Institute
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3D Diagnostics Towards Better Reserving

Chris Smerald

24 October 2018

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Better Sensing and Responding to Change:

The Trio in Your Triangle -Understanding Age, Period and Cohort Effects.

24 October 2018

Agenda

- Digression du GAR
- APC: The ~~Revolution in Your Rectangle~~ Trio in Your Triangle:
  - Age
  - Period
  - Cohort
- AP, AC, or PC
- Data Polishing and Modelling Anecdotes

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Digression: Good Actuarial Report Complexity Heuristic

Role	Users, Knowledge, Experience	Uses + Scope	Expectations
Decision Making	Reserving actuaries, lots of logic, statistics, and ”know pain”	**Make better model and parameter selections. This reserving cycle**	**Practical insights you can apply Satisfy your curiosity**
Risk Management	Capital actuaries ditto	**Risk Modelling insights**	**Improved knowledge of risk drivers for later application**
Governance	Assurance actuaries ditto	**Tools for challenge and review**	**Practical things you can use to help sense check**
Work	**Inputs**	**Modelling**	**Communication and Documentation**
Pint of View on Delivery	Something real	Easy to understand so you could possibly apply it	**Want it to be clear and simple. Please, no big formulas and proofs**
Complicating Factors	Israel data context new to audience. Not used to using economic series or critical thinking in three dimensions	Concepts new. Hard to get your head around DY; AY to some degree. Process complex	Many user needs and expectations. Time limitation. Lots of background
Simplifying Factors	It’s only a triangle	Audience familiar with the data transformations used	It’s a 45 min slot. There is only so much that they will expect
Simplify	Avoid too much Israel context	Exhibits not formulas Reduce # covariates; (# AY’s?)	Storytelling; same thing from multiple perspectives.
Assurance	Get their feedback	Do logic and formula checks	Do a test run with someone in advance

Enter the Trio in Your Triangle: Age, Period and Cohort

Table 1: Suicide rates among men in the USA, 1910-2004

Age	Time period 1910-1914	1915-1919	1920-1924	1925-1929	1930-1934	1935-1939	1940-1944	1945-1949	1950-1954	1955-1959	1960-1964	1965-1969	1970-1974	1975-1979	1980-1984	1985-1989	1990-1994	1995-1999	2000-2004	Cohort
10–14	0.61	0.63	0.53	0.55	0.54	0.67	0.64	0.79	0.59	0.69	0.86	0.92	1.03	1.28	1.53	2.22	2.34	2.25	1.89	1990-1994
15–19	6.33	4.94	3.57	4.07	4.92	4.83	3.90	3.81	3.77	4.19	5.77	7.22	10.29	12.87	13.94	16.89	17.94	14.98	12.46	1985-1989
20–24	21.79	15.30	10.98	11.79	13.33	13.02	10.72	10.09	9.55	9.97	12.06	14.94	21.62	27.49	25.54	26.21	26.39	23.57	20.70	1980-1984
25–29	27.91	21.10	14.50	16.02	18.49	17.77	14.35	12.26	12.22	12.16	14.39	16.86	21.56	26.88	26.25	25.36	25.09	22.79	20.29	1975-1979
30–34	32.38	26.98	19.03	18.93	22.78	22.04	17.69	15.19	13.61	14.28	17.03	18.09	20.55	23.39	23.93	24.24	24.65	22.61	20.52	1970-1974
35–39	36.72	28.78	24.75	25.86	29.44	26.91	21.44	19.80	17.27	16.79	19.71	20.69	21.39	22.69	22.71	23.41	24.14	23.56	22.14	1965-1969
40–44	42.96	33.50	27.44	30.97	36.66	32.62	24.22	24.08	21.98	22.49	23.26	23.53	23.06	22.81	22.31	21.97	23.28	23.88	24.16	1960-1964
45–49	50.28	37.32	33.38	38.29	48.22	40.27	28.78	27.52	27.49	27.05	28.00	25.77	26.64	24.63	22.69	22.55	22.61	23.01	24.44	1955-1959
50–54	59.49	41.79	39.22	46.18	58.44	48.27	35.06	33.27	32.32	33.30	33.90	30.18	27.84	25.72	24.13	23.80	22.88	21.52	23.29	1950-1954
55–59	67.68	53.98	42.47	49.09	66.72	53.24	42.78	39.15	35.75	37.57	37.91	34.31	31.61	27.40	25.97	25.65	23.68	21.87	22.40	1945-1949
60–64	67.93	52.21	49.89	57.34	74.63	57.42	45.46	43.80	42.13	40.38	36.67	35.58	31.31	29.50	25.48	26.26	24.77	20.84	20.37	1940-1944
65–69	64.48	59.30	51.52	64.44	76.92	59.21	47.83	43.32	44.21	41.17	36.41	34.23	33.56	31.26	27.87	29.97	26.74	23.86	21.15	1935-1939
70–74	64.30	57.64	55.65	65.68	82.67	58.22	52.77	48.20	48.41	44.92	40.72	36.84	36.92	36.44	35.06	39.18	34.03	29.77	26.40	1930-1934
75–79							57.54	56.13	50.15	51.12	48.09	42.56	41.16	44.53	42.92	50.78	46.05	37.83	33.36	1925-1929
80–84							61.17	58.68	59.30	58.64	55.67	47.69	48.12	45.77	49.91	63.00	59.18	49.22	42.18	1920-1924
	1830-183	1835-183	1840-184	1845-184	1850-185	1855-185	1860-186	1865-186	1870-187	1875-187	1880-188	1885-188	1890-189	1895-189	1900-190	1905-190	1910-191	1915-191		Cohort

Enter the Trio in Your Triangle: Age, Period and Cohort

CS1

Suicide Rates as a Triangle

Birth \ Age year \	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
1900-1904	0.61	4.94	10.98	16.02	22.78	26.91	24.22	27.52	32.32	37.57	36.67	34.23	36.92	44.53	49.91
1905-1909	0.63	3.57	11.79	18.49	22.04	21.44	24.08	27.49	33.30	37.91	35.58	33.56	36.44	42.92	63.00
1910-1914	0.53	4.07	13.33	17.77	17.69	19.80	21.98	27.05	33.90	34.31	31.31	31.26	35.06	50.78	59.18
1915-1919	0.55	4.92	13.02	14.35	15.19	17.27	22.49	28.00	30.18	31.61	29.50	27.87	39.18	46.05	49.22
1920-1924	0.54	4.83	10.72	12.26	13.61	16.79	23.26	25.77	27.84	27.40	25.48	29.97	34.03	37.83	42.18
1925-1929	0.67	3.90	10.09	12.22	14.28	19.71	23.53	26.64	25.72	25.97	26.26	26.74	29.77	33.36	55.67
1930-1934	0.64	3.81	9.55	12.16	17.03	20.69	23.06	24.63	24.13	25.65	24.77	23.86	26.40	46.05	
1935-1939	0.79	3.77	9.97	14.39	18.09	21.39	22.81	22.69	23.80	23.68	20.84	21.50	44.92		
1940-1944	0.59	4.19	12.06	16.86	20.55	22.69	22.31	22.55	22.88	21.87	20.37	41.17			
1945-1949	0.69	5.77	14.94	21.56	23.39	22.71	21.97	22.61	21.52	22.40	40.38				
1950-1954	0.86	7.22	21.62	26.88	23.93	23.41	23.28	23.01	23.29	37.57					
1955-1959	0.92	10.29	27.49	26.25	24.24	24.14	23.88	24.44	33.27						
1960-1964	1.03	12.87	25.54	25.36	24.65	23.56	24.16	27.49							
1965-1969	1.28	13.94	26.21	25.09	22.61	22.14	23.88								
1970-1974	1.53	16.89	26.39	22.79	20.52	22.71									
1975-1979	2.22	17.94	23.57	20.29	20.55										
1980-1984	2.34	14.98	20.70	18.49											
1985-1989	2.25	12.46	14.94												
1990-1994	1.89	5.77													
1995-1999	0.79														

Warning: Social Sciences may use more data than us!

Slide 6

CS1 Christopher Smerald, 18/10/2018

A Social Sciences Modelling View

$$E[\boldsymbol{Suicides}_{Birth=i,Age=j}]$$

$$= \#\boldsymbol{Births}_{Y=i}$$

$$* \text{ Base Year Suicide Rate}$$

$$* \prod_{0}^{j} \boldsymbol{Age\ Effect}_{t}$$

$$* \prod_{i}^{i+j} \boldsymbol{Period\ (semi-decade) Effect}_{t}$$

$$* \prod_{0}^{i} \boldsymbol{Cohort\ (Formative)\ Effect}_{t}$$

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An Actuarial Science View on Modelling

$$E[\boldsymbol{Incremental\ Loss}_{AY=i,DY=j}]$$

$$= \boldsymbol{Exposure}_{AY=i}$$

$$* \text{ Pure Loss Rate} \rightarrow \text{Year 0}$$

$$* \prod_0^j \boldsymbol{Development\ factor}_t$$

$$* \prod_i^{i+j} \boldsymbol{CY}\ effect_t$$

$$* \prod_0^i \boldsymbol{AY}\ effect_t$$

Earned Premium or Etc.

Base Loss Rate

Loss Dev.

CY Trend

AY "Level" Changes

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Age Period Cohort: The Development Distribution

Suicide Rates as a Triangle

Birth \ Age year \	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
1900-1904	0.61	4.94	10.98	16.02	22.78	26.91	24.22	27.52	32.32	37.57	36.67	34.23	36.92	44.53	49.91
1905-1909	0.63	3.57	11.79	18.49	22.04	21.44	24.08	27.49	33.30	37.91	35.58	33.56	36.44	42.92	63.00
1910-1914	0.53	4.07	13.33	17.77	17.69	19.80	21.98	27.05	33.90	34.31	31.31	31.26	35.06	50.78	59.18
1915-1919	0.55	4.92	13.02	14.35	15.19	17.27	22.49	28.00	30.18	31.61	29.50	27.87	39.18	46.05	49.22
1920-1924	0.54	4.83	10.72	12.26	13.61	16.79	23.26	25.77	27.84	27.40	25.48	29.97	34.03	37.83	42.18
1925-1929	0.67	3.90	10.09	12.22	14.28	19.71	23.53	26.64	25.72	25.97	26.26	26.74	29.77	33.36	55.67
1930-1934	0.64	3.81	9.55	12.16	17.03	20.69	23.06	24.63	24.13	25.65	24.77	23.86	26.40	46.05	
1935-1939	0.79	3.77	9.97	14.39	18.09	21.39	22.81	22.69	23.80	23.68	20.84	21.50	44.92		
1940-1944	0.59	4.19	12.06	16.86	20.55	22.69	22.31	22.55	22.88	21.87	20.37	41.17			
1945-1949	0.69	5.77	14.94	21.56	23.39	22.71	21.97	22.61	21.52	22.40	40.38				
1950-1954	0.86	7.22	21.62	26.88	23.93	23.41	23.28	23.01	23.29	37.57					
1955-1959	0.92	10.29	27.49	26.25	24.24	24.14	23.88	24.44	33.27						
1960-1964	1.03	12.87	25.54	25.36	24.65	23.56	24.16	27.49							
1965-1969	1.28	13.94	26.21	25.09	22.61	22.14	23.88								
1970-1974	1.53	16.89	26.39	22.79	20.52	22.71									
1975-1979	2.22	17.94	23.57	20.29	20.55										
1980-1984	2.34	14.98	20.70	18.49											
1985-1989	2.25	12.46	14.94												
1990-1994	1.89	5.77													
1995-1999	0.79														

Loss Development Selections: Mixing Age and Period

% Rep	AY	12:24	24:36	36:48	48:60	60:72	72:84	84:96	96:108	108:120	120:132	132:144	144:156	156:168	168:180	180:192	192:204	204:216	216:228	228:240	240:252	252:264	264:276	276:288
99.98%	1992	3.061	1.544	1.285	1.208	1.121	1.091	1.061	1.030	1.022	1.015	1.007	1.009	1.004	1.008	1.002	1.004	1.002	1.000	1.001	1.000	1.000	1.000	1.000
99.96%	1993	3.234	1.581	1.323	1.191	1.142	1.083	1.067	1.034	1.028	1.014	1.014	1.006	1.006	1.005	1.002	1.005	1.001	1.000	1.002	1.000	1.001	1.002	
99.92%	1994	3.241	1.497	1.323	1.204	1.125	1.091	1.074	1.040	1.017	1.014	1.014	1.009	1.013	1.003	1.003	1.003	1.000	1.001	1.003	1.000	1.000		
99.86%	1995	2.945	1.474	1.329	1.210	1.141	1.083	1.088	1.027	1.023	1.020	1.010	1.007	1.007	1.004	1.001	1.002	1.000	1.001	1.000	1.001			
99.77%	1996	2.902	1.532	1.349	1.205	1.115	1.105	1.080	1.034	1.022	1.017	1.018	1.008	1.008	1.002	1.003	1.002	1.002	1.002	1.003				
99.59%	1997	3.084	1.556	1.304	1.182	1.123	1.116	1.075	1.034	1.029	1.020	1.022	1.017	1.005	1.005	1.001	1.002	1.000	1.001					
99.47%	1998	3.215	1.530	1.258	1.191	1.154	1.099	1.065	1.043	1.034	1.023	1.010	1.019	1.008	1.005	1.002	1.001	1.001						
99.40%	1999	2.975	1.477	1.285	1.209	1.132	1.092	1.060	1.035	1.023	1.022	1.010	1.006	1.007	1.002	1.001	1.002							
99.25%	2000	3.154	1.465	1.289	1.176	1.123	1.107	1.074	1.038	1.030	1.014	1.013	1.005	1.006	1.006	1.002								
99.07%	2001	3.214	1.502	1.300	1.188	1.145	1.118	1.063	1.058	1.028	1.015	1.014	1.009	1.004	1.002									
98.66%	2002	3.301	1.536	1.296	1.226	1.145	1.103	1.076	1.035	1.031	1.021	1.016	1.008	1.009										
97.98%	2003	4.768	1.569	1.375	1.206	1.140	1.103	1.079	1.050	1.044	1.030	1.025	1.010											
97.28%	2004	3.347	1.707	1.317	1.180	1.153	1.078	1.069	1.035	1.034	1.018	1.012												
95.81%	2005	3.918	1.609	1.324	1.216	1.145	1.101	1.082	1.041	1.031	1.015													
94.05%	2006	3.907	1.585	1.282	1.237	1.149	1.105	1.071	1.038	1.018														
91.24%	2007	3.506	1.538	1.330	1.224	1.142	1.141	1.069	1.044															
87.66%	2008	3.516	1.583	1.340	1.235	1.173	1.119	1.085																
81.54%	2009	3.431	1.670	1.370	1.247	1.158	1.148																	
72.67%	2010	5.659	2.005	1.471	1.282	1.207																		
62.44%	2011	5.793	2.083	1.444	1.294																			
49.81%	2012	6.249	2.076	1.449																				
35.37%	2013	6.044	2.214																					
17.92%	2014	7.100																						
2.93%	2015																							
5	**Yr Wtd**	6.121	1.973	1.408	1.253	1.164	1.122	1.075	1.041	1.031	1.019	1.015	1.007	1.007	1.004	1.002	1.002	1.001	1.001	1.002	1.000	1.000	1.001	1.000

