	1,299	1,160	1,458	681	23	8,712
Total	36,974	9,355	7,163	8,526	3,508	127	65,653

6.2.5. In Table 6.4 we show the corresponding number of years of Claim. This is calculated by taking the count of days claimed as described in the algorithm above, and dividing by 365 (366 if the year is leap year).

Table 6.4: Years of Claim, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52.

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	1,323.0	572.9	403.6	472.5	140.4	6.0	2,918.4
1992	1,373.8	654.0	510.6	666.4	221.5	6.8	3,433.0
1993	1,436.1	658.5	551.6	693.0	242.2	8.9	3,590.3
1994	1,487.1	677.8	596.2	751.4	275.4	9.7	3,797.6
1995	1,531.1	742.4	787.3	938.8	383.1	6.4	4,389.1
1996	1,603.9	772.4	848.8	1,010.4	446.2	7.7	4,689.4
1997	1,635.8	858.1	885.1	1,164.4	550.9	17.4	5,111.7
1998	1,638.6	865.4	928.3	1,227.7	603.7	17.7	5,281.3
Total	12,029.3	5,801.6	5,511.5	6,924.6	2,863.4	80.4	33,210.8

6.2.6. We denote the number of years of claim at Age x in Year Y as $C(x,Y)$. This relates to the calendar year Y. However, the exposure calculated in Stage 1 was adjusted to relate to the period d prior to the calendar year. We adjusted in effect by reducing the exposure by the rate of growth and we can do the same for the years claimed, by multiplying $C(x,Y)$ by $\exp(-r(x,Y).d)$, where $r(x,Y)$ is the growth rate of the In force for that Age in that Year and cell. Whilst this adjustment is simple and approximate, and for example takes no further account of the known Claim dates, it does maintain consistency with the calculations in Stage 1 and the limitations in the form of the available data prevent a more exact approach. Hence the adjusted exposure for Age x for Year Y, $R2(x)$, is given by:

$$R2(x,Y) \quad = \quad R1(x,Y) - C(x,Y).\exp(-r(x,Y).d)$$

If either F_0 or F_1 is zero, so that we cannot calculate $r(x,Y)$, we use the average growth rate for all Ages for that Year and cell.

6.2.7. For certain cells we find that the time spent claiming, adjusted is this way, is greater than the Stage 1 exposure, i.e. $C(x,Y).\exp(-r(x,Y).d) > R1(x,Y)$. This occurs only when the numbers of cases are small, usually at the extremity of the age range. It is not clear why this occurs, but there may be inconsistencies between the In force files and the Claims files supplied by particular offices, and the discrepancies are so small that it not worth investigating in detail. In such cases we set the adjusted time spent claiming to equal $R1(x,Y)$ so that $R2(x,Y)$ equals zero and carry on. This is an additional adjustment to that described in paragraph 6.1.16, where we set to zero all items where the Stage 1 exposure is zero.

6.2.8. In Table 6.5 we show the adjusted totals for the years of claim, i.e. $C(x,Y).\exp(-r(x,Y).d)$, allowing for the adjustments noted in paragraph 6.2.7, and in Table 6.6 we show the adjusted total exposures ($R2$) in both cases for all Ages together, for Males, Occupation Class 1.

Table 6.5: Adjusted years of Claim, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52.

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	1,323.1	572.6	399.5	468.4	132.9	6.0	2,902.5
1992	1,373.6	655.4	509.6	660.6	219.7	5.9	3,424.7
1993	1,435.6	659.4	550.8	702.5	240.5	9.0	3,597.7
1994	1,487.0	678.1	592.8	743.5	262.4	9.7	3,773.6
1995	1,530.5	742.8	782.9	927.9	373.2	6.4	4,363.8
1996	1,603.0	772.0	846.9	1,001.7	434.9	7.7	4,666.1
1997	1,634.1	857.3	879.7	1,154.3	531.8	17.2	5,074.4
1998	1,637.8	864.7	921.3	1,223.1	594.2	17.3	5,258.5
Total	12,024.6	5,802.3	5,483.6	6,882.0	2,789.6	79.1	33,061.3

Table 6.6: Exposed to risk (*R2*), at Stage 2, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52.

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	23,156.3	26,017.8	27,371.8	33,261.5	15,002.0	88.2	124,897.6
1992	22,055.6	26,061.1	30,635.9	43,378.3	19,220.4	248.6	141,599.9
1993	20,924.8	26,421.8	33,477.5	48,386.8	24,586.6	237.5	154,035.1
1994	19,906.4	24,700.2	32,373.3	47,601.1	26,079.2	231.8	150,892.0
1995	19,088.3	24,283.6	42,499.2	58,496.1	32,226.2	234.0	176,827.4
1996	18,207.9	23,724.0	42,031.5	58,632.1	33,357.7	244.1	176,197.4
1997	17,640.7	26,434.9	43,746.4	62,656.8	38,089.8	1,419.2	189,987.7
1998	16,795.2	29,160.9	55,244.6	75,405.3	47,579.9	1,355.6	225,541.5
Total	157,775.2	206,804.4	307,380.3	427,818.0	236,141.9	4,059.0	1,339,978.7

6.2.9. In Table 6.7 we show the ratios, as a percentage, that the time deducted because of claiming bears to the exposure before deduction (*R1*).

Table 6.7: Ratio 100 × Adjusted time claiming / *R1*, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP0 to DP52.

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	5.40	2.15	1.44	1.39	0.88	6.38	2.27
1992	5.86	2.45	1.64	1.50	1.13	2.31	2.36
1993	6.42	2.43	1.62	1.43	0.97	3.63	2.28
1994	6.95	2.67	1.80	1.54	1.00	4.00	2.44
1995	7.42	2.97	1.81	1.56	1.14	2.66	2.41
1996	8.09	3.15	1.98	1.68	1.29	3.05	2.58
1997	8.48	3.14	1.97	1.81	1.38	1.20	2.60
1998	8.89	2.88	1.64	1.60	1.23	1.26	2.28
Total	7.08	2.73	1.75	1.58	1.17	1.91	2.41

6.2.10. We note that the reduction in the exposure because of deducting the time spent claiming is 2.41% overall, ranging from 7.08% for DP1 to 1.17% for DP52 (ignoring the less common DPs). This difference is not surprising. However, the percentage has increased over the Years, more noticeably for DP1 and DP52 than for the other Deferred Periods.

6.2.11. We should note also that most DP1 Claims are backdated to commence on the day of Sickness, although to be eligible for a Claim they should have lasted for at least the Deferred Period of six days.

6.3. *Stage 3: Sicknesses not (yet) claiming*

6.3.1. Since all policies (except for those in DP0) have a Deferred Period before a Claim may start, the In force population includes some insured who are already Sick, but have not been Sick for long enough for a Claim to have commenced, as well as others who have gone on to claim. They cannot become Sick again at the same time, so they should be excluded from the exposed to risk. For those cases where a Claim has not been made we do not know who they are or for how long they have been Sick, but we can make some estimates.

6.3.2. This involves estimating three quantities. Our first adjustment is to take out the period Sick, during the Deferred Period when they were not yet claiming, for those whose Sickness lasted long enough for a Claim to have arisen. Our second is to take out the estimated period spent Sick by those whose Sicknesses were shorter than the Deferred Period, so were not eligible to claim. Our third is to take out estimated periods Sick but not claiming for those whose Sickness lasted for the Deferred Period, but who failed to claim, having recovered within the run-in period.

6.3.3. Each recorded Inception has, in principle, been Sick long enough to have reached the end of the Deferred Period, so has been Sick for the duration of that Deferred Period. We say “in principle” because the data includes some cases where a Claim does not start on the expected day at the end of the Deferred Period. See Section 6.8 of CMI Working Paper 6 for an analysis of this for Claim records for the period 1975 to 1998. If the Claim also terminates within the Deferred Period, the case has been excluded from this investigation. There are also cases where the recorded date of Claim is after the end of the Deferred Period, sometimes a year later than one might expect. We assume that these might be coding errors, but they have not been investigated, and are not excluded. The numbers of all of these are small, so we assume in these calculations that each Claim has in fact occurred at the end of the specified Deferred Period.

6.3.4. We can calculate the years spent in the Deferred Period quite simply, by multiplying the recorded number of Inceptions (cum Duplicates at this stage) by the appropriate number of days, and then dividing by 365. We have attributed each Inception according to the estimated age at the date of Commencement of Sickness, denoted Age 1 in paragraph 4.3, and we attribute the number of days in the Deferred Period to the same age without allowing for the fact that the insured may have passed a birthday during that period. For DP1 we allow for the fact that eligible Claims start at the Commencement of Sickness, so we have already counted all the days of Sickness in counting the days of claim. We make no allowance for the growth (positive or negative) of the exposure, because the dates of Commencement of Claim for the recorded Inceptions occur during a particular calendar year and the days Sick during the Deferred Period extend backwards from that date, possibly into the previous calendar years, so match up already with our estimate of the exposure.

6.3.5. The numbers of years to be deducted from the exposure because of Sickness during the Deferred Period, for those whose Sickness lasted long enough for a Claim to have arisen, are shown in Table 6.8. They are necessarily zero for DP1, and larger for the longer DPs than for the shorter ones, even though there are fewer Inceptions for the longer DPs.

Table 6.8: Assumed years Sick before claiming, for observed Inceptions, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52.

Year	DP1	DP4	DP13	DP26	DP52	DPs 0+2+8	Total
1991	0.0	34.9	32.7	64.3	49.9	0.2	182.0
1992	0.0	34.1	45.9	64.8	91.7	0.2	236.7
1993	0.0	31.5	43.9	80.3	40.9	0.4	197.0
1994	0.0	28.0	45.9	101.2	72.8	0.5	248.4
1995	0.0	35.4	63.6	130.1	128.6	0.3	358.1
1996	0.0	28.4	54.6	117.2	100.7	0.2	301.1
1997	0.0	30.9	51.9	119.7	126.7	0.1	329.2
1998	0.0	32.5	53.9	108.2	97.7	0.3	292.7
Total	0.0	255.8	392.2	785.8	709.1	2.3	2,145.2

6.3.6. We can compare these values with those shown in Table 6.5 for the years actually claiming. They are all very much shorter, but proportionately longer for the longer Deferred Periods.

6.3.7. Our second adjustment requires us to estimate the number of Sicknesses that have occurred, whether or not there has been a Claim, based on the recorded number of Inceptions. At this point we start making use of the auxiliary functions described in Section 5. Up to now we have needed only the actual data. We show results in the tables below assuming both the Type 2 recovery rates and the Type 3 rates for DP4. We have, as yet, made no assumptions about the Termination rates for DP0, DP2 and DP8, so we omit them from now on, and we also omit totals for any table that shows values that depend on these rates.

6.3.8. We calculate, for each Age and each Deferred Period, the estimated number of Sicknesses by the formula for E[S|I] shown in paragraph 5.7.2:

$$\text{Estimated Sicknesses} = (\text{Number of Inceptions (Age, DP)} + 1) \,/\, p(\text{Age, DP}) - 1$$

where p(Age, DP) is the probability of a Sickness lasting for the Deferred Period and then a Claim being made, which equals $\pi(x+½, d) \times \eta(x+½, d)$, as shown for quinquennial ages in Table 5.7. The resulting number of estimated Sicknesses is shown in Table 6.9. These include Sicknesses that result in Inceptions. We make one exception to the formula: if the exposure for any Age is zero, the estimated number of Sicknesses is also taken as zero. But if the number of Inceptions for any Age is zero, the formula produces a (relatively small) number of estimated Sicknesses.

Tables 6.9: Estimated number of Sicknesses, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52, using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	8,751.2	1,450.8	5,952.6	3,578.6	1,048.4	34,286.7
1992	7,354.3	1,495.3	7,319.7	4,008.2	1,313.7	38,980.5
1993	8,390.6	1,287.4	6,778.1	3,929.4	984.4	29,624.0
1994	6,360.0	1,188.8	6,885.8	5,241.0	1,279.8	27,155.4
1995	6,725.6	1,461.4	8,197.6	5,831.1	1,714.5	33,241.6
1996	5,931.2	1,105.2	8,427.5	5,475.8	1,493.7	22,379.4
1997	5,201.5	1,199.6	7,639.6	5,256.2	1,782.4	22,765.2
1998	4,894.9	1,240.4	8,465.5	5,573.2	1,581.3	24,081.8
Total	53,609.4	10,428.9	59,666.3	38,893.5	11,198.3	232,514.6

6.3.9. The estimated numbers of Sicknesses are very much larger than the numbers of Inceptions, shown in Table 4.2a, and should perhaps be compared with the exposure. Table 6.10 shows the estimated number of Sickness per 100 years of exposure, using the years of exposure (*R2*) shown in Table 6.6.

Table 6.10: Estimated number of Sicknesses per 100 years of exposure, using exposed to risk *R2*, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP0 to DP52, using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	37.8	5.6	21.7	10.8	7.0	131.8
1992	33.3	5.7	23.9	9.2	6.8	149.6
1993	40.1	4.9	20.2	8.1	4.0	112.1
1994	31.9	4.8	21.3	11.0	4.9	109.9
1995	35.2	6.0	19.3	10.0	5.3	136.9
1996	32.6	4.7	20.1	9.3	4.5	94.3
1997	29.5	4.5	17.5	8.4	4.7	86.1
1998	29.1	4.3	15.3	7.4	3.3	82.6
Total	34.0	5.0	19.4	9.1	4.7	112.4

6.3.10. The estimated number of Sicknesses per year of exposure varies very much between Deferred Periods; there is a very great difference between the results for DP4 with Type 2 and Type 3 rates. This shows how uncertain our backwards estimation process is, but in fact it has little effect on the resulting Claim Inception rates, as we see in Section 7. The method we have used, described in Section 5.7, uses only the number of Inceptions, and assumes an unlimited exposure from which the estimated Sicknesses might be drawn. The method used previously (as set out for example in CMIR 12) estimated the number of Sicknesses from the exposure and the graduated Sickness rates; this required a recursive process of estimation, which our method avoids, and it ignores the observed Inceptions. Some method that combined both the given exposure and the given Inceptions might be better.

6.3.11. We also calculate, for Deferred Periods with a run-in period, the estimated number of Sicknesses that reach the end of the Deferred Period, and for which no Claim is made. We use the formula shown in paragraph 5.7.5 to calculate E[N] and then deduct the number of recorded Inceptions, I. The numbers are shown in Table 6.11. For DP4 with Type 3 rates the numbers are larger than the number of Inceptions, shown in Table 4.2a.

Table 6.11: Estimated number of eligible Sicknesses for which no Claim is made, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP4 and DP13, using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP4(2)	DP13(2)	Total(2)	DP4(3)
1991	64.0	16.2	80.2	549.7
1992	65.2	20.7	86.0	569.3
1993	57.7	19.9	77.6	493.1
1994	53.8	20.2	74.1	464.1
1995	65.7	26.2	91.9	565.6
1996	50.8	24.3	75.1	427.7
1997	55.6	22.8	78.4	468.1
1998	57.0	23.6	80.6	478.8
Total	469.8	173.9	643.7	4,016.6

6.3.12. We now calculate the expected number of days Sick, for each Age and Deferred Period, for those whose Sickness terminated in the Deferred Period. We use the estimated numbers of Sicknesses (Table 6.10), excluding the actual numbers of Inceptions and the estimated numbers of eligible Sicknesses for which no Claim is made (Table 6.11), and multiply by the expected numbers of days Sickness among those Claims that terminate before the end of the Deferred Period, as shown for quinquennial ages in Table 5.5. The resulting periods, in years, are shown in Table 6.12.

Table 6.12: Estimated years Sick for those whose Sickness terminated in the Deferred Period, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52, using Type 2 Termination rates; also Type 3 rates for DP4

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	33.4	22.3	121.3	108.2	52.4	380.0
1992	27.9	23.3	152.3	117.8	69.1	426.4
1993	31.7	19.8	143.2	121.3	48.9	331.7
1994	23.3	18.6	144.7	160.6	64.4	310.8
1995	24.5	22.7	179.5	183.2	89.3	379.1
1996	21.3	16.9	177.3	175.0	77.5	260.5
1997	18.5	18.4	162.2	166.5	91.6	272.4
1998	17.0	18.8	174.0	169.3	79.7	283.8
Total	197.5	160.9	1,254.4	1,201.9	573.0	2,644.6

6.3.13. We can compare these also with the values shown in Table 6.5 for the years actually claiming. For DP1 they are quite small, because we are including only Sicknesses that recover with one week, but for the other Deferred Periods they are a significant fraction of the Table 6.5 numbers.

6.3.14. Our third adjustment is for those with Deferred Periods that include an assumed run-in period, DP4 and DP13. This is for the periods spent Sick, both during the Deferred Period and during the run-in, when a Sickness that would be eligible is not claimed for. To calculate this we take the estimated numbers of non claims and multiply by the sum of the Deferred Period and the expected number of days Sick, all during the run-in period, among those not claiming (shown in Tables 5.6(a), (b) and (c), as "*ec*"). The resulting values are shown in Table 6.13.

Table 6.13: Estimated years Sick for those whose Sickness terminated in the run-in period, but who did not claim, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP4 and DP13, using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP4(2)	DP13(2)	DP4(3)
1991	6.5	4.5	64.4
1992	6.6	5.7	66.4
1993	5.8	5.5	57.8
1994	5.4	5.6	54.2
1995	6.6	7.2	66.1
1996	5.1	6.7	50.2
1997	5.6	6.3	55.0
1998	5.7	6.5	56.2
Total	47.3	47.9	470.4

6.3.15. We can see from Table 6.13 that the periods are quite small, especially for DP13 and DP4(2) where we are assuming a four-week run-in period.

6.3.16. We can put together all three adjustments and show the totals in Table 6.14. We deduct these from the exposures shown in Table 6.6 (*R2*) and get an adjusted exposure, which we denote *R3*, as shown in Table 6.15. Note that Table 6.14 allows also for any adjustments that are necessary to stop the exposure in any cell becoming negative. These are very small.

6.3.17. We can see that the difference in the total exposures for DP4 using Type 2 Termination rates or Type 3 rates is not large. The difference between some of the adjustments is quite large but the adjustments themselves are not large in proportion to the original total.

6.3.18. In Table 6.16 we show the ratios, as a percentage, that the time deducted because of Sickness not claimed for bears to the original exposure before any deductions (*R1*). These can be compared with the percentages shown in Table 6.7. They are all much smaller, except for DP4(3), but they are not negligible.

