Institute and Faculty of Actuaries

The Link Between Classical Reserving and Granular Reserving Through Double Chain Ladder and its Extensions

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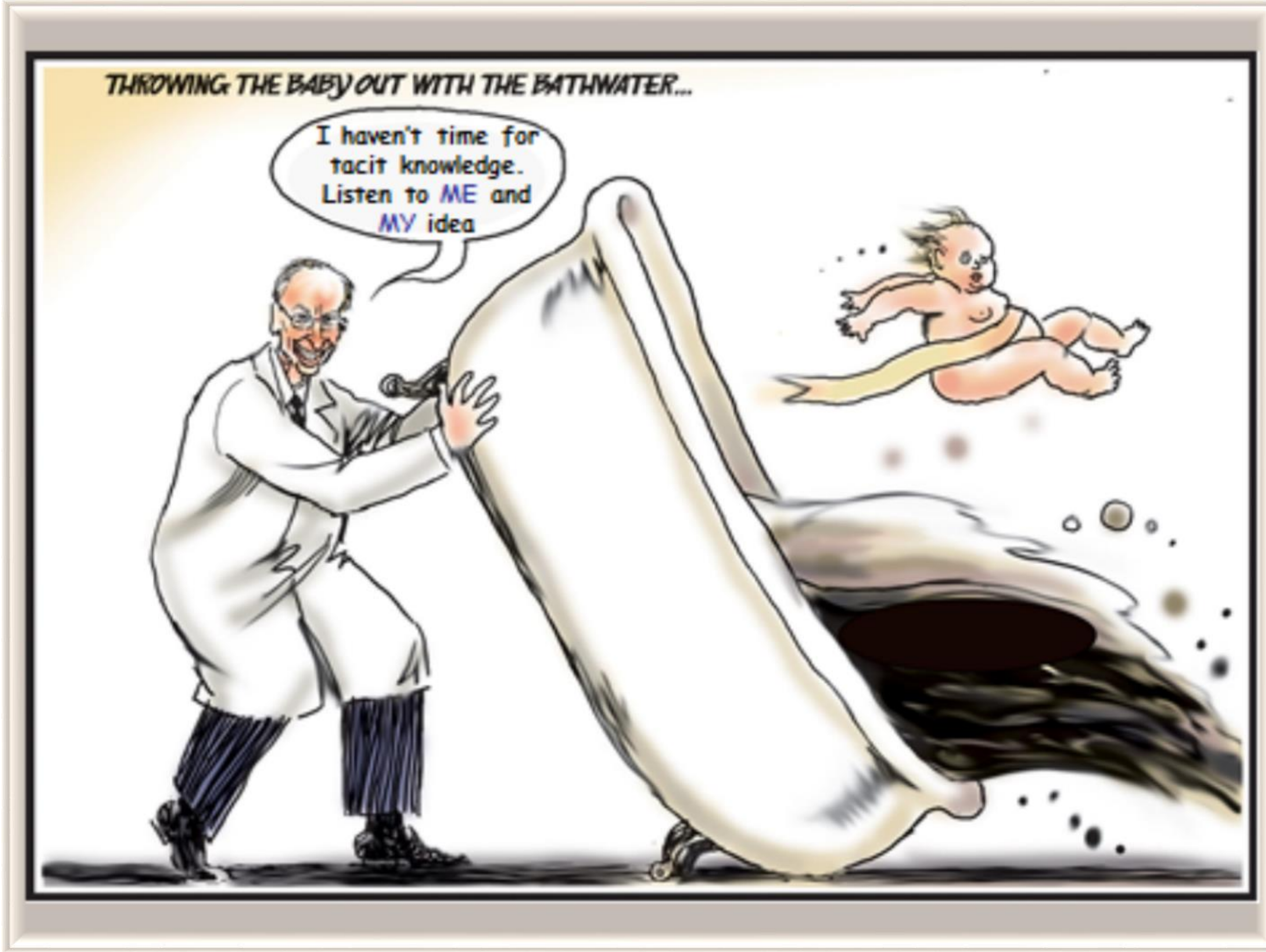
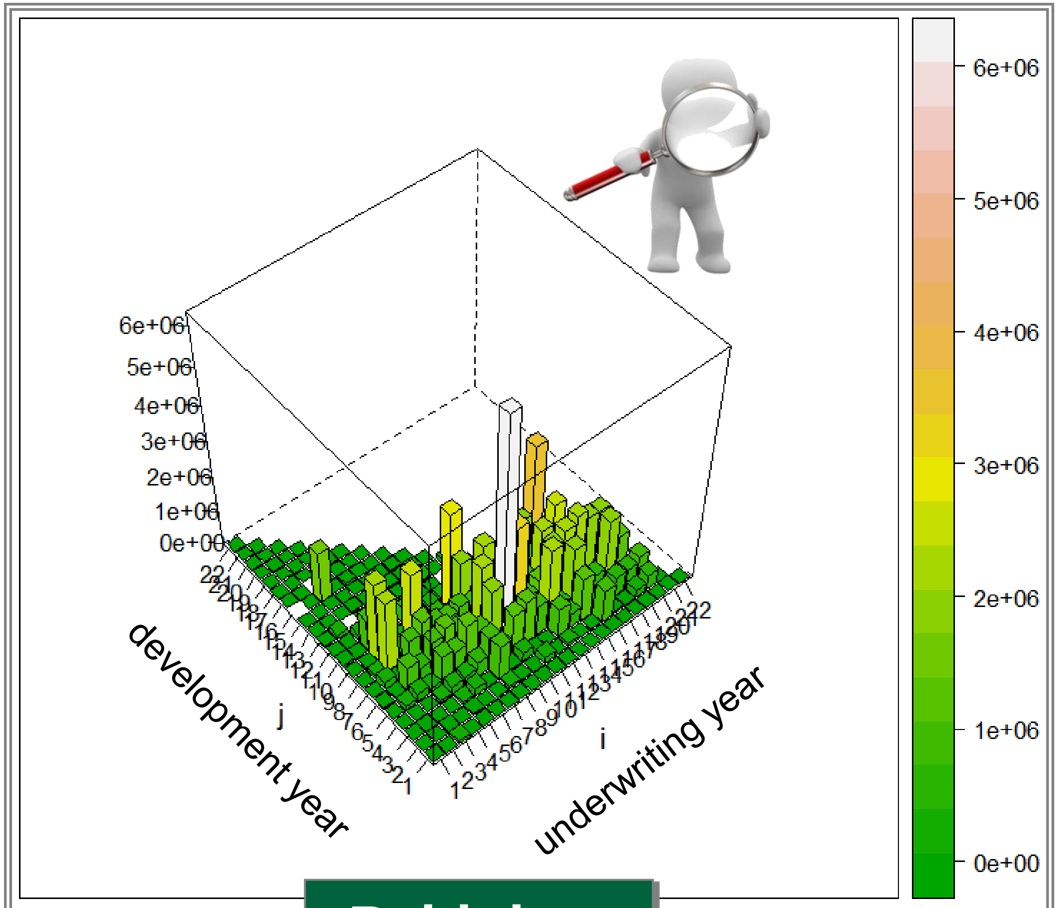
Scene 1

Scene 2


THROWING THE BABY OUT WITH THE BATHWATER...
I haven't time for tacit knowledge. Listen to ME and MY idea

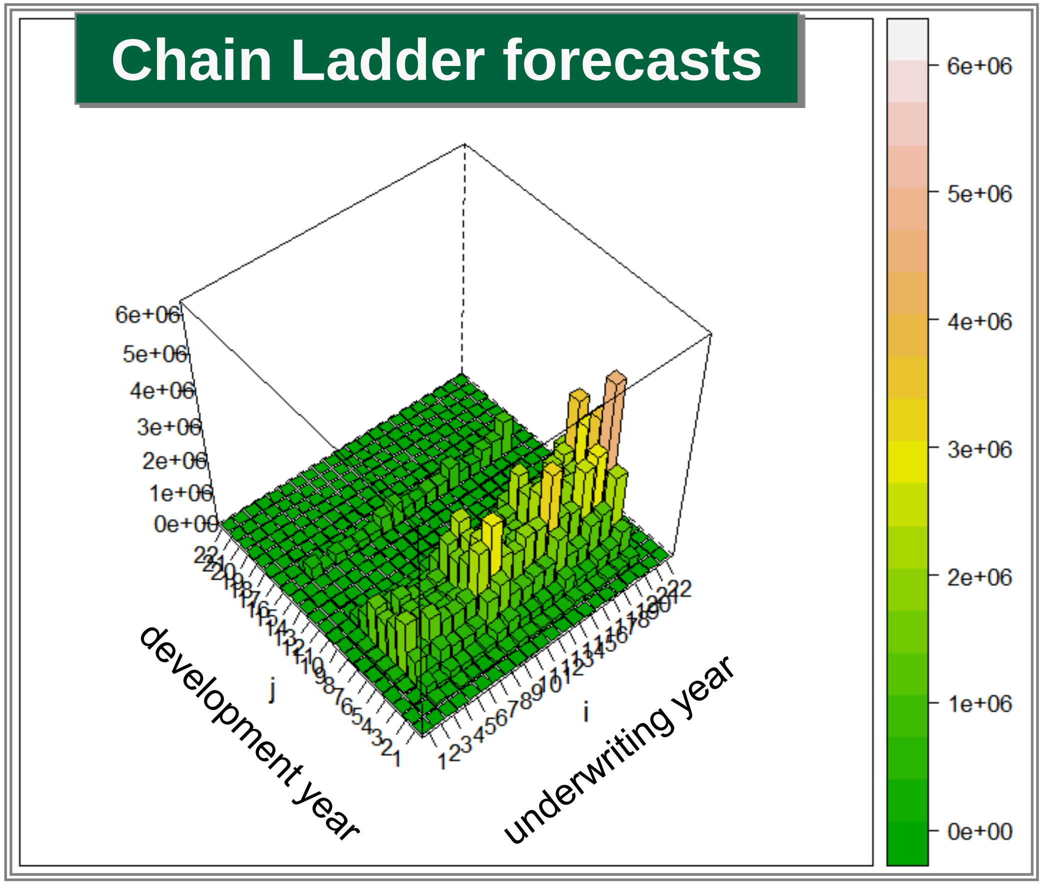
Scene 3

Aggregate or continuous?

