Mortality and Deprivation

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joint work with Jie Wen and Andrew J.G. Cairns

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Actuarial Research Centre, IFoA

ARC workshop - Edinburgh - April 2018

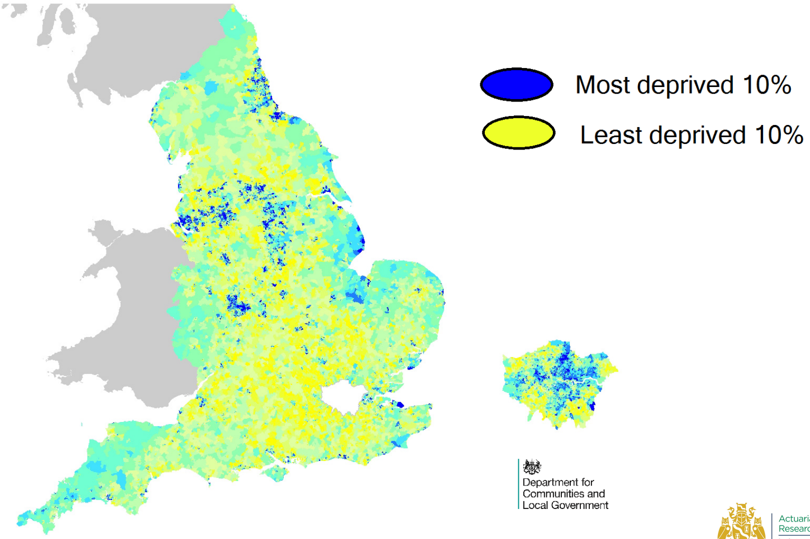
Index of Multiple Deprivation

The IMD is a weighted combination of seven indices of deprivation:

- Income (22.5%)
- Employment (22.5%)
- Education (13.5%)
- Health (13.5%)
- Crime (9.3%)
- Barriers to Housing and Services (9.3%)
- Living environment (9.3%)

source: GOV.UK

Index of Multiple Deprivation (IMD) areas



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Data

- We consider mortality rates for males in England for the ten IMD deciles (2015).
- ages: 40-89, years: 2001-2015
- source: Office for National Statistics

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Model for the Number of Death in Different Groups

$$D_{xti} \quad \sim \quad \text{Poisson}\left(m_{xti}E_{xti}\right)$$

For each period (calendar year) t, age x and socio-economic group i we have

D_{xti}: Number of deaths,

E_{xti}: Central exposure-to-risk

m_{xti}: force of mortality

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Aim: to compare different models for the force of mortality m_{xti}.

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Aim: to compare different models for the force of mortality m_{xti}.

We define socio-economic groups with reference to the Index of Multiple Deprivation for England.

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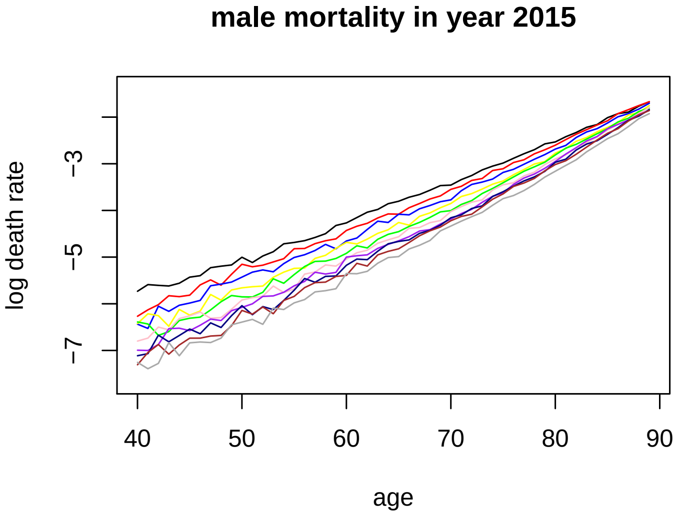
Death rates by IMD decile



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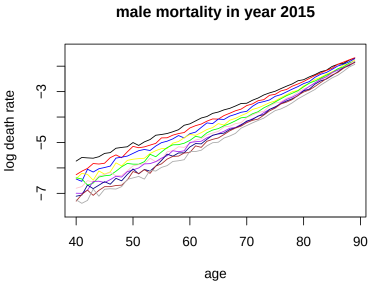

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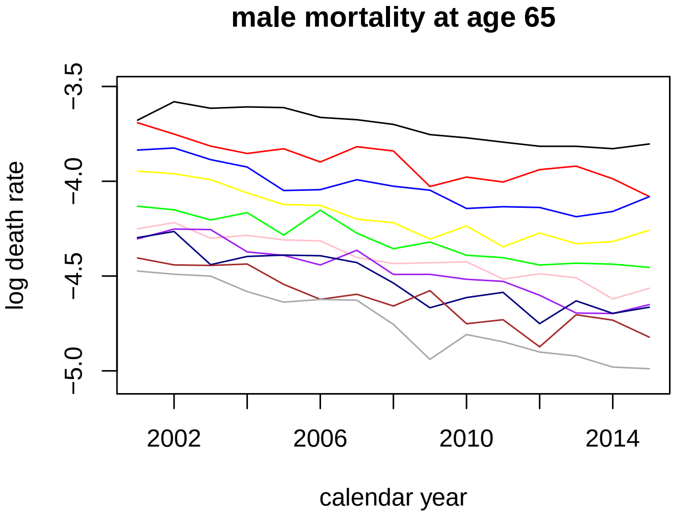

Models

All considered models are variants of group specific Lee-Carter type models with the extension to a second age-period effect by Renshaw & Haberman (2003):

$$\log m_{xti} \quad = \quad \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} + \gamma_{ci}$$

where $c = t - x$ is the cohort (year of birth).

