ECONOMIC APPLICATIONS TO ACTUARIAL WORK: PERSONAL PENSIONS AND FUTURE RATES OF RETURN

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ABSTRACT

This paper considers certain applications of economics to actuarial work, concentrating on three issues of public interest. The background to the first two issues is the Government's encouragement of personal pensions particularly through stakeholder pensions. The paper shows how the use of an economics model enables investigations to be made, firstly on the demand for personal pensions by different income groups and secondly on the likely inequality of benefits between generations due to variations in the rates of return. The conclusion on the first issue is that personal pensions are of little benefit to low-earners, i.e. those the Government most wants to benefit, and recommends a way of rectifying this; the main conclusion on the second issue is that households will take action to mitigate much of the inequality. The third issue, that of estimating future rates of return, is one of key financial importance. The paper reviews relevant economic papers in this area and considers how the actuarial profession may improve communication with the economics discipline and with its members.

KEYWORDS

Equity Risk Premium; Financial Risk; Intergenerational Risk Sharing; Liquidity Constraints; Personal Pensions; Future Rate of Return; Real Interest Rate

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Nothing questioneth nothing learneth. — Thomas Fuller, *Gonomologia* no. 2241

1. Introduction

1.1 This paper considers three specific issues of concern to the actuarial profession and of public interest and, more generally, the application of economics to actuarial matters.

1.2 The first two issues concern key financial risks to a large section of the public, and are therefore of important public interest. They have grown in importance from the introduction by the Government of stakeholder pensions and the encouragement, generally, of personal pension schemes. The Government's main aim in this area has been to mitigate the cost to the Exchequer of the support to the retired by an ageing population. However, this pension policy has been put in place without any detailed analysis of the public's willingness to invest in such pensions, particularly the segment of the public that the Government most wants to influence, that is lower earners.

1.3 In investigating the first pension fund issue, we consider the effect on the demand for these funds of uncertainty of earnings, the main income risk for low earners. One method of 'insuring' against this risk is to save, in good times, for possible bad times, but such savers face a dilemma between maximising tax advantages by contributing to pension funds and the need to use the savings at short notice, that is the need for liquidity. Using a partial equilibrium model, we show, because of pension funds' illiquidity and the yearly maximum contribution limits as a percentage of earnings, that low-earners are least likely to want to, and to be able to, contribute to pension funds. We go on to recommend a way that could increase low-earners' pension fund demands to 5% or more of earnings.

1.4 The second pension fund issue, that of uncertainty of the eventual pension due to uncertainty of the rate of return on defined contribution and personal pension plans, is accentuated by the move towards such schemes away from defined benefit ones. These latter, implicitly, have intergenerational risk sharing built into their design. The trend just described will increase the degree of intergenerational inequality, since a long period of below average stock market returns will lead to a generation not having saved adequately for its retirement, despite having invested in pension schemes at what would normally have been a sufficient level.

1.5 Using a fully articulated partial equilibrium model of the United Kingdom tax and pension system, we estimate the possible degree of intergenerational inequality. We show that, when household behaviour is taken into account, the level of intergenerational pensioner inequality is significantly reduced. We also consider whether any inequality can be economically mitigated by the purchase of performance guarantees for personal pension funds, and conclude, from our model, that their cost is so prohibitive that even the most risk-averse household is likely to have little interest in them.

1.6 The third specific issue is that of estimating future rates of return. In the past few years, this has become a matter of wide public interest, as a result of millions of individuals having to make decisions, in respect of which a key factor is the likely future rate of return on their investment. We are thinking here, especially, of those who are considering taking out, or have already effected, with-profits endowment assurances for the repayment of mortgage loans, where the likely future rate of return is critical to the decision as to what type of repayment vehicle to use or to judging whether the rate of funding into an existing endowment assurance is likely to be adequate. Historically, estimates of future rates of return have been required principally by Governments, companies and other organisations. Today, the area has a much higher profile than it has had in the past, because of the direct effect on the very large number of individuals affected, especially if the estimates that they are advised to use, implicitly or explicitly, prove significantly wrong in practice.

1.7 A range of work in the area of future rates of return has been developed and written by the actuarial profession, both in techniques and ideas. There is also a large body of economic research, which appears not to be known to the majority of actuaries, and which we consider that actuaries can use to improve methods of estimation. This paper includes a summary of some of the key relevant research. It also puts forward recommendations in this area on how the profession could improve communication between it and the economics discipline, and how it can best keep its members abreast of developments, within and outside the profession, and be proactive with regard to future developments.

1.8 This paper has a main section on each of the specific issues: Section 2 on the demand for personal pension funds; Section 3 on return guarantees on personal pension funds and their ability to reduce intergenerational risk; and Section 4 on estimating future rates of return. Section 5 gives more general consideration to the application of economics to actuarial matters, and highlights areas for debate; while Section 6 summarises conclusions.

2. The Demand for Personal Pensions

2.1 *Introduction*

We turn now to the two personal pension issues that we have investigated, starting with the demand for personal pension funds by contributors with different levels of income.

2.2 *Model Households*

2.2.1 To investigate this issue, we have used the partial equilibrium model described in Appendices 1 and 2. In our model of household savings' behaviour, households have two motives to save, namely: to insure themselves against future unpredictable falls in their earnings, 'the precautionary motive'; and to save for their retirement, 'the retirement motive'. They can save in either of two assets, a liquid asset and a non-liquid tax-advantageous asset, in practice a pension fund. We shall refer to the liquid asset as liquid wealth holdings, and the non-liquid asset as the personal pension plan. (The stochastic process describing the time evolution of a household earnings, and the probability distribution describing asset returns are covered in the appendices).

2.2.2 Households with a strong precautionary motive will not want to save in a personal pension plan. Any savings in this plan are tied up until retirement, and so cannot be used to increase household consumption in the event of an unpredicted fall in earnings. These households will, therefore, predominantly save in the liquid wealth asset.

2.2.3 Households with a strong retirement motive will want to save in the personal pension plan, as this has tax advantages. Under current tax law, there are tax rebates available on contributions to these plans, but there are maximum contribution rates, defined as percentages of pre-tax earnings, which limit the amount that can be saved in this asset at any time. Further, it is obligatory that, on retirement, the fund, apart from a lump sum payment of 25% of the fund, be annuitised. Our model does this using a current life table (the values are given in Table A2.1) and a real risk-free rate of return of 3% p.a.

2.2.4 The two principal tax advantages of the personal pension plan are, firstly, the 25% tax free lump sum payment on retirement and, secondly, the tax rebate on contributions. The latter one is particularly advantageous for higher rate tax payers (in the model, those earning above 1.3 times average household income), as the rebate is available at the higher rate, but it is likely that the tax payers will be subject only to the basic rate on their pension.

2.2.5 There is also one further tax advantage, which is made more important, because there is no motive in the model to leave bequests. If there is no bequest motive, households will annuitise all their assets, liquid and non-liquid, at retirement, so as to reduce the risk of running out of resources should they live too long. However, under present tax law, the annuity received is taxed according to the source of the annuity funds, and this gives rise to some double taxation of liquid assets. The original earnings that gave rise to these liquid savings were taxed at the applicable rate, and then, when annuitised, the interest component is taxed again. In contrast, all savings in pension funds are taxed only once, and that is when the pension is received.

2.2.6 We have assumed that both types of assets are invested principally in the same equities; thus both investments have the same uncertain rate of return. This rate of return is assumed to be independently log-normally distributed; it was calibrated on the basis of the average return and volatility of the FTSE All-Share Index over the last 30 years. We also assumed that the returns on both assets are not taxed. This is obviously a simplification, but is a reasonable assumption for all but the relatively wealthy.

2.2.7 Finally, with regard to taxation, while the majority of individuals save less than the present ISA annual contribution limit, an ISA or equivalent being the model's assumed liquid asset, we recognise that there may be additional tax benefits to personal pension plans for the higher rate taxpayers that are not captured by this model.

2.2.8 To run the model, it is necessary to start with an initial income wealth and population distribution of our 30,000 households. Given this initial starting point, the model can then be run for any length of period, with new generations being born and the older ones dying in line with the model assumptions. As, to some extent, the initial population and wealth distribution is arbitrary, it is desirable to analyse the model only once the effects of this initial distribution have decayed away, or, equivalently, that the model has reached a long-run equilibrium. We chose, therefore, to run the model from our initial distributions for 300 years, and analyse the behaviour of the generation born in this 300th year. As the observed behaviour of this generation depends, to a large extent, on the realised returns over its life, we repeated this experiment 4,500 times using a different set of random innovations each time, and recorded from each simulation only the behaviour of the generation born in the 300th period. The results in this section, where we are analysing the behaviour of an average cohort, are derived by averaging the behaviour over the ensemble of these 4,500 cohorts. In the next section, where we examine the distribution of annuities across generations, we treat each of the 4,500 cohorts separately, and look at the distribution of a particular statistic derived from each experiment.

2.3 *Data Analysis*

2.3.1 In Figure 1 we have plotted the average household level of earnings, consumption and annuity payments by age, as well as the average

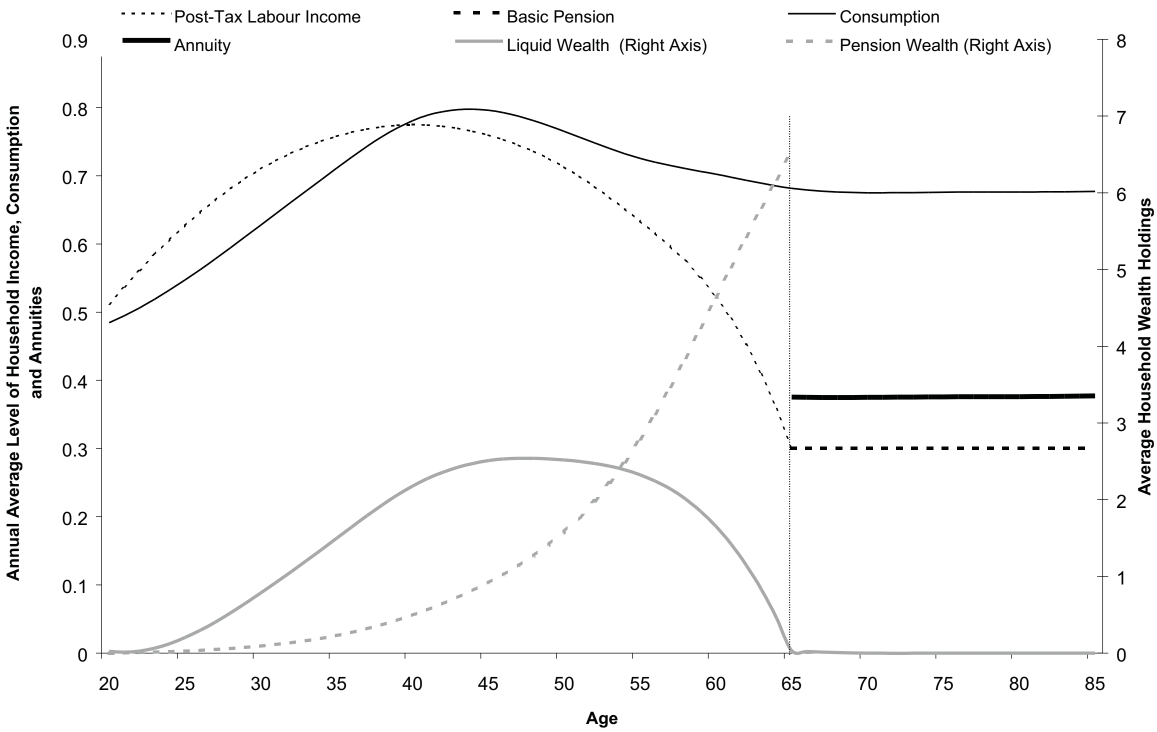

Figure 1. The average household profile of income, consumption and wealth over its lifetime