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Age Development

AY	12:24	24:36	36:48	48:60	60:72	72:84
1992	3.061	1.544	1.285	1.208	1.121	1.091
1993	3.234	1.581	1.323	1.191	1.142	1.083
1994	3.241	1.497	1.323	1.204	1.125	1.091
1995	2.945	1.474	1.329	1.210	1.141	1.083
1996	2.902	1.532	1.349	1.205	1.115	1.105
1997	3.084	1.556	1.304	1.182	1.123	1.116
1998	3.215	1.530	1.258	1.191	1.154	1.099
1999	2.975	1.477	1.285	1.209	1.132	1.092
2000	3.154	1.465	1.289	1.176	1.123	1.107
2001	3.214	1.502	1.300	1.188	1.145	1.118
2002	3.301	1.536	1.296	1.226	1.145	1.103
2003	4.768	1.569	1.375	1.206	1.140	1.103
2004	3.347	1.707	1.317	1.180	1.153	1.078
2005	3.918	1.609	1.324	1.216	1.145	1.101
2006	3.907	1.585	1.282	1.237	1.149	1.105
2007	3.506	1.538	1.330	1.224	1.142	1.141
2008	3.516	1.583	1.340	1.235	1.173	1.119
2009	3.431	1.670	1.370	1.247	1.158	1.148
2010	5.659	2.005	1.471	1.282	1.207	1.167
2011	5.793	2.083	1.444	1.294	1.232	
2012	6.249	2.076	1.449	1.247		
2013	6.044	2.214	1.511			
2014	7.100	2.276				
2015	6.870					
2015	45,471	312,386				
2016	49,843					

Gross / Pvt Owned Vehicles < 3.5 Tons / Israel

Paid: LDF's

Expos	AY	0 - 12*	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108
1,508,931	1992	7.234	3.061	1.544	1.285	1.208	1.121	1.091	1.061	1.030
1,740,390	1993	5.801	3.234	1.581	1.323	1.191	1.142	1.083	1.067	1.034
2,016,977	1994	5.378	3.241	1.497	1.323	1.204	1.125	1.091	1.074	1.040
2,114,667	1995	5.367	2.945	1.474	1.329	1.210	1.141	1.083	1.088	1.027
2,356,514	1996	4.622	2.902	1.532	1.349	1.205	1.115	1.105	1.080	1.034
2,761,648	1997	4.036	3.084	1.556	1.304	1.182	1.123	1.116	1.075	1.034
2,971,558	1998	3.773	3.215	1.530	1.258	1.191	1.154	1.099	1.065	1.043
2,799,135	1999	4.182	2.975	1.477	1.285	1.209	1.132	1.092	1.060	1.035
2,874,732	2000	3.893	3.154	1.465	1.289	1.176	1.123	1.107	1.074	1.038
3,109,641	2001	3.480	3.214	1.502	1.300	1.188	1.145	1.118	1.063	1.058
3,420,265	2002	3.088	3.301	1.536	1.296	1.226	1.145	1.103	1.076	1.035
1,898,513	2003	1.898	4.768	1.569	1.375	1.206	1.140	1.103	1.078	1.050
3,057,578	2004	2.903	3.347	1.707	1.317	1.180	1.153	1.078	1.069	1.035
3,116,301	2005	2.441	3.918	1.609	1.324	1.216	1.145	1.101	1.082	1.041
3,244,092	2006	2.314	3.907	1.585	1.282	1.237	1.149	1.105	1.071	1.038
3,377,090	2007	2.454	3.506	1.538	1.330	1.224	1.142	1.141	1.069	1.044
3,648,870	2008	1.988	3.516	1.583	1.340	1.235	1.173	1.119	1.085	1.034
3,842,099	2009	1.793	3.431	1.670	1.370	1.247	1.158	1.148	1.086	
4,089,308	2010	0.664	5.659	2.005	1.471	1.282	1.207	1.167		
4,443,322	2011	0.588	5.793	2.083	1.444	1.294	1.232			
4,711,514	2012	0.511	6.249	2.076	1.449	1.247				
4,913,443	2013	0.488	6.044	2.214	1.511					
5,109,890	2014	0.398	7.100	2.276						
5,319,006	2015	0.427	6.870							
5,645,939	2016	0.441								

Down the Line: Looking for Small Change

Paid: Incremental LDF (Current Increment / Cumulative Prior)*

	1: Expos	2: Expos	Sum	2: 1	3: 1	Sum	3: 2	4: 2	Sum	4: 3	5:3	Sum	5: 4	6:4	Sum
1992	5,002.0	10,309.3	15,311.3	2.06	1.67	3.73	0.54	0.44	0.98	0.28	0.27	0.55	0.21	0.15	0.36
1993	3,850.4	8,601.7	12,452.1	2.23	1.88	4.11	0.58	0.51	1.09	0.32	0.25	0.58	0.19	0.17	0.36
1994	3,266.9	7,320.6	10,587.5	2.24	1.61	3.85	0.50	0.48	0.98	0.32	0.27	0.59	0.20	0.15	0.35
1995	3,081.8	5,992.7	9,074.4	1.94	1.40	3.34	0.47	0.49	0.96	0.33	0.28	0.61	0.21	0.17	0.38
1996	2,481.8	4,719.2	7,201.0	1.90	1.54	3.45	0.53	0.54	1.07	0.35	0.28	0.63	0.20	0.14	0.34
1997	2,177.6	4,538.4	6,716.0	2.08	1.71	3.80	0.56	0.47	1.03	0.30	0.24	0.54	0.18	0.15	0.33
1998	1,989.6	4,407.3	6,396.9	2.22	1.70	3.92	0.53	0.39	0.93	0.26	0.24	0.50	0.19	0.18	0.37
1999	1,934.7	3,820.6	5,755.3	1.97	1.42	3.39	0.48	0.42	0.90	0.28	0.27	0.55	0.21	0.16	0.37
2000	1,730.0	3,726.9	5,456.9	2.15	1.47	3.62	0.46	0.42	0.89	0.29	0.23	0.52	0.18	0.14	0.32
2001	1,499.6	3,319.8	4,819.4	2.21	1.61	3.83	0.50	0.45	0.95	0.30	0.24	0.54	0.19	0.17	0.36
2002	1,288.5	2,964.8	4,253.3	2.30	1.77	4.07	0.54	0.46	0.99	0.30	0.29	0.59	0.23	0.18	0.40
2003	854.8	3,220.7	4,075.5	3.77	2.71	6.48	0.57	0.59	1.16	0.38	0.28	0.66	0.21	0.17	0.37
2004	1,270.8	2,982.8	4,253.7	2.35	2.37	4.71	0.71	0.54	1.25	0.32	0.24	0.55	0.18	0.18	0.36
2005	1,122.3	3,274.6	4,396.9	2.92	2.39	5.31	0.61	0.52	1.13	0.32	0.29	0.61	0.22	0.18	0.39
2006	1,115.7	3,243.6	4,359.3	2.91	2.29	5.19	0.59	0.45	1.03	0.28	0.30	0.59	0.24	0.18	0.42
2007	1,231.9	3,086.7	4,318.6	2.51	1.89	4.39	0.54	0.51	1.05	0.33	0.30	0.63	0.22	0.17	0.40
2008	1,083.1	2,725.0	3,808.1	2.52	2.05	4.56	0.58	0.54	1.12	0.34	0.31	0.65	0.23	0.21	0.45
2009	1,000.3	2,431.8	3,432.1	2.43	2.30	4.73	0.67	0.62	1.29	0.37	0.34	0.71	0.25	0.20	0.44
2010	542.0	2,524.8	3,066.8	4.66	5.69	10.34	1.00	0.94	1.95	0.47	0.42	0.89	0.28	0.27	0.55
2011	528.5	2,533.1	3,061.6	4.79	6.27	11.06	1.08	0.92	2.01	0.44	0.42	0.87	0.29	0.30	0.59
2012	503.9	2,645.0	3,148.9	5.25	6.72	11.97	1.08	0.93	2.01	0.45	0.36	0.81	0.25		
2013	513.8	2,591.7	3,105.4	5.04	7.34	12.38	1.21	1.13	2.35	0.51					
2014	474.9	2,897.0	3,371.9	6.10	9.06	15.16	1.28								
2015	547.9	3,216.3	3,764.2	5.87											
2016	634.2														

* e.g., 3: 2 means Year 3 Incremental Paid / Cumulative Through Year 2"

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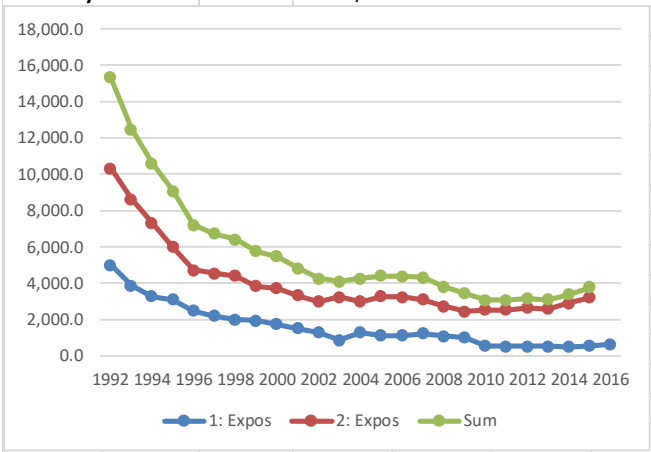
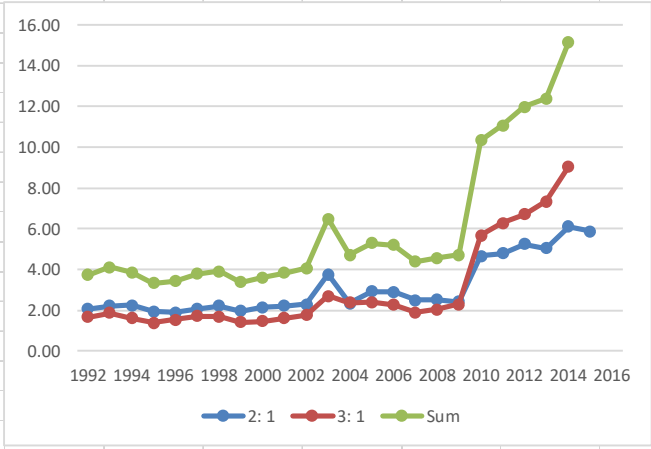
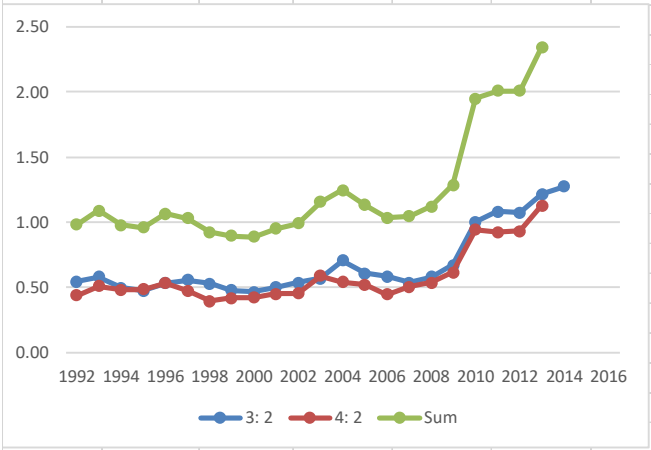
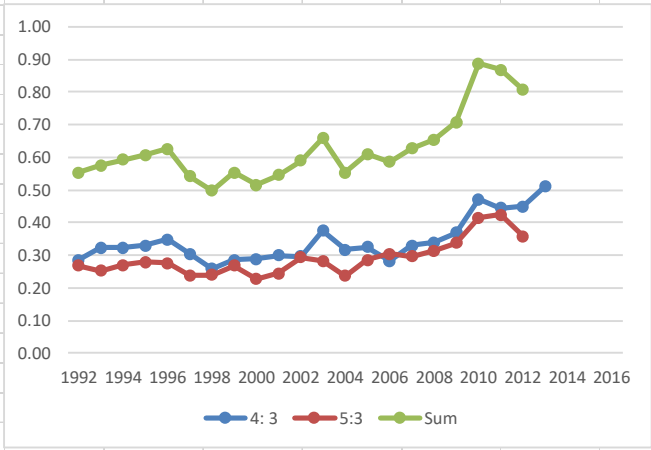
Down the Line (LDF – 1): -Early Swap, -Tail Lift

