

TRANSACTIONS

OF THE

FACULTY OF ACTUARIES

The Compilation of Price Index Numbers and Yield Statistics Relative to Stock Exchange Securities. By A. C. MURRAY, F.F.A., Secretary, Scottish Equitable Life Assurance Society.

[Read before the Faculty, 16th April 1930. A Synopsis of the Paper will be found on page 119.]

INTRODUCTORY.

JUST a year ago, Mr. C. M. Douglas read a Paper to us entitled: "The Statistical Groundwork of Investment Policy" (*T.F.A.*, xii., 173). In the discussion which followed, a suggestion was made that the Faculty might continue the work started by the author, by undertaking the compilation and publication of a service of index numbers relating to prices and yields of British Stock Exchange Securities for the use of our members and others. The proposal was taken up with enthusiasm by our President, and as a result an Investment Research Committee was appointed with the calculation of index numbers as a first object.

The Committee strongly recommended that the Institute of Actuaries should be asked to co-operate in the project, and it was with great pleasure that the Institute's ready acceptance of our Council's invitation was received. It is hoped that publication of the service may be commenced at an early date.

As a member of the Research Committee, I have had the advantage of discussing the subject at some length, not only with the other members of the Faculty Committee, but also with our friends in the south. It follows that my views have been considerably influenced by the opinions of my colleagues, and I wish to acknowledge at the outset the great assistance which I have received from our discussions. At the same time I have, of course, no desire to suggest that any other members of the

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Committees agree with what I shall say in support of the methods advocated, and on other points ; so that while realising very fully my indebtedness to those gentlemen, I accept personal responsibility for all that this Paper contains. The conclusions of the Paper coincide at most points with those come to by the Committees, and the general lines on which the calculations are proceeding are on the basis of the methods here indicated.

The questions which I propose to discuss are the purposes of an index number of Stock Exchange Securities, and the most suitable form of index number for the purposes decided upon. No attempt will be made to deal with matters of detail, the object being to arrive at conclusions on questions of principle. Thus, although subdivision of securities into groups is discussed, the groups which would be suitable are not detailed.

As will be seen, the main difficulties of the problem arise from two sources. The first is the variety of purposes for which the index must cater. The more limited the intended uses of the index, the easier it is to compile suitable statistics, although, even with the most limited practical objective, it is doubtful whether the ideal index is possible. In any case, if we go too far in the limitation, the value of the results is, of course, correspondingly diminished.

The second difficulty is a statistical one, and is increased by the proposed separate index numbers for different classes of securities, necessitating the use of a relatively small number of prices in each calculation.* The most popular use of security index numbers is probably for the purpose of measuring changes in market values from one date to another. It will be found, however, that this particular application is in some cases open to serious objection, and that whatever method of calculation be adopted, the *extent* of the average appreciation or depreciation in the securities used will be approximately represented only in classes where the securities forming the index show some uniformity in their individual price movements.

VARIOUS TYPES OF SECURITY PRICE INDEX NUMBERS.

Security index numbers have been compiled for the purpose of representing speculative activity. Such an index is naturally concerned mainly with ordinary shares, and for the purpose in view a variable list composed of a fixed number of the best

* The minimum number may be taken as four or five.

selling shares, either "unweighted" or weighted according to volume of transactions, may be used. The index gives some indication of market conditions, and conveys an idea of the results which would have been achieved by an investor who had continually changed his securities so as to hold always those most actively traded in. This index shows very wide fluctuations, because in a general upward movement the prices of the shares in greatest demand form the index. In a falling market, when sellers predominate, the shares most readily thrown on the market form the basis of the calculations.

Another form of index, which serves a different purpose, is formed from a list of investments selected because of their representative character. Such an index, when weighted according to nominal amounts, is intended to indicate the changes in the aggregate market value of all the investments from which the index list was chosen. If the index is based on simple weights it should represent the average of the price movements of all the investments.

We have then these three types of index—the first dealing with speculative activity; the second, market valuation; and the third, price movements.

The differences in the results produced by these methods are, of course, very great. For example, the *Financial Times* indexes of New York Industrials, compiled by Irving Fisher, show in the "Traders" index during 1929 a rise of 58% between January and September, while the corresponding rise in the "Investors" index is 14%. The former index is based on most active stocks and the latter on a fixed list. It is perhaps unnecessary to give further examples. It is obvious that the three types of index, calculated for quite different purposes, will give different results, and consequently the type to which an index belongs must be kept very carefully in view when making use of it.

It should be mentioned, however, that a knowledge of the type of index is not in itself sufficient. The user of a security index should have information as to the method of selection of securities and the method of calculation. He must know whether accrued interest is allowed for, and how issues of bonus shares or share issues on bonus terms are treated.

It is clear that the first task of the compiler of an index is to decide which type will be most suitable, and to enable him to do so he must define as precisely as possible the purposes which the index is intended to serve.

PURPOSES TO BE SERVED.

As a first requirement, our index should be suitable for the use of large investors, such as Insurance Companies, Financial Trust Companies, and so on. In fact, what we want is an Investors' Index rather than a Speculators' Index. The purposes to be served may, I think, be divided under three general headings as follow :—

- (a) Uses directly connected with current investment activities.
- (b) The investigation of the effect of economic, financial and political events on the level of security prices.
- (c) Assistance in measuring approximately changes in the value of a portfolio of securities.

(a) and (b) will be concerned more with the direction of the curve of prices and changes in direction than with the extent of the movements.

The practical usefulness of reliable price index numbers in connection with the everyday work of Insurance Officials and others concerned with the investment of large funds is very great, provided separate indexes are available for the various classes of securities involved. The utility of the price index, however, is limited (particularly in the case of Ordinary Shares) unless yield statistics are available in addition. I accordingly assume throughout that both sets of figures are to be calculated, and the compilation of the yield statistics is dealt with later.

The usefulness of such figures in connection with daily problems of investment is similar in character to the usefulness of a reliable book of reference. Constantly it will be possible to avoid troublesome investigation by a glance at the statistics. The past prices of an individual security, so far as available from other sources, may be compared with the general movement of the prices as a whole in the group to which it belongs; the current yield on any investment may be compared with the average yield of the appropriate group; a comparison of average yields obtainable in different classes of investments may be made, and the past history of such yields can be seen at once. Some indication of the effect of past events on the prices of the various classes of securities is available. A comparison by classes of the price movements of securities held, with the appropriate index numbers, may prove the success of the methods of selecting securities purchased, or draw attention to a weakness in the tests applied in their selection.

Other useful purposes coming under heading (a) could be cited, but I do not think it necessary to go further. Investment Departments of Insurance Offices are continually faced with questions relating to past movements of prices and yields, the complete answer to which often cannot be ascertained in time to be of any use. The answer would in many cases be supplied at once by the indexes. Only by practical experience will the full benefits of an extensive service become apparent, and I suggest that most of us in course of time will regard such figures as an essential part of our investment equipment.

Heading (b) is concerned with problems of the nature discussed at length in Mr. Douglas's paper already referred to.

The third general use mentioned is self-explanatory. Provided index numbers are available for a sufficiently large number of separate groups of securities, it should be possible to estimate roughly from time to time the changes in the value of securities held, although, as has been mentioned, the results should not be applied to valuation purposes in groups which show wide divergence amongst the movements of individual securities.

In referring to valuation, I do not suggest, of course, that a skilled investor's holdings will show in all classes the same movements as the index, but in times of wide movements even a very rough idea of appreciation or depreciation, obtained without making a full valuation of the securities, would be of considerable value.

SELECTION OF SECURITIES AND WEIGHTING (PRICES).

The natural sequence in which to consider problems of compilation would be (1) selection of securities; (2) weighting of securities; and (3) formula of calculation. As the problem of weighting, however, is bound up with that of selection, and as the method of selection advocated here arises directly from consideration of the question of weighting, I shall deal with the latter first.

Fisher's very thorough treatise on index numbers—*The Making of Index Numbers*—is essentially devoted to problems involving price and quantity, the quantity element producing the weight to be applied to the price. In fact, the author goes so far as to suggest that there is no problem where index numbers have yet been employed where weights are not involved, and he shows throughout the book a contempt for any unweighted index,

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although it is admitted, in Appendix IX. to the 3rd Edition (p. 525), written in reply to criticisms, that "undoubtedly a *simple* "index made from *well chosen* samples may have somewhat more "value than I thought when this book was written."