level of holdings in the two assets. The graph, therefore, plots the average behaviour of a particular cohort in the model. The time path of average post-tax earnings is exogenous to the model, and has been calibrated from data from the Family Resource Survey (1997). It has the characteristic hump-shape, rising at first as earnings rise, and falling after the mid forties, principally because the number of hours worked fall. The level is normalised so that the average level of pre-tax earnings of all working households is 1; for comparison purposes only, the average household level of pre-tax earnings was around £20,000 in 1998. Total taxation on earnings, national insurance plus income taxation, is fixed at 33% for all income levels. After retirement, earnings include allowance for the basic state pension, fixed at 25% of average pre-tax household earnings.

2.3.2 The consumption profile is also hump-shaped, but for different reasons. Consumption rises at first due to three causes: firstly, preferences; secondly, the combined effect of rising earnings and the no-borrowing constraint; and thirdly, the increase in household size. The trend reverses as households start to become smaller again from the mid-forties. However, consumption declines much less, as households smooth their consumption by using some of their savings. During retirement, households are able to sustain a level of consumption due to their combined income from the basic state pension and their personal pensions. The contribution of these two sources is approximately equal for the average household.

2.3.3 The consumption profile is close to the observed age profile found in the survey data. The only discrepancy is that observed household consumption tends to fall more dramatically on retirement. However, it is possible that this could change in the future, as more and more household members retire having had access to the private pension market over the whole or most of their working life.

2.4 *The Savings Dilemma*

2.4.1 Of particular interest is how households save so as to achieve their optimal consumption profile. When adult members of households are young, they save predominantly in the liquid wealth asset. The savings can, therefore, be used either to smooth household consumption levels during working life, should household earnings fall, or, if earnings do not fall, to fund their consumption levels during their late working life, in which case they are able to contribute more to their pension fund during their later working years. The personal pension fund does not offer a higher rate of return than the liquid asset, so contributing earlier to the pension fund rather than the liquid asset does not offer any investment return advantages, although it does reduce liquidity.

2.4.2 As household members approach retirement, they become less concerned about a fall in earnings, as there is less working life remaining. They wish, however, to take advantage of the tax advantages of personal pensions.

Table 1. Breakdown of assets by post-tax earnings quintiles

Age		Quintile				
		Lowest	2nd	3rd	4th	Top
45-50	Average post-tax earnings	0.26	0.43	0.61	0.86	1.45
	Average total assets	1.30	2.36	3.52	5.11	8.85
	Percent held in personal pensions	0.8%	4.6%	13.8%	26.9%	45.1%
50-55	Average post-tax earnings	0.23	0.40	0.56	0.80	1.41
	Average total assets	1.30	2.46	3.63	5.37	9.55
	Percent held in personal pensions	3.8%	14.1%	26.7%	40.9%	57.2%
55-60	Average post-tax earnings	0.20	0.34	0.49	0.71	1.30
	Average total assets	1.15	2.45	3.80	5.63	10.19
	Percent held in personal pensions	8.6%	28.6%	42.9%	55.5%	70.5%
60-65	Average post-tax earnings	0.15	0.27	0.40	0.58	1.10
	Average total assets	1.34	3.35	5.29	8.11	15.48
	Percent held in personal pensions	21.7%	49.7%	62.5%	72.9%	84.2%
At	Average post-tax earnings	0.28	0.40	0.54	0.75	1.33
retirement	Average total assets	0.56	2.46	4.89	8.61	18.71
	Percent held in personal pensions	67.7%	85.2%	96.5%	99.3%	99.3%

Nevertheless, they cannot leave all their contributions to the personal plan until just before retirement, because of the maximum contribution limits. Households do wish, though, to leave their contributions to their personal plans as late as possible; for most households, contributions start about the age of 50.

2.4.3 However, the more a household is concerned about its liquidity level, the more it will delay contributions to its personal pension plan. Such households will be predominantly those with a low current level of wealth, which will be those with a history of low levels of earnings. Table 1 gives a breakdown of household asset holdings by quintile groups of income. Clearly, low-income households hold a far lower percentage of their assets in personal pension plans. Part of the reason for this is that these households can maintain their living standard after retirement with just the basic state pension, and so they will wish to hold liquid assets in order to raise their consumption levels before retirement. However, these households also do not want to reduce their liquidity further by tying up too large a percentage of their already relatively low level of assets holdings.

2.5 *Recommendations*

2.5.1 Households save in personal pension plans because of the tax benefits. However, because of the maximum contribution limits, to take full advantage of the tax benefits, households are forced to start saving in their pension plans, typically around the age of 45-50. If the maximum contribution limits were raised, then households would delay their contributions till later, in order to sustain their liquidity for longer.

2.5.2 It is the low-income households that are the most affected by the contribution constraints. If the households have no liquidity, then they will consume all their current earnings and rely on the basic state pension after retirement. If they have any liquidity, they will want to use it to sustain their consumption and rely on the basic state pension after retirement. Either way, they will not save in personal pension plans. However, if their earnings do rise unexpectedly later, they will not be able to take full advantage of the tax benefits, because of the maximum contribution limits. Conversely, high-income families are not so concerned with their liquidity, and, therefore, can take full advantage of the tax benefits by starting to contribute earlier to their personal pension plans.

2.5.3 It may be noted that, until recently, there had been a pension contribution allowance 'carry-forward' period for a maximum of six years, but in April 2001 all carry-forward was rescinded. This carry-forward enabled individuals who contributed to a personal pension plan below their full entitlement during the carry-forward period to add any unused entitlement to their current limit, although no individual was allowed to contribute more than his or her earnings.

2.5.4 To assist low-income households without significantly benefiting better-off ones, we recommend reinstating the carry-forward facility, and extending it to, say, 25 years. Under our proposal, and taking into account the ages of contribution already obtained from our model, households could claim a tax rebate at the basic rate on any wealth or earnings that, on retirement, they took as an annuity, thus allowing households to keep assets liquid for longer, without their losing pension contribution tax rebates. Though this is how we have implemented the proposal in our model, it must be emphasised that this is not how such a proposal would necessarily be implemented in practice, as it is not realistic or desirable to expect people to keep records of their earnings over 25 years.

2.5.5 There are, however, a number of ways that this proposal could be implemented in practice. The amount on which individuals would be eligible to receive a tax rebate, conditional on it being annuitised, could be assessed on the basis of their National Insurance Record; these records are already used to assess individuals' eligibility to state pensions based on their contributions over 40 years. It would also, probably, be necessary to impose some limit to this amount; one based on the upper earnings limit would be the easiest, so as to reduce the cost of the programme and to target it, particularly at the low income earners, who are the most liquidity constrained. Another, even easier, approach to implement is simply to allow everyone a one-off opportunity to receive a tax rebate on an amount of wealth, conditional on it being annuitised, up to a pre-defined maximum amount. Given these implementation proposals, the term 'carry-forward' might be considered a misnomer, but, in terms of how this proposal is implemented in our economic model, the phrase 'carry-forward' is accurate.

Table 2. Change in asset holding by income quintile group under a 25-year carry-forward rule

Quintile	Lower	2nd	3rd	4th	5th
Change in asset holdings	19.73%	4.72%	3.52%	0.97%	0.00%

2.5.6 In Table 2 we show the percentage change in asset holdings at retirement after tax refunds by income quintile groups, when there is this 25-year carry forward rule. As before, we assume everyone annuitises all their wealth at retirement, and takes full advantage of the tax relief. In Table 2 there is only a significant change in the wealth holdings of the lower income quintile group, who no longer have a dilemma of whether to save for retirement or for earnings insurance.

2.5.7 An alternative way of reporting the difference under the two carry-forward rules is to draw a graph of the distribution of annuities amongst households at retirement, see Figure 2. Again, it is very clear that it is those with a low lifetime income who are encouraged to save more by this change in rule. At the poorer end of the distribution, a significant proportion of population have increased the size of their annuity. At the richer end, there is very little difference. The distributions are not smooth, as they were generated from a sample of 30,000 households only.

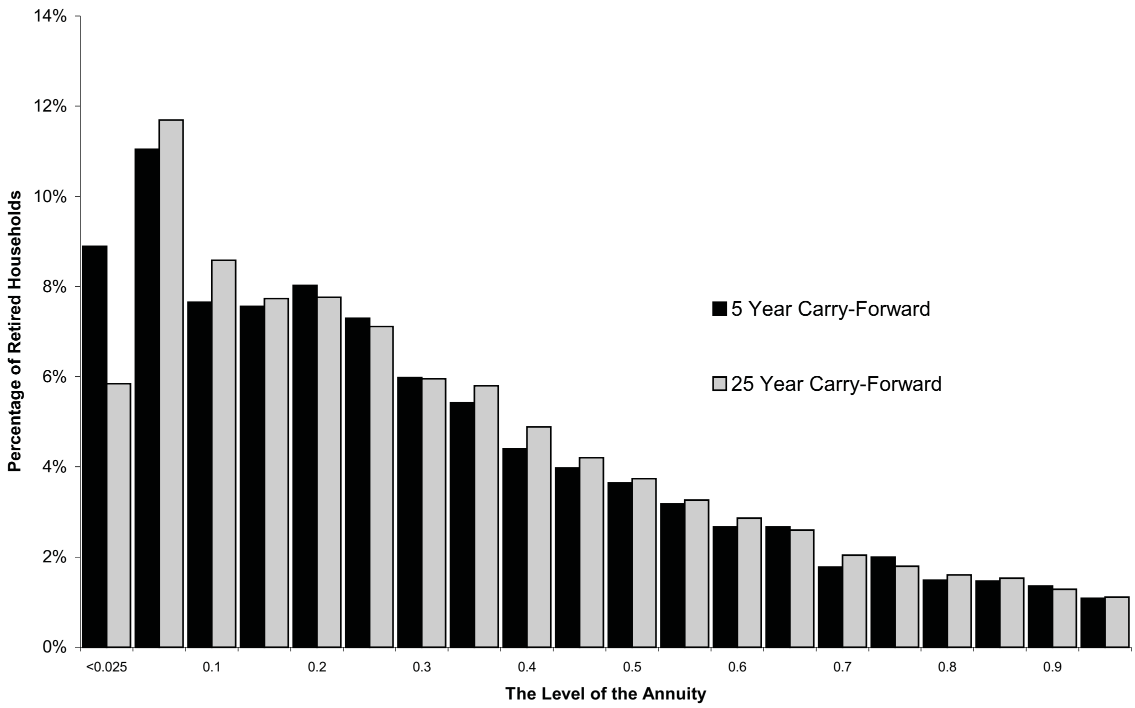

Figure 2. The distribution of annuity payment levels under different carry-forward periods

2.6 *Benefit of Recommendations*

2.6.1 Section 2 has aimed to illustrate the dilemma for low-income households between saving to satisfy the desire for insurance against poor earnings and saving to satisfy the desire for a better standard of living after retirement. This dilemma is a direct result of the interaction between the tax benefits of the personal pension plan and the statutory maximum contribution limits on any such plan. We have also demonstrated that a relaxation of these limits, by the extension of the carry-forward rule, benefits predominantly low-income households. It also encourages them to save more for retirement, as can seen from the taller grey bars in Figure 2. This is by no means the only way for introducing more liquidity into the market for personal pensions; it is perhaps the simplest. One straightforward enhancement, for instance, is the inclusion of a maximum amount of earnings that can be carried forward.