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Specific versions include models with:

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fixed age effects : constant $\beta^1_{xi} = 1$ and linear $\beta^2_{xi} = x - \bar{x}$, where $\bar{x}$ is the mean age in the data set. (Plat, 2009)

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common period effects : $\kappa^k_{ti} = \kappa^k_t$ (Li and Lee, 2005)

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Specific versions include models with:

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fixed age effects : constant $\beta^1_{xi} = 1$ and linear $\beta^2_{xi} = x - \bar{x}$, where $\bar{x}$ is the mean age in the data set. (Plat, 2009)

common period effects : $\kappa^k_{ti} = \kappa^k_t$ (Li and Lee, 2005)

group specific trends in common period effects : $\kappa^k_{ti} = \kappa^k_t + \eta_i(t - \bar{t})$

and variations with and without cohort effects.

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Models

$$
\begin{array}{llcll}
\text{m1:} & \log m_{xti} & = & \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti} & \text{(Plat, 2009)} \\
\text{m2:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_x \kappa^1_{ti} + \beta^2_x \kappa^2_{ti} & \text{(Kleinow, 2015)} \\
\vdots & & & & \\
\text{m6:} & \log m_{xti} & = & \alpha_x + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti} & \text{m1 + common } \alpha \\
\vdots & & & & \\
\text{m9:} & \log m_{xti} & = & \alpha_x + \eta_i(x - \bar{x}) + \kappa^1_t & \\
& & & +d^0_i + d^1_i(t - \bar{t}) & \\
& & & +(x - \bar{x})\left(\kappa^2_t + d^2_i(t - \bar{t})\right) & \\
\vdots & & & & \\
\text{m12:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_{xi} \kappa^1_{ti} + \beta^2_{xi} \kappa^2_{ti} & \text{(Renshaw\&Haberman, 2003} \\
\vdots & & & & \\
\text{m14:} & \log m_{xti} & = & \alpha_x + \beta^1_x \kappa^1_{ti} + \beta^2_x \kappa^2_{ti} & \text{m2 + common } \alpha \\
\text{m15:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_x \kappa^1_t + \beta^2_{xi} \kappa^2_{ti} & \text{(Li\&Lee, 2005)}
\end{array}
$$

+ variants with common or group-specific cohort effect, γ_c or γ_{ci}.

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Estimation and Identifiability

- Maximum Likelihood estimation based on $D_{x,t,i} \sim \text{Poisson}\left(\mu_{x,t,i} E^{c}_{x,t,i}\right)$ is applied to obtain estimated parameter values.
- All suggested models have some identifiability issues, that is, different parameter values lead to the same fitted mortality rates m_{xti}, and, therefore to the same value of the likelihood function.
- To obtain unique parameter values we apply model-specific constraints.

Questions for this talk

$$\log m_{xti} \quad = \quad \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} + \gamma_{ci}$$

- What parameters should be chosen to be group specific and which parameters are common?
- Should age-effects be estimated?
- Should we include cohort effects (common or group specific)?
- What parameters show the greatest differences between IMD groups?
- Are the groups clustered?

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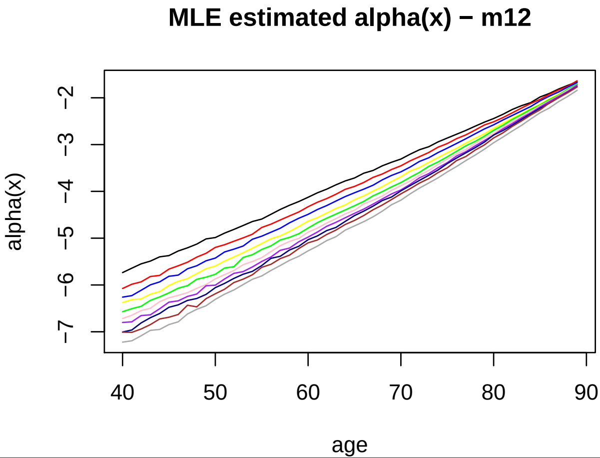
Parameter estimates - m12 - the most general model

$$\log m_{xti} = \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} \text{ (Renshaw\&Haberman, 2003)}$$



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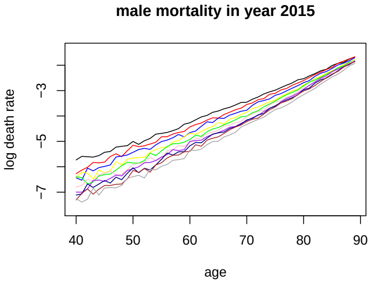