Table 6.14: Years Sick and not claiming, cum Duplicates, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52, using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	33.4	63.7	158.4	172.5	102.0	479.3
1992	27.9	64.0	203.9	182.4	160.4	526.7
1993	31.7	57.2	192.5	201.4	89.8	420.7
1994	23.3	52.1	196.1	261.1	136.8	392.4
1995	24.5	64.6	250.0	312.8	216.3	480.6
1996	21.3	50.4	238.1	292.1	177.2	338.8
1997	18.5	54.9	220.0	286.1	217.6	358.0
1998	17.0	57.1	234.3	277.5	176.9	372.4
Total	197.5	464.0	1,693.4	1,986.0	1,277.1	3,369.0

Table 6.15: Exposed to risk (*R3*), at Stage 3, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52, using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	23,122.9	25,954.1	27,213.4	33,089.0	14,900.0	25,538.5
1992	22,027.7	25,997.1	30,432.0	43,195.9	19,060.0	25,534.4
1993	20,893.1	26,364.6	33,285.0	48,185.4	24,496.8	26,001.1
1994	19,883.1	24,648.2	32,177.2	47,339.9	25,942.4	24,307.8
1995	19,063.8	24,219.0	42,249.2	58,183.4	32,009.8	23,803.1
1996	18,186.6	23,673.6	41,793.4	58,340.0	33,180.6	23,385.2
1997	17,622.2	26,380.0	43,526.3	62,370.6	37,872.2	26,076.9
1998	16,778.3	29,103.8	55,010.3	75,127.7	47,403.1	28,788.5
Total	157,577.7	206,340.4	305,686.9	425,831.9	234,864.8	203,435.5

Table 6.16: Ratio 100 × Adjusted time Sick but not claiming / *R1*, for each Year 1991-98, for Standard*, Males, Occupation Class 1, DP1 to DP52, using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	0.14	0.24	0.57	0.51	0.67	1.80
1992	0.12	0.24	0.65	0.41	0.83	1.97
1993	0.14	0.21	0.57	0.41	0.36	1.55
1994	0.11	0.21	0.59	0.54	0.52	1.55
1995	0.12	0.26	0.58	0.53	0.66	1.92
1996	0.11	0.21	0.56	0.49	0.52	1.38
1997	0.10	0.20	0.49	0.45	0.56	1.31
1998	0.09	0.19	0.42	0.36	0.37	1.24
Total	0.12	0.22	0.54	0.46	0.53	1.58

6.4. *Stage 4: Duplicates*

6.4.1. So far we have based all our calculations for the exposure on the cumD files, but we now have to switch to exD figures. In most cases we multiply the exposures, *R3*, by the ratio of the number of exD Inceptions to the number of cumD Inceptions. However, for some combinations of Age, Year and Deferred Period there are no Inceptions, either cumD or exD, so we multiply the exposure, *R3*, by the overall ratio of exD Inceptions to cumD Inceptions for all Ages for that Year and Deferred Period. The inverse of these overall ratios are shown in Table 4.2c. The resulting assumed exD exposures, *R4*, are shown in Table 6.17.

6.4.2. When these calculations were first carried out, we observed that in some "cells", the numbers of cumD cases was non-zero, but the number of exD cases was zero. This was inconsistent, and it caused us to revisit the whole question of how Duplicates were defined. The new method, Method 10, set out in the Duplicates Note, avoids this problem.

Table 6.17: Exposed to risk (*R4*), at Stage 4, adjusted for exD Inceptions,
for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52,
using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	9,397.5	19,794.8	20,495.1	23,100.0	9,892.9	19,466.2
1992	8,994.7	19,450.0	22,599.6	34,944.0	11,768.4	19,093.4
1993	8,650.7	21,355.8	27,280.3	37,629.9	21,736.4	21,057.9
1994	8,337.6	18,653.0	25,387.3	35,241.7	20,858.4	18,389.0
1995	7,971.8	18,284.1	33,652.4	46,730.9	26,128.5	17,962.3
1996	7,374.0	18,747.4	31,433.9	43,140.7	26,475.8	18,516.1
1997	7,368.8	20,113.0	35,107.2	53,092.7	29,887.7	19,877.4
1998	6,857.2	21,893.6	48,616.6	63,927.5	42,703.2	21,655.0
Total	64,952.2	158,291.7	244,572.4	337,807.3	189,451.3	156,017.5

6.5. *Stage 5: Scaling down*

6.5.1. The next stage is to adjust the exposures by multiplying by $\pi.\eta$, which equals the probability of survival to the end of the Deferred Period, times the probability that a Claim is then made, or $\pi(x+½,0,d) \times \eta(x+½,d)$. The totals for the resulting values, *R5*, are shown in Table 6.18.

6.5.2. We note that *R5* (and *R6* in the final stage) are not true measures of exposure. This is because, although up to *R4* we are measuring in policy years, the multiplication by $\pi(x+½,0,d) \times \eta(x+½,0,d)$ in this stage destroys the units. The calculations in Stages 1, 2, 3, 4 and 6 (which could equally well be performed before Stage 5) are all the necessary steps to produce the true exposure, *R*, but Stage 5 converts this to $R.\pi.\eta$ which is required to allow us to get an estimator of the Sickness rate, σ, as explained in paragraph 3.1.

Table 6.18 Exposed to risk (*R5*), at Stage 5, for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52,
using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	4,238.5	5,829.3	950.6	1,559.9	1,097.7	543.1
1992	4,167.8	5,968.9	1,025.1	2,174.5	1,374.9	594.8
1993	4,031.6	6,612.1	1,245.4	2,241.3	2,196.6	659.7
1994	3,924.4	5,915.6	1,118.6	2,226.1	2,049.1	614.9
1995	3,816.1	5,923.8	1,559.0	2,773.2	2,517.0	641.3
1996	3,620.7	6,008.0	1,542.5	2,645.0	2,644.7	614.6
1997	3,695.5	6,602.6	1,808.2	3,176.1	2,939.7	719.9
1998	3,470.5	6,736.5	2,081.3	3,502.7	4,043.3	662.7
Total	30,965.1	49,596.8	11,330.8	20,298.8	18,863.0	5,051.0

6.6. *Stage 6: Policy expiry age adjustment*

6.6.1. The final stage relates only to the high ages in the data. If an IP policy with 52 weeks Deferred Period ceases at age 65 then any Sickness that starts when the policyholder is aged 64 cannot produce a Claim. Our recorded Claim Inceptions are classified by age last birthday at the Commencement of Sickness, and we can observe that there are no recorded Inceptions at age 64 last birthday in the DP52 data. The CMI Individual IP dataset contains information on policy expiry age (actually expiry year) for some but not all of the data records. However, it is consistent with the data, and not implausible, to assume that it is age 65 for men and age 60 for women. There are a few Inceptions and In force at higher ages recorded, but they are very few. We have therefore adjusted the exposure at age 64 for males and age 59 for females by reducing the numbers by the fraction *d*. We have also deleted any exposure at a higher age than this chosen maximum. This means that for DP52 there is no exposure above age 63 (that is, the DP52 exposure for age 64 and above has been set to zero).

6.6.2. The final adjusted exposures, *R6*, are shown in Table 6.19. They are little different from those shown in Table 6.18. In addition the number of eligible Inceptions is reduced slightly, in fact by only 1 for Males, Occupation Class 1, in DP1 in 1991. At this stage we add the years together, and pass over the number of Inceptions, ex Duplicates, together with the value of *R6*, for each Age and Deferred Period, to the graduation process.

Table 6.19 Exposed to risk (*R6*), at Stage 6, for each Year 1991-98,
for Standard*, Males, Occupation Class 1, DP1 to DP52,
using Type 2 Termination rates; also Type 3 rates for DP4.

Year	DP1	DP4(2)	DP13(2)	DP26	DP52	DP4(3)
1991	4,234.4	5,820.4	936.8	1,508.7	1,081.9	540.1
1992	4,165.9	5,959.7	1,013.4	2,122.5	1,342.7	591.8
1993	4,030.4	6,604.0	1,230.2	2,213.6	2,148.0	657.1
1994	3,922.8	5,904.1	1,111.1	2,183.4	2,011.4	611.2
1995	3,815.1	5,912.2	1,544.8	2,708.3	2,468.5	637.4
1996	3,619.0	5,998.6	1,528.0	2,612.3	2,591.3	611.5
1997	3,694.2	6,595.2	1,801.1	3,127.0	2,900.6	717.4
1998	3,469.1	6,730.5	2,065.0	3,449.8	3,985.6	660.8
Total	30,950.8	49,524.8	11,230.5	19,925.7	18,529.9	5,027.3

6.7. *Example*

6.7.1. This whole process is quite complicated, though it requires rather little source data. Because it is unfamiliar, we describe in detail in Appendix B how it is done for one specific Age (40), for one Year (1991), for one Deferred Period (DP13), for Males, Occupation Class 1, the Class that we have used throughout this Section.

6.7.2. Calculations which are the same in principle as those described in Appendix B, but vary in detail because values may be zero, are carried out for every Age, every Year, every Deferred Period, every Occupation Class, and each Sex. For Females and for Occupation Classes other than Class 1 the same factors as for Males, Occupation Class 1, are used. This is inconsistent, but in the absence of other Termination rates it is the best we can do.

7. GRADUATIONS

7.1. *Preliminary comments*

7.1.1. We now have available the numbers of events (Inceptions) and the (central) exposed to risk at each Age, for all the Years 1991 to 1998 added together, within each Deferred Period. For the graduations we use only the data for Males, Occupation Class 1. From the numbers of events and the exposed to risk we can get crude rates of Sickness, one for each Age, and we can therefore proceed to graduating these rates using a suitable formula. Some brief preliminary comments are desirable.

7.1.2. The process is exactly like the graduation of mortality data. For a general description of the statistical techniques and tests used in our graduation process we refer the reader to the paper "*On Graduation by Mathematical Formula*" by Forfar, McCutcheon and Wilkie (1988). [Forfar, D O, McCutcheon, J J and Wilkie, A D, *J.I.A.* **115**, 1-149 and *T.F.A.* **41**, 97-269, and discussion thereon *J.I.A.* **115**, 693-708.]

7.1.3. On the previous occasion (CMIR 12) it was found convenient to fit a formula to $\sigma(x)$ of the form $\sigma(x) = \exp(\text{polynomial in } x)$, and for three of the Deferred Periods a polynomial of order three was necessary, e.g. $\sigma(x) = \exp(a + bx + cx^2 + dx^3)$. For DP26 a polynomial with one term fewer was sufficient.

7.1.4. On this occasion we use a wider range of possible graduation formulae, the GM(r,s) series, familiar in the graduation of mortality data. These are of the form:

$$\sigma(x) = (\text{polynomial in } x \text{ of degree } r-1) + \exp(\text{polynomial in } x \text{ of degree } s-1)$$

The formulae used previously fit into this series as GM(0,s) formulae, with no "r" term.

7.1.5. We do not, however, use polynomials in the Age, x, directly. First we scale the Age by putting:

$$t = (x - 40) / 25$$

so that $t = -1$ when $x = 15$, $t = 0$ when $x = 40$ and $t = +1$ when $x = 65$. Next, we use Chebycheff polynomials in t. These are defined by:

$$C_0(t) = 1$$
$$C_1(t) = t$$

and thereafter

$$C_{n+1}(t) = 2t\, C_n(t) - C_{n-1}(t)$$

The advantage of using these Chebycheff polynomials is that, over the range (–1, +1) they are orthogonal. Thus, if the data is spread reasonably evenly over the age range that corresponds with $t = (-1, +1)$, in this case $x = (15, 65)$, then, when we add one term to the polynomial, the coefficients of the earlier terms do not change very much, and it is possible to estimate the statistical significance of the added term just by comparing its estimated value with the standard error of that estimate, and test whether its value is significantly different from zero. This test would not be possible if we were to use powers of x unadjusted.

7.1.6. We use maximum likelihood to find the best parameters of the "best fitting" formula for each combination of r and s. As we assume the number of Sicknesses is Poisson distributed with parameter $R.\sigma(x)$, and use the log likelihood, the exercise simplifies to the problem of maximising the function:

$$L(\theta) = \sum_x (I(x).\ln \sigma(x) - R(x).\sigma(x))$$

where θ is the set of parameters being fitted. We then calculate the standard errors of the estimates of these parameters by inverting the information matrix.

7.1.7. We then choose an appropriate formula taking into account a number of criteria:

(a) the value of the maximum likelihood, allowing for the number of parameters used;

(b) whether the value of any extra parameter is statistically significant;

(c) whether the various non-parametric tests, count of positives and negatives runs test and Kolmogorov-Smirnov test, are satisfied;

(d) whether the results of the parametric tests, serial correlations, Pearson χ^2 test, testing $X^2 = \sum(A-E)^2 / E$ against a χ^2 distribution, and also the Poisson Deviance test, described in Section 7.2, are satisfactory;

(e) whether the curve produces sensible values outside the range of the given data.

7.1.8. Considering (e) further: we have data from about age 18 to age 65, but there is rather little at the youngest ages, and the data shows peculiar features at the highest age. Yet we would like the graduated Sickness rates to give plausible values from say age 16 to age 70. At present most Individual IP policies expire at (or before) age 65, but there is nothing to stop offices writing policies to a later age, and current attitudes to lengthening the working lifespan suggest that this may become common; in which case offices could make use of plausible rates of Sickness (and of recovery and death from 'in Claim') beyond age 65.

7.1.9. The principles that we have used are a combination of the criteria just described. In general we choose the graduation with the fewest parameters that fits the data according to the non-parametric tests and also behaves sensibly between ages 16 and 70. If an extra parameter makes a big difference to the log likelihood or to the deviance, we may use it, provided it still gives sensible values over the desired age range.

7.2. *Poisson deviance*

7.2.1. The deviance of a model is defined as the difference between the maximum log likelihood of the fitted model, and the log likelihood of a saturated model in which there are enough parameters so that the expected number of events is equal to the actual number at all Ages (cells). If we can assume that the number of events is normally distributed, then the deviance (the "usual" or "Pearson" deviance) is the traditional value denoted X^2 in paragraph 7.1.7(d).

7.2.2. Alternatively, if the events are assumed to be Poisson distributed than a Poisson deviance is defined and is equal at each Age to:

$$2 \times (A \times \ln(A/E) - (A - E)) \qquad \text{if } A \neq 0$$
$$2 \times E \qquad \text{if } A = 0$$

The total deviance is then the sum of these values over all Ages. The Poisson deviance, like the Pearson deviance, is then distributed as χ_n^2 with the same number of degrees of freedom as usual.

7.2.3. The Poisson deviance is quite similar to the Pearson deviance if the values of A and E are reasonably large. But when the values of A and E are small the Poisson deviance gives more suitable results, and has the advantage that grouping to get the numbers large enough is not necessary. One can still group if desired, but it is not essential. We use both definitions of the deviance in what follows.

7.2.4. We note that, in calculating the Pearson deviance we allow for the continuity adjustments. That is, in calculating the difference between the actual number of events, *A*, and the expected number, *E*, we allow for the fact that the actual number is necessarily an integer, so move the difference ½ nearer to zero, so that the adjusted difference, "adj *D*" is calculated:

if	$A - E > 0.5$	then	adj $D = A - E - 0.5$
if	$-0.5 \leq A - E \leq 0.5$	then	adj $D = 0$
if	$A - E < -0.5$	then	adj $D = A - E + 0.5$

We then use this adjusted difference in the calculation of X^2 and in the calculation of the serial correlation coefficients.

7.2.5. For the calculation of X^2 we group the data so that each age group has at least 5 expected events. For the Poisson deviance and all the other tests this grouping is not done. At this stage it makes little difference, because most ages have enough data. But when we investigate smaller experiences the difference is noticeable.

7.3. *DP1*

7.3.1. We start with the data for DP1, Deferred Period one week. The exposed to risk runs from ages 22 to 64 and the Inceptions from age 23 to 64. We have already excluded the exposure and the one Inception at and above age 65. This gives us 43 consecutive years of useable data. In this data there are 9,696 Inceptions and 30,950.8 years of (adjusted) exposure. This gives a crude rate of Sickness of 0.3133 per year (the logarithm of which is –1.1606).

7.3.2. We start by fitting a number of GM(*r*,*s*) combinations and then choosing those that have a small number of parameters and fit the data adequately. For DP1, as for the other

DPs, only GM(0,*s*) formulae are necessary. This gives the same sort of formula as last time, with ln(σ(*x*)) = polynomial in *x* (or *t*). We find that the choice for DP1 is between GM(0,4) and GM(0,5). The statistics for these (and the graduations for the other Deferred Periods) are shown in Table 7.1, and the graduated rates are graphed in Figure 7.1, which shows also the crude rates, and the high and low "gates", which show the confidence interval for the observed value of σ(*x*) at a 95% level.

7.3.3. We can see that several of the test statistics for GM(0,4) show unsatisfactory values, whereas GM(0,5) appears to fit much better. There are 11 runs for GM(0,4) but there are 24 for GM(0,5). The three serial correlation coefficients, for which we show "T" values, are significantly high for GM(0,4), but all are satisfactory for GM(0,5). The values of X^2 and of the Poisson Deviance are large for GM(0,4), but satisfactory for GM(0,5). The maximised log likelihood for GM(0,5) is 22.3 better than for GM(0,4), a very big difference. Yet when we look at the graphs we do not see a great difference between the GM(0,4) and GM(0,5) rates, except at the extremes, where the GM(0,5) rates fall very uncomfortably as we go down to age 15 or up to age 70. For that reason we prefer the GM(0,4) rates.

7.3.4. In Table 7.1 we identify ages where the deviance is particularly large (greater than 7.5), and show whether the crude rates at those ages are particularly low or particularly high. The GM(0,4) graduation shows that at age 26 the crude rate is particularly low; there are 39 Inceptions as compared with 59.4 expected; likewise at age 46 where there are 345 Inceptions as compared with 417.2 expected. These ages can be identified on the graph. The little ripples wherein GM(0,5) differs from GM(0,4) give expected values of 50.5 and 393.2 for these ages, with lower deviances. But the ripples that help the curve for GM(0,5) to fit the data better in the middle of the range cause it to diverge rather extremely at the ends.