2.6.2 The economic benefits to low-income households of such a relaxation have been shown to be significant. However, in this model we have used a fairly conservative estimate of the level of earnings volatility of low-income households. Survey data, see Dutta, Sefton & Weale (1999) for example, suggest that the earnings volatility of low-income households is higher than the population average. In this case, the benefits could be considerably greater.

2.6.3 The work just described has been carried out without reference to the policy of means testing or targeting of benefits, a policy that has become particularly central to government thinking over the last few years. Had means testing been taken into account, the benefit of personal pensions to low earners would have been even less. However, with the introduction of the pension credit in 2003, the returns to a small private pension will increase again.

3. Personal Pension Return Guarantees

3.1 *Intergenerational Inequality*

3.1.1 In this section we look at the of distribution of pension wealth across the generations. We therefore focus on the investment risk faced by households. Households in the same generation all have identical investment opportunities; the modelled difference between them is due to their earnings history or earnings risk. The various generations modelled will be differentiated, not by the distribution of earnings across their generation, which will be the same, but by their unique history of investment returns.

3.1.2 Recently, there has been a surge of research effort devoted to examining the implications of this investment risk on intergenerational inequality. This is a direct result of the steady trend, in nearly all developed countries, away from defined benefit (DB) pension schemes to defined contribution (DC) or personal pension plans. DB schemes have built into their design an almost perfect insurance for their members against investment risk. However, in DC schemes, this risk is no longer shared across generations. Therefore, if a generation were reliant on DC schemes and were to experience an unlucky string of poor returns on their investments, then, in retirement, they could be left poor relative to their ascendants or descendants.

3.1.3 Miles & Timmermann (1999), Feldstein & Ranguelova (1998) and Feldstein, Ranguelova & Samwick (1999) looked at this issue, and they all found that investment risk can cause a significant amount of intergenerational inequality. However, they all assumed that individuals saved a constant share of their income in every period. As Miles & Timmermann point out, this is likely to underestimate the actual levels of inequality, as households tend to save more in the final years of their working life. A reduction in the effective investment period will increase the dispersion between generations.

3.1.4 However, this assumption does not take into account that households will adjust their saving behaviour in the light of past returns. Thus, if they have experienced poor returns in the past, they are likely to invest more later so as to 'catch up'. Conversely, if they were lucky, then they will reduce their subsequent contributions and enjoy the good life now. This behavioural response will dampen intergenerational inequality during retirement.

3.2 *Use of the Model*

3.2.1 Using our model, we were able to investigate the level of intergenerational inequality, allowing for both household behavioural responses and a realistic tax environment. The approach is described in ¶2.2.8. It amounts to running 4,500 experiments with a different set of random innovations, and examining the behaviour of the generation born in the 300th period. In this section we describe the distribution of annuities across these 4,500 cohorts.

3.2.2 Figure 3 plots the distribution of the average level of the annuity across the generations. This distribution has a mean of 0.47, a median of 0.43 and a standard deviation of 0.17. This distribution is heavily skewed, with some generations doing very well from their investments and with only about 6% of generations having an average annuity less than one standard deviation below the generational mean.

3.2.3 The distribution is heavily skewed, because of the tied nature of the investment in personal pensions. If a generation is particularly lucky with its early investment, it can reduce its current level of contributions, but cannot withdraw money from the pension fund. Therefore, the behaviour of these lucky generations is partly constrained. In contrast, the only action an unlucky generation will wish to take is to increase its level of contributions; its actions will only be constrained by the maximum contribution limits.

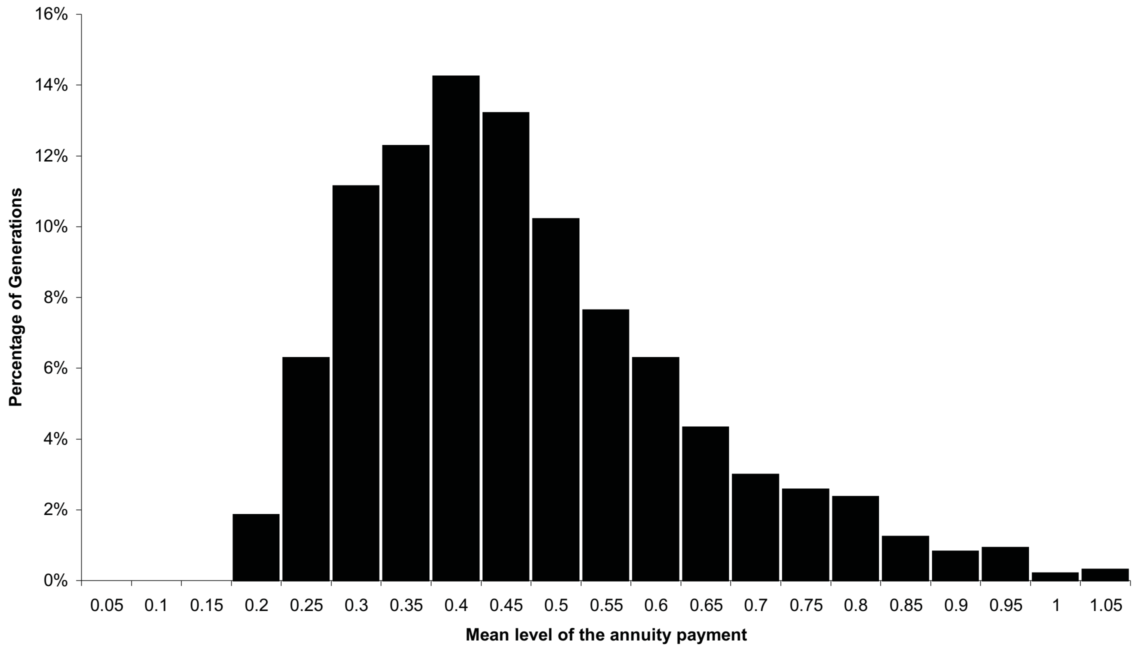

Figure 3. The distribution of the mean annuity payment across generations

3.3 *Return Guarantees*

3.3.1 Both Miles & Timmermann and Feldstein & Ranguelova suggest a role for the derivative market in insuring households against poor returns in the equity market. Households could choose to invest in so-called capital protected equity funds. These funds guarantee a certain rate of return on any investment. The guarantee is paid for by forgoing some of the gains in periods when the equity market returns are above the guaranteed rate. These guarantees are generally matched by put options on each equity in the fund. These put options can be exercised in periods when stock market returns are poor, and so insure against such periods.

3.3.2 The guarantee or put options can be paid for in a variety of ways, for example, by forgoing all returns above a given return (a bull spread) or a constant proportion of the returns above a given rate. We chose the latter of these two examples, although, often, capital protected funds use a mixture of both.

3.3.3 Figure 4 is a diagram of the return payoff of this particular type of fund as a function of the return on the underlying equity. If we denote as s the proportion of returns forgone in periods of high equity returns (the cost of the guarantee), and $r_{guaranteed}$ as the guaranteed level of returns, then the diagram illustrates that, if the actual return of the underlying assets was less than $r_{guaranteed}/s$, then the investor receives a return of $r_{guaranteed}$ and, if the return is greater than $r_{guaranteed}/s$, then the investor receives an additional $(1 - s)$ of this extra return. We calculated the cost of the guarantee, s, using the Black-

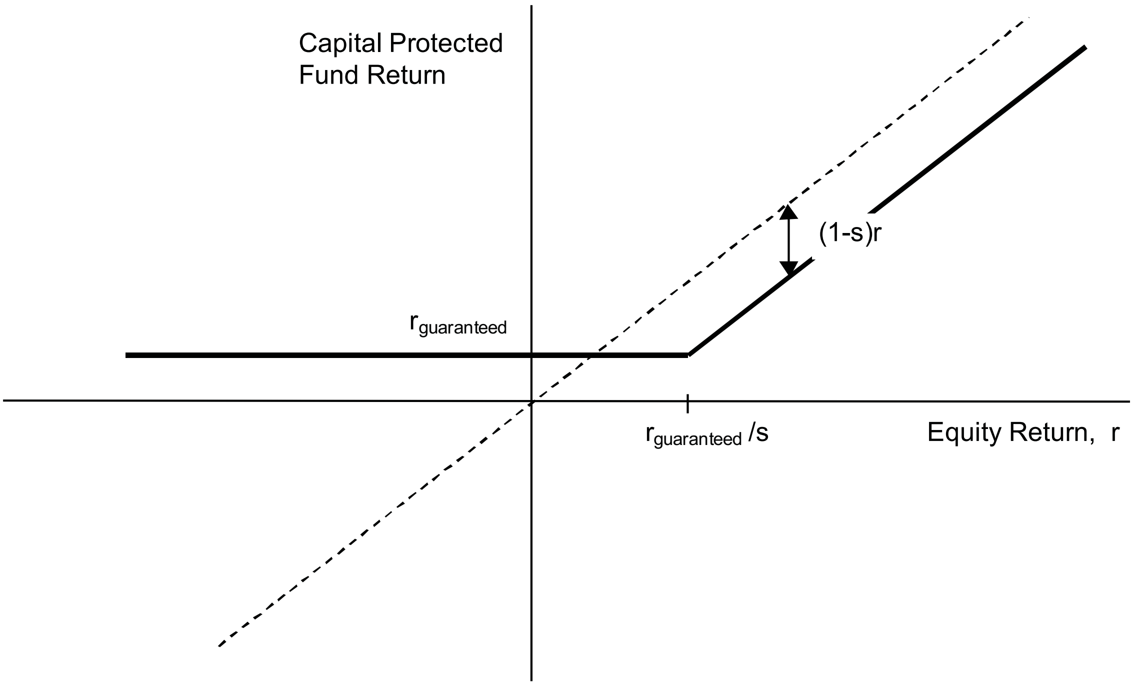

Figure 4. A plot of the capital protected fund payoff against the market return

Scholes option pricing formula. This cost is a function only of the volatility of the underlying equity, the risk free rate and the guaranteed return on the protected fund.