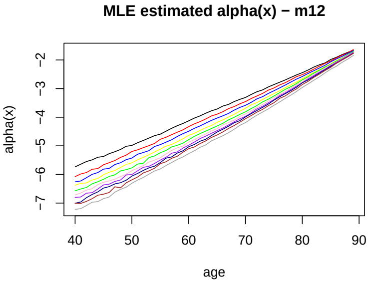

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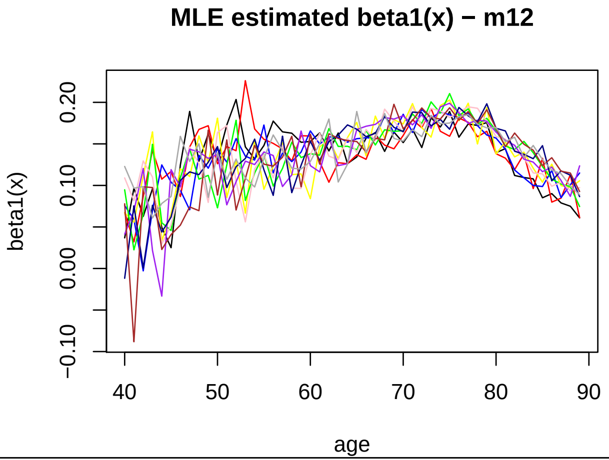

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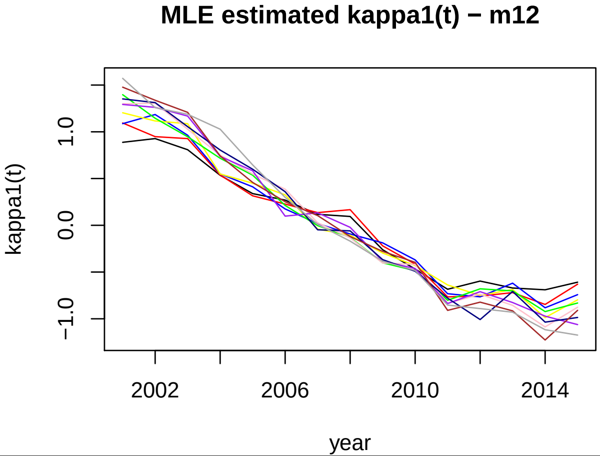

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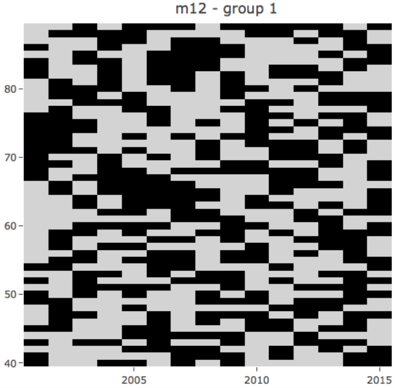

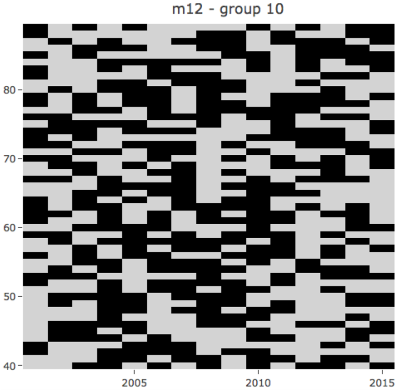

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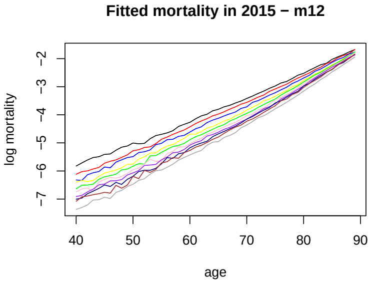

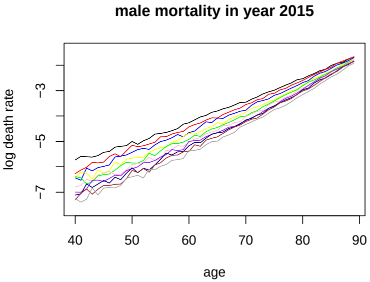

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Parameter estimates - m12 - the most general model

Bayesian Information Criterion: $k \log n - 2\log(L)$

	Log-likelihood	parameters	constraints	d.o.f.	BIC
m12	-30,131.47	1800	40	1760	75,966.82

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- good fit
- very large number of parameters

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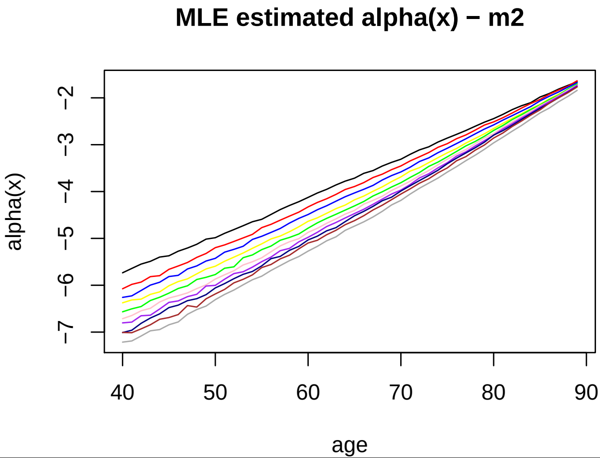
Parameter estimates - m2 - common beta

$$\log m_{xti} = \alpha_{xi} + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2 \text{ (Kleinow, 2015)}$$