So far as commodity price indexes are concerned, there certainly seems little likelihood of an index number involving simple weights being the most suitable one. If the purpose of the index is to measure the cost of living, the weights should depend on the consumption of the commodities involved; if for the purpose of measuring trading results, the weights should depend on the volume of transactions; and so on. If, however, we consider the hypothetical (and impossible) case of an individual consumer who spends equal amounts of money on each commodity, an index applicable to his cost of living should give equal weight to all the commodities involved. Thus a rise of 10% in the price of bread should affect the index number in the same way as a rise of 10% in the price of beer, or any other commodity which he consumes. Such a case is, of course, quite impracticable, but in security index numbers we have, I think, an analogous situation demanding such treatment.

The case is that of an investor who invests an equal amount of money in each of several securities in a group. The index number which could correctly be applied to his holdings would be one which gives equal weight to every security involved. For the purposes stated, namely everyday use, research, and as a basis for approximate valuation, an unweighted index is what would be required; the only qualification being that the research is to be related to *investment* problems. Weighted indexes of security prices would be necessary as a basis for research concerned with economic problems, such as refer to changes in aggregate market values, for example.

With certain exceptions an Insurance Company does not tend to make its purchases proportionate to nominal amounts nor to market capitalisation, and still less does it consider volume or number of dealings in relation to the amount to be invested. The exceptions are that securities of very small market capitalisation and securities in which there is practically no market will generally be excluded from consideration altogether. Amongst securities of larger amount, Insurance Companies' purchases will tend *within each class* to approximate more nearly to equal amounts of money in each than to any other distribution which a system of weighting could cater for. What I have said of Insurance

Offices in this connection applies equally, I think, to other large investors.

The conclusion is that the most suitable statistics for our purposes should provide a separate unweighted price index for each class, representing the movements of the securities of large amount within the class. Practical considerations do not admit of a dividing line between "large" and "small" amounts which will apply to all groups. The criterion should be large market capitalisation rather than large nominal amount, and, where a company is concerned, should apply to the amount of the particular stock in question and not to the total capital of the company.

In coming to this conclusion as regards weights, we have advanced some distance with the problem of selection by the exclusion of securities of small market capitalisation. From the securities of large market capitalisation available, we may either select by a mechanical formula or make a deliberate selection, choosing securities in such a way that the selected list is a fair representation of the whole, and providing for such features as are discussed in the next paragraph.

We will find, even amongst the securities of large amount, which we are now considering, that lack of dealings renders some securities less suitable than others. There are in certain groups several companies so closely related in their working arrangements and capital position that they would be more correctly described as *one* company, and might weight the index unfairly if all were included. There are questions of geographical distribution of the companies in each group. For example, a rigid formula of selection applied to British Companies might give undue weight to English Companies relative to Scottish Companies, or vice versa. Also within a given group there may be a certain number of companies with special characteristics. Amongst Investment Trust Companies, for instance, some specialise in a particular form of investment. It would be impossible for a rule of thumb method of selection to secure the correct representation in the index of all features such as these.

On the other hand, there is an appeal in an index constructed from data which have been selected by a mechanical formula, as there can be no question or doubt in the mind of the user as to whether the personal views of the compilers have biased the figures one way or the other. Further, those using the index should have as much information as possible regarding the manner in which the figures have been built up. A statement of the formula

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will show exactly how securities were chosen. If the formula of selection is known, its limitations will be appreciated.

While on the whole I personally would incline to an index based on intelligent selection, there is undoubtedly a great deal to be said for a fixed rule.

Whichever method be adopted, it will be necessary in the Fixed Interest classes to exclude redeemable securities having only a short period to run, as price movements of these will be restricted in view of approaching maturity. A minimum duration of say twenty years would meet the case. Assuming the securities are to be used for yield statistics as well as for a price index, a further condition is necessary, namely, that the price and nominal rate of interest should be such as to make the difference between the running yield and the redemption yield of little importance, thus permitting us to deal with the running yield throughout and so saving much labour. The exclusion of the shorter Debentures has some bearing on one of the uses to which I have said the index might be applied. I refer to approximate valuation; if the investments in respect of which a rough indication of the change in value is required include shorts and semi-shorts, the index will not, of course, be directly applicable to the classes in which those appear. Their exclusion from the index, however, is essential.

In the foregoing remarks subdivision of securities into groups has been implied. The statement that large investors will tend to invest equal sums in each security is true only within homogeneous classes. From one class to another wide differences will exist. But, apart from the question of the relation between the index and the distribution of investments held, there is a more fundamental reason for the subdivision. The more homogeneous the securities in the group, the stronger will be the tendency to general movement among the prices. The first and most obvious division of securities, is into two groups—Fixed Interest and Variable Dividend—but within those two large groups very wide divergencies amongst sub-groups take place. In fact, the data on which the large general groups are based are of so heterogeneous a character that the resulting indexes are of little use for our purposes. Such an index cannot be employed for our first or third objects and is of limited value when applied to the second.

The following index numbers of Stocks and Shares of British Companies show, in a short period of eight months, variations in the indexes of various groups of Ordinary Shares ranging from +26.1%

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to -12.8% , and in the Fixed Interest group from -2.9% to -21.3% . The use of general index numbers combining such results may lead to conclusions which are true of Fixed Interest Stocks or Ordinary Shares as a whole, but which are quite erroneous when applied to subdivisions of the broad groups.

INDEX NUMBERS AS AT 30TH AUGUST 1929.*
BASED ON 100 AT 31ST DECEMBER 1928.

Class.	Home Rails.		Shipping.		Electric Supplies.		Building Material.	
	Debs.	Ord.	Pref.	Ord.	Pref.	Ord.	Pref.	Ord.
Number of Securities.	4	4	5	4	4	5	4	7
Index.	—	106.2	—	87.2	—	126.1	—	95.2
	93.0	—	78.7	—	96.4	—	97.1	—

While in the case of Fixed Interest bearing stocks, it may not be necessary to subdivide to the extent indicated by the groups selected in the above Table, some subdivision is essential. There seems no alternative to subdivision of the Ordinaries into each of the separate industries involved, if the resulting index is to be of maximum value.

An index service formed on the foregoing basis will represent the price movements of securities of large market capitalisation in each group, giving equal weight in each index to every security included. By dealing only with securities of large market capitalisation we tend to eliminate the more speculative ventures, which usually are of relatively small amount. Amongst the sounder investments of small amount in a group, the movement of prices should approximate to the movement of those in the index. Consequently, while we are dealing with an index of securities of large amount, we obtain also an approximation to the movements of the whole group, excluding the more speculative types—for example, many small companies floated in the boom of 1928. In this way we may, I think, claim that, although based on the requirements of large investors, the index is applicable to all purposes which may be fulfilled by an index of unweighted prices of the less speculative investments.

* Compiled by geometric averages from representative securities of large amount. Accrued Interest (net) allowed for.

METHOD OF AVERAGING.

It has been stated that a price index prepared for general use should be based on a simple formula which can be easily understood. This consideration has no doubt been responsible to some extent for the extensive use of an arithmetic average. The truth of the matter, however, so far as our problem is concerned, is that the arithmetic method and the *meaning* of the results which it produces are the very reverse of simple. In fact, in extreme cases the results have no practical meaning at all. Accordingly, I propose to eliminate "simplicity" as a test of the suitability of a formula, but to retain as a factor speed of calculation.

The methods of averaging which are available are :—

- (1) Arithmetic.
- (2) Geometric.
- (3) Harmonic.
- (4) Median.
- (5) Mode.

In each case an average of price ratios is intended; thus, the formulae for the arithmetic and geometric index numbers at time y are

$$\frac{1}{n} \left[\frac{P_y}{P_o} + \frac{P'_y}{P'_o} + \frac{P''_y}{P''_o} + \dots \right] \text{ and } \left[\frac{P_y}{P_o} \times \frac{P'_y}{P'_o} \times \frac{P''_y}{P''_o} \times \dots \right]^{\frac{1}{n}}$$

respectively, where $P_o, P'_o, \dots$ represent prices at time o (the base), $P_y, P'_y, \dots$ prices at time y , and n the number of securities. The ratio of the arithmetic index number at time z to the index number at time y is

$$\frac{\frac{P_z}{P_o} + \frac{P'_z}{P'_o} + \frac{P''_z}{P''_o} + \dots}{\frac{P_y}{P_o} + \frac{P'_y}{P'_o} + \frac{P''_y}{P''_o} + \dots}$$

The corresponding ratio in the geometric index is

$$\frac{\left[\frac{P_z}{P_o} \times \frac{P'_z}{P'_o} \times \dots \right]^{\frac{1}{n}}}{\left[\frac{P_y}{P_o} \times \frac{P'_y}{P'_o} \times \dots \right]^{\frac{1}{n}}} = \left[\frac{P_z}{P_y} \times \frac{P'_z}{P'_y} \times \dots \right]^{\frac{1}{n}}$$

and this, being independent of base date prices, will always be the same whatever base be chosen.