3.3.4 Table 3 gives the cost of the guarantee for different levels of the guaranteed rate, the risk free rate and volatility of equity returns, all quoted at an annual rate. It is worth noting also that, in practice, the cost of the guarantee is likely to be higher than those calculated in Table 3, because of transaction and administrative costs, as well as of possible derivative risk premiums. Although the returns in the table depend, in the obvious way, on the period of the guarantee, the cost of the guarantee s, in terms of return foregone, remains

Table 3. The cost of the capital protected fund

Risk free rate of return	Guaranteed rate of return	Annual standard deviation of equity returns	Cost of guarantee s	Guaranteed rate/$(1-s)$
1.03	1.00	0.17	0.06	1.07
1.03	1.01	0.17	0.10	1.12
1.05	1.00	0.17	0.03	1.03
1.05	1.01	0.17	0.04	1.05
1.03	1.00	0.24	0.12	1.14
1.03	1.01	0.24	0.17	1.22
1.05	1.00	0.24	0.07	1.07
1.05	1.01	0.24	0.09	1.11

the same for whatever period. In our model, we assumed that the guarantee was over a five-year period, at the end of which the return of the fund was calculated and a new guarantee was purchased. The results, though, will be independent of this choice; this follows directly from the assumption that the equity returns were independently lognormally distributed.

3.3.5 In our first simulation, we assumed that the risk-free rate of return was 3%, and the protected fund guaranteed a return of 1% p.a. As with all the other simulations, we assumed a market volatility of 17%. Under these assumptions, we found there was no demand for capital protected personal pension funds. The undesirability of capital protected personal pensions is a direct consequence of the size of the equity risk premium, the differential between the expected return on equity and the risk-free rate and household risk preferences.

3.3.6 Households would have had to be extremely risk adverse to benefit from the capital guarantee. In the simulation that we chose, a coefficient of relative risk aversion is equal to 2. This is on the high side, despite the use by Auerbach & Kotlikoff (1987) of a coefficient of 4. For example, Cooley & Prescott (1995) use 1 and Blundell, Browning & Meghir (1994) use 1.3. To create a demand for capital protected personal pension funds, it is necessary to assume coefficients of risk aversion as implausibly high as 10-12. For a comprehensive review on this issue, see Kocherlakota (1996). However, the fundamental notion is that, using consensus values for describing individuals' attitude to risk, and given the observed level of equity returns relative to any risk-free instruments, it is hard to generate a demand for risk-free instruments. Or, equivalently, to create a demand for risk-free instruments, individuals would need to be far more risk-averse than any estimates suggest.

3.3.7 Our result is supported by a recent observation on capital protected funds in the *Guardian* (25 March 2000): "All this leaves the capital-protected ISAs looking decidedly unloved ... capital-protected funds have suffered because they are expensive to run." Further support for the result can be found in the recent performance of capital protected funds. Barclays B2 launched a capital protected fund in 1999, only to withdraw it a year later, as its costs were thought to be too high to be attractive to investors.

3.3.8 In order to increase the attractiveness of the capital protection, we ran the model assuming that there was a government subsidy of 3% of the capital sum added to any investment in a capital protected pension fund at the beginning of every period. This had the effect of reducing the cost of the guarantee, so that s fell from 0.1 to 0.05; even this subsidy only created a small demand for these capital protected funds, all amongst the working population older than age 60. Moreover, this demand was entirely focused on the top two quintile earning groups, who wished to invest only an average of 15% of their total pension fund in these capital protected schemes.

4. Estimating Future Rates of Return

4.1 *Background*

4.1.1 'Making financial sense of the future' to professional standards requires extensive and disparate training and qualities, as do the implications of: "We can provide solutions to *any* (our italics) problem involving financial risk", which is set out in Vision and Values (1999). To fulfil this, each different aspect of financial risk ought to be considered. This section of our paper considers one key aspect, that of estimating future rates of return.

4.1.2 Parallel to the actuarial profession's development of Vision and Values, much publicity has been given to the need, in the light of falling with-profits bonus rates and falling illustration rates of return, for those who are considering taking out a mortgage loan, to decide what type of repayment vehicle to use, and, for the millions who have already effected endowment assurances for the repayment of mortgage loans, to judge whether the rate of funding into their existing endowment assurance is likely to be adequate. The likely future rate of return is critical to borrowers' decisions.

4.1.3 Another high profile area is that of fund transfers or mergers, where assumptions on future rates of return can be critical to the value of entitlement to surpluses. More information on both of these areas is included in Appendix 3. Also, as the value of the equity risk premium is an important factor in determining project discount rates, the range of the equity risk premium's forecast alone can often leave in doubt whether a project is likely to achieve significant profits or losses.

4.1.4 A range of actuarial work has been carried out in the field of rates of return and their prediction for the future, with, by far the major actuarial contribution in the field having been made by A. D. Wilkie, starting with his contribution to the Maturity Guarantees Working Party (1980). The resulting stochastic model developed by Wilkie, and put forward by the Working Party, included parameters to predict dividends and dividend yields and, using these yields, share prices. This model, which has become known as the Wilkie model, has been updated many times over the years, the latest *B.A.J.* update being Wilkie (1995b). That paper examined a large number of economic variables, which extended, for example, to long and short-term interest rates and inflation rates. The paper, among other things, set out and reviewed comprehensive data, considered alternative time series, deduced and examined model parameters, and used them to show projected funnels of doubt.

4.1.5 In Wilkie (1995a), the author used his model to estimate the future equity risk premium. A key aspect of Wilkie's interpretation of the results was that he concluded that a proportion of the rate of return since 1923 was due to the change in expected inflation, which Wilkie considered was zero in 1923 and 5% p.a. at the time he wrote his paper. This led to the conclusion of an 'average conditions' geometric mean, gross of tax, equity risk premium of 2% p.a., also justified as 1% yield gap and 1% real dividend increase. Also, we refer to Wilkie (1993) in ¶4.2.4.5.

4.1.6 Section 4.2 of the paper reviews some relevant economic papers in the area of future rates of return, after which the paper identifies some key issues raised, and makes recommendations to the profession for improving communication and giving advice to members.

4.2 *Recent Economic Papers*

4.2.1 *Papers reviewed*

The economic papers reviewed include Bliss (1999), which looks at the real interest rate and Dimson, Marsh & Staunton (2000). The latter reviews rates of return in the 20th century and considers what returns might be in the 21st century. Additionally, Cochrane (1999) provides a review of financial economists' understanding of changes in stock market value, and Campbell & Cochrane (1999) covers an investigation into the link between a macroeconomic factor, change in consumption, and stock market level and dividend yield volatility.

4.2.2 *Bliss's real interest rate*

4.2.2.1 Bliss's paper reviews the most important econometric models that have been developed to predict the long run real interest rate. Two of the key models are Diamond's, the details of which are given by Blanchard & Fisher (1989), which effectively assumes all savings are via pension funds, and Solow's (1956), whose model is more appropriate to a world where inherited wealth is passed on from one generation to the next. Bliss considers that, while many models provide useful insight into the factors affecting the real interest rate, none of them is sophisticated enough to be of practical use today. Bliss, therefore, suggests that various models are combined, in particular Diamond's with Solow's.

4.2.2.2 In the various models, the factors affecting the real interest rate are savings rates, demographics and technological progress. Bliss thinks that these will continue to be the 'grand forces', even in a more complex model, although he adds inflation as a key factor over the last 25 years.

4.2.3 *Dimson, Marsh & Staunton's risk and return*

4.2.3.1 Dimson, Marsh & Staunton have broadened and refined research into investment returns and the equity risk premium over the whole of the 20th century, and across the stock markets of the 12 largest developed countries, thereby covering, by value, almost all of the world stock markets. For the U.K., this involved constructing entirely new equity and bond indices for 1900-2000. In their paper, the authors also consider, in a reasoned way, the implications of their findings for the value of the equity risk premium in the 21st century.

4.2.3.2 We highlight here a number of aspects of Dimson, Marsh & Staunton's paper, and start with their predictions for the 21st century. For these, the authors, while starting with the historic position, consider how far the factors underlying the equity risk premium in the last century are likely to subsist or change over this one, and draw reasoned conclusions.

4.2.3.3 In making these conclusions the authors considered:

(1) likely changes in the equity risk premium, on account of the risks having altered or because of changes in savers' levels of risk aversion; and
(2) eliminating the effect of once-off non-repeatable market influences.

4.2.3.4 Looking further at these considerations, key factors identified include:

(1) a fall in the required rate of return due to diminished investment risk, for example, due to a more secure business environment or due to diversification domestically or internationally; and
(2) extensive technological change and generally unprecedented growth in productivity, along with globalisation and improvements in corporate governance. The authors single out these as the major factors, which caused the 'unanticipated growth in corporate cash flow'.

4.2.3.5 The authors consider that the above factors were not built into equity prices at the beginning of the second half of the last century, but are taken into account in current prices, and the resulting implied once-off increase in returns cannot be expected to be included in the expected 21st century equity risk premium.

4.2.3.6 For equities, bonds and bills in the 20th century, Dimson, Marsh & Staunton calculate a range of statistics for nominal and real rates of return, including arithmetic and geometric means. Their research found that the 20th century equity risk premium over the bill rate varied from 2.8% p.a. for Denmark to 7.7% p.a. for France, with the worldwide average being 5.7% p.a. and the U.K. having 4.9% p.a. They argue that, as a result of globalisation and, we would add, the coming closer together of several countries' stock markets, many differences between the various countries can be accounted for by one-off non-repeatable market influences. Hence, the starting average for any country when considering future equity risk premiums should be the worldwide average, although this does not preclude adjustments to allow for distinct country attributes, for example Japan's. The authors do not make specific predictions for the expected equity risk premium over the 21st century, but indicate that its geometric mean is 'almost certainly less than 5-6% p.a.'.

4.2.4 *Cochrane's new facts in finance*

4.2.4.1 'New' in the title of this paper is taken from around the early 1980s. Before then, Cochrane says, financial economists considered that fixed-interest and equity rates of return were essentially unpredictable, that is that prices followed 'random walks'. The paper reviews the significant changes in financial economists' views since the early 1980s and identifies areas of predictability. It should be noted, though, that this does not necessarily mean that the predictable areas are great enough to exploit profitably, after taking account of transaction costs. Also, exploitable differences may soon disappear, due to exploitation.