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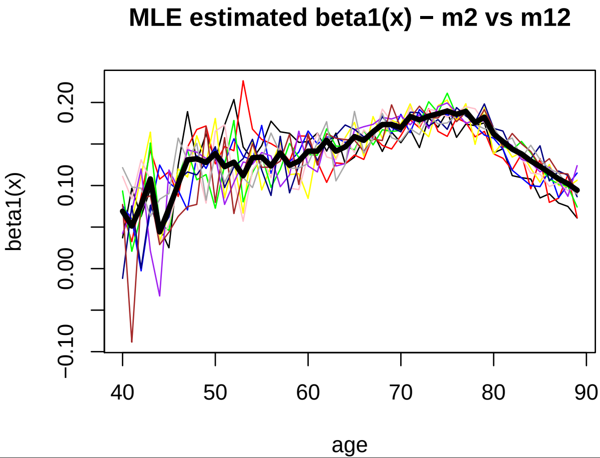

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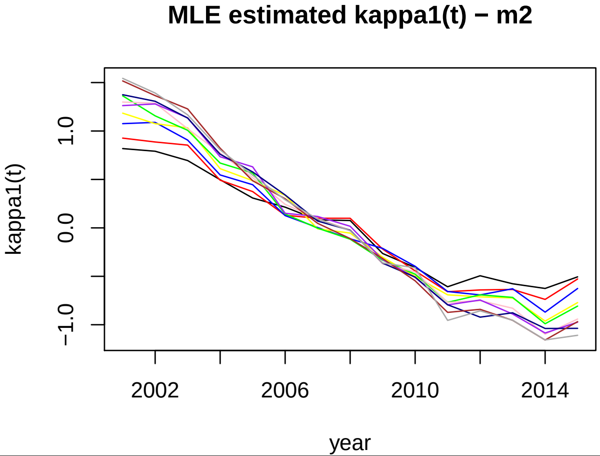

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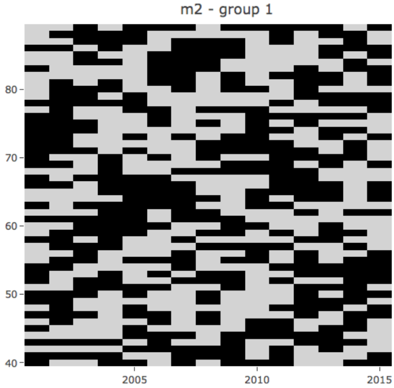

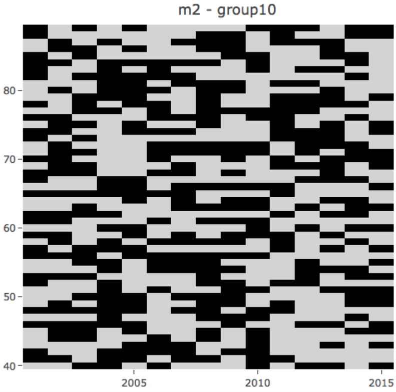

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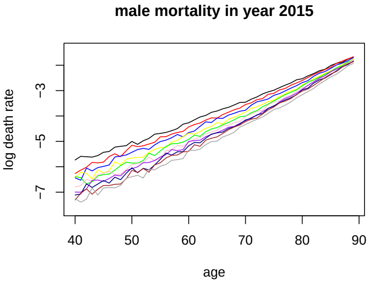

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Parameter estimates - m2 - common beta

Bayesian Information Criterion: $k \log n - 2 \log(L)$

	Log-likelihood	parameters	constraints	d.o.f.	BIC
m2	-30,591.20	900	22	878	69,016.49
m12	-30,131.47	1800	40	1760	75,966.82

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Parameter estimates - m2 - common beta

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- reasonable fit
- smaller number of parameters than m12
- BIC improved

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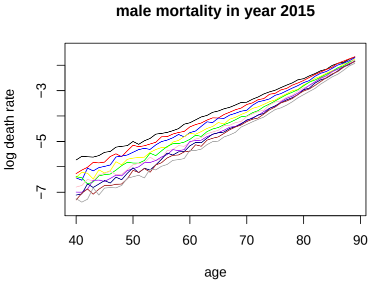
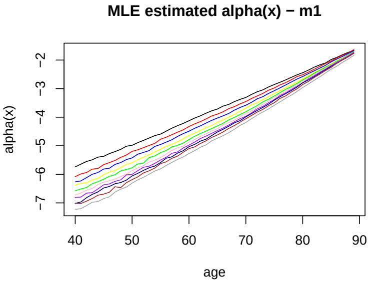
$$\log m_{xti} = \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti} \text{ (Plat, 2009)}$$