It will be observed that in adopting either of these methods, prices of £100 of stock and prices of shares of £1 nominal or any other denomination may be used in the same calculation without any need for adjustment to common nominal amounts.

For our purpose the Median and Mode appear to have nothing to recommend them, and are, in fact, entirely unsuitable in most cases in view of the small numbers of securities involved by reason of the division into classes.

Let us consider Arithmetic and Geometric averages. For an unweighted index of security prices, it appears that we must endeavour to fulfil as far as possible the following conditions :—

- (1) The relation between any two numbers in the index should be the same whatever base be chosen. It follows that the index number reckoned forward and backward should show the same percentage change in opposite directions.
- (2) The index should show approximately the result as between any two points of an investment of equal amounts of money in each security involved in the index.
- (3) The formula should admit of speedy calculation.
- (4) The formula should admit of the addition, deletion, or exchange of securities included without vitiating the results, and without undue labour.
- (5) The formula should admit of a simple method of combining group indexes to form one index in respect of several groups ; the new index complying with conditions (1), (2) and (4), at least to the extent that the group indexes themselves comply.

Test (1) requires, for example, that if December 1928 be the base, the same percentage change should be shown, say, between April 1930 and May 1931 as if April 1930 or any other date had been taken as the base. Similarly, if on the base December 1928 the index number as at April 1935 is twice the base number, then the formula should show the December 1928 figure as one-half of that for April 1935, when the latter date is the base. The arithmetic method does not fulfil this test. Speaking generally, different results are obtained from different bases, and it follows that a reverse calculation will not necessarily produce the reciprocal of a forward calculation between the same two dates. The geometric method, on the other hand, fulfils Test (1) in all respects. The extent of the error involved in the arithmetic method is dependent on the variation in individual price movements as will be discussed later.

The second test requires, for example, that if the index rises from 100 to 120 over a period, an investment of £10,000 made at

the first date and spread equally over the securities used in the index should show approximately an appreciation of £2,000 at the second date. In the special case where the first date of comparison is the base date of the calculations, the arithmetic method will fulfil Test (2) exactly, but this special case is of little importance. The more uniform the variations in the individual prices the more nearly will Test (2) be met by the use of either arithmetic or geometric averages, and in the extreme case where the percentage movements in all individual prices are identical between consecutive dates of calculation, the results by the arithmetic method (whatever date be chosen as base) and the geometric method will be the same, and these results will satisfy Test (2) exactly at all points. Immediately, however, we depart from hypothetical conditions and deal with actual cases, either formula may fail to give the desired approximation.

In the case of a group of Ordinary Shares represented by a small number of securities showing wide divergencies amongst their individual price movements, discrepancies will appear whichever method be employed. Where the divergencies are extreme, either method is liable to give results which are absurd in relation to Test (2), as a simple example involving two securities will show at once.

In the Fixed Interest classes, however, the geometric method will usually give a close approximation to the investment results desired, while in the Ordinary Groups, approximations near enough to be of value will be obtained in most cases. (See Table II., page 112.)

Combining this test with Test (1) the geometric results referred to will be consistent as between any two dates reckoned forward and backward, and are independent of base. It is also very important that in all cases, extreme or otherwise, the figures produced by geometric averages will have a specific meaning; the ratio of any two numbers in the index giving the geometric average of individual price ratios between the two dates compared. Further, as a true investment result would be shown by the arithmetic average of the ratios of the *same* individual prices, it follows, the geometric average being less than the arithmetic, that the geometric method has the advantage that the Test (2) "error" is always in the same direction. Appreciation will never be overstated nor depreciation understated.

Applying Tests (1) and (2) simultaneously to the arithmetic method, we may or may not achieve reasonable results in regard

to Test (2) in view of the failure of the method when Test (1) is applied—that is on account of the dependency on the base chosen. The difference, as compared with investment results, may be positive or negative, and we cannot, without an investigation of individual prices, say which. Where individual prices tend to move together, the arithmetic method may be regarded as showing between any two dates an *approximation* to the arithmetic average of individual price ratios between the two dates of comparison, that is, an approximation to investment results. But when there are marked divergencies in individual price movements, the approximation no longer holds and, further, no practical meaning can be assigned to the results.

It will be observed that a fundamental difference between the two methods is that the geometric average of individual price ratios is the same thing as the ratio of the geometric averages of individual prices, whereas there is no such relationship in the case of arithmetic averages.

Long-dated British Government Securities form an ideal group for an index, and the percentage changes in individual stocks correspond so closely that Test (2) would be met equally well either by the arithmetic or geometric method. In order to examine the results in a group at the other end of the scale, I have chosen the most unfavourable class of Ordinary Shares included in the index numbers published in *T.F.A.* xii., 198—namely *Iron, Coal and Steel*. Mr. Douglas has kindly supplied me with the net prices, and I have recalculated the index as at December of each year from 1924 to 1928 inclusive, adopting as base each of those dates in turn and employing the arithmetic formula. To facilitate comparison, instead of submitting the index numbers, I have shown in the following Table the percentage change in the index between various dates. Column (1) gives the figures for base December 1923 as calculated by Mr. Douglas, and column (7) shows movements as obtained by the geometric formula. The results by the arithmetic method, printed in italics, show comparisons in each case with the base date of the calculation and, therefore, give the actual result at the second date of an investment made at the first date of equal sums in every security in the index.

Before considering the figures, I would emphasise that taking this Table as a whole it represents a group, the movements in which over the period are such as to make any index unreliable, at any rate for valuation purposes. At the same time, the figures are useful here as an illustration. For purposes of valuation one

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would require, in an erratic group like this, a series of indexes showing results by the arithmetic method with all dates from the commencement successively forming the base! Nothing less would exactly meet the case.

TABLE I.—IRON, COAL AND STEEL.

Ordinary Shares.

Dates compared.		Percentage changes in Price Index Numbers.						
		Arithmetic Indexes.						Geo-metric Indexes.
From	To	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dec.	Dec.	Base : 1923	1924	1925	1926	1927	1928	
1924	1925	- 18·2	- 18·3	- 19·3	- 18·3	- 18·1	- 17·8	- 18·8
	1926	- 26·2	- 26·5	- 27·1	- 46·5	- 42·4	- 46·9	- 35·0
	1927	- 26·2	- 26·6	- 26·3	- 44·3	- 42·4	- 46·3	- 33·7
	1928	- 22·5	- 22·9	- 22·3	- 43·9	- 41·3	- 46·0	- 32·9
1925	1926	- 9·9	- 10·0	- 9·6	- 34·5	- 29·6	- 35·4	- 19·9
	1927	- 9·8	- 10·1	- 8·7	- 31·9	- 29·7	- 34·7	- 18·3
	1928	- 5·3	- 5·6	- 3·7	- 31·3	- 28·4	- 34·3	- 17·4
1926	1927	+ 0·1	- 0·1	+ 1·0	+ 4·0	- 0·1	+ 1·2	+ 1·9
	1928	+ 5·0	+ 4·9	+ 6·5	+ 4·9	+ 1·8	+ 1·7	+ 3·1
1927	1928	+ 5·0	+ 5·0	+ 5·4	+ 0·8	+ 1·9	+ 0·4	+ 1·2

Prices on which this Table is based are given in Appendix I.

Examining the horizontal columns of the Table, we find that in every line the "error"—*i.e.* the difference between the figure in italics and the others—is greater at some point in the arithmetic results than in the geometric column. Some of the most glaring discrepancies in the arithmetic index are shown by adopting a base subsequent to the dates compared, but this is merely chance, and those differences or greater differences might equally well arise with a base prior to the dates of comparison. The worst result in the

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Table is shown in the 1925 to 1928 arithmetic comparison. On the base 1928, the fall in the index is nearly ten times that produced on the base 1925!