Figure 7.1. Crude rates, "gates" and graduated values of σ(*x*) for DP1, using GM(0,4) and GM(0,5)

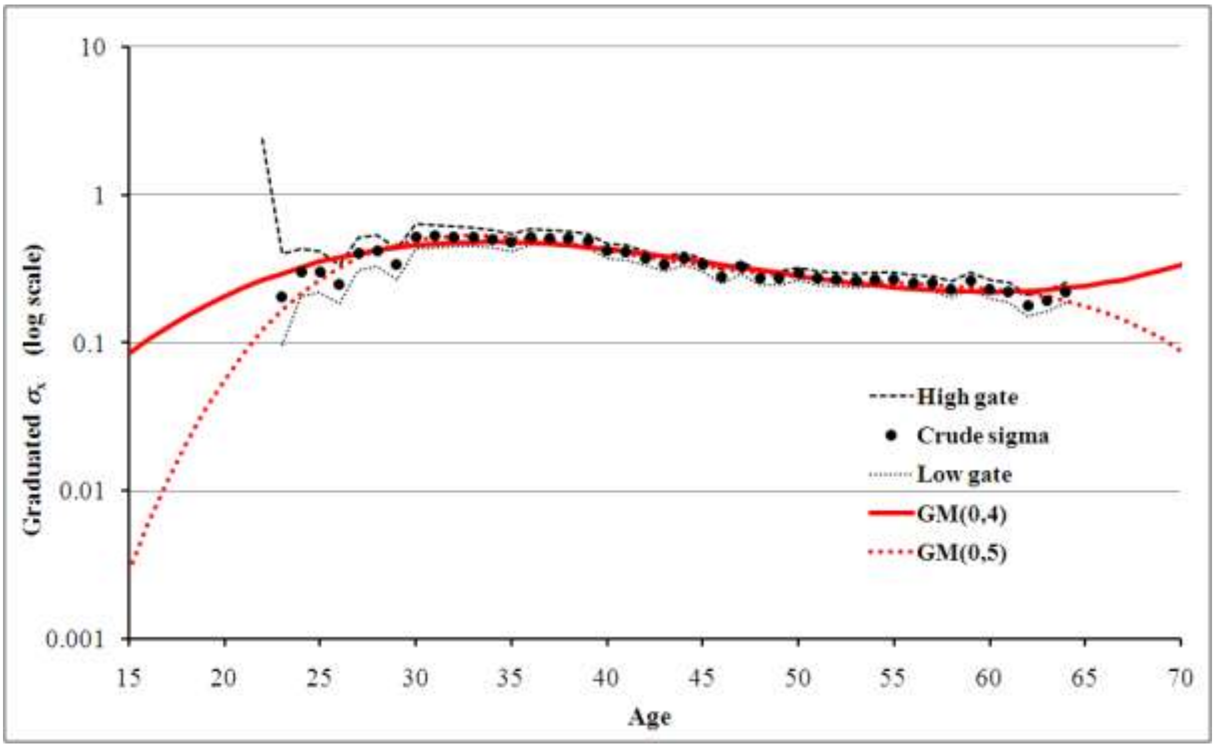


Table 7.1. Statistics for graduations of σ(x) for DP1 to DP52

Deferred Period	DP1	DP1	DP4 (2)	DP4 (3)	DP13(2)	DP26	DP52
Age range	22-64		17-64	18-64	18-64	18-64	18-64
Inceptions	9,696	9,696	2,304	2,304	1,125	1,107	502
Exposures (*R6*)	30,950.8	30,950.8	49,524.8	5,027.3	11,230.5	19,925.7	18,529.9
Crude rate	0.3133	0.3133	0.0465	0.4583	0.1002	0.0556	0.0271
GM(r,s)	(0,4)	(0,5)	(0,2)	(0,4)	(0,2)	(0,2)	(0,2)
Log likelihood	–20,608.4	–20,586.1					
Parameters							
b_0	–1.416038	–2.091355	–3.200943	–0.779340	–2.008366	–2.786287	–3.816687
SE(b_0)	0.0529	0.1199	0.0264	0.0778	0.0487	0.0526	0.0757
b_1	0.238522	1.414159	0.474149	–0.292921	–0.614523	–0.216186	0.507200
SE(b_1)	0.0996	0.2122	0.0505	0.1507	0.0871	0.0917	0.1399
b_2	–0.588151	–1.569621		–1.018972			
SE(b_2)	0.0621	0.1660		0.0997			
b_3	0.333549	0.874816		0.474466			
SE(b_3)	0.0379	0.0924		0.0671			
b_4		–0.310795					
SE(b_4)		0.0473					
+/-	20/23	22/21	25/23	24/23	24/23	21/26	15/32
p(+ or -)	0.3804	0.5	0.4427	0.5	0.5	0.2800	0.0093
runs	11	24	21	23	21	13	19
p(runs)	0.0003	0.3789	0.1553	0.3842	0.1881	0.0006	0.2589
p(K-S)	0.4194	1.0000	0.9232	0.9894	1.0000	0.5919	0.7663
Serial Correlations							
T(r_1)	3.04	–0.38	0.45	0.63	1.10	1.32	0.45
T(r_2)	2.28	–1.12	0.04	0.10	–0.24	0.20	1.01
T(r_3)	2.08	0.80	–1.89	–1.64	1.65	–0.16	1.53
Poisson Deviance	93.43	48.79	74.93	78.32	42.12	59.56	47.43
p(Deviance)	0.000002	0.1128	0.0045	0.0008	0.5945	0.0717	0.3738
X^2 (grouped)	86.98	44.78	64.72	68.96	32.88	44.25	32.05
degrees of freedom	38	37	40	38	37	35	30
$p(X^2)$	0.000010	0.1778	0.0080	0.0016	0.6624	0.1359	0.3654
Low ages	26,46		51, 60	51, 60			
High ages			57	57			45

7.4. *DP4*

7.4.1. The two methods for DP4 that we have used produce very different amounts of adjusted exposures, so must give values of $\sigma(x)$ at very different levels. They also turn out to be rather different shapes.

7.4.2. The basic statistics and the test results for DP4 based on exposure derived using Type 2 Termination rates are shown in Table 7.1 under the heading DP4(2). There is exposure from ages 17 to 64. There were no Inceptions to be omitted above that age. This gives us 2,304 Inceptions and 49,524.8 years of (adjusted) exposure, giving a crude rate of 0.0465. A GM(0,2) formula (a Gompertz, with a positive slope) fits quite satisfactorily, and the results are shown in Figure 7.2a. The values of the Poisson deviance and of X^2 are rather high, although all the other tests are satisfied. Inspection shows that there are low numbers of Inceptions at ages 51 and 60 (actual 60, expected 86.5 at age 51; and actual 45, expected 72.4 at age 60) and high numbers at age 57 (actual 90, expected 64.7).

7.4.3. The basic statistics and the test results for DP4 based on exposure derived using Type 3 Termination rates are also shown in Table 7.1 under the heading DP4(3). There is exposure from ages 18 to 64; at age 17 the deductions for estimated Claims make the exposure zero. This gives us 2,304 Inceptions and 5,027.3 years of (adjusted) exposure, giving a crude rate of 0.4583, rather higher than for DP1 and very much higher than for DP4(2). A GM(0,4) formula is the lowest order one that fits reasonably, and the results are shown in Figure 7.2b. The values of the Poisson deviance and of X^2 are rather high, as for DP4(2), although again all the other tests are satisfied, and the curve has a sensible shape at both ends as it approaches ages 15 and 70. As with DP4(2) there are low numbers of Inceptions at ages 51 and 60 (actual 61, expected 88.4 at age 51; actual 45, expected 69.2 at age 60) and high numbers at age 57 (actual 90, expected 61.9), but we can see from the graph that no reasonably low order formula will get very close to all the crude rates.

Figure 7.2a. Crude rates, “gates” and graduated values of σ(*x*) for DP4(2), using GM(0,2)

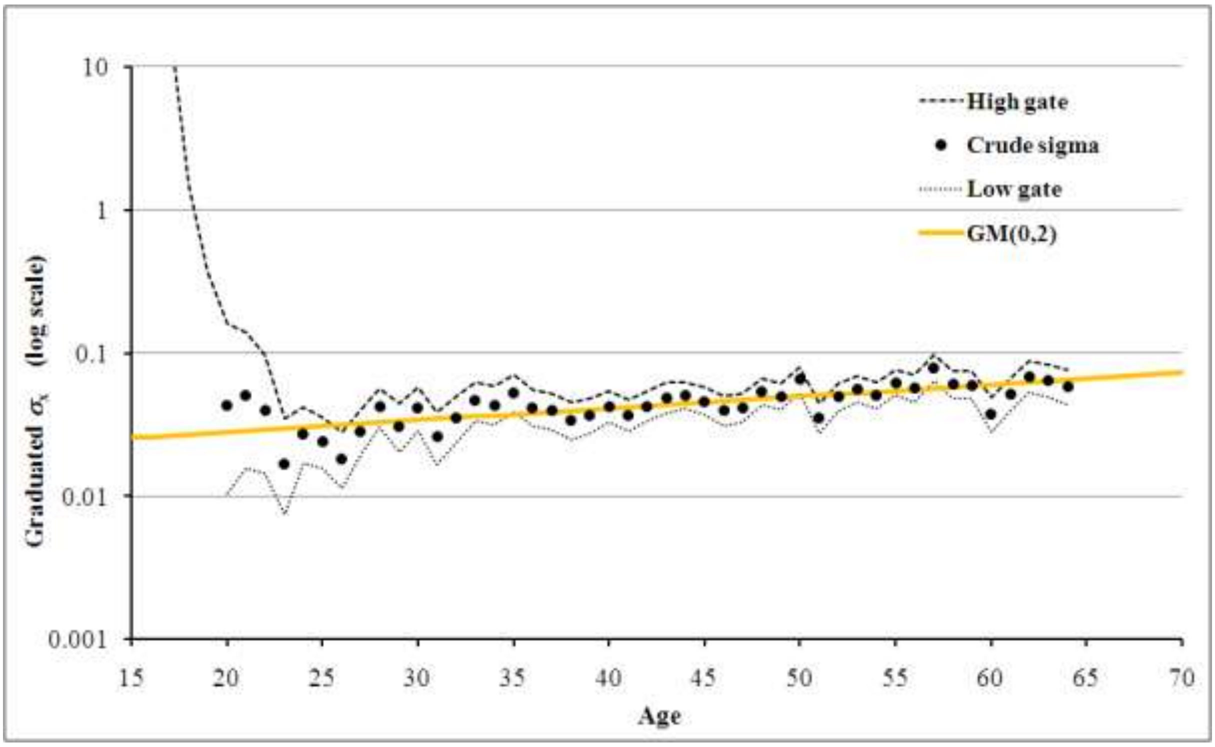
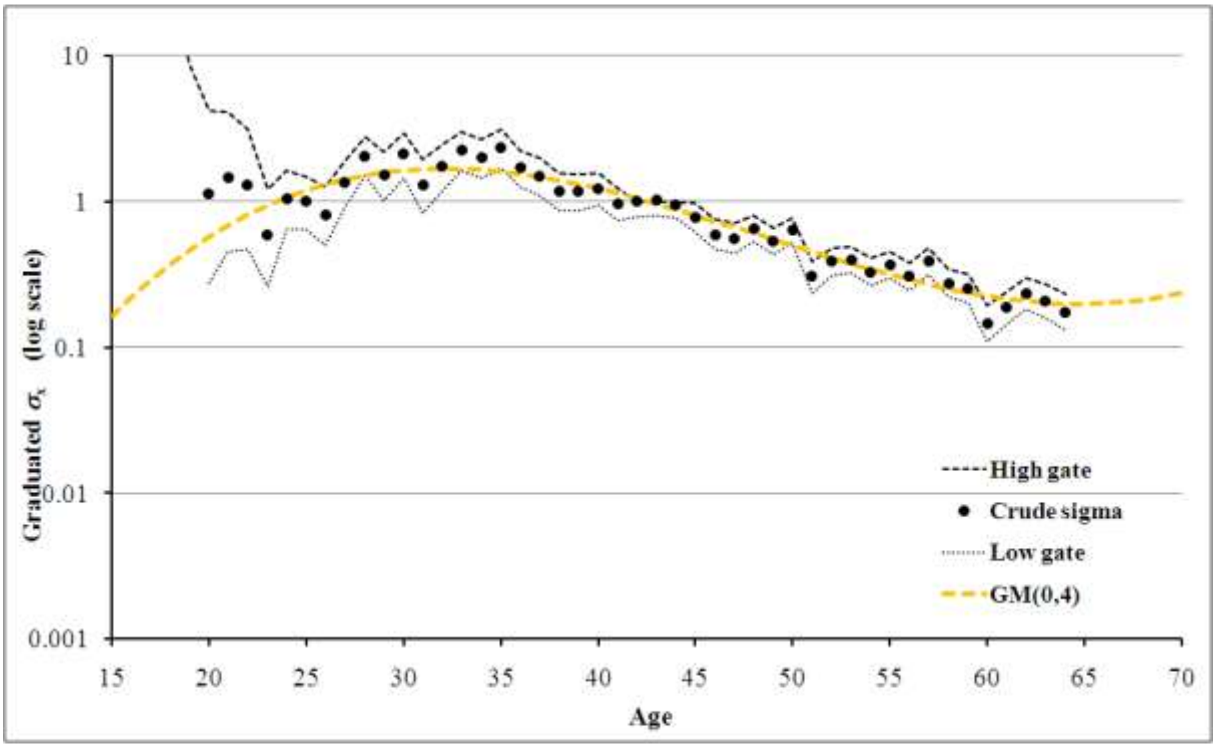


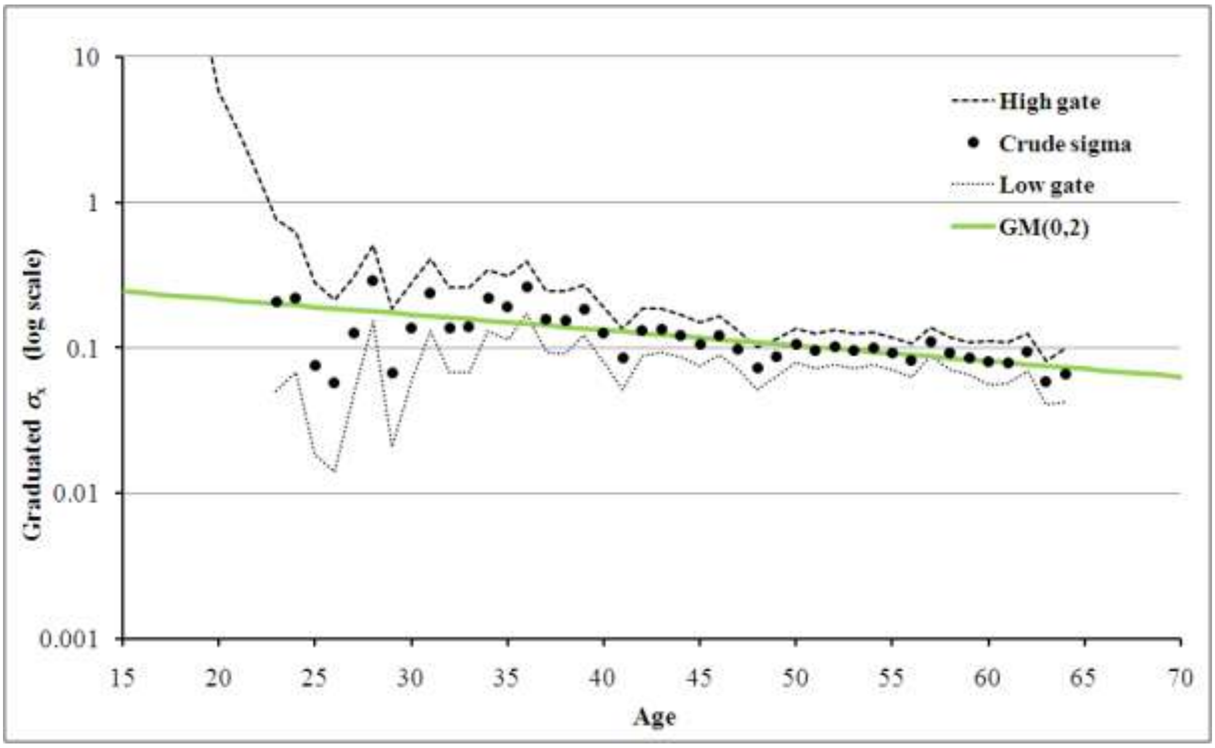
Figure 7.2b. Crude rates, “gates” and graduated values of σ(*x*) for DP4(3), using GM(0,4)



7.5. *DP13*

7.5.1. The basic statistics and the test results for DP13 are shown in Table 7.1. There is exposure from ages 18 to 64; there were no Inceptions above age 64 to be excluded. We have 1,125 Inceptions and 11,230.5 years of (adjusted) exposure, giving a crude rate of 0.1002, well below those for DP1 and DP4(3), but above that for DP4(2). We find that a GM(0,2) formula (a Gompertz with a negative slope) fits adequately, in spite of there being quite enough data to justify a higher order formula if that had been necessary, and the results are shown in Figure 7.3. All the statistical tests are satisfactory, and there are no ages with extreme values. The shape of the curve is quite reasonable as it goes outside the range of the data.

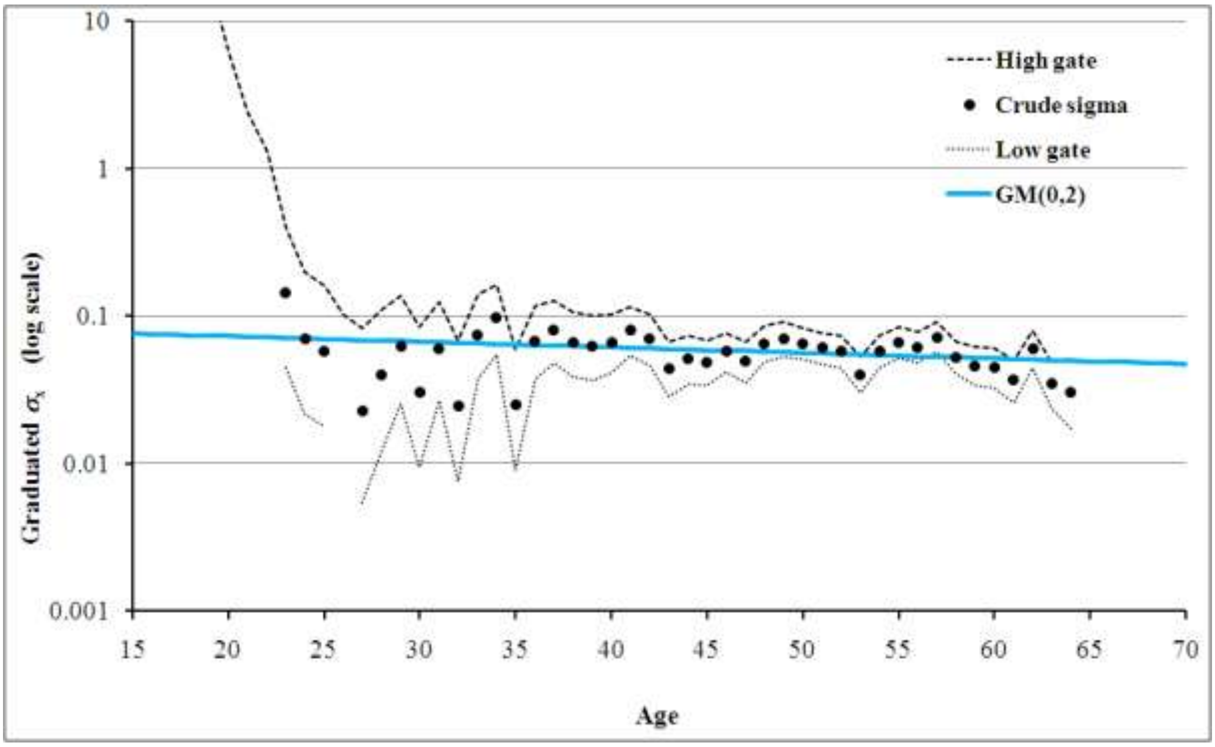
Figure 7.3. Crude rates, "gates" and graduated values of σ(*x*) for DP13, using GM(0,2)



7.6. *DP26*

7.6.1. The basic statistics and the test results for DP26 are shown in Table 7.1. There is exposure from ages 18 to 64, and no Inceptions above age 64. We use ages 18 to 64, and have 1,107 Inceptions and 19,925.7 years of (adjusted) exposure, giving a crude rate of 0.0556, lower again than DP13. We find that again a GM(0,2) formula (a Gompertz also with a negative slope) fits reasonably, and the results are shown in Figure 7.4. The number of runs (13) is rather low, but the values of the Poisson deviance and of X^2 are satisfactory. Inspection shows no ages with extreme numbers of Inceptions.

