A MARKET-BASED APPROACH TO PRICING WITH-PROFITS GUARANTEES

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ABSTRACT

This paper addresses one of the fundamental issues faced by offices in the transacting of with-profits business, namely the provision of equity-related performance along with maturity guarantees that increase over the lifetime of the policy. The approach commonly followed of using capital to cover the office's investment mismatch risk is considered from an individual policy viewpoint, and, through stochastic modelling, the degree of security represented by different reserve levels identified. An alternative approach using derivatives to provide similar levels of security is then presented and the range of policy proceeds resulting from the two methods compared. While it is recognised that the latter approach may have limited application in practice, the ideas presented could form the basis of a new approach to pricing guarantees under with-profits business that would be consistent with the pricing of current derivative-backed guaranteed products.

KEYWORDS

With-Profits; Guarantees; Policyholders' Reasonable Expectations; Capital; Risk; Derivatives; Statutory Reserves; Stochastic Modelling; Low Inflation; Bonus Rates; Asset Shares

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The real voyage of discovery consists not in seeking new lands but seeing with new eyes. M. Proust

1. INTRODUCTION

1.1 *The 'With-Profits' Concept*

1.1.1 'With-profits' can mean many different things, with policyholders' reasonable expectations (PRE) varying from what is virtually a smoothed managed fund to sharing in all aspects of an office's value creation activities.

1.1.2 Central to all variations on the with-profits theme, however, is the granting of guarantees, whether they take the form of a (conventional) sum assured augmented by periodic reversionary bonus declarations or, in the case of unitised contracts, a guaranteed rate of growth (or, at the very least, a guarantee of non-reduction) in unit prices.

1.1.3 In addition, most (if not all) policyholders do not anticipate receiving just the guaranteed amounts at maturity. Despite warnings that terminal bonuses are not guaranteed, the with-profits concept tends to carry with it the hope that investment performance over the life of the policy will be such that the guaranteed return will not be all that is received.

1.1.4 Thus the strategies followed by offices transacting with-profits business have to reconcile a conflict between the need to meet the guarantees and the requirement for investment freedom in the interests of total policyholder returns — and that in the context of various practical complications.

1.2 *Some Practical Considerations*

1.2.1 The date on which a policy will terminate is not always known with certainty. For example, it may be surrendered or the maturity date may be at the policyholder's option. The latter is a particularly relevant complication, because guarantees will apply at more than one date.

1.2.2 The levels of initial guarantees as well as the size of subsequent reversionary bonus declarations (or unit price increases) have to be set, and the bonus philosophy followed can influence significantly an office's financial management.

1.2.3 Another key aspect of with-profits business is the smoothing of payouts to reduce the effect of market volatility on policy proceeds. The particular approach to smoothing affects, not just policyholder payouts, but can prove very significant to the financial health of the office, particularly in times of falling asset shares due to sustained poor investment returns.

1.2.4 Finally, while this section is not meant to be exhaustive, mention should certainly be made of the financial effects of the structure of the portfolio of business an office has, as well as the rate at which its new business is allowed to grow.

1.3 *Scope of the Paper*

1.3.1 In writing this paper, the authors have aimed to present some fundamental points as clearly as possible. As a consequence, important simplifications have been made.

1.3.2 The investigation has been restricted to guarantees vesting at maturity, and how, and at what price, they can be provided. In addition, the modelling has been carried out on individual policies rather than on portfolios.

1.3.3 The concentration on maturity guarantees is not meant to imply that practical considerations, such as those in Section 1.2, do not matter, which, of course, they do. However, extending the study to take account of these aspects could have so complicated the investigations that the fundamental points might have been lost or, at best, the issues clouded. These practical considerations will certainly throw up additional issues, but these will tend to be of a more particular nature than the generic points that this paper seeks to present. The situation with flexible maturity dates is particularly interesting, but is outwith the scope of this paper, being better dealt with in the context of portfolio studies.

1.3.4 Considering individual policies seemed the obvious way to highlight the risk/return trade-off implicit in all with-profits business, and demonstrates how this varies with duration, guarantee level and asset-backing strategy. The authors recognise that the presence of a portfolio (depending, of course, on its structure and, particularly, how widely spread anticipated outstanding terms are) could affect the degree of security aimed at for each policy in isolation. Such an investigation has been left to other papers (although some further comments are presented in Appendix A). Nevertheless, the results presented in this paper are a necessary first step in quantifying any benefit that may be gained from being part of a portfolio and the extent to which it could be passed on to policyholders.

1.3.5 In particular, it should be noted that, where an office has launched a tranche-based with-profits product (or ended up with something similar through its pattern of sales), it would be relatively straightforward to apply the concepts discussed in this paper. Where all the policies in a tranche share the same investment experience (through having similar start and end dates), there can be no diversification of investment risk over time. Thus, the risk to the office associated with the guarantee under any particular policy is the same whether that policy is viewed in isolation or as part of a portfolio.

1.4 *Outline of the Paper*

1.4.1 Further comments on the operation of with-profits business are presented in Section 2, highlighting, particularly, the relationship between investment strategy (i.e. risk profile) and capital requirements.

1.4.2 Section 3 introduces the investment model used for the stochastic simulations presented in later sections and highlights some of the issues to be taken into account when interpreting the results and drawing conclusions.

1.4.3 In order to compare the price of the capital-backed approach with that of the derivative-based alternative presented in Section 6, it is necessary to quantify the extent of capital support required under the former method. Section 4 presents the results of a stochastic investigation into this, with Section 5 discussing various approaches to charging for the capital levels derived.

1.4.4 Section 7 compares the approaches described in Sections 5 and 6, showing, in particular, the range of policyholder returns that the stochastic model projects.

1.4.5 Section 8 makes some concluding remarks.

2. The Capital-Backed Approach to With-Profits Business

2.1 *Two Familiar Alternatives*

2.1.1 As described above, the authors see the writing of with-profits business essentially as the provision of some form of increasing investment guarantee with ultimate payouts that reflect equity performance to some (ideally significant) extent. Two approaches to delivering such a combination of investment guarantees with equity upside are commonly presented.

2.1.2 In the first, an additional amount of money (i.e. over and above the sum of the individual policy asset shares) is used to 'support' the investment strategy being followed for the asset shares (in particular, the extent to which the funds are invested in equity-type assets). These additional assets may also be available to facilitate the smoothing of payouts over time.

2.1.3 In what is sometimes known as the 'revolving fund' approach, no additional assets are assumed to exist. Instead, each cohort of with-profits business is intended to be self-supporting in terms of both investment mix and payout smoothing.

2.2 *Variations on a Theme*

2.2.1 Whichever approach is followed obviously has consequences for the running of the office and, in particular, its capital management strategy. This encompasses such issues as policy design, guarantees granted, bonus declarations and investment mix.

2.2.2 From the standpoint of providing investment guarantees with additional equity upside, however, we see these two strategies as being variations on the same theme, rather than two dramatically different approaches.

2.2.3 Essentially, the office is choosing to bear the balance of the investment risk of backing guarantees, at least in part, with equity-type investments. It does this by having sufficient capital available to meet the guarantees (or whatever is considered to represent PRE), even under adverse investment experience.

2.2.4 When an individual policy is being considered, as it is in this paper, it is immaterial whether the capital support comes from an estate or not. For the purposes of this paper, therefore, we will not distinguish between the existence and non-existence of an estate, but refer to both of these strategies as the 'capital-backed' approach.

2.3 *The Need to Quantify Capital Requirements*

2.3.1 Obviously, the more capital there is available (either from an estate or from asset shares, or both), the greater the investment risk that can be run. However, quantifying, for a given amount of capital, just how far the investment mix of the asset shares can be biased towards equity-type investments, although very topical, is not straightforward.

2.3.2 Where an office is involved in capital-raising activities, whether through demutualisation, reinsurance financing or securitisation, to improve the financial position of a block of with-profits business, the extent to which the investment mix can be less constrained is often the key reason for the exercise.

2.3.3 At the opposite end of the spectrum, as it were, the recent report of the Closed Fund Alternative Working Party (Hairs *et al.*, 1999) highlighted similar issues. In this case, though, the problem is quantifying the extent to which the investment mix for a closed book of business should be constrained during its run-off, including the possibility that the investment constraints could be reduced initially.

2.3.4 A third example can be found in the identification of 'orphan surplus' within a with-profits fund. In order to demonstrate that a pool of assets is not needed to back PRE in respect of the affected policies, one has to quantify the assets that are required.

2.3.5 Finally, for an ongoing business where no restructuring or capital raising is envisaged, the possible approaches to charging asset shares for whatever capital support is given will be limited if no mechanism exists to quantify the capital involved. For the purposes of this paper too, it is necessary to quantify what capital support is needed to back a particular guarantee, in order that an appropriate charge for it can be compared with the price of the derivative-based approach which will be introduced later.

2.4 *Methods of Quantifying Capital Requirements*

2.4.1 Perhaps the most common approach to this problem, particularly since the publication of the Maturity Guarantees Working Party's report in 1980, is to carry out some form of stochastic simulation.

2.4.2 An alternative would be to perform a series of deterministic projections on specially chosen 'worst-case' scenarios.

2.4.3 The former requires the choice of an investment model along with all the relevant assumptions and parameters. It also involves the identification of the probability of failure to meet guarantees which the office is prepared to tolerate.

2.4.4 The latter approach requires similar choices, although, in this case, volatility in investment returns and acceptable risks of failure are combined (perhaps implicitly) in the selection of the deterministic worst cases to be withstood.

2.4.5 In this paper stochastic projections have been used.

3. Some Comments on the Stochastic Modelling used

3.1 *A Word of Explanation*

The authors recognise that there is a wide range of experience and views on stochastic methods in the United Kingdom actuarial profession. In order to keep the paper accessible to a wide audience, the following chapter presents a brief overview of the model used and the key points to bear in mind when drawing conclusions from the results.

3.2 *Means and Tails*

3.2.1 The expected cost of a liability to an office and the reserve needed to be $x\%$ sure of being able to meet it are not necessarily the same, or even close, particularly where x is close to 100. While both involve assumptions about future experience, the former focuses on the mean (and how it might change over time), whereas the latter is concerned with both the mean and the variance (in particular the length of the lower tail) of the likely distribution of outcomes.

3.2.2 For risks such as mortality, a reserve based on a prudent assumption of future mean experience is usually sufficient, provided that due allowance is made for possible future changes in experience. (These may result from, say, changes in underwriting practices or, perhaps more significantly, general changes in human mortality.) Claims volatility is rarely an issue, for several reasons:

(a) The distribution of possible mortality rates at any age is closely grouped around the mean.
(b) Where lives are independent, volatility in mortality experience decreases with portfolio size.
(c) Reassurance can be used to cater for 'outlying risks', where the office does not have sufficient exposure for the mean to be relevant.

3.2.3 Where the distribution of possible outcomes is wider, however, reserves that give a high probability of being sufficient to meet the associated obligations can be considerably more than the mean. For example, in their study of Canadian maturity guarantees (for segregated fund business, akin to U.K. unit-linked business), Boyle & Hardy (1996) presented figures showing reserves up to fifteen times the expected cost of the corresponding guarantees.

3.2.4 Not only is the distribution of possible investment returns widely spread out, but the risk is such that the variance in possible outcomes does not decrease with an increase in the number of similar policies in the portfolio (where 'similar' particularly refers to the dates on which guarantees fall due). Appropriate diversification of assets may reduce the volatility of the portfolio, but the size of the portfolio does not. In other words, the more policies with a particular guarantee, the greater the cost to the office — in direct proportion. Given the current experience with guaranteed annuity rates granted when interest rates were higher (along with mean mortality rates), this is not a point which needs elaboration.

3.2.5 Thus, in modelling the consequences of an office providing with-profits guarantees through the capital-backed approach, we will be particularly concerned with the tails (principally the lower tail) of the distribution of possible investment performance.

3.2.6 This is akin to restricting attention to deterministic worst-case investment scenarios, but a stochastic approach to this problem has the advantage that the rest of the distribution of investment outcomes can then be used to give an indication of the likely spread of maturity values that a policyholder may receive.

3.2.7 Of course, this then leads into one of the main criticisms of stochastic modelling; that the results from using a model are just that — results from one particular model of the future — and are simply a function of the underlying assumptions one has made.

3.3 *The Choice of Model*

3.3.1 For the purpose of this paper, it is not necessary that the 'right' model of future investment returns be used, even if such a thing exists. Rather, 'something sensible' will suffice, so long as the particular characteristics of the model are borne in mind when interpreting the results.

3.3.2 There are many different models of investment performance from which to choose, although not all are fully in the public domain.

3.3.3 Perhaps the best known model, at least in actuarial circles, is that developed by Wilkie. It is well documented and used widely. Its particular characteristics are well understood, and it is easily adaptable to different economic outlooks.

3.3.4 It is this model, as presented in Wilkie (1995), but adjusted to reflect a low inflation environment, which the authors have used for this piece of research.

3.3.5 The authors recognise that the use of a different model would have given different numerical results, but are confident that the main messages from the paper would have remained unchanged. (The results of sensitivity are available from the authors.) In any case, it is not the absolute size of the numbers that matters. Rather, so long as the modelling is internally consistent (for example, in terms of asset share projections, statutory valuation bases, guarantees and bonus declarations), it is their shape and the relationships between them that are important.

3.3.6 Before concluding this section, it should be noted that Wilkie's model is what is known as 'autoregressive', or 'mean reverting'. In other words, the further away from the mean of its distribution one year's observation is, the more likely it is that next year's observation will be closer to the mean. The effect this has in practice will be considered further in various sections.

3.4 *Adjusting the Model for Low Inflation Conditions*

3.4.1 Wilkie developed his model from a study of U.K. investment returns over the period 1923-1994. By investigating possible relationships between certain key variables, he built up a complex structure of inter-relatedness. He then incorporated a random element in order that appropriate projections of future investment returns could be made.

3.4.2 Wilkie's model is not limited to the U.K. market and its historic performance. By using appropriate values for the 37 parameters within the model, other markets, or the U.K. in a variety of different economic conditions, can be simulated.

3.4.3 The parameters cover both mean returns expected (in terms of forces of interest) and the corresponding standard deviations, along with a correlation structure encompassing the principal economic measures. In his 1995 paper, Wilkie presented a selection of input parameter assumptions based on historical analyses for a variety of countries. Using the U.K. set as a basis for future projections gives a (geometric) mean equity return of 11% p.a., mean inflation of 4.8% p.a. and mean return on Consols of 8%. The authors felt that these means were too high for use in a study of guarantee pricing in current conditions, and that assumptions that reflected the expectation of continuing low inflation would be more appropriate. "An important feature of the way I believe this model should be used", writes Wilkie in the same paper, "is that those using it should form their own opinions about the choice of appropriate mean values".

3.4.4 The authors chose to alter just the parameters relating to mean returns, and to leave the corresponding standard deviations and correlation structure unchanged. While it may seem appropriate to have altered the latter in some way, it was not clear to the members of the group that returns were currently, or could be expected to be in the future, any less volatile than in the past. On the other hand, increasing the standard deviations from the historically derived figures would only have made the capital requirements presented below even greater. Rather than 'over-sell' the conclusions in the paper, the authors felt that it was better to leave different views of future variance to be incorporated when interpreting the results.