It is easily shown that, regardless of the base, the arithmetic method will give correct investment results between any two points, provided the proportionate changes between the dates of comparison are the same in each security; and the close agreement amongst all the 1924 to 1925 arithmetic comparisons is due to a sufficiently close agreement in individual price movements between those dates. The satisfactory geometric result for the same comparison is, of course, due to the same reason. It can also be shown that, provided only the changes in individual security prices between the base and the first date are all in the same proportion, a correct investment result between the dates of comparison will be obtained by the arithmetic method. At certain points in the Table there is sufficient agreement between base and first date to give reliable results—for example, in the case of any comparison starting from 1924 on base 1923. The difference in individual price movements from 1925 to 1926 was very marked, but, as mentioned, this does not upset the comparisons referred to. It will be observed that in the 1926 to 1927 comparison, the arithmetic method twice indicates a fall, whereas the investment comparison shows a rise.

Most of the figures in columns (4), (5) and (6) show greater differences from the investment result than does column (7) and, further, these arithmetic results have no practical meaning.

It will be seen from the foregoing that the selection of a suitable base is important in using the arithmetic method, but it must be remembered that the most suitable base at the commencement may conceivably be the least suitable at a future date.

Having discussed favourable and unfavourable extremes—*British Government Securities* and *Iron, Coal and Steel*—it will be useful to examine the behaviour of the geometric index in intermediate groups. With this in view Table II. is given, dealing with all the groups adopted by Mr. Douglas and showing for each the difference between investment results and the geometric index over a period of five years—from December 1923 to December 1928.

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TABLE II.

GROUP.	Number of Securities.	December 1923 to December 1928.	
		Investment Result. Capital Profit (+) Loss (-) per cent.	Geometric Index. Per cent. alteration in Index.
Government and Corporation			
<i>British Funds</i> . . .	5	+ 2·7	+ 2·7
<i>Dominion Government</i> . . .	5	+ 2·8	+ 2·8
<i>Corporation Stocks</i> . . .	10	- 1·2	- 1·2
<i>Dominion Corporation</i> . . .	5	+ 2·9	+ 2·9
Debentures.			
<i>Investment Trust</i> . . .	5	- 2·6	- 2·6
<i>Electric Light and Power</i> . . .	5	+ 0·5	+ 0·5
<i>Industrial</i> . . .	8	- 0·6	- 0·6
<i>Iron, Coal and Steel</i> . . .	5	- 10·0	- 11·0
<i>Shipping</i> . . .	3	- 1·2	- 1·2
Preference.			
<i>Investment Trust</i> . . .	5	+ 3·1	+ 3·0
<i>Electric Light and Power</i> . . .	5	+ 4·4	+ 4·3
<i>Industrial</i> . . .	8	+ 1·1	+ 0·8
<i>Iron, Coal and Steel</i> . . .	5	- 26·6	- 37·8
Ordinary.*			
<i>Investment Trust</i> . . .	5	+ 102·2	+ 101·7
<i>Electric Light and Power</i> . . .	5	+ 67·6	+ 57·5
<i>Industrial</i> . . .	8	+ 52·5	+ 46·7
<i>Iron, Coal and Steel</i> . . .	5	- 33·4	- 42·4
<i>Shipping</i> . . .	3	+ 23·4	+ 16·6

For all the Fixed Interest groups, except *Iron, Coal, and Steel Preference*, these results are entirely satisfactory. Amongst the Ordinaries, as might have been expected, *Investment Trusts* show close agreement. The *Electric Light and Power* and *Industrial* group

* The price changes in individual securities included in the Ordinary Share groups are shown in Appendix II.

results are quite good, but the remaining two, again as might have been anticipated, are not so satisfactory. The Table, covering a period of five years, confirms, I think, what has been claimed for the geometric method, and also emphasises the necessity for caution in using the figures as a measure of appreciation and depreciation in the more erratic groups.

Tests (4) and (5) are, I think, equally well fulfilled by either method. In laying down Test (5) I should mention that I do not visualise a combined index of all securities so much as the combination of groups of similar character, such as *Insurance Companies*, *Investment Trusts*, and *Banks*.

The harmonic mean suffers from similar disadvantages to the arithmetic, and in addition involves more work.

The geometric method involves less work than the arithmetic [Test (3)], and my conclusion is that the geometric index is the most suitable for our purpose, but I would add that when extreme variations in the price changes of individual securities take place in a group, that group index must not be used for valuation purposes, but only to show direction of movement.

Several tests of a more theoretical character might have been added to those laid down, but, as they would hinder rather than aid satisfactory practical conclusions being arrived at, they have been omitted. It is appreciated, for instance, that the geometric method breaks down on a price becoming zero or negative, but I do not think this has any bearing on the suitability of the method, even allowing that both of these contingencies are remotely possible.

For completeness, reference should be made to a method which is sometimes advocated and which Fisher has named "Aggregative." Using the symbols already adopted, but adjusting the prices so that the price per cent. is taken in each case, this method is represented by the formula

$$\frac{P_y + P'_y + P''_y + \dots}{P_0 + P'_0 + P''_0 + \dots}$$

It has the fatal drawback, when applied to our problem which does not involve weights, that if between any two dates, not excluding the base, the percentage movements of individual securities are interchanged, a different result will be obtained. The requirement that a rise of ten per cent., say in a security price, should affect the index equally, irrespective of which security it applies to, seemed too obvious to set down as one of the tests, and I feel that the

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aggregative method fails fundamentally. The effect of the method would be to represent purchases of an equal *nominal* amount of each security, which clearly is inapplicable to any practical problem.

Summarising the conclusions reached, the essential features of the scheme will be :—

- (1) The Index to be an index of securities of large amount.
- (2) Securities to be subdivided into homogeneous groups and a separate index calculated for each group.
- (3) Security prices to be unweighted.
- (4) Geometric mean of price ratios to be employed.

We may now consider some minor problems.

PRICES TO BE USED.

The period of time to elapse between calculations must be decided largely on the basis of the amount of work which can be undertaken. If practicable, monthly figures for Fixed Interest Stocks, and weekly figures for Ordinaries would meet most requirements. The use for each security of the average of prices recorded during the period rather than prices obtaining at the end of the period has been advocated. I think it preferable, however, to have an index representing the position from one fixed date to another. The most suitable quotation to use is the middle price from the London Stock Exchange Official List. With regard to the base figure, there seems no reason to depart from the most usual figure, namely, 100.

BONUS ISSUES AND ACCRUED INTEREST.

Bonus issues, or issues on Bonus terms, taking place after the commencement of the index in respect of Ordinary or Deferred Shares must, of course, be allowed for, and might be dealt with by taking the price after such an issue of $(1 + \frac{1}{n})$ shares, where the free issue is $\frac{1}{n}$ of one share. Thus, if $n = 1$, that is, where one share is given for one held and that free, the price of two shares would be taken subsequently to the price being quoted ex rights. To take another example: if a share standing at 40s. receives one new share, at a price of 30s. for one held, the result is that two shares are involved at 70s. and the number of shares related to the previous price of 40s. is $\frac{40}{70}$ of two shares or $(1 + \frac{1}{7})$ shares,

and subsequently to the issue the price of $1\frac{1}{7}$ shares would be taken. Allowance for accrued interest should be made, particularly in view of the subdivision into small groups. The effect of ignoring it may be important. Net accrued interest should be deducted rather than gross.

YIELDS.

So far, I have dealt mainly with the problems connected with price index numbers. As mentioned, however, if the scheme is to have a maximum utility, some indication of the yields of investments at different points of time is necessary. Provided the securities available are sufficiently long-dated, the running yield—that is, without allowance for redemption—is most suitable.