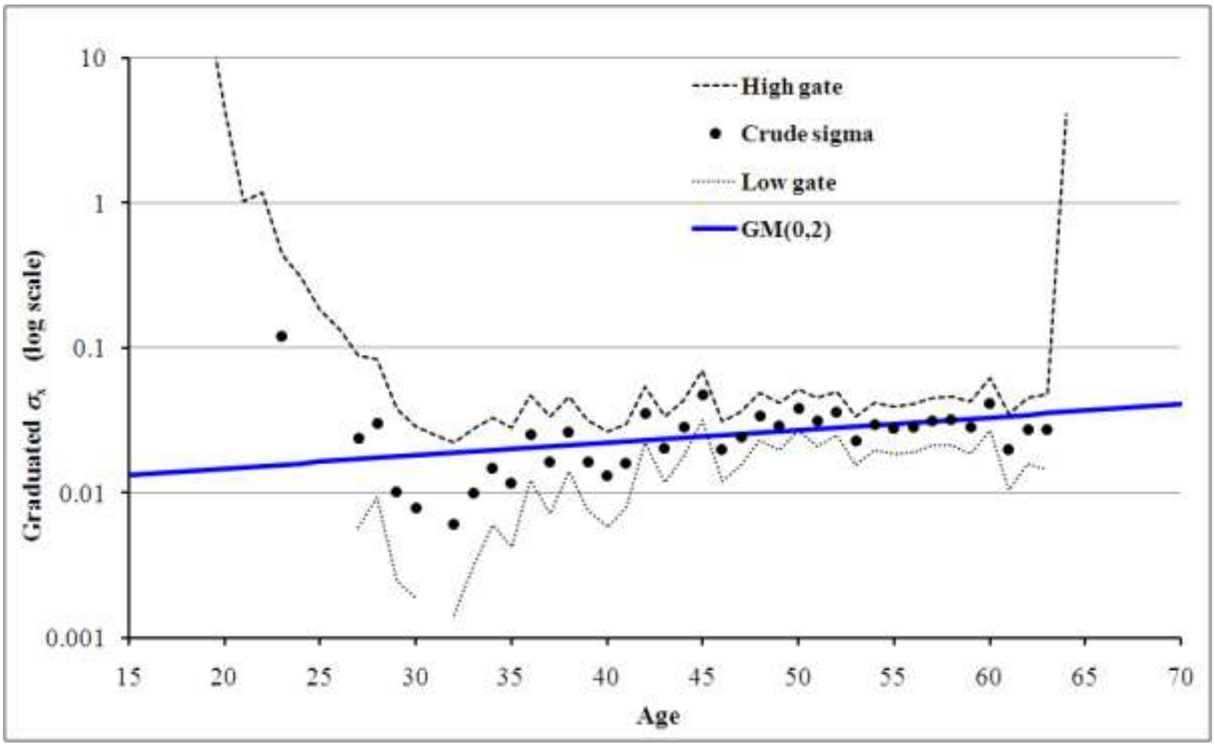
Figure 7.4. Crude rates, “gates” and graduated values of σ(*x*) for DP26, using GM(0,2)



7.7. *DP52*

7.7.1. The basic statistics and the test results for DP52 are shown in Table 7.1. There is exposure from ages 18 to 64. We have 502 Inceptions and 18,529.9 years of (adjusted) exposure, giving a crude rate of 0.0271, much lower than that for DP26. We find that again a GM(0,2) formula (a Gompertz but this time with a positive slope) fits reasonably, with almost all the tests satisfied, and the results are shown in Figure 7.5. However, there are 15 positive and 32 negative deviations; but 10 of the ages have zero Inceptions, so that negative deviations are unavoidable. Inspection shows high numbers of Inceptions at age 45 (23 actual, 11.8 expected).

Figure 7.5. Crude rates, "gates" and graduated values of σ(x) for DP52, using GM(0,2)



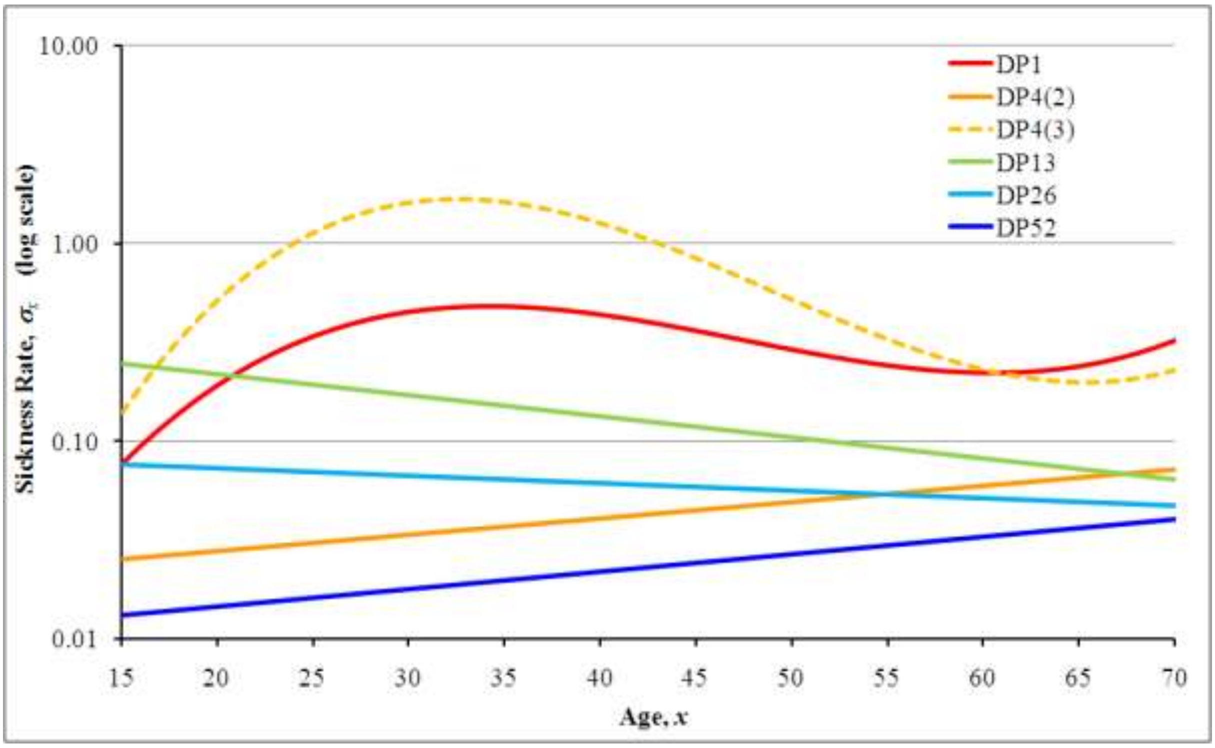
7.8. *All DPs, σ.π.η factors, and the intensity of Claim incidence*

7.8.1. In Table 7.2 we show values of the graduated Sickness rates, σ(x), at quinquennial ages, from 15 to 70, and in Figure 7.6 we show graphs of the graduated Sickness rates for all Deferred Periods. We can observe that the rates for DP4(3) are mostly rather higher than for DP1, but are a similar shape. All the others are straight lines (on a logarithmic scale), with DP13 mostly lower than DP1 or DP4, but higher than DP26 which in turn is higher than DP52. DP4(2) is very much lower than DP4(3), and crosses the lines for some of the longer DPs. Note that the graphs in Figures 7.1 to 7.5 are not identical with those shown in Figure 7.6, because the former show values for ages offset by half a year (15.5, 16.5, etc) since the data are classified by age last birthday, i.e. for ages x to x+1, whereas the latter shows values of σ(x) for integral values of x (15, 16, etc).

Table 7.2. Graduated Sickness rates, σ(x), for Deferred Periods DP1 to DP52.

Age, x	DP1	DP4(2)	DP4(3)	DP13(2)	DP26	DP52
15	0.076058	0.025347	0.138089	0.248120	0.076528	0.013248
20	0.191259	0.027868	0.515147	0.219424	0.073289	0.014663
25	0.338827	0.030641	1.134084	0.194048	0.070188	0.016228
30	0.450844	0.033688	1.613854	0.171606	0.067218	0.017961
35	0.480375	0.037039	1.626117	0.151759	0.064374	0.019878
40	0.436972	0.040724	1.270781	0.134208	0.061650	0.022001
45	0.361790	0.044775	0.843693	0.118686	0.059041	0.024349
50	0.290673	0.049228	0.521262	0.104960	0.056543	0.026949
55	0.241607	0.054125	0.328283	0.092821	0.054150	0.029826
60	0.221505	0.059509	0.230847	0.082086	0.051859	0.033011
65	0.238803	0.065429	0.198540	0.072593	0.049664	0.036535
70	0.322768	0.071937	0.228760	0.064197	0.047563	0.040436

Figure 7.6. Sickness rates, σ(x), for DP1 to DP52, with both DP4(2) and DP4(3).



7.8.2. What is important, however, is not just the values of σ(x), the assumed intensity of Sickness, but the values of $\sigma(x).\pi(x,d).\eta(x,d)$, which in combination gives the intensity of going Sick, surviving as Sick to the end of the Deferred Period, and then claiming, or put more simply, the intensity of Claim incidence. In Tables 7.3, 7.4 and 7.5 we show these factors, successively, $\pi(x,d)$ then $\eta(x,d)$, then $\sigma(x).\pi(x,d).\eta(x,d)$. The values of $\pi(x,d)$ are close to those shown in Table 5.2, but are for integral values of x, not ages x+½, and likewise the values of $\eta(x,d)$ are close to the complement of the probability of not claiming shown in Table 5.4, but are for integral ages.

Table 7.3. Values of $\pi(x,d)$, probability of survival from age x to end of Deferred Period d.

Age, x	DP1	DP4(2)	DP4(3)	DP13(2)	DP26	DP52
15	0.414480	0.407425	0.060571	0.062358	0.072242	0.129311
20	0.323193	0.305479	0.024847	0.024555	0.030077	0.066919
25	0.277522	0.249734	0.013398	0.012405	0.015824	0.041516
30	0.270674	0.232363	0.010829	0.009306	0.012076	0.034197
35	0.296715	0.246244	0.013107	0.010555	0.013595	0.037770
40	0.351440	0.287361	0.021347	0.016476	0.020670	0.052208
45	0.429200	0.352097	0.040305	0.030614	0.037022	0.081370
50	0.520634	0.434081	0.077313	0.059097	0.068710	0.129802
55	0.613958	0.523624	0.138299	0.107895	0.120942	0.198304
60	0.698714	0.610166	0.222033	0.177795	0.193115	0.280540
65	0.768672	0.685458	0.318400	0.261398	0.276453	0.363675
70	0.822119	0.744967	0.413036	0.345497	0.356630	0.431699

Table 7.4. Values of $\eta(x,d)$, assumed probability of claiming at the end of the Deferred Period.

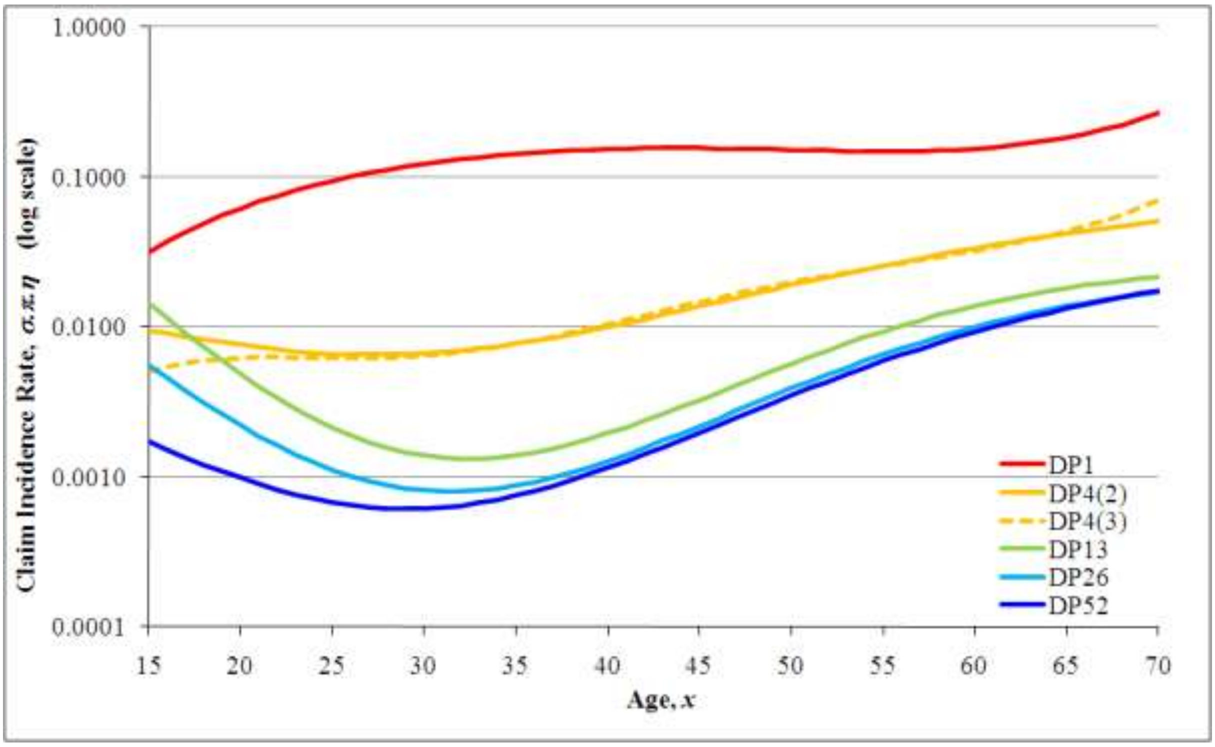
Age, x	DP1	DP4(2)	DP4(3)	DP13(2)	DP26	DP52
15	1.0	0.921471	0.597710	0.924868	1.0	1.0
20	1.0	0.890951	0.486157	0.899315	1.0	1.0
25	1.0	0.865668	0.408829	0.880212	1.0	1.0
30	1.0	0.850536	0.369140	0.871271	1.0	1.0
35	1.0	0.847324	0.363259	0.873091	1.0	1.0
40	1.0	0.854746	0.385729	0.883618	1.0	1.0
45	1.0	0.869595	0.430745	0.899420	1.0	1.0
50	1.0	0.888101	0.491424	0.917008	1.0	1.0
55	1.0	0.907008	0.559665	0.933723	1.0	1.0
60	1.0	0.924113	0.627371	0.948039	1.0	1.0
65	1.0	0.938300	0.688134	0.959410	1.0	1.0
70	1.0	0.949266	0.738144	0.967926	1.0	1.0

Table 7.5. Values of $\sigma(x).\pi(x,d).\eta(x,d)$, assumed intensity of Claim incidence.

Age, x	DP1	DP4(2)	DP4(3)	DP13(2)	DP26	DP52
15	0.031525	0.009516	0.004999	0.014310	0.005529	0.001713
20	0.061813	0.007585	0.006223	0.004846	0.002204	0.000981
25	0.094032	0.006624	0.006212	0.002119	0.001111	0.000674
30	0.122032	0.006658	0.006451	0.001391	0.000812	0.000614
35	0.142535	0.007728	0.007743	0.001399	0.000875	0.000751
40	0.153570	0.010003	0.010464	0.001954	0.001274	0.001149
45	0.155280	0.013709	0.014647	0.003268	0.002186	0.001981
50	0.151334	0.018978	0.019804	0.005688	0.003885	0.003498
55	0.148336	0.025706	0.025410	0.009351	0.006549	0.005915
60	0.154768	0.033555	0.032156	0.013836	0.010015	0.009261
65	0.183561	0.042081	0.043500	0.018205	0.013730	0.013287
70	0.265354	0.050872	0.069744	0.021469	0.016962	0.017456

7.8.3. In Figure 7.7 we plot the values of $\sigma(x).\pi(x,d).\eta(x,d)$. We see that the rates for DP1 are consistently much the highest, as we would expect. The values for DP4(2) and DP4(3) are quite close together, except at the extreme ages. The values for DP13, DP26 and DP52 are in the expected sequential order, though they overlap those for DP4 at the very youngest ages. However, all the rates at young ages are based just on the projections of the formulae fitted to the ages where there is significant data. As this feature is of little practical significance, we do not consider it necessary to set constraints on the graduation so as to impose any further order on the pattern of Claim incidence rates by Deferred Period.

Figure 7.7. Values of $\sigma(x).\pi(x,d).\eta(x,d)$, the intensity of Claim incidence, for DP1 to DP52, both DP4(2) and DP4(3).



7.8.4. A full table of the intensities of Claim incidence, for individual exact ages over the range 20 to 70, is provided in Appendix D.

7.9. *Type 2 or Type 3 rates*

7.9.1. Although the DP4(2) and DP4(3) variants produced markedly different estimates of the Sickness intensities, the estimates of Claim incidence intensities for DP4 are close together except at the extreme ages. This confirms that, although the need to estimate π and η does introduce uncertainty into the graduated Sickness intensities, little of that extra uncertainty flows through to the estimates of Claim incidence intensities. The Committee plans to use only the DP4(2) variant for Claim incidence in its future work.

8. COMPARISON WITH 1975-78 RATES OF SICKNESS AND CLAIM INCIDENCE

8.1. The graduation of the CMI Individual IP experience for 1975-78 is set out in CMIR 12 (with a short Erratum published in CMIR 13). The resulting graduated rates are commonly referred to as the "SM1975-78" tables, where the "SM" reflects the subset of data used which was the Standard, Males experience. Occupation Class was not recorded in the data for years prior to 1991, but the Standard experience excluded cases recorded as having an "occupational rating", so that the subset of data used for SM1975-78 broadly corresponds to the Standard*, Males, Occupation Class 1 subset adopted for the graduation of the 1991-98 experience, as set out in this paper.

8.2. When the 1975-78 rates were graduated there was insufficient data to produce rates for DP52, so Sickness rates, σ(x), were graduated only for DP1 to DP26. We show the values of σ(x) at exact quinquennial ages from 15 to 70, in Table 8.1. The values can be compared directly with those for the corresponding Age and Deferred Period in Table 7.2. We also plot the graduated Sickness rates for both 1975-78 and 1991-98 in Figure 8.1.

8.3. The graduated Sickness rates for DP1 in 1991-98 are higher for middling ages, lower for the more extreme ages, than in 1975-78. For DP4 one set of rates, DP4(3), is very much higher than in 1975-78, the other, DP4(2) is very much lower. For DP13 and DP26 the rates are lower in 1991-98 than in 1975-78, apart from the very youngest ages for DP13.

8.4. The formulae for Sickness rates for the 1975-78 graduations and for the current 1991-98 graduations are essentially the same, although they are expressed in different ways, so the values of the parameters are quite different. We explain this, and show how to convert from one formula to the other, in Appendix C.

8.5. We also compare the values of Claim incidence intensities, σ(x).π(x,d).η(x,d), for 1991-98 and for 1975-78. We show sample values of Claim incidence intensities for 1975-78, at exact quinquennial ages from 15 to 70, in Table 8.2, and plot the graduated intensities for both 1975-78 and 1991-98 in Figure 8.2. These 1975-78 values have been derived from the graduated Sickness Inception and Termination rates of the SM1975-78 tables in the same way as shown in Section 7.8 for the 1991-98 rates and so can reasonably be compared on a like-for-like basis with those for the corresponding Age and Deferred Period in Table 7.5. However, the following points should be noted before considering the comparison:

- For DP1 a Deferred Period of 6 days has been assumed, in line with the basis for the 1991-98 graduations. Had a Deferred Period of 7 days been assumed (as was the case for previous investigations, the derived Claim incidence intensities would have been lower (85.4% of the value shown at age 20, rising almost linearly to 95.5% at age 60).
- For DP4, DP13 and DP26, the 1975-78 values assume a 4-week run-in period for Claim recovery rates. These run-in period adjustments differ from those in the 1991-98 graduations, particularly for DP26 (for which there is no run-in period in the new rates).
- For DP52 no formal basis has been published for SM1975-78. However, rates were required to enable experience analyses for DP52 to be produced, and the main elements of the basis adopted are specified in CMIR 15 (page 4): the Sickness Inception intensity for DP52 has been taken to be $0.68926\sigma_x$ where σ_x is the graduated intensity for DP26. The factor 0.68926 was chosen so that the ratio of actual to expected Claims for the Males, individual policies, Standard experience 1975-78, DP52 was 100%.
- It is not clear what allowance, if any, should be made for a run-in period in Claim recovery rates for DP52 for 1975-78. The values shown in Table 8.2 and Figure 8.2 for

DP52 take account of the 0.68926 scalar factor but make no allowance for any run-in period. The rates adopted as 'SM1975-78' for published results for DP52 did assume a run-in period (although the precise details are unclear) so that the rates used were 95.5% of the value shown at age 20, rising almost linearly to 99.4% at age 60.