Year on year Offset: Gross / Pvt Owned Vehicles < 3.5 Tons / Israel

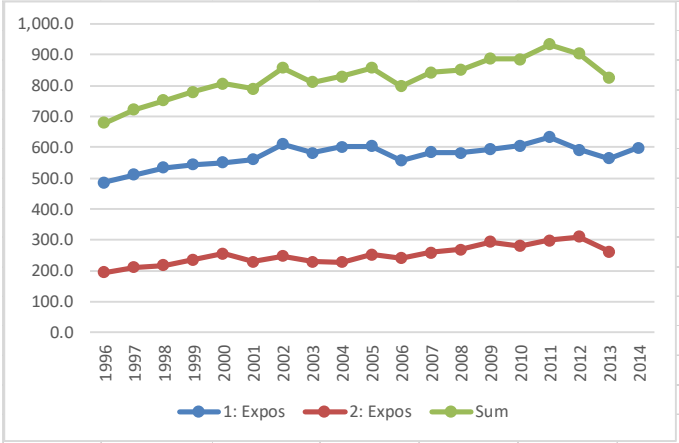


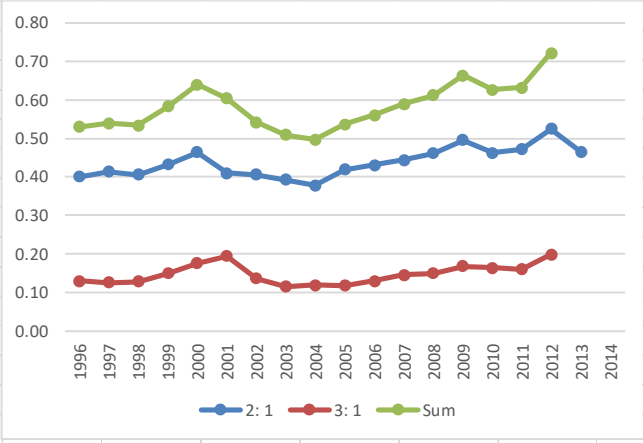


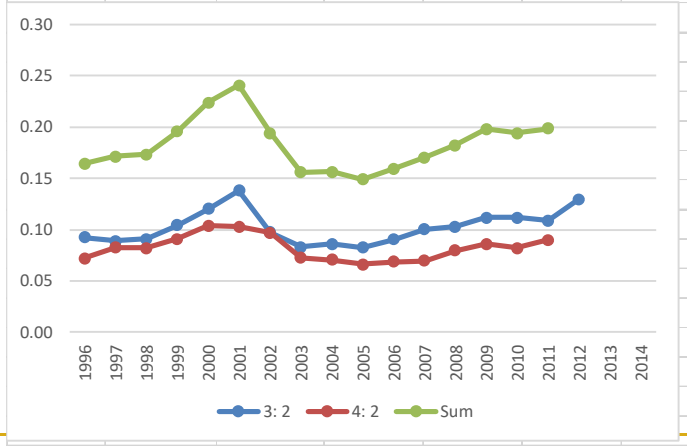


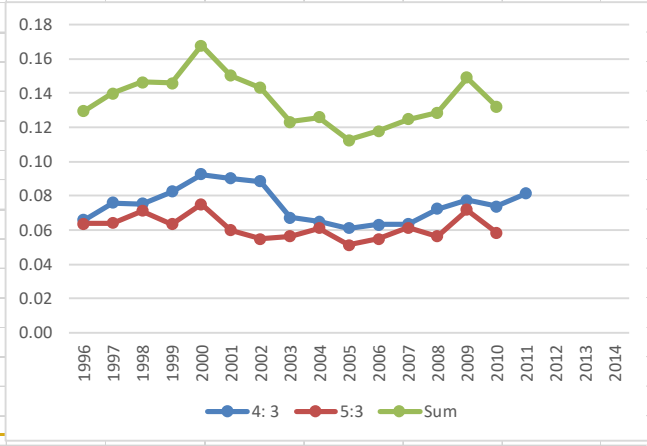


UK Motor Subset: Some Lift







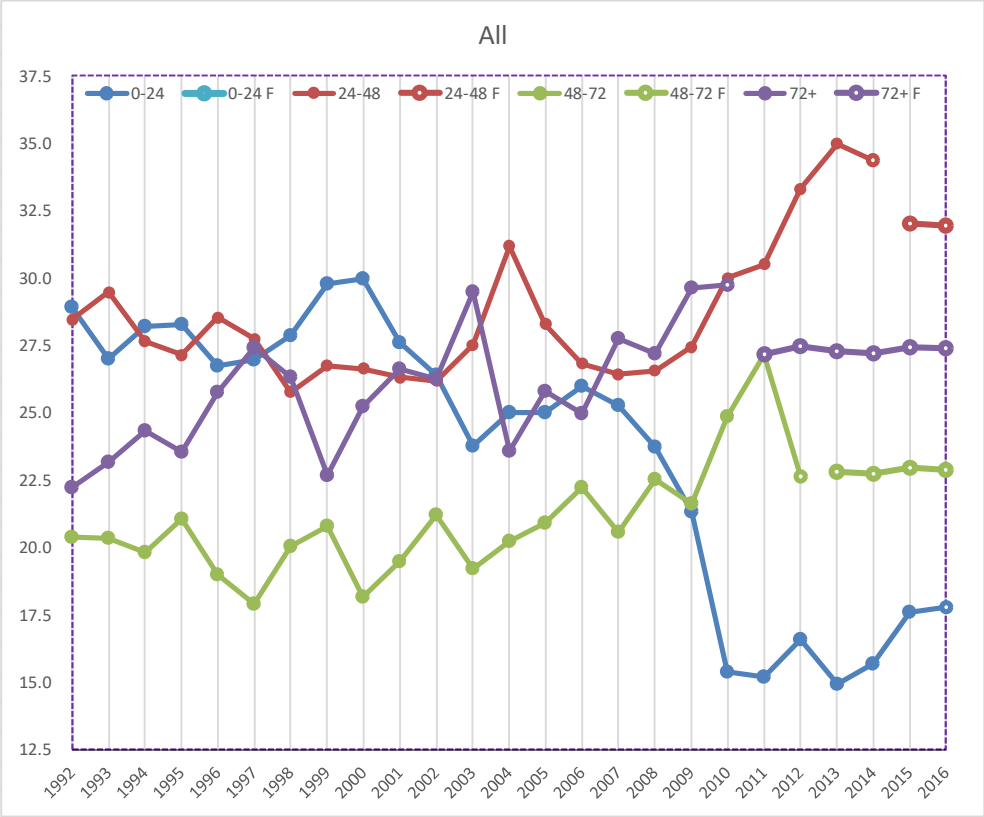


Biased Measure: Percent of Ultimate –Israel Data

Gross / Pvt Owned Vehicles < 3.5 Tons / Israel

Paid Loss: Incremental Hybrid Ult

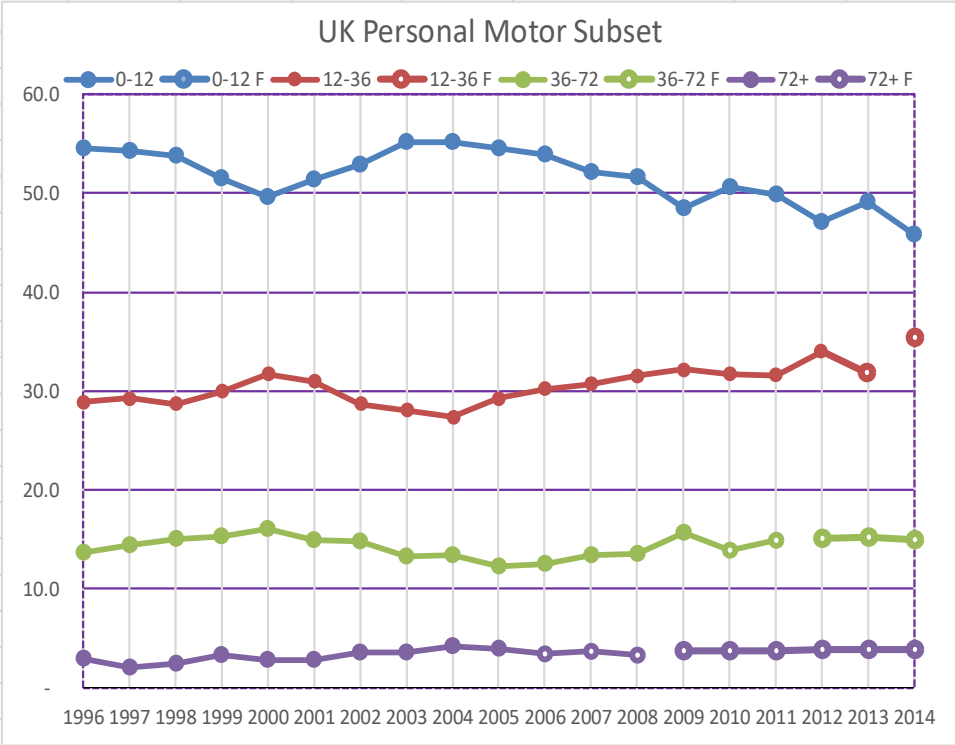
AY	0-24	0-24 F	24-48	24-48 F	48-72	48-72 F	72+	72+ F
1992	28.9		28.5		20.4		22.2	
1993	27.0		29.5		20.3		23.2	
1994	28.2		27.7		19.8		24.3	
1995	28.3		27.1		21.1		23.5	
1996	26.7		28.5		19.0		25.7	
1997	26.9		27.7		17.9		27.4	
1998	27.9		25.8		20.0		26.3	
1999	29.8		26.7		20.8		22.7	
2000	30.0		26.6		18.2		25.2	
2001	27.6		26.3		19.5		26.6	
2002	26.4		26.2		21.2		26.2	
2003	23.8		27.5		19.2		29.5	
2004	25.0		31.2		20.2		23.6	
2005	25.0		28.3		20.9		25.8	
2006	26.0		26.8		22.2		25.0	
2007	25.3		26.4		20.6		27.7	
2008	23.7		26.6		22.5		27.2	
2009	21.3		27.4		21.6		29.6	
2010	15.4		30.0		24.9		29.7	
2011	15.2		30.5		27.1			27.2
2012	16.6		33.3		22.6			27.5
2013	14.9		35.0			22.8		27.3
2014	15.7		34.4			22.7		27.2
2015	17.6			32.0		22.9		27.4
2016	17.8			31.9		22.9		27.4



Biased Measure: % of Ult. –UK Personal Motor Subset

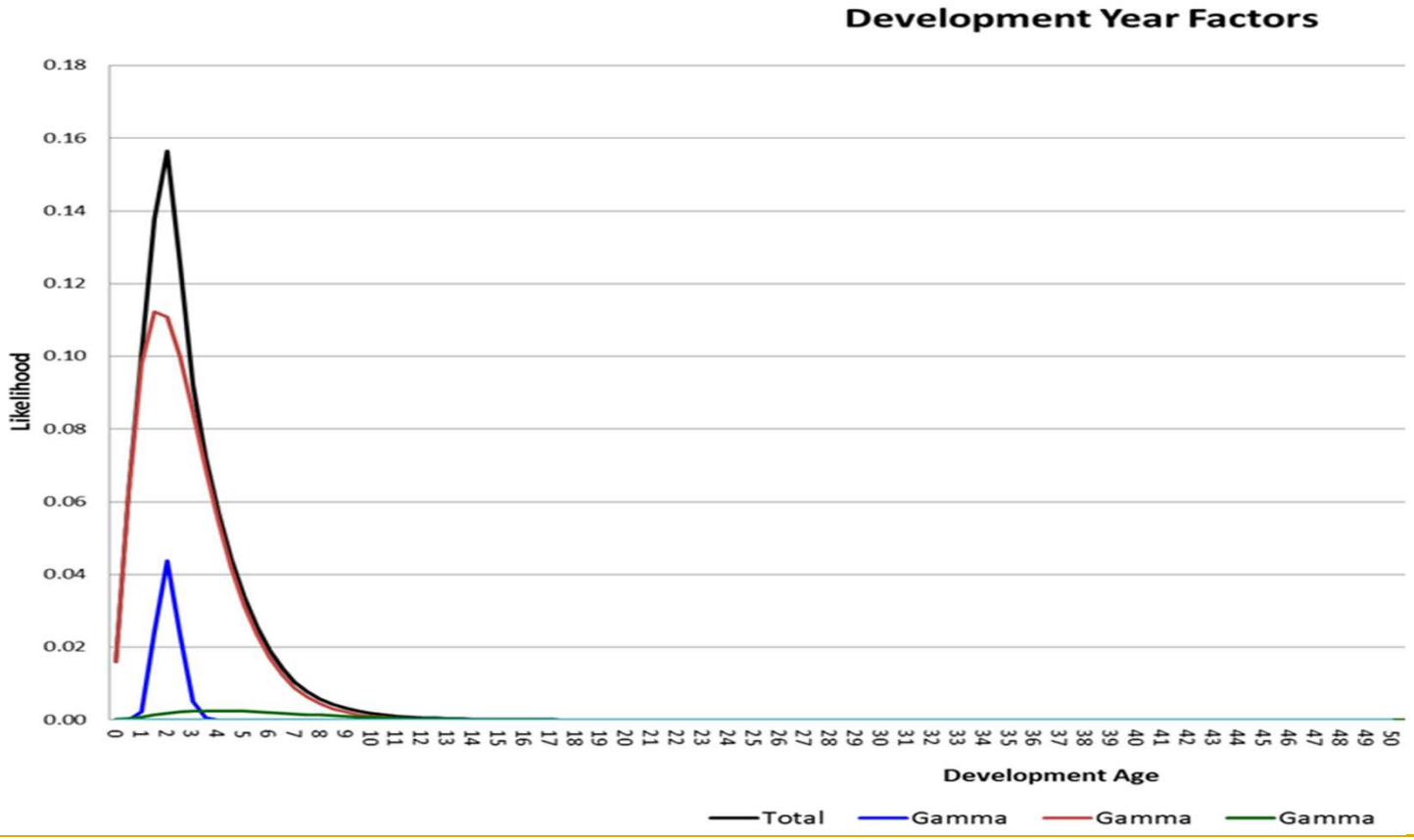
Paid Loss: Incremental Hybrid Ult

AY	0-12	0-12 F	12-36	12-36 F	36-72	36-72 F	72+	72+ F
1996	54.6		28.9		13.7		2.9	
1997	54.3		29.2		14.4		2.0	
1998	53.8		28.7		15.1		2.4	
1999	51.4		30.0		15.2		3.4	
2000	49.6		31.7		16.0		2.8	
2001	51.3		31.0		14.9		2.8	
2002	52.9		28.7		14.8		3.6	
2003	55.2		28.1		13.2		3.5	
2004	55.1		27.4		13.3		4.1	
2005	54.5		29.3		12.3		3.9	
2006	53.9		30.2		12.5		3.4	
2007	52.2		30.7		13.4		3.6	
2008	51.6		31.6		13.6		3.2	
2009	48.5		32.1		15.7			3.7
2010	50.6		31.7		13.9			3.7
2011	49.9		31.5		14.9			3.7
2012	47.1		34.0			15.1		3.9
2013	49.1		31.8			15.3		3.9
2014	45.8			35.4		14.9		3.8