4.2.4.2 An important point that Cochrane emphasises is that the predictability is longer term. An analogy is given with weather forecasting. While we know that in mid-January in London, as well as in Cochrane's Chicago, the temperature will be higher six months later, and we can make a useful forecast of the likely increase, we do not know if the temperature, one week after the mid-January date, will have increased or decreased, and, more importantly, the 1/26th of the six-month increase estimate is of no help at all in the one-week prediction. Of course, the timescales of market changes are very much longer; Cochrane refers to studies on certain indicators with 'very predictable' results on five-year time horizons.

4.2.4.3 Cochrane's paper reviews investigations into the predictability in, among other areas, bond and equity returns and as a result of volatility of markets, and concludes that, despite belief to the contrary up to 15 to 20 years ago, in the last couple of decades predictability at long horizons has been proved in each of these areas.

4.2.4.4 Looking briefly at bond prices, the traditional explanation behind an upward sloping yield curve is, that over many years, the rise in short-term interest rates improves the rate of return from rolling over short-term bonds to equal that from holding long-term bonds. Now it is held that, given the same yield curve, long-term bonds can expect to yield more over one year than shorter-term ones.

4.2.4.5 Turning to equity returns, over the last dozen or so years Fama & French (1988, 1989, 1993, 1995 & 1996) have carried out a range of investigations into, and developed, a number of formulae which can be applied to adjust the equity risk premium. Cochrane explains that these include using prices relative to dividends, book values, earnings, sales and other divisors, in respect of which five-year returns on shares seem very predictable. There are, therefore, implications on the likely level of future equity risk premium, from a given date, both by where it is in the economic cycle and also by the level of stock market prices at that date relative to the long-term trend in the level of prices. It should be added that, in respect of share price changes relative to dividends, Wilkie (1993) showed the same for the U.K. as Fama & French did for the United States of America.

4.2.4.6 Cochrane argues in his paper that each of the relationships just described are variations on a common theme, each using price variables to infer market expectations of future returns; each case suggests that financial markets offer rewards in the form of average returns for holding risks related to recessions and financial distress in addition to risks associated with overall market movement.

4.2.4.7 Throughout his paper, Cochrane emphasises the importance of being able to link equity performance with macro-economic factors. As examples of investigations into such links, Cochrane cites Jagannathan & Wang (1996) and Reyfman (1997), who used earnings, Chen, Roll & Ross (1986), who, among other variables, considered industrial production and inflation, and Cochrane (1996), who considered investment growth. In particular, there is the research carried out by Campbell & Cochrane, for which see Section 4.2.5.

4.2.5 *Campbell & Cochrane's consumption-based explanation*

4.2.5.1 One of the most interesting of the recent developments has been the detailed investigation by Campbell & Cochrane into the relationship, from 1890 to 1995, between a real macro-economic factor, change in consumption, and stock market volatility and the equity risk premium. They did this both by the direct prediction of share prices from changes in consumption, and also by first predicting dividend yields from the latter, and then using dividend levels to determine prices. They obtained similar results by both methods.

4.2.5.2 Their underlying economic reason for the relationship was that investors fear shares primarily because they perform poorly at the bottom of an economic cycle, such fear being unrelated to the risks associated with the long-term average growth in consumption. It is also of interest that, in this research, Campbell & Cochrane used a fixed real interest rate, which meant, in effect, that changes to consumption were linked directly to equity rates of return.

4.2.5.3 The authors concluded that, while the model did not explain changes in stock market prices better that some purely financial models, the model did do reasonably well and moreover: "the model gives some hope that finance can search for fundamental risk factors that explain at least the time-series behaviour of aggregate stock returns rather than just relate some asset returns to other asset returns leaving fundamental issues such as the equity risk premium as free parameters."

4.3 *Issues*

4.3.1 *Review of the Wilkie model*

4.3.1.1 Section 4.3 identifies a number of issues, mostly of a continuing nature, raised by the foregoing, as well as some other issues pertinent to future rates of return. It starts by taking up some issues already brought to the attention of the actuarial profession in detailed reviews of the Wilkie model.

4.3.1.2 The Wilkie model has been subject to independent reviews by the profession, although many of the comments in the process apply more generally. The latest review of the Wilkie (1995b) version was considered by

Huber (1997). This identified a number of problems with certain technical aspects of the model, and drew attention to its inconsistency with 'certain orthodox financial economic theories'.

4.3.1.3 A wide ranging review of an earlier version of the model was made by the Working Party of Geoghegan *et al.* (1992). In our view, subsequent updates to the model have not invalidated the key Working Party comments. Among other things, the Working Party gave extended consideration to the model's applications in the pension fund, life offices and investment fields, and the practical problems in the areas of skills, training and actuarial judgement associated with the use of the model and its applications.

4.3.1.4 The Working Party had available comments on the model by Professor Harvey of the London School of Economics, and was unable to recommend the Wilkie or *any model* (our italics) for "widespread actuarial use", and, in justification of this, quoted from Harvey's own conclusion as follows:

> "A stochastic investment model is undoubtedly useful in giving actuaries some idea of the kind of movements to be expected in key financial areas. The Wilkie model represents an important first step in this direction. However, it is not definitive, and it is important to recognise that the whole approach can be developed and improved in many ways. Wilkie built his model based on Box-Jenkins methods; and approaches based on other kinds of econometric and time series modelling procedures may be better and simpler.
>
> If the actuarial profession is to take stochastic models seriously, and to use them in an intelligent and critical way, it is necessary for those in the profession to understand the way such models are built, and the nature and strength of their weaknesses. This leads to the conclusions that econometrics and time series analysis should figure to a much larger extent in actuarial training ... ."

4.3.1.5 The Working Party called for more research in the area that it reviewed and, during the discussion on the Working Party's report, G. T. Pepper said that he hoped that the Working Party would "sit continuously with a mandate to keep in touch with modern time series developments and to communicate them to the profession." It may be noted that Wilkie, in (1995b), as well as Appendix B of the Working Party's report, considered more elaborate forms of time series, although there will inevitably be a time, if the model is to be kept up to date, when others will need to take over such work.

4.3.2 *Model aspects*

4.3.2.1 Firstly, let us take the just mentioned issue raised by Pepper. To our knowledge, no working party was set up to keep in touch with time series. Moreover, while Pepper specifically raises time series, Harvey refers to economic modelling procedures in general, and we would suggest that the actuarial profession communicates to its members on the wider basis.

4.3.2.2 Such models, though, need reviewing in other ways. For instance, with the passage of time, additional years' data become available, and it would seem, to us, helpful and beneficial that these were communicated to members, along with model updates, from a central source. Further, from time to time, improvements in past data are published, and these can be dealt with a similar way to new data.

4.3.2.3 There is also the need to consider the interpretation of model results. Wilkie, in his 1995a paper, in particular as a result of his view on the effect of inflation on equity returns, concluded that the gross of tax geometric mean value of the estimated equity risk premium was around 2% p.a. On the other hand, there are several papers, with reasoned predicted equity risk premium geometric means of 5-6% p.a. or higher quoted in Dimson, Marsh & Staunton's paper; these authors' views has been quoted in ¶4.2.3.6 of our paper. There is a substantial difference between Wilkie's conclusion and those of the various other authors just mentioned. Since Wilkie produced his results, expected inflation has reduced from around 5% p.a. to 2.5% p.a. It would be of value to update Wilkie's work to reflect this and data in respect of the intervening years, and to try to come to a conclusion on the relative merits of the different results and their interpretation.

4.3.2.4 To assist in all of the above, we suggest that the actuarial profession regularly brings to the attention of its members relevant economic research, and sets this alongside actuarial work in a similar area.

4.3.3 *Practical advice*

As already mentioned, the Geoghegan *et al.* report considered the use of the Wilkie and similar models in a wide range of actuarial areas, but felt unable to recommend any model for any of the uses considered. While we understand why the Working Party came to their conclusion, there is, in our opinion, a range of practical advice that can be given by those whose speciality is in the field of rates of return models and associated areas to those whose expertise lies elsewhere, but need to use the models and their results for a number of specific actuarial uses. Some of that advice may come from training, but other parts could be more helpfully supplied by central communication to members.

4.3.4 *Areas of research*

4.3.4.1 We suggest here a number of areas where the actuarial profession might wish to consider encouraging or facilitating research. These are:

(a) consideration of differences between historic and future bond rates;
(b) estimation of the rate of inflation; and
(c) the relationship between macro-economic factors and rates of return.

4.3.4.2 While there is a long history of short and long-term bond rates, there has been limited consideration of how the means and distributions of future values might differ from those found from parameters based on historical values. This is of some importance, and consideration could usefully be given to this, perhaps by facilitating, in some way, Bliss's (1999) suggestion of combining Solow's and Diamond's models.

4.3.4.3 Frequently, an inflation rate is determined from the difference between conventional and index-linked gilt yields. While the Treasury, through the Debt Management Office, can influence the market, and will take into account its view of inflation, risk factors, for instance solvency considerations and how far the type of security matches a saver's liabilities, are considerations for investors. Appendix 4 sets out, as at 5 February and 16 June 2000, and as at 9 March 2001, three dates taken at random, a comparison between conventional and index-linked gilt yields; these imply inflation rates of 3.0%, 2.7% and 2.5% p.a. respectively. These figures may be compared to the Bank of England target of 1.5% to 2.5% p.a., with underlying inflation, at the time of writing, within the range. Moreover, we would have considered that the likely future inflation rate would have increased during this period, due to more expansionary than expected spending plans. It is important to have a reliable way of deciding on appropriate predicted inflation rates consistent with other assumptions, and we consider that there would be benefit to research how far determining the inflation rate from the difference between conventional and index-linked yields is satisfactory.

4.3.4.4 The work of Campbell & Cochrane (1999) encourages us to think that there are real macro-economic factors that will explain stock market movements and equity rates of return more usefully than purely financial factors. The actuarial profession could consider facilitating research in this field.

4.4 *Consideration of the Way Forward and Recommendations*

4.4.1 *Control of investigations*

4.4.1.1 In Section 4.3 we identified a number of issues where reviews should be carried out, most on a continuing basis. If these are all to take place in a timely way, then there needs to be some overall control. In the field of mortality, a large body of work is carried out and controlled by the Continuous Mortality Investigation Bureau, and we would recommend that an equivalent be set up in the field of finance. This could be called the Continuous Finance Investigation (CFI) and be under the control of a Board appointed by the actuarial profession.