8.6. Based on the comparison in Figure 8.2 we see that for DP1 the Claim Inception rates are reasonably similar (after aligning the assumed Deferred Period at 6 days) over the age range 30 to 55, with the new rates being lower at the extreme ages. For DP4 the new rates are around 30% lower at most ages, but are higher at younger ages. For DP13 the comparison is similar to DP4 at most ages, but with greater divergence at younger ages. For DP26 the 1991-98 rates are around 50% higher at most ages (and rather more at younger ages). For DP52, the 1991-98 rates are substantial higher at almost every age, and much closer to the DP26 rates than was the case for the 1975-78 rates. The typical shape of the rates with age has changed so that, in general terms, the new rates rise relative to the 1975-78 rates (and in absolute terms) below age 30, and tail down relative to the 1975-78 rates above age 55.

Table 8.1. Graduated Sickness rates, $\sigma(x)$, for 1975-78
SM1975-78 for Deferred Periods DP1 to DP26.

Age, *x*	DP1	DP4	DP13	DP26
15	0.331677	0.142954	0.199932	0.217046
20	0.348555	0.195319	0.217195	0.168851
25	0.345181	0.227268	0.214488	0.137919
30	0.328230	0.234645	0.198247	0.118281
35	0.305354	0.223976	0.176573	0.106507
40	0.283178	0.205942	0.156034	0.100695
45	0.266736	0.190054	0.140850	0.099956
50	0.260019	0.183416	0.133721	0.104178
55	0.267282	0.192867	0.137471	0.114003
60	0.295195	0.230238	0.157565	0.130987
65	0.356911	0.325112	0.207302	0.158018
70	0.481350	0.565797	0.322336	0.200150

Table 8.2. Values of $\sigma(x).\pi(x,d).\eta(x,d)$, assumed intensity of Claim incidence, for 1975-78, derived from SM1975-78 for Deferred Periods DP1 to DP52.

Age, *x*	DP1	DP4	DP13	DP26	DP52
15	0.115441	0.002548	0.000510	0.000152	0.000040
20	0.130242	0.004650	0.000834	0.000196	0.000056
25	0.138470	0.007227	0.001239	0.000265	0.000083
30	0.141354	0.009966	0.001722	0.000376	0.000127
35	0.141172	0.012704	0.002305	0.000560	0.000203
40	0.140543	0.015599	0.003060	0.000874	0.000342
45	0.142112	0.019221	0.004148	0.001429	0.000604
50	0.148712	0.024765	0.005910	0.002452	0.001116
55	0.164094	0.034762	0.009114	0.004412	0.002159
60	0.194539	0.055391	0.015660	0.008324	0.004377
65	0.252478	0.104387	0.030871	0.016467	0.009293
70	0.365495	0.242423	0.071882	0.034157	0.020644

Figure 8.1. Graduated Sickness rates, σ(x), for DP1 to DP52, for 1991-98 and 1975-78.

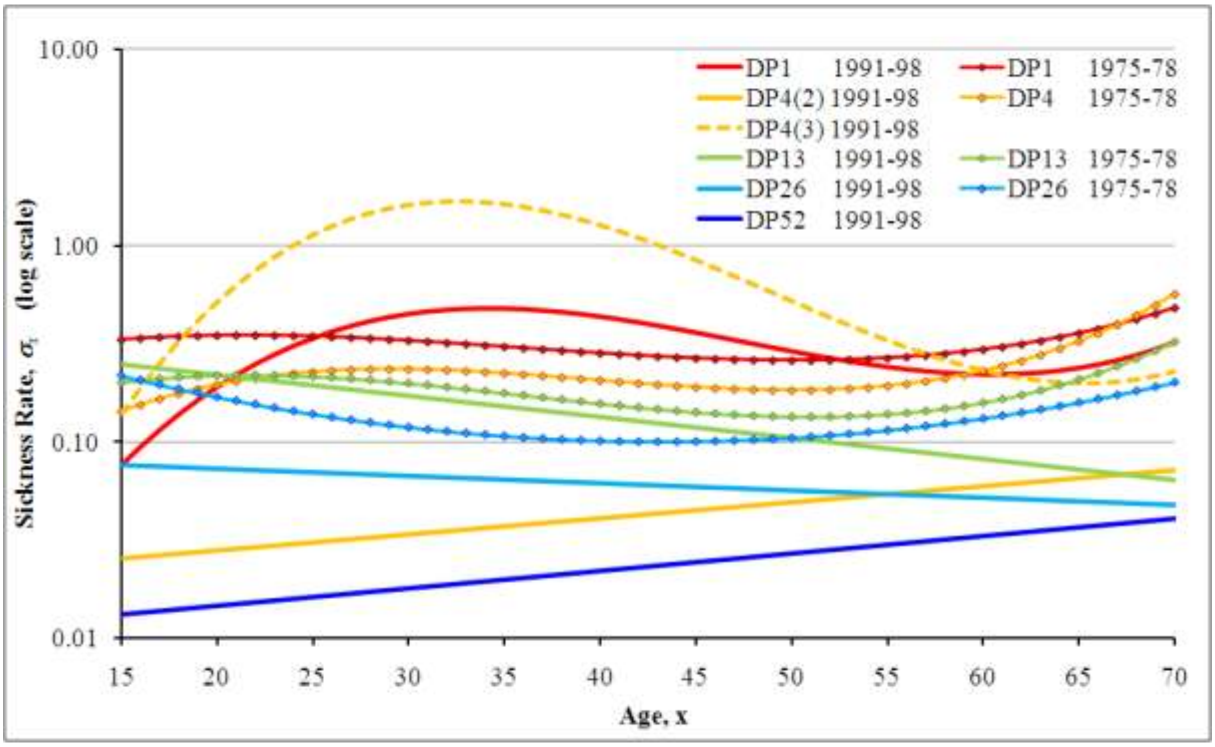
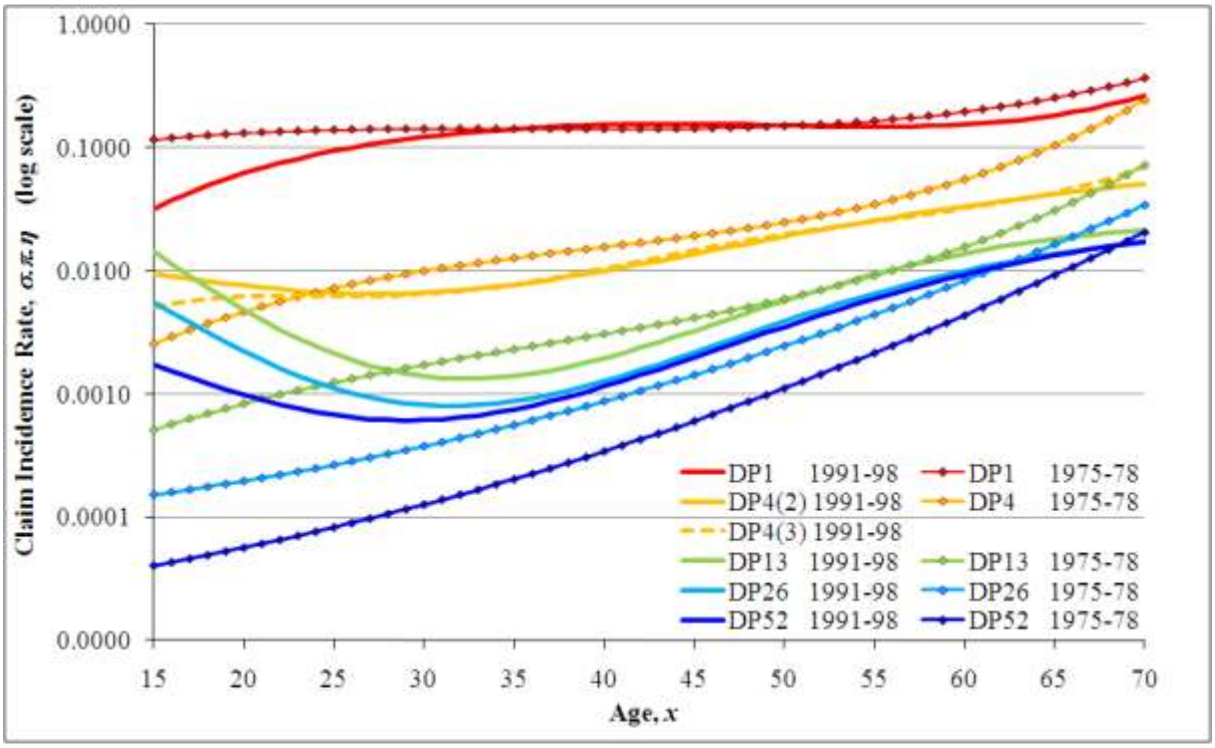


Figure 8.2. Values of $\sigma(x).\pi(x,d).\eta(x,d)$, the intensity of Claim incidence, for DP1 to DP52, for 1991-98 and 1975-78.



9. COMPARISON OF RESULTS ON NEW AND PREVIOUS METHODOLOGY

9.1. We are now in a position to compare the actual Inceptions in one experience with those expected using the new graduated Sickness rates, σ. But first we use our new methods to compare the results for the three quadrennia from 1991 to 2002, as already published in CMIRs 18, 20 and 22, using the SM1975-78 basis for the comparison. The comparisons in the CMIRs use the cum Duplicates numbers of Inceptions, adjusting the standard errors to allow for Duplicates, so we do the same, using the cum Duplicates Claim numbers throughout.

9.2. We first compare the numbers of Inceptions, cum Duplicates. In Tables 9.1(a) and (b) we show the numbers, using our new methods, for Males and Females separately, for each quadrennium, for each Occupation Class (OC) and each Deferred Period, including DP0, DP2 and DP8.

9.3. In Tables 9.2(a) and (b) we show the numbers of Inceptions as given in CMIRs 18, 20 and 22, for the same groupings. In the CMIRs the numbers for DP0, DP2 and DP8 are included with those for DP1, DP4 and DP13 respectively.

9.4. Careful comparison shows that, except for DP1, the numbers are reasonably close, but seldom identical, except where the numbers are fairly small. Generally the numbers using the new method are lower, but this is not always the case. The differences can be attributed to the different ways in which we have treated exclusions, including omitting False One-day Claims and Odd Deferred Periods, and treating Early Terminations a little differently. For DP1, reducing the Deferred Period from seven to six days increases the numbers of Claims treated as Inceptions considerably.

Table 9.1(a) Numbers of Inceptions, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Males, using the new methodology.

1991-94									
OC	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	All DPs
1	0	13,254	23	1,676	3	675	622	255	16,508
2	0	0	0	290	0	126	52	11	479
3	0	0	0	435	2	99	45	12	593
4	0	1	0	441	15	106	39	3	605
5	121	70	0	2,693	71	1,519	628	252	5,354
All OCs	121	13,325	23	5,535	91	2,525	1,386	533	23,539

1995-98									
OC	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	All DPs
1	0	10,710	18	1,658	2	898	953	455	14,694
2	0	7	0	288	9	246	134	55	739
3	0	2	0	487	17	243	116	42	907
4	0	0	0	387	16	190	74	32	699
5	54	0	0	868	0	705	350	173	2,150
All OCs	54	10,719	18	3,688	44	2,282	1,627	757	19,189

1999-2002									
OC	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	All DPs
1	0	7,839	10	1,307	15	1,068	1,027	486	11,752
2	0	8	1	310	14	423	200	84	1,040
3	0	1	0	529	37	324	140	62	1,093
4	0	0	0	257	23	306	85	28	699
5	14	0	0	797	0	749	540	187	2,287
All OCs	14	7,848	11	3,200	89	2,870	1,992	847	16,871

Table 9.1(b) Numbers of Inceptions, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Females, using the new methodology.

1991-94									
OC	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	All DPs
1	0	1,348	2	486	1	169	162	55	2,223
2	0	1	0	75	0	35	21	1	133
3	0	0	0	27	0	4	0	0	31
4	0	0	0	1	0	0	0	0	1
5	0	6	0	223	6	163	165	53	616
All OCs	0	1,355	2	812	7	371	348	109	3,004

1995-98									
OC	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	All DPs
1	0	1,126	0	402	2	224	268	164	2,186
2	0	1	0	75	4	71	58	32	241
3	0	0	0	18	0	18	23	14	73
4	0	0	0	0	0	2	0	0	2
5	0	0	0	94	0	118	78	57	347
All OCs	0	1,127	0	589	6	433	427	267	2,849

1999-2002									
OC	DP0	DP1	DP2	DP4	DP8	DP13	DP26	DP52	All DPs
1	0	695	1	334	12	352	438	259	2,091
2	0	0	0	94	9	117	96	99	415
3	0	0	0	6	1	20	27	15	69
4	0	0	0	3	0	9	3	2	17
5	0	0	0	104	0	189	224	79	596
All OCs	0	695	1	541	22	687	788	454	3,188

Table 9.2(a) Numbers of Inceptions as reported in CMIRs 18, 20 and 22, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Males.

1991-94						
OC	DP1	DP4	DP13	DP26	DP52	All DPs
1	11,905	1,694	676	623	246	15,144
2	0	290	126	52	12	480
3	0	436	100	46	20	602
4	2	442	120	39	3	606
5	191	2,751	1,699	655	254	5,550
All OCs	12,098	5,613	2,721	1,415	535	22,382

1995-98						
OC	DP1	DP4	DP13	DP26	DP52	All DPs
1	9,551	1,674	902	954	452	13,533
2	7	288	255	134	55	739
3	3	487	260	115	43	908
4	0	389	200	75	33	697
5	56	938	831	391	175	2,391
All OCs	9,617	3,776	2,448	1,669	758	18,268

1999-2002						
OC	DP1	DP4	DP13	DP26	DP52	All DPs
1	7,194	1,315	1,080	1,026	492	11,107
2	8	312	434	200	83	1,037
3	1	529	359	140	62	1,091
4	0	257	324	86	28	695
5	14	778	693	519	176	2,180
All OCs	7,217	3,191	2,890	1,971	841	16,110

Table 9.2(b) Numbers of Inceptions, as reported in CMIRs 18, 20 and 22, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Females.

1991-94						
OC	DP1	DP4	DP13	DP26	DP52	All DPs
1	1,266	491	172	165	56	2,150
2	1	75	35	21	6	138
3	0	27	5	3	4	39
4	0	1	0	2	1	4
5	8	227	178	174	53	640
All OCs	1,275	821	390	365	120	2,971

1995-98						
OC	DP1	DP4	DP13	DP26	DP52	All DPs
1	1,069	406	227	272	165	2,139
2	1	75	75	59	33	243
3	0	18	18	25	21	82
4	0	0	3	4	0	7
5	0	110	140	101	55	406
All OCs	1,070	609	463	461	274	2,877

1999-2002						
OC	DP1	DP4	DP13	DP26	DP52	All DPs
1	663	340	369	441	264	2,077
2	0	94	126	96	101	417
3	0	6	22	27	15	70
4	0	4	10	6	4	24
5	0	101	178	217	73	569
All OCs	663	545	705	787	457	3,157

9.5. We next compare the ratios 100 × A/E (Actual Inceptions / Expected Inceptions). In Tables 9.3(a) and 9.3(b) we show the values of these ratios for each quadrennium, for Males and Females, for each Occupation Class and each Deferred Period. If the number of Inceptions is less than 10 we do not show any ratio. We omit DP0, DP2 and DP8. The Expected Inceptions have been calculated using the new methods but using the graduated SM1975-78 Sickness intensities, $\sigma(x)$, and Termination intensities, $\rho(x, z)$ and $\nu(x, z)$ (for calculation of exposure, including the $\pi(x,d)$ and $\eta(x,d)$ factors). The basis adopted is set out in Section 8, with sample Claim Inception intensities shown in Table 8.2.

9.6. In Tables 9.4(a) and (b) we show the ratios 100 × A/E for Inceptions as reported in CMIRs 18, 20 and 22, for the same groupings. These are also calculated using the SM1975-78 rates throughout but using the previous methodology. Where the number of Inceptions is less than 10, no values are shown.

9.7. The ratios are generally similar, though there are larger differences when the numbers are fairly small, and also for DP52, where the ratios using the new methods are fairly consistently higher than those shown in the CMIRs. In spite of the considerable increase in the numbers of Inceptions for DP1, the ratios are still about the same. This is because the adjusted exposure has been calculated using the values of $\pi(x+½,d)$ for a six-day Deferred Period rather than a seven-day one (most importantly in Stage 5: Scaling down), so the adjusted exposure is correspondingly larger.

Table 9.3(a) Ratios 100 × *A*/*E* for Inceptions, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Males, using the new methods, on the basis of the SM1975-78 rates throughout.

		1991-94			
OC	DP1	DP4	DP13	DP26	DP52
1	100.5	71.8	98.0	143.0	292.9
2		114.1	154.2	158.7	403.8
3		162.9	222.8	215.1	683.7
4		238.2	289.9	356.1	
5	58.2	109.1	126.2	142.3	272.3
All OCs	100.1	100.5	122.8	147.3	287.7

		1995-98			
OC	DP1	DP4	DP13	DP26	DP52
1	95.7	68.2	87.2	153.5	329.8
2		73.7	101.1	135.6	475.8
3		82.7	153.1	235.1	583.6
4		156.1	181.4	284.1	996.7
5		55.3	105.7	170.0	322.6
All OCs	95.2	70.5	103.6	162.6	354.5

		1999-2002			
OC	DP1	DP4	DP13	DP26	DP52
1	82.7	51.7	72.5	128.9	269.5
2		61.1	110.2	152.2	300.2
3		62.7	123.2	223.0	420.9
4		100.6	158.4	225.2	394.6
5		58.1	100.7	259.9	338.8
All OCs	82.0	58.2	93.9	161.1	296.9

Table 9.3(b) Ratios 100 × *A*/*E* for Inceptions, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Females, using the new methods, on the basis of the SM1975-78 rates throughout.

		1991-94			
OC	DP1	DP4	DP13	DP26	DP52
1	120.4	148.0	209.8	414.1	661.6
2		202.7	353.7	921.1	
3		388.8			
4					
5		140.1	182.5	503.4	807.3
All OCs	120.1	152.7	205.2	468.6	718.8

		1995-98			
OC	DP1	DP4	DP13	DP26	DP52
1	125.7	107.0	144.0	320.8	660.2
2		107.7	210.5	456.8	821.1
3		170.5	241.4	646.5	1,192.7
4					
5		83.1	169.1	332.9	1,022.6
All OCs	125.4	103.4	162.1	346.2	752.4

		1999-2002			
OC	DP1	DP4	DP13	DP26	DP52
1	95.1	65.5	122.1	295.8	533.3
2		81.7	170.0	349.7	728.7
3			125.7	432.7	409.2
4					
5		85.5	164.3	695.8	852.9
All OCs	94.5	70.4	139.8	366.6	601.2

Table 9.4(a) Ratios 100 × A/E for Inceptions using SM1975-78, as reported in CMIRs 18, 20 and 22, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Males.