3.4.5 The parameters changed were the following:

	Value used (%)	Wilkie 1995 value (%)
Mean annual force of inflation (QMU)	2.40	4.70
Mean annual dividend yield (YMU)	2.20	3.75
Mean real dividend growth rate p.a. (DMU)	2.55	1.60
Mean long term real interest rate p.a. (CMU)	2.50	3.05

It can be seen that, with a dividend yield similar to current levels, an increase had to be assumed in the mean annual rate of real growth in dividends in order to get a mean equity return that was felt to be reasonable.

3.4.6 The resulting projected mean inflation rate and mean nominal rates of return by asset class are the following:

	Annual rate (%)	Wilkie 1995 rate (%)
Inflation	2.5	4.8
Cash	4.2	6.6
Consols (i.e. undated gilts)	5.0	8.0
Equities	7.6	11.0

3.4.7 It is obvious that, for unchanged guarantee levels (and volatility of returns), higher assumed means would result in lower capital requirements in Section 4. Nevertheless, the authors feel that the above returns are not unreasonable long-term assumptions for the purposes of this paper. Those readers who would have preferred slightly higher assumptions are referred to sensitivity results available from the authors.

3.5 *Using the Model in Practice*

3.5.1 In the interests of simplicity, but without losing more generic applicability, this research has chosen to distinguish between only two asset classes, namely gilts and U.K. equities. While the authors recognise that offices usually include cash, debenture and property holdings in the assets backing their with-profits liabilities, it was felt that restricting attention to just two asset classes should be sufficient for the purpose of this paper. The proportion of a fund invested in equities will be referred to as the 'equity backing ratio' (EBR).

3.5.2 Equity returns have been modelled as described in the previous section. For fixed-interest returns, however, a simpler approach has been used.

3.5.3 Wilkie's model does not attempt to project a yield curve shape for fixed-interest returns. Instead, returns on cash and undated gilts are produced, and the derivation of dated gilt performance left to the user. A variety of approaches can be used for this, ranging from the assumption of a straight line to more complicated shapes, possibly even varying over time.

3.5.4 In considering the amount of capital required to back a particular guarantee, the key assumptions are the mean and variance of the assets in which the capital is invested. Gilts investment involves some uncertainty, since the market prices of future purchases are unknown. However, stochastic modelling of gilts returns would complicate the interpretation of the results without adding much to the conclusions.

3.5.5 Were it considered appropriate to model some degree of negative correlation between fixed-interest and equity returns, then the authors accept that a deterministic approach to gilts would result in modelled fund return volatility being too high. Given, however, that past U.K. experience has tended to show a positive correlation between equity and bond total returns, the approach adopted (which, in effect, is an assumption of exact matching of outstanding terms) should be sufficient for this paper. (In fact, the authors found that the model produced a slightly negative correlation between undated gilt returns and equity returns over a ten-year period. Over very short periods, however, slightly positive correlation was observed.)

3.5.6 Thus, for all the results shown in Section 4 onwards, any gilts holdings are projected deterministically with an annual rate of return of 5.0%.

3.5.7 Due to the assumption of a higher mean return for equities than gilts, the EBR of a fund can be expected to increase over time. In what follows, there will be times when this will be allowed to happen and times when it will not. In the latter case, the assumption of annual rebalancing of the investment mix will be referred to as 'rebasing'.

3.5.8 Rather than alter the underlying model to allow for, say, monthly time periods, this study retains a yearly time period. A consideration of day-to-day office solvency would, of course, require some more dynamic approach. For an investigation into guarantee pricing, however, annual simulations are sufficient. The only slight drawback is that, in modelling the maturity values of regular premium business, part of the effect of what is known as 'pound-cost averaging' is lost by the use of annual rather than of monthly premiums.

3.5.9 In what follows, when reference is made to the investment growth projected, this will be expressed in terms of an annualised growth rate which would have produced the same maturity value as the corresponding stochastic projection.

3.5.10 As already mentioned, of particular relevance to this study is the size of the lower tail of the distribution of modelled equity returns. In other words, attention will be given to the bottom 'so many' simulations from each set. Where only 1,000 simulations were run for each scenario, it was found that the sparseness of the tails could give rise to anomalous comparisons. As the '100% EBR' line in Figure 3.1 shows, even with 10,000 10-year simulations, that the results are not completely smooth.

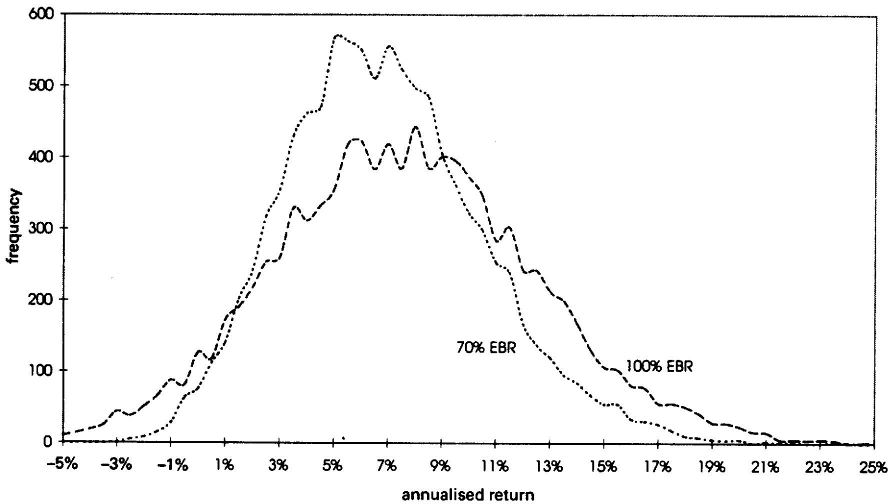

Figure 3.1. Distribution of annualised returns from 10,000 simulations

With 20,000 simulations smoothness improved, but not sufficiently to justify the considerable increase in calculation time. In order to keep the computing time manageable, all the results that follow are based on 10,000 simulations.

3.6 *Interpreting some Initial Results*

3.6.1 As can be seen from Figure 3.1, when the EBR is reduced to 70% the results are intuitively reasonable. Despite the absence of rebasing the investment mix each year, the distribution of annualised return over 10 years is less spread out, but with a lower mean.

3.6.2 *Distribution percentiles*

3.6.2.1 An alternative approach to presentation is to consider the percentiles of the distribution of annualised returns. Figure 3.2 shows how some of the more significant percentiles vary with the EBR assumed for the same 10-year example (again without rebasing). Various observations can be made from this.

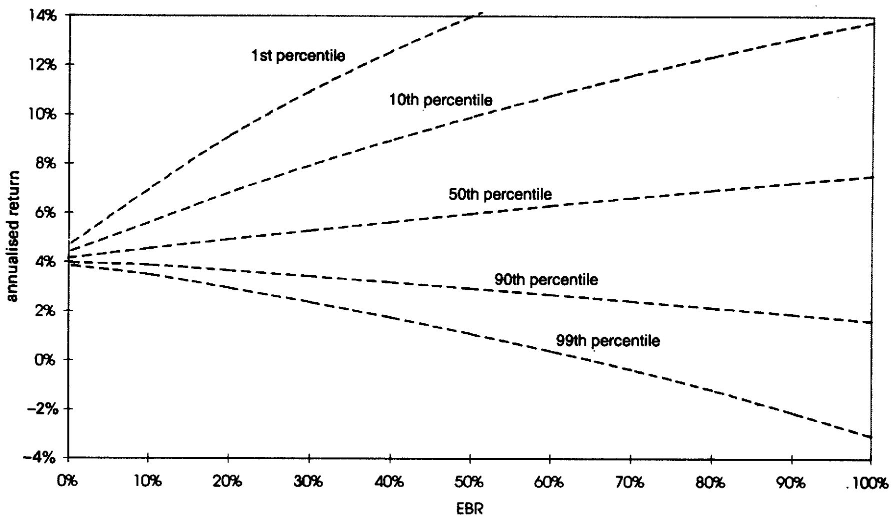

Figure 3.2. Distribution of annualised returns from 10,000 simulations

3.6.2.2 Not surprisingly, the median annualised rate of return increases with EBR, as, in fact, does the mean annualised return, reflecting the different mean returns assumed for gilts and equities.

3.6.2.3 The extent of the volatility in equity returns produced by the Wilkie model (and the parameter assumptions used for this paper) can be seen in the way that the percentiles become more and more spread out as EBR increases. (As a result, for EBRs of 70% or more, a negative return over 10 years was recorded in at least 1% of simulations.) Were the model not auto-regressive, the spread in modelled returns would be even greater.

3.6.2.4 Another influence on the spread of modelled returns may be the small degree of correlation between equity and gilt total returns in the model, since, for the purposes of this graph, the gilts portion was projected stochastically on the assumptions of perfect matching of outstanding term, a straight line yield curve and a cash return of 4.2%.

3.6.2.5 The median gilt return is a little over 4% (much less than the mean of 5% given in ¶3.4.6) as a result of these yield curve and term assumptions.

3.6.2.6 The relative importance of the volatility in returns from gilts compared with that for equity returns can be seen clearly by comparing the spread of the percentiles on the left hand side of the graph with that on the right. This confirms that the decision to model gilts deterministically throughout the modelling does not significantly detract from the conclusions drawn.

3.6.3 *Implications for guarantee levels*

3.6.3.1 Expressing the results shown in Figure 3.2 in terms of the ability to meet particular levels of guarantees makes interesting reading. In Figure 3.3 the percentage of runs that failed to produce an annualised rate of return over 10 years above each of 0%, 1%, 2%, ..., 5% is plotted for EBRs of 50% and above.

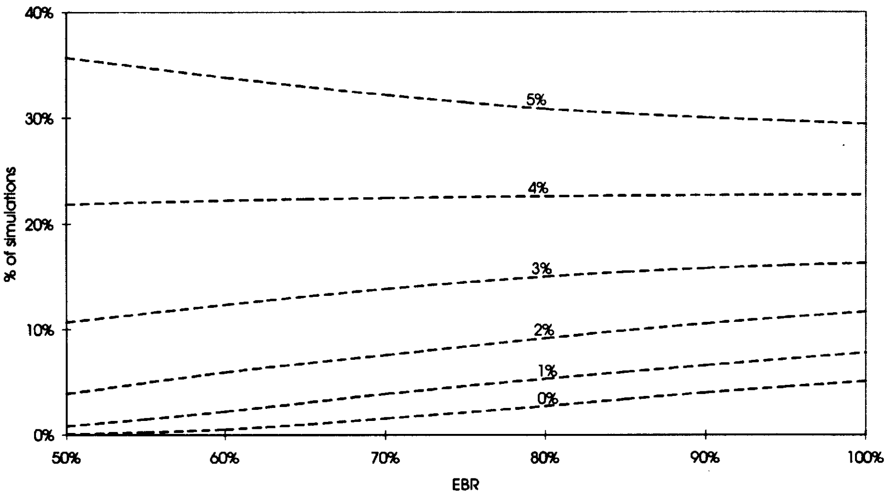

Figure 3.3. Proportion of annualised returns from 10,000 simulations which fail to meet a defined level

Several aspects of the results should be noted.

3.6.3.2 The extent of upward slope of each line reduces with increase in target rate of return. Perhaps a better way of expressing this is to say that increasing the target rate of return has a proportionately greater effect the smaller the EBR assumed.

3.6.3.3 As EBR increases, so does the mean rate of return (see ¶3.6.2.2), thereby reducing the proportion of simulations which fail to produce an annualised return of at least 5%.

3.6.3.4 The increased volatility that accompanies the increased EBR, however, results in a greater spread of annualised returns, thereby increasing the number of simulations that fail to make each of the lower target rates.

3.6.3.5 The number of simulations that fail to return, over 10 years, even the original amount invested is non-zero for almost all the EBRs featured above. Indeed, the proportion of simulations failing to make what many investors would consider modest rates of return is quite striking. While it must be stressed that these results are 'only' the results of the model and its assumed parameters, they, nevertheless, give some food for thought.

3.6.4 *Rebasing 'distortions'*

In all the stochastic projections that follow the asset mixes of the funds being projected are never rebased to the starting EBR. Doing so would result in artificially high returns because of the use of an autoregressive model. Each year the disinvestment would be from the better performing asset into the poorer performer (at least, as far as the immediately preceding year was concerned). Where the former had exceeded the mean of its distribution that year, it would be more likely to perform less well in the following year. Conversely, where the poorer performer had underperformed the mean of its distribution, its performance would be more likely to improve. While such a situation may represent an investor's dream, the authors felt that it was rather too optimistic to model!

4. Capital Backing and Probabilities of Shortfall

4.1 *Introduction*

4.1.1 This section presents the results of the stochastic modelling of capital requirements introduced in Sections 2.3 and 2.4. It starts by considering, in some detail, single premium conventional with-profits business (CWP), for a variety of EBRs and durations in force. A summary of the results for corresponding annual premium examples is then presented, with the main consideration placed in Appendix B. The section concludes by extending the analysis to single premium unitised with-profits cases (UWP).

4.1.2 The tables that follow show results for policies with 10-year original terms. The equivalent figures for policies with other original terms are available from the authors. They tell a similar story, and so would add little to the conclusions of the paper.

4.1.3 The key quantity to measure is the probability that there are insufficient assets at maturity to meet the guarantee — the probability of shortfall.

4.1.4 The analysis of the conventional examples starts by deriving the probabilities of shortfall in the case where the assets at the start of the projection period are equal to the sum of the minimum reserve (including resilience reserve) and the required minimum margin under current U.K. reserving regulations.

While the absolute numbers should be treated with caution, the shape of the results is well worth considering.

4.1.5 Amounts of starting assets required to meet a target probability of shortfall are then derived, and, in order to enable chargeable support to be identified, expressed as percentages of corresponding asset shares. Where the percentage is greater than 100, it is assumed that the additional assets (i.e. the capital support that is needed) will be invested with the same EBR as the associated asset share.

4.1.6 In order to produce meaningful results, all the various assumptions that need to be made must be consistent. How this has been addressed is the subject of Section 4.2.

4.2 *Ensuring Consistency and Transparency*

4.2.1 Results are given for every duration t, from outset to the anniversary one year before maturity (i.e. $0 \leq t \leq 9$). What is shown, for each t, is the result of the projection (basically, stochastic for equities, deterministic for gilts) from time t until time 10.

4.2.2 For some of the tables of projections, all that is required is to know the level that the maturity guarantee has reached by time t. For other tables of results, corresponding asset shares are also required.

4.2.3 Generating these values stochastically at each time t could have been done, but introducing such additional complexity would not have added anything to the points being illustrated, just made the results much more complicated to interpret! A simpler approach has, therefore, been used to generate the starting values.