Parameter estimates - m1

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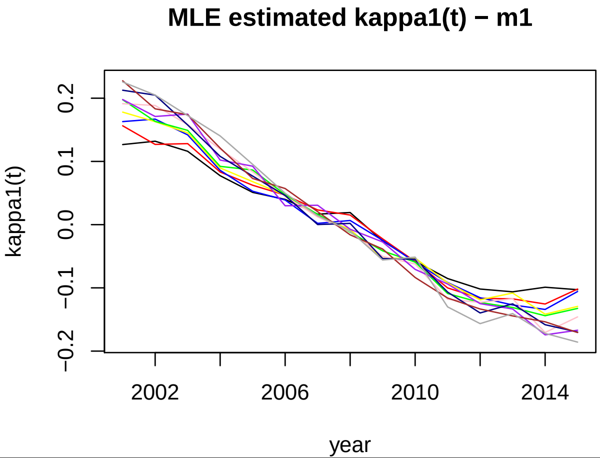

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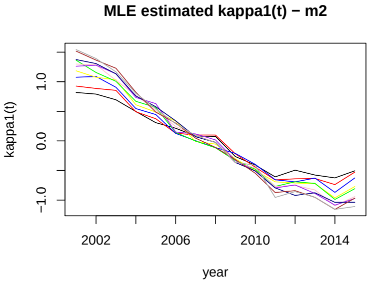

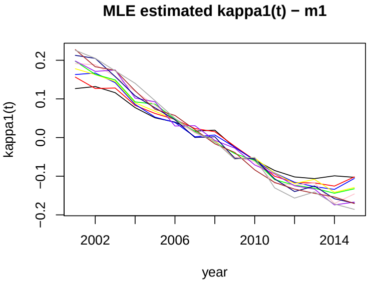

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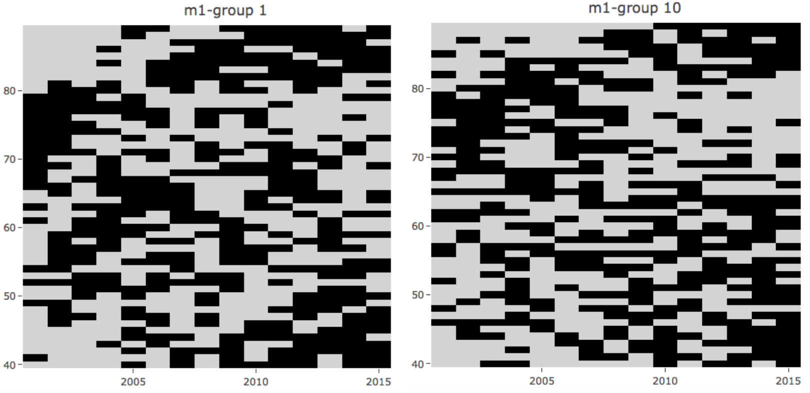

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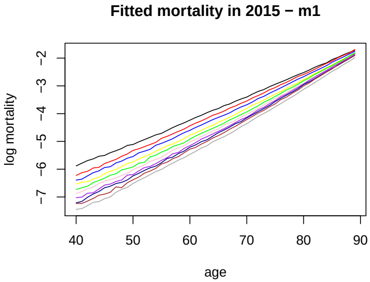

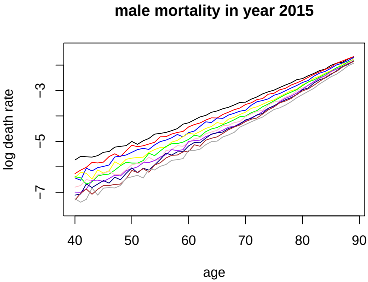

Parameter estimates - m1

Bayesian Information Criterion: $k \log n - 2 \log(L)$

	Log-likelihood	parameters	constraints	d.o.f.	BIC
m1	-31,403.22	800	20	780	69,766.10
m2	-30,591.20	900	22	878	69,016.49
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- model does not fit as well as m2 and m12 (lower likelihood)
- number of parameters reduced further
- but BIC does not improve

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Next steps

$$
\begin{array}{llcll}
\text{m1:} & \log m_{xti} & = & \alpha_{xi} + \kappa^1_{ti} + (x - \bar{x})\kappa^2_{ti} & \text{(Plat, 2009)} \\
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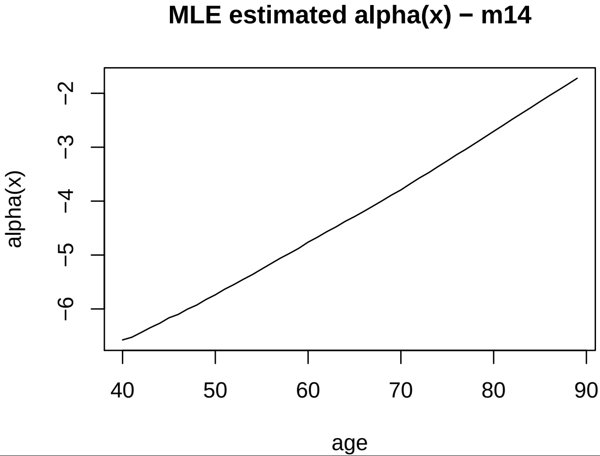
\end{array}
$$

Try a common alpha for m1 and m2.