4.4.1.2 Inevitably, a number of aspects of the CFI would be different from the CMI and, in particular, there would not be any equivalent of its substantial data gathering from insurance companies and its accompanying processing costs. However, there are a number of qualities of the CMI that could usefully be carried forward to a CFI. These include the issuing, from time to time, of sequentially numbered reports, authoritatively written, regularly updated, with analyses of past experience and reasoned estimates of changes that might be expected in the future. All this could provide actuaries with sound assistance and support in their dealings with clients and principals.

4.4.2 *Scope of CFI*

4.4.2.1 It is envisaged that the scope of the investigation, while concentrating on equities and the equity risk premium, would include rates of return from all major types of investment, and that investigation results would show the relationships with key economic factors such as inflation rates.

4.4.2.2 Some detailed aspects of the scope of a CFI follow from Section 4.3. These include:

(1) to encourage and/or facilitate the use of improvements in modelling procedures, such as, but not limited to, the handling of time series, as they become available;
(2) to encourage and/or facilitate updating the model with the latest data, and, where appropriate, with improvements in data from earlier years;
(3) consequent to (i) and (ii) or otherwise, to review the interpretation of historical results and consider their implications for the future; and
(4) to report, from time to time, on (i) to (iii).

4.4.2.3 It is envisaged that an important aspect of the CFI's work would be the communication to members of the actuarial profession, via CFI reports, of relevant economic papers and research, where appropriate, to be put alongside related actuarial research.

4.4.2.4 One way suggested to us to assist this communication is the charging of one of the many working parties set up for each of the main annual conferences, general, investment, life and pensions, with preparing a survey of relevant economic research, and that, parallel to this, it might also be beneficial to invite more economists to address actuarial conferences.

4.4.2.5 We would also like to see included the practical advice referred in Section 4.3.3.

4.4.2.6 In addition to reviewing published papers, we consider it appropriate for a CFI Board to facilitate or commission its own research. Possible areas of such research were given in Section 4.3.4.

4.4.2.7 It is our view that actuarial research is not well known outside the profession, and the Board should consider how this could be beneficially communicated to economists and other relevant professionals.

4.4.2.8 There may be limited expertise in this field within the profession, and it may, therefore, be advantageous if some of the work for the CFI Board was carried out in conjunction with a reputed economic research organisation

4.4.3 *Other aspects*

4.4.3.1 It is envisaged that the CFI is expanded to include related areas and other financial issues arising from Vision and Values (1999), or of concern and importance to the actuarial profession and its members. For instance it could include the continuous investigation of selected U.K. equities, suggested in Clarkson (1999).

4.4.3.2 Another advantage of the approach to future rates of return put forward is that it could eliminate the historic difference in approach to setting rates of return assumptions between different areas of work, for instance between that of life offices and that of self-administered pension funds. Traditionally, work on the former has generally used nominal rates and the latter real ones, while more recently the approach has been affected by statutory regulation, such as is embodied in the profession's guidance note, GN29. While differences can arise from the nature of the liabilities, which the assets match, they can lead to anomalies in the implied inflation rate and equity risk premium.

4.4.3.3 One specific need for the recommendations that we have just made arose in writing this paper, as, in considering the issues set out in Sections 2 and 3, it was necessary to make an assumption about the future real risk free rate of return. Following the recommendation by Dimson, Marsh & Staunton (2000), we took an international approach. At the time of writing, looking first at nominal rates, long-term U.S. bonds yield 5.75% p.a. and those in France and Germany average a similar level. Japan's 1.8% p.a. nominal yield is due to unique factors, and was given a low weight.

4.4.3.4 The U.K.'s long-term bond nominal yield, at the time of writing, at 5.00% p.a. is out of line with North America and Europe but has been given due weight, as we are considering U.K. issues. Our average inflation rate estimate for the various countries is 2.25% p.a. Using this figure, we arrive at real interest rates of around 2.75% p.a. for the U.K. and 3½% p.a. in North America and the key countries of continental Europe. Before coming to a conclusion, we considered the worldwide 2.3% p.a. average real bond yield over the last 50 years, calculated by Dimson, Marsh and Staunton, and the reasons leading to that value. Giving due weight to that and the yields and size of market of the various countries worldwide, we decided to use a 3% p.a. real interest rate, but consider that the preceding reasoning illustrates the need for the recommendations that we have made on estimating future rates of return.

5. Economic Applications and Areas for Debate

5.1 *Economic Applications*

5.1.1 It can be seen from Sections 2 to 4 that the economics discipline can, in a range of ways, assist areas of actuarial interest.

5.1.2 Sections 2 and 3 show how an economics model, in this case a general equilibrium model, can be used to investigate certain public interest aspects of pension funds. Examples of other areas where a general equilibrium model can be expected to give helpful results include:

(a) an examination of the efficacy of various aspects of means testing;

(b) investigating the rules on the flexibility of taking annuities, taking account of best advice and dependence on benefits; and
(c) investigating the implications of alternative mixes of state and personal pensions.

5.1.3 A general equilibrium model is just one type of many models available to econometricians, and we would consider that it would be advantageous if actuaries were familiar with the capabilities of a range of economic models.

5.1.4 In Section 4, in the area of estimation of future rates of return, we saw that there was a large number of papers written about investigations, which we felt that the majority of actuaries were unaware of, and which were highly relevant to being able to come to the most appropriate conclusions on estimates of future rates of return. In our opinion, it should not be surprising that economists have carried out these investigations. Estimation of future rates of return, while not be the most pressing economics problem, is, nevertheless, of significant relevance to the running of the economy, in so far it is of importance to Governments and to industry and commerce, as well as of increasing financial importance to the general public. It is, therefore, an area of significant interest to economists, where more useful and relevant research can be expected

5.1.5 Additionally, it should not be surprising that there are other areas that economists have researched, which are also areas of actuarial interest. For instance, in the U.K. and the U.S.A., a large proportion of savings is accounted for by pension funds and life assurance funds. They have significant importance in those countries' investments, and, therefore, are the subject of much consideration by economists.

5.1.6 We have included a number of recommendations, and have drawn a number of conclusions in this paper, following from the specific issues about which we have written. We consider that an additional recommendation follows from the above, one on actuarial education. The recommendation is that, in its continuing investigations into members' educational needs, the actuarial profession should consider, in the light of the importance of the links between actuarial work and economics, financial economics and economic models, whether and how these areas should be given greater weight in actuarial student training and/or in post-qualification courses.

5.2 *Areas for Debate*

5.2.1 We have, in earlier sections of the paper, put forward recommendations, following from our conclusions on the three specific issues in this paper, and on the general issue of the application of the economics discipline to actuarial work. To help initiate discussion on the issues, we have set out below some of the relevant questions for debate, although we are sure that there are many others that can usefully be asked.

5.2.2 On the issue of demand for personal pension funds, we put forward the following questions:

(a) Is there agreement that the current pension tax concessions benefit the better off proportionally more than low earners, and are, in practice, of little benefit to low-income households?

(b) Is there support for the reintroduction and substantial extension of the 'carry-forward' rules, to be implemented in a practicable way, such as has been put forward in this paper?

(c) Are there any alternative proposals that would have a similar effect?

5.2.3 On the issue of intergenerational inequality of rates of return from personal pension funds, we put forward the following questions:

(a) How far should intergenerational inequality be of concern?

(b) Do so-called guaranteed equity funds give the public poor value for money?

5.2.4 On the issue of estimating future rates of return, we suggest the following questions for debate:

(a) What is it thought that knowledgeable members of the public consider to be actuaries' relative strengths, particularly in the fields of mortality and financial risk?

(b) Is the present position on the estimation of future rates of return satisfactory, especially in the context of the Vision and Values of the actuarial profession, including its emphasis on financial risk and its wish to be of higher profile in its stance on current issues affecting the public?

(c) If the present position is considered unsatisfactory, what steps should be taken to rectify the position? In particular, is there support for the setting up by, or under the auspices of, the profession of a Continuous Financial Investigation — possibly wholly or partly in conjunction with a body which has economic model experience — concentrating initially on the estimation of future rates of return, and with the object of issuing as authoritative, and as broad, reports as those of the CMI?

5.2.5 On the value of economics to actuarial work, we suggest:

(a) Is it considered that the current awareness and knowledge of what the economics discipline can offer actuaries is satisfactory?

(b) If the present position is considered unsatisfactory, how best should it be rectified?

6. Conclusions

6.1 This paper has investigated three specific issues of important public interest: that of the demand for pension funds by contributors with different levels of earnings, due to uncertainty of those earnings; that of the equity between different generations of pension fund contributors due to uncertainty in the different levels of achieved rates of return; and that of the estimation of future rates of return

6.2 Looking first at the demand for personal pensions, we examined the dilemma that savers have between, on the one hand, the preservation of liquidity, that is delaying pension fund contributions, and thereby not being able to take full advantage of tax benefits, and, on the other hand, trying to maximise tax benefits and being exposed to liquidity problems. It has been shown in the paper, using a general equilibrium model, that, to minimise likely liquidity problems to a reasonable level, pension fund saving is likely to be deferred until savers are in their late forties, and that the dilemma is particularly acute for those on relatively low incomes. The latter, which is regressive, could be overcome by relaxation of the carry-forward rules, possibly capped to ensure those on low incomes benefit very much more proportionally. It has been shown that the relaxation is a way of helping low-income families (defined as those in the bottom two earnings quartiles) save at least 5% of their income, and possibly much more, for their retirement.

6.3 The other personal pension issue investigated was the intergenerational inequality due to rate of return risk. Here we have shown, using a fully articulated partial equilibrium model, that inequality is not likely to be as great as has previously been estimated, because household behaviour is likely to mitigate possible intergenerational differences. We have also shown that, even for the most risk adverse households, the cost of derivatives is too prohibitive to be of interest.

6.4 Turning to the estimation of future rates of return, the paper reviewed recent relevant research and identified issues, most of a continuing nature, that it is considered need to be addressed. To do this in a controlled way, it was recommended that a Continuous Financial Investigation be set up and a Board, appointed by the actuarial profession, be given the responsibility to monitor, initiate and commission work in this area, and to issue reports to members that are as comprehensive and as authoritative as those of the CMI. It is considered that our recommendations for the estimation of future rates of return would achieve a significant improvement over the current position, giving support to the profession's members in this area.

6.5 In this paper we have shown that there are a number of areas in which economic research already published and economic models can be of use to actuaries, and that this is likely to be the case in many other areas. We therefore conclude that it would be advantageous if actuarial education took much greater account of the economics discipline, particularly in those areas just mentioned.

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APPENDIX 1

THE GENERAL EQUILIBRIUM MODEL

A1.1 *Our Model*

A1.1.1 A general equilibrium model is a generalisation of the overlapping generations in an economy, as described by Diamond (1965). This was extended to look at the demand for personal pensions. In the model for this paper, households can choose to save in either of two accounts. Both accounts are presumed to consist predominantly of equity, and, as such, their returns are risky. However, savings in the first account are perfectly liquid, and can be withdrawn at any time. By contrast, any savings in the second account are tax deductible, but can be withdrawn only at retirement; further, at retirement a proportion of these assets must be taken as an annuity. Otherwise our model is similar to the models used in Huggett (1996) and Sefton, Dutta & Weale (1998), and so incorporates all the principal factors that can affect individuals' saving behaviour, namely uninsurable income and capital return risk, wealth constraints, retirement, state pensions and an annuity market.