4.2.4 For each current duration t, the asset share has been accumulated from outset deterministically using a 7.6% annual return on equities and 5% p.a. return on gilts, weighted by the appropriate EBR (thereby assuming rebasing of the asset mix each year). The projection from time t onwards does not incorporate rebasing of the EBR, thereby avoiding the artificial inflation of returns described in ¶3.6.4.

4.2.5 The levels of initial sums assured for the conventional policies (taken to be the accumulation of premiums to maturity at 1% p.a.) have been set reasonably low compared with the mean rates of return being assumed.

4.2.6 The same two-tier reversionary bonus rates have been assumed for both single and annual premium conventional cases for all EBRs. At 1% of sum assured and 2% of attaching bonus, the rates are obviously more onerous (compared with deterministic asset share growth) the lower the EBR. Nevertheless they seem reasonably plausible, as well as broadly consistent with the other assumptions being made. These rates of reversionary bonus are assumed to apply at every policy anniversary up to and including time t. During the stochastic projection no further bonuses are assumed to be declared. (The authors recognise that this may not sit well with PRE in all situations, particularly in the cases where the bonus rates are relatively high compared with modelled returns, but is sufficient for the purposes of the paper.)

4.2.7 As mentioned in ¶4.1.4, the initial sets of results are not based on the accumulated asset share at time *t*, but take as the asset value the sum of the statutory minimum reserve (including resilience reserve) and the required minimum margin. In calculating the reserve, the EBR of the asset share has been allowed for in deriving the appropriate yield to use. No attempt has been made to maximise the yield by efficient hypothecation of assets. Thus, while the reserves may be slightly higher in some cases than the minimum an office would be permitted to publish, they are consistent with the EBRs assumed at the start of their projection to time 10.

4.2.8 The authors would not disagree with readers who feel that these assumptions are somewhat simple and idealistic. They have been deliberately set that way, since the more easily what has been modelled can be grasped, the more straightforward it will be to think through the implications for more complicated scenarios.

4.3 *Modelling the Strength of Statutory Minimum Reserves for Conventional Business*

4.3.1 *Introduction*

As stated in ¶4.1.1, the results given in this section concentrate on single premium business. The equivalent annual premium results are summarised briefly with the various tables presented in Appendix B.

4.3.2 *Shortfall probabilities*

4.3.2.1 Table 4.1 presents the ‘probabilities of shortfall’, i.e. the probabilities that the accumulation, for 10-*t* years, of the sum of the statutory reserve (including resilience reserve) and required minimum margin at time *t* is less than the sum assured and bonuses attaching at time *t* (i.e. the amount at time *t* which is guaranteed to be paid at maturity). While the absolute values of these figures are probably not completely unrealistic, it is the relationship between them that is of particular interest.

4.3.2.2 For most durations, as EBR increases, so does the probability of shortfall. In other words, the statutory reserving basis is not uniformly strong across different investment strategies (for the same level of guarantees).

4.3.2.3 For the two latest durations, the shape of each row is not so straightforward, with the probabilities for the higher EBRs being less than those for the lower ones. The reason lies partly in the assumptions made and partly in the way that the reserving basis being used allows for equity investment.

4.3.2.4 Reserves are calculated using a discount rate which is the weighted average (in line with EBRs) of 4.9% and 2.4% (being 97.5% of 5% and 2.5%). The gilts proportion of the reserve is then projected forward at 5%, thereby creating a small surplus at maturity. The equity proportion, on the other hand, is stochastically projected using a distribution with a much higher mean. The greater

Table 4.1. Probability of shortfall at maturity
(CWP, with resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	2.7%	3.2%	3.7%	4.2%	4.4%	4.9%
1	3.9%	4.6%	5.2%	5.7%	6.1%	6.5%
2	5.1%	5.8%	6.3%	6.9%	7.3%	7.7%
3	5.8%	6.3%	6.7%	7.2%	7.7%	8.0%
4	6.6%	7.0%	7.3%	7.7%	8.1%	8.5%
5	7.6%	7.9%	8.1%	8.5%	8.8%	9.1%
6	8.3%	8.4%	8.6%	8.8%	9.0%	9.2%
7	8.0%	8.0%	8.0%	8.1%	8.3%	8.4%
8	7.3%	7.0%	6.9%	6.9%	7.0%	7.1%

the EBR, the greater the volatility in maturity value, but also the greater the expected return and the lower the valuation yield. It therefore becomes more likely that growth at least equal to the valuation rate of interest will be projected as EBR increases. These different 'forces' are at work in all the rows, and, as the outstanding duration changes, so does their relative strength.

4.3.2.5 As might have been expected, for each EBR, the probabilities of shortfall increase (at least initially) as *t* increases and the duration over which equities are held to back a guarantee reduces. Intuitively this seems reasonable, particularly for results from an autoregressive model. Basically, the shorter the outstanding duration, the fewer the years over which, it is hoped, good performance from equities may be received to compensate for any poorer years.

4.3.2.6 That the probabilities do not continue to increase as outstanding duration decreases is testament to the security provided by the resilience reserve.

4.3.3 *Removing the resilience reserve*

4.3.3.1 Table 4.2 presents the equivalent probabilities of shortfall when the statutory minimum reserves are calculated without the resilience reserves.

4.3.3.2 As expected, the probabilities of shortfall increase with expired duration, thereby indicating the extent of strengthening in the previous table provided by the resilience test.

4.3.3.3 All the figures in Table 4.2 are greater than the corresponding ones in Table 4.1, but the extent of the increase reduces as EBR increases. In other words, the resilience test, when combined with the modelling assumptions, seems to be more onerous on gilt investment than on equity. The use of a weighted average valuation yield (which results in the post-resilience-scenario reserve not moving in line with either of the two asset types in the backing assets) will certainly be affecting the results. Nevertheless, it does not appear unreasonable, particularly for the earlier durations, that 300 basis points on top of an annual gilt yield of 5% should prove stronger than assuming a one off 25% drop in equity values (this being the resilience test that generally proved the most stringent).

Table 4.2. Probability of shortfall at maturity (CWP, without resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	10.1%	10.8%	11.2%	11.5%	11.6%	11.9%
1	13.6%	14.1%	14.7%	15.0%	15.1%	15.4%
2	16.3%	16.8%	17.1%	17.3%	17.5%	17.6%
3	18.4%	18.9%	19.2%	19.4%	19.7%	19.8%
4	20.5%	20.9%	21.3%	21.4%	21.5%	21.7%
5	23.4%	23.5%	23.7%	23.8%	23.9%	24.1%
6	26.0%	26.1%	26.2%	26.3%	26.4%	26.4%
7	29.7%	29.5%	29.5%	29.4%	29.5%	29.5%
8	32.7%	32.2%	32.0%	31.8%	31.8%	31.8%
9	35.8%	34.8%	34.2%	33.9%	33.7%	33.7%

4.3.3.4 There is again a change in shape towards the later rows as the different discount rates and projection assumptions inter-relate to favour, slightly, the higher EBR cases.

4.3.3.5 The greater similarity between the different columns is the result of using only the dividend yield (rather than total return) for the equity-related contribution to the weighted-average valuation yield, thereby creating a form of resilience reserve within the 'basic' statutory minimum reserve.

4.3.4 *Asset share comparison*

4.3.4.1 It is worth considering the relationship between the statutory reserves calculated (including the resilience reserve) and the deterministic asset shares, shown in Table 4.3.

4.3.4.2 For each duration the proportions increase with EBR, although not quickly enough to avoid the probabilities of shortfall also rising.

4.3.4.3 With some of the proportions greater than 100%, the results suggest that with-profits business cannot be considered always to provide capital to an office, even on a published free assets basis.

4.3.4.4 The use of minimised reserves (see ¶4.2.7) would have made very little difference to the figures (the greater the proportion of gilts, the greater the effect, but, for the 50% EBR, the absolute reductions in Table 4.3 figures range from only 3.5% down to 1.8%) and certainly would not have changed the previous conclusion.

4.3.5 *Some tentative conclusions*

4.3.5.1 Even though relatively few results have been presented so far, certain tentative conclusions can be made.

4.3.5.2 First, that the statutory valuation basis, including the three resilience tests as well as the required minimum margin, is not uniformly strong over either policy duration or reserve investment mix. (While the probabilities of shortfall at

Table 4.3. Statutory reserve as a percentage of deterministically accumulated asset share (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	88.4	92.5	96.9	101.6	106.4	111.3
1	87.1	90.8	94.7	98.9	103.4	107.8
2	85.7	89.1	92.7	96.4	100.4	104.5
3	84.4	87.5	90.7	94.0	97.6	101.3
4	83.2	85.9	88.7	91.7	94.9	98.2
5	81.9	84.3	86.9	89.5	92.3	95.2
6	80.7	82.8	85.0	87.3	89.8	92.3
7	79.6	81.4	83.3	85.3	87.3	89.5
8	78.5	80.0	81.6	83.2	85.0	86.8
9	77.4	78.6	79.9	81.3	82.7	84.2

maturity presented here may seem rather high, the absolute numbers are dependent, to an extent, on the model and assumptions chosen. In addition, as mentioned in Appendix A, the risk to an office of a given policy, within the context of its portfolio, is not necessarily the same as when that policy is viewed in isolation.)

4.3.5.3 Second, that the notion sometimes presented, of with-profits business providing capital to an office need not always be the case.

4.3.5.4 Of course, in many other cases, there is capital available within the asset share that could (subject, in practice, to the smoothing policy of the office) be considered to be available to help meet guarantees, and hence reduce the probabilities of shortfall, as shown in Table 4.3.

4.3.6 *Annual premium business*

4.3.6.1 As can be seen from Appendix B, the results for corresponding annual premium cases are similar, and confirm the tentative conclusions above.

4.3.6.2 The general shape of the results is the same, although the annual premium case gives rise to a greater spread in the size of the shortfall probabilities.

4.3.6.3 Unlike the single premium case, though, the annual premium statutory reserves almost always exceed the corresponding modelled asset shares.

4.3.7 *Comparing the single and annual premium examples*

4.3.7.1 A comparison of the annual and single premium results must be made with care, because the levels of initial guarantee are different and, particularly, because, in the annual premium case, part of the guarantee relates to money not yet received.

4.3.7.2 There are also durational differences to consider, with the annual premium contract being of shorter discounted mean term than the corresponding single premium.

4.3.7.3 The greater range of shortfall probabilities for the annual premium case highlights an interesting consequence of using a net premium valuation. The sum assured modelled is based on a 1% interest rate, but the valuation rate (reflecting the different equity proportions) is higher, and so the net premiums used in the statutory reserves are smaller than the gross figures. Thus, effectively, the annual premium statutory reserves include an expense provision which, in the stochastic projections, is used to help offset investment volatility and hence reduce shortfall probabilities.

4.3.7.4 Thus, not only could the investment risks that an office runs between its single and annual premium portfolios be very different, but the published reserves need not give the whole picture. The implications for inter-office comparisons, on the basis of Form 9 free asset considerations, are clear.

4.4 *Capital-Backing for Conventional Business with a Standardised Shortfall Probability*

4.4.1 *The choice of standardised shortfall probability*

4.4.1.1 There is probably no 'right answer' to what probabilities of shortfall should be tolerated for with-profits business. For the purposes of the comparison between the capital-backed and derivative-based approaches to providing the guarantee, a consistent figure is required which, so long as it is 'sensible', should not influence the conclusions unduly. In keeping with the work done by the Maturity Guarantees Working Party, the authors have chosen to use a probability of 1%.

4.4.1.2 It could be argued that 99% security is too high to target for individual policies within a portfolio, since the resulting shortfall probability for the portfolio as a whole is likely to be less than 1%. The authors would not disagree, but stress that, as Appendix A develops, the extent of any portfolio benefit is a function of the structure of the portfolio and, particularly, the spread of guarantee vesting dates. It has already been noted, in ¶1.3.5, that some tranches of policies might give rise to no portfolio benefits.

4.4.1.3 The quantification of possible portfolio benefits is outwith the scope of this paper, as is a study of the issues surrounding their distribution.

4.4.1.4 Obviously the capital requirements presented below would have been different if another shortfall probability had been used, but the same is true for all the other modelling assumptions. The authors hope that the generic points developed will prove of interest, even if the size of some of the numbers is more than some readers would feel comfortable with.

4.4.2 *Single premium CWP business capital support*

4.4.2.1 Table 4.4 presents the results for the 10-year single premium case introduced in Section 4.3. What is shown, for each duration t, is the assets, expressed as a percentage of the deterministic asset share at time t, required to give a 99% chance of having sufficient assets at time 10 to meet the maturity guarantee applying at time t.
It is worth drawing attention to several points.

4.4.2.2 As might have been expected, for a given duration, the amount of capital required for '1% security' increases with EBR, reflecting the increased volatility in the assets while the guarantee level is left unchanged.

Table 4.4. Proportion of asset share needed for 1% shortfall probability (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	93.5%	101.2%	110.3%	121.1%	134.3%	150.7%
1	95.0%	103.1%	112.7%	124.5%	139.0%	157.4%
2	94.6%	102.4%	111.7%	122.9%	136.8%	154.4%
3	94.5%	102.0%	111.0%	121.9%	135.5%	152.6%
4	93.2%	100.0%	108.0%	117.7%	129.6%	144.5%
5	92.4%	98.7%	106.1%	115.1%	126.0%	139.5%
6	90.9%	96.5%	103.1%	110.9%	120.2%	131.7%
7	88.8%	93.4%	98.8%	105.1%	112.5%	121.4%
8	86.3%	89.9%	94.0%	98.7%	104.2%	110.7%
9	80.9%	82.7%	84.7%	86.9%	89.5%	92.3%

4.4.2.3 Most of the percentages are greater than 100, some considerably so. On these assumptions, it is only for lower EBRs that any surplus capital is available from asset shares to fund other aspects of the office's activity.

4.4.2.4 Apart from a small initial rise, the percentages decrease as duration increases, with the rate of decrease greater the larger the EBR. This is a consequence of the guarantee growing from outset to time t at a lower rate than the deterministic asset share. For each extra year of the pre-stochastic starting value accumulation, the asset share is modelled to grow at the appropriate weighted average of 5% and 7.6%, whereas the guarantee is modelled to grow at a little over 1%.

4.4.2.5 Table 4.4 will be used later in the paper when comparing the effects of different approaches to the charging for capital support with the costs of the derivative-backed alternative.

4.4.3 *An alternative presentation*

4.4.3.1 As an aside, when the amounts of capital-backing underlying Table 4.4 are re-expressed as percentages of the corresponding minimum statutory reserves (with resilience and required minimum margin), the following picture emerges. While this table is effectively a combination of Tables 4.3 and 4.4, it does give rise to some interesting points in its own right.

4.4.3.2 For a given duration, the larger the EBR, the greater the required uplift on the statutory minimum reserve (including resilience reserve and required minimum margin), which is consistent with the previous conclusions about the relative treatment of gilts and equities in the statutory approach.