Parameter estimates - m14 - common alpha and beta

$$\log m_{xti} = \alpha_x + \beta_x^1 \kappa_{ti}^1 + \beta_x^2 \kappa_{ti}^2$$



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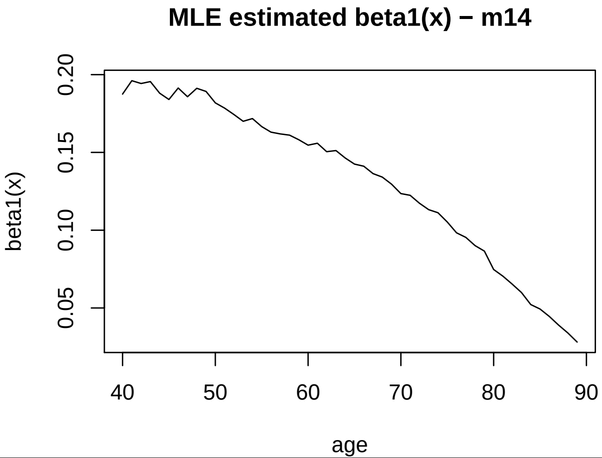

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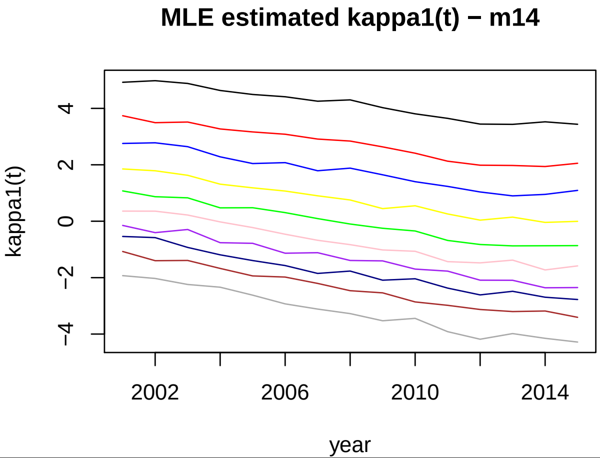

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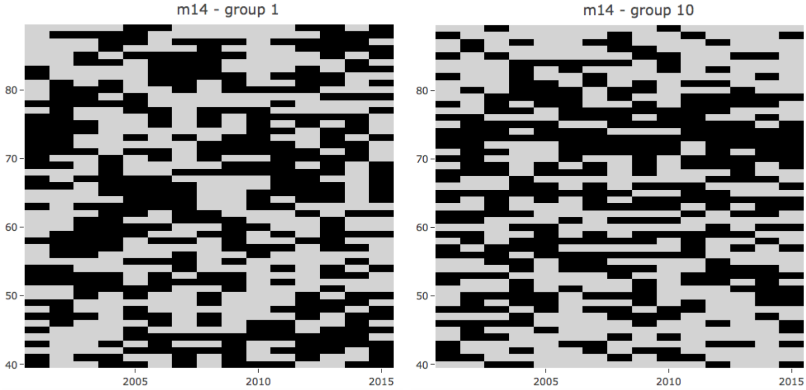

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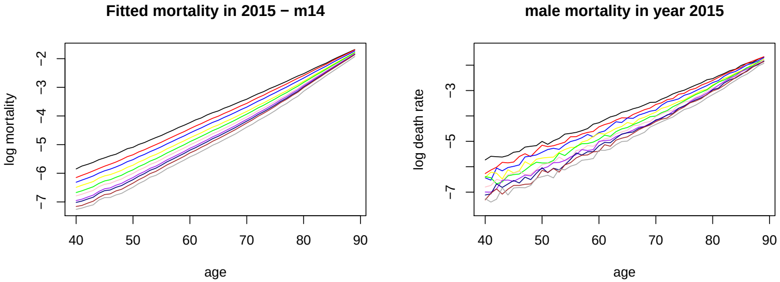

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Parameter estimates - m14 - common alpha and beta

Bayesian Information Criterion: $k \log n - 2 \log(L)$

	Log-likelihood	parameters	constraints	d.o.f.	BIC
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- higher likelihood than m1 with fewer parameters
- best BIC

Parameter estimates - m6 - M1 with common al pha

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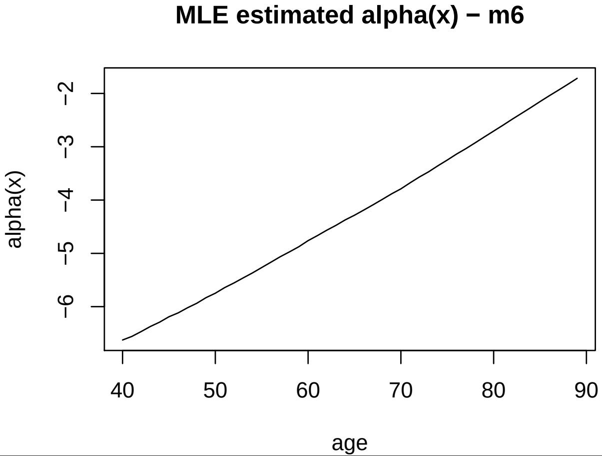

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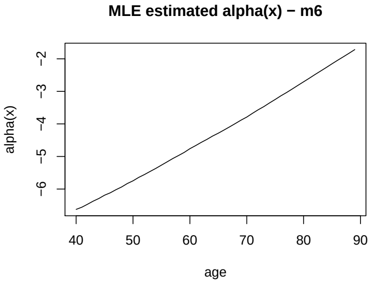