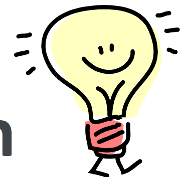


Paid data

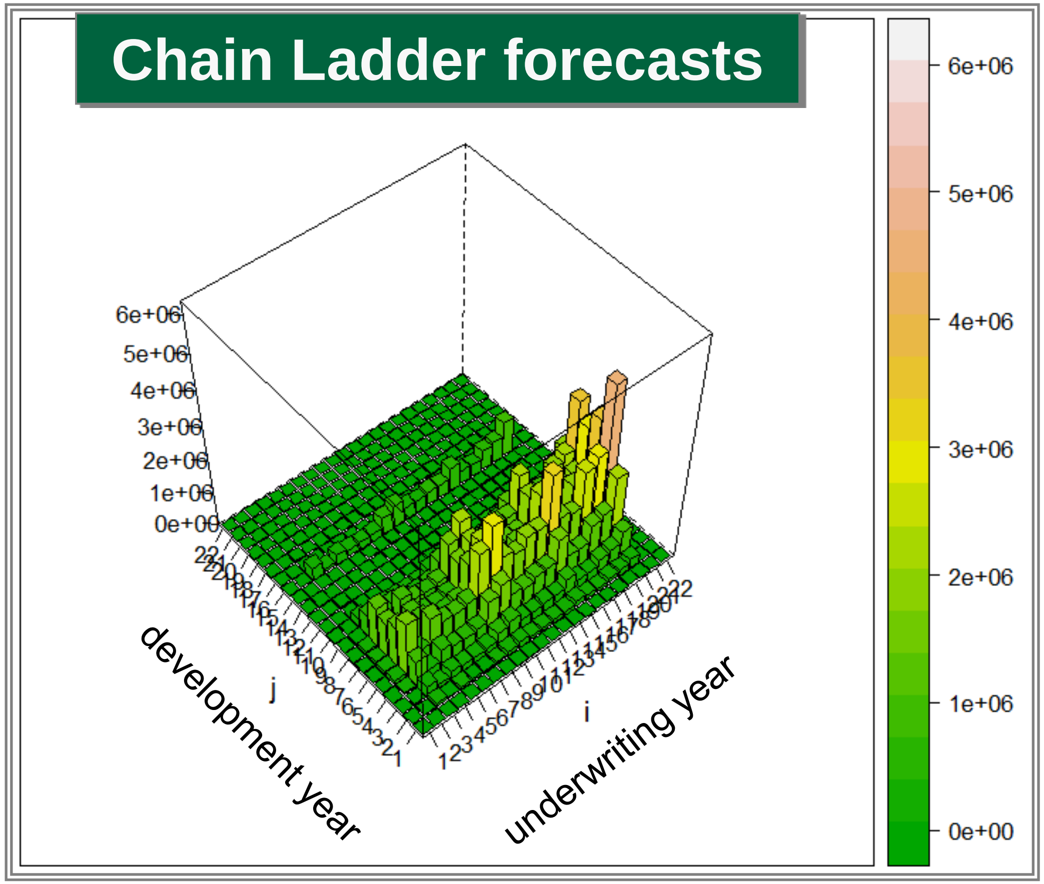
What did we learn?



…Chain Ladder is a structured histogram



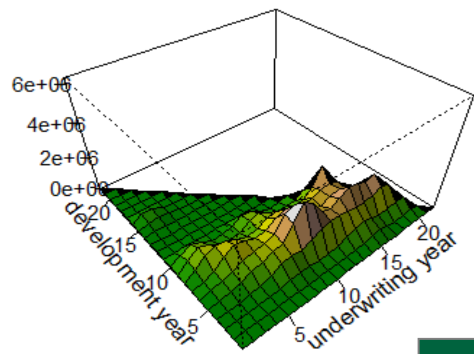
What do we know?



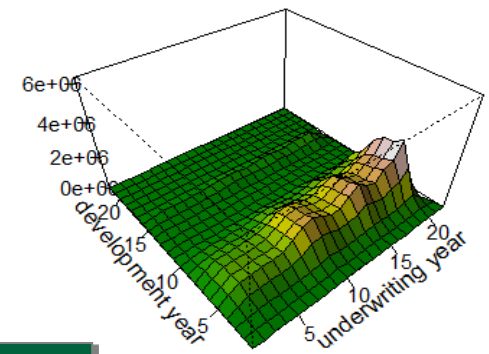
…histograms are not efficient!!! 

What should we do?

Local Linear-unstructured

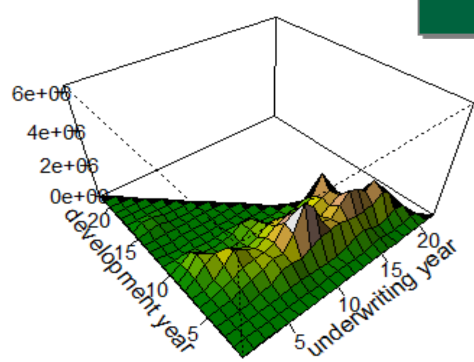


Local Linear

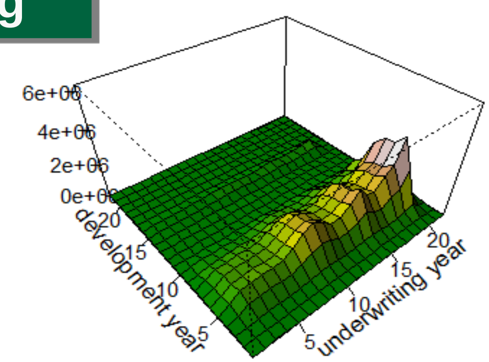


Kernel smoothing

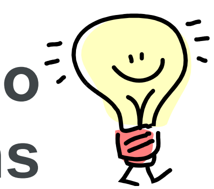
MBC-unstructured



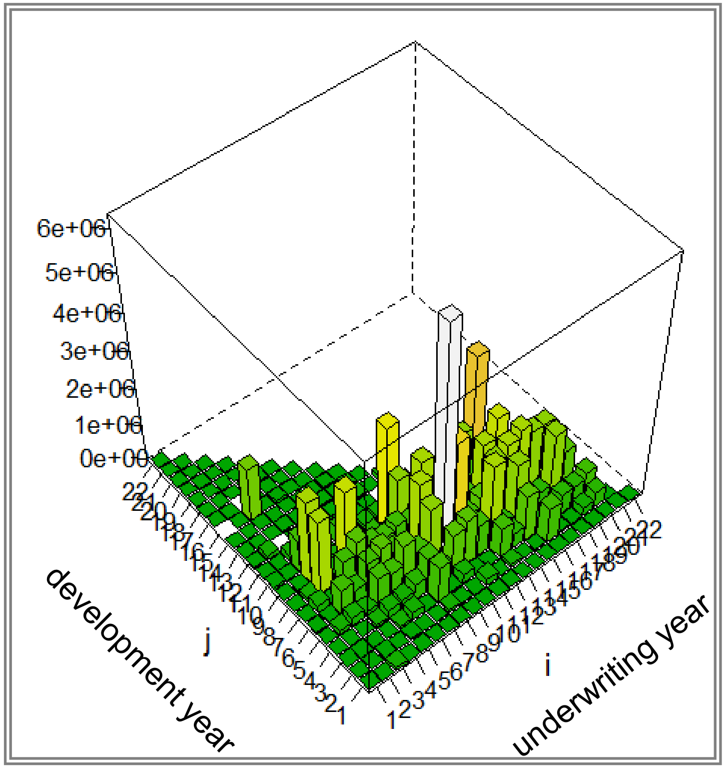
MBC

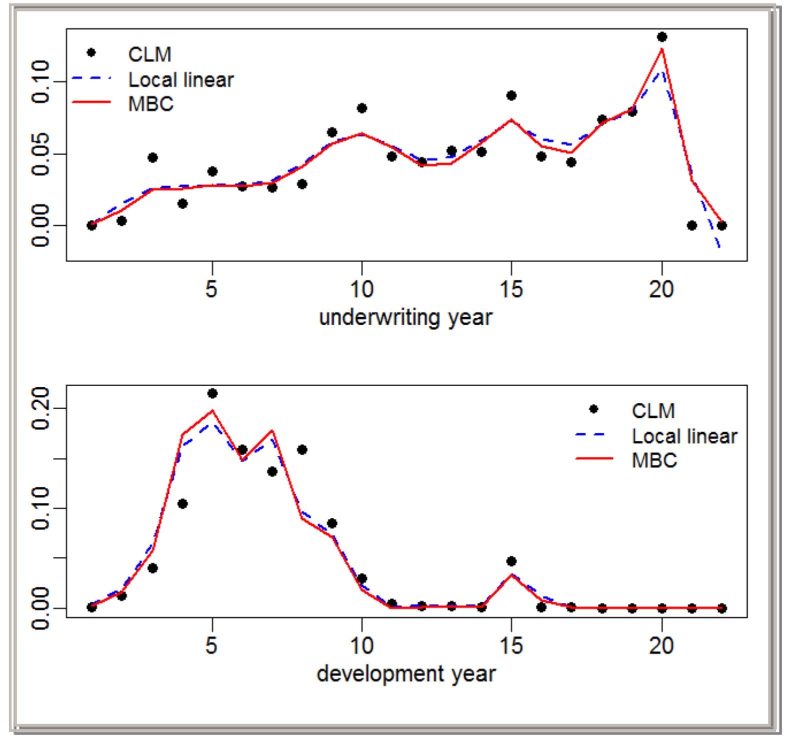


…we should do what statisticians normally do when they want to improve histograms

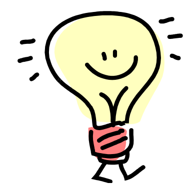


What is Chain Ladder really modelling?





…time effects



When going granular…

It would be surprising if the most important thing would not be **time effects** also here

Conclusion

Remember your **(continuous) Chain Ladder** when going granular

Warning when going granular

When you built a system from many small systems you get BIAS

Where to go from here?