and Faculty of Actuaries

Getting at the Distribution



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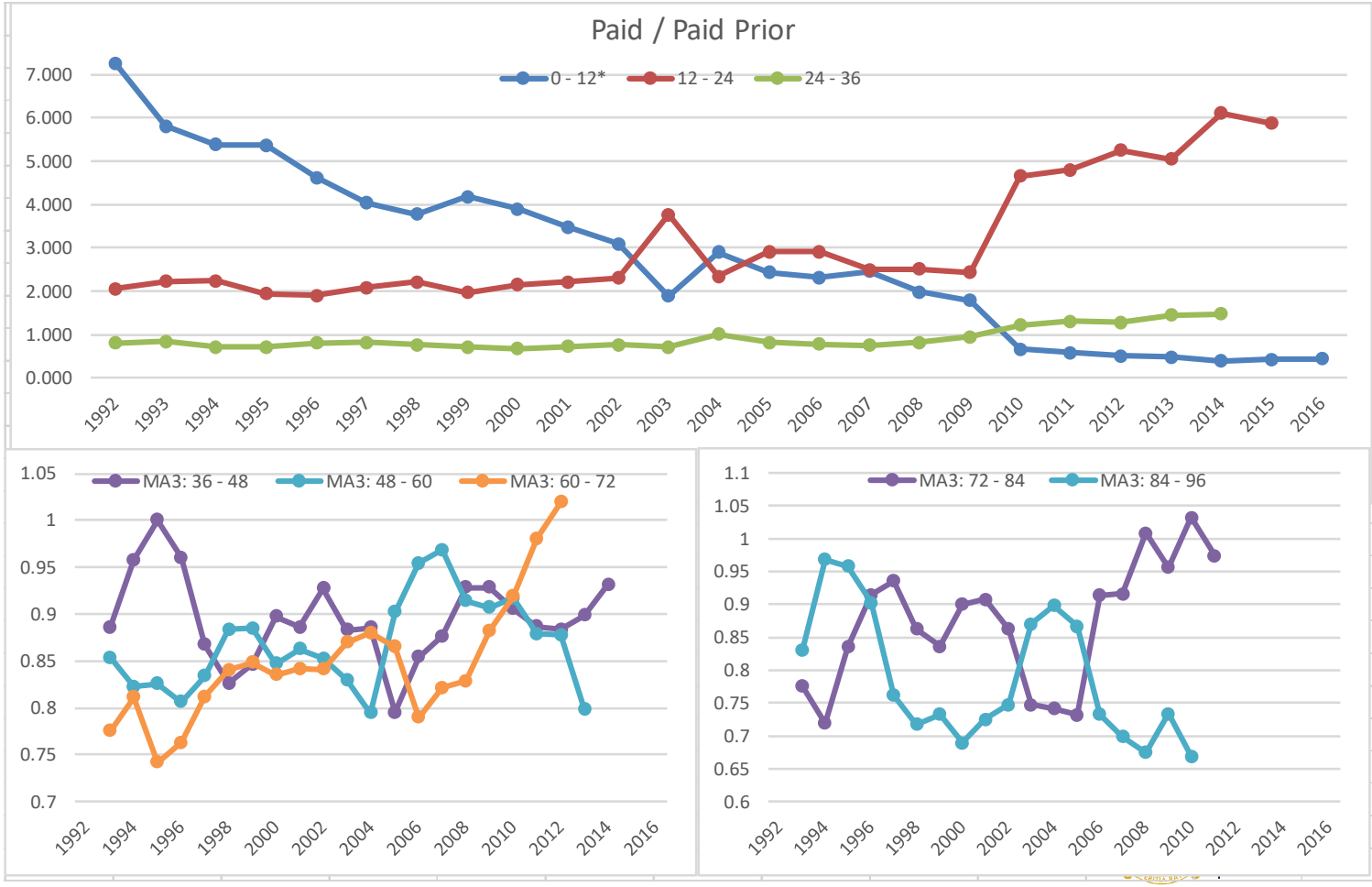
Unbiased: Decay Rate Incremental = $Paid_i$ / $Paid_{i-1}$

Decay 2: 1

AY	0 - 12*	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - 132	132 - 144	144 - 156	156 - 168
1992	7.234	2.061	0.808	0.808	0.940	0.704	0.842	0.729	0.528	0.754	0.706	0.469	1.305	0.491
1993	5.801	2.234	0.841	0.878	0.783	0.886	0.667	0.874	0.544	0.849	0.517	0.986	0.433	0.983
1994	5.378	2.241	0.719	0.972	0.838	0.735	0.817	0.889	0.582	0.447	0.822	0.994	0.662	1.482
1995	5.367	1.945	0.718	1.023	0.847	0.812	0.675	1.143	0.333	0.872	0.916	0.523	0.646	1.032
1996	4.622	1.902	0.812	1.006	0.791	0.678	1.012	0.843	0.453	0.670	0.787	1.103	0.460	1.004
1997	4.096	2.084	0.825	0.891	0.781	0.798	1.055	0.722	0.487	0.881	0.706	1.121	0.775	0.280
1998	3.775	2.215	0.770	0.745	0.929	0.999	0.740	0.722	0.711	0.808	0.896	0.447	1.920	0.449
1999	4.182	1.975	0.718	0.882	0.940	0.764	0.792	0.769	0.618	0.676	0.986	0.401	0.607	1.244
2000	3.895	2.154	0.880	0.911	0.784	0.822	0.977	0.769	0.990	0.861	0.496	0.896	0.902	1.515
2001	3.480	2.214	0.729	0.897	0.810	0.918	0.932	0.991	0.980	0.919	0.948	0.973	0.617	0.401
2002	3.888	2.901	0.769	0.849	0.989	0.784	0.810	0.810	0.495	0.921	0.976	0.814	0.484	1.185
2003	1.898	3.708	0.719	1.090	0.794	0.821	0.841	0.837	0.685	0.922	0.725	0.849	0.419	0.984
2004	2.983	2.947	1.008	0.769	0.747	1.064	0.987	0.997	0.949	1.002	0.929	0.880	0.790	
2005	2.441	2.918	0.818	0.890	0.882	0.819	0.797	0.900	0.935	0.790	0.901	0.717		
2006	2.914	2.907	0.780	0.769	1.078	0.779	0.812	0.742	0.985	0.498	1.128			
2007	2.454	2.506	0.753	0.942	0.902	0.778	1.132	0.557	0.683	0.538				
2008	1.988	2.516	0.814	0.922	0.925	0.910	0.804	0.798	0.439					
2009	1.793	2.431	0.946	0.922	0.914	0.796	1.090	0.669						
2010	0.664	4.659	1.220	0.941	0.881	0.941	0.974							
2011	0.588	4.793	1.308	0.854	0.957	1.019								
2012	0.511	5.249	1.281	0.866	0.798									
2013	0.488	5.044	1.455	0.932										
2014	0.398	6.100	1.485											
2015	0.427	5.870												
2016	0.441													

Loss "Decay"


Paid / Paid Prior
0 - 12*
12 - 24
24 - 36
7.000
6.000
5.000
4.000
3.000
2.000
1.000
0.000
1992 1993 1994 1995 1996 1997 1998 1999 2000 2001 2002 2003 2004 2005 2006 2007 2008 2009 2010 2011 2012 2013 2014 2015 2016
MA3: 36 - 48
MA3: 48 - 60
MA3: 60 - 72
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012 2014 2016
MA3: 72 - 84
MA3: 84 - 96
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012 2014 2016

Prior Active

Prior Active = Prior OS + Prior Paid

AY	Exposure	0 - 12	12 - 24	24 - 36	36 - 48	48 - 60
1992	1,508,931	876,365	1,546,937	1,420,602	1,304,747	1,086,607
1993	1,740,390	1,021,137	1,600,587	1,615,077	1,377,203	1,119,774
1994	2,016,977	948,778	1,652,505	1,528,460	1,343,548	1,072,021
1995	2,114,667	1,077,560	1,613,715	1,480,554	1,290,078	1,041,414
1996	2,356,514	1,121,348	1,596,440	1,471,037	1,332,244	1,076,485
1997	2,761,648	1,105,176	1,628,007	1,560,527	1,406,603	1,102,523
1998	2,971,558	1,011,616	1,688,360	1,588,247	1,353,160	1,100,296
2005	3,116,301	973,049	1,638,753	1,549,062	1,307,191	1,073,158
2006	3,244,092	970,312	1,597,376	1,435,702	1,225,844	1,045,414
2007	3,377,090	1,029,493	1,552,008	1,439,730	1,318,503	1,084,013
2008	3,648,870	868,284	1,320,064	1,315,651	1,212,495	992,479
2009	3,842,099	810,807	1,325,143	1,352,643	1,222,035	1,023,613
2010	4,089,308	590,275	1,090,085	1,222,371	1,126,899	971,154
2011	4,443,322	529,195	1,111,000	1,267,973	1,153,469	961,722
2012	4,711,514	4				
2013	4,913,443	4				
2014	5,109,890	4				
2015	5,319,006	4				
2016	5,645,939					

	0 - 12*	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96
Average:	0.056	0.470	0.244	0.215	0.207	0.209	0.223	0.220
PA CV	69.5%	12.1%	20.2%	9.6%	7.5%	13.0%	17.2%	13.3%

Incremental Paid / Prior Active

AY	0 - 12*	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96
1992	0.145	0.513	0.235	0.207	0.212	0.179	0.198	0.189
1993	0.116	0.442	0.237	0.206	0.189	0.206	0.177	0.217
1994	0.108	0.512	0.212	0.222	0.212	0.195	0.201	0.235
1995	0.107	0.410	0.196	0.219	0.213	0.214	0.189	0.283
				0.230	0.201	0.169	0.228	0.248
				0.205	0.202	0.191	0.282	0.201
2008	0.040	0.420	0.225	0.208	0.209	0.233	0.244	0.255
2009	0.036	0.413	0.239	0.216	0.218	0.207	0.285	0.254
2010	0.013	0.429	0.283	0.238	0.227	0.248	0.316	
2011	0.012	0.473	0.295	0.221	0.232	0.284		
2012	0.010	0.526	0.322	0.243	0.219			
2013	0.010	0.514	0.349	0.276				

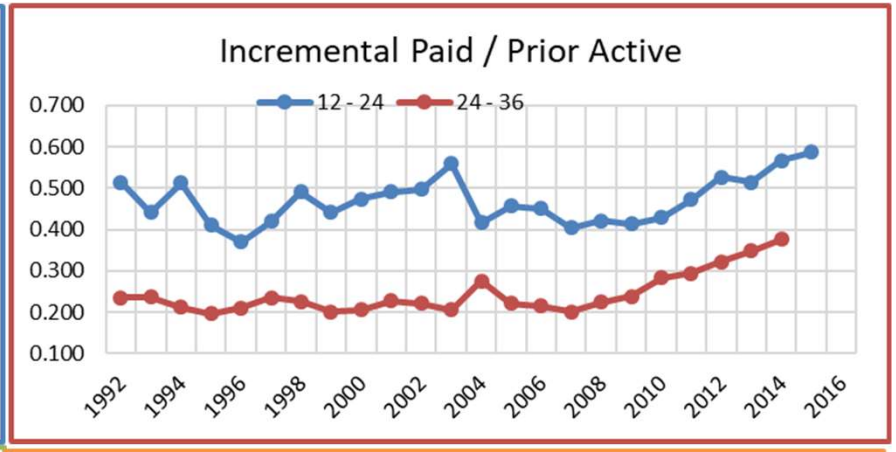
Gross / Pvt Owned Vehicles < 3.5 Tons / Israel

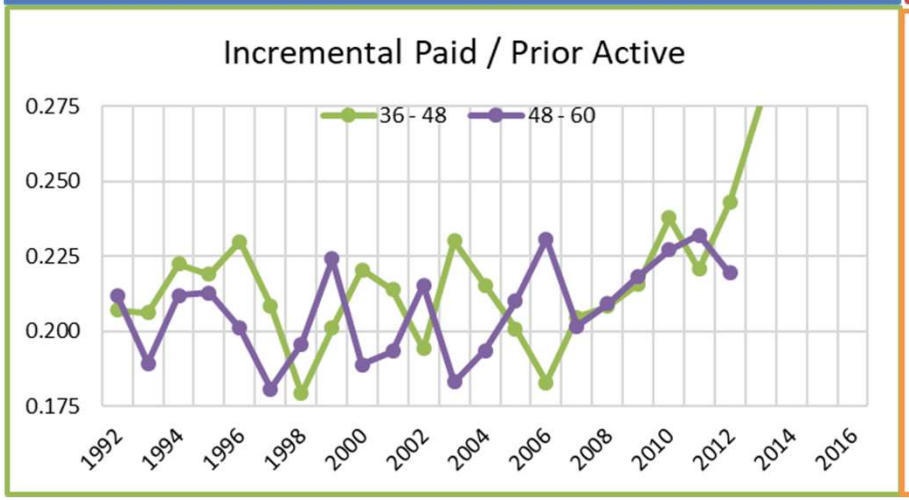
	0 - 12*	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120
Pmt Decay CV	69.5%	44.3%	27.6%	**9.0%**	10.0%	**11.5%**	17.3%	17.2%	24.6%	22.9%
PrAct Decay CV	69.5%	**12.1%**	**20.2%**	9.6%	**7.5%**	13.0%	**17.2%**	13.3%	**15.9%**	**18.4%**
LDF CV	69.5%	44.3%	37.5%	19.7%	14.6%	19.2%	21.5%	**11.8%**	19.3%	25.0%

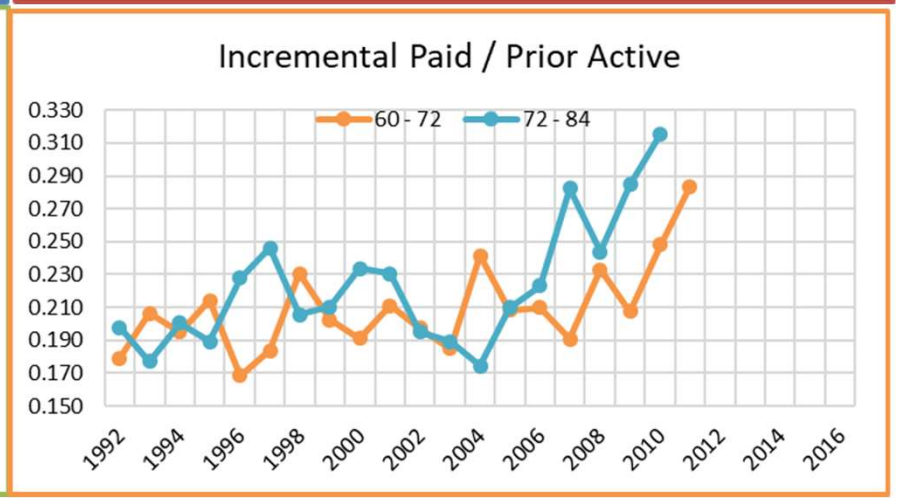
Gross / Pvt Motor / UL

	0 - 12*	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120
Average Decay:	0.435	0.437	0.337	0.821	0.838	0.583	0.520	0.561	0.648	0.741
Pmt Decay CV	10.9%	8.8%	13.2%	**9.9%**	12.5%	14.0%	23.2%	28.4%	47.2%	114.6%
PrAct Decay CV	10.9%	**7.3%**	**8.8%**	10.4%	**9.2%**	**11.3%**	**21.0%**	33.9%	48.8%	72.3%
LDF CV	10.9%	8.8%	15.8%	13.6%	10.7%	11.7%	23.2%	**18.5%**	**35.0%**	**68.2%**

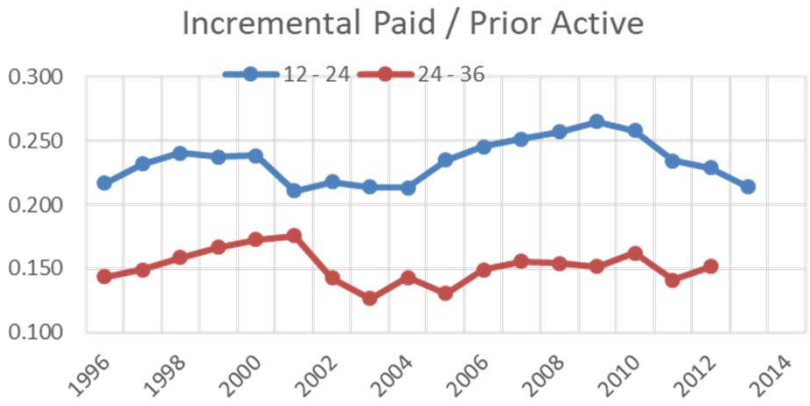
Israel Motor: Paid/ Prior Active (BOS + Prior Incremental Paid)