4.4.3.3 For each EBR, as outstanding duration decreases, the statutory reserve does not grow as fast as the asset share, and, unlike the asset share (on the

Table 4.5. Ratio of 1% reserve to statutory minimum reserve (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	105.8	109.4	113.8	119.2	126.2	135.4
1	109.1	113.5	119.0	125.9	134.4	146.0
2	110.4	114.9	120.5	127.4	136.3	147.8
3	111.9	116.7	122.4	129.7	138.8	150.7
4	112.0	116.4	121.8	128.3	136.6	147.2
5	112.7	117.0	122.2	128.6	136.5	146.5
6	112.6	116.5	121.2	126.9	133.9	142.7
7	111.6	114.8	118.6	123.3	128.8	135.6
8	109.9	112.3	115.2	118.6	122.6	127.4
9	104.5	105.1	106.0	107.0	108.2	109.6

modelling assumptions being used), never reaches the level required to give 99% security of meeting the guarantee.

4.4.4 *Some comments on the equivalent annual premium results*

4.4.4.1 As was the case with the statutory reserving results, the annual premium results (which are given in Appendix B) tell a similar story to the single premium case.

4.4.4.2 One significant difference to note is that the capital support required over the asset share case does not reduce so quickly with increasing duration in the annual premium case. This is a consequence of the guarantee being related to all the premiums payable, whereas the asset share, at any time, only reflects the premiums that have been paid to date (which has the effect of reducing the rate at which the asset share 'out-grows' the guarantee). This would tend to suggest that, all other things being equal, single premium conventional with-profits business should be less capital intensive than annual premium business (even without allowing for any difference in initial expense levels).

4.4.4.3 The additional strength within the net premium reserves already noted can also be seen.

4.5 *Extending the Standardised Shortfall Probability Capital-Backing to Unitised With-Profits Business*

4.5.1 At the time of writing, discussions are continuing regarding what changes should be made to the statutory valuation regulations, particularly as far as unitised with-profits business (UWP) is concerned. Rather than produce results on a basis that could be overtaken by events, the extent of capital backing required, in order to restrict the probability of shortfall at maturity to 1%, is expressed, in this section, only in relation to the deterministically-modelled asset shares. This approach ties in with the comparative work presented later. An alternative presentation, expressing the capital support at time t as a percentage of the amount then guaranteed is included, by way of an interesting aside, in Section 4.6.

4.5.2 Only the results for single premium cases are presented in this section.

4.5.3 *Some comments on the approach to modelling used*

4.5.3.1 As in the conventional case, asset shares are assumed to accumulate to time *t* at the appropriate weighted average of the assumed returns on equities and gilts.

4.5.3.2 Unit values are assumed to grow at the gilt rate of return, namely, 5% p.a., irrespective of the EBR of the underlying assets. Allocation rate complexities are ignored, as are bid/offer spreads. Thus, premiums are assumed to be invested fully in units.

4.5.4 *Single premium UWP business capital support*

4.5.4.1 In Table 4.6 the maturity guarantee, which (for all single premium UWP projections) starts equal to the asset share at time 0, is assumed to grow at 5% (being a combination of the guaranteed rate plus discretionary bonus) until time *t*, and then at the guaranteed rate thereafter (i.e. no future discretionary bonus is assumed).

4.5.4.2 Table 4.6 shows the assets, expressed as a percentage of the deterministic asset share at time *t*, that are required to give a 99% chance of having sufficient assets at time 10 to meet the maturity guarantee applying at time *t*. While the shape is broadly similar to the corresponding conventional table (Table 4.4), various features are worth noting.

4.5.4.3 Compared with the conventional results, the percentages here start smaller, reflecting the smaller initial guarantee (the conventional case started by guaranteeing an accumulation rate throughout the term of 1% p.a.).

4.5.4.4 It takes three years for the quicker-growing UWP guarantee to 'overtake' the conventional guarantee, purely a consequence of the various assumptions made.

4.5.4.5 For the last few durations the percentages decrease, reflecting the larger number of past years in which the return on the asset share has exceeded the guarantee rate and the shorter future period of investment volatility.

4.5.4.6 Table 4.6 will be used later in the paper when comparing the effects of different approaches to the charging for capital support with the costs of the derivative-backed alternative.

4.6 *Alternative Presentation for UWP Business and More Onerous Guarantee Levels*

4.6.1 The capital support requirements in Section 4.5 are presented as percentages of the corresponding asset shares, as is appropriate for a study into possible policyholder charges. An alternative approach would be to express the same results as percentages of the corresponding unit values. While, in isolation, such a presentation would not naturally lead to an identification of capital support benefits, it would enable a comparison to be made with the reserving levels

Table 4.6. Proportion of asset share needed for 1% shortfall probability (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	84.7%	91.6%	99.8%	109.6%	121.6%	136.4%
1	89.4%	97.0%	106.1%	117.1%	130.8%	148.2%
2	92.6%	100.2%	109.2%	120.2%	133.8%	151.1%
3	96.1%	103.8%	112.9%	124.0%	137.7%	155.2%
4	98.5%	105.7%	114.2%	124.4%	137.0%	152.7%
5	101.4%	108.4%	116.6%	126.4%	138.3%	153.2%
6	103.8%	110.2%	117.6%	126.5%	137.2%	150.3%
7	105.3%	110.8%	117.1%	124.6%	133.4%	144.0%
8	106.3%	110.7%	115.7%	121.6%	128.3%	136.3%
9	103.5%	105.8%	108.4%	111.2%	114.5%	118.1%

currently being used by companies. In addition, it might also throw some light on aspects of the current discussions concerning statutory reserving requirements for UWP business.

4.6.2 Table 4.7 shows the required assets as a proportion of the unit value in the single premium case with a 0% guarantee.
As might have been expected, the figures increase with duration relative to those in Table 4.6, reflecting the slower growth in unit value compared with asset shares. (The underlying capital needs are, of course, the same in both tables.) The general shape of each column is still the same, but with a smaller decrease for later durations, since here no 'terminal bonus cushion' is being assumed.

Table 4.7. Proportion of unit value needed for 1% probability of shortfall (guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	84.7%	91.6%	99.8%	109.6%	121.6%	136.4%
1	90.5%	98.5%	108.0%	119.5%	133.7%	151.9%
2	94.9%	103.2%	113.1%	125.0%	139.9%	158.6%
3	99.7%	108.5%	118.9%	131.5%	147.2%	167.0%
4	103.4%	112.1%	122.3%	134.6%	149.6%	168.4%
5	107.9%	116.7%	127.0%	139.4%	154.5%	173.1%
6	111.7%	120.4%	130.4%	142.3%	156.6%	174.1%
7	114.8%	122.8%	132.1%	142.9%	155.7%	170.9%
8	117.3%	124.5%	132.8%	142.2%	153.1%	165.8%
9	115.7%	120.8%	126.5%	132.7%	139.6%	147.2%

4.6.3 Had the equivalent of Table 4.7 been shown for different guaranteed rates of future unit growth, the general shape would have been the same, but the percentages, particularly for longer outstanding durations, considerably larger.

Figures 4.1 and 4.2 show this for a variety of different guaranteed unit growth rates for, respectively, the 50% EBR and 100% EBR cases.

4.6.4 These results confirm what is intuitive. Guarantees are expensive in terms of capital requirements. This is particularly so at high EBRs, because of the greater volatility of investment returns. However, higher guarantees are relatively less onerous at longer durations, because of the shorter future periods of volatile returns.

4.6.5 While portfolio cross-subsidies may enable lower levels of capital support to be held without compromising shortfall security, the general message still remains — unit growth rates (and unit growth rate guarantees) must be kept affordable.

4.7 *Conclusion*

4.7.1 The main purpose of this section is to prepare for the comparison, in Section 7, by identifying the extent of capital support (in addition to the asset share) required by some reasonable levels of guarantee. The key tables are, therefore, Tables 4.4 and 4.6.

4.7.2 Clearly, many assumptions have been made in deriving these tables. While these may be rather artificial, the authors are confident that they are not unreasonable in current conditions.

4.7.3 Extrapolation to other situations should not be difficult — for example the case of past bonus declarations being higher than those projected in future. Another example would be where historic investment returns have enabled more substantial terminal bonus cushions to be built up than were modelled above.

4.7.4 Many of the results are for conventional business. This is appropriate, because the bulk of the in force with-profits business in the U.K. is conventional, and there may be particular relevance to demutualisation arrangements. While most new business is unitised, its statutory reserving basis is under review.

4.7.5 By covering all policy anniversaries, the results presented are applicable to both in force and new business.

4.7.6 In studies such as the one presented here, it is often not the actual size of the results which matters, but their relative shape. This is particularly so for the case for the statutory reserve experiments in Section 4.3. The reader should not conclude from those results that the authors are implying that published reserves are not appropriate, rather that a comparison of 'Form 9' free reserves may not give a complete picture of offices' relative positions.

5. Charging for Capital Backing

5.1 *Introduction*

5.1.1 The purpose of Section 4 was to quantify the extent to which capital support could be required by with-profits policies over their lifetime, in order to maintain a given level of exposure to equity investment, particularly in a low

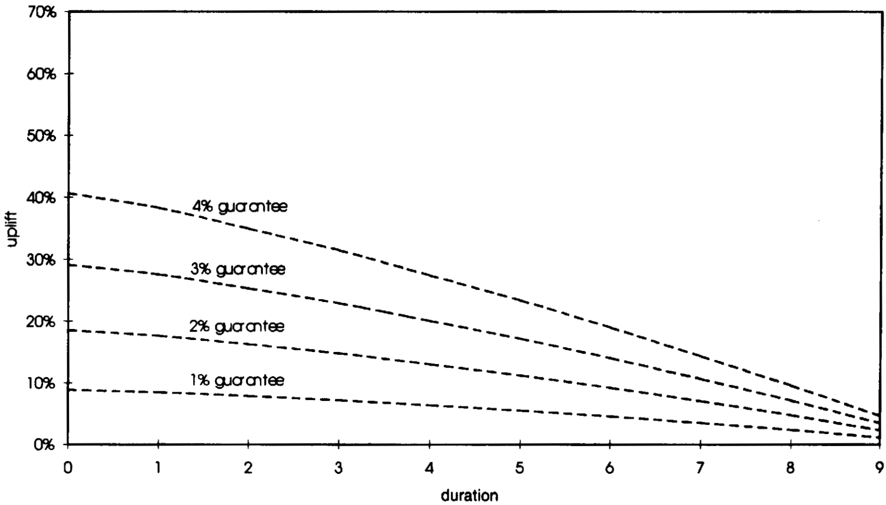

Figure 4.1. Additional percentage of unit value needed to maintain 1% probability of shortfall for non-zero guarantee rates (50% EBR)

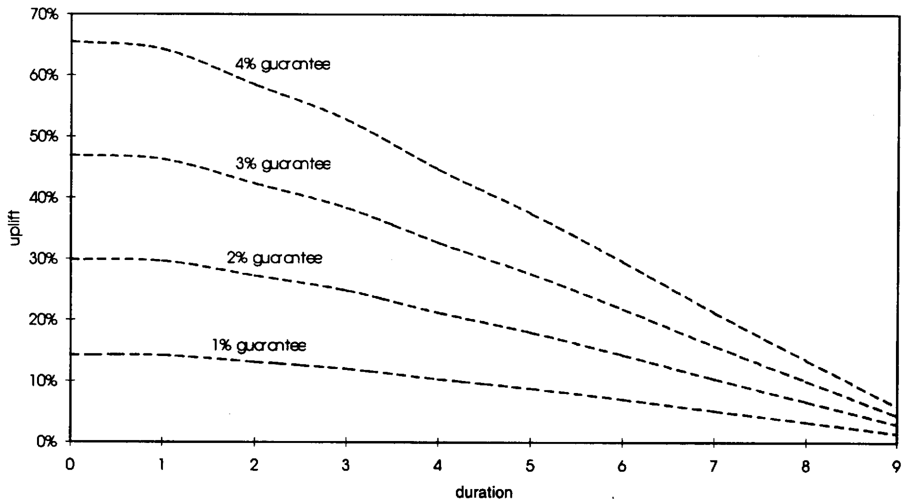

Figure 4.2. Additional percentage of unit value needed to maintain 1% probability of shortfall for non-zero guarantee rates (100% EBR)

inflation environment. This section follows on from this, and considers different approaches that an office might use, should it be considered appropriate to charge the policyholder for such support. (Obviously, for the purpose of this paper, were the capital not to be charged for, then there would be little point in considering the cost of the derivative-backed approach to be presented later.)

5.1.2 To avoid numerous tables of results, those illustrated in Sections 5.4 and 5.5 have been limited to single premium 10-year policies for both the CWP case, introduced in Section 3, and the UWP case with 0% p.a. guarantee rate in the future.

5.1.3 Whenever the effects of charges on policyholder returns are shown, the capital support involved is that needed to target a 1% shortfall probability on an individual policy basis.

5.2 *Market Practice*

5.2.1 *Current approaches*

5.2.1.1 An indication of the variety of approaches currently followed by offices in the U.K. towards with-profits business and, in particular, the charges levied on asset shares can be found in the asset share survey carried out from time to time by Tillinghast Towers Perrin (1997). The following points were drawn from these results.

5.2.1.2 Currently the majority of offices do not charge for capital backing of with-profits business, although there is an increasing trend in offices that do. No indication is given about the number of offices that measure the capital support required for different cohorts of business.

5.2.1.3 It is less common to charge for capital requirements of UWP business than CWP.

5.2.1.4 The charges are for, amongst other things, covering the cost of guarantees, covering the cost of smoothing, providing an extra return on capital or building up an estate.

5.2.1.5 Charges are deducted via reductions in investment return on asset shares or reductions in surrender or maturity values.

5.2.2 *Likely trends*

5.2.2.1 The results in Section 4 suggest that, in an environment of sustained low inflation and low expected investment returns, the amount of capital required to maintain an acceptable level of guarantee shortfall risk could be quite large (depending, of course, on the definition of 'acceptable' and the extent that any portfolio cross-subsidy was passed on to policyholders).

5.2.2.2 Added to this, the capital of life offices is coming under increasing pressure, due to such issues as reserving for guaranteed annuity options, lightening annuity mortality rates and the possible new UWP reserving requirements.

5.2.2.3 As capital becomes a more precious resource, the trend towards monitoring the use to which it is put, and the value created through it, is likely to continue.

5.2.2.4 Capital is required for a variety of purposes by a life office. Money used to back with-profits guarantees is not then available to support the writing of new business. Nor is it available to be invested without constraints, in order to maximise its expected return and hence build up the ongoing strength of the fund. In addition, capital put aside to back guarantees (or smoothing policy) may be called upon at times and so it is not just opportunity cost that is the issue. It seems natural, therefore, that more specific charging of asset shares for whatever capital support is allocated will receive more attention in the future.

5.2.2.5 Paragraph 5.2.2.4 looks at the situation where capital support is provided by the estate of the office. Where the capital is supplied by other asset shares, similar considerations apply. For one asset share to gain, another may be losing. Of course, views will differ as to the appropriateness of seeking to identify such cross-subsidy. Considering the effect of different approaches to charging for the access to another's capital need not imply that the charges must be levied in practice. Rather, the exercise could be regarded as a way of quantifying the benefits of membership of such an arrangement.