A. Double Chain Ladder

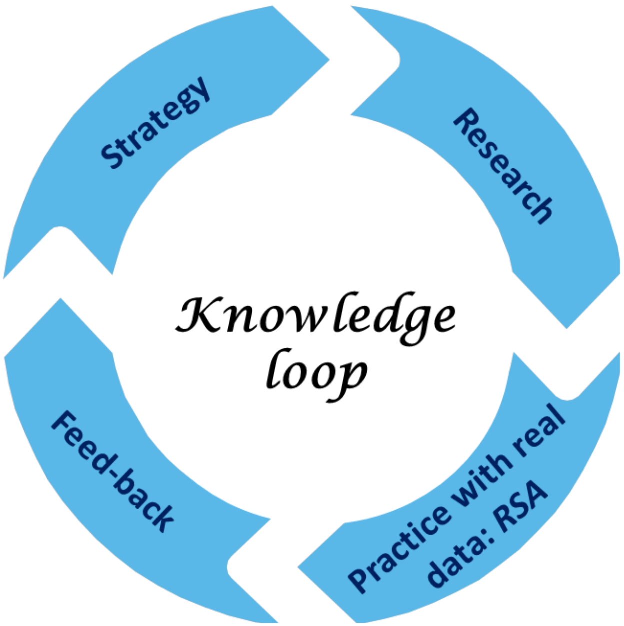
B. Double Chain Ladder and Bornhuetter-Ferguson

C. Adding Prior Knowledge to Double Chain Ladder

D. Continuous versions of A, B and C

… doing mathematical statistical theory on optimizing reserving type of structured models

Background



2010 Including Count Data in Claims Reserving
2011 Cash flow simulation for a model of outstanding liabilities based on claim amounts and claim numbers
2012 Double Chain Ladder

2012 Statistical modelling and forecasting in Non-life insurance

2013 Double Chain Ladder and Bornhuetter-Ferguson

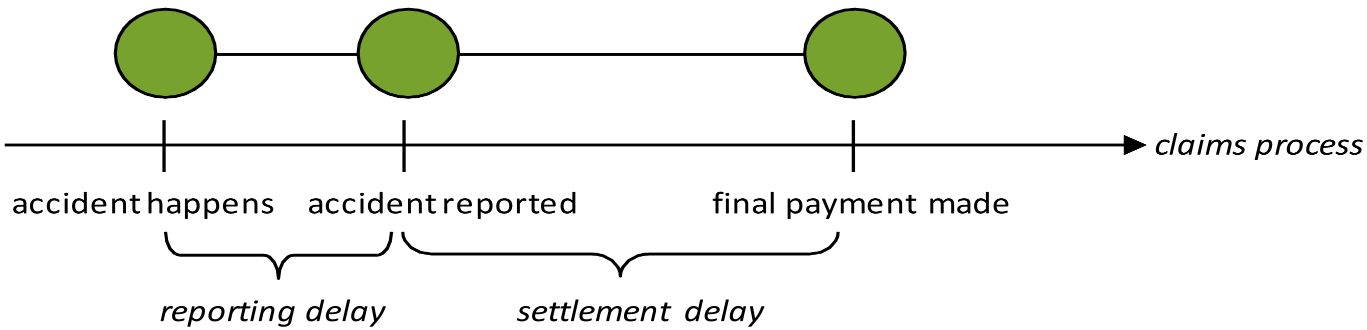
2013 Double Chain Ladder, Claims Development Inflation and Zero Claims

2014 RBNS preserving Double Chain Ladder (submitted)

Our aim: a package implementing recent research developments

The problem: the claims reserving exercise

The life of an individual claim in the general claims process:



Three categories of claim:

- Incurred but not reported, **IBNR**
- Reported but not settled, **RBNS**
- Reported and paid.

The problem: the claims reserving exercise

The objectives:

- How large **future claims payments** are likely to be
- The **timing** of future claim payments
- The **distribution** of possible outcomes: future **cash-flows**.

Framework: Double Chain Ladder

What is Double Chain Ladder?

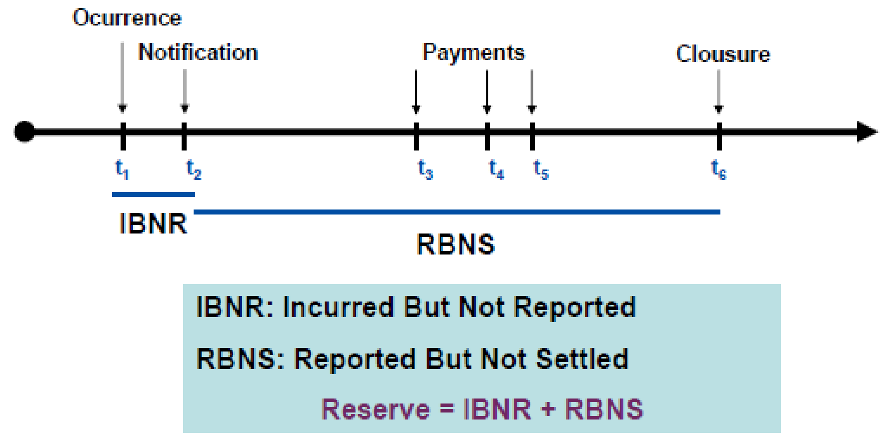
A firm statistical model which breaks down the chain ladder estimates into individual component

Why?

- Connection with classical reserving (tacit knowledge)
- RBNS and IBNR claims
- The distribution: full cash-flow

What is required?

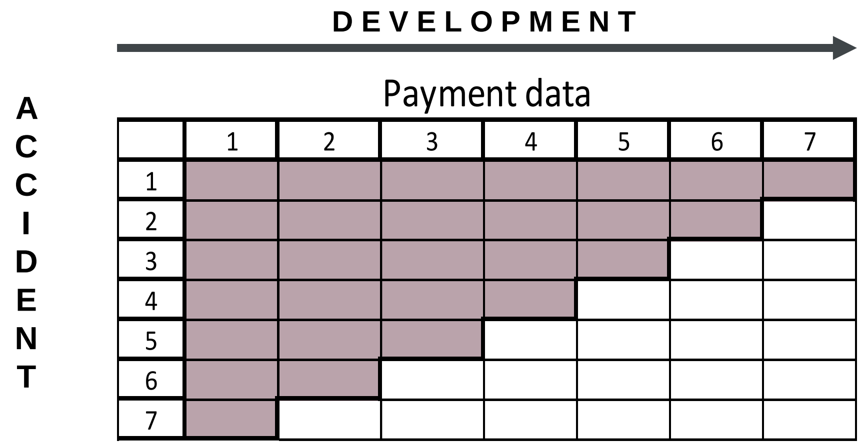
It works on **run-off triangles** (adding expert knowledge if available).



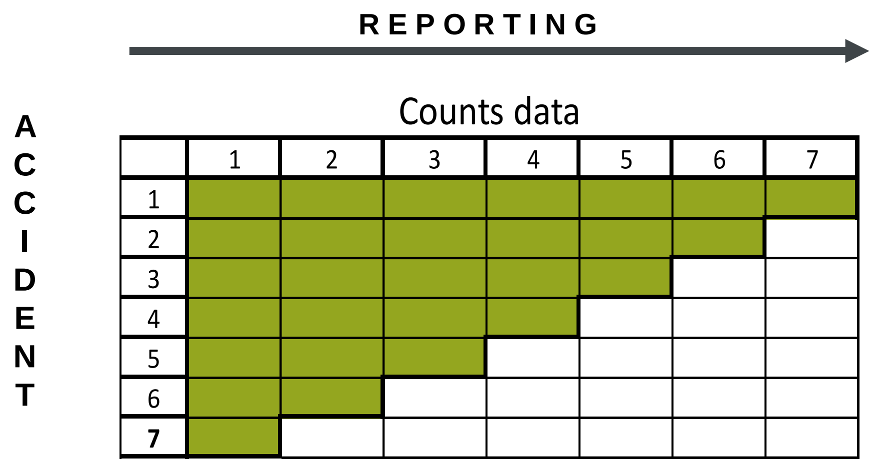
The modelled data: two run-off triangles

We model annual/quarterly run-off triangles:

- Incremental aggregated payments **(Paid triangle)**

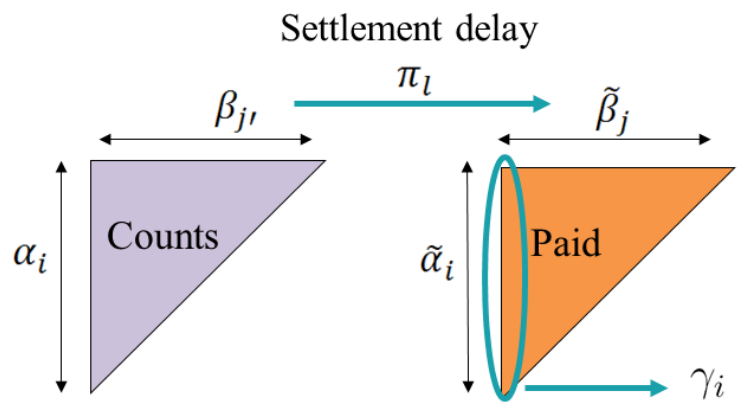


- Incremental aggregated counts data, which is assumed to be fully run off. **(Counts triangle)**



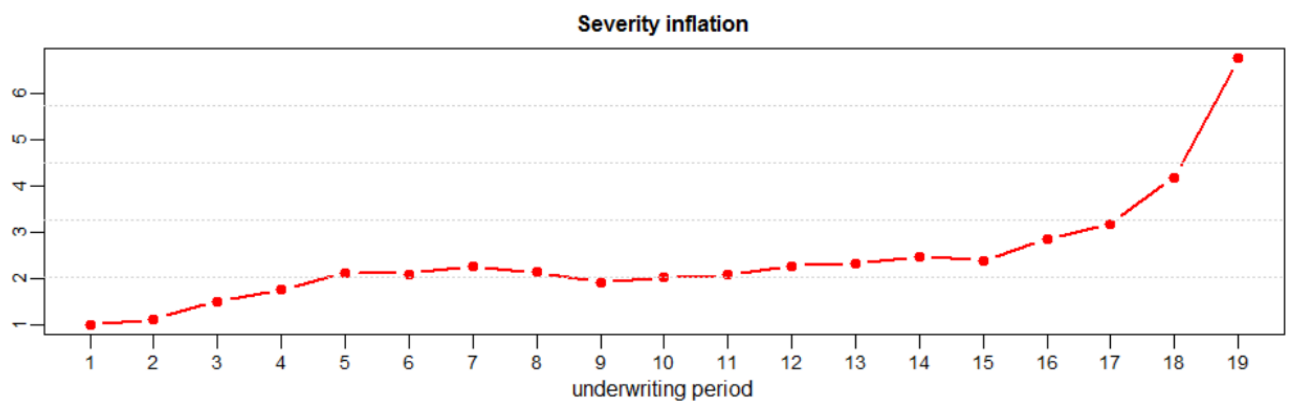
The Double Chain Ladder Model

Parameters involved in the model:

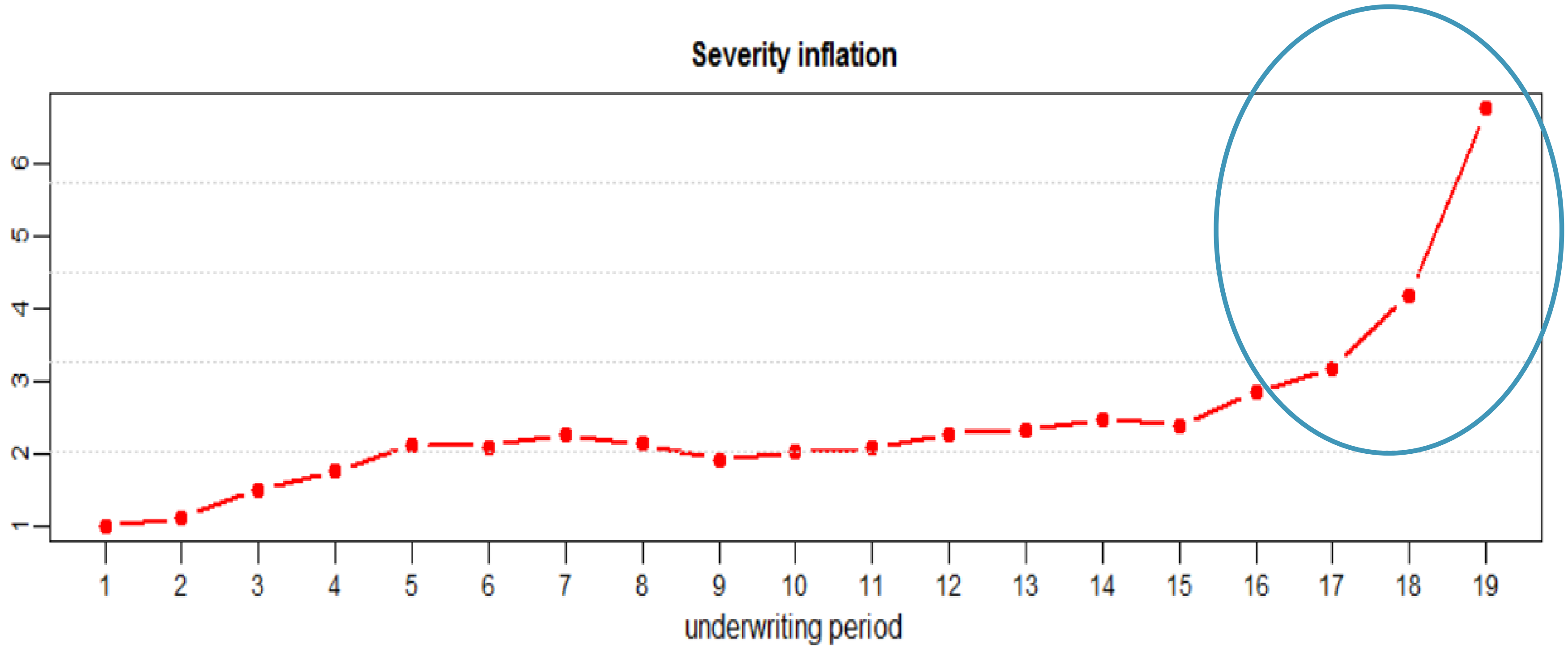


Severity mean: $\gamma_i \times \mu$

variance: $\gamma_i^2 \times \sigma^2$



The Double Chain Ladder Model



The Double Chain Ladder Model

The payments triangle

A/Y	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	51,645	513,057	747,581	554,656	426,090	211,996	212,916	16,199	9,091	36,933	0	0	0	0	0	0	0	0	0
2	143,607	1,006,115	910,371	735,878	593,070	765,997	614,642	245,482	116,065	14,633	36,298	164,636	14,601	66,826	6,415	11,400	0	0	
3	345,758	1,467,254	1,291,694	1,236,995	1,127,060	779,156	391,920	844,780	94,346	229,871	12,232	11,210	20,826	84,329	9,938	16,898	0		
4	408,108	1,875,253	1,809,624	1,859,877	1,806,412	1,422,161	761,701	306,739	109,769	139,582	53,448	36,557	0	6,731	0	0			
5	711,788	3,253,701	2,695,979	2,592,550	3,376,797	2,100,946	923,045	434,936	124,256	29,942	23,026	324	58,834	31,180	12,306				
6	941,448	3,614,819	3,273,886	4,479,163	3,841,136	2,032,530	1,241,700	471,996	120,135	59,047	5,081	295	9,393	0					
7	1,221,479	5,814,000	5,904,668	7,112,406	5,320,976	2,425,835	856,998	196,958	133,568	40,099	11,797	65,669	98,728						
8	1,684,782	8,163,947	7,609,088	7,722,323	6,298,256	1,981,161	830,186	580,355	197,501	124,446	63,687	28,557							
9	2,253,183	9,479,779	7,696,767	8,260,492	5,871,622	2,339,555	1,099,429	363,351	147,355	43,520	13,782								
10	2,042,830	8,791,743	9,169,217	7,864,324	5,894,987	1,977,707	722,425	245,391	59,786	-1,390									
11	1,570,388	9,961,564	9,669,606	8,024,282	6,120,733	2,391,815	617,560	97,794	70,961										
12	1,455,847	9,182,448	8,261,734	8,373,519	4,994,670	1,885,764	882,915	241,387											
13	1,128,853	7,675,536	8,515,497	6,467,241	4,505,204	1,502,376	460,521												
14	1,380,818	11,547,624	8,890,421	7,964,029	4,951,038	1,980,364													
15	2,195,835	12,381,318	10,390,839	7,516,444	4,968,713														
16	2,068,049	14,178,820	11,164,349	7,740,463															
17	1,747,083	11,599,608	8,808,101																
18	3,294,583	15,210,026																	
19	4,664,157																		

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2	143,607	1,006,115	910,371	735,878	593,070	765,997	614,642	245,482	116,065	14,633	36,298	164,636	14,601	66,826	6,415	11,400	0	0	
3	345,758	1,467,254	1,291,694	1,236,995	1,127,060	779,156	391,920	844,780	94,346	229,871	12,232	11,210	20,826	84,329	9,938	16,898	0		
4	408,108	1,875,253	1,809,624	1,859,877	1,806,412	1,422,161	761,701	306,739	109,769	139,582	53,448	36,557	0	6,731	0	0			
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17	1,747,083	11,599,608	8,808,101																
18	3,294,583	15,210,026																	
19	4,664,157																		

The Double Chain Ladder Model

The reserve per underwriting year

	Reserve	Proportion of total reserve
1	3	0.00
2	899	0.00
3	622	0.00
4	3,549	0.00
5	23,190	0.00
6	44,363	0.00
7	160,618	0.00
8	272,556	0.00
9	383,729	0.00
10	406,193	0.00
11	550,067	0.00
12	654,305	0.00
13	996,606	0.01
14	2,572,927	0.01
15	5,473,730	0.03
16	15,460,375	0.08
17	21,755,702	0.11
18	44,471,818	0.23
19	98,978,239	0.51

The Double Chain Ladder Model

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9	383,729	0.00
10	406,193	0.00
11	550,067	0.00
12	654,305	0.00
13	996,606	0.01
14	2,572,927	0.01
15	5,473,730	0.03
16	15,460,375	0.08
17	21,755,702	0.11
18	44,471,818	0.23
19	98,978,239	0.51

= 85% of total reserve

The Double Chain Ladder Model

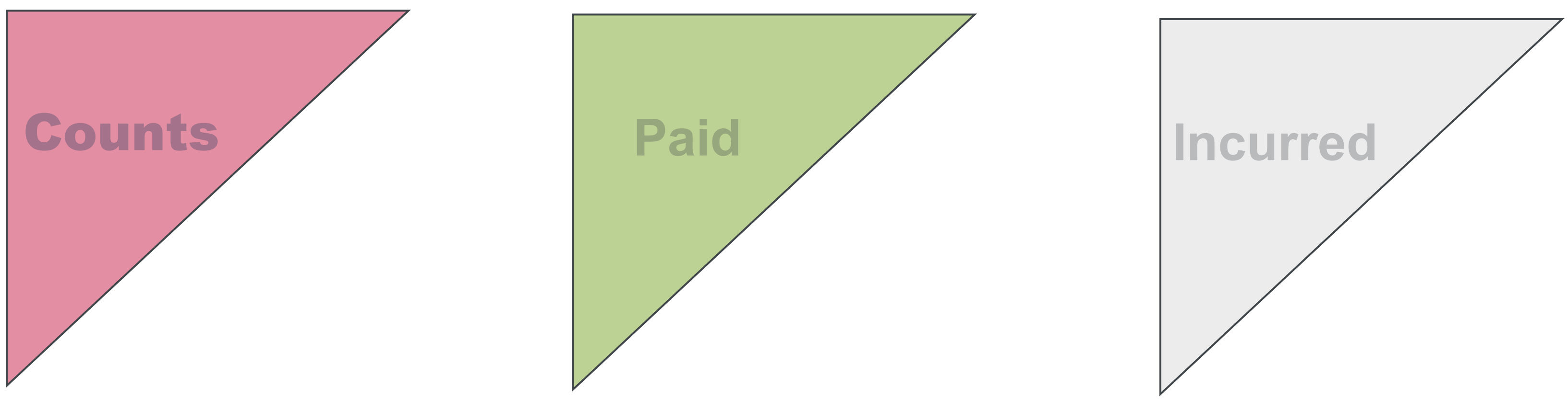
Summary of the major drawback of classical Chain Ladder (and thus the basic Double Chain Ladder method):

- The lack of data in the most recent underwriting years yields to a severity inflation estimation being too instable and thus not trustable in those most recent years

- Even worse, these most recent underwriting years account for the major part of the reserve.

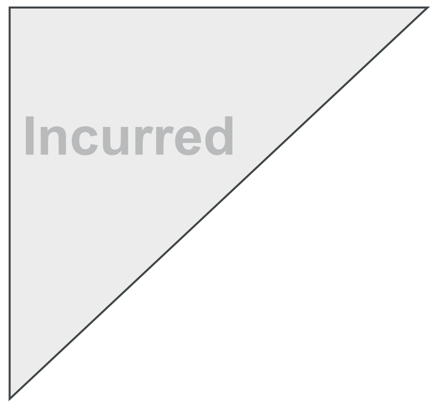
Double Chain-Ladder and Bornhutter-Ferguson (BDCL) and RBNS Preserving Double Chain Ladder (PDCL)

Solution: Incorporate **Case estimates (expert knowledge)**:



Double Chain-Ladder and Bornhutter-Ferguson (BDCL) and RBNS Preserving Double Chain Ladder (PDCL)

The incurred triangle:



- It is not data, but a mixture of data and expert knowledge
- It contains payments and case estimates of RBNS claimes.

Double Chain-Ladder and Bornhutter-Ferguson (BDCL) and RBNS Preserving Double Chain Ladder(PDCL)

- From the incurred data one can extract a more robust estimator of the severity inflation in the DCL model (BDCL)

- From the incurred triangle, one can extract the RBNS part estimated by the case department (PDCL)

- The RBNS case estimates differ from the DCL RBNS estimates.

Double Chain-Ladder and Bornhutter-Ferguson (BDCL) and RBNS Preserving Double Chain Ladder (PDCL)

The values of the severity inflation estimates in the most recent calendar years result in a big difference between DCL and case estimates based RBNS numbers

BDCL and PDCL are two alternative strategies to the volatile paid chain ladder model

Double Chain Ladder provides the technical starting point for all these adjustments of the chain ladder method

	RBNS via DCL	RBNS via case estimates	diff/ultimate
1	3	0	0.000001079
2	899	0	0.000165047
3	622	4,011	0.000425492
4	3,549	-9,524	0.001233359
5	23,190	36,500	0.000811942
6	42,428	5,000	0.001858854
7	157,798	1,381	0.005326865
8	267,289	92,278	0.004922006
9	376,629	57,627	0.008405282
10	393,672	190,335	0.005469987
11	531,115	241,142	0.007420978
12	624,839	1,444	0.017349014
13	948,925	1,210,062	0.008355894
14	2,477,369	2,719,667	0.006167349
15	5,326,310	6,123,466	0.018570090
16	15,212,072	9,249,185	0.117815545
17	21,262,118	13,099,480	0.185892648
18	42,552,010	24,828,096	0.281437273
19	74,097,651	31,454,377	0.411446240

Double Chain-Ladder and Bornhutter-Ferguson (BDCL)

What does Double Chain Ladder and Bornhutter-Ferguson (BDCL) do?