The natural procedure in the compilation of a yield index would be to use the same securities as are selected for the price index numbers and to employ similar methods of construction. So far as the securities to be employed are concerned, this plan presents no difficulties, provided that in making the selection the conditions as to duration, etc., which have been laid down, are given effect to. The methods of construction suggested for the price index, however, will be found unsuitable for an index of yields, which, in fact, becomes quite an independent problem when we consider that an average of actual yields on a given date has a definite meaning, whereas an average of the corresponding prices has none. We may therefore, if we wish, discard the idea of an index in the generally accepted sense and compile instead a series of yield averages. That is, we would deal with averages of actual yields instead of averages of yield ratios. If an arithmetic average be employed the results will show at each date the yield obtainable by the investment of equal amounts of money in each security involved. The relation amongst the resulting figures would be the same as would be produced by a yield index calculated by the aggregative method already referred to. The only difference between the method here suggested and the aggregative index would be that, in the latter series, the initial figure would be adjusted to 100 and subsequent figures adjusted similarly. That is to say, the actual averages would in fact form an aggregative index, the base being the average of yields at the base date. Regarded as such the results would be open to the obvious objection to an unweighted aggregative index already mentioned. The figures should not be so regarded, however. The

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uses to which yield statistics will be put are of a different nature to those applicable to a price index, which are essentially related to price movements over a period—problems of yield, on the other hand, being concerned with the yield obtainable at different points of time.

I consider that arithmetic averages of actual yields constitute the best method of treating the problem, and in view of the intention to compare yields at different dates, when perhaps different rates of tax apply, prefer gross figures to net.

An arithmetic *index* of yields would show the average of the ratios of each individual yield at any time to the corresponding yield at the base date. Such statistics would be inferior for all practical purposes to the proposed yield averages. A geometric yield index is equally objectionable, and in any case is impracticable, as non-payment of interest or the passing of a dividend would produce a nominal rate of 0 which would make the index number 0, or render it indeterminate. In presenting yield figures it would be desirable to state the number of securities involved in each calculation, and also the number of non-payers included in these, in order to indicate how far low averages are due to non-payment of interest or dividends.

CLASSES WHERE IRREDEEMABLE AND LONG-DATED SECURITIES ARE NOT AVAILABLE.

In these notes, so far, I have assumed that it is possible to obtain in the Fixed Interest groups which are to be treated a sufficient number of irredeemable or long-dated securities. The objection to combining shorts and longs in one index has been mentioned. Amongst the important groups it will be found that in most cases the required condition is fulfilled.

We have, however, two important classes which cannot be dealt with on the foregoing lines. These are *Dominion Government Securities* and *Foreign Government Securities*. A feature of most investments in these classes is that they are redeemable at a fixed date or within a defined period, and there are relatively few really long-dated stocks or bonds.

Even if long-dated securities were available, neither class is suitable for the purpose of a comprehensive index. It is meaningless to combine prices of the bonds of Denmark, Mexico, Japan, Greece, and so on, in one index. Almost equally unsuitable is the combination of British Dominions. Price index numbers for each country separately are impracticable on account of the

lack of long-dated securities. At the same time it is desirable to include these classes in the scheme. The most satisfactory plan would appear to be to calculate, in the case of each country, an average of yields, allowing for redemption, and to exclude securities having less than say ten years to run. The precise method of redemption adopted should be allowed for in the calculation in each case. Here again, I think, for the purpose of our statistics, gross yields are preferable to net.

Bonds having an *option* for redemption at an early date are suitable so long as quoted at a price considerably below the redemption price, and so long as the final date is at least ten years off.

Countries defaulting on interest payments would necessarily be excluded.

EXISTING INDEX NUMBERS.

Mr. Douglas referred in his paper last winter to the Security Index Numbers already published in this country, as did Mr. Recknell in the discussion on the paper. These indexes, published by *The Bankers Magazine*, *The Financial Times*, *The Investors Chronicle* and *Money Market Review*, and the *London and Cambridge Economic Service*, all have their particular uses. There is, however, an almost complete absence of yield statistics; the only figures available being a yield index based on four Fixed Interest Stocks, included in the *London and Cambridge Service*. Further, in each case there are reasons why the price index is not suitable for the particular objects here set out.

In January of this year a new production was added to the list. That is the daily index of the *Financial News*, which keeps us informed from day to day of the average price movements of a selection of thirty leading British Industrial Ordinary Shares. This is a useful addition to current statistics on which the *Financial News* is to be congratulated, although probably most of us would have preferred some subdivision of the securities with separate figures for each class.

In conclusion, I wish to thank Mr. C. M. Douglas, F.F.A., not only for supplying me with the net prices employed in his own calculations, but also for kindly reading the draft of this Paper and making many helpful suggestions. I am also indebted to Mr. C. M. Gulland, F.F.A., for having assisted with the numerical calculations necessary for Tables I. and II.

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APPENDIX I.

Net Prices on which Table I., page 110, is based.

	1923	1924	1925	1926	1927	1928
<i>Security.</i>						
Armstrong, Whitworth	86·875	75·20	62·50	16·25	20·00	16·04
Cammell, Laird & Co.	76·25	61·25	50·00	46·25	37·50	40·00
Dorman, Long & Co. .	77·50	67·50	52·50	61·25	51·25	57·50
Guest, Keen & Nettlefolds	200·00	180·00	175·00	188·54	186·25	190·00
Vickers	70·00	60·00	41·25	45·00	60·00	65·00

APPENDIX II.

*Price changes in Individual Securities employed
in the fourth section (Ordinary Shares) of Table II., page 112.*

<i>Investment Trusts.</i>		<i>Electric Light and Power.</i>	
	%		%
Debenture Corporation .	+ 94·90	City of London . .	- 4·23
Foreign, American & General	+ 87·49	County of London .	+ 160·74
Investment Trust Cor- poration	+ 91·32	Metropolitan . .	+ 106·16
Mercantile Investment & General Trust . .	+ 111·80	Midland Counties .	+ 50·15
United States Debenture Corporation . .	+ 125·51	Yorkshire	+ 25·36
<i>Industrial.</i>		<i>Iron, Coal and Steel.</i>	
Associated Portland Cement	+ 38·38	Armstrong, Whitworth .	- 81·54
John Barker & Co. .	+ 38·17	Cammell, Laird & Co. .	- 47·54
Bleachers' Association .	+ 20·08	Dorman, Long & Co. .	- 25·81
British Aluminium .	+ 138·61	Guest, Keen & Nettle- folds	- 5·00
British Oil & Cake Mills	+ 20·99	Vickers	- 7·14
Fine Cotton Spinners .	+ 13·57		
General Electric Co .	+ 122·65	<i>Shipping.</i>	
Gordon Hotels . .	+ 27·70	Cunard Steamship . .	+ 81·51
		P. and O. Steam Navigation	+ 7·14
		Royal Mail Steam Packet	- 18·53

SYNOPSIS

Reference is made in the introductory remarks to the appointment of Investment Research Committees by the Faculty and Institute of Actuaries to undertake, in the first instance, the compilation of a comprehensive service of Index Numbers relating to British Stock Exchange Securities which will be suitable for Insurance Companies, Trust Companies, and so on. The Committees have been at work for some time, and it is hoped that publication of the results will be commenced at an early date.

The Paper discusses the purposes of such indexes and the most suitable form of index numbers for the purposes decided upon. The difficulties arise from two sources; the variety of uses for which the index must cater, and the requirement that the results must be comparable between *any two* dates in the series.

At every point in compilation the objects of the results must be carefully kept in view. These objects are divided under three headings: (a) uses directly connected with current investment activities; (b) investigation of the effect of economic financial and political events on the level of security prices; and (c) assistance in measuring approximately changes in the value of a portfolio of investments. It is suggested that securities should be divided into homogeneous classes, and a separate index calculated for each class. The wide groups "Fixed Interest" and "Variable Dividend" are not suitable for the purposes involved. The weighting system should approximate to the distribution likely to be found in an Insurance or Trust Company's investments.

Within each class companies' holdings will tend to approximate more nearly to equal amounts of money in each security than to any other distribution for which a system of varied weights would cater. Simple weights are accordingly advocated.

Various methods of averaging are discussed and reasons given for preferring the geometric average of price ratios to other methods. The dangers of adopting the arithmetic average—where the resulting figures are always dependent on the base prices—are illustrated in Table I. The geometric results are independent of base. Even using the geometric average great care must be exercised in applying the results to valuation purposes in groups where individual price movements show wide divergencies. The "error" involved by the geometric method, however, is always in the same direction. Capital appreciation is never overstated and depreciation never understated.

The foregoing remarks refer to price indexes. To obtain maximum utility from the results yield statistics also are required. The very different nature of the problem of yields is stated and an arithmetic average of individual yields in each group is advocated. While the price index should be related to 100 at the starting-point it is obviously preferable to publish actual yield averages.