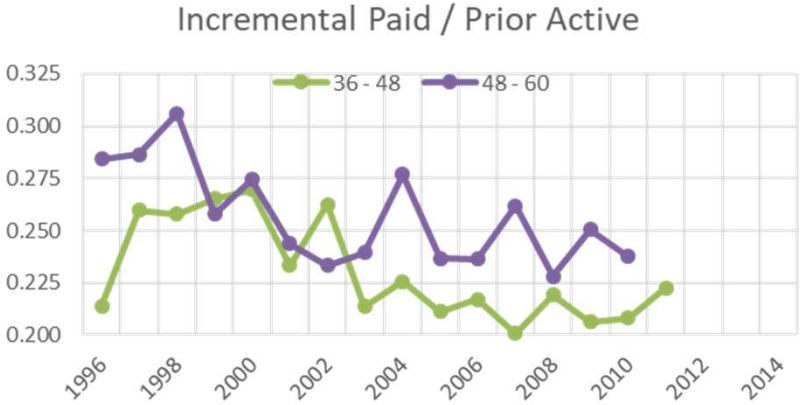


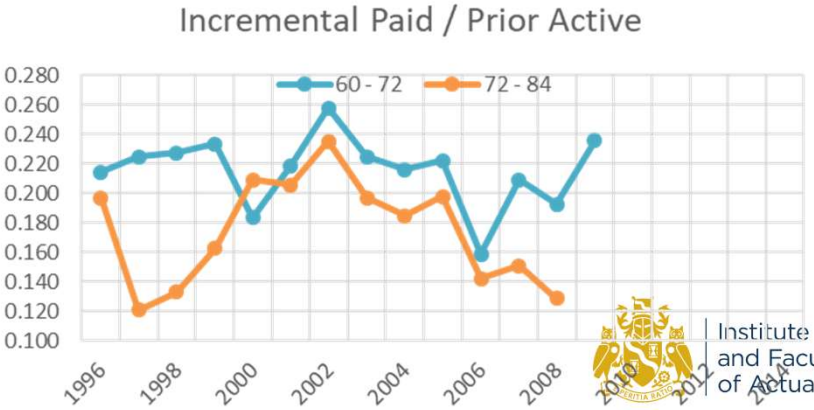




UK Motor: Paid/ Prior Active (BOS + Prior Incremental Paid)







Institute and Faculty of Actuaries

Age Period Cohort: The Calendar Effect

Suicide Rates as a Triangle

Birth \ Age year \	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
1900-1904	0.61	4.94	10.98	16.02	22.78	26.91	24.22	27.52	32.32	37.57	36.67	34.23	36.92	44.53	49.91
1905-1909	0.63	3.57	11.79	18.49	22.04	21.44	24.08	27.49	33.30	37.91	35.58	33.56	36.44	42.92	63.00
1910-1914	0.53	4.07	13.33	17.77	17.69	19.80	21.98	27.05	33.90	34.31	31.31	31.26	35.06	50.78	59.18
1915-1919	0.55	4.92	13.02	14.35	15.19	17.27	22.49	28.00	30.18	31.61	29.50	27.87	39.18	46.05	49.22
1920-1924	0.54	4.83	10.72	12.26	13.61	16.79	23.26	25.77	27.84	27.40	25.48	29.97	34.03	37.83	42.18
1925-1929	0.67	3.90	10.09	12.22	14.28	19.71	23.53	26.64	25.72	25.97	26.26	26.74	25.77	33.36	55.67
1930-1934	0.64	3.81	9.55	12.16	17.03	20.69	23.06	24.63	24.13	25.65	24.77	23.86	26.40	46.05	
1935-1939	0.79	3.77	9.97	14.39	18.09	21.39	22.81	22.69	23.80	23.68	20.84	21.50	44.92		
1940-1944	0.59	4.19	12.06	16.86	20.55	22.69	22.31	22.55	22.88	21.87	20.37	41.17			
1945-1949	0.69	5.77	14.94	21.56	23.39	22.71	21.97	22.61	21.52	22.40	40.38				
1950-1954	0.86	7.22	21.62	26.88	23.93	23.41	23.28	23.01	23.29	37.57					
1955-1959	0.92	10.29	27.49	26.25	24.24	24.14	23.88	24.44	33.27						
1960-1964	1.03	12.87	25.54	25.36	24.65	23.56	24.16	27.49							
1965-1969	1.28	13.94	26.21	25.09	22.61	22.14	23.88								
1970-1974	1.53	16.89	26.39	22.79	20.52	22.71									
1975-1979	2.22	17.94	23.57	20.29	20.55										
1980-1984	2.34	14.98	20.70	18.49											
1985-1989	2.25	12.46	14.94												
1990-1994	1.89	5.77													
1995-1999	0.79														

Actual / Expected: Age by Calendar Period

Actual / Expected: Paid Loss for Period Ending:

Age	Dec-00	Dec-01	Dec-02	Dec-03	Dec-04	Dec-05	Dec-06	Dec-07	Dec-08	Dec-09	Dec-10	Dec-11	Dec-12	Dec-13	Dec-14	Dec-15	Dec-16
24 - 25 Yrs																	0.649
23 - 24 Yrs																0.349	(0.499)
22 - 23 Yrs															0.761	4.208	1.365
21 - 22 Yrs														0.290	1.317	0.574	1.179
20 - 21 Yrs													0.098	0.446	0.508	0.597	0.257
19 - 20 Yrs												0.492	1.006	1.601	(0.051)	1.889	1.336
18 - 19 Yrs											0.285	0.310	0.588	1.059	1.942	1.112	1.032
17 - 18 Yrs										2.294	1.710	0.627	0.434	2.479	0.537	0.954	0.544
16 - 17 Yrs									2.639	3.062	1.895	1.205	1.117	1.018	0.477	1.214	(0.278)
15 - 16 Yrs								0.994	1.117	1.661	0.671	1.644	0.790	1.280	0.333	0.905	0.723
14 - 15 Yrs							1.956	1.297	0.829	0.975	0.434	1.323	1.258	0.386	1.434	0.553	1.172
13 - 14 Yrs						0.638	0.816	1.859	0.991	1.164	0.666	1.173	1.052	0.847	0.577	1.308	0.839
12 - 13 Yrs					1.259	0.803	1.214	0.929	1.122	2.255	2.528	0.818	0.624	1.211	1.070	1.389	1.269
11 - 12 Yrs				0.465	0.894	0.884	0.692	1.176	1.401	0.634	0.649	0.832	0.946	1.064	1.597	0.773	0.698
10 - 11 Yrs			0.822	0.752	0.738	1.099	0.885	1.037	1.178	1.169	0.770	0.807	1.085	1.562	0.944	0.808	1.090
9 - 10 Yrs		0.731	0.914	0.564	0.753	0.706	0.923	1.061	0.745	0.977	0.926	1.005	1.353	1.121	1.013	0.607	0.746
8 - 9 Yrs	0.758	0.842	0.987	0.675	0.824	0.819	1.026	0.869	0.954	1.395	0.854	1.148	0.874	1.003	0.954	1.085	0.853
7 - 8 Yrs	# 0.907	0.994	1.189	1.067	0.987	0.847	0.825	1.007	0.835	1.013	0.984	0.934	1.101	0.960	0.933	1.141	1.157
6 - 7 Yrs	# # 0.770	0.717	0.872	0.942	0.808	0.802	0.902	0.974	0.835	0.810	0.872	0.842	0.891	1.153	0.985	1.191	1.358
5 - 6 Yrs	# # # 0.921	0.747	0.774	0.947	0.879	0.801	0.906	0.909	0.835	0.994	0.917	0.951	0.884	1.062	0.948	1.209	1.348
4 - 5 Yrs	# # # 0.892	0.787	0.800	0.933	0.789	0.800	0.940	0.825	0.802	0.912	0.994						0.972
3 - 4 Yrs	# # # 0.880	0.752	0.867	0.879	0.857	0.831	0.956	0.939	0.903	0.806	0.892						1.170
2 - 3 Yrs	# # # 0.837	0.813	0.799	0.791	0.810	0.765	1.017	0.874	0.873	0.784	0.802						1.153
1 - 2 Yrs	# # # 1.317	1.366	1.262	1.225	1.236	1.174	1.242	1.292	1.211	1.146	1.020	0.867	0.844	[illegible]	0.832	0.903	0.998
0 - 1 Yrs	# # # 3.248	2.919	2.726	1.681	2.561	2.180	2.275	2.475	2.332	2.149	0.953	0.902	0.903	0.[illegible]4	0.758	0.871	0.941
Over 2 Yrs	0.860	0.790	0.859	0.868	0.840	0.811	0.946	0.937	0.883	0.927	0.878	0.902	0.940	1.027	0.990	1.074	1.128
Total	**1.005**	**0.948**	**0.986**	**0.942**	**0.930**	**0.910**	**1.031**	**1.035**	**0.972**	**0.993**	**0.901**	**0.897**	**0.926**	**1.008**	**0.961**	**1.045**	**1.105**
Up to 2 Yrs	1.634	1.636	1.510	1.269	1.554	1.339	1.403	1.487	1.383	1.314	1.010	0.873	0.853	0.911	0.820	0.898	0.989

Benchmark Period

and Faculty of Actuaries

For Calendar Periods, We Want Confidence

Avoid this "Square" Fatty School Offer.

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{n}}$$

σ = lower case sigma
$\sum$ = capital sigma
$\bar{x}$ = x bar

Instead, Opt For This Lean Trendy Range

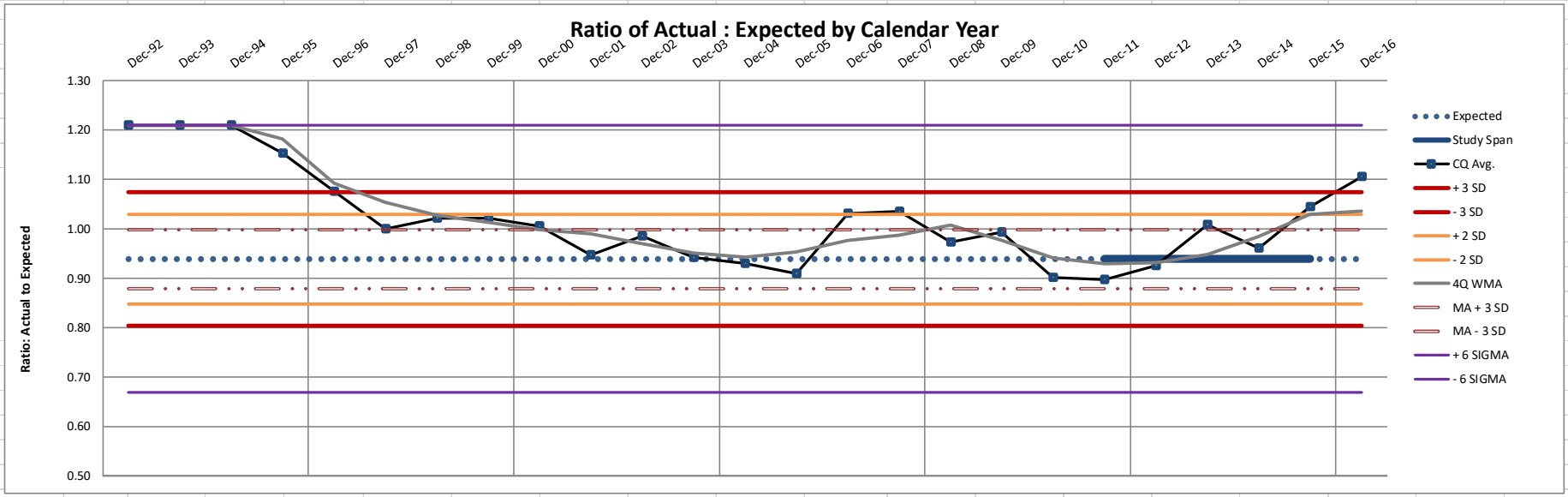
$$\hat{\sigma} = \frac{\bar{R}}{d_2(n)}$$

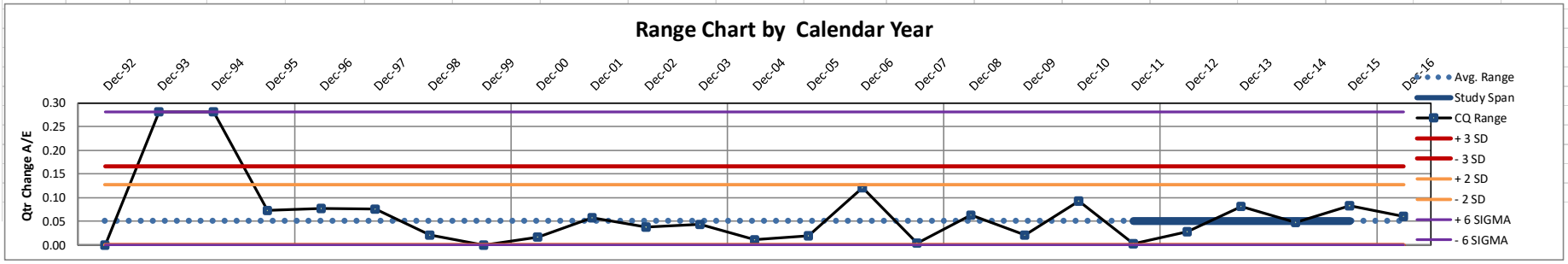
1. Calculate $\boldsymbol{\sigma}$ in a skinny, trendy, series way
2. $2\boldsymbol{\sigma}$ and $3\boldsymbol{\sigma}$ bands around baseline
3. Cap at $6\boldsymbol{\sigma}$
4. Plot series
5. Plot Series Variance Too