A1.1.2 The model was solved numerically using the algorithm described in Sefton (2000), developed slightly to cope with three states rather than two, and two control variables rather than one.

A1.2 *The Population*

A1.2.1 In our model, the economy consists of 30,000 households. We treat the household as the basic economic unit, and describe its lifespan and composition. We index each household by i and denote the age of the household by τ. Each household begins life at $\tau = 0$, and consists of a man and a woman, who are both 20 years old. The household dies when the second of the spouses dies. The life of each adult is uncertain, but we assume that the maximum age of each spouse is 90 ($\tau = 70$). We denote the conditional probability of the household dying at the end of period τ, given that it has survived to the beginning of that period, as ψ_τ implying, of course, that $\psi_{70} = 1$; the probabilities ψ_τ take into account the different male and female life expectancies. It therefore follows that the probability that a household will survive another n years from period τ, $\phi_{n,\tau}$, is simply the cumulative product of the conditional probabilities $\phi_{n,\tau} = \Pi_{j=\tau}^{\tau+n-1}(1 - \psi_j)$.

A1.2.2 During its existence, the size of the household varies for two reasons, firstly, because children are born, and secondly, because one of the spouses may die. We use figures for the number of children belonging to mothers of different ages to calculate the number of children in each household as a function of the age of the latter. In order to do this, we assume that no child ever dies and every child leaves home at 19. However, we do not model the fertility decision, and so we simply assume that the size of every household is equal to the average size for that age of household.

A1.2.3 Secondly, from the life tables, we can calculate the average number of adults as a property of the age of each household. We then convert the number of adults and children in each household to an adult equivalent, using the standard McClements scale to take account of economies of scale in household management; this scale is summarised in Appendix 2. We denote the McClements equivalent size of the household at age τ as m_τ. This effective household size influences the utility that the household derives from any particular level of consumption.

A1.2.4 To ensure that the population of households stays constant, we assume that, on a death of a household, a descendent household is born immediately. This descendent household inherits, not only the remaining physical assets of its parent household, but also a proportion of its human capital characteristics.

A1.3 *The Income Process*

A1.3.1 As well as facing uncertainty about its lifespan, each household faces considerable uncertainty about its future labour income $y_{i\tau}$. This is equal to the economy-wide constant wage rate s_{wage}, multiplied by the earning power of the individual household $h_{i\tau}$. We use the subscript t to denote the time dependence of all aggregate prices and quantities, where relevant, and τ to denote the time dependence of all household specific quantities; clearly the two are related by the specific birth-date of the household t_i^{birth}, so that $t = t_i^{birth} + \tau$. The wage rate is known, but earning power is expected to follow the following stochastic auto-regressive process:

$$\log h_{i\tau+1} - \log \bar{h}_{\tau+1} = \rho(\log h_{i\tau} - \log \bar{h}_\tau) + \varepsilon_{i\tau}$$

where $\varepsilon_{i\tau}$ is an uncorrelated innovation processes drawn from the normal distribution $N(\mu_\tau, \sigma^2)$ and $\bar{h}_\tau$ is the mean level of earning power for a household of age τ.

A1.3.2 This is the model of income dynamics studied in detail in Atkinson, Bourguignon & Morrisson (1992), and used by Huggett (1996) in his equilibrium model of the U.S. economy. As Atkinson *et al.* state, this process has a number of desirable properties. Firstly, as earning power is lognormally distributed for the youngest cohort, it remains so for every cohort thereafter. This is useful, as the log normal distribution has, for a long time, been used as a reasonable fit of the earnings distribution. Secondly, it can be easily calibrated to fit the observed earnings distribution. We used data from the British Household Panel Survey (BHPS, 1991-1995) data on household gross labour income to calibrate the coefficients of our income process. These coefficients are detailed in Appendix 2.

A1.3.3 The initial level of earning power of the descendent household, subscripted by j, is related to the initial earning power of its parent household, subscripted by i. This captures persistence in the level of earning power between generations, due either to nature or nurture (Becker & Tomes, 1979). We use the simple mean reversion model that has been estimated recently by Zimmerman (1992), and Dearden, Machin & Reed (1997) on data for the U.S.A. and the U.K., that is:

$$\log h_{j1} = \lambda \log h_{i1} + (1 - \lambda^2)^{\frac{1}{2}} \tilde{\varepsilon}_i - (1 - \lambda) \frac{\tilde{\sigma}^2}{2}$$

where the parameter λ represents the amount of persistence between generations. The innovation $\tilde{\varepsilon}_i$ is distributed normally, $N(0, \tilde{\sigma}^2)$. The coefficients in the equation are chosen so that, in the steady state, the distribution of the logarithm of earning power amongst households at the beginning of their working life will be $N(-\tilde{\sigma}^2, \tilde{\sigma}^2)$.

A1.4 *The Household Consumption Plan*

A1.4.1 We use the standard notation $c_{i\tau}$, $w_{i\tau}$ and $v_{i\tau}$ to denote the ith household's total consumption during, and wealth in the liquid and non-liquid accounts at the beginning of, any period, respectively. Each household makes its consumption decisions so to maximise its utility over the rest of its expected lifetime.

A1.4.2 Our specification assumes that utility is derived only from consumption, and not from bequests. This implies that all bequests are accidental. The assumption is in keeping with recent empirical work by Altoniji, Hayashi & Kotlikoff (1992), which finds little evidence for any altruism in the bequest behaviour. Huggett (1996) demonstrated that it was possible to replicate the levels of observed aggregate wealth without including a bequest motive in the model. The utility resulting from any consumption stream is described by a constant elasticity of substitution (CES) utility function:

$$U_{i\tau} = \left(E_\tau \left(\sum_{k=0}^{70-\tau} \phi_{k,\tau} (1 - \delta) \left(\frac{c_{i\tau+k}}{m_{i\tau+k}} \right)^{(1-\gamma)} \delta^k \right) \right)^{\frac{1}{1-\gamma}}$$

where the parameters δ and γ are the discount factor and the coefficient of relative risk aversion respectively and E_τ is the expectation operator with respect to the innovations in income and the return to assets.

A1.4.3 A household's consumption is limited by the constraint that, on its death, it must not be in debt. As there is a finite probability of dying in any period, this constraint actually implies that, at all times, a household's liquid wealth must always be positive or zero. Up to retirement, at the end of year $\tau^{\text{Ret}} = 45$ (when the adults are aged 65), a household can either consume or save its post-tax income in either its liquid asset or non-liquid asset; savings $s_{i\tau}$ in the non-liquid asset are tax deductible.

A1.4.4 However, there is a maximum amount that can be saved in the non-liquid asset. This maximum level is age dependent and defined as a proportion of the household's labour income. These proportions κ_τ are fixed by U.K. tax law, and rise steadily from 17.5% at the start of the household's working life to 40% just before retirement. See Table A2.4. We can, therefore, write down the household's budget constraints:

$$w_{i\tau} \geq 0, c_{i\tau} \geq 0, \kappa_\tau y_{i\tau} \geq s_{i\tau} \geq 0$$
$$w_{i\tau+1} = (1 + (1 - t^K)r_{t+1})(w_{i\tau} + (1 - t^L - t^{SI})y_{i\tau} - c_{i\tau} - (1 - t^{SI})s_{i\tau})$$

and

$$v_{i\tau+1} = (1 + (1 - t^K)r_{t+1})\left(v_{i\tau} + \frac{s_{i\tau}}{(1 - t^L)}\right) \text{ for all } \tau < 45 = \tau^{\text{Ret}}.$$

where t^L, t^{SI} and t^K are the constant labour income, social insurance and capital income tax rates.

A1.4.5 We have assumed that the post-tax return $(1 - t^K)r_t$ on both assets is the same. In practice, the returns on these two assets are taxed differently. However, in the U.K., because dividends to private pension funds are taxed at the basic rate of 20%, and because most individuals do not pay any capital gains tax, as the capital gain tax threshold is relatively high, this assumption roughly holds.

A1.4.6 Returns for any period t, at the beginning of that period, are uncertain. We have denoted this by subscripting the returns to period t with the subscript $t + 1$, thus implying that the returns for that period r_{t+1} are not known until the next period. We assume returns in any period are uncorrelated with returns in any other period, and, further, that they are log normally distributed. Thus $\log(r_{t+1}) \sim N(\mu^r, (\sigma^r)^2)$, where we have calibrated μ^r, σ^r to be consistent with the last 30 years of return data of the U.K. FT All-Share Index.

A1.5 *Retirement and Pensions*

A1.5.1 At the beginning of its retirement τ^{Ret}, it is optimal for each household to take as an annuity all its wealth, liquid and non liquid. For the sake of simplicity, we assume that households, where one or both the householders reach retirement age, all buy a level indexed-linked last survivor annuity. Relaxing this assumption would have significantly increased the difficulty of solving the households' optimisation problem, without significantly increasing the efficacy of the model. The annuity from the non-liquid asset is taxed at the full income tax rate, whereas only the interest component b of the annuity from liquid assets is taxed. The final complication is that a proportion a of the non-liquid asset is received tax-free, the annuity from which is taxed as that from the liquid asset. After retirement, the household's income consists of the annuity, that is private pension $p^p_{i\tau}$, the state pension p^s_t, as well as the post-tax return to its any wealth saved after retirement. Thus after retirement in year t_i^{Ret} at age τ^{Ret} the budget constraint is:

$$w_{i\tau^{\mathrm{Ret}}+1} = (1 + (1 - t^K)r_{\tau+1})\big(p^p_{i\tau^{\mathrm{Ret}}} + p^s_t - c_{i\tau^{\mathrm{Ret}}}\big)$$

$$w_{i\tau+1} = (1 + (1 - t^K)r_{\tau+1})(w_{i\tau} + p^p_{i\tau} + p^s_t - c_{i\tau}) \qquad \text{for all } \tau > \tau^{\mathrm{Ret}}$$

where:

$$p^p_{i\tau} = (1 - t^L)(1 - a)\frac{v_{i\tau^{\mathrm{Ret}}}}{\chi(r^f)} + \left(a\frac{v_{i\tau^{\mathrm{Ret}}}}{\chi(r^f)} + \frac{w_{i\tau^{\mathrm{Ret}}}}{\chi(r^f)}\right)\big((1 - b) + (1 - t^L)b\big)$$

and

$$\chi(r^f) = \left(\sum_{n=\tau^{\mathrm{Ret}}}^{70} \frac{\phi_{n-\tau^{\mathrm{Ret}},\tau^{\mathrm{Ret}}}}{(1 + r^f)^{n-\tau^{\mathrm{Ret}}}}\right).$$

A1.5.2 There are a few comments needed on these equations. Firstly, according to U.K. tax law, the interest component of an annuity is the remainder of the annuity after the capital content has been subtracted. The capital content is calculated as the purchase price of the annuity divided by the actuarial value of the annuity. The actuarial value of the annuity is the sum of all future payments discounted for mortality, but not for interest, that is $\chi(0)$. We calculate the annuity rates using our mortality figures given in Table A2.1. However, in reality, they are calculated using pre-prescribed tables. Secondly, the annuity rate is calculated according to actuarial principles, on the basis of pre-tax risk free rate of return r^f.