5.3 *Alternative Approaches to Charging for Capital Support*

5.3.1 *Opportunity cost*

5.3.1.1 In order to keep the research as widely applicable as possible, the authors chose to investigate two theoretical approaches to charging asset shares for the opportunity cost of their access to capital. In both cases it was anticipated that the charges would only be levied on asset shares which were less than the appropriate capital requirements for 1% shortfall probabilities derived in Section 4.

5.3.1.2 The first approach is to apply a charge equal to a proportion of the whole asset share (the 'asset share charging approach'). This is a broad-brush approach that does not specifically reflect the extent to which capital support is required at any given time. However, it is straightforward to apply, and is equivalent to the favoured method mentioned in the Tillinghast report. For the purposes of the comparative modelling work, the charge was set at (a rather arbitrary) 0.15% p.a.

5.3.1.3 The second method investigated is a more targeted approach (the 'capital support charging approach'). In this case, the charge is applied only to the capital support which is deemed to be allocated (i.e. the excess of the appropriate reserve derived in Section 4 over the asset share). It is assumed that capital support set aside is invested in the same equity proportion as the asset share, and that otherwise it would have been invested fully in equities. Capital is also presumed to be limited. The charge, therefore, has two parts:

(a) the loss of expected future equity out-performance over gilts; based on the amount of excess capital invested in gilts, plus

(b) a further charge (1% in the results shown below) as a percentage of the total excess capital to 'pay' for its use.

5.3.2 *Risk cost*

5.3.2.1 The above charges only reflect the opportunity cost of use of capital. They do not cover the cost of the guarantee itself.

5.3.2.2 In general, the extra capital support will not be required at maturity to meet the guarantee. Rather, it is held to allow the investment mismatch risk to be taken over the outstanding duration of the policy. Nevertheless, within the 99% of cases where (after allocating the appropriate supporting capital) there should be no shortfall, there will be some which require to draw on some of the support capital in order to meet the guarantee.

5.3.2.3 This was certainly the case in the modelling work carried out for this paper. For each guarantee level considered, the expected amount of capital support (i.e. in excess of asset shares) used per policy to meet guarantees was calculated, and then deducted as a 'one-off' charge at maturity.

5.3.2.4 As far as the bottom 1% of runs are concerned, the charge in ¶5.3.2.3 takes account of the cost of losing all the capital support allocated. Of course, further money will also be required in order to meet the guarantees in these worst cases. Depending on the source of this further capital, it could be argued that an additional charge should be levied. While the authors appreciate this, such additional charges have been ignored in order to maintain consistency with the derivative alternative presented in Section 6. Ignoring this across all charging methods will not affect the relative size of the results.

5.4 *The Impact of Charges on Capital Support Requirements for CWP Example*

5.4.1 The capital requirements developed in Section 4 were based simply on modelled investment returns (with no charges). Obviously, were the asset shares to grow slightly more slowly, as a result of annual charges, the capital support needs would be correspondingly greater.

5.4.2 The extent of increase in capital requirements is an indication of the severity of the charges involved. Thus, in the tables that follow, it is this 'uplift percentage' which is shown.

5.4.3 There is a feature of the asset share charging approach which is worth noting at this point. In cases where capital support is required, this charging method involves making deductions in proportion to the size of the asset share, and not in proportion to the capital support. As a result, any increase in capital support does not get reflected in increased charges.

5.4.4 The results under the two charging approaches are presented separately.

5.4.5 *Asset share charging approach*

5.4.5.1 Table 5.1 presents the results for the 10-year single premium CWP case when the asset share charging approach (at a rate of 0.15% p.a.) is used. The figures shown are the ratios of the post-charging reserves required to deliver a

1% shortfall probability to the pre-charging ones. Where a pre-charging reserve (for 1% shortfall probability) is less than the corresponding asset share, no capital backing is needed and therefore no charges are imposed. Thus, that reserve is unchanged from Table 4.4 and 100% is shown. Certain features of Table 5.1 are worth noting and are covered in ¶¶5.4.5.2 to 5.4.5.6.

Table 5.1. Capital required if asset share charging employed, as a proportion of (reserves for 1% shortfall probability with no charges) (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	101.4%	101.5%	101.5%	101.6%	101.7%
1	100.0%	101.2%	101.2%	101.2%	101.2%	101.3%
2	100.0%	101.3%	101.3%	101.4%	101.5%	101.6%
3	100.0%	101.0%	101.0%	101.0%	101.0%	101.1%
4	100.0%	100.0%	101.0%	101.0%	101.1%	101.1%
5	100.0%	100.0%	100.8%	100.9%	100.9%	101.0%
6	100.0%	100.0%	100.6%	100.6%	100.7%	100.7%
7	100.0%	100.0%	100.0%	100.5%	100.6%	100.6%
8	100.0%	100.0%	100.0%	100.0%	100.4%	100.4%
9	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

5.4.5.2 The ratios which are greater than 100% are simply a function of outstanding duration and the size of the 99th percentile (relative to the guarantee) from the corresponding projection to maturity. In particular, the amount of capital support needed to achieve a 1% shortfall probability is irrelevant.

5.4.5.3 In the main the percentages fall as duration increases reflecting the fewer number of years over which charges are applied.

5.4.5.4 The percentages increase gradually with EBR. This is because, the larger the EBR, the smaller is the 99th percentile of the projection from a given duration. The smaller the 99th percentile at maturity, the greater the proportionate effect of the accumulated charges and hence the slightly higher ratio above.

5.4.5.5 The relatively low increase in percentages with EBR is a consequence of what was mentioned in ¶5.4.5.2. Charging asset shares (in effect) only in proportion to the term over which capital support is required, and not in proportion to the amount of capital involved, is bound not to reflect well the relative benefits being gained for different EBRs. For example, at duration 0 the extra capital needed for an EBR of 60% is 1.2% of asset share, increasing to 50.7% of asset share for an EBR of 100% (see Table 4.4). The charges, as reflected by an increase of reserves ranging from 1.4% for 60% EBR to 1.7% for 100% equities, do not bear a similar relation to each other.

5.4.5.6 The same point applies for portfolios where all asset shares enjoy the same EBR, irrespective of their outstanding duration.

5.4.6 *Capital support charging approach*

5.4.6.1 Table 5.2 contains the same results for the capital support charging approach. This table differs from Table 5.1 in several ways.

Table 5.2. Capital required if capital support charging employed, as a proportion of (reserves for 1% shortfall probability with no charges) (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	100.3%	102.2%	104.1%	105.8%	107.2%
1	100.0%	100.6%	102.2%	103.6%	104.9%	105.8%
2	100.0%	100.5%	102.3%	104.1%	105.9%	107.6%
3	100.0%	100.3%	101.5%	102.6%	103.6%	104.3%
4	100.0%	100.0%	101.1%	102.1%	103.1%	103.9%
5	100.0%	100.0%	100.7%	101.5%	102.3%	102.9%
6	100.0%	100.0%	100.2%	100.8%	101.2%	101.6%
7	100.0%	100.0%	100.0%	100.3%	100.6%	100.9%
8	100.0%	100.0%	100.0%	100.0%	100.1%	100.3%
9	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

5.4.6.2 The uplift over capital needs ignoring charges is greater for higher EBRs and less for lower ones. In fact, the increases in reserves look, in some way, proportional to the extra capital required over and above the asset share. There are several effects at work.

5.4.6.3 The charges imposed have two components (opportunity cost and risk cost) which are both based on the excess of the 1% reserve over the asset share. The corresponding effects on maturing asset shares will also be proportional to the pre-charging capital support needs. In other words, the smaller the maturing asset share before charges, the bigger the effect of the charges, and hence more capital support is needed to maintain the shortfall probability. This increase in capital support will, in turn, lead to a further increase in charges, leading to a further increase in reserves, and so on. In a sense, therefore, the impact of this charging approach is geared.

5.4.6.4 The opportunity cost component of the charge is proportional to the percentage invested in gilts. This portion of the charge, therefore, decreases as EBR increases. This goes some way to mitigating the effect of the much higher capital requirements at higher EBRs.

5.4.6.5 A third effect is a consequence of the risk cost component and the sheer size of the capital support amounts for high EBRs. Even without the opportunity cost component, a 1% p.a. charge applied to capital support of, say, 30% of an asset share is much more onerous than a 0.15% p.a. charge on the whole of the asset share.

5.5 *The Impact of Charges on Policyholder Returns for CWP Example*

5.5.1 From the discussion in the previous section, it will be obvious that the effect of the asset share charging approach on policyholder returns is more straightforward than that of the capital support alternative. For that reason, this section concentrates on the latter.

5.5.2 *Policyholder return percentiles*

5.5.2.1 Before quantifying the effect of charges on policyholder returns in specific cases, it is worth looking at the spread of possible returns. Figure 5.1

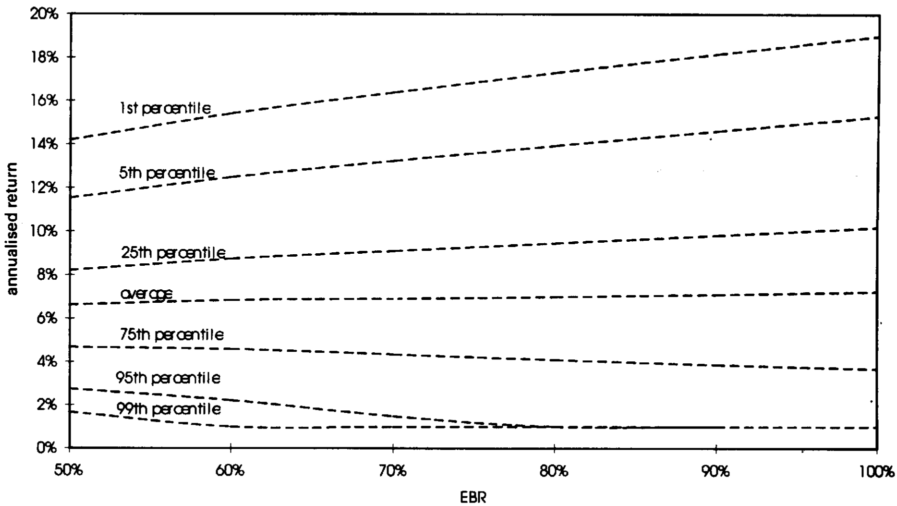

Figure 5.1. Spread of projected returns for different EBRs (CWP with capital support charging approach)

gives an indication of that spread for the 10-year single premium CWP case at duration 0. The picture at other durations would not be dissimilar.

5.5.2.2 The following comments can be made:

(a) The volatility of potential return reduces as EBR reduces, as expected, since all the volatility of return is modelled within the equity component, as discussed in Section 3.

(b) The returns at the lower tails are better for low EBRs than for high EBRs. This is also as expected, since the lower tails of the distribution of equity returns exhibit annualised returns below that assumed for gilts. In addition, charges are lower, as a consequence of lower capital support requirements.

(c) The guarantees bite (in more than 1% of cases) for EBRs of around 60% and above, as can be seen from the shape of the 99th percentile.

(d) Equity outperformance at higher percentiles of the distribution of returns is sufficient to outweigh charges, and mean returns are better at higher EBRs than at lower ones.

5.5.3 *Policyholder mean returns*

5.5.3.1 While the relative effect will vary for different percentiles, an indication of the impact of the different charging approaches can be got from looking at their effect on mean returns (over the 10-year period).

5.5.3.2 Table 5.3 presents the appropriate actual maturity accumulations from the modelling work carried out. As stated above, these projections start from duration 0. In other words, the stochastic projection (of equities) is over the whole term of the policy. Table 5.4 presents the same information, but expressed in terms of annualised returns (or reductions in annualised return or yield (RIY)) over 10 years.

Table 5.3. Comparison of mean accumulations on the different charging bases

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
No charging	18,993	19,439	19,879	20,319	20,758	21,197
Asset share charging	18,993	19,215	19,635	20,042	20,439	20,829
Capital support charging	18,993	19,392	19,521	19,671	19,864	20,126

Table 5.4. Mean returns and comparative reductions in yield on the different charging bases

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
Yield (no charging)	6.6%	6.9%	7.1%	7.3%	7.6%	7.8%
Asset share RIY	0.0%	0.1%	0.1%	0.1%	0.2%	0.2%
Capital support RIY	0.0%	0.0%	0.2%	0.3%	0.5%	0.6%

5.5.3.3 These tables confirm several of our previous observations:

(a) Obviously, in the cases where no extra capital is required, no charges are taken and mean returns are the same.

(b) The shapes of the RIYs differ for the two approaches to charging, with the result that the mean payout on asset share charging improves relative to that on capital support charging as EBR increases (reflecting the increased capital needs as a result of the greater investment risk).

(c) The returns at high equity backing ratios under the capital support approach are substantially lower than under the asset share charging method. This probably says more about the 0.15% chosen as the level for the latter than about the deductions under the former.

(d) Despite the drain on policyholder returns from the charge for the capital needed, it is still worth the policyholders' while having a high EBR, at least in terms of average returns relative to lower EBRs.

5.6 *The Impact of Charges on Capital Support Requirements for UWP Example*

5.6.1 Tables, 5.5 and 5.6 are the UWP equivalents of Tables 5.1 and 5.2, and are included for completeness. The percentage uplifts relate to the figures shown in Table 4.6, and so apply to the case where no future unit growth (from the duration in question) is allowed for.

5.6.2 There is little comment to add beyond what has been said for CWP business. The uplift in reserves again can be seen to reflect the charging method used, particularly the correspondence between the charges under the capital support approach and the amount of capital support required.

Table 5.5. Capital required if asset share charging employed, as a proportion of reserves for 1% shortfall probability with no charges, (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	100.0%	100.0%	101.5%	101.6%	101.7%
1	100.0%	100.0%	101.2%	101.2%	101.2%	101.3%
2	100.0%	101.3%	101.3%	101.4%	101.5%	101.6%
3	100.0%	101.0%	101.0%	101.0%	101.0%	101.1%
4	100.0%	100.9%	101.0%	101.0%	101.1%	101.1%
5	100.8%	100.8%	100.8%	100.9%	100.9%	101.0%
6	100.6%	100.6%	100.6%	100.6%	100.7%	100.7%
7	100.5%	100.5%	100.5%	100.5%	100.6%	100.6%
8	100.3%	100.3%	100.3%	100.4%	100.4%	100.4%
9	100.2%	100.2%	100.2%	100.2%	100.2%	100.2%

Table 5.6. Capital required if capital support charging employed, as a proportion of reserves for 1% shortfall probability with no charges (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	100.0%	100.0%	100.0%	101.8%	103.5%	104.9%
1	100.0%	100.0%	101.0%	102.5%	103.8%	104.8%
2	100.0%	100.0%	101.8%	103.6%	105.4%	107.1%
3	100.0%	100.6%	101.8%	102.9%	103.8%	104.6%
4	100.0%	100.8%	101.9%	103.0%	103.9%	104.7%
5	100.2%	101.0%	101.9%	102.7%	103.4%	104.0%
6	100.4%	100.9%	101.4%	101.9%	102.3%	102.5%
7	100.4%	100.8%	101.2%	101.5%	101.7%	101.9%
8	100.3%	100.5%	100.7%	100.8%	100.9%	101.0%
9	100.1%	100.1%	100.2%	100.2%	100.2%	100.2%

5.6.3 The relative overall shape of the two tables differs slightly compared with those in Section 5.4, but only because of the differing shapes of capital requirements derived in Sections 4.4 and 4.5.