		1991-94			
OC	DP1	DP4	DP13	DP26	DP52
1	97.9	72.4	97.3	141.0	276.4
2		112.4	149.6	152.9	378.0
3		160.9	210.1	209.1	918.0
4		233.0	285.7	325.0	
5	37.1	110.8	137.0	146.3	270.8
All OCs	95.4	101.3	129.1	147.7	282.0

		1995-98			
OC	DP1	DP4	DP13	DP26	DP52
1	92.0	68.7	86.2	151.6	321.3
2		72.9	100.6	133.5	461.0
3		82.0	147.2	225.5	552.0
4		154.2	174.2	273.7	887.0
5	25.5	59.7	123.6	187.2	321.7
All OCs	90.0	71.9	108.1	164.1	346.8

		1999-2002			
OC	DP1	DP4	DP13	DP26	DP52
1	81.4	52.0	71.0	127.2	269.1
2		61.1	106.9	149.7	293.3
3		62.3	121.5	216.4	400.0
4		99.1	154.9	219.4	378.4
5	11.9	56.6	92.4	244.1	314.8
All OCs	79.7	57.8	91.1	156.9	290.1

Table 9.4(b) Ratios 100 × *A*/*E* for Inceptions, using SM1975-78, as reported in CMIRs 18, 20 and 22, cum Duplicates, for each quadrennium, each Occupation Class and each Deferred Period, for Females.

			1991-94		
OC	DP1	DP4	DP13	DP26	DP52
1	120.9	140.9	200.4	367.4	596.0
2		193.2	306.0	678.0	
3		336.0			
4					
5		139.4	183.1	478.0	671.0
All OCs	120.8	146.9	198.4	429.9	669.0

			1995-98		
OC	DP1	DP4	DP13	DP26	DP52
1	126.4	104.1	136.7	301.9	618.0
2		104.0	199.0	420.0	761.0
3		152.0	202.0	573.0	1266.0
4					
5		95.1	192.3	388.5	854.0
All OCs	126.0	103.1	161.4	342.0	700.0

			1999-2002		
OC	DP1	DP4	DP13	DP26	DP52
1	94.9	65.3	119.8	281.6	518.7
2		79.7	165.4	326.5	706.3
3			122.9	375.0	375.0
4			217.4		
5		80.3	149.3	609.6	715.7
All OCs	94.7	69.4	134.1	341.6	572.0

10. Other Experiences 1991-98

10.1. *Introduction*

10.1.1. We now present an analysis of the experience of Males for other Occupation Classes, and of Females. We compare the number of actual Inceptions with those expected using the Sickness rates and other factors derived from the data for Males, Occupation Class 1. This is not a perfect comparison. The factors we use are the "$\sigma.\pi.\eta$" factors, which we have derived not just from the numbers of Inceptions for Males, OC1, but also using the Termination rates and assumed proportion of those claiming on the basis of the rates for Males, OC1. Further, the adjustments to the exposed to risk are based on the factors for Males, OC1. Strictly we should use factors applicable to the relevant Sex and Occupation Class throughout. If the numbers of Inceptions were large enough we could have graduated the experiences for the different Sex and Occupation Class categories, but, again, we would need the Termination rates for each category to go through this process correctly. However, the comparisons of actual to expected Inceptions shown in CMIRs 18, 20 and 22 are on the same somewhat inconsistent basis that we shall use here, and a fully consistent basis would require a fuller investigation of all the factors for each Sex and Occupation Class category.

10.1.2. We analyses each combination of Sex, Occupation Class and Deferred Period separately and independently. In some cases the number of Inceptions is very small. If the number is less than 10 then this group is not analysed. This would apply to DP0, DP2 and DP8 frequently, but these Deferred Periods are omitted here. It also applies to DP1 for Males and Females, Occupation Classes 2, 3 and 4, and also to DP1 for Females, Occupation Class 5 (Class "Not Given") and to Females, Occupation Class 4 for all Deferred Periods.

10.1.3. We do not expect that the expected number of Inceptions, derived from the graduated rates for Males, Occupation Class 1, will agree with the observed numbers, although we present results for such comparisons. We also, therefore, test the assumption that the actual number of Inceptions is compatible with an adjusted expected number of Inceptions. We make the adjustment by ratioing the expected numbers for each Age by the overall ratio of actual to expected Inceptions so that the adjusted total expected number equals the total actual number (for that combination of Sex, Occupation Class and Deferred Period). That is, we test whether a constant proportional change in the basic rates of Sickness would fit the data. Sometimes this is a reasonable assumption; sometimes it is seen to be unsatisfactory. We denote the original expected by "E" and the adjusted expected by "E^*".

10.1.4. Because the numbers of Inceptions are generally much lower than for Males, Occupation Class 1, we calculate all the statistical tests on the basis of grouped ages, where ages are grouped so that the expected number of Inceptions in each cell is at least 5.0.

10.1.5. We present and use the results for DP4(2) throughout. The results for DP4(3) are quite similar.

10.1.6. For completeness we show in Table 10.1 the results for Males, Occupation Class 1. In this and in the following tables, the upper part of the table shows results using the original expected, E, and the lower part shows results for the adjusted expected, E^*. For the Deferred Periods shown, DP1, DP4, DP13, DP26 and DP52, the data for which has been graduated, the results show a figure of $100 \times A/E$ of 100.0, as we would expect.

Table 10.1. Statistics showing experience for Males, Occupation Class 1 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	9,696	2,304	1,125	1,107	502
Exposure (years) (*R6*)	30,950.8	49,524.8	11,230.5	19,925.7	18,529.9
Expected	9,696.0	2,304.0	1,125.0	1,107.0	502.0
100 × *A*/*E*	100.0	100.0	100.0	100.0	100.0
Using *E*:					
No. of groups	42	42	39	37	32
+/–	20/22	22/20	22/17	20/17	12/20
p(+ or –)	0.44	0.44	0.26	0.37	0.11
runs	11	19	19	12	15
p(runs)	0.0004	0.22	0.41	0.0100	0.42
p(K-S)	0.42	0.92	1.00	0.59	0.77
Serial Correlations:					
T(r_1)	3.00	0.42	0.77	1.24	0.30
T(r_2)	2.26	0.01	–0.05	0.13	0.69
T(r_3)	2.07	–1.91	1.61	0.03	1.09
Poisson Deviance	92.87	71.70	39.39	55.67	39.66
p(Deviance)	0.0000	0.0029	0.45	0.0250	0.17
X^2	86.98	64.72	32.88	44.25	32.05
$p(X^2)$	0.0001	0.0137	0.74	0.19	0.46
Using *E**:					
No. of groups	42	42	39	37	32
+/–	20/22	22/20	22/17	20/17	12/20
p(+ or –)	0.44	0.44	0.26	0.37	0.11
runs	11	19	19	12	15
p(runs)	0.0004	0.22	0.41	0.0100	0.42
p(K-S)	0.42	0.92	1.00	0.59	0.77
Serial Correlations:					
T(r_1)	3.00	0.42	0.77	1.24	0.30
T(r_2)	2.26	0.01	–0.05	0.13	0.69
T(r_3)	2.07	–1.91	1.61	0.03	1.09
Poisson Deviance	92.87	71.70	39.39	55.67	39.66
p(Deviance)	0.0000	0.0029	0.45	0.0250	0.17
X^2	86.98	64.72	32.88	44.25	32.05
$p(X^2)$	0.0001	0.0137	0.74	0.19	0.46

10.1.7. The exposure measure shown in Table 10.1 (and the similar Tables 10.2 to 10.12) is the R6 measure developed in Section 6. This measure starts from the policy years exposure, adjusted from an “all lives” to a “healthy lives” basis by deducting an estimate of the total time spent in Sickness during the exposure period. But note in particular, that the exposure is then scaled down by multiplying by $\pi.\eta$, which equals the probability of survival to the end of the Deferred Period, times the probability that a Claim is then made.

10.2. *Males: Occupation Class 2*

10.2.1. The results for Males, Occupation Class 2, are shown in Table 10.2. The numbers of Inceptions for DP1 are too few to analyse. The other Deferred Periods show adequate numbers, which reduce as the Deferred Period increases. The ratio of Actual to Expected Inceptions is well above 100% for DP4, DP13 and DP52, at 124.0%, 127.4% and 140.8%, but it is below 100% for DP26 at 94.5%.

10.2.2. We consider first the three Deferred Periods where the experience is above that expected. In each case the number of groups with positive differences between *A* and *E* is well above the number expected, although, given the numbers of positive and negative differences, the numbers of runs are not unreasonable. The values of the Poisson deviance and the Pearson X^2 are much too high. When we consider the adjusted expected, the numbers of positives and negatives are satisfactory, but for both DP4 and DP13 the Kolmogorov-Smirnov tests show low probabilities and the serial correlation coefficients are high.

10.2.3. Detailed inspection shows that, even on the adjusted basis, for DP4 the actual Inceptions are high compared with those expected for most ages up to and including age 43 and low for most ages above that. For DP13, the actual Inceptions are high for most ages up to 44 and low for most ages above that. So a proportionate adjustment does not do well. For DP52, which has many fewer Inceptions, only 62, the adjusted basis looks quite reasonable.

10.2.4. For DP 26, for which the ratio 100 × A/E is 94.5, the tests on the unadjusted basis show quite good results. The adjusted basis appears wholly satisfactory. Inspection shows, however, that on both bases the actual Inceptions are rather high for ages up to 46 and are lower than the expected for ages above that, which is a similar pattern to that of DP4 and DP13.

Table 10.2. Statistics showing experience for Males, Occupation Class 2 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	7	510	329	174	62
Exposure (years) (*R6*)	178.1	9,178.4	2,470.6	3,278.2	1,709.0
Expected		411.2	258.3	184.0	44.0
100 × *A*/*E*		124.0	127.4	94.5	140.8
Using *E*:					
No. of groups		37	28	23	7
+/–		30/7	23/5	11/12	4/3
p(+ or –)		0.0001	0.0005	0.5	0.5
Runs		9	9	8	6
p(runs)		0.07	0.50	0.0443	0.20
p(K-S)		0.0088	0.0183	0.11	0.86
Serial Correlations:					
$T(r_1)$		3.29	2.82	1.55	–0.95
$T(r_2)$		2.89	2.21	1.76	–0.54
$T(r_3)$		1.07	1.45	0.74	0.51
Poisson Deviance		68.73	66.75	27.35	19.44
p(Deviance)		0.0012	0.0001	0.24	0.0069
X^2		70.21	74.33	19.62	22.40
$p(X^2)$		0.0008	0.0000	0.66	0.0022
Using *E**:					
No. of groups		39	31	22	10
+/–		20/19	16/15	12/10	5/5
p(+ or –)		0.5	0.5	0.42	0.5
Runs		16	8	8	4
p(runs)		0.0981	0.0014	0.07	0.17
p(K-S)		0.0046	0.0096	0.12	0.78
Serial Correlations:					
$T(r_1)$		3.10	2.60	1.48	–0.41
$T(r_2)$		2.85	2.59	1.64	0.26
$T(r_3)$		1.26	1.51	–0.07	–0.91
Poisson Deviance		52.13	52.37	26.22	9.46
p(Deviance)		0.08	0.0096	0.24	0.49
X^2		44.18	48.79	19.68	6.49
$p(X^2)$		0.26	0.0221	0.60	0.77

10.3. *Males: Occupation Class 3*

10.3.1. The results for Males, Occupation Class 3, are shown in Table 10.3. The number of Inceptions for DP1 is only 2, so no analysis is done. The numbers for the other Deferred Periods are reasonable, though DP52 has only 55 Inceptions. The ratios of actual to expected are well above 100% for all Deferred Periods. For all the Deferred Periods presented all the statistical tests show that there is a big difference between actual and expected.

10.3.2. The results on the adjusted basis for DP4 show that, although the numbers of positive and negative deviations are about equal, there are only two runs. The actual Inceptions are greater than the adjusted expected for every age group up to age 44 and below for every age group above that. For DP13 a similar feature is seen; for most of the age groups up to age 45 the actual numbers are high; for most above that age the actual numbers are low. For DP26 and DP52, for which the actual numbers of Inceptions are smaller, the tests appear satisfactory, but inspection shows that the same feature is evident, with relatively high numbers of Inceptions at many younger ages and relatively low numbers at higher ages. But note that this is after the adjustment to the expected. On the unadjusted basis, the numbers of actual Inceptions are higher than the numbers expected at almost all ages.

10.4. *Males: Occupation Class 4*

10.4.1. The results for Males, Occupation Class 4, are shown in Table 10.4. The number of Inceptions for DP1 is only one, so no analysis is done. The numbers for the other Deferred Periods are less than for Occupation Class 3, and for DP52 are only 34. The ratios of actual to expected are very high, ranging from 196.0% for DP26 to 267.0% for DP4. Out of 61 age groups in total, only two show a negative deviation.

10.4.2. The results on the adjusted basis show similar features to what we see for Occupation Class 3. For DP4 the deviations are mostly positive up to age 44 and mostly negative thereafter; for DP13 the same is true up to age 48. For DP26 and DP52 the deviations are more mixed up, and the statistical tests are all satisfied.

Table 10.3. Statistics showing experience for Males, Occupation Class 3 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	2	873	319	149	55
Exposure (years) (*R6*)	45.3	12,739.9	1,572.4	1,775.0	1,093.7
Expected		570.9	169.0	100.2	27.9
100 × *A*/*E*		152.9	188.8	148.8	197.1
Using *E*:					
No. of groups		39	23	15	5
+/–		34/5	22/1	12/3	4/1
p(+ or –)		0.0000	0.0000	0.0176	0.19
runs		8	3	7	2
p(runs)		0.15	0.5	0.36	0.40
p(K-S)		0.0000	0.0017	0.16	0.41
Serial Correlations:					
T(r_1)		3.66	2.54	2.19	0.67
T(r_2)		3.74	2.74	0.38	–1.00
T(r_3)		3.50	1.61	0.05	–0.80
Poisson Deviance		251.98	175.19	48.79	33.32
p(Deviance)		0.0000	0.0000	0.0000	0.0000
X^2		315.51	253.00	59.97	45.38
p(X^2)		0.0000	0.0000	0.0000	0.0000
Using *E**:					
No. of groups		40	31	22	9
+/–		21/19	15/16	8/14	5/4
p(+ or –)		0.44	0.5	0.14	0.5
runs		2	10	6	3
p(runs)		0.0000	0.0137	0.0134	0.0714
p(K-S)		0.0000	0.0001	0.0402	0.2492
Serial Correlations:					
T(r_1)		4.00	1.42	2.03	1.24
T(r_2)		4.17	2.39	0.99	–0.11
T(r_3)		3.73	2.38	0.74	–0.31
Poisson Deviance		114.87	85.94	39.96	24.55
p(Deviance)		0.0000	0.0000	0.0109	0.0035
X^2		106.10	88.63	33.79	14.72
p(X^2)		0.0000	0.0000	0.05	0.10

Table 10.4. Statistics showing experience for Males, Occupation Class 4 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	1	757	286	108	34
Exposure (years) (*R6*)	14.1	6,417.4	1,125.5	970.3	584.7
Expected		283.6	122.0	55.1	14.9
100 × *A*/*E*		267.0	234.4	196.0	227.8
Using *E*:					
No. of groups		31	19	9	2
+/–		31/0	17/2	9/0	2/0
p(+ or –)		0.0000	0.0004	0.0020	0.25
runs		1	5	1	1
p(runs)		1	0.5	1	1
p(K-S)		0.0011	0.0452	0.80	1.00
Serial Correlations:					
$T(r_1)$		3.24	2.77	0.91	–0.71
$T(r_2)$		2.79	2.03	–0.37	0.00
$T(r_3)$		1.00	1.02	–0.79	0.00
Poisson Deviance		627.29	212.73	49.34	17.86
p(Deviance)		0.0000	0.0000	0.0000	0.0001
X^2		978.51	321.96	62.59	22.00
$p(X^2)$		0.0000	0.0000	0.0000	0.0000
Using *E**:					
No. of groups		38	30	17	6
+/–		20/18	16/14	10/7	4/2
p(+ or –)		0.44	0.43	0.31	0.34
runs		8	10	10	3
p(runs)		0.0000	0.0207	0.45	0.40
p(K-S)		0.0000	0.0035	0.53	0.55
Serial Correlations:					
$T(r_1)$		3.17	2.45	–0.07	–0.79
$T(r_2)$		2.10	2.98	0.49	–0.30
$T(r_3)$		2.63	1.33	–0.65	0.05
Poisson Deviance		100.68	65.19	26.22	15.92
p(Deviance)		0.0000	0.0002	0.07	0.0142
X^2		93.99	59.15	16.97	7.81
$p(X^2)$		0.0000	0.0012	0.46	0.25

10.5. *Males: Occupation Class "5" (Class "Not Given")*

10.5.1. The results for Males, Occupation Class "5", are shown in Table 10.5. Occupation Class "5" includes all those policies which the office concerned cannot classify by Occupation Class, so it contains a mixture of Classes. The numbers of Inceptions for all Deferred Periods are quite large and for DP4 and DP13 are larger even than for Occupation Class 1. The ratio of actual to expected for DP1, at 59.2% is particularly low. It is not obvious why this should be. For the other Deferred Periods the ratios are above 100, but for DP26 and DP52 are not very much above, at 104.4% and 103.5%.

10.5.2. For DP26 and DP52 the statistical tests indicate that the numbers of actual Inceptions are consistent with those expected on the original basis. For DP4 and DP13 there are very significant differences between Class 5 and the graduated rates on both bases. On the adjusted basis the actual numbers are high up to age 42 for DP4 and age 44 for DP13, and lower thereafter.

10.6. *Males: All Occupation Classes*

10.6.1. We combine the results for Males for all Occupation Classes in Table 10.6. For the Deferred Periods shown, the ratios of $100 \times A/E$ are close to or a bit above 100.

10.7. *Females: Occupation Class 1*

10.7.1. The results for Females, Occupation Class 1, are shown in Table 10.7. Note that we omit data for Females from age 60 upwards. The numbers of Inceptions for all Deferred Periods shown are reasonably large, though much less than for Males, Occupation Class 1. The ratios are of actual to expected are all above 100%, being respectively 120.2%, 165.4%, 187.4%, 204.3%, 189.5% for DPs 1, 4, 13, 26 and 52. The statistical tests show that the Female experience is significantly higher than that for Males.

10.7.2. When we look at results on the adjusted basis, we find that the results appear to fit adequately only for DP26 and DP52. For DP1 the ratios of actual to expected are below 100% for most ages up to 45 and above 100% for every age above that, being quite heavy, ranging from 130.5% to 173.0%, for each age from 50 to 57. For DP4 the pattern is different: the numbers of actual Inceptions are well above those expected on the adjusted basis for ages 31 to 36 and 55 to 58, and are well below those expected for ages 23 to 29. For DP13 the actual Inceptions are generally high up to age 45 and are well below those expected for ages 48 to 57. Thus the three Deferred Periods show different patterns.