Institute and Faculty of Actuaries

Paid **Gross / Pvt Owned Vehicles < 3.5 Tons / Israel**

All Accident Years Combined



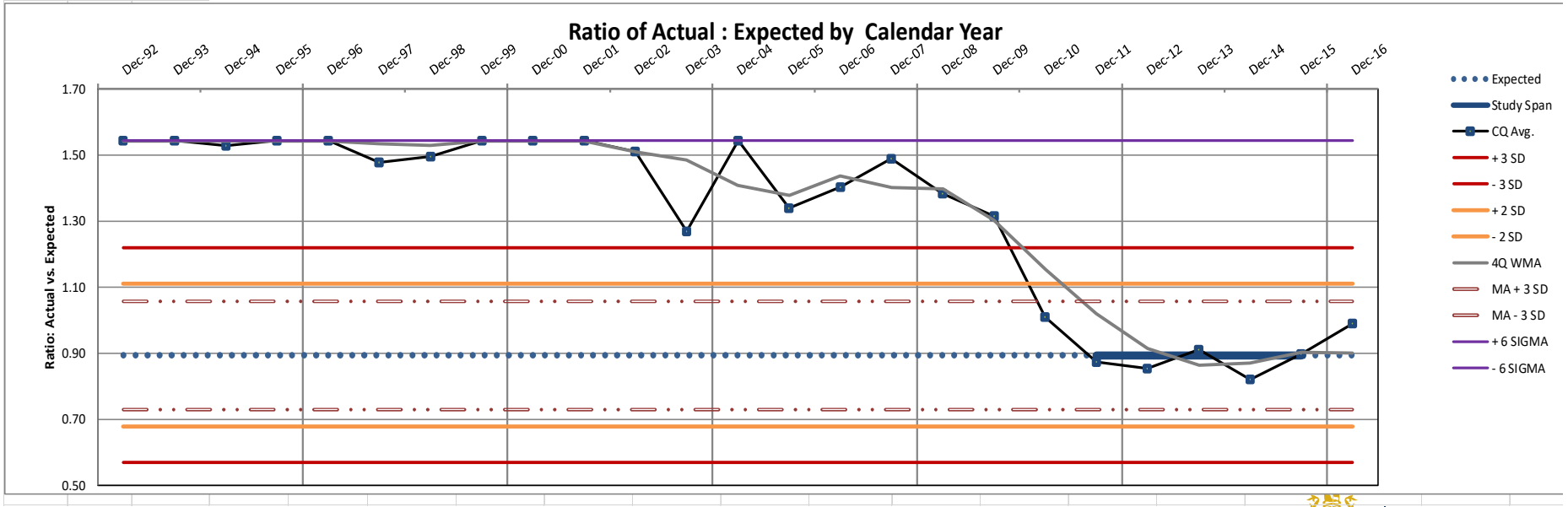


Israel Paid "Early"

Paid

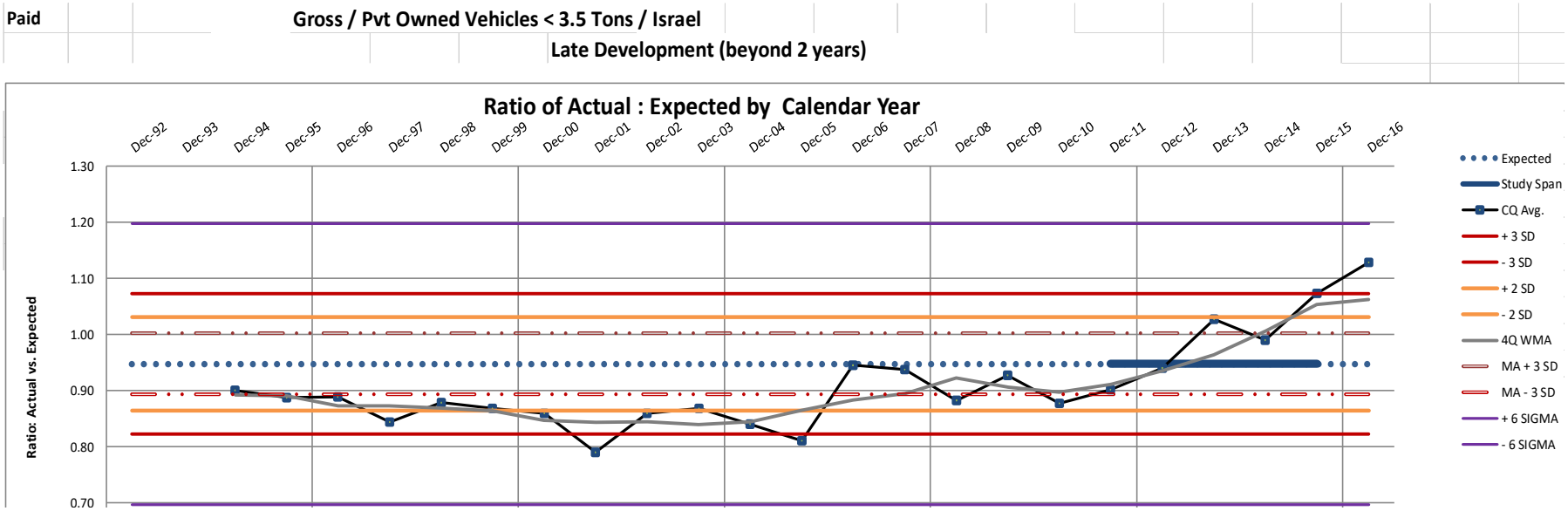
Gross / Pvt Owned Vehicles < 3.5 Tons / Israel

Early Development (up to 24 months)



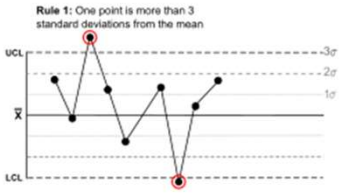
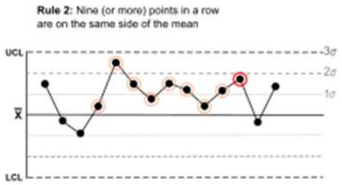
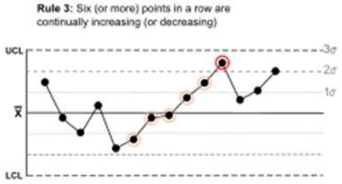
Institute and Faculty of Actuaries

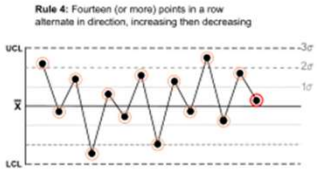
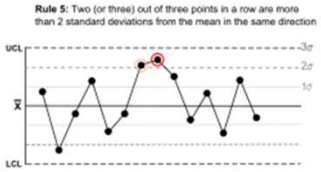
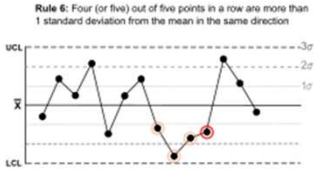
Israel Paid “Late”



Institute and Faculty of Actuaries

Nelson's Rules (https://en.wikipedia.org/wiki/Nelson_rules)

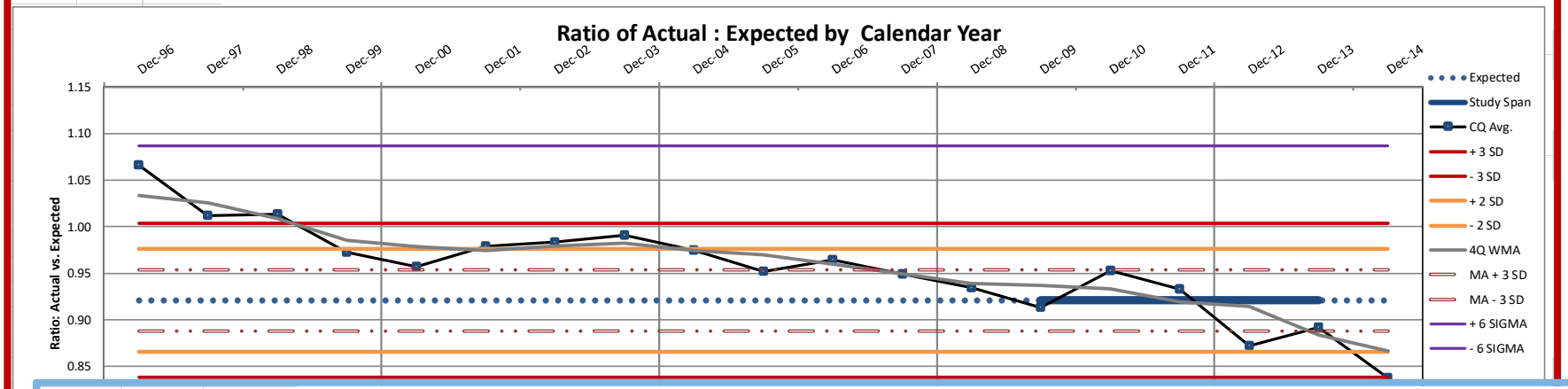
Rule	Description	Chart Example	Problem Indicated
Rule 1	One point is more than 3 standard deviations from the mean.		One sample (two shown in this case) is grossly out of control.
Rule 2	Nine (or more) points in a row are on the same side of the mean.		Some prolonged bias exists.
Rule 3	Six (or more) points in a row are continually increasing (or decreasing).		A trend exists.

Rule	Description	Chart Example	Problem Indicated
Rule 4	Fourteen (or more) points in a row alternate in direction, increasing then decreasing.		This much oscillation is beyond noise. Note that the rule is concerned with directionality only. The position of the mean and the size of the standard deviation have no bearing.
Rule 5	Two (or three) out of three points in a row are more than 2 standard deviations from the mean in the same direction.		There is a medium tendency for samples to be mediumly out of control. The side of the mean for the third point is unspecified.
Rule 6	Four (or five) out of five points in a row are more than 1 standard deviation from the mean in the same direction.		There is a strong tendency for samples to be slightly out of control. The side of the mean for the fifth point is unspecified.

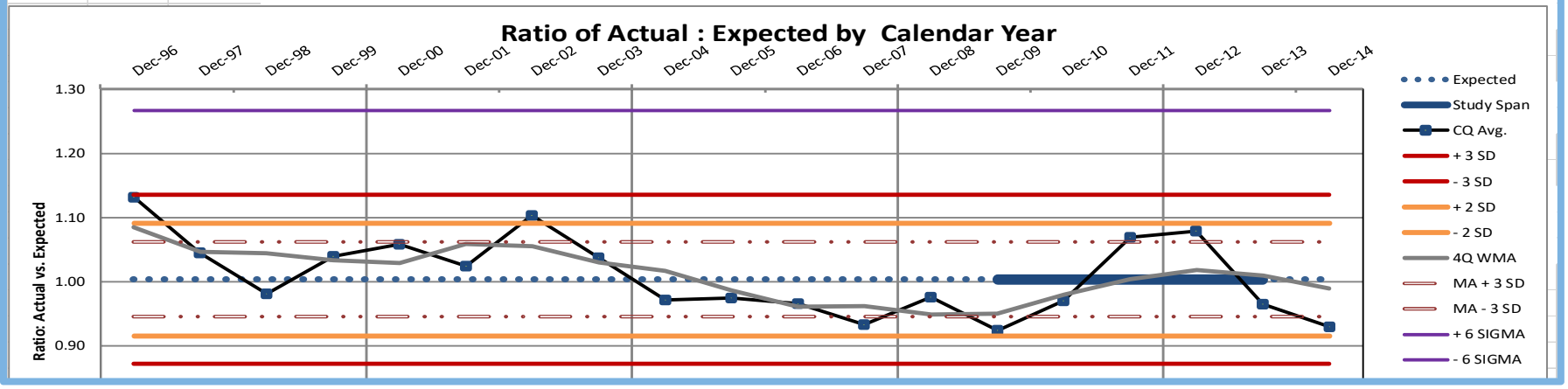
Paid

Gross / UK Pvt Motor /

Early Development (up to 24 months)



Inc'd



A Full Panel View

Institute and Faculty of Actuaries

Age Period Cohort: The Origin (Accident Period) Effect

Suicide Rates as a Triangle

Birth \ Age year \	10-14	15-19	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
1900-1904	0.61	4.94	10.98	16.02	22.78	26.91	24.22	27.52	32.32	37.57	36.67	34.23	36.92	44.53	49.91
1905-1909	0.63	3.57	11.79	18.49	22.04	21.44	24.08	27.49	33.30	37.91	35.58	33.56	36.44	42.92	63.00
1910-1914	0.53	4.07	13.33	17.77	17.69	19.80	21.98	27.05	33.90	34.31	31.31	31.26	35.06	50.78	59.18
1915-1919	0.55	4.92	13.02	14.35	15.19	17.27	22.49	28.00	30.18	31.61	29.50	27.87	39.18	46.05	49.22
1920-1924	0.54	4.83	10.72	12.26	13.61	16.79	23.26	25.77	27.84	27.40	25.48	29.97	34.03	37.83	42.18
1925-1929	0.67	3.90	10.09	12.22	14.28	19.71	23.53	26.64	25.72	25.97	26.26	26.74	29.77	33.36	55.67
1930-1934	0.64	3.81	9.55	12.16	17.03	20.69	23.06	24.63	24.13	25.65	24.77	23.86	26.40	46.05	
1935-1939	0.79	3.77	9.97	14.39	18.09	21.39	22.81	22.69	23.80	23.68	20.84	21.50	44.92		
1940-1944	0.59	4.19	12.06	16.86	20.55	22.69	22.31	22.55	22.88	21.87	20.37	41.17			
1945-1949	0.69	5.77	14.94	21.56	23.39	22.71	21.97	22.61	21.52	22.40	40.38				
1950-1954	0.86	7.22	21.62	26.88	23.93	23.41	23.28	23.01	23.29	37.57					
1955-1959	0.92	10.29	27.49	26.25	24.24	24.14	23.88	24.44	33.27						
1960-1964	1.03	12.87	25.54	25.36	24.65	23.56	24.16	27.49							
1965-1969	1.28	13.94	26.21	25.09	22.61	22.14	23.88								
1970-1974	1.53	16.89	26.39	22.79	20.52	22.71									
1975-1979	2.22	17.94	23.57	20.29	20.55										
1980-1984	2.34	14.98	20.70	18.49											
1985-1989	2.25	12.46	14.94												
1990-1994	1.89	5.77													
1995-1999	0.79														

Israel Motor Exposure Candidates

Structural Questions:

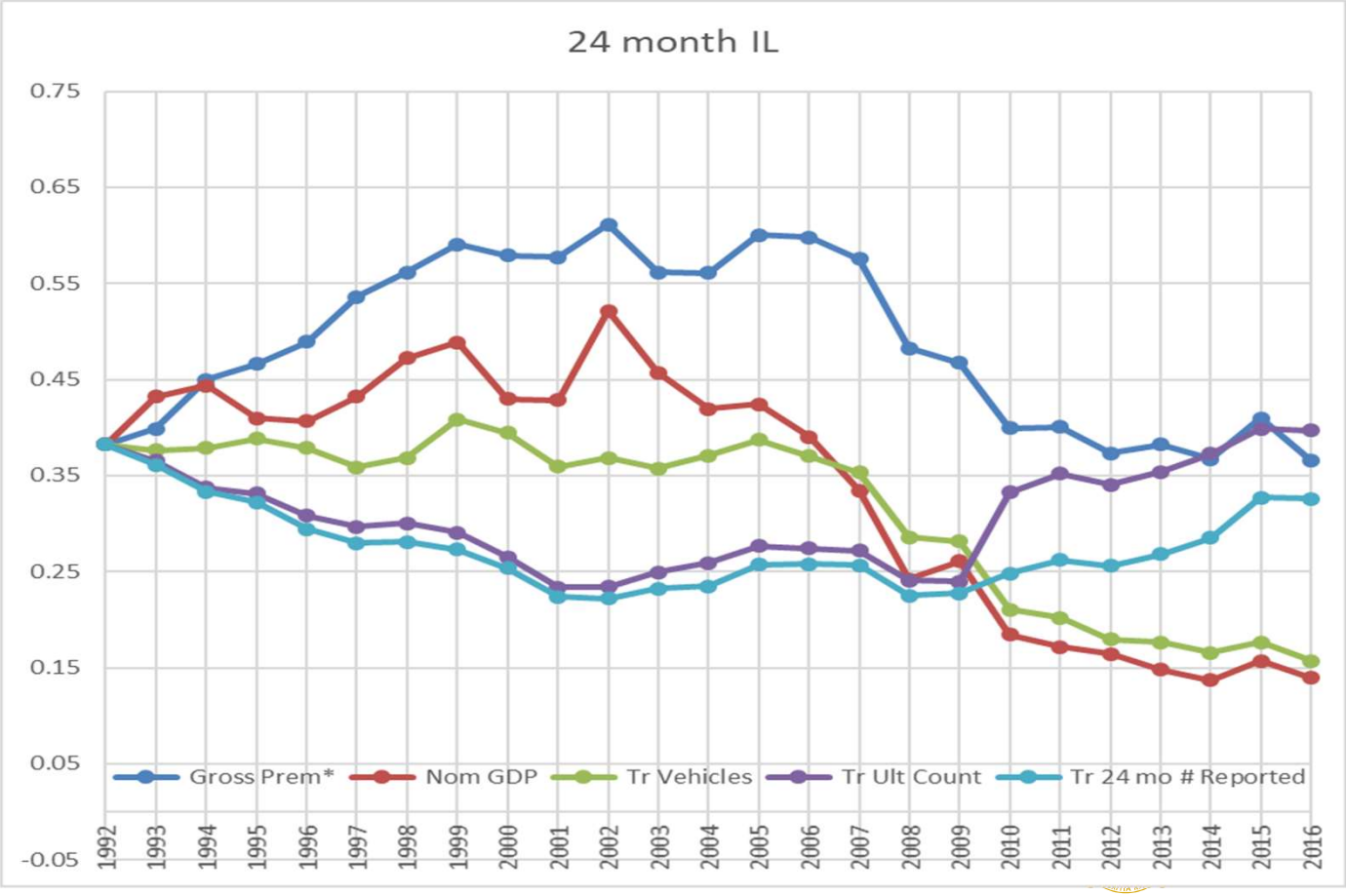
-GDP vs. Prem. Link

-Invert Vehicles vs. # Rep from 2009-16

-Can signal flagging be automated?

-Is Prem. On-Level Effective?

-Autoregressive?



UK Motor Subset: Exposure Candidates

Structural Questions:

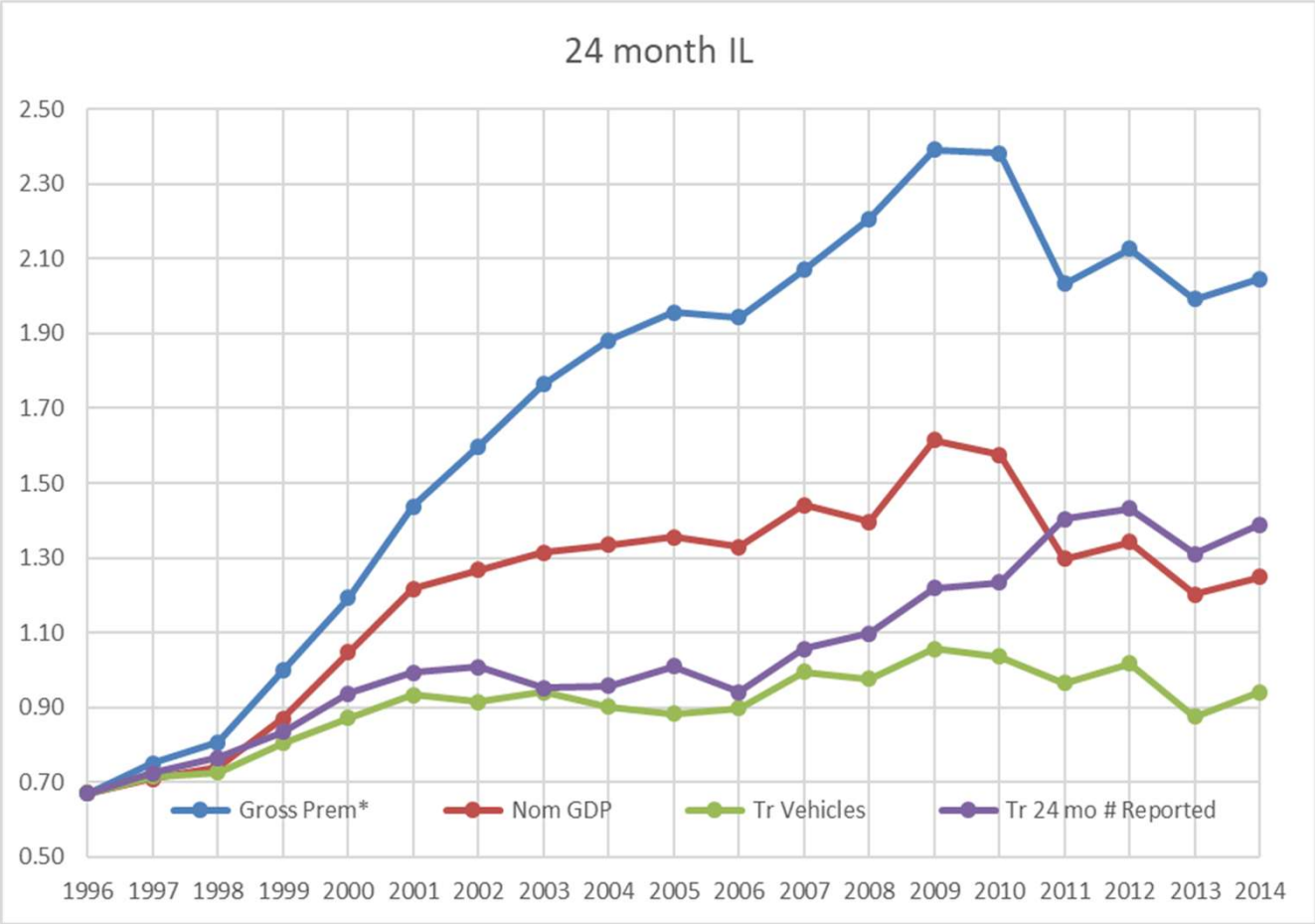
-GDP vs Tr. Vehicles

-Invert Vehicles vs. # Rep from 2009-16

-Is Inflation = CPI?

-Is Prem. On-Level Effective?

-Autoregressive?



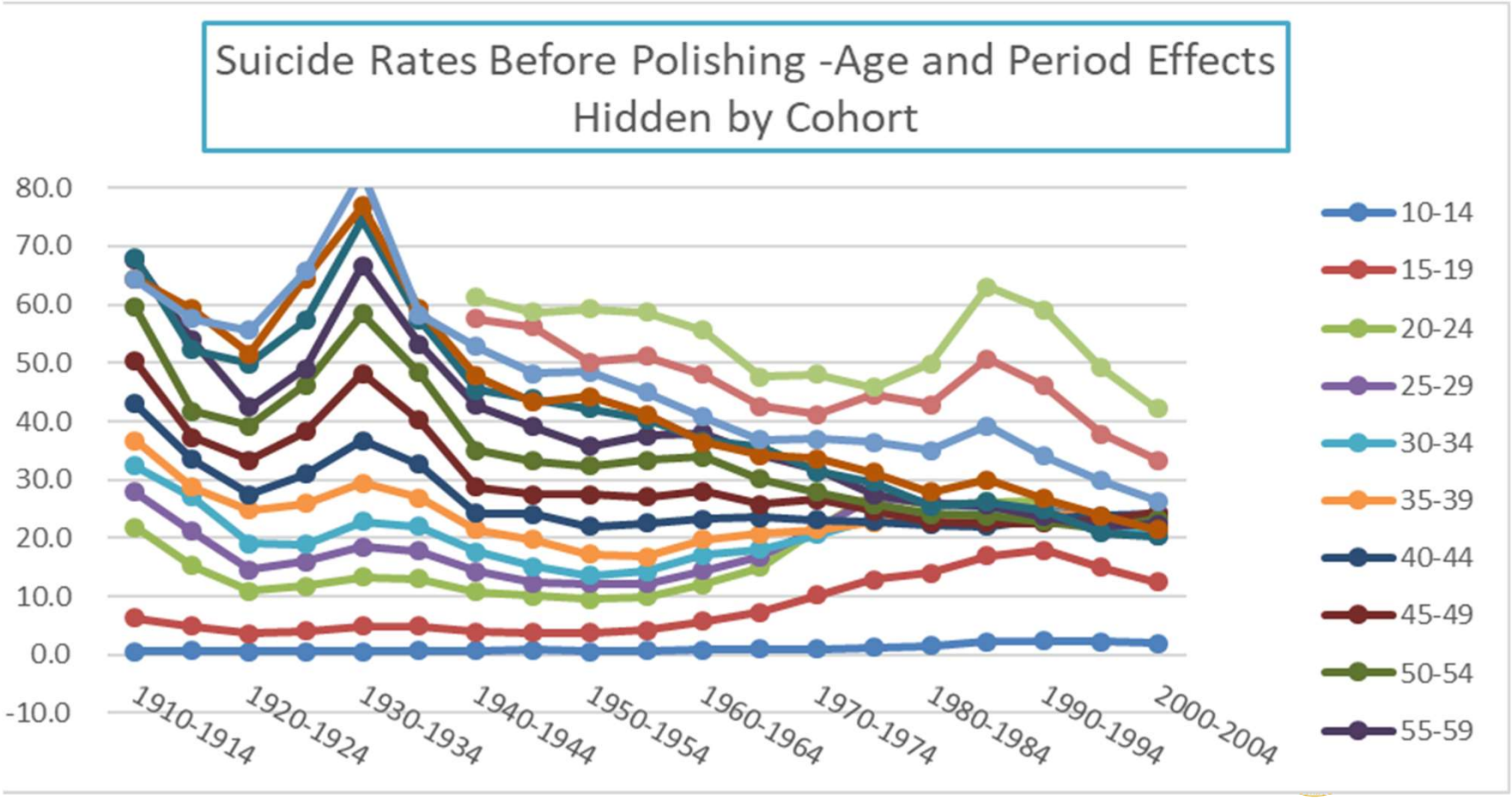
Age Period Cohort

Age **Period Cohort**

Age Period **Cohort**

- Stable Age Effects Required for Most Approaches
  - 0-1 Lag Development Detection
  - Decay Approaches Needed
  - Split Triangles Needed
- Period Effects
  - Spikes, Shifts, Trends Lead Change Elsewhere
  - Small Aging Changes Are Likely Period Effects
- Cohort Effects Take Time to Establish
  - Predictive Models Needed

Institute and Faculty of Actuaries


Suicide Rates Before Polishing -Age and Period Effects Hidden by Cohort
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0.0
-10.0
1910-1914
1920-1924
1930-1934
1940-1944
1950-1954
1960-1964
1970-1974
1980-1984
1990-1994
2000-2004
10-14
15-19
20-24
25-29
30-34
35-39
40-44
45-49
50-54
55-59

Age Period Effects

Table 2: Suicide rates among men in the USA, 1910-2004

Age	1910 - 1914	1915 - 1919	1920 - 1924	1925 - 1929	1930 - 1934	1935 - 1939	1940 - 1944	1945 - 1949	1950 - 1954	1955 - 1959	1960 - 1964	1965 - 1969	1970 - 1974	1975 - 1979	1980 - 1984	1985 - 1989	1990 - 1994	1995 - 1999	2000 - 2004	Row median
10–14	0.610	0.630	0.530	0.550	0.540	0.670	0.640	0.790	0.590	0.690	0.860	0.920	1.030	1.280	1.530	2.220	2.340	2.250	1.890	0.790
15–19	6.330	4.940	3.570	4.070	4.920	4.830	3.900	3.810	3.770	4.190	5.770	7.220	10.290	12.870	13.940	16.890	17.940	14.980	12.460	5.770
20–24	21.790	15.300	10.980	11.790	13.330	13.020	10.720	10.090	9.550	9.970	12.060	14.940	21.620	27.490	25.540	26.210	26.390	23.570	20.700	14.940
25–29	27.910	21.100	14.500	16.020	18.490	17.770	14.350	12.260	12.220	12.160	14.390	16.860	21.560	26.880	26.250	25.360	25.090	22.790	20.290	18.490
30–34	32.380	26.980	19.030	18.930	22.780	22.040	17.690	15.190	13.610	14.280	17.030	18.090	20.550	23.390	23.930	24.240	24.650	22.610	20.520	20.550
35–39	36.720	28.780	24.750	25.860	29.440	26.910	21.440	19.800	17.270	16.790	19.710	20.690	21.390	22.690	22.710	23.410	24.140	23.560	22.140	22.710
40–44	42.960	33.500	27.440	30.970	36.660	32.620	24.220	24.080	21.980	22.490	23.260	23.530	23.060	22.810	22.310	21.970	23.280	23.880	24.160	23.880
45–49	50.280	37.320	33.380	38.290	48.220	40.270	28.780	27.520	27.490	27.050	28.000	25.770	26.640	24.630	22.690	22.550	22.610	23.010	24.440	27.490
50–54	59.490	41.790	39.220	46.180	58.440	48.270	35.060	33.270	32.320	33.300	33.900	30.180	27.840	25.720	24.130	23.800	22.880	21.520	23.290	33.270
55–59	67.680	53.980	42.470	49.090	66.720	53.240	42.780	39.150	35.750	37.570	37.910	34.310	31.610	27.400	25.970	25.650	23.680	21.870	22.400	37.570
60–64	67.930	52.210	49.890	57.340	74.630	57.420	45.460	43.800	42.130	40.380	36.670	35.580	31.310	29.500	25.480	26.260	24.770	20.840	20.370	40.380
65–69	64.480	59.300	51.520	64.440	76.920	59.210	47.830	43.320	44.210	41.170	36.410	34.230	33.560	31.260	27.870	29.970	26.740	23.860	21.150	41.170
70–74	64.300	57.640	55.650	65.680	82.670	58.220	52.770	48.200	48.410	44.920	40.720	36.840	36.920	36.440	35.060	39.180	34.030	29.770	26.400	44.920
75–79							57.540	56.130	50.150	51.120	48.090	42.560	41.160	44.530	42.920	50.780	46.050	37.830	33.360	46.050
80–84							61.170	58.680	59.300	58.640	55.670	47.690	48.120	45.770	49.910	63.000	59.180	49.220	42.180	55.670

