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Progress Update for the Working Party: ‘Uncertainty of the 99.5th Percentile and Other Tail Statistics’ (“99.5th & OTS WP”)

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Agenda

- Scope & Workstreams
- Preliminary Results: Parameter Uncertainty
- Plans for 2018/19

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Workstreams

- Workstream 0
  - Gather existing work so far into a single reference document
- Workstream 1
  - Investigate parameter and model uncertainty
  - Focus for 2017/18 has been parameter uncertainty
- Workstream 2
  - Create a survey to gauge practitioners thoughts around percentile and tail statistic estimation
- Workstream 3
  - Investigate industry data and build benchmarks if possible

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Workstream 1

Parameter Uncertainty

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Key assumptions, Caveat, Request

- Assumptions
  - No model error
  - No need to adjust data
    - Missing
    - Trends (inflation, exposure)
    - To ultimate
  - Copula forms and strengths are known
- **Caveat:** Results draft & subject to peer-review & replication
- **Request:** Any other questions you'd like us to investigate?

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Methodology: Proxy parameter-setting process to build up a distribution of estimates

For a single ‘line of business’:

1. Simulate true distribution (5k, stratified)
2. Sample *n* data points (5k)
3. Fit using MLE assuming true distribution known (i.e. no model error)
4. Simulate from the fitted distribution (5k, stratified, for each of the 5k fitted)
5. Calculate statistics of interest of the fitted distribution (i.e. 5k of simulated statistics)

For multiple lines of business:

- Do the same but for *m* lines. Assume all lines identical.
- Correlate assuming copula and copula strength is known
- Calculate statistics on the aggregate distribution

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Parameter Combinations

- Distributions: Gamma, Lognormal, Pareto
- CoVs: 5%, 20%, 40%, 100%, 200%
- Data pts: 5, 10, 15, 20, 25, 100
- LoB Counts: 1, 5, 10, 20, 50, 100 (for selected combinations)
- Copulas & strengths:
  - Gaussian: 0, 0.25, 0.5, 0.95
  - Gumbel: 1, 1.2, 1.53, 5.5

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Base Distribution: Lognormal, 20%CV, 10 data pts

Distribution	Mean	95th P	99.5th P
True	100.0	135.8	163.5
E[MLE]	100.0	133.1	158.6
Bias (%)	0.0%	-2.0%	-3.0%
SD[MLE]	6.3	12.8	20.9
CV[MLE]	6.3%	9.6%	13.2%
95% CI as %True	[-12%, 13%]	[-19%, 18%]	[-25%, 25%]

- MLE estimation of percentiles is biased for Lognormal (and under)
- Bias is small (but not insignificant) next to CV[MLE]
- Both bias and CV grow as percentile increases
- 95%CI means 95% of sims in [-x%,+y%] of True stat

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Vary sample size: Examine Bias (Lognormal, 20%CV)

Distribution	Mean		95th P		99.5th P	
True		100.0		135.8		163.5
Sample Size	E[MLE]	Bias %	E[MLE]	Bias %	E[MLE]	Bias %
5	100.0	0.0%	130.2	-4.1%	153.4	-6.2%
10	100.0	0.0%	133.1	-2.0%	158.6	-3.0%
20	100.0	0.0%	134.5	-1.0%	161.1	-1.5%
100	100.0	0.0%	135.6	-0.1%	163.1	-0.2%

- Bias decreases proportionately up to n=20
  - (n=100 seems to do better than this)

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Vary sample size: Examine CV[MLE] and 95% CI (Lognormal, 20%CV)

Distribution	Mean		95th P		99.5th P	
True		100.0		135.8		163.5
Sample Size	CV[MLE]	95% CI	CV[MLE]	95% CI	CV[MLE]	95% CI
5	8.8%	[-16%, 19%]	13.5%	[-26%, 24%]	18.5%	[-33%, 34%]
10	6.3%	[-12%, 13%]	9.6%	[-19%, 18%]	13.2%	[-25%, 25%]
20	4.5%	[-8%, 9%]	6.8%	[-13%, 13%]	9.2%	[-18%, 18%]
100	2.0%	[-4%, 4%]	3.0%	[-6%, 6%]	4.1%	[-8%, 8%]

- CV and Confidence Intervals decrease proportionately to $\sqrt{n}$
  - As predicted by MLE theory
- Adjust values accordingly in later slides if required
- Need about 65 data pts for +/-10% LN(20%CV)

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Vary CV: Examine CV[MLE] and 95% CI (Lognormal, 10 data pts)

Distribution	Mean		95th P		99.5th P	
True		100.0		135.8		163.5
CV	CV[MLE]	95% CI	CV[MLE]	95% CI	CV[MLE]	95% CI
5%	1.6%	[-3%, 3%]	2.4%	[-5%, 4%]	3.3%	[-7%, 6%]
20%	6.3%	[-12%, 13%]	9.6%	[-19%, 18%]	13.2%	[-25%, 25%]
40%	12.6%	[-22%, 26%]	18.9%	[-34%, 38%]	26.2%	[-43%, 55%]
100%	31.6%	[-46%, 75%]	43.1%	[-59%, 101%]	62.6%	[-70%, 157%]
200%	64.3%	[-66%, 174%]	72.0%	[-74%, 189%]	112.4%	[-84%, 327%]