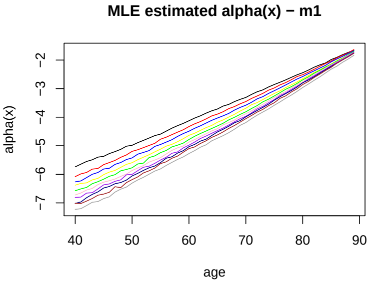

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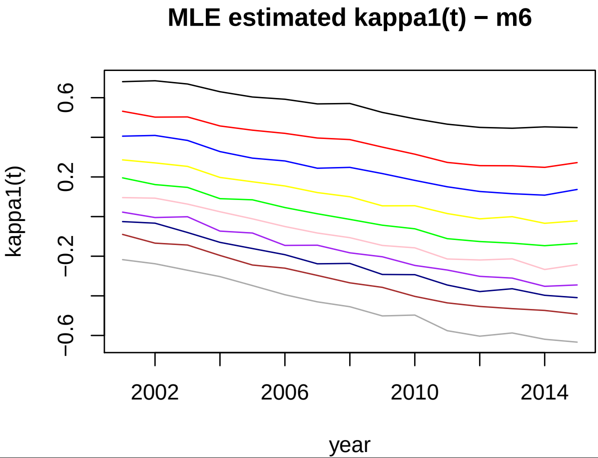

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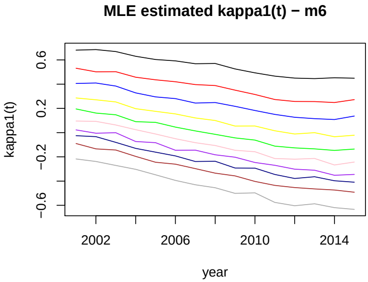

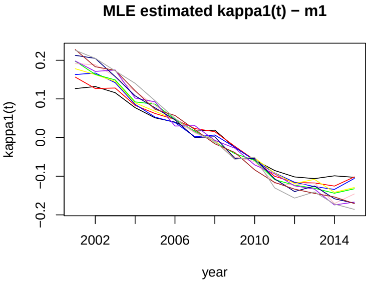

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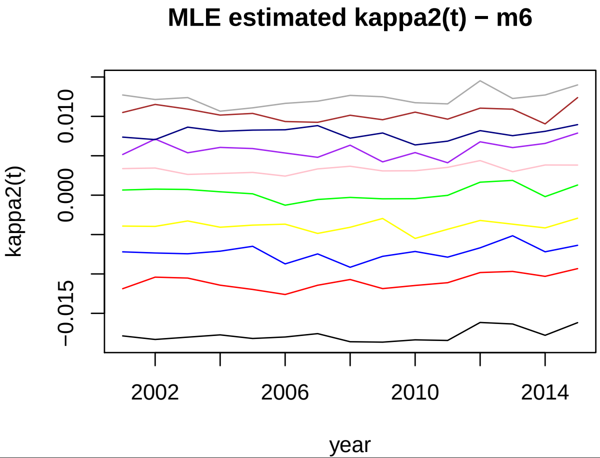

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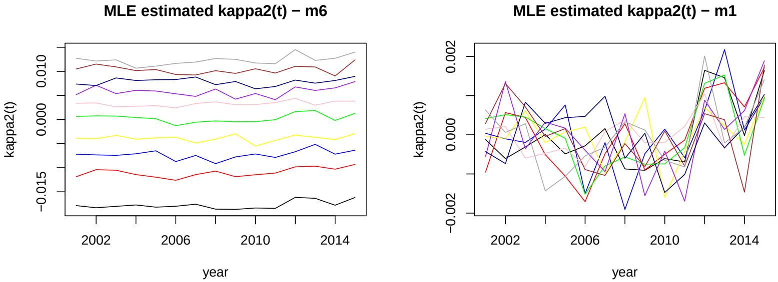

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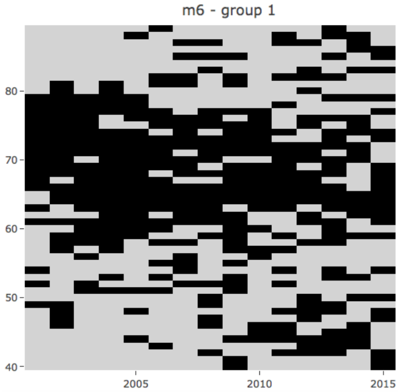

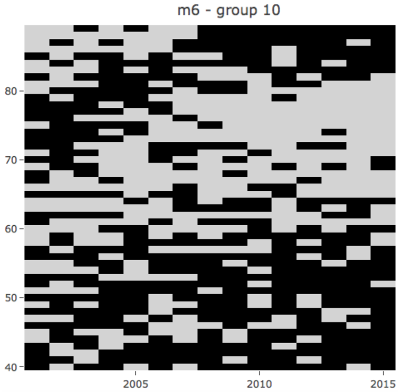

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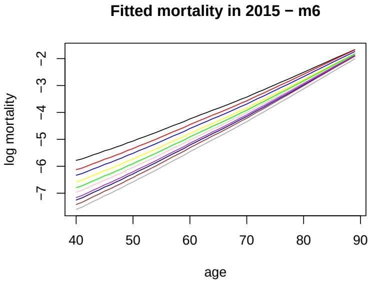