DISCUSSION

Mr. C. M. Douglas.—The Paper submitted to us on this occasion deals with a subject which in more than one direction is leading the Faculty into new territory, that is territory which, although familiar to many of us, has not previously been the subject of our occupation. For that reason it is greatly desirable that we approach this Paper with all our critical faculties on the alert. Unfortunately, the Research Committee have not yet completed their preliminary investigations and we are therefore still without their Report, but in the meantime Mr. Murray has generously prepared for us a detailed account of one of the initial problems which confronted that Committee. While we must wait just a little longer before we can learn the full scope and intentions of the Committee's proposals, this fundamental problem and the Committee's solution of it, as expounded in the present Paper, merits our careful attention.

For my own part I have always been sympathetic to the general objects of investment research. In earlier days of study I was frequently struck by the different treatment given to the two sides of our Balance Sheet. To the one side of Liabilities there was directed the most devoted care and attention; large and skilled staffs were employed in estimating the individual risk as each was placed on the Books, in assessing the premiums requisite for such risks and in valuing from year to year the total liabilities incurred. In fact, many actuaries appeared to have devoted their whole career to the one detailed problem of constructing an efficient mortality table. In contrast to this the Assets side appeared to me, at that time, to receive an almost casual regard. The selection and assessment of price was carried out by only one or two individuals under all the limitations of their individual and personal experiences, strengthened by the recommendations of a stockbroker anxious to do business. When it came to placing a value on these several Assets from year to year the middle market price was chosen in the sort of indifferent air that one price was as good as another. All this was in such vivid contrast to the rigid treatment of Liabilities, the careful selection of rules and formulae which were applied in their valuation, that I used to wonder if the relative importance of the two sides were really quite so disproportionate as such treatment would suggest. I was, of course, answered in some measure by the position which arose after the war.

In Mr. Murray's Paper we have the fundamental problem stated, namely, the compilation of index numbers. In all questions arising under Liabilities we have the statistical background of our mortality tables. In the selection of an individual risk we relate the report of the skilled examiner to that general background, and if the risk is a special one we have the support of our group mortality table for relative comparison. Similarly, it is on the mortality table that we base our prices in accepting the individual risk, and we employ the mortality table again when valuing the liabilities from year to year. In a different but somewhat kindred manner the index numbers will form the statistical background in all questions relating to our Assets.

Mr. Murray, however, is in no way concerned with the uses of such tables, and if he does refer to these uses it is only in so far as they affect his choice of method. To carry the parallel further, Mr. Murray is concerned with the construction and graduation of the mortality table or, in other words, the creation of our tools. This problem he deals with very

thoroughly and most efficiently. He has not been lacking in courage in suggesting to us the difficulties attending such work, and in pioneer study of this kind it is all to his credit that we are shown the disadvantages as well as the advantages of any method which he selects. In dealing with the question of averaging I think every one will agree that, keeping the objects in view, the geometric is the most serviceable. In fact, the geometric is rapidly coming into general use for all statistical work of this nature, the arithmetic having been found in practice to be too particular and restrictive for employment in any continuous time series. In Table I. the vagaries of an arithmetic average are well shown, and as in actual practice we are investing at all times, none of these individual arithmetic figures would have any utility value for general application. The geometric averages are given in Column 7, and these, it will be found, afford a reasonable approximation to the average of all the arithmetic figures. That is exactly what we want.

The problem of a yield index is undoubtedly a different one from an index of price, and I agree with Mr. Murray that for general purposes it is better to give a series of actual yields. If it should be required to produce a curve with a base of 100 at any point it would be a simple operation to run off the requisite figures from the data provided, and the objections to an arithmetic average are not present in the same force since the yield figure itself is a rate per centum.

There is one point on which I would ask the Faculty to record a decision, and that is the use of the word *indexes*. On an earlier occasion, when discussing this subject myself, I used the form *indices* and now Mr. Murray gently rebukes me by employing the English plural throughout his Paper. Since reading the Paper in draft, I have inspected the *New English Dictionary*, which is our latest and most authoritative guide. This work gives ten or more meanings of the word index, and states that in current use the plural is *indices* in senses 8, 9, and usually in other senses except 5, in which *indexes* is usual. The sense which is excepted is that which refers to a table of contents or reference list, while of the two senses particularly requiring *indices*, number 8 is the mathematical sense and number 9 is described as, "in various sciences, a number or formula expressing some "property, form, ratio, etc., of the thing in question." This is the only sense which can properly embrace the present word, so it seems certain that the *N.E.D.* requires the Latin plural, *indices*.

Although the majority of American investigators use the word *indexes*, I have so far been unable to find any of our own authorities, statisticians or mathematicians using other than the form *indices*. There is another and a non-mathematical science in which the similar word *codex* recurs. It is interesting to note that the *N.E.D.* gives no alternative to the usual *codices*.

It might be argued that the volume of the *N.E.D.* in question, having been published in 1901, does not allow for the changes of the past thirty years and the malevolent effect of American usage on our own language during recent years. The only more recent publication of any authority is that charming volume of Fowler, called *Modern English Usage*, published in 1926. On the subject of Latin plurals generally he states that *dogmas*, *formulas*, *indexes*, etc., are fitter for popular writing, while scientific treatises tend to *dogmata*, *formulae*, *indices*, etc., and continues by saying, "All that "can safely be said is that there is a tendency to abandon the Latin plurals "and that when one is really in doubt which to use, the English form should "be given preference." Nobody of course attempts to stutter out *specieses*, *theses* or *baseses*, but I submit that the point is of importance in this early stage of the work, and I hope that Mr. Murray will see his way to instruct the Editor to effect the change before final publication. In the

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meantime it would be of value to have the opinions of our more senior members.

Mr. J. Dunlop.—I am rather at a disadvantage in that I am a member of the Committee and therefore cannot say very much in criticism of Mr. Murray's Paper, as it is really in anticipation of the Committee's Report. There are, however, two points I might refer to. The first is in regard to the selection of securities to be included in the *indices* or *indexes*. Mr. Murray rather infers that the selection must always be in the form of sampling, that all the securities will never be taken into an index. I think the only real objection against taking all the securities in is a mechanical one. I am not suggesting they should all come in with equal weight—it is quite obvious that they should not—and I am not going to suggest how they should be weighted, but I do feel where you are finding an index for such a group as oil, you should take every company that operates with oil in the district or country you are concerned with. The most frequently quoted securities index number in the world—the Dow-Jones average—is calculated by taking every equity stock quoted on the New York Stock Exchange, and it is produced every morning from the previous day's prices. I am not defending that index. I merely mention it to demonstrate the mechanical possibility.

The other point that I would like to refer to is the question of yield statistics. Mr. Murray has confined his remarks entirely to yields on dividends. Now, in regard to equity stocks, I am sure he will agree with me that yield on earnings is probably equally important, and I think that statistics properly made up on earnings would be invaluable to many people. If one attempted to make them up from the reports of Continental Companies it would be quite hopeless—their Inland Revenues are more credulous than ours—but if one referred to American statistics some extraordinarily interesting results would be obtained from the various industrial groups. In this country I am not quite so sure it is an easy task to undertake, because, as every one knows, our reports are not uniform in style, and there would be difficulties as to exactly how you were to determine what the company has earned. But, after all, we are dealing with averages, and I think that these differences by the time they are averaged would be comparatively small, and that they are not sufficient to nullify the value of yield statistics on earnings.

Mr. B. Murrie.—I should like to congratulate Mr. Murray, and the Faculty, upon his intensely interesting Paper. I think we are all pretty well agreed that the service, when once established, will be very useful, especially, perhaps, in connection with sound, fixed-dividend securities. I suppose we may take it as certain that securities of that class will never go entirely out of fashion. But, at the present time, it seems that the class in regard to which we particularly want guidance is the class of industrials and other securities with fluctuating yields and fluctuating prices, and it is just here that, as I think we may gather from the Paper, the service will not perhaps be of quite so much use as in the case of the other class. There are two points that I should like to say a few words upon, and as I may seem rather critical on these, I ought to apologise for following the usual custom of being silent upon the many excellent points.