5.6.4 Furthermore, had higher UWP guarantee rates been considered here, the difference in the two tables would have been even greater, on account of the higher capital requirements (and hence larger charges on the capital support method) involved.

5.7 *Concluding Comments*

5.7.1 Some of the above results will be considered further in Section 7, when the performance of the capital-backed approach to providing with-profits guarantees is compared with the derivative-based alternative introduced in Section 6.

5.7.2 *Comparison of approaches*

5.7.2.1 It is apparent from the above that to charge for access to capital via a simple annual percentage of asset share does not achieve a good fit between the incidence of the capital support needed and the charge for it. Nor would it necessarily produce equitable charges where asset share EBRs were not all set equal within the portfolio.

5.7.2.2 This could be problematic if, as is usually the case, there are other potential uses of capital or capital is limited. If different cohorts of business have substantially different capital needs, not reflecting this in different levels of charge could result in quite significant smoothing across them.

5.7.2.3 The capital support approach may be a more natural method to use for this purpose, in that it better targets the real opportunity costs. However, it is not so easy to explain to policyholders, particularly those whose asset shares need a fair amount of capital support, and are therefore subject to quite large charges as a result.

5.7.3 *Are higher EBRs worth having?*

5.7.3.1 At the charging levels used above for the capital support approach, the higher capital strain from high EBRs resulted in substantially higher charges that reduced the benefit of long-term equity outperformance over gilts, but not to the extent that mean overall returns did not increase with EBR. Of course, were a different opportunity cost to be used, then it is quite possible that the opposite could be the case.

5.7.3.2 It is not just mean returns which are relevant, however. The volatility in possible return also needs to be considered, since this will affect how much, if any, terminal bonus may be payable. Figure 5.1 showed something of the spread of possible policyholder returns and, in particular, the greater volatility resulting from higher EBRs. Thus, in deciding the extent to which with-profits asset shares should be invested in real assets, PRE considerations must encompass acceptable levels of volatility in terminal bonuses.

5.7.4 *Giving and receiving*

5.7.4.1 This section has concentrated on the situation where asset shares need capital support. There is also the 'other side of the coin', namely, where asset shares are greater than the amount needed to maintain a given shortfall probability, and so can be net providers of capital support.

5.7.4.2 Where the policies receiving support have to pay for it, it could be argued that any policy providing support should receive at least some of the benefit. Were this to be the case, the extent of cross-subsidy implicit in the asset share approach might require particular consideration.

5.7.4.3 The authors accept that it is possible to manage a with-profits fund at a much higher level than is set out above, without seeking to identify precisely the degree to which any particular policy is benefiting at any particular time. However, such an approach does not lend itself to a comparison with the market price of alternative investment structures, which may be viewed by policyholders as simpler means to the same end.

6. The Derivative-Based Alternative

6.1 *Introduction*

This section of the paper contains the development of the derivative-based alternative for providing with-profits guarantees. Derivatives can be used to provide as complete a portfolio hedge as practicalities permit. However, this is costly, and security for the office would be obtained at the expense of lower policyholder returns. In order to ensure a fair comparison of costs, the derivatives used must offer (only) the same level of financial security as the capital backing derived in Section 4. Once the option strategy is in place, no further capital should be needed (beyond the asset share less option premium) in order to meet the guarantees 99% of the time. Estimated market prices of such derivatives are presented later in this section. Discussion of the resulting policyholder payouts, particularly compared with those under the capital-backed approach, is kept until Section 7.

6.2 *The Put Spread Strategy*

6.2.1 As the gilts proportion of any asset share is assumed to be perfectly matched by outstanding duration, it is only the maturity value of the equity portion which is uncertain. The derivatives purchased, therefore, only relate to the equity component.

6.2.2 Purchasing a European put option with the strike price determined by the relationship between the amount of guarantee at maturity not covered by gilts and the current level of equities within the asset share (less the amount of equities sold to purchase the options) would give 100% protection, subject to:

(a) the assets on which the option is written being a perfect hedge for the equity portfolio in which the asset share is invested; and
(b) counterparty risk.

A study of the latter is outwith the scope of this paper. The former will be ignored for the time being, but discussed later in the paper.

6.2.3 Comparing the cost of this 100% protection approach with the capital-backed alternative would not be consistent, due to the, albeit small, differences in the risks being run. In order to be consistent with the 99% criterion used in setting the capital requirements in Sections 4 and 5, the degree of security provided by the derivatives must be reduced in some way.

6.2.4 Buying 99% of the required put option would, of course, not give what is required, but, rather, less than 100% security in more than 1% of cases. Selling a second European put option, on the other hand, could do what is needed. If the strike price of this option were determined by the relationship between the amount of the 99th percentile projected asset share at maturity not covered by gilts and the current level of equities within the asset share (adjusted for the amount of equities used to purchase the options), then its sale would remove the security against equity downside not offered by the capital-backed approach.

6.2.5 Thus, for each policy whose asset share is insufficient to offer the desired level of security, two options need to be considered. The one to be sold will have a lower strike price than the one bought, and so only offset part of the cost of the latter rather than make any new money. In all other respects (i.e. in terms of underlying asset, term to expiry, and so on) the options will be identical. This type of arrangement is often referred to as a 'put spread'.

6.2.6 Figure 6.1 shows the maturity profile for a put spread strategy with a high strike price of 80% of the initial value and a low strike of 60%. The payoff is zero for values above 80%, where (it is assumed for this example) no capital is required to support the guarantee. As the value of the underlying equity drops below 80% of the initial value, there is an increasing return from the options to cover the shortfall versus the guarantee. This return reaches a maximum at a strike of 60%, which, in this example, represents the 1% lower tail of likely equity performance.

6.2.7 Another possible derivative strategy would be to hold cash and sell a 'call spread'. In the example above this would involve holding sufficient cash to provide 20% payoff at maturity, selling a call option with a strike price of 60% and buying a call option with a strike price of 80%. This arrangement would have the same economic effect as the put spread and, in theory (assuming 'put/call parity'), the same cost. For the remainder of the paper, only the put spread will be considered.

6.3 *Modelling the Cost of the Put Spread Strategy*

6.3.1 The cost of the put spread has been calculated using the standard Black-Scholes equation for European options on current market conditions at the time of writing (August 1999). Whether or not the market prices are consistent with the approach taken to the stochastic modelling of the long-term future is not important for the purposes of this paper; it is relevant to use current market

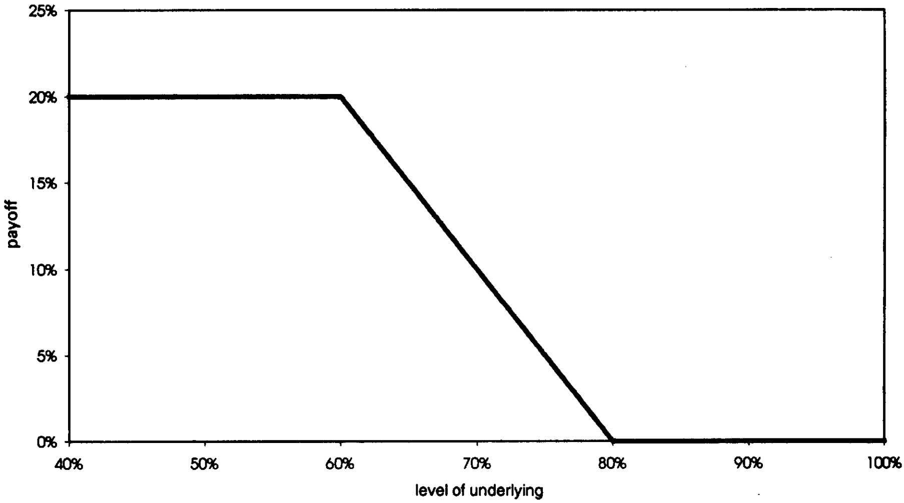

Figure 6.1. Profile of payoff from a put spread at expiry

conditions for the options, as they would be purchased 'up front', thereby locking in the conditions at the time.

6.3.2 *Black-Scholes input parameters*

6.3.2.1 There are several input parameters required in order to calculate the price of each put option. The underlying equities have been assumed to be FTSE 100 equities. This could introduce a mismatch, in practice, between the options and the underlying with-profits fund, and is discussed in Section 6.5. The strike prices have been determined, as described above, as proportions of the spot price of the equities available within the asset share. The term to expiry of each option has been taken to be the same as the outstanding duration of the policy being considered. The dividend yield assumption is consistent with the economic assumptions given in Section 3.4. The appropriate discount rates have been derived from the current (August 1999) term structure of sterling swap rates in the market.

6.3.2.2 The implied volatility is a function of term to expiry and strike price. A mid-market volatility surface has been used to derive the volatility for each put option. One of the features of this surface is the large skew in volatility between different strike prices. At low strike prices the volatility is much higher than at high ones. This feature of the current U.K. market has the benefit of making the sold put option more valuable than if the volatility was flat, cheapening the overall structure.

6.3.3 *A minor practical point*

The net cost of the derivative spread is deducted from the asset share at the start of each stochastic projection in proportion to its EBR at the time. The EBR of the initial asset share therefore remains unchanged. There is a possible second order effect on the statutory reserves in this situation due to the potential impact on allowable yield from switching into a derivative. This small effect has been ignored since the paper does not specifically consider the cost of 'statutory capital' (i.e. the situation where an asset share is less than the corresponding statutory reserve).

6.4 *Cost of the Put Spread Strategy*

6.4.1 The results which follow are based on the same 10,000 simulations (and, in particular, their 99th percentiles) as were used for Sections 4 and 5. Each figure is obtained by comparing the 99th percentile projected return with the corresponding maturity guarantee. Where the former is smaller, the two are used to set the lower and upper strike prices of the put options and the overall put spread cost calculated.

6.4.2 Table 6.1 presents the one-off costs of the appropriate put spreads for the 10-year CWP single premium example. For ease of comparison, the costs are expressed as percentages of the underlying asset share (before the options are purchased). Table 6.2 presents the equivalent information for the corresponding UWP example (where no future increase in unit prices is assumed).

6.4.3 While the main discussion of these results (and their effect on policyholder returns) is reserved until Section 7, the following observations are made at this point:

(a) The shape of the costs is broadly in line with the extra capital required to support the guarantee. This is to be expected, since the relationship between capital required (guarantee at maturity) and asset share (return at maturity) is the basis of the assessment of the strike prices of the put options.

(b) There is not a precise correlation — the put spread becomes relatively cheaper as term to expiry decreases. This is due, partly, to falling time value within the options, and, partly, to the put options moving from at or slightly in the money (for early durations) to an increasingly out of the money position (for later durations). The latter is the result of the deterministic accumulation of starting asset shares at a rate in excess of the build up of guarantees.

(c) The pattern of costs for both CWP and UWP examples generally reflects the level of underlying guarantees. The effect of a higher UWP guarantee assumption can be seen in Table 6.3.

Table 6.1. Cost of put spread as a proportion of asset share (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	0.0%	0.2%	1.6%	3.2%	4.8%	6.5%
1	0.0%	0.5%	1.9%	3.4%	5.1%	6.7%
2	0.0%	0.4%	1.7%	3.1%	4.6%	6.1%
3	0.0%	0.3%	1.5%	2.8%	4.1%	5.5%
4	0.0%	0.0%	1.0%	2.1%	3.3%	4.5%
5	0.0%	0.0%	0.7%	1.7%	2.7%	3.7%
6	0.0%	0.0%	0.3%	1.1%	1.9%	2.8%
7	0.0%	0.0%	0.0%	0.5%	1.1%	1.8%
8	0.0%	0.0%	0.0%	0.0%	0.3%	0.8%
9	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Table 6.2. Cost of put spread as a proportion of asset share (UWP, guarantee rate = 0%)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	0.0%	0.0%	0.0%	1.4%	2.9%	4.5%
1	0.0%	0.0%	0.9%	2.3%	3.8%	5.4%
2	0.0%	0.0%	1.3%	2.7%	4.1%	5.7%
3	0.0%	0.5%	1.7%	3.1%	4.4%	5.9%
4	0.0%	0.8%	1.9%	3.1%	4.3%	5.5%
5	0.2%	1.1%	2.1%	3.2%	4.3%	5.4%
6	0.5%	1.3%	2.2%	3.1%	4.1%	5.0%
7	0.7%	1.3%	2.0%	2.8%	3.6%	4.3%
8	0.8%	1.3%	1.8%	2.4%	2.9%	3.5%
9	0.4%	0.7%	0.9%	1.2%	1.5%	1.8%

6.4.4 The cost of the put spread can be quite high, in excess of 5% of asset share at 100% EBR. This is not as bad as it seems, though, since this is a one off cost, and not incurred each year.

6.4.5 The costs are not dissimilar to the results for capital support charging in Section 5. While the different impact of the two methods will be analysed in Section 7, it is interesting to note that the cost of the options spread over the remaining years of the term appears to be of the same order of magnitude as the RIY under capital support charging.

6.5 *Practical Considerations*

6.5.1 The option strategy suggested above has been deliberately kept simple to aid transparency and comparability. At the same time, however, it is important to be aware of the additional complexity that would be involved in practice.

Table 6.3. Cost of put spread as a proportion of asset share (UWP, guarantee rate = 2% per annum)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	0.6%	2.1%	3.8%	5.5%	7.3%	9.0%
1	1.2%	2.8%	4.5%	6.2%	8.0%	9.8%
2	1.5%	3.0%	4.6%	6.3%	7.9%	9.6%
3	1.8%	3.2%	4.7%	6.2%	7.8%	9.3%
4	1.8%	3.1%	4.4%	5.7%	7.1%	8.4%
5	1.9%	3.1%	4.2%	5.4%	6.5%	7.7%
6	1.9%	2.9%	3.9%	4.8%	5.8%	6.8%
7	1.7%	2.5%	3.3%	4.0%	4.8%	5.6%
8	1.5%	2.0%	2.6%	3.1%	3.7%	4.3%
9	0.7%	1.0%	1.2%	1.5%	1.8%	2.1%

6.5.2 In reality, with-profits funds are invested in a range of asset classes, the returns on which are volatile. Full hedging of downside risk with put options would require tailored options for each asset class.

6.5.3 Among the broad categories of asset class are U.K. equities, overseas equities, U.K. fixed interest and overseas fixed interest. These can be hedged in turn by FTSE 100 options, various overseas index options (e.g. S & P 500 for the United States of America, Eurostoxx 50 or individual European index options for Europe and so on), gilt options and overseas bond options (on, for example, Bunds and U.S. Treasuries). Nevertheless, there is a potential mismatch or basis risk between the coverage of put options and the underlying investments. Options tend to relate to blue chip indices, whereas with-profits investment performance reflects the actual basket of assets held.