Table 10.5. Statistics showing experience for Males, Occupation Class “5” for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	68	3,372	2,106	889	379
Exposure (years) (*R6*)	372.8	55,576.1	15,194.0	15,335.2	13,107.6
Expected	114.9	2,624.9	1,550.2	851.5	366.1
100 × *A*/*E*	59.2	128.5	135.9	104.4	103.5
Using *E*:					
No. of groups	16	44	42	34	27
+/–	1/15	32/12	36/6	21/13	16/11
p(+ or –)	0.0003	0.0018	0.0000	0.11	0.22
runs	2	12	7	13	13
p(runs)	0.13	0.0122	0.0166	0.10	0.41
p(K-S)	1.00	0.0000	0.0000	0.36	0.68
Serial Correlations:					
$T(r_1)$	–0.29	5.56	4.08	1.80	–0.60
$T(r_2)$	–0.13	5.29	3.14	–0.69	–0.17
$T(r_3)$	–0.07	5.11	3.27	–0.67	–0.32
Poisson Deviance	44.92	756.79	390.43	53.85	31.13
p(Deviance)	0.0001	0.0000	0.0000	0.0165	0.27
X^2	23.58	1038.18	497.39	48.09	24.13
$p(X^2)$	0.10	0.0000	0.0000	0.06	0.62
Using *E**:					
No. of groups	11	45	43	35	27
+/–	6/5	24/21	24/19	16/19	15/12
p(+ or –)	0.5	0.38	0.27	0.37	0.35
runs	8	4	9	15	13
p(runs)	0.26	0.0000	0.0000	0.16	0.37
p(K-S)	1.00	0.0000	0.0000	0.35	0.67
Serial Correlations:					
$T(r_1)$	–0.62	5.84	4.94	2.02	–0.63
$T(r_2)$	–0.56	5.63	4.18	–0.38	–0.30
$T(r_3)$	–1.44	5.41	4.23	–0.51	–0.42
Poisson Deviance	19.00	562.23	212.04	49.84	30.68
p(Deviance)	0.06	0.0000	0.0000	0.0496	0.28
X^2	11.26	644.38	225.06	42.78	22.96
$p(X^2)$	0.42	0.0000	0.0000	0.17	0.69

Table 10.6. Statistics showing experience for Males, All Occupation Classes for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	9,774	7,816	4,165	2,427	1,032
Exposure (years) (*R6*)	31,562.1	133,436.6	31,593.0	41,284.4	35,024.8
Expected	9,878.0	6,194.5	3,224.5	2,297.7	955.0
100 × *A*/*E*	98.9	126.2	129.2	105.6	108.1
Using *E*:					
No. of groups	42	45	44	41	35
+/–	19/23	37/8	39/5	27/14	21/14
p(+ or –)	0.32	0.0000	0.0000	0.0298	0.16
runs	9	6	5	15	15
p(runs)	0.0000	0.0001	0.0031	0.08	0.21
p(K-S)	0.36	0.0000	0.0000	0.14	0.20
Serial Correlations:					
T(r_1)	3.00	5.24	5.12	1.78	1.57
T(r_2)	2.26	5.13	3.77	1.21	1.65
T(r_3)	2.10	4.62	3.78	0.18	2.03
Poisson Deviance	92.68	951.49	538.47	88.54	58.46
p(Deviance)	0.0000	0.0000	0.0000	0.0000	0.0077
X^2	86.11	1172.62	665.66	82.81	52.61
p(X^2)	0.0001	0.0000	0.0000	0.0001	0.0283
Using *E**:					
No. of groups	42	45	44	41	35
+/–	21/21	25/20	24/20	21/20	18/17
p(+ or –)	0.5	0.28	0.33	0.5	0.5
runs	11	2	7	19	17
p(runs)	0.0004	0.0000	0.0000	0.26	0.37
p(K-S)	0.36	0.0000	0.0000	0.13	0.18
Serial Correlations:					
T(r_1)	3.01	5.60	5.60	1.89	1.27
T(r_2)	2.26	5.54	4.68	1.41	1.30
T(r_3)	2.08	5.10	4.68	0.38	1.84
Poisson Deviance	91.61	559.89	287.49	81.40	52.41
p(Deviance)	0.0000	0.0000	0.0000	0.0002	0.0295
X^2	85.93	595.88	307.19	71.75	43.22
p(X^2)	0.0001	0.0000	0.0000	0.0021	0.16

Table 10.7. Statistics showing experience for Females, Occupation Class 1 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	1,596	769	351	366	197
Exposure (years) (*R6*)	3,748.4	11,783.6	1,606.7	3,029.8	4,369.4
Expected	1,327.7	464.8	187.3	179.1	104.0
100 × *A*/*E*	120.2	165.4	187.4	204.3	189.5
Using *E*:					
No. of groups	37	37	27	26	15
+/–	28/9	34/3	25/2	23/3	15/0
p(+ or –)	0.0013	0.0000	0.0000	0.0000	0.0000
runs	10	5	5	4	1
p(runs)	0.0329	0.16	0.5	0.0438	1
p(K-S)	0.0000	0.0008	0.0252	0.43	1.00
Serial Correlations:					
T(r_1)	4.19	4.07	1.58	2.00	0.69
T(r_2)	4.19	2.28	2.69	0.39	–0.10
T(r_3)	3.84	1.62	2.09	0.81	–0.08
Poisson Deviance	182.25	250.68	159.26	197.93	73.74
p(Deviance)	0.0000	0.0000	0.0000	0.0000	0.0000
X^2	202.94	304.18	207.91	258.36	85.06
$p(X^2)$	0.0000	0.0000	0.0000	0.0000	0.0000
Using *E**:					
No. of groups	37	38	36	35	26
+/–	17/20	24/14	17/19	17/18	13/13
p(+ or –)	0.37	0.07	0.43	0.5	0.5
runs	8	10	17	19	17
p(runs)	0.0001	0.0018	0.31	0.5	0.16
p(K-S)	0.0000	0.0001	0.0037	0.20	0.98
Serial Correlations:					
T(r_1)	4.16	4.67	0.97	1.80	0.42
T(r_2)	4.11	3.00	2.48	0.86	0.22
T(r_3)	3.73	1.97	1.41	1.16	–0.37
Poisson Deviance	131.35	85.67	59.32	56.72	15.08
p(Deviance)	0.0000	0.0000	0.0085	0.0115	0.96
X^2	124.82	72.94	52.82	42.50	9.45
$p(X^2)$	0.0000	0.0006	0.0349	0.18	1.00

10.8. *Females: Occupation Class 2*

10.8.1. The results for Females, Occupation Class 2, are shown in Table 10.8 The numbers of Inceptions for all Deferred Periods are quite small, with those for DP1 being only 2. The ratios of actual to expected are very large, ranging from 193.1% for DP4 to 297.4% for DP26. On the adjusted basis the results look acceptable, except that for DP13 the actual Inceptions for ages 37 to 39 are very high, being 15 compared with an expected number of 5.3, so the contribution to the deviance from this group is very high. But the numbers are rather small.

10.9. *Females: Occupation Class 3*

10.9.1. The results for Females, Occupation Class 3, are shown in Table 10.9. The numbers of Inceptions for all Deferred Periods are now very small, being fewer than 100 in total, and DP1 having none at all. The ratios of actual to expected are very high, ranging from 284.1% for DP13 to 393.2% for DP52. With such small numbers the adjusted basis appears to fit adequately, but one cannot really tell.

10.10. *Females: Occupation Class 4*

10.10.1. The Claim Inceptions and exposure for Females, Occupation Class 4, are shown in Table 10.10. Since there are only 5 Inceptions in total, no further analysis has been done.

10.11. *Females: Occupation Class "5" (Class "Not Given")*

10.11.1. The results for Females, Occupation Class "5", are shown in Table 10.11. DP1 has only 6 Inceptions, but the numbers for the rest are reasonable. The ratios of actual to expected are high, ranging from 160.6% for DP4 to 261.8% for DP26. These are all very significantly high. On the adjusted basis the results for DP26 and DP52 are acceptable. For DP4 the ratios are above 100% for most ages up to 44 and below 100% for most ages above that. For DP13, age 37 shows a particularly high ratio, with 15 Inceptions against 4.5 expected, thus contributing heavily to the deviance. When particular ages show very high numbers of Inceptions it is always possible that there are unidentified Duplicates in the data; we do not know.

10.12. *Females: All Occupation Classes*

10.12.1. We combine the results for Females for all Occupation Classes in Table 10.12. For these Deferred Periods the ratios are all well above 100, and are all much larger than the corresponding ratios for Males.

Table 10.8. Statistics showing experience for Females, Occupation Class 2 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	2	147	98	77	38
Exposure (years) (*R6*)	9.9	1,909.7	303.7	437.5	702.9
Expected		76.1	36.4	25.9	16.3
100 × *A*/*E*		193.1	269.3	297.4	233.4
Using *E*:					
No. of groups		12	6	4	3
+/–		12/0	6/0	4/0	3/0
p(+ or –)		0.0002	0.0156	0.06	0.13
runs		1	1	1	1
p(runs)		1	1	1	1
p(K-S)		1.00	0.34	0.99	1.00
Serial Correlations:					
$T(r_1)$		0.18	0.10	–0.43	–0.29
$T(r_2)$		0.61	–0.57	–0.97	–0.58
$T(r_3)$		–1.02	–0.80	0.40	0.00
Poisson Deviance		60.71	92.27	73.55	22.72
p(Deviance)		0.0000	0.0000	0.0000	0.0000
X^2		72.27	158.60	116.96	28.88
$p(X^2)$		0.0000	0.0000	0.0000	0.0000
Using *E**:					
No. of groups		18	15	12	6
+/–		7/11	7/8	6/6	2/4
p(+ or –)		0.24	0.5	0.5	0.34
runs		5	3	6	5
p(runs)		0.0175	0.0023	0.39	0.20
p(K-S)		0.99	0.08	0.84	0.99
Serial Correlations:					
$T(r_1)$		0.27	1.82	1.22	–0.16
$T(r_2)$		0.85	0.65	0.71	0.22
$T(r_3)$		–0.41	0.20	–0.91	–0.72
Poisson Deviance		11.67	31.25	12.47	2.82
p(Deviance)		0.86	0.0081	0.41	0.83
X^2		8.50	28.73	8.88	1.68
$p(X^2)$		0.97	0.0174	0.71	0.95

Table 10.9. Statistics showing experience for Females, Occupation Class 3 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	0	38	23	22	16
Exposure (years) (*R6*)	1.2	289.5	70.9	103.5	169.8
Expected		11.9	8.1	6.0	4.1
100 × *A*/*E*		318.5	284.1	364.7	393.2
Using *E*:					
No. of groups		2	1	1	4
+/–		2/0	1/0	1/0	2/2
p(+ or –)		0.25	0.5	0.5	0.5
runs		1	1	1	4
p(runs)		1	1	1	0.33
p(K-S)		1.00	1.00	1.00	0.93
Serial Correlations					
$T(r_1)$		–0.71	0.00	0.00	–0.70
$T(r_2)$		0.00	0.00	0.00	0.57
$T(r_3)$		0.00	0.00	0.00	–0.86
Poisson Deviance		36.08	18.23	24.99	5.21
p(Deviance)		0.0000	0.0000	0.0000	0.27
X^2		53.17	25.64	39.65	3.20
$p(X^2)$		0.0000	0.0000	0.0000	0.53
Using *E**:					
No. of groups		7	4	4	3
+/–		4/3	2/2	2/2	1/2
p(+ or –)		0.5	0.5	0.5	0.5
runs		3	2	4	3
p(runs)		0.20	0.33	0.33	0.33
p(K-S)		0.98	0.31	0.93	1.00
Serial Correlations:					
$T(r_1)$		0.42	0.49	–0.70	–1.15
$T(r_2)$		–1.25	–0.70	0.57	0.29
$T(r_3)$		–0.91	–0.79	–0.86	0.00
Poisson Deviance		9.44	9.41	5.21	1.52
p(Deviance)		0.22	0.05	0.27	0.68
X^2		6.06	6.39	3.20	0.84
$p(X^2)$		0.53	0.17	0.53	0.84

Table 10.10. Statistics showing experience for Females, Occupation Class 4 for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions	0	1	3	1	0
Exposure (years)	0.0	27.8	7.2	9.6	8.4

Table 10.11. Statistics showing experience for Females, Occupation Class "5" for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	6	307	272	234	104
Exposure (years) (*R6*)	23.5	4,616.8	1,167.9	1,529.2	1,636.8
Expected		191.2	137.4	89.4	40.4
100 × *A*/*E*		160.6	197.9	261.8	257.7
Using *E*:					
No. of groups		25	20	12	7
+/–		21/4	16/4	12/0	7/0
p(+ or –)		0.0005	0.0059	0.0002	0.0078
runs		4	6	1	1
p(runs)		0.0115	0.23	1	1
p(K-S)		0.14	0.13	1.00	1.00
Serial Correlations:					
T(r_1)		1.47	1.59	–0.87	–1.14
T(r_2)		0.76	0.45	–0.68	0.06
T(r_3)		1.18	1.66	0.53	–0.50
Poisson Deviance		96.97	156.76	179.32	81.50
p(Deviance)		0.0000	0.0000	0.0000	0.0000
X^2		116.79	219.82	263.83	118.64
p(X^2)		0.0000	0.0000	0.0000	0.0000
Using *E**:					
No. of groups		35	29	27	15
+/–		20/15	14/15	13/14	6/9
p(+ or –)		0.25	0.5	0.5	0.30
runs		14	10	13	9
p(runs)		0.10	0.0291	0.35	0.43
p(K-S)		0.06	0.0335	0.88	0.89
Serial Correlations:					
T(r_1)		1.61	2.57	0.11	–0.66
T(r_2)		0.55	1.34	–1.63	0.01
T(r_3)		2.03	0.56	0.15	–0.40
Poisson Deviance		45.64	70.02	41.92	25.58
p(Deviance)		0.1075	0.0000	0.0335	0.0427
X^2		37.85	58.26	31.54	16.99
p(X^2)		0.3403	0.0010	0.2496	0.3192

Table 10.12. Statistics showing experience for Females, All Occupation Classes for 1991-98, using data grouped so that the number expected in each group is at least 5.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD	1,604	1,262	747	700	355
Exposure (years) (*R6*)	3,782.9	18,627.4	3,156.4	5,109.5	6,887.3
Expected	1,338.8	745.1	370.1	301.0	164.8
100 × *A*/*E*	119.8	169.4	201.8	232.6	215.4
Using *E*:					
No. of groups	37	39	37	32	22
+/–	28/9	38/1	36/1	31/1	22/0
p(+ or –)	0.0013	0.0000	0.0000	0.0000	0.0000
runs	10	3	2	2	1
p(runs)	0.0329	0.5	0.05	0.06	0.5
p(K-S)	0.0000	0.0142	0.0001	0.46	0.98
Serial Correlations:					
$T(r_1)$	4.17	3.81	2.83	0.84	0.54
$T(r_2)$	4.17	2.35	2.94	0.78	0.02
$T(r_3)$	3.84	2.39	2.63	1.26	–0.27
Poisson Deviance	179.01	379.51	411.30	457.32	180.42
p(Deviance)	0.0000	0.0000	0.0000	0.0000	0.0000
X^2	198.25	469.09	583.74	655.03	230.36
$p(X^2)$	0.0000	0.0000	0.0000	0.0000	0.0000
Using *E**:					
No. of groups	37	40	39	37	32
+/–	17/20	21/19	20/19	17/20	14/18
p(+ or –)	0.37	0.44	0.5	0.37	0.30
runs	8	9	7	17	11
p(runs)	0.0001	0.0001	0.0000	0.26	0.0266
p(K-S)	0.0000	0.0025	0.0000	0.18	0.88
Serial Correlations:					
$T(r_1)$	4.14	4.14	3.72	1.25	0.87
$T(r_2)$	4.08	2.25	3.83	1.62	0.80
$T(r_3)$	3.72	2.02	3.40	0.94	–1.03
Poisson Deviance	129.62	83.64	117.83	82.69	21.76
p(Deviance)	0.0000	0.0001	0.0000	0.0000	0.91
X^2	122.63	74.50	107.23	71.35	15.34
$p(X^2)$	0.0000	0.0008	0.0000	0.0006	0.99

10.13. *Results for single years, 1991-98*

10.13.1. It is possible also to analyse the data for each Year from 1991 to 1998 separately. We do this only for Males, Occupation Class 1, and in Table 10.13 we show some results. We show first the numbers of Inceptions each Year for each Deferred Period. We see that these have been falling considerably for DP1 and rising considerably for DP13, DP26 and DP52. This reflects the exposure, not the experience. We then show the ratio of the number of actual Inceptions to the number expected for each Year. These vary quite a lot, but there is no obvious pattern. Most Years show some ratios above 100, some below. However, all the ratios for 1995 are above 100 and all those for 1998 are below, so we can describe these as relatively heavy and relatively light Years.

10.13.2. We also indicate the significance of the results. Where the test of numbers of positives and negatives (note: after grouping) shows clear significance ($p(pos) < 0.01$) we put the ratio in *italic*. Where the deviance (note: before grouping) shows extreme significance ($p(Dev) < 0.001$) we put the ratio in **bold**. In some cases both symbols apply. Again, there is no clear pattern in these results.

Table 10.13. Statistics showing experience for Males, Occupation Class 1
for each Year from 1991 to 1998.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Inceptions, exD					
1991	1,469	308	91	76	29
1992	1,254	304	124	94	46
1993	1,456	300	130	114	35
1994	1,172	254	122	144	50
1995	1,251	328	182	185	96
1996	1,111	259	154	156	70
1997	1,016	284	150	175	94
1998	967	267	172	163	82
All Years	9,696	2,304	1,125	1,107	502
100 × *A/E*					
1991	106.2	117.9	98.4	91.7	96.0
1992	94.1	110.9	122.0	79.8	122.3
1993	***113.5***	97.9	105.1	92.6	59.8
1994	94.3	91.5	108.7	119.1	91.5
1995	104.9	117.0	117.7	122.8	*144.9*
1996	100.1	91.9	101.4	107.7	99.9
1997	91.2	90.6	84.5	100.6	120.0
1998	**93.1**	86.4	*81.6*	84.5	77.1
All Years	100.0	100.0	100.0	100.0	100.0

10.14. *Summary*

10.14.1. It is convenient to put together the results for Males and Females, for all Occupation Classes, and all Deferred Periods in a single table, as we show in Table 10.14. We also show the ratio of the value of 100 × A/E for Females to that for Males, which shows that the Inception rates for Females are always higher than those for Males, ranging from 20% higher to more than three times the level.

Table 10.14. Ratios 100 × A/E for Males and Females, Occupation Classes 1 to 4 and "5", 1991 to 1998.

Deferred Period	DP1	DP4(2)	DP13	DP26	DP52
Males					
Class 1	100.0	100.0	100.0	100.0	100.0
Class 2		124.0	127.4	94.5	140.8
Class 3		152.9	188.8	148.8	197.1
Class 4		267.0	234.4	196.0	227.8
Class "5"	59.2	128.5	135.9	104.4	103.5
All Occupation Classes	98.9	126.2	129.2	105.6	108.1
Females					
Class 1	120.2	165.4	187.4	204.3	189.5
Class 2		193.1	269.3	297.4	233.4
Class 3		318.5	284.1	364.7	393.2
Class 4					
Class "5"		160.6	197.9	261.8	257.7
All Occupation Classes	119.8	169.4	201.8	232.6	215.4
Ratio of Females to Males					
Class 1	1.20	1.65	1.87	2.04	1.90
Class 2		1.56	2.11	3.15	1.66
Class 3		2.08	1.50	2.45	2.00
Class 4					
Class "5"		1.25	1.46	2.51	2.49
All Occupation Classes	1.21	1.34	1.56	2.20	1.99

APPENDICES

Appendix A: Derivation of the negative binomial distribution for the number of Sicknesses that do not become Inceptions

A.1 In this Appendix we show the derivation of the negative binomial distribution for the number of Sicknesses that do not become Inceptions.