On page 100 Mr. Murray states the purposes to be served, and the third purpose is, "Assistance in measuring approximately changes in the value of a portfolio of securities." Now, Mr. Murray has told us that he would not weight the prices in any way, and we are probably all agreed that no system of weighting would be likely to bring results any nearer to actual facts. But, when we look at any heavy list of investments, we find

weighting very much in evidence. For example, we may find in one group £10,000 invested in one security and perhaps £100,000 in another. In the case of fixed interest investments this perhaps does not matter much, but in the other class it will be apt to affect the results very considerably. Assuming that we have the net prices available, it seems to me that an exact valuation might very expeditiously be made by the help of an adding machine. You would have a column headed £10,000, and in that you would put the prices for your lots of that amount, and by changing the decimal place, you would include the £1,000 and the £100,000 lots; also, by a simple multiplication, you could easily include the £2000 and the £5000 lots. Other columns would be added with suitable headings. Summing the columns and multiplying the totals by the respective headings, you would obtain an exact result with very little labour. The service figure might come in handy as a rough check.

Mr. Murray mentions as one of the uses of the service that it enables one to compare the current yield on any investment with the average yield of any appropriate group. This is quite a useful test, but it cannot be taken as at all conclusive as to the advisability of taking, or continuing to hold, the investment in question. In a broad free market, occasional differences are often the result of underlying causes which it would be useful to know. This leads me on to say that, however useful the service may be, we must still attach great importance to getting it supplemented by the advice of people of experience both in industrial affairs and in the stock markets generally.

Mr. A. E. King.—Being one of those serving on the Investment Research Committee, I do not propose to touch upon the methods which they have suggested should be adopted. I would like to say, however, that, like the last speaker, I should not myself care to employ the index to give me an idea of how the value of a portfolio of securities is moving. I would be as diffident in doing this as in employing a Model Office in connection with the valuation of the liabilities of a Life Office. I am aware that the author uses the word "approximate" in dealing with this matter on page 100 of his Paper, but if an approximate method were wanted, I would rather have a quick valuation made of the principal securities in the portfolio or of the principal groups of securities. If, however, one is prepared to use the index for the purpose mentioned, I think it should be used with considerable caution.

I had rather hoped that some of the members would have given attention in the discussion to the section of the Paper headed "Purposes to be served" dealt with on page 100, as this is one of the most vital sections. Although most of us present may agree that there is good purpose to be served by the index being set up, there are, I think, a good many people who are not so sure. Any one, however, who attempts research work on investment will agree that the need of a good investment index is constantly arising. I remember that when Mr. Douglas gave us his valuable paper about twelve months ago, one subject discussed was how fixed interest security prices varied with the prices of ordinary stocks in various circumstances, e.g. when general money rates were moving upwards or downwards, and it was pointed out that quite different conclusions on this subject had been arrived at by various people. Some held that prices for the two groups moved up and down together; others that there was a definite "lag." I think myself that this matter could have been dealt with much more easily had a proper investment index been available in respect of various groups of securities, commercial and otherwise. Then, again, I had an opportunity recently of seeing a series of French graphs which showed movements of indexes of groups of ordinary stocks, in

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various countries, and how they varied with one another and with commodity indexes, etc. I had no information of how these group of stocks were composed—which took away some value from the graphs—but it was interesting to note that there were two British curves plotted down for ordinary stocks differing widely from each other; this was very unsatisfactory.

While, therefore, we may all be willing to admit that the question of a Security Price Index is perhaps a small corner of the large subject of Investment Research, it was because the need of a thoroughly reliable index was so often felt that the Investment Research Committee decided to take up this subject first of all.

Mr. E. M. Campbell, W.S.—I should like to say what a privilege it is to be allowed to be present at this meeting of the Faculty of Actuaries, and to have had the opportunity of reading Mr. Murray's Paper before coming here, as well as hearing Mr. Murray himself expound it to us. As a member of the very much less learned Society of Writers to the Signet, I do not know that I have anything of very special value to contribute to the discussion—or, indeed, that I am competent to do so. I would only like to say from what Mr. Murray says, and from what other speakers have said in the course of the discussion, it is evident that index numbers must be used with the very greatest reserve and caution. No automatic results can follow, and we should beware of any sweeping conclusions. They must be interpreted with a detailed knowledge of how the index is composed, and with its various limitations. But, subject to these limitations, I would say they are of very great value to people in charge of large funds for investment, and controlling the policy of corporations. They are particularly valuable, I think, because they may give us some clue in predicting the future. As regards the question raised by Mr. Douglas as to whether we are to use the word *indexes* or *indices*, I possibly in a moment of rashness would like to put in a plea for our using popular expressions. The man in the street is very easily frightened of a subject of this kind, but I venture to think it is important for the man in the street, so therefore do not let us bamboozle him by any more technical expressions than are absolutely necessary.

Mr. G. J. Lidstone, LL.D.—I should like, as one of Mr. Murray's former colleagues as well as one very much interested in the subject, to congratulate him on the very admirable, clear and lucid way in which his Paper has been prepared. The subject is a very intricate and difficult one, but he has robbed it of a great deal of its terrors by the clear way he has dealt with it, and the atmosphere of common sense with which he has inspired it. I am quite sure it will be found of the greatest value to all of us, not only in studying the subject generally but also in appreciating the scope and uses of the Indexes, for which we are all anxiously waiting. I am not going to plunge into the orthographical discussion which was rather boldly started by Mr. Douglas. I merely remark that I think if Mr. Murray decides to follow the advice given by Mr. H. W. Fowler (and impartially quoted by Mr. Douglas) he will be in very good company.

Mr. C. M. Gulland.—While I think most of us are agreed that the Indexes to be published must prove of great and increasing value, the yield averages or "indicators," as they may be called, will be of immediate service, as no complete statistics of this nature for this country are in existence. This is particularly the case as regards the Dominion and Foreign Government yield indicators, and these should prove invaluable for purposes of comparison with new investments, etc., as although lists of the yields from such securities are, of course, issued periodically by various brokers and finance houses, we have generally no knowledge of

the methods of calculation adopted for the redemption yields or of the assumptions involved—in fact, the yields frequently differ considerably from those produced by what we are accustomed to regard as the correct methods of calculation.

Mr. Murray in the course of his Paper states that the necessary calculation by the geometric method involves less work than by the arithmetic method, but I notice that Professor Irving Fisher in his book, *The Making of Index Numbers*, to which reference has already been made by Mr. Murray, puts the time occupied by the latter at about four-fifths of the former. Professor Fisher does not, I think, state the actual numerical processes used, but in order to elucidate this question I carried out a short investigation, calculating a series of typical index numbers by both methods: (1) geometric, using the rules adopted by the Committee, and (2) arithmetic: (a) using logarithms; (b) using a calculating machine; the reciprocals of the base prices being tabulated and thus multiplication being more rapid, substituted for division, the products being set down separately and added afterwards; and (c) using a machine as in (b), but omitting to clear the slide between each calculation and thus making the additions automatically.

The average time taken by (2) (a)—arithmetic with logarithms—was about twice that by the method adopted by the Committee, while (2) (b) required about 15 per cent. longer than the geometric. The arithmetic, using the last method mentioned, does actually require, I found, only about three-quarters of the time taken by the geometric, but, of course, no record is made of the individual price ratios which may be required if the arithmetic or "Investment Result" method is to be put to the fullest use. If the calculations are to be referred to at any future date a record of these intermediate steps would appear to be almost essential.

This question of the time required may seem a minor point, but where the indexes involved are very numerous, as in this case, it becomes of considerable importance.

Mr. G. H. Recknell wrote as follows:—The subject of Mr. Murray's Paper is of particular interest to me, because the problems and difficulties which it expounds have been so much discussed recently in connection with the investment index, which the Faculty and the Institute have so happily combined to produce.

Assuming that the objects to be achieved by the proposed index are clearly thought out, the most important questions to be determined are the method of (a) weighting; (b) selection; and (c) averaging. Taking "weighting" first, my personal predilection is in favour of a system of weights in order to give due effect to movements in prices of the leaders of each group, but I confess to having weakened considerably on this issue. The starting-point in the "selection" of securities is obviously the elimination of all the smaller and unimportant companies, and, of those remaining, it is within my own experience that actual investment in each does tend to be for approximately equal amounts. Further, the particular method of selection decided upon, namely the shares of all companies with an ordinary share market capitalisation in excess of £2,000,000, in itself introduces a measure of weighting.