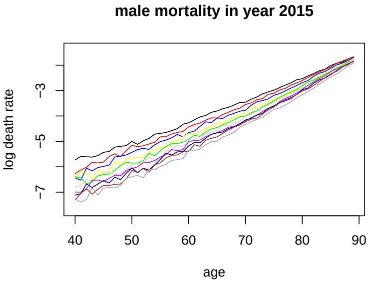

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- smallest number of parameters
- second best model in terms of BIC
- better BIC than m1

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Ranking the Models - Goodness of Fit - Bayesian Information Criterion: $k \log n - 2\log(L)$

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m3	-31,962.82	530	2	528	68,636.80
m4	-31,700.90	665	11	654	69,237.21
m5	-31,474.15	675	12	663	68,864.03
m6	-32,052.10	350	2	348	67,209.29
m7	-31,467.01	665	11	654	68,769.44
m8	-31,460.51	675	12	663	68,836.75
m9	-32,184.56	120	6	114	65,386.30
m10	-31,536.52	550	4	546	67,944.81
m11	-30,323.70	1350	22	1328	72,496.68
m12	-30,131.47	1800	40	1760	75,966.82
m13	-30,558.20	1150	20	1130	71,199.00
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m15	-30,462.15	1215	22	1193	71,569.04

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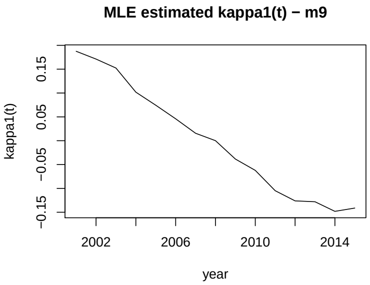
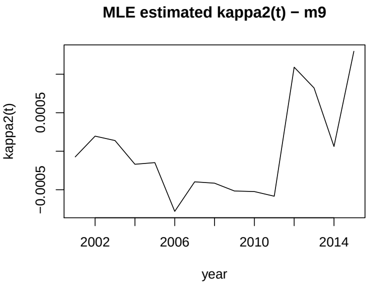
$$\log m_{xti} = \alpha_x + \eta_i(x - \bar{x}) + \kappa_t^1 + d_i^0 + d_i^1(t - \bar{t}) + (x - \bar{x})\left(\kappa_t^2 + d_i^2(t - \bar{t})\right)$$





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Lowest BIC - M9

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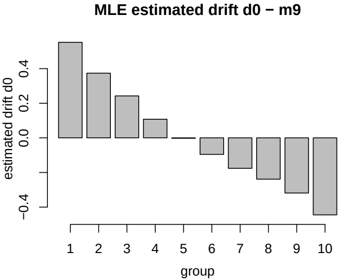

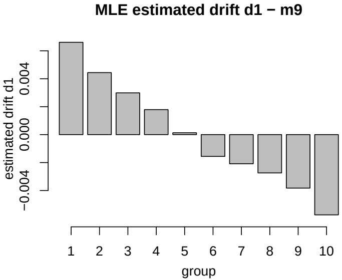

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- Lowest BIC, few parameters
- introduces constant group specific improvement rates
- for ever increasing mortality differentials

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Models

$$
\begin{array}{llcll}
\text{m1:} & \log m_{xti} & = & \alpha_{xi} + \kappa^1_{ti} + (x-\bar{x})\kappa^2_{ti} & \text{(Plat, 2009)} \\
\text{m2:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_x\kappa^1_{ti} + \beta^2_x\kappa^2_{ti} & \text{(Kleinow, 2015)} \\
\vdots & & & & \\
\text{m6:} & \log m_{xti} & = & \alpha_x + \kappa^1_{ti} + (x-\bar{x})\kappa^2_{ti} & \text{m1 + common } \alpha \\
\vdots & & & & \\
\text{m9:} & \log m_{xti} & = & \alpha_x + \eta_i(x-\bar{x}) + \kappa^1_t & \\
 & & & +d^0_i + d^1_i(t-\bar{t}) & \\
 & & & +(x-\bar{x})\left(\kappa^2_t + d^2_i(t-\bar{t})\right) & \\
\vdots & & & & \\
\text{m12:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_{xi}\kappa^1_{ti} + \beta^2_{xi}\kappa^2_{ti} & \text{(Renshaw\&Haberman, 2003} \\
\vdots & & & & \\
\text{m14:} & \log m_{xti} & = & \alpha_x + \beta^1_x\kappa^1_{ti} + \beta^2_x\kappa^2_{ti} & \text{m2 + common } \alpha \\
\text{m15:} & \log m_{xti} & = & \alpha_{xi} + \beta^1_x\kappa^1_t + \beta^2_{xi}\kappa^2_{ti} & \text{(Li\&Lee, 2005)}
\end{array}
$$

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Conclusions

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- Cohort effect do not improve the fit for those models
- If a cohort effect is included it should be a common cohort effect

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