- CV[MLE] seems to be approximately linear to CV of underlying distribution

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Vary Distribution: Examine CV[MLE] and 95% CI (CV 20%, 10 data pts)

Distribution	Mean		95th P		99.5th P	
True	100.0		G=135.0; LN=135.8; P=136.7		G=159.3; LN=163.5; P=200.0	
CV	CV[MLE]	95% CI	CV[MLE]	95% CI	CV[MLE]	95% CI
Gamma	6.4%	[-12%, 13%]	8.8%	[-18%, 16%]	11.3%	[-23%, 20%]
Lognormal	6.3%	[-12%, 13%]	9.6%	[-19%, 18%]	13.2%	[-25%, 25%]
Pareto	6.3%	[-9%, 15%]	15.7%	[-24%, 35%]	29.2%	[-39%, 68%]

- CV[MLE] seems to increase with tail severity
- Note 95P close for all distributions, 99.5P different for Pareto

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Increase 'Lines of Business': (Lognormal, 20% CV, 10 data pts, Gaussian Copula 0.25)

Distribution	Mean		95th P		99.5th P	
True Values by LoB Count	100 | 500 | 1,000 | 2,000 | 5,000 | 10,000		136 | 612 | 1,197 | 2,390 | 5,932 | 11,816		164 | 691 | 1,331 | 2,604 | 6,439 | 12,888	
#LoBs	CV[MLE]	95% CI	CV[MLE]	95% CI	CV[MLE]	95% CI
1	6.3%	[-12%, 13%]	9.6%	[-19%, 18%]	13.2%	[-25%, 25%]
5	2.8%	[-5%, 5%]	3.6%	[-8%, 6%]	4.6%	[-10%, 7%]
10	2.0%	[-4%, 4%]	2.5%	[-6%, 4%]	3.1%	[-7%, 5%]
20	1.4%	[-3%, 3%]	1.7%	[-5%, 2%]	2.2%	[-5%, 4%]
50	0.9%	[-2%, 2%]	1.1%	[-4%, 1%]	1.5%	[-3%, 2%]
100	0.6%	[-1%, 1%]	0.8%	[-3%, 0%]	1.1%	[-3%, 1%]

- Assumes Copula and strength is known
- Holding underlying CV constant: generally LoBs ↑, individual CVs ↑
- Gaussian copula tail-independence cause of 95th and 99.5th about the same for 50 and 100 LoB counts?
- Note sim error greater here than for 1 LoB
- CV[MLE] reduces like $\sqrt{m}$

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Increase 'Lines of Business': (Lognormal, 20% CV, 10 data pts, Gumbel Copula 1.2)

Distribution	Mean		95th P		99.5th P	
True Values by LoB Count	100 | 500 | 1,000 | 2,000 | 5,000 | 10,000		136 | 619 | 1,221 | 2,442 | 6,023 | 12,214		164 | 745 | 1,516 | 3,044 | 7,376 | 14,782	
#LoBs	CV[MLE]	95% CI	CV[MLE]	95% CI	CV[MLE]	95% CI
1	6.3%	[-12%, 13%]	9.6%	[-19%, 18%]	13.2%	[-25%, 25%]
5	2.8%	[-6%, 6%]	3.7%	[-8%, 6%]	5.2%	[-11%, 9%]
10	2.0%	[-4%, 4%]	2.6%	[-6%, 4%]	3.9%	[-10%, 4%]
20	1.4%	[-3%, 3%]	1.8%	[-5%, 2%]	2.8%	[-9%, 2%]
50	0.9%	[-2%, 2%]	1.2%	[-3%, 2%]	2.0%	[-5%, 3%]
100	0.6%	[-1%, 1%]	0.9%	[-4%, 0%]	1.6%	[-5%, 2%]

- Gumbel 1.2 gives about 0.25 Rank Correlation
- Assumes Copula and strength is known
- Note True 95th percentiles reasonably similar but up to 15% increase in 99.5th capital from changing copula for these params
- Greater CV[MLE] and range with Gumbel

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Summary of preliminary results

- Reiterating the following assumptions:
  - Underlying distributions and copulas (and copula strengths) are known
  - No adjustments needed to data (inflation, to ultimate etc)
  - MLE used to estimate params
  - 95% CI is reasonable
  - Uncertainty is CV of the implied MLE dists

- **Results**
  - Uncertainty (of estimates) decreases per $\sqrt{n}$, number of data points
  - Uncertainty increases linearly in the CV of underlying distribution
  - For distributions 'like' LN(20%CV) need about 65 data points before 95% of 99.5P estimates +/-10% of True 99.5P
  - Uncertainty reduces like $\sqrt{m}$ for LoBs*
  - Uncertainty (per CV) of 95P not massively less than 99.5P
- Out of interest: Two examples of model error being more significant at 99.5P vs 95P:
  - Gamma, LN and Pareto (20%CV) all close at 95P
  - Normal (0.25) and Gumbel (1.2) close at 95P

2018/9 work

- Workstream 1
  - Replicate using open-source (any R volunteers?)
  - Compound distributions
  - Copula strength and form estimation
  - Model error
    - How often does a different distribution give a better fit to sample data vs the true distribution?
    - How does this vary if data points (e.g. extremes) are removed?
- Workstream 2
  - Work towards creating a survey

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Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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