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Data Polishing: Iterative Removal of Row / Column Medians

Table 3: ITERATION 1 Suicide rates among men in the USA, 1910-2004

Age	1910 - 1914	1915 - 1919	1920 - 1924	1925 - 1929	1930 - 1934	1935 - 1939	1940 - 1944	1945 - 1949	1950 - 1954	1955 - 1959	1960 - 1964	1965 - 1969	1970 - 1974	1975 - 1979	1980 - 1984	1985 - 1989	1990 - 1994	1995 - 1999	2000 - 2004
10–14	-0.18	-0.16	-0.26	-0.24	-0.25	-0.12	-0.15	0.00	-0.20	-0.10	0.07	0.13	0.24	0.49	0.74	1.43	1.55	1.46	1.10
15–19	0.56	-0.83	-2.20	-1.70	-0.85	-0.94	-1.87	-1.96	-2.00	-1.58	0.00	1.45	4.52	7.10	8.17	11.12	12.17	9.21	6.69
20–24	6.85	0.36	-3.96	-3.15	-1.61	-1.92	-4.22	-4.85	-5.39	-4.97	-2.88	0.00	6.68	12.55	10.60	11.27	11.45	8.63	5.76
25–29	9.42	2.61	-3.99	-2.47	0.00	-0.72	-4.14	-6.23	-6.27	-6.33	-4.10	-1.63	3.07	8.39	7.76	6.87	6.60	4.30	1.80
30–34	11.83	6.43	-1.52	-1.62	2.23	1.49	-2.86	-5.36	-6.94	-6.27	-3.52	-2.46	0.00	2.84	3.38	3.69	4.10	2.06	-0.03
35–39	14.01	6.07	2.04	3.15	6.73	4.20	-1.27	-2.91	-5.44	-5.92	-3.00	-2.02	-1.52	-0.02	0.00	0.70	1.43	0.85	-0.57
40–44	19.08	9.62	3.56	7.09	12.78	8.74	0.34	0.20	-1.90	-1.39	-0.62	-0.35	-0.82	-1.07	-1.57	-1.91	-0.60	0.00	0.28
45–49	22.79	9.83	5.89	10.80	20.73	12.78	1.29	0.03	0.00	-0.44	0.51	-1.72	-0.85	-2.86	-4.80	-4.94	-4.88	-4.48	-3.05
50–54	26.22	8.52	5.95	12.91	25.17	15.00	1.79	0.00	-0.95	0.03	0.63	-3.09	-5.43	-7.55	-9.14	-9.47	-10.39	-11.75	-9.98
55–59	30.11	16.41	4.90	11.52	29.15	15.67	5.21	1.58	-1.82	0.00	0.34	-3.26	-5.96	-10.17	-11.60	-11.92	-13.89	-15.70	-15.17
60–64	27.55	11.83	9.51	16.96	34.25	17.04	5.08	3.42	1.75	0.00	-3.71	-4.80	-9.07	-10.88	-14.90	-14.12	-15.61	-19.54	-20.01
65–69	23.31	18.13	10.35	23.27	35.75	18.04	6.66	2.15	3.04	0.00	-4.76	-6.94	-7.61	-9.91	-13.30	-11.20	-14.43	-17.31	-20.02
70–74	19.38	12.72	10.73	20.76	37.75	13.30	7.85	3.28	3.49	0.00	-4.20	-8.08	-8.00	-8.48	-9.86	-5.74	-10.89	-15.15	-18.52
75–79							11.49	10.08	4.10	5.07	2.04	-3.49	-4.89	-1.52	-3.13	4.73	0.00	-8.22	-12.69
80–84							5.50	3.01	3.63	2.97	0.00	-7.98	-7.55	-9.90	-5.76	7.33	3.51	-6.45	-13.49
Column Median	19.08	8.52	3.56	7.09	12.78	8.74	1.29	0.03	-0.95	-0.10	-0.62	-2.46	-1.32	-1.52	-3.13	0.70	0.00	-4.48	-3.05

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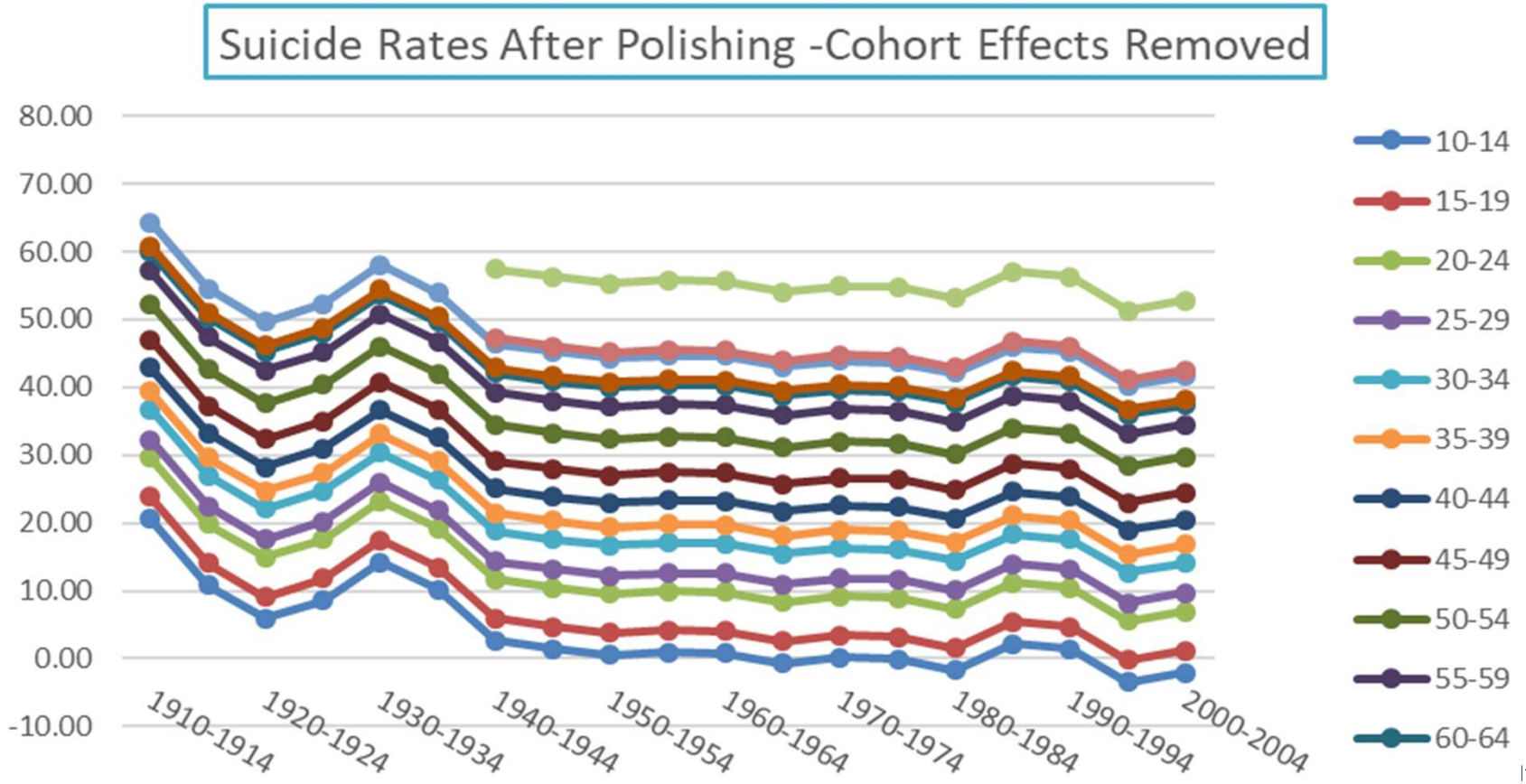
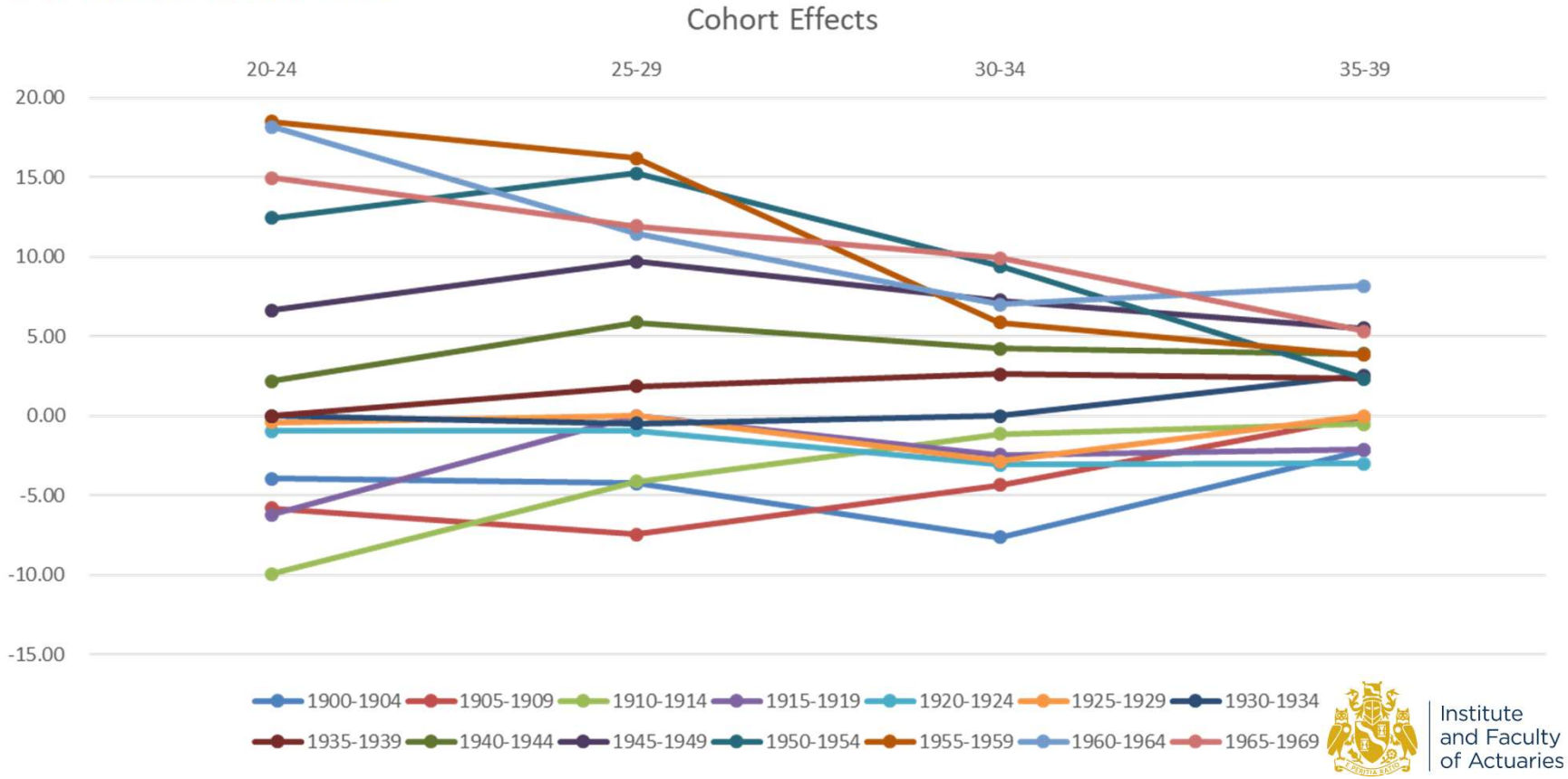

Suicide Rates After Polishing -Cohort Effects Removed
80.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
-10.00
1910-1914
1920-1924
1930-1934
1940-1944
1950-1954
1960-1964
1970-1974
1980-1984
1990-1994
2000-2004
10-14
15-19
20-24
25-29
30-34
35-39
40-44
45-49
50-54
55-59
60-64

Cohort Effects



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An Actuarial Science View on Modelling

$$E[\boldsymbol{Incremental\ Loss}_{AY=i,DY=j}]$$

$$= \boldsymbol{Exposure}_{AY=i}$$

$$* \text{ Pure Loss Rate} \rightarrow \text{Year 0}$$

$$* \prod_0^j \boldsymbol{Development\ factor}_t$$

$$* \prod_i^{i+j} \boldsymbol{CY}\ effect_t$$

$$* \prod_0^i \boldsymbol{AY}\ effect_t$$

Earned Premium or Etc.

Base Loss Rate

Loss Dev.

CY Trend

AY "Level" Changes

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A Simple Example

Expos * Base Rate * Development (Age) * CY (Period) * AY (Cohort)

Exposure		Constant	AY Direction		CY Direction	
AY	OLEP	Pure ELR	Trends	Factor.	Trends	Factor
2011	1,000	75.00%	1.000	1.000	1.000	1.000
2012	1,500	na	1.070	1.070	1.020	1.020
2016	2,000	na	1.020	1.091	1.040	1.061
2014	1,750	na	**0.970**	**1.059**	**1.030**	**1.093**
2015	1,500	na	0.970	1.027	1.010	1.104
2016	1,500	na	0.970	0.996	1.000	1.104
2017	1,500	na	0.970	0.966	1.005	1.109
2018	na	na	na	na	**1.005**	**1.115**
2019	na	na	na	na	1.005	1.120
2020	na	na	na	na	1.005	1.126
2021	na	na	na	na	1.005	1.131

Dev. Year	0	1	2	3	4
Decay	17.3%	175.0%	75.0%	75.0%	75.0%
Factor	17.3%	30.2%	22.7%	17.0%	12.8%
Cum	17.3%	47.5%	70.2%	87.2%	100.0%

Incremental Loss / Forecast Triangle

AY	0	1	2	3	4	Ult	U L/R	-
2011	130	231	162	147	111	782	78.2%	
2012	223	365	330	250	178	1,346	89.8%	14.8%
2016	257	542	410	292	220	1,721	86.1%	-4.1%
2014	**263**	**464**	**330**	**249**	**188**	**1,495**	**85.4%**	**-0.8%**
2015	244	405	305	230	173	1,358	90.5%	6.0%
2016	214	376	283	214	169	1,256	83.7%	-7.5%
2017	208	366	276	219	157	1,227	81.8%	-2.3%

Loss Development Factors

	0-1	1-2	2-3	3-4
2011	2.785	1.450	1.281	1.166
2012	2.638	1.562	1.272	1.152
2016	3.111	1.514	1.242	1.147
2014	**2.768**	**1.454**	**1.236**	
2015	2.660	1.470		
2016	2.759			

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Questions

Comments

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