- It shows that the severity inflation can be estimated from the incurred data
- It replaces the fragile severity inflations coming from DCL by the more robust severity inflation of the incurred data
- The paper argues that BDCL extracts the most important information from the incurred data
- A number of emperical studies have shown that BDCL indeed is a robust and reliable reserving method.

Classical Incurred reserving embedded in Double Chain Ladder (IDCL)

What does IDCL do?

- First a classical incurred reserving method is run
- Then severity inflation parameters are changed in DCL model, such that this model exactly produces the classial incurred reserving forecasts
- Now we have the classical incurred forecasts, but embedded in the DCL model
- One can therefore compare classical reserving of paid and incurred methods through the Double Chain Ladder framework.

RBNS Preserving Double Chain Ladder (PDCL)

What does RBNS preserving Double Chain Ladder (PDCL) do?

- PDCL preserves the RBNS case estimates
- The DCL parameters estimates are also adjusted accordingly
- Therefore, PDCL estimates the exact RBNS case estimates but also corrects the IBNR estimates taking advantage of the RBNS information
- PDCL is one among many potential extensions of double chain ladder that we are working at. It is not fully implemented in theory or software. The final implementation avaits a few more research results, fx on bootstrapping.

DCL, BDCL, IDCL and PDCL embedded in the same framework

Validation is now possible!

- Classical paid reserves could not be compared to classical incurred reserving techniques via statistical validation
- In the new Double Chain Ladder framework these two methods can now be validated and compared
- BDCL and PDCL and all the other potential extensions of DCL can be validated in the same way.

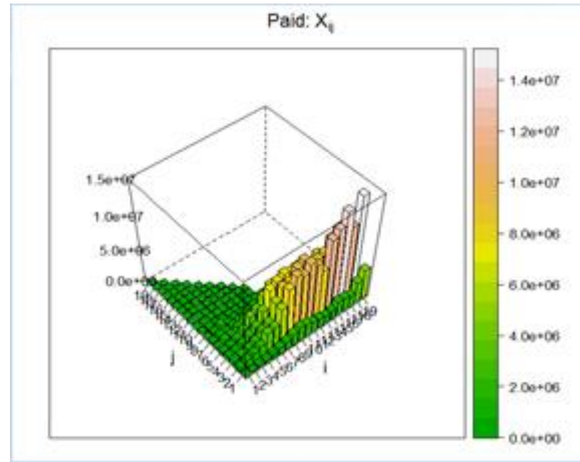
Visualizing the data: the histogram

DEVELOPMENT

A C C I D E N T

Payment data

	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

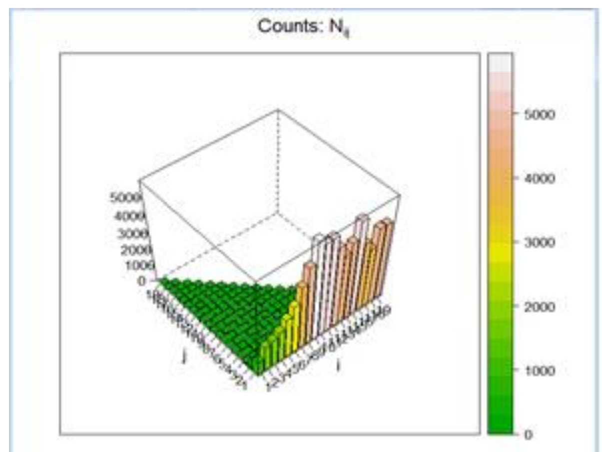


REPORTING

A C C I D E N T

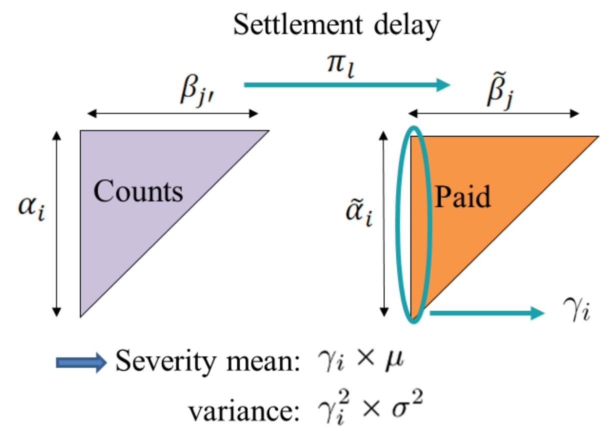
Counts data

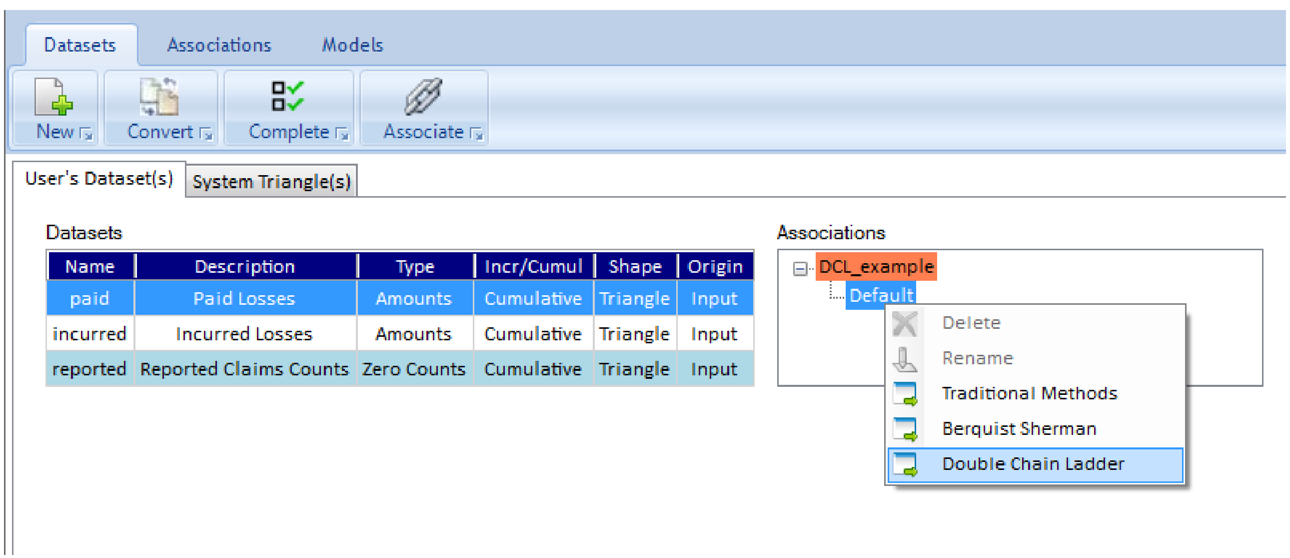
	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							



DCL estimation in RES-TIMATOR®

- Data required in Res-timator® to run DCL:
  - Paid Cumulative Losses
  - Incurred Cumulative Losses
  - Reported Cumulative Counts
- Create an association using these datasets. The software will convert automatically the associated datasets into their incremental form
- Run DCL to view results.





The kernel: DCL parameter estimation in RES-TIMATOR® (1)

The parameters under DCL tab enables the user to visualize the break down of the double chain ladder parameters (distributional model). The Res-timator® provides the option to edit the parameters (percentage change in inflation, maximum delay and delay probability) based on his insights.

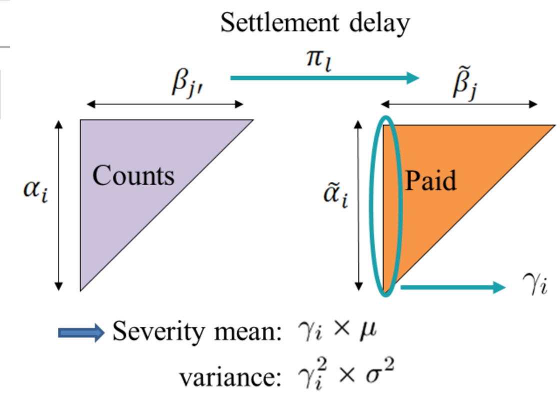
Age to Age Factors | DCL | BDCL | Validation | Bootstrapping

Reserves | Parameters

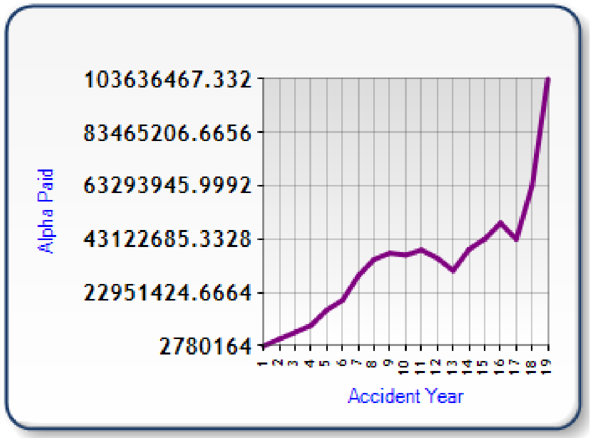
A/Y	Alpha Counts	Alpha Paid	Claim Severity Inflation	Percentage Change in Inflation(%)	Inflation Adjusted Mean	Inflation Adjusted Variance	Devel Period	Beta Counts	Beta Paid	Delay Probability
1	1,078.000	2,780,164.000	1.000	-	2,579.078	283,538,878.919	0	0.760	0.045	0.059
2	1,890.000	5,446,036.000	1.117	11.729	2,881.586	353,953,911.543	1	0.210	0.248	0.310
3	2,066.000	7,964,267.000	1.495	33.782	3,855.035	633,490,777.640	2	0.019	0.220	0.203
4	2,353.000	10,595,962.000	1.746	16.816	4,503.304	864,462,659.526	3	0.006	0.201	0.200
5	3,015.000	16,386,921.643	2.107	20.696	5,435.292	1,259,300,657.011	4	0.002	0.153	0.139
6	3,727.358	20,125,249.314	2.094	-0.659	5,399.493	1,242,766,550.240	5	0.001	0.068	0.044
7	5,057.486	29,341,294.389	2.250	7.450	5,801.728	1,434,823,340.980	6	0.001	0.031	0.023
8	6,482.961	35,529,146.749	2.125	-5.536	5,480.550	1,280,359,798.058	7	0.001	0.015	0.009
9	7,727.447	37,921,082.119	1.903	-10.457	4,907.469	1,026,593,997.459	8	0.000	0.005	0.002
10	7,134.404	37,161,287.389	2.020	6.142	5,208.898	1,156,579,377.362	9	0.000	0.004	0.003
11	7,318.549	39,077,085.659	2.070	2.509	5,339.616	1,215,356,399.084	10	0.000	0.001	0.000
12	6,152.041	35,962,170.550	2.267	9.479	5,845.740	1,456,675,178.539	11	0.000	0.002	0.003
13	5,241.984	31,305,635.909	2.316	2.165	5,972.274	1,520,418,188.716	12	0.000	0.002	0.002
14	6,149.972	39,250,378.017	2.475	6.867	6,382.393	1,736,403,746.384	13	0.000	0.003	0.003
15	7,027.988	43,190,100.318	2.383	-3.710	6,145.625	1,609,962,454.353	14	0.000	0.001	0.000
16	6,724.910	49,240,592.725	2.839	19.147	7,322.335	2,285,508,549.607	15	0.000	0.001	0.001
17	5,260.153	43,160,527.850	3.182	12.060	8,205.427	2,870,027,081.548	16	0.000	0.000	0.000
18	5,869.307	63,192,265.905	4.175	31.217	10,766.882	4,941,553,970.830	17	0.000	0.000	0.000
19	5,953.168	103,636,467.332	6.750	61.692	17,409.138	12,919,276,518.963	18	0.000	0.000	0.000