6.5.4 For example, an office's U.K. equity portfolio may be closer to the FTSE All-Share Index than the FTSE 100. While this mismatch can be reasonably large, the largest companies are becoming a bigger proportion of the full market. For example, the ratio of market capitalisation between the FTSE 100 and FTSE All Share companies is around 76% at the time of writing (August 1999). Companies already use futures and options on blue chip indices for tactical asset allocation purposes, albeit over short terms. The performance of these two indices can easily diverge over the short term, but over longer terms they will tend to match each other reasonably well.

6.5.5 Long-term options, like those modelled in this paper, are only available as over the counter (OTC) derivatives from investment banks and the like. Current admissibility limits restrict the level of concentration of investment in OTCs. Liquidity and diversification of counterparty risk could be issues, due to the limited number of providers and the fact that the value of the spread can become an increasingly significant part of the portfolio. One method of mitigating counterparty exposure is to receive cash collateral from the counterparty, to be kept on default. This has the added benefit of also reducing the exposure for admissibility purposes.

6.5.6 In with-profits funds the levels of guarantees at various outstanding terms change all the time due to, amongst other things, new money, bonus declarations and policy exits. An office hedging its guarantees in the future will soon find the level of guarantee to be hedged has changed. The overall hedge could be reviewed on a regular basis and updated for the change in term structure and notional amount of guarantees written.

6.6 *Comment and Conclusions*

6.6.1 The main conclusions will be developed in Section 7, when the use of derivatives is compared with the capital support approach.

6.6.2 While the simple strategy presented above is somewhat theoretical, it is possible to devise more complex strategies which, while not a perfect hedge, would give some comfort that the guarantees would be met in the future and therefore free up some of the capital which would otherwise be required.

6.6.3 The derivative strategy involves spending money as insurance against reasonably unlikely events, whereas the reserving strategy locks up capital that is ultimately expected, in most circumstances, to be released to the office. The cost of the put spread is broadly proportional to the amount of reserves that would be required if excess capital were being used to back the guarantees. Thus, if the capital locked up is charged for, the two approaches are not as different as they seem.

7. Comparison of the Derivative and Capital-Backed Strategies

7.1 *Introduction*

7.1.1 Sections 4, 5 and 6 have presented two broad approaches which a life office could use, in conjunction with appropriate charges, to provide guarantees under with-profits policies. Overall there are four separate situations to compare, the first three of which are variants of the capital-backed strategy:

(a) capital-backed with no charges;
(b) capital-backed with charges proportional to asset share;
(c) capital-backed with charges proportional to the level of support; and
(d) the derivative-backed approach.

7.1.2 The comparison deals first with mean policyholder returns and then goes on to consider the possible range of payouts under each approach. For simplicity, attention is restricted to the 'duration 0' case.

7.1.3 When option (b) was first introduced in Section 5, the charging level illustrated was 0.15% of asset share p.a. In order to illustrate the sensitivity of results to this choice, an alternative, 0.25% p.a., is also considered below. Similarly, for option (c), the Section 5 choice of 1% over equity return for opportunity cost p.a. is augmented by a second possibility, 2% over equity return p.a. By interpolation or extrapolation, the level of charges equivalent to the put spread can therefore be derived.

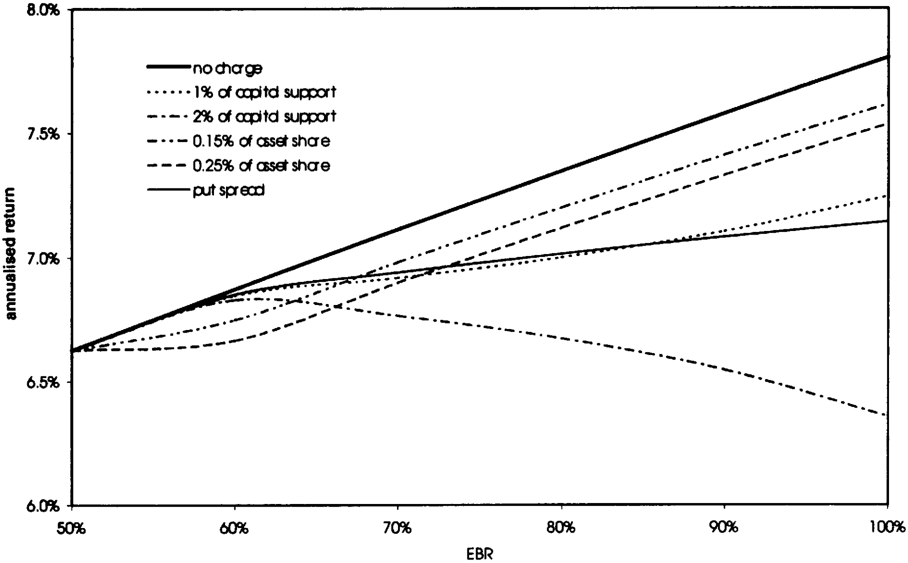

Figure 7.1. Mean returns on different charging methods, 10-year, single premium, CWP, duration zero

7.2 *Comparison of Mean Returns for Conventional Business*

7.2.1 Figure 7.1 illustrates, for a range of EBRs, the mean annualised returns under each of the charging approaches for the 10-year single premium CWP example at duration 0.

7.2.2 Not surprisingly, all the plots for methods allowing for charges lie on or below that for the no charge comparator.

7.2.3 The lines plotting returns for the two levels of option (b) (i.e. charging in proportion to the asset share) show the limitations of this method of charging for use of capital. At the lower EBRs, where capital backing is modest, the impact of charges is the highest of any method. For higher EBRs, the charges seem light compared with the cost of the put spread approach. In fact, as EBRs increase, the charges do not reflect the fact that the capital support is increasing exponentially.

7.2.4 As may be obvious, the problem highlighted in ¶7.2.3 is actually a consequence of keeping constant the level of asset share being charged, regardless of EBR and the consequential capital needs. This point will be picked up later.

7.2.5 On the other hand, the mean returns under the capital support charging approach follow a more intuitive pattern. There is little or no charge at low EBRs, due to little or no need for extra capital. In the case of the 1% charge (in

Table 7.1. Reductions in yield due to charges on the range of methods with reference to mean returns (CWP, duration zero)

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
Yield with no charges	6.6%	6.9%	7.1%	7.3%	7.6%	7.8%
Reductions in yield due to:						
Put spread cost	0.0%	0.0%	0.2%	0.3%	0.5%	0.7%
Asset share charge 0.15% p.a.	0.0%	0.1%	0.1%	0.1%	0.2%	0.2%
Asset share charge 0.25% p.a.	0.0%	0.2%	0.2%	0.2%	0.2%	0.3%
Capital support charge 1% p.a.	0.0%	0.0%	0.2%	0.3%	0.5%	0.6%
Capital support charge 2% p.a.	0.0%	0.0%	0.3%	0.7%	1.0%	1.4%

addition to the opportunity cost of holding gilts) for use of capital, the mean returns increase with EBR, as does the extent of divergence from the 'no charges' line. Interestingly, however, the resulting line seems to track quite closely that for the put spread alternative.

7.2.6 The results in ¶7.2.5 seem reasonable. As capital requirements increase with EBR, so does the corresponding charge. Nevertheless, expected payouts still increase with EBR, justifying the preference for equity investment.

7.2.7 This is not the situation when the level of charge is increased to 2% (plus the opportunity cost of holding bonds). The high capital requirements at high EBRs, combined with this higher level of opportunity cost charge, result in deductions which cause the mean policyholder return to fall as EBR increases. While the 2% charge may be a little 'over-cooked', its inclusion does illustrate the fact that, if the cost of capital is high, it makes little sense for an office to aim for a high EBR, as policyholders cannot be expected to benefit from equity outperformance.

7.2.8 It is, perhaps, not surprising that the 1% of capital support line should approximately track that for the put spread approach. As explained in Section 6, under each approach, the deductions from asset share (whether through annual charges for capital or the one-off cost of the derivative) are, in some sense, proportional to the amount of capital support required. The fact that they are so close together suggests that the 1% charge on this capital is roughly equivalent to the estimated market costs of opting for the derivative-backed alternative, at least on the set of assumptions used.

7.2.9 An alternative way of presenting the financial effects of each of the charging alternatives is to express them as reductions in annualised policyholder return (RIY). These are shown in Table 7.1, from which can be seen again the similar impact on policyholder returns of the put spread and the capital support (with an opportunity cost of 1% over equity return) approaches.

7.3 *Comparison of Spread of Returns for Conventional Business*

7.3.1 Figure 7.2 illustrates, for an EBR of 100%, the spread of modelled

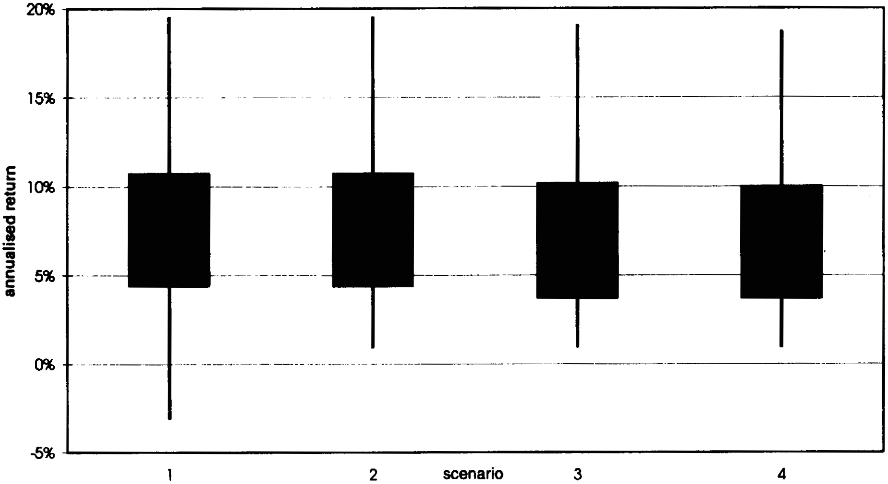

Figure 7.2. Spread of returns to policyholder and impact of guarantees and charges (CWP, duration zero)

policyholder returns under various scenarios. Again, the situation modelled is the 10-year CWP single premium at duration 0.

7.3.2 The spread of payouts is illustrated using the 25th and 75th percentile payouts (the 'boxes') to cover 50% of the expected range of returns and the 1st and 99th percentiles (the 'whiskers') to display the wider range of outcomes.

7.3.3 The scenarios modelled are the following:

(1) no charges and no guarantees – this simply represents the range of projected asset shares without any allowance for guarantees or their costs;
(2) no charges, but allowing for the with-profits guarantee;
(3) with-profits guarantees with charging for use of capital (capital support charging approach); and
(4) with-profits guarantees hedged using a put spread.

7.3.4 Figure 7.2 illustrates several important points:

(a) Offering a guarantee reduces the volatility of returns by putting a floor on the policyholder payout. This is illustrated in the difference between scenarios 1 and 2.
(b) Charging for the guarantee (through capital support charges) acts as a ceiling for returns. The 1st, 25th and 75th percentiles all fall while the 99th percentile remains at the guarantee floor, i.e. the spread of returns reduces.
(c) Buying put spread protection at the outset has much the same effect on policyholder payouts as charging for the guarantee at the levels modelled.

(d) An EBR of 100% was chosen for the figure in order to emphasise the points being made. Other EBRs would have exhibited similar features, but with less variability due to lower investment volatility and hence lower guarantee charges.

7.4 *Comparison of Mean Returns for Unitised With-Profits Business*

7.4.1 Figure 7.3, the UWP equivalent of Figure 7.1, shows the mean returns under the same six charging alternatives for the 10-year, single premium case at outset, where no unit growth is assumed at all. The patterns reflect the features identified for CWP, but with a few differences.

7.4.2 UWP guarantees tend to build up at different rates from those for CWP business. Their initial values are typically less than the CWP equivalents, being based on premiums paid rather than premiums payable. Of course, where a generous growth rate has been guaranteed (whether just on units bought or on all future purchases too), the situation can be very different. In the example modelled in Figure 7.3, the guarantee is set at the premium amount, less than the CWP example (which guaranteed 1% p.a. return), and this is reflected in the way the charges start to apply further to the right, around an EBR of 70%.

7.4.3 For EBRs above 70%, the plotted lines follow similar patterns to the CWP example, but with not such pronounced differences between them. This is due to the generally lower capital requirements, which are a function of the 0% guarantee rate used.

7.4.4 Again the mean returns on the put spread and the 1% capital support charge are very similar, as can be seen from Table 7.2, the UWP equivalent of Table 7.1.

7.4.5 An analysis of the spread of payouts reveals a similar pattern to that shown for CWP business in Figure 7.2.

7.5 *Conclusions*

7.5.1 In this section it has been demonstrated that, if an office wishes to charge for the capital support required to back a particular level of guarantees or a specific EBR, then a charge as a fixed proportion of asset share (or via a fixed reduction in asset share investment return) does not match the actual incidence of capital support particularly effectively.

7.5.2 A better approach appears to be applying the charge only in proportion to the actual support amount, and basing the level of the charge on what it is actually costing to tie capital up in this way (i.e. the loss of some equity return and whatever other opportunities are considered relevant).

7.5.3 It is, however, recognised that the asset share charging method is easy to apply in practice. Given a particular EBR for a block of business, an appropriate level of asset share charge could be derived from Figure 7.1. In other words, the charge implied by the put spread for that EBR could be converted into a charge as a proportion of asset share. This charge would need to be reassessed if the office wished to alter the block's EBR. The capital support charging

Table 7.2. Reductions in yield due to charges on the range of methods with reference to mean returns (UWP, guarantee rate = 0%, duration zero)

	Equity backing ratio					
	50%	60%	70%	80%	90%	100%
Yield with no charges	6.6%	6.9%	7.1%	7.3%	7.5%	7.7%
Reductions in yield due to:						
Put spread cost	0.0%	0.0%	0.0%	0.1%	0.3%	0.5%
Asset share charge 0.15% p.a.	0.0%	0.0%	0.0%	0.1%	0.1%	0.2%
Asset share charge 0.25% p.a.	0.0%	0.0%	0.0%	0.2%	0.2%	0.2%
Capital support charge 1% p.a.	0.0%	0.0%	0.0%	0.2%	0.3%	0.4%
Capital support charge 2% p.a.	0.0%	0.0%	0.0%	0.3%	0.6%	0.9%

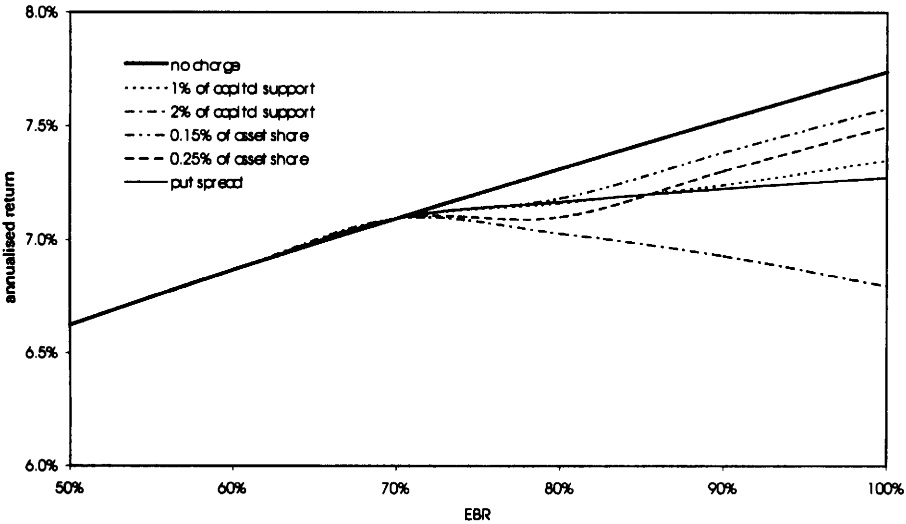

Figure 7.3. Mean returns on different charging method (UWP, guarantee rate = 0%, duration zero)

method is more robust, in that the charging automatically copes with a change in EBR (so long as the appropriate capital requirements are known). While the figures shown relate only to a 10-year policy at outset, similar considerations apply for other terms and durations.