A.2 In a fixed interval of time let $S \geq 0$ denote the number of independent Sicknesses that occur, let $I \geq 0$ be the number of those Sickness which (after Deferred Period d, which may be up to d after the end of the fixed period) become Inceptions, and let $J \equiv S - I$, the number of Sicknesses that do not become Inceptions (either because they recover or die before the end of the Deferred Period, or because they recover or die during the run-in period and do not claim). Further, let each Sickness have a probability p of becoming an Inception, with $q \equiv 1 - p$. We know p (or at least infer its value from the assumed Claim Termination rates) and we observe I. We wish to derive the distribution of J or equivalently of S, given I.

A.3 We see that $I|S \sim$ Binomial(S, p), and $J|S \sim$ Binomial(S, q). Let $f_{J|I}(j) \equiv P[J = j|I]$ be the probability function of $J|I$. If $j < 0$ then the underlying event cannot occur, and so $f_{J|I}(j) = 0$ for $j < 0$. If $j \geq 0$ then the underlying event can occur only if there are $S = I + j$ Sicknesses, of which j do not result in Inceptions, and the probability of this event is, from the binomial distribution for $J|S = I + j$, proportional to:

$$\frac{(I+j)!}{j!} q^j$$

that is,

$$f_{J|I}(j) = K\frac{(I+j)!}{j!} q^j \qquad j = 0, 1, 2, \ldots$$

where the (normalising) constant of proportionality K depends on I but not on j. From its functional dependence on j this probability function is easily identified as that of a negative binomial distribution with parameters $I + 1$ and p, and so $K = p^{I+1} / I!$.

A.4 The negative binomial distribution with parameters n and p is sometimes defined as the distribution of the number of failures in an infinite sequence of independent Bernoulli trials, each with probability of success p, before the observation of the n^{th} success. Dispensing with the ordering, one could restate this as follows: the negative binomial distribution with parameters n and p may be defined as the distribution of the number of failures in an infinite number of independent Bernoulli trials, each with probability of success p, given that the observed number of successes is $n - 1$. In the Sickness context, the number of failures is J and the observed number of successes is I. Thus $J|I$ has a negative binomial distribution with parameters $I + 1$ and p.

A.5 Using the negative binomial formula we get successive probabilities:

$$P[J = 0 \mid I] = p^{I+1}$$

$$P[J = 1 \mid I] = (I + 1)\, p^{I+1}.q$$

$$P[J = 2|I] = \frac{(I+2)(I+1)}{2} p^{I+1} q^2$$

and generalising:

$$\mathrm{P}[J = j|I] = \frac{(I+j)!}{I!j!} p^{I+1} q^j = {}_{I+j}C_j p^{I+1} q^j$$

A.6 We also get the mean of J, $E[J|I] = (I + 1)\, q / p = (I + 1) / p - (I + 1)$, and the variance $(I + 1)\, q / p^2$. So the mean of $S = I + J$, or $E[S|I] = I + (I + 1)\, q / p = (I + 1) / p - 1$, and the variance (which we do not use) is unchanged.

Appendix B: Specimen calculations

B.1 In this Appendix we show details of how the calculations described in Section 6 are carried out for one example: Age 40, Year 1991, DP13, for Males, OC1.

B.2 We start with the number of policies In force at the beginning and end of 1991, and the number of cumD and exD Inceptions during the year, all counted from the source files, and using the Age Definitions described in paragraph 6.1.2 for In force and paragraph 4.3 for Inceptions. We also get from the source files the number of days claiming at Age 40 during 1991, using the method described in paragraph 6.2.2. This is all the source data that we need.

In force at start of 1991	F_0	1,064
In force at end of 1991	F_1	978
Inceptions cumD during 1991	Ic	4
Inceptions exD during 1991	Ix	2
Days claiming during 1991	DS	1,132

B.3 We now calculate the rate of growth of the In force, $r = \ln F_1 - \ln F_0 = -0.0843$. The In force has decreased over the year, so this value is negative. The Deferred Period is 13 weeks, or 91 days, and the fraction of a year that this represents is taken as $d = 91/365 = 0.2493$, just less than one quarter. Using the rate of growth we could estimate the In force at 91 days before the start of the year and the end of the year by multiplying F_0 and F_1 by $\exp(-r.d) = 1.0212$, giving 1,086.59 and 998.77. However, we can go directly to an estimate of the average In force over this period as $F_0.\exp(-r.d).(\exp(r) - 1) / r = 1042.06$. A straight linear method would give 1042.44, calculated as $(F_0 + F_1) / 2 - d.(F_1 - F_0)$. The mixture of exponential and linear would give 1042.68, calculated as $(F_0 + F_1) / 2 \exp(-r.d)$. There is little difference here, but the method we have used seems to give better results when the numbers are quite small.

B.4 We now need values calculated using the methods described in Section 5. We define the run-in period as 4 weeks or 28 days = 0.0767 of a year and denote the Deferred Period plus the run-in period as d_2. We need the following:

Probability of Sickness surviving DP, Type 2 rates:	$\pi(x+½,0,d)$	0.017435	Table 5.2
Probability of Sickness lasting for run-in, Type 1 rates:	$\pi_1(x+½,d,d_2)$	0.907119	Table 5.3
Probability of Sickness lasting for run-in, Type 2 rates:	$\pi_2(x+½,d,d_2)$	0.802820	Table 5.3
Expected days Sick for Terminations in DP, Type 2	$e(x+½,0,d)$	7.1349	Table 5.5
Expected days Sick for Terminations in run-in, Type 1	$eb(x+½,d,d_2)$	16.6651	Table 5.6(c)
Expected days Sick for Terminations in run-in, Type 2	$ebc(x+½,d,d_2)$	12.4129	Table 5.6(c)

B.5 From these numbers we can calculate first the probabilities of a Sickness that has lasted for the Deferred Period (A) lasting for the run-in period, $pa = \pi_2 = 0.802780$; (B) terminating in the run-in period and Claim is made, $pb = \pi_2 / \pi_1 - \pi_2 = 0.082202$, and (C) terminating in the run-in period with no Claim being made $pc = 1 - pa - pb = 1 - \pi_2 / \pi_1 = 0.114978$. (We omit age and duration subscripts). The value of η is $1 - pc = 0.885022$.

B.6 We can also calculate the expected days of Sickness within the run-in period for those in categories (B) and (C). The value of *eb* is already calculated and *ec* is readily calculated from the formula in paragraph 5.6.3, $ec = \{(pb + pc).ebc - pb.eb\} / pc = 9.3729$ days. Later we convert these periods to years by dividing by 365.

B.7 Using the cumD Claims file we have calculated the time spent claiming, in days, as 1,132. Some of this may arise from the four cumD Inceptions at Age 40, unless they reach Age 41 before the end of Deferred Period. Other days may arise from Inceptions at Age 39 that reach 40 and are still Sick before the end of the year. Others from Inceptions in previous years that have reached Age 40 during this year. We have not inspected the files in this detail, except by computer. We convert the number of days to years giving 3.10 years. We need to shift this back by a period *d* by multiplying by $\exp(-r.d)$ to get 3.17 years. This gives an estimate of the time spent claiming in the last 13 weeks of the previous year, and in the current year but deducting the assumed time spent claiming in the last 13 weeks of the current year.

B.8 We next need the times spent Sick but not claiming. The four Inceptions necessarily spend 91 days each in the Deferred Period. At this stage we assume that all the records do correctly record dates of Claim in accordance with the assumed Deferred Period, in this case 13 weeks. Although we know that some Claim records are given with dates that are inconsistent, we ignore these discrepancies. Some of the time spent in the Deferred Period by the Inceptions may have been in the previous year; but we have no record of the time spent in the Deferred Period towards the end of the year by Claims that become Inceptions in the following year. We assume that these balance. This gives us 364 days spent in the Deferred Period, or 0.997 of a year.

B.9 Now we estimate the number of Sicknesses, using the formula in paragraph 5.7.2, and abbreviating E[S|I] to $S = (I + 1) / p - 1$, where p is the probability of a Sickness turning into a Claim, given by $\pi(x+½,0,d) \times \eta(x+½,0,d) = 0.017435 \times 0.885022 = 0.015430$. This gives us the estimated Sicknesses as $(4+1) / 0.015430 - 1 = 323.04$. The number of Sicknesses that reach the end of the Deferred Period and do not claim, $E[N|I] = N = (I + 1)/(1 - pc) - 1 - I = 5/0.885022 - 1 - 4 = 0.65$. This gives us the estimated number of Sicknesses that fail to reach the end of the Deferred Period as $323.04 - 4 - 0.65 = 318.39$.

B.10 The average time spent Sick but not claiming by those Sicknesses that do not reach the end of the Deferred Period is $e(x+½,0,d)$ or 7.1349 days. The time in years among the estimated Sicknesses is thus $318.39 \times 7.1349 / 365 = 6.22$ years. The average time spent Sick but not claiming by those who do reach the end of the Deferred Period, but still do not claim equals the Deferred Period plus $ec = 91 + 9.3729 = 100.3729$ days. The total time spent Sick but not claiming by these is therefore $0.65 \times 100.3729 / 365 = 0.18$ years.

B.11 The various deductions that we have give us an account, all in years:

Initial exposure, *R1*	1,042.06
less actual Claims	3.17
Stage 2 exposure, *R2*	1,038.90
less Deferred Period for Inceptions	1.00
less period Sick for those who recover within DP	6.22
less period Sick for non-claimers in run-in	0.18
Stage 3 exposure, *R3*	1,031.50

B.12 Our next adjustment is to multiply *R3* by the ratio of exD Inceptions to cumD Inceptions, giving *R4* = 1,031.50 ×2 / 4 = 515.75. We then multiply by the probability that a Sickness results in a Claim, $\pi(x+½,0,d) \times \eta(x+½,0,d)$, already calculated above as 0.015430, giving us *R5* = 515.75 × 0.015430 = 7.96.

B.13 The expiry age adjustment does not apply at this age 40, so here *R6* = *R5* = 7.96.

B.14 Finally we add together the figures for the eight years 1991 to 1998 giving us 18 exD Inceptions, and 143.04 notional years exposed. We say notional, because, although up to *R4* we are in fact measuring in years, the multiplication by $\pi(x+½,0,d) \times \eta(x+½,0,d)$ destroys the units. As described in paragraph 3.1, the final calculation just allows us to get an estimator of the Sickness rate, σ.

B.15 Note that we carry more decimal places in the computer calculations than shown above, so the results may appear to not round correctly.

Appendix C: Conversion from 1975-78 formulae to 1991-98 formulae

C.1 For the 1975-78 graduations, the graduated values of $\sigma(x)$ were expressed as the exponential of a polynomial in powers of x. For the 1991-98 graduations we have expressed them as GM(0,s) formulae. These are fundamentally the same. In this Appendix we explain the correspondence between them, and how to convert from one format to the other.

C.2. In the GM(0,s) formula we use a polynomial in the form of the Chebysheff factors of t, a transformed version of x. Going up to the fourth power, or GM(0,5), we can express the polynomial in any of three forms as:

$$a + b.x + c.x^2 + d.x^3 + e.x^4$$

or

$$g + h.t + j.t^2 + k.t^3 + l.t^4$$

or

$$n.\mathrm{C}_0(t) + p.\mathrm{C}_1(t) + q.\mathrm{C}_2(t) + r.\mathrm{C}_3(t) + s.\mathrm{C}_4(t)$$

where:

$$t = (x - u) / v$$

hence $$x = v.t + u$$

and:

$$C_0(t) = 1,$$
$$C_1(t) = t,$$
$$C_2(t) = 2t^2 - 1,$$
$$C_3(t) = 4t^3 - 3t,$$
$$C_4(t) = 8t^4 - 8t^2 + 1$$

C.3 We can convert from one representation to another, most conveniently by using the t form as an intermediate step. Thus to convert from the x form to the t form we put:

$$g = a + b.u + c.u^2 + du^3 + e.u^4$$
$$h = b.v + 2c.u.v + 3d.u^2.v + 4e.u^3.v$$
$$j = c.v^2 + 3d.u.v^2 + 6e.u^2.v^2$$
$$k = d.v^3 + 4e.u.v^3$$
$$l = e.v^4$$

To go in the other direction, from the t form to the x form we put:

$$a = g - h.u/v + j.u^2/v^2 - k.u^3/v^3 + l.u^4/v^4$$
$$b = h/v - 2j.u/v^2 + 3k.u^2/v^3 - 4l.u^3/v^4$$
$$c = j/v^2 - 3k.u/v^3 + 6l.u^2/v^4$$
$$d = k/v^3 - 4l.u/v^4$$
$$e = l/v^4$$

To convert from the t form to the Chebysheff form we put:

$$n = g + j/2 + 3l/8$$
$$p = h + 3k/4$$
$$q = j/2 + l/2$$
$$r = k/4$$
$$s = l/8$$

To go in the other direction, from the Chebysheff form to the t form we put:

$$g = n - q + s$$
$$h = p - 3r$$
$$j = 2q - 8s$$
$$k = 4r$$
$$l = 8s$$

C.4 In Tables C.1 and C.2 we show the original coefficients, which are in the x form for the 1975-78 graduations, and the Chebysheff form for the 1991-98 graduations, together with the coefficients in the other two forms and the intermediate form.

Table C.1. Coefficients of the graduated Sickness rates for the 1975-78 graduations in x form, t form and Chebysheff form.

	DP1	DP4	DP13	DP26
x form				
a	–1.796000	–4.256000	–2.722000	–0.481900
b	0.080830	0.239200	0.129000	–0.084340
c	–0.002686	–0.006498	–0.004240	0.0009749
d	0.00002498	0.00005476	0.00003888	0.0
t form				
g	–1.261680	–1.580160	–1.857680	–2.295660
h	–0.353650	–0.444800	–0.589400	–0.158700
j	0.194750	0.045750	0.266000	0.609313
k	0.390313	0.855625	0.607500	0.0
Chebysheff form				
n	–1.164305	–1.557285	–1.724680	–1.991004
p	–0.060916	0.196919	–0.133775	–0.158700
q	0.097375	0.022875	0.133000	0.304656
r	0.097578	0.213906	0.151875	0.0

Table C.2. Coefficients of the graduated Sickness rates for 1991-98 graduations in Chebysheff form, t form and x form.

	DP1	DP4(2)	DP4(3)	DP13	DP26	DP52
Chebysheff form						
n	–1.416038	–3.200943	–0.779340	–2.008366	–2.786287	–3.816687
p	0.238522	0.474149	–0.292921	–0.614523	–0.216186	0.507200
q	–0.588151		–1.018972			
r	0.333549		0.474466			
t form						
g	–0.827887	–3.200943	0.239632	–2.008366	–2.786287	–3.816687
h	–0.762125	0.474149	–1.716319	–0.614523	–0.216186	0.507200
j	–1.176302		–2.037944			
k	1.334196		1.897864			
x form						
a	–8.084687	–3.959581	–10.005045	–1.025129	–2.440389	–4.628207
b	0.529947	0.018966	0.775228	–0.024581	–0.008647	0.020288
c	–0.012129		–0.017836			
d	0.00008539		0.00012146			

Appendix D: Values of σ(x).π(x,d).η(x,d), the intensity of Claim incidence, for Males, Occupation Class 1, 1991-98, DP1 to DP52, both DP4(2) and DP4(3).

Age Exact	DP1	DP4(2)	DP4(3)	DP13	DP26	DP52
20	0.061813	0.007585	0.006223	0.004846	0.002204	0.000981
21	0.068354	0.007313	0.006266	0.003997	0.001875	0.000893
22	0.074898	0.007080	0.006273	0.003338	0.001613	0.000820
23	0.081391	0.006888	0.006258	0.002825	0.001405	0.000760
24	0.087783	0.006736	0.006233	0.002427	0.001240	0.000711
25	0.094032	0.006624	0.006212	0.002119	0.001111	0.000674
26	0.100102	0.006551	0.006204	0.001881	0.001010	0.000645
27	0.105962	0.006518	0.006218	0.001698	0.000933	0.000626
28	0.111586	0.006525	0.006260	0.001561	0.000877	0.000614
29	0.116950	0.006571	0.006336	0.001461	0.000837	0.000611
30	0.122032	0.006658	0.006451	0.001391	0.000812	0.000614
31	0.126811	0.006786	0.006609	0.001349	0.000800	0.000625
32	0.131269	0.006955	0.006815	0.001329	0.000801	0.000644
33	0.135386	0.007168	0.007070	0.001332	0.000814	0.000671
34	0.139146	0.007425	0.007378	0.001355	0.000838	0.000706
35	0.142535	0.007728	0.007743	0.001399	0.000875	0.000751
36	0.145537	0.008079	0.008165	0.001463	0.000925	0.000805
37	0.148145	0.008481	0.008647	0.001549	0.000988	0.000871
38	0.150353	0.008933	0.009191	0.001658	0.001067	0.000949
39	0.152159	0.009440	0.009797	0.001792	0.001161	0.001041
40	0.153570	0.010003	0.010464	0.001954	0.001274	0.001149
41	0.154594	0.010623	0.011192	0.002145	0.001407	0.001273
42	0.155250	0.011302	0.011978	0.002369	0.001563	0.001416
43	0.155563	0.012042	0.012819	0.002629	0.001743	0.001581
44	0.155560	0.012844	0.013711	0.002928	0.001950	0.001768
45	0.155280	0.013709	0.014647	0.003268	0.002186	0.001981
46	0.154764	0.014637	0.015624	0.003653	0.002454	0.002222
47	0.154059	0.015629	0.016634	0.004086	0.002756	0.002491
48	0.153215	0.016684	0.017670	0.004569	0.003094	0.002793
49	0.152288	0.017800	0.018729	0.005102	0.003470	0.003128
50	0.151334	0.018978	0.019804	0.005688	0.003885	0.003498
51	0.150414	0.020215	0.020894	0.006325	0.004340	0.003905
52	0.149589	0.021509	0.021996	0.007013	0.004834	0.004349
53	0.148923	0.022857	0.023113	0.007749	0.005368	0.004832
54	0.148483	0.024257	0.024248	0.008530	0.005941	0.005354
55	0.148336	0.025706	0.025410	0.009351	0.006549	0.005915
56	0.148557	0.027200	0.026608	0.010207	0.007191	0.006513
57	0.149221	0.028735	0.027860	0.011091	0.007863	0.007149
58	0.150413	0.030309	0.029184	0.011996	0.008560	0.007820
59	0.152227	0.031917	0.030606	0.012914	0.009279	0.008525
60	0.154768	0.033555	0.032156	0.013836	0.010015	0.009261
61	0.158159	0.035220	0.033873	0.014754	0.010760	0.010025
62	0.162542	0.036909	0.035802	0.015658	0.011511	0.010813
63	0.168090	0.038618	0.038001	0.016540	0.012260	0.011623
64	0.175011	0.040343	0.040538	0.017392	0.013002	0.012449
65	0.183561	0.042081	0.043500	0.018205	0.013730	0.013287
66	0.194059	0.043830	0.046997	0.018973	0.014438	0.014131
67	0.206904	0.045586	0.051164	0.019689	0.015121	0.014977
68	0.222604	0.047347	0.056180	0.020347	0.015773	0.015818
69	0.241807	0.049110	0.062273	0.020941	0.016389	0.016646
70	0.265354	0.050872	0.069744	0.021469	0.016962	0.017456