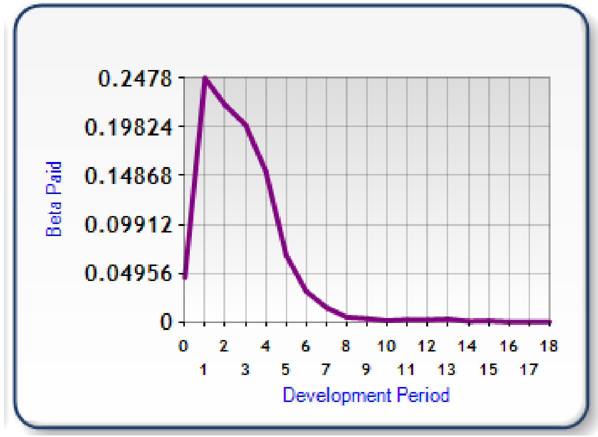
Update Values

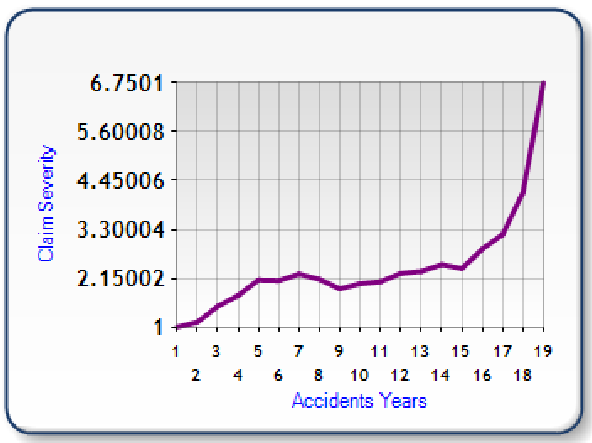
Adjusted Severity Mean	Variance
2579.07800300238	283,538,878.919

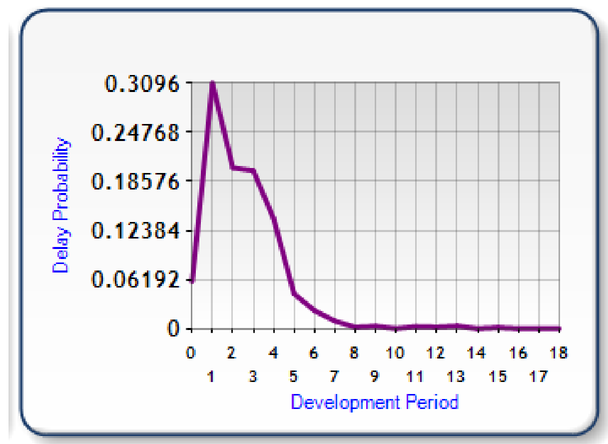


The kernel: DCL parameter estimation in RES-TIMATOR® (2)









The kernel: BDCL parameter estimation in RES-TIMATOR® (1)

The BDCL method is a modification of the DCL method. The purpose is to stabilize the results by using the more stable incurred underwriting year inflation in place of a less stable paid data underwriting year inflation. The main idea is to implement DCL also on the incurred triangle in order to obtain the inflation from the incurred data.

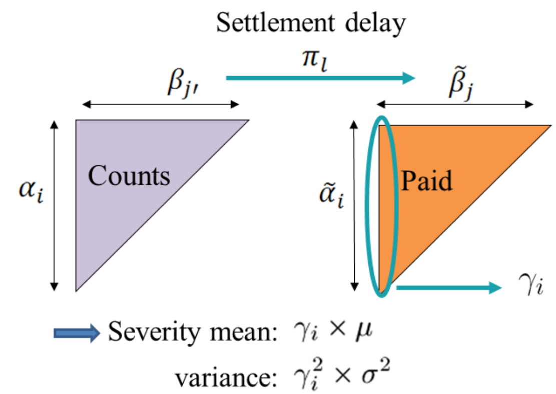
Age to Age Factors | DCL | BDCL | Validation | Bootstrapping

Reserves | Parameters

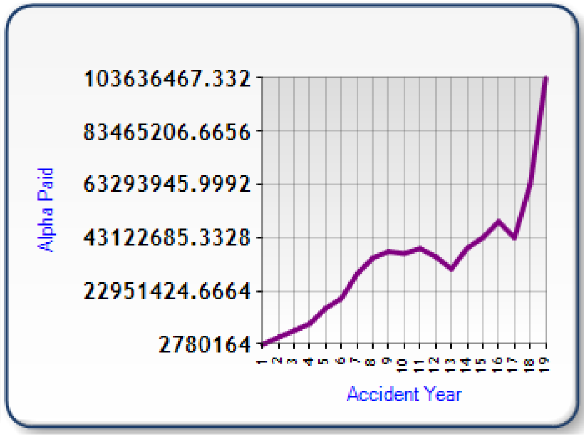
A/Y	Alpha Counts	Alpha Paid	Alpha Incurred	Claim Severity Inflation	Inflation Adjusted Mean	Inflation Adjusted Variance
1	1,078.000	2,780,164.000	2,780,164.000	1.000	2,579.078	347,074,657.955
2	1,890.000	5,446,036.000	5,446,036.000	1.117	2,881.586	433,268,387.209
3	2,066.000	7,964,267.000	7,968,278.000	1.495	3,856.976	776,225,517.982
4	2,353.000	10,595,962.000	10,586,438.000	1.745	4,499.257	1,056,271,246.313
5	3,015.000	16,386,921.643	16,389,768.528	2.108	5,436.236	1,542,022,115.704
6	3,727.358	20,125,249.314	20,104,253.401	2.091	5,393.859	1,518,074,910.949
7	5,057.486	29,341,294.389	29,211,995.279	2.240	5,776.162	1,740,895,190.057
8	6,482.961	35,529,146.749	35,375,620.408	2.116	5,456.868	1,553,749,101.179
9	7,727.447	37,921,082.119	37,621,542.584	1.888	4,868.704	1,236,860,615.721
10	7,134.404	37,161,287.389	36,922,588.250	2.007	5,175.440	1,397,618,057.418
11	7,318.549	39,077,085.659	38,699,914.578	2.050	5,288.078	1,459,115,320.371
12	6,152.041	35,962,170.550	35,120,206.661	2.214	5,708.877	1,700,573,234.465
13	5,241.984	31,305,635.909	31,185,544.952	2.307	5,949.363	1,846,864,178.794
14	6,149.972	39,250,378.017	38,743,288.383	2.443	6,299.936	2,070,934,063.161
15	7,027.988	43,190,100.318	41,885,602.845	2.311	5,960.005	1,853,476,780.975
16	6,724.910	49,240,592.725	41,407,141.777	2.387	6,157.460	1,978,322,540.956
17	5,260.153	43,160,527.850	33,838,374.273	2.494	6,433.153	2,159,443,102.786
18	5,869.307	63,192,265.905	41,623,670.356	2.750	7,091.962	2,624,380,002.450
19	5,953.168	103,636,467.332	43,816,396.189	2.854	7,360.398	2,826,809,801.505

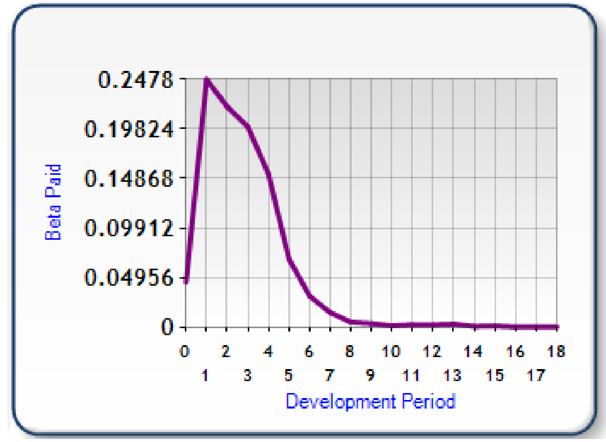
Devel Period	Beta Counts	Beta Paid	Beta Incurred	Delay Probability
0	0.760	0.045	0.824	0.059
1	0.210	0.248	0.217	0.310
2	0.019	0.220	0.001	0.203
3	0.006	0.201	0.030	0.200
4	0.002	0.153	-0.032	0.139
5	0.001	0.068	-0.023	0.044
6	0.001	0.031	-0.009	0.023
7	0.001	0.015	-0.004	0.009
8	0.000	0.005	-0.003	0.002
9	0.000	0.004	-0.001	0.003
10	0.000	0.001	-0.001	0.000
11	0.000	0.002	0.000	0.003
12	0.000	0.002	0.000	0.002
13	0.000	0.003	0.000	0.003
14	0.000	0.001	0.001	0.000
15	0.000	0.001	-0.001	0.001
16	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	0.000

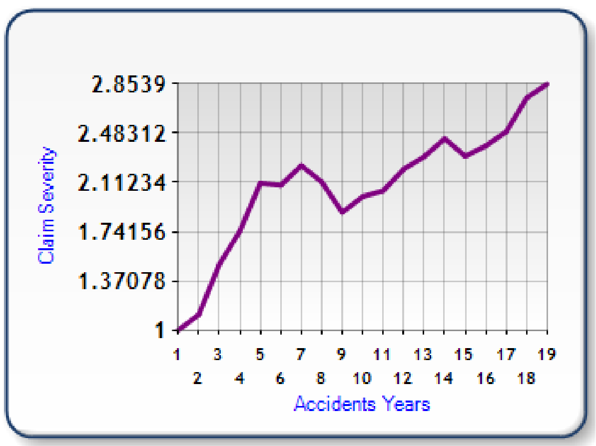
Adjusted Severity Mean	Variance
2579.07800300238	347,074,658.0

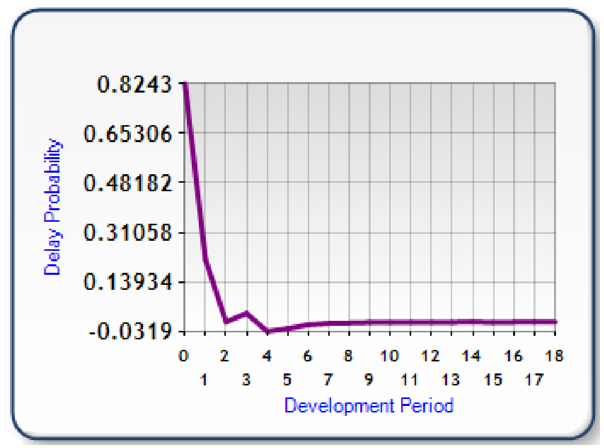


The kernel: BDCL parameter estimation in RES-TIMATOR® (2)









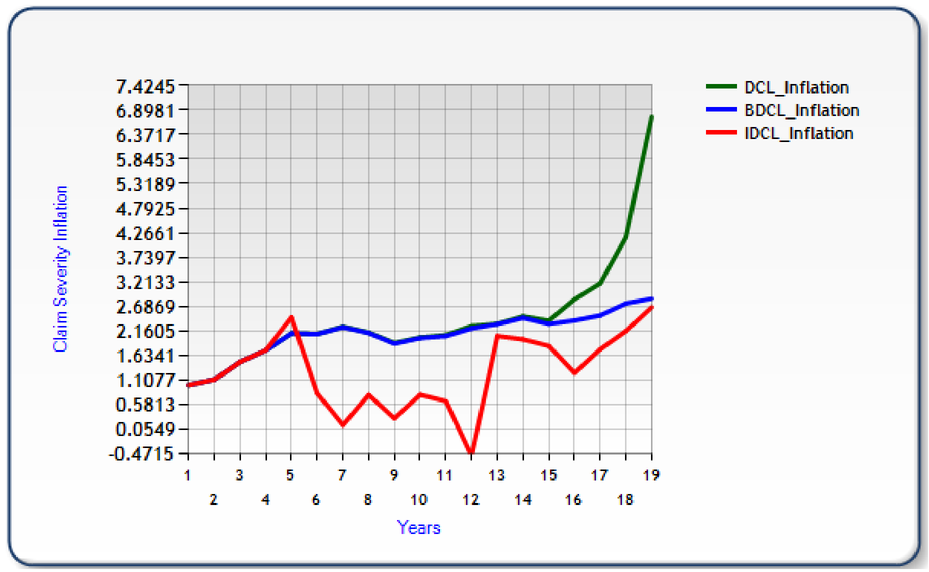
The kernel: IDCL parameter estimation in RES-TIMATOR®

The IDCL method is as well a modification of the DCL method. Practitioners often regard the incurred reserve to be more realistic. In this respect, IDCL method is introduced to estimate DCL model in such a way to completely replicate the Chain Ladder reserve from incurred data. The main idea is to rescale the inflation parameter as estimated from DCL and replace it in the final calculations while the rest of the parameters remain unchanged. Note that in Res-timator® there is the option to update the inflation in IDCL method.

Percentage Change in Inflation(%)	Inflation Adjusted Mean	Inflation Adjusted Variance
-	2,579	64,489,973,847
11.729	2,882	80,505,638,540
33.782	3,855	144,085,367,897
16.816	4,503	196,619,153,313
40.544	6,329	388,373,894,645
-66.427	2,125	43,776,590,574
-82.575	370	1,329,168,363
452.106	2,044	40,515,781,049
-64.079	734	5,227,928,374
179.893	2,055	40,955,784,321
-17.594	1,694	27,811,778,749
-179.780	-1,351	17,701,636,189
-491.460	5,289	271,261,739,524
-3.464	5,106	252,792,806,963
-7.012	4,748	218,585,548,508
-31.530	3,251	102,476,939,788
40.381	4,564	201,949,052,636
22.049	5,570	300,821,140,194
23.637	6,887	459,836,637,486

Update Values

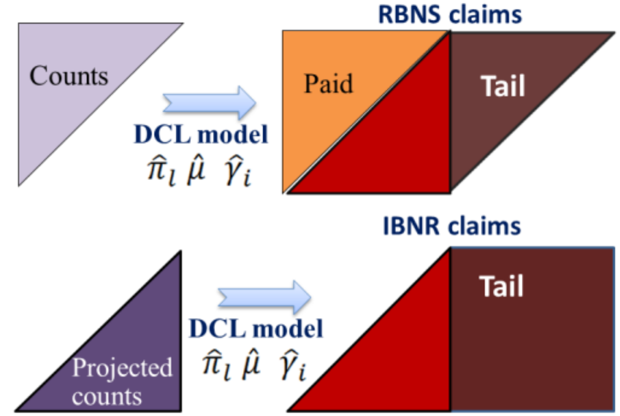
Inflation comparison chart from the 3 different DCL versions:



The best estimate: RBNS/IBNR split using DCL

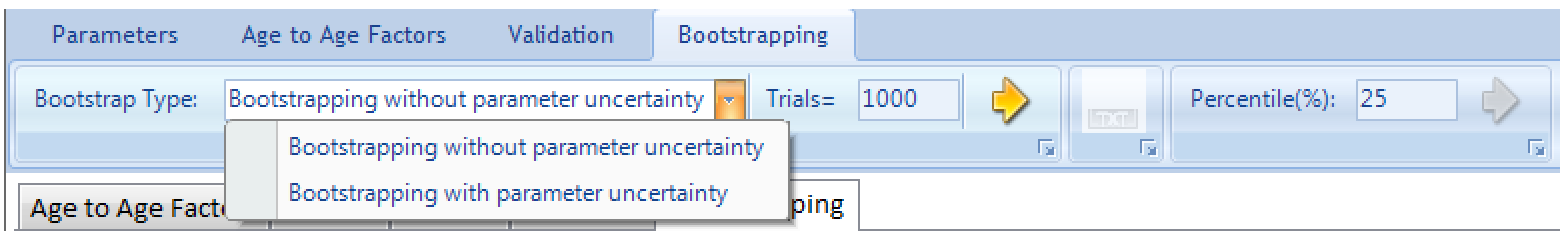
Both IBNR and RBNS cash flows are derived by pointwise prediction separately and can be presented under the reserve tab based either on accident, development or calendar year:

A/Y	IBNR	RBNS	Total Unpaid
1	0	3	3
2	0	899	899
3	0	622	622
4	0	3,549	3,549
5	0	23,190	23,190
6	1,935	42,428	44,363
7	2,821	157,798	160,618
8	5,268	267,289	272,556
9	7,099	376,629	383,729
10	12,521	393,672	406,193
11	18,951	531,115	550,067
12	29,466	624,839	654,305
13	47,680	948,925	996,606
14	95,557	2,477,369	2,572,927
15	147,419	5,326,310	5,473,730
16	248,303	15,212,072	15,460,375
17	493,584	21,262,118	21,755,702
18	1,919,808	42,552,010	44,471,818
19	24,880,587	74,097,651	98,978,239
Total	27,911,000	164,298,489	192,209,489



The full cash-flow: Bootstrapping in RES-TIMATOR® using DCL (1)

- The Bootstrapping provides the distribution of IBNR, RBNS and Total (RBNS+IBNR) Reserves by calendar year
- There are two types of Bootstrapping, one with parameter uncertainty and one without
- Number of simulations (trials) is specified by the user.



The full cash-flow: Bootstrapping in RES-TIMATOR® using DCL (2)

- A table showing a summary of the distribution: mean, std. deviation, quantiles.

Simulated Avg Triangles | Statistics

Statistics

	Simulated IBNR	Simulated RBNS	Simulated Total Unpaid
Mean	27,568,486	164,299,168	191,867,654
Std	4,450,262	9,490,450	10,411,597
Min	14,479,796	133,832,392	160,809,114
Max	50,077,806	195,326,570	226,202,537
Skewness	0	0	0
Kurtosis	1	0	0
Quantile	24,478,435	157,868,894	184,863,560

Standard Deviation Per Calendar Year

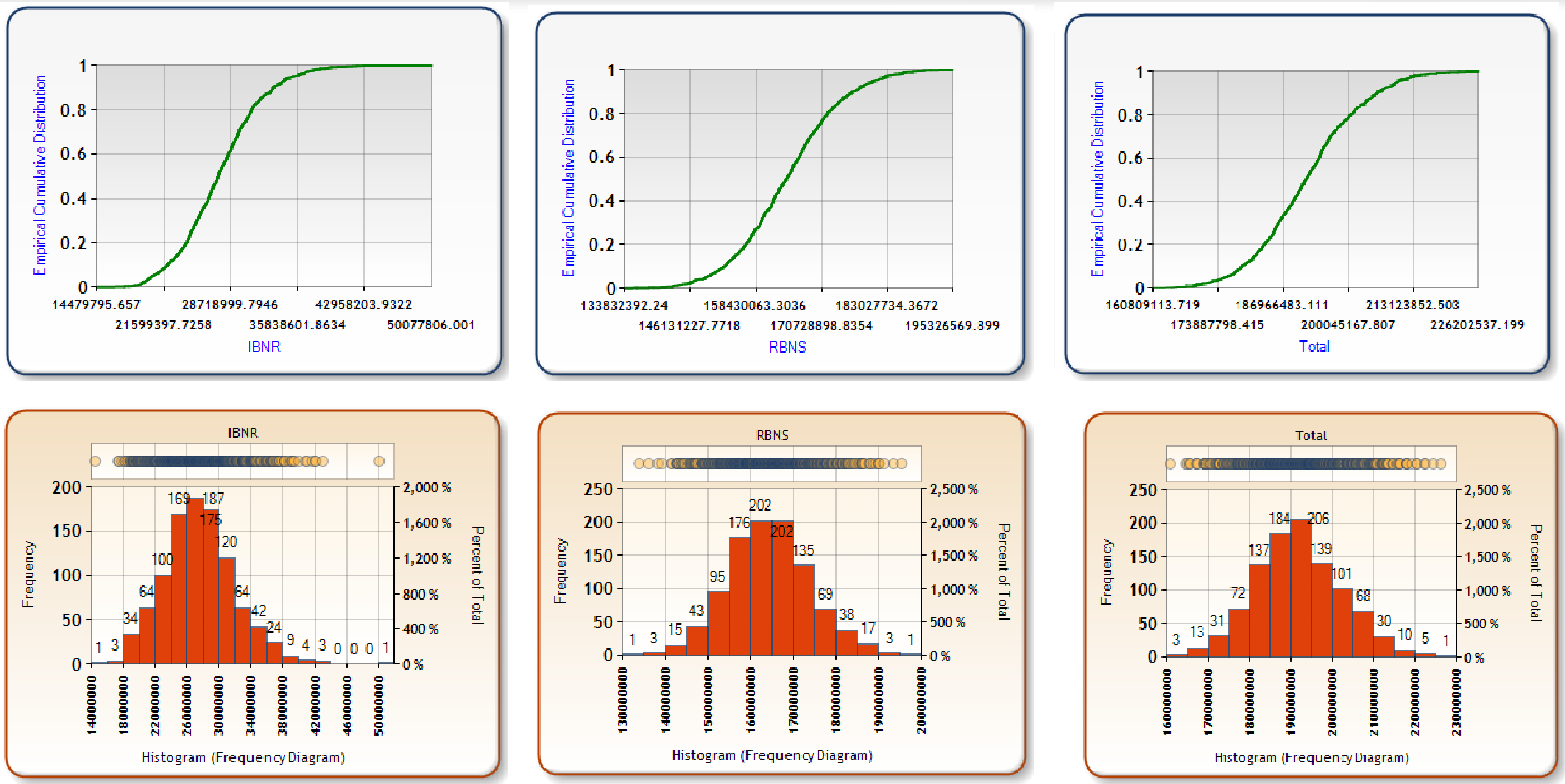
Calendar Year	Simulated IBNR	Simulated RBNS	Simulated Total
20	981,616	5,472,884	5,568,782
21	2,355,680	4,510,953	5,051,992
22	2,006,700	4,224,980	4,625,988
23	2,100,884	3,352,119	3,998,378
24	1,778,527	1,914,353	2,568,713
25	1,104,180	1,398,972	1,747,716
26	741,180	922,168	1,203,915
27	580,283	642,394	849,063
28	352,317	577,484	672,324
29	332,103	399,866	515,850
30	145,628	510,562	540,834
31	242,683	497,457	554,181
32	207,729	483,743	514,802
33	277,026	210,024	350,506
34	83,844	303,409	316,092
35	137,124	11,256	137,944
36	69,280	56,038	88,770
37	50,548	0	50,548
38	18,144	0	18,144
39	83	0	83
40	2,569	0	2,569
41	5,184	0	5,184
42	0	0	0
43	0	0	0
44	0	0	0
45	0	0	0
46	0	0	0
47	0	0	0
48	0	0	0
49	0	0	0

Mean Per Calendar Year

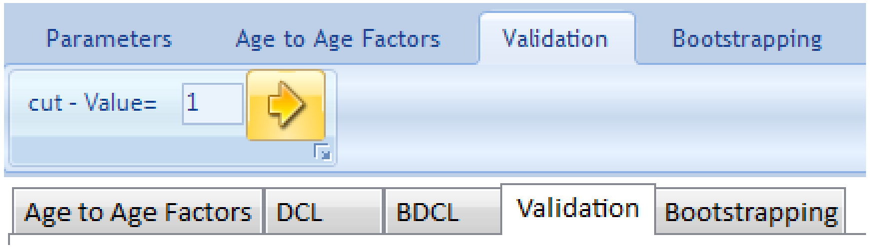
Calendar Year	Simulated IBNR	Simulated RBNS	Simulated Total
20	1,369,142	59,491,836	60,860,978
21	7,380,337	41,370,584	48,750,921
22	5,521,709	31,218,200	36,739,909
23	5,596,397	17,644,786	23,241,183
24	4,011,567	6,431,716	10,443,283
25	1,696,331	3,257,159	4,953,490
26	873,997	1,485,377	2,359,374
27	482,318	751,691	1,234,008
28	176,249	639,114	815,364
29	143,798	437,571	581,369
30	39,582	550,628	590,209
31	79,815	435,412	515,227
32	53,355	357,750	411,105
33	89,736	94,939	184,676
34	13,548	126,297	139,845
35	30,381	981	31,361
36	5,857	5,128	10,986
37	2,887	0	2,887
38	1,214	0	1,214
39	4	0	4
40	98	0	98
41	164	0	164
42	0	0	0
43	0	0	0
44	0	0	0
45	0	0	0
46	0	0	0
47	0	0	0
48	0	0	0
49	0	0	0

The full cash-flow: Bootstrapping in RES-TIMATOR® using DCL (3)

- Cumulative empirical distribution and histogram of IBNR, RBNS and Total Reserve.

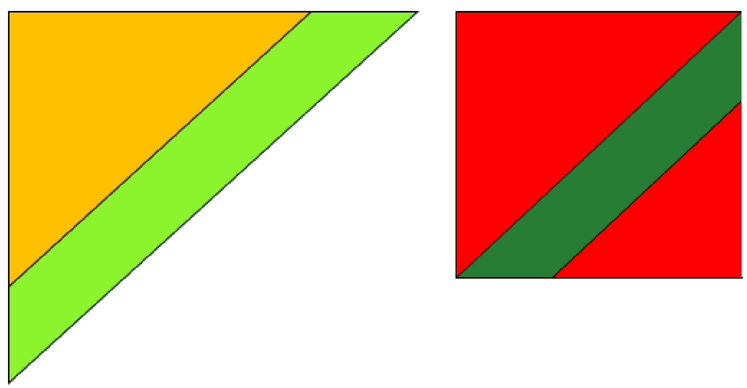


Validation in RES-TIMATOR® (1)



Testing results against experience:

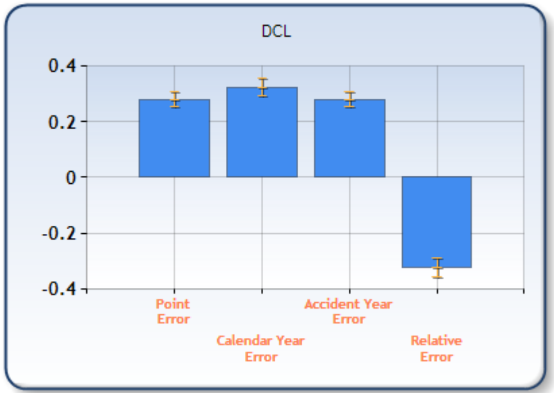
1. Cut c=1,2,…. diagonals (periods) from the observed triangle, where there is the functionality to edit the cut-value.
2. Apply the estimation methods.
3. Compare forecasts and actual values.



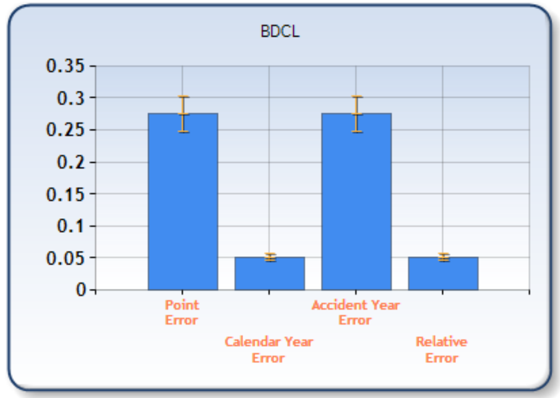
Validation in RES-TIMATOR® (2)

For comparison reasons, it provides point, relative, calendar and accident year error:

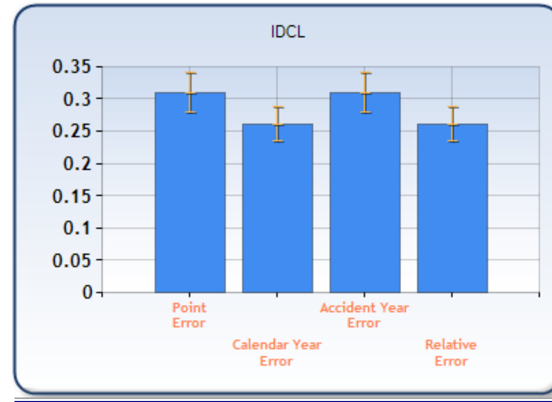
1-period cut:



	Point Error (%)	Calendar Year Error (%)	Accident Year Error (%)	Relative Error (%)
DCL	27.986804	32.349439	27.986804	-32.349439

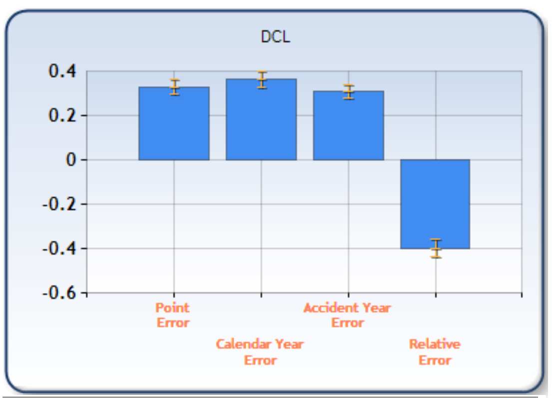


	Point Error (%)	Calendar Year Error (%)	Accident Year Error (%)	Relative Error (%)
BDCL	27.563240	5.069029	27.563240	5.069029

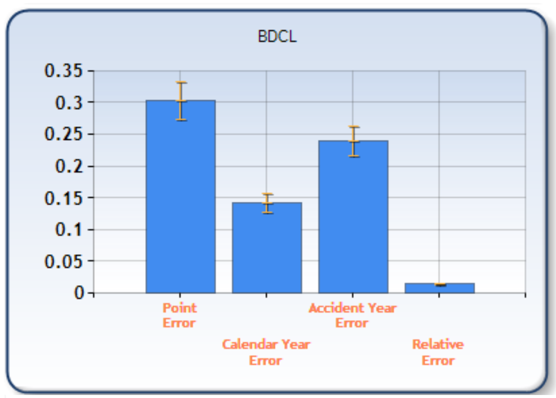


	Point Error (%)	Calendar Year Error (%)	Accident Year Error (%)	Relative Error (%)
IDCL	31.018852	26.164268	31.018852	26.164268

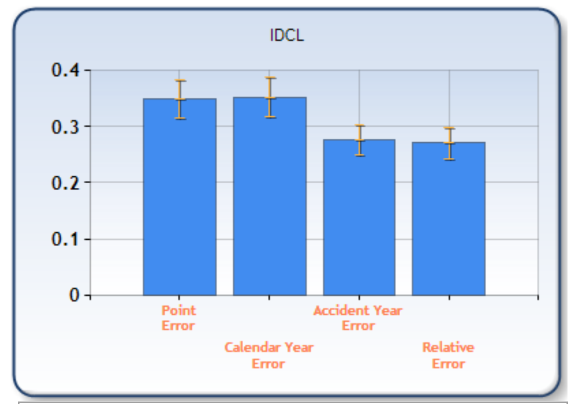
4-period cut:



	Point Error (%)	Calendar Year Error (%)	Accident Year Error (%)	Relative Error (%)
DCL	32.745875	36.230231	30.865219	-40.017321



	Point Error (%)	Calendar Year Error (%)	Accident Year Error (%)	Relative Error (%)
BDCL	30.278552	14.203014	23.948456	1.382489



	Point Error (%)	Calendar Year Error (%)	Accident Year Error (%)	Relative Error (%)
IDCL	34.859977	35.136012	27.617377	27.013036

Summary: Process Embedded in RES-TIMATOR®

DATA IMPORT

↓

RUN DCL

↓

RESULTS

Estimate parameters	Best estimates	Validation	Bootstrapping

Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.