A1.6 *The Government*

A1.6.1 The Government must fund a fixed exogenous level of public consumption from its income tax and capital tax revenues. The level of the state pension is fixed to be a constant proportion P of the mean income level $p^s_t = P\bar{y}_t$, where $\bar{y}_t$ is the average level of earnings of those in work at time t. This state pension is paid for entirely from social insurance tax payments. The social insurance fund is kept in balance in every period, thus the social insurance tax rate t^{SI}_t is fixed so that:

$$\sum_{i \in V^{\mathrm{Work}}(t)} t_t^{SI} y_{i\tau} = \sum_{i \in V^{\mathrm{Ret}}(t)} p_t^s$$

where $V^{\mathrm{Work}}(t)$ and $V^{\mathrm{Ret}}(t)$ are the sets of working and retired households alive at time t.

APPENDIX 2

CALIBRATION OF THE MODEL

A2.1 *Purpose of Calibration*

A2.1.1 In order to speed up the computation of the partial equilibria, we assume that every period on the model was equal to five actual years. However, where applicable, we record all parameter values and results at their equivalent annual rate. The process of calibration was iterative. For given values of the parameters, we calculated the steady state distribution of wealth consumption and income across our households. These distributions were compared to those derived from data surveys (Family Expenditure Survey, 1997; Inland Revenue Statistics, 1998). However, we also required the parameter values to be well within the ranges of previously published estimated values.

A2.2 *The Utility Function and the Economic Environment*

A2.2.1 The two sources for the demographic constants were the U.K. life tables and the table of live births by age of mother; both are published by the Office of National Statistics in Annual Abstract of Statistics (1994). Households are assumed to live for a maximum 70 years, so as to model an adult life from 20 to 90 years old. All individuals retire at the age of 65, after 45 working years, and receive a state pension worth 30% of average earnings in 1991. The number of households remains constant at 30,000, as, on a death of a household, a descendent household is born immediately. This descendent household, not only inherits the remaining assets of its parent household, but also a proportion of its human capital characteristics. We described in the text the assumptions necessary to calculate the constants in Table A2.1.

Table A2.1. The values of the demographic constants used in the model

Household age at period end	5	10	15	20	25	30	35	40	45	50	55	60	65	70
Age of adults at period end	25	30	35	40	45	50	55	60	65	70	75	80	85	90
P(dying) during the five year period	0.000	0.000	0.000	0.000	0.000	0.001	0.002	0.006	0.016	0.040	0.087	0.173	0.313	1.000
Average number of adults	2.000	1.994	1.988	1.980	1.963	1.953	1.927	1.884	1.817	1.715	1.581	1.427	1.273	1.142
Average number of children	0.137	0.579	1.161	1.589	1.471	0.977	0.429	0.107	0.011	0.000	0.000	0.000	0.000	0.000
McClements family size	1.003	1.051	1.155	1.284	1.359	1.300	1.152	1.017	0.939	0.890	0.837	0.777	0.716	0.665

A2.2.2 The Before Housing Costs McClements equivalent scale is used by the U.K. Department of Social Security in all their household income statistics. The scale is normalised, so that a two-adult household has a value of one. To calculate the score, add the following scores: 0.61 for the first adult, 0.39 for the second, and, for each dependant aged 0-1 add 0.09, aged

2-4 add 0.18, aged 5-7 add 0.21, aged 8-10 add 0.23, aged 11-12 add 0.25, aged 13-15 add 0.27, and for each dependant over 16 add 0.36.

A2.2.3 The preference parameters, the discount factor and the coefficient of risk aversion (δ, γ), are calibrated from the recent studies of Attanasio & Weber (1993) of micro-data for the U.K. These agree closely with those in Auerbach & Kotlikoff (1987) and Prescott (1986). We have taken issue with Huggett (1996), who uses a discount factor greater than one, and we have, instead, adopted a discount rate of 0.95 p.a.

A2.3.4 Households make their consumption decision based on the economic environment in which they live. This environment is summarised in this model by the distribution of the risky return on equity, the risk free return rate, the level of the state pension, wage rate, income tax and capital tax rates. These values are given in Table A2.2.

Table A2.2. Parameter values for preferences (annual rates for γ and δ)

Parameter			Parameter		
Discount factor	δ	0.95	Risk free rate of return	r^f	0.03
Coefficient of risk aversion	γ	2.0	Wage rate	s_{wage}	0.76
Capital tax rate	t^K	0.0	Mean pre-tax income	$\bar{y}$	1.0
Income tax rate	t^L	0.2	Implied social insurance tax	t^{SI}	0.132
Mean of log of equity returns	μ^r	0.061	Mean equity return		0.071
S.E. of log of equity returns	σ^r	0.119	S.E. of equity returns		0.17
			Implied annuity rate	χ	0.071

A2.3.5 For reference, we have also given the value of two derived parameters. The first is the annuity rate implied by the risk free rate and mortality rates in Table A2.1, and the second is the social insurance tax rate, determined by the model, that is just sufficient to pay completely for the given level of the state pension.

A2.4 *The Income Process*

A2.4.1 We used data from the British Household Panel Survey (1991-1995) data on household gross labour income to calibrate the coefficients of our income process. These coefficients are detailed in the table A2.3. The mean estimates of earning power are given in Table A2.4, where they have been normalised, so that the mean household's earning power in the first period of working life is 1. We have also included in the table the variance of the distribution of each cohort's log of earning power, the Gini coefficient of the distribution of earning power, and the maximum allowable contribution rates to the non-liquid asset.

Table A2.3. Value of model parameters determining the income process

Parameter	ρ	σ^2	λ	$\tilde{\sigma}^2$
Value	0.993	0.013	0.6	0.203

Table A2.4. Descriptive statistics of the earnings distribution and maximum contribution rates by age

Household age at period end	5	10	15	20	25	30	35	40	45
Age of adults	25	30	35	40	45	50	55	60	65
Mean earning power, $\bar{h}_\tau$	1.000	1.225	1.386	1.482	1.515	1.486	1.394	1.241	1.028
Var(h_τ)	0.203	0.251	0.296	0.338	0.378	0.414	0.448	0.480	0.500
Gini(exp(h_τ))	0.250	0.277	0.299	0.319	0.335	0.349	0.361	0.371	0.379
Max. contribution rates, κ_τ	0.175	0.175	0.175	0.2	0.2	0.25	0.3	0.35	0.4

APPENDIX 3

RECENT HIGH PROFILE AREAS AFFECTED BY RATES OF RETURN

A3.1 *Introduction*

This appendix considers two recent high profile areas requiring assumptions for rates of return.

A3.2 *Mortgage Repayment Funding*

A3.2.1 The first area is that of mortgage repayment funding, an issue that affects millions of actual and potential individual borrowers, as well as insurance companies and lending institutions. At present, the two main repayment choices are regular repayments of capital and interest and, as an alternative, the funding of the loan repayment using a with-profits or unit-linked endowment assurance. To make a financial comparison between the two methods, it is necessary to use a future mortgage interest rate — essentially a future short-term interest rate plus an expense and profit margin — and a future rate of return on investments, principally U.K. equity and fixed interest.

A3.2.2 Looking specifically at future rates of return on investments, Appendix 4 sets these out on two equity risk premium bases, that of the 2% p.a. from Wilkie (1995a), and that of 4.5% p.a., an average taken from a number of recent publications. These lead to net returns on investments of 5.5% p.a. and 7.3% p.a. respectively, a substantial difference from the individual investor's point of view.

A3.3 *Fund Transfers and Mergers*

A3.3.1 Another example of the importance of the assumptions on future rates of return can be found in the area of fund transfers and mergers, where the benefits being transferred are entitled to part, or all, of a fund's surplus. Where, as is common, investment strategy is constrained by the available surplus, then it can be argued that it is necessary to review future earnings as well as market values in considering the value of that surplus, compared to other classes of business entitled to the same or different surpluses. Here again, the level of future rates of return can be critical, and, generally, best estimates rather than prudent levels are required, for example, because different generations of policyholders can look at what is prudent from different angles, and can come to conflicting conclusions. Where best estimates are required, then the estimation process needs to be, and seen to be, much more robust.

A3.3.2 In some cases there are two completely independent parties involved, in which case each party would use its own assumptions, and the balance between the parties would be the result of negotiations. Often, though, the parties are not entirely independent. There is, in such cases, the temptation to use assumptions that are prudent for just one of the parties involved. Moreover, lack of independence is not normally resolved by employing an independent actuary, as his or her brief is normally to comment on whether the deal is not disadvantageous to one or more of the parties rather than whether or not the benefits of the deal are balanced between the parties.

APPENDIX 4

SOME FINANCIAL STATISTICS

A4.1 This paragraph sets out yields on U.K. Government securities relating to 5 February and 16 June 2000 and 9 March 2001, the dates having been chosen at random.

	5.2.00	16.6.00	9.3.01
	% p.a.	% p.a.	% p.a.
FTSE Actuaries Government securities 15-year index	5.20	4.85	4.68
Index-linked gilt 2 ½% 2016 yield — see note (1)	2.14	2.04	2.16

Note (1): The index-linked yields set out are those for 5.2.00, 16.6.00 and 9.3.01, assuming inflation rates of 3.0%, 2.7% and 2.5% p.a. respectively, from which it can be seen that, using a current approach, the implied inflation rates are also 3.0%, 2.7% and 2.5% p.a. for the same dates.

A4.2 This paragraph considers the estimated future rate of return on equities, using a current approach, on two bases for the equity risk premium, basis (1) being from Wilkie (1995a) and basis (2) an average of a number of recent publications.

Basis	(1)	(2)
	% p.a.	% p.a.
Equity risk premium	2.00	4.75
Gross return on long-term gilts at time of writing	5.00	5.00
Gross rate of return on equities	7.00	9.75
Return on gilts net of 20% tax rate	4.00	4.00
Net of tax return on equities — see note (2)	5.84	8.17
Net rate of return on a 80% equity/20% gilt with profit/managed fund	5.47	7.34

Note (2): The net rate has been calculated using a tax rate of 20% on the FTSE All-Share dividend yield of 2.3% and 15% on the balance, which assumes equities are sold every seven years, discounting at the net gilt rate.

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