7.5.4 In particular, the structure and level of charge (i.e. 100 basis points over expected equity return) used in the capital support charging method do not look unreasonable. When a larger opportunity cost was incorporated (of 200 basis points), the impact on policyholder returns at high EBRs was extreme. This does not necessarily mean that this level of charge is too high. Rather, where capital is expensive (or in short supply), the results suggest that it is not always in the policyholders' interests to draw on it to support investment risk.

7.5.5 It has also been demonstrated in this section that, where an office chooses to hedge the investment mismatch risk with a suitable put spread, then the cost of doing so seems, irrespective of EBR, to track that of the capital-backed approach, with a required opportunity cost of 100 basis points above expected equity return. Of course, the profit margin of derivative providers could alter this relationship.

7.5.6 Were these costs to be translated into policyholder charges, then, while their incidences would be very different, their effects would be very similar (for policies held to maturity). A consideration of the situation for lapses and, indeed, for whole portfolios, is outwith the scope of this paper.

7.5.7 It may appear rather a coincidence that the put spread and capital charging method costs look so similar. The pricing of derivatives is based on risk-free discount rates derived from market swap rates. These, typically, are based on gilt rates plus an appropriate credit spread, and so are likely to lie somewhere between the long-term expected returns on gilts and equities. However, as previously discussed, the strike prices in the put spread (and hence the cost of the spread) are closely related to the amount of capital support which would otherwise be needed. It seems reasonable, therefore, that the costs of the two approaches could move in line as EBRs (and hence capital support needs) change, provided that the level of charge on support capital is consistent with market prices.

8. Conclusion

8.1 With-profits guarantees do not come free. Either an asset share's EBR is tightly constrained or some additional source of security must be found. This can come from additional capital, whether internal (e.g. other asset shares or an office's estate) or external.

8.2 This paper has presented another alternative: purchasing appropriate derivatives (with policyholder monies) which give the required degree of protection. While the current derivatives market may not provide a perfect match to the investment exposure of every U.K. with-profits fund, the approach presented could be used to derive a market-consistent charging level for capital backing.

8.3 It would appear, from a recent survey, that the majority of U.K. offices do not charge for capital backing of with-profits business. It may be that this will change, as capital becomes more of an issue in the industry. The authors hope that this paper will help offices to prepare.

8.4 The authors accept that some with-profits offices may continue to consider it inappropriate to charge each policy for the specific support it receives.

Nevertheless, it is hoped that this paper will still prove useful to such offices in indicating the extent of cross-subsidy which could be taking place.

ACKNOWLEDGEMENTS

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The views expressed are, of course, those of the authors and not necessarily those of their employers. However, the authors would like to thank their employers for accommodating time demands, particularly of writing the paper.

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The following list includes, not only the works referred to in the paper, but other publications that might prove helpful as a starting point for further research.

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APPENDIX A

SOME COMMENTS ON PORTFOLIOS

A.1 This paper has considered the investment risk and capital requirements at an individual policy level. As noted in Section 1.3, this was in order to highlight various aspects of the risk/return trade-off implicit in with-profits business. Results derived from hypothetical portfolios can be limited in their application and even disguise some of the underlying forces at work — hence the single policy approach. The authors believe the methods outlined in the paper can be applied in the context of a portfolio, but certain aspects need careful consideration.

A.2 In the comparison of the cost of capital-backed and derivative-backed security presented, the amount of capital support required has been calculated on the basis of a 'stand-alone' shortfall probability. One of the key considerations when portfolios are considered is the extent to which the capital needed to secure a given level of security for the portfolio as a whole might differ from the sum of the corresponding individual policy requirements.

A.3 In most cases, (particularly where there is a good spread of outstanding durations within the portfolio) it could be expected that the portfolio-based requirement will be less than the corresponding sum of individual policy requirements, through the diversification of investment risk over time. As it is most unlikely that every maturing asset share will lie at or below the bottom percentile of possible outcomes, in many cases it can be expected that capital will be released by some maturities which could then be used to offset the risk of future shortfalls.

A.4 Translating this portfolio cross-subsidy into a per-policy effect is not straightforward, however. While a given policy might still enjoy 99% security against shortfall (as well as lower capital support to be charged for) through being part of a portfolio, the nature of the risk being run is now changed. Rather than focusing on just an investment risk, the security afforded is now dependent on what happens in the rest of the portfolio. Thus, the situation is no longer equivalent to the derivative-backed approach presented in Section 6, and hence cost comparisons are not wholly valid. (Were an alternative, portfolio-based, derivative arrangement to be developed, it is not obvious how the cost could be translated into appropriate individual policy charges.)

A.5 It is even possible to construct a portfolio of policies with differing durations, where the investment risk is such that the portfolio requires a greater level of additional capital than the sum of the capital required at an individual policy level. While the authors did not investigate such portfolios in any depth, their existence emphasises the importance, for any office, of modelling its own particular portfolio, and it is hoped that the generic 'per-policy' results presented in this paper may help to shed light on some of the underlying issues.

A.6 Where a policy's asset share is less than the appropriate statutory minimum reserve, the derivation of a suitable charge (should the office consider it appropriate to apply it) for the capital support given is quite straightforward. In this case, however, the presence of a portfolio may provide a large number of asset shares capable of 'lending' the capital in question, but it will not influence the cost/risk in the way described above — statutory reserves must be calculated policy by policy, with future valuation strain eliminated. Where the capital support deemed to be required to ensure statutory solvency can be maintained in a range of adverse investment conditions more onerous than those represented by the three prescribed resilience scenarios, the presence of a portfolio may change things, but it depends on its nature and, in particular, what 'post-scenario' cross-subsidies exist.

APPENDIX B

RESULTS FOR ANNUAL PREMIUM BUSINESS

B.1 As for the single premium example, the sum assured for the 10-year annual premium case investigated below has been set to guarantee initially an accumulation rate of 1% p.a. (i.e. for an annual premium of £1,000, the sum assured is £10,567). The modelling of annual premium business involves dealing with a few additional issues. How they have been treated in this work is as follows:

(a) Premiums have been assumed to be payable annually in advance.

(b) The reserves at time t have been calculated before the payment of the premium then due (and so the reserve at time 0 is simply the required minimum margin of 0.3% of the sum assured).

(c) Premiums are always invested in line with the EBR at the top of the appropriate column, even though, during the stochastic part of the projection, the underlying asset share EBR could be more or less than at time t.

B.2 Again as for the single premium example, the probabilities of shortfall for the reserves, including the resilience allowance, are presented first. While there are some obvious similarities to Table 4.1, there are also significant differences.

Table B.1. Probability of shortfall at maturity (with resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	1.9%	4.2%	6.5%	8.5%	10.4%	12.1%
1	1.8%	3.9%	5.9%	7.7%	9.6%	11.0%
2	1.8%	3.8%	5.8%	7.5%	9.3%	10.7%
3	2.1%	3.9%	5.8%	7.3%	9.1%	10.2%
4	2.2%	3.9%	5.8%	7.5%	9.1%	10.3%
5	2.6%	4.6%	6.3%	7.8%	9.3%	10.6%
6	3.2%	5.1%	6.6%	8.0%	9.4%	10.5%
7	3.7%	5.1%	6.4%	7.8%	8.8%	9.6%
8	3.2%	4.5%	5.6%	6.5%	7.3%	8.0%
9	1.6%	2.1%	2.5%	2.9%	3.3%	3.7%

B.3 The probabilities of shortfall again rise with EBR, indicating a ‘softening’ of reserve strength as EBR increases. The reason for this lies in the size of the net premiums, which increase with the decreasing valuation yield.

B.4 The probabilities also rise and fall with term expired, demonstrating the effect of the resilience test.

B.5 This time, however, the probabilities cover a much greater range than in the single premium case, showing a considerable increase towards the top right hand corner of the table. This is a consequence of the use of a net premium valuation with a relatively low rate of interest. The lower the valuation rate (i.e. the higher the EBR), the higher the net premium being valued, and hence the lower the reserves relative to the single premium situation.

B.6 When the resilience reserve is excluded, the resulting probabilities of shortfall are as shown in Table B.2. In comparing with Table 4.2, some of the same observations can be made as in the single premium case. It can also be noted that at short durations the resilience test has relatively little effect because of the much more gradual build up of the reserve than in a single premium policy.

Table B.2. Probability of shortfall at maturity (without resilience reserve)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	1.9%	4.2%	6.5%	8.5%	10.4%	12.1%
1	2.7%	5.3%	7.4%	9.7%	11.4%	13.2%
2	4.1%	6.6%	9.2%	11.4%	13.1%	14.7%
3	5.6%	8.4%	10.9%	13.0%	15.0%	16.5%
4	7.8%	10.9%	13.5%	15.5%	17.1%	18.4%
5	10.8%	14.2%	16.7%	18.5%	20.0%	21.4%
6	14.2%	17.2%	19.6%	21.4%	22.8%	24.1%
7	17.8%	21.0%	23.4%	25.2%	26.8%	27.9%
8	21.3%	24.3%	26.4%	28.3%	29.5%	30.6%
9	23.6%	26.4%	28.6%	30.5%	31.9%	33.1%

B.7 Expressing the statutory reserves as a percentage of modelled asset shares highlights a very different situation from that in the single premium example, as Table B.3 demonstrates.

Table B.3. Statutory reserve as a percentage of the deterministically accumulated asset share (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	–	–	–	–	–	–
1	115.9	118.6	121.4	124.3	127.2	130.2
2	112.6	115.1	117.8	120.5	123.4	126.3
3	110.2	112.7	115.2	117.8	120.6	123.4
4	108.2	110.5	112.9	115.4	118.0	120.8
5	106.3	108.5	110.8	113.1	115.6	118.2
6	104.5	106.5	108.7	110.9	113.3	115.8
7	102.6	104.7	106.7	108.8	111.0	113.3
8	100.7	102.7	104.7	106.7	108.7	110.9
9	98.9	100.8	102.7	104.6	106.6	108.6

B.8 As for the single premiums, for each duration the proportions increase with EBR, although (and to an even greater extent) not quickly enough to avoid the probability of shortfall also rising.

B.9 Almost all the proportions are greater than 100%, giving further confirmation that with-profits business cannot always be considered to provide capital to an office, even on a published free assets basis.

B.10 The extent to which the use of 'optimised' reserves (see ¶4.2.7) would have reduced these proportions is dependent upon the investment mix assumed for the additional assets required, over and above the asset shares, to cover the reserves. It has not been investigated.

B.11 Now that some results for annual premium business have been considered, the tentative conclusions presented in Section 4.3.5 have received further support, namely that:

(a) The statutory valuation basis, including the effects of the three resilience tests as well as the required minimum margin, is not uniformly strong over either policy duration or reserve investment mix.

(b) The notion sometimes presented, of with-profits business providing capital to an office, need not always be the case, particularly for annual premium business.

B.12 The relationship between the asset share and the reserve required for a 1% probability of shortfall at maturity in the annual premium case is shown in Table B.4. This is analogous to Table 4.4 for the single premium case.

Table B.4. Proportion of asset share needed for 1% shortfall probability (CWP)

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	–	–	–	–	–	–
1	104.5%	111.0%	118.5%	126.9%	136.7%	148.2%
2	105.3%	111.8%	119.2%	127.7%	137.4%	148.9%
3	106.7%	113.5%	121.1%	130.0%	140.3%	152.4%
4	107.7%	114.5%	122.3%	131.3%	141.8%	154.3%
5	108.3%	115.2%	123.0%	132.0%	142.6%	155.2%
6	108.3%	114.8%	122.3%	130.9%	141.1%	153.0%
7	107.3%	113.3%	120.1%	127.8%	136.8%	147.4%
8	105.9%	111.1%	116.9%	123.6%	131.3%	140.2%
9	100.3%	103.5%	107.1%	111.0%	115.3%	120.1%

B.13 A similar picture emerges to that seen for the single premium examples, namely, probabilities increasing with EBR for a given t and initially rising, but mainly falling, with duration for a given EBR.

B.14 The only significant difference to note is the much slower rate of decrease in the proportions as duration increases. This is a consequence of the guarantee being related to all the premiums payable, whereas the asset share at any time only reflects the premiums that have been paid to date. This would tend to suggest that, all other things being equal, single premium conventional with-profits business should be less capital-intensive than annual (even without allowing for any difference in initial expense levels).

B.15 No proportions are shown for duration 0, since it is assumed that no premiums have yet been received and the asset share has been set to zero.

B.16 The annual premium table analogous to Table 4.5 is Table B.5. From this can be drawn conclusions similar to the single premium case, but with some important differences.

Table B.5. Ratio of 1% reserve to statutory minimum reserve (with resilience reserve and required minimum margin (CWP))

	Equity backing ratio					
Duration	50%	60%	70%	80%	90%	100%
0	–	–	–	–	–	–
1	90.1%	93.6%	97.6%	102.1%	107.5%	113.8%
2	93.5%	97.1%	101.2%	105.9%	111.4%	117.9%
3	96.8%	100.7%	105.1%	110.3%	116.4%	123.5%
4	99.5%	103.6%	108.3%	113.8%	120.2%	127.7%
5	101.9%	106.2%	111.0%	116.7%	123.3%	131.2%
6	103.6%	107.8%	112.5%	118.0%	124.5%	132.2%
7	104.7%	108.2%	112.6%	117.5%	123.3%	130.0%
8	105.2%	108.1%	111.7%	115.9%	120.7%	126.3%
9	101.4%	102.8%	104.3%	106.1%	108.2%	110.6%

B.17 The effect of the more stringent treatment of gilts can again be seen in percentages which rise with EBR.

B.18 The percentages for the earlier durations are considerably smaller than in Table 4.5, reflecting the additional strength within the net premium reserves, first noted in Section B.4, which applies for all EBRs since, in all cases, the valuation rate of interest is greater than the 1% used to accumulate the premiums to arrive at the sum assured (and therefore the valuation net premiums will always be less than the office premium).

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