Having once eliminated the companies with a small capitalisation, I take the view that the ultimate list should depend not on individual choice, but on a mechanical formula. A rigid formula eliminates personal discretion, free play to which would inevitably produce as many different lists of securities as there were "selectors." A formula logically and intelligently constructed inspires greater confidence in the mind of the user of the index, and indicates, not in terms of general principles but in precise language, the exact ideas of the compiler.

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With regard to the method of averaging, Mr. Murray's investigations are extremely valuable and their results are striking. The choice, as he shows, rests between an arithmetic and a geometric method, but the results of the tests are convincingly in favour of the geometric mean; and a significant confirmation of this is in the tendency for official commodity price indexes to break away from the at first sight more facile arithmetic mean. Incidentally, the eccentric behaviour of the Iron, Coal and Steel group in Table I. throws a great deal of doubt on the value of a conglomerate index based on a large number of dissimilar securities—the kind of thing which is so easy to quote and is so freely made use of by the financial press.

It might be inferred that once these prime questions of selection, of weighting and of averaging are settled, the rest is easy. But in truth there remain a great variety of minor details to be thought out, other than those alluded to in the Paper, before compilation can begin. Since the beginning of our joint discussions a new index has appeared—the daily index of ordinary shares published by the *Financial News*. But it makes no attempt at scientific refinements, being based on the arithmetical average of thirty dissimilar ordinary shares, and its main virtue is in the frequency of its publication. It is hoped to commence publication of the Actuaries' Index in May, and the event may well be epochal for it will forge the first permanent and continuous link between the Faculty and the Institute. Moreover, other developments in the field of investment research are already in mind, such, for instance, as the construction of average yields on ordinary shares based on earnings, an addition to the index which would clearly greatly enhance its practical value.

The President (Mr. Charles Guthrie).—We have had a very interesting and valuable discussion, and I think the Faculty has reason to congratulate itself upon having received, in each of the past two sessions, papers bearing on the subject—the very important subject to our profession—of investment. We are indebted to Mr. Douglas for having, in the first instance, brought into prominence the necessity of having a satisfactory statistical basis in connection with investment work; and we have this evening listened with interest to Mr. Murray's carefully thought out statement of the manner in which such statistical basis should be compiled. In the short intervening space between these two papers much practical work has been done, and I think we can say that seldom has any author of a paper seen so large a part of his suggestions carried into practice in so short a time as has been the good fortune of Mr. Douglas. The Investment Research Committee, which your Council appointed rather less than a year ago, has worked indefatigably from that day to the present date, and in the latter period of their labours I am glad to say they have had the hearty co-operation and valued assistance of the corresponding Committee of the Institute. The work of this joint body, and the well thought out scheme which they are now in a position to put into practical working, seem to me to be one of the best object lessons of the great value of thorough co-operation and genuine team work. The work in connection with the preparation of an index on the lines set out in Mr. Murray's Paper has so far advanced that the respective Councils of the Institute and Faculty have authorised the Presidents to issue a letter to all the Life Offices and also to Banks, Investment Trusts, Stockbrokers and others to whom such a compilation must be most useful, setting forth the lines upon which it is proposed to issue the index under the auspices of the two bodies, and asking them to subscribe to the same. This letter is already issued or should be in the course of a few days, and I think it will interest you to have some particulars of this communication which has

been authorised and signed by the President of the Institute and myself.*

We look with confidence to the results of this invitation, and the Committees are equally confident that in course of time those subscribers who use the material with intelligence and care will find it of ever-increasing value.

I should like to take this opportunity of saying that I consider that the value of this work is not only in the provision of material which will become invaluable in the ever-increasingly important province of our work relating to investments, although that in itself would be something to congratulate ourselves upon. It has had other results to which, as a member of our profession, I attach great importance. The opportunity has been taken of a most useful and cordial co-operation between the two actuarial bodies, which I think I am justified in saying has been greatly appreciated by all those who have had the privilege of taking part in it. Further, one can feel justified in hoping that another result in the future should be the practical proof to the community of investors of the great value to them of the work of our actuarial bodies. Still further I look forward to the work of these Investment Research Committees continuing and extending in the future along other lines of an equally valuable nature, both theoretical and practical. I think at this stage, although Mr. Murray has not yet replied, it is usual for the Chairman to express what I am sure is in the minds of you all—our appreciation of the value of this Paper which he has put before us, not only as explaining a new movement in this particular domain of our profession, but also as being expressed, as Dr. Lidstone has said, in a very clear and understandable manner, which I am sure will be valuable, not only to those about to use the Index, but also very valuable as statistics to the Faculty. I would like to express on your behalf, therefore, to Mr. Murray our very great appreciation of his Paper.

Mr. Murray replied as follows:—I am very grateful indeed for the many kind things that have been said about the Paper, and I am glad to find some enthusiasm for the research which is taking place. With regard to the discussion generally, I agree with most of what has been said, although I prefer the word *indexes* to *indices*. There is authority for the use of *indexes*, and as Mr. Campbell has said, it is a more suitable word to employ in a case like this where the results are intended for general use. It is interesting to notice that in spite of the general move over to the geometric method referred to by Mr. Douglas, the new *Financial News* Index is worked out on an arithmetic basis. I agree with what Mr. Dunlop and Mr. Recknell have said about the desirability of calculating yield statistics based on earnings in the case of Ordinary Shares. The *dividend* yield, however, should never be ignored. I think there is no doubt that the American slump of last year was in part due to the fact that Common Stocks had risen to inflated prices owing to purchases made regardless of the actual dividend return to be received. It would be most satisfactory to calculate both indexes, and I hope that it may be found possible to do so at a later date. I understood Mr. Dunlop to say, with regard to selection, that he thought the best course would be to take every company in the group and use weights. For the reasons stated in the Paper, I feel sure that for our purposes this would be inferior to what is being done, and in any case there would be great difficulty in deciding on the weights to be used. In papers dealing with investment work mention is invariably made in the discussion of the importance of the selection of individual securities, and it is very desirable that this

* A copy of the Letter will be found on page 129.

should be emphasised. I entirely agree with what Mr. Robert Murrie says about selecting individual investments, and, of course, I do not for a moment suggest that one should simply look at the yield of a security, compare it with the yield of its group and come to conclusions on this comparison alone. A comparison such as this is merely one of the very many factors involved in selecting investments. I have tried to make it clear that I do not anticipate too much from index numbers as an aid to the approximate valuation of securities. At the same time, whatever the distribution of the investments held in a Fixed Interest group where the prices move with reasonable uniformity, useful approximations will undoubtedly be obtained if the index is applied with care and allowance made for profit or loss on sales, and so on during the period under review. When we come to deal with Ordinary Shares I agree that even greater care will be necessary. It is probable, however, that in insurance companies the distribution will often approximate to equal amounts of cash in each security within certain subdivisions. Where the distribution is on these lines, and where prices tend to move together, the index will give an indication of the position which should be of value, even if only to point to the necessity for a complete valuation.

Mr. Campbell said we must beware of sweeping conclusions. I agree. We must not look on the index number as a tool which operates mechanically. It does not. Used intelligently it *assists* in a difficult task and avoids much individual investigation. It tells what is happening at the moment and what has happened in the past, and these are factors to be considered along with many others in any view of the future. It is interesting that Mr. Gulland's investigation confirms what I have said about speed of calculation. The one method of operation to which he refers, where the arithmetic method would be quicker than the geometric, is not satisfactory, because it loses track of individual prices. It is, of course, very desirable for various reasons to retain particulars of the periodical prices for each security. It is very pleasing to know that Dr. Lidstone and others who have spoken see some real usefulness in the scheme, because one feels, as Mr. King says, that there may be people who do not agree right away that it is going to be of great value. Personally I have no doubts whatever. In conclusion, I wish to thank you again very much for your appreciation of the Paper.

Subsequently **Mr. Murray** wrote:—In the discussion on my Paper on Index Numbers of Stock Exchange Securities, in taking exception to the suggestion to employ the index to obtain an idea of the change in value of securities held, Mr. A. E. King emphasised that I had used the word "approximate," but he did not refer to the word "assistance." The third use referred to in the Paper (page 100) is "Assistance in measuring approximately changes in the value of a portfolio of securities."

I introduced the word "assistance" partly because I do not believe the index could be used for this purpose in all groups. In certain groups, however, as a rough measure, I think we will find when we can make the test, that the index used with caution will be of some real value. For anything like an accurate valuation of a portfolio, I would not, of course, recommend the